

Coupling IDDES with an algebraic transition model in a high-order discontinuous Galerkin solver

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ABSTRACT

This work aims at designing a transitional hybrid RANS/LES on top of a high-order discretization framework. To do so, we employ the Improved Delayed Detached Eddy Simulation (IDDES) based on the Spalart–Allmaras closure and we equip it with an algebraic transition RANS branch, namely the SA-BCM model. In order to incorporate the transition model into the hybrid setup as seamlessly as possible we recalibrate one of the model coefficients and modify the IDDES shielding function to force the treatment of transitional regions in RANS mode. The numerical study makes use of an incompressible modal discontinuous Galerkin solver. The performance of the proposed implementation is evaluated by investigating a number of test cases, including the flow past aerodynamic profiles at various angle of attack and a sphere in supercritical conditions. The results are promising as the solver is able to accurately predict different transition mechanisms. Furthermore, they display the advantages associated with the proposed changes, some of which are not restricted to algebraic approaches, but can be extended to all transition models based on an intermittency function.

1. Introduction

In recent years hybrid RANS/LES turbulence models caught the attention of the CFD community thanks to the suitability for performing scale-resolving simulations at a fairly reasonable computational cost. The most widely used class of hybrid RANS/LES models is based on the Detached Eddy Simulation (DES) of Spalart (1997). Over the years many improvements of the original version from 1997 have been proposed, the main ones being the Delayed Detached Eddy Simulation (DDES) by Spalart et al. (2006) and the Improved DDES (IDDES) of Shur et al. (2008). In a recent paper (Alberti et al., 2025), we analyzed the performance of both variants in a high-order modal discontinuous Galerkin (DG) solver using a series of benchmark flow problems, including fully-developed turbulent channel flows, a backward-facing step flow and an airfoil in conditions of massive separation. The results demonstrated that, even in this hybrid turbulence modeling context, a high-order numerical approach can significantly reduce the number of degrees of freedom required to achieve a specific level of accuracy. However, we also found that the shielding function of DDES may weaken under certain conditions and hence IDDES usually outperforms the former thanks to its additional protection layer, originally proposed to embed a wall-modeled LES (WMLES) functionality. Consequently, the current work is carried out only with the IDDES variant.

By default, DES-like hybrid models are incapable of predicting laminar-to-turbulent evolution, therefore the correct simulation of transitional flows lies upon the inclusion of some sort of “transition capturing” strategy. A possible way to proceed is to compute the model constant C_{DES} dynamically (Yin et al., 2015; Yin and Durbin, 2016), however a simpler way is to replace the fully turbulent RANS model with a transitional modification. The introduction of a transitional RANS background model in DES-like approaches was an idea exploited by many research groups (Hodara and Smith, 2017; Zhou et al., 2019; Tangermann and Klein, 2023; Feng et al., 2024; Möller et al., 2024). In our framework, the intermittency function that makes the turbulence model transitional is computed not relying on additional transport equations but instead on algebraic relations. Specifically, the fully-turbulent Spalart–Allmaras (SA) model (Spalart and Allmaras, 1992) is replaced with the SA-BCM algebraic transition model of Cakmakcioglu et al. (2020). The SA-BCM model has been extensively validated for our particular DG scheme on a test suite (Crivellini et al., 2023) and it has also shown enhanced predictions compared to the fully-turbulent SA for two-dimensional RANS simulations of flapping foils (Alberti et al., 2023). Moreover, an attempt to use it in combination with DDES on a high-order scheme was already made by Yalçın et al. (2021). The fact that SA-BCM does not add partial differential equations (PDEs) to the overall system makes it particularly well suited for hybrid turbulence

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models where keeping computational cost low is a central aspect. However, a possible drawback of its algebraic structure may be found in some limitations when dealing with very sensitive processes, such as those driven by the growth and breakdown of Tollmien–Schlichting waves, i.e. natural transition. Although Yalçın et al. (2021) suggest a few design tweaks to smoothen the incorporation of SA-BCM into DES-like closures, here we retain just the one involving the multiplication of both the terms of eddy viscosity production and dissipation by the intermittency function. The other two adjustments that we propose, which constitute the novelties of this work, regard the recalibration of the model constant χ_2 in the SA-BCM model and forcing the transitional region to work in RANS mode. About the last novelty, the idea is to handle the transition process through the correlations validated for RANS. Indeed, it is highly unlikely that the coefficients of SA-BCM, as well as other transition RANS models, are able to guarantee equally accurate predictions when the solver is working in LES mode.

The resulting solver is tested by simulating several external flows at moderate Reynolds number, in an attempt to evaluate its performance in different scenarios. First, the flow over an Eppler 387 airfoil at angles of attack $0^\circ, 8^\circ, 14^\circ$ and 16° is presented, where the model is challenged in the detection of laminar separation and/or massively separated regions. Next, there is the flow impinging on a sphere in supercritical conditions, which allows the model to be validated with respect to a natural transition process. The last case study is reserved to the flow past a NACA0012 airfoil with an incidence of 11.5° , where the flow oscillates periodically between a state with a short laminar separation bubble (LSB) at the leading edge and one featured by global stall.

The outline of the paper is as follows. Section 2 briefly recalls the IDDES model, discusses the embedding of the transition model and presents the temporal and DG spatial schemes; Section 3 illustrates the predictive capabilities of the transitional IDDES by showing the numerical results. In Section 4, we conclude by highlighting the main takeaways and exploring directions for future research.

2. Numerical method

2.1. Hybrid RANS/LES turbulence model

Our simulations are based on the SA-IDDES of Shur et al. (2008). The equations, in their incompressible compact form and neat of gravitational effects, read as

$$\mathbf{P} \frac{\partial \mathbf{w}}{\partial t} + \frac{\partial}{\partial x_i} (\mathbf{F}_{c,i} + \mathbf{F}_{v,i}) = \mathbf{s}, \quad (1)$$

with $\mathbf{P} = \mathbf{I} - \mathbf{I}^{(1,1)}$, the unknown vector $\mathbf{w} = [p, u_i, \tilde{v}]^T$, and the convective $\mathbf{F}_{c,i} = [u_i, u_j u_i + \delta_{ij} p, \tilde{v} u_i]^T$ where δ_{ij} identifies the Kronecker delta and the viscous flux $\mathbf{F}_{v,i} = [0, -(v + \nu_t)(\partial u_j / \partial x_i + \partial u_i / \partial x_j), -(v + \tilde{\nu}) \partial \tilde{v} / \partial x_i]^T$. The right-hand side is composed by the source term $\mathbf{s} = [0, 0, s_{\tilde{v}}]^T$, with

$$s_{\tilde{v}} = \underbrace{C_{b1} \tilde{S} \tilde{v}}_{s_{\tilde{v}P}} - \underbrace{C_{w1} f_w \left(\frac{\tilde{v}}{\tilde{d}} \right)^2}_{s_{\tilde{v}D}} + \underbrace{\frac{C_{b2}}{\sigma} \frac{\partial \tilde{v}}{\partial x_i} \frac{\partial \tilde{v}}{\partial x_i}}_{s_{\tilde{v}C}}, \quad (2)$$

where $s_{\tilde{v}P}$, $s_{\tilde{v}D}$, $s_{\tilde{v}C}$ indicate the production, destruction and cross term, respectively. The unknowns of Eq. (1) are the velocity vector u_i , the pressure divided by the density $p = P/\rho$ and the pseudo-eddy viscosity $\tilde{v}$, with ν being the molecular viscosity of the fluid and $\nu_t = f_{v1} \tilde{v}$ the actual eddy viscosity.

For a complete description of the source term in Eq. (2), obtained through the definition of the modified vorticity magnitude $\tilde{S}$ plus all closure functions and constants the reader may refer to Spalart and Allmaras (1992). Compared to Spalart and Allmaras (1992), however, trip terms are dropped in Eq. (2) and so the SA model functions in a fully turbulent mode. Moreover, to avoid potential liability related to $\tilde{v} < 0$ and ensure an increasing eddy viscosity, Eq. (2) uses $\tilde{v} = \max\{0, \tilde{v}\}$.

Another difference with respect to the original version of Spalart and Allmaras (1992), in analogy with Crivellini et al. (2013), comes from the function r which is limited to a value $r_{max} = 10$ through the relation

$$r^* = \frac{\tilde{v}}{\tilde{S} \kappa^2 d_w^2}, \quad r = \begin{cases} r^*, & 0 \leq r < r_{max}, \\ r_{max}, & r < 0 \quad \text{or} \quad r \geq r_{max}, \end{cases} \quad (3)$$

which solves the issues of numerical instability caused by negative values of r .

The quantity $\tilde{d}$ represents the hybrid length scale of DES-like models and in IDDES is given by

$$\tilde{d} = \tilde{f}_d (1 + f_e) d_w + (1 - \tilde{f}_d) C_{DES} \Delta, \quad (4)$$

where Δ is the subgrid length scale and $C_{DES} = 0.65$. The parameter C_{DES} was initially calibrated for decaying homogeneous isotropic turbulence using a high-order finite-difference scheme (Shur et al., 1999). However, since it has been also validated within a modal DG scheme for the Taylor–Green vortex flow by Basile et al. (2022), and has already shown good performance even in our solver (Alberti et al., 2025), we retain the same value here. In Eq. (4), $\tilde{f}_d$ is the shielding function and reads

$$\begin{aligned} \tilde{f}_d &= \max \{1 - f_{dt}, f_B\}, \\ f_{dt} &= 1 - \tanh \left[(8r_{dt})^3 \right], \\ f_B &= \min \{2 \exp(-9\alpha^2), 1\}, \quad \alpha = 0.25 - d_w/h_{max}, \end{aligned} \quad (5)$$

with the elevating function f_e assembled as in Shur et al. (2008). From Eqs. (4) and (5) one can see that IDDES is able to function in two different modes depending on the inflow turbulent content: when $(1 - f_{dt}) > f_B$ (e.g. laminar inflow or very coarse mesh) it works as a DDES; on the other hand, if $(1 - f_{dt}) < f_B$ (e.g. unsteady inflow conditions or fine mesh) it works as a WMLES. In both modes, RANS regions are identified by $\tilde{f}_d = 1$ whereas LES activates for $\tilde{f}_d = 0$.

We select $\Delta = \tilde{\Delta}_w$ (Mockett et al., 2015) as the subgrid length scale, since in one of our recent publications (Alberti et al., 2025) it proved to be among the best performing filters, allowing for a proper destruction of the eddy viscosity in the gray area. We point out that $\tilde{\Delta}_w$ replaces the entire filter width and not just the “outer” scale of Δ_{IDDES} , cfr. Probst and Reuß (2015). Removing the min/max blending in the subgrid length scale does not undermine the basic IDDES functionality, which is mainly associated with the shielding function $\tilde{f}_d$ of Eq. (5). The reason why we decided to replace the whole filter is thoroughly discussed in Alberti et al. (2025), where we show that Δ_{IDDES} (which is denoted as $\Delta_{h_{un}}$ there) is responsible for a detrimental fast decay of the eddy viscosity in the gray area, at least in our DG framework. In accordance to what was found in Alberti et al. (2025), Δ as well as all length scales used in the turbulence model are normalized using $k+1$ as a scaling factor to account for the high-order DG approximation. The model is not equipped with any low-Reynolds correction since we already demonstrated that high-order DG methods are well-suited for implicit LES (ILES) and, consequently, excessive eddy viscosity tends to be detrimental.

2.1.1. Transition modeling

In cases where laminar-to-turbulent evolution has a non-negligible impact on the overall flow dynamics, the replacement of the fully turbulent SA model with a transition modification may be a useful practice. In this regard, algebraic transition models represent ideal candidates because they do not add further equations in the global system of PDEs. The algebraic transition models taken into consideration in the present work modify the source term as

$$s_{\tilde{v}} = \gamma_{BC} \left[C_{b1} \tilde{S} \tilde{v} - C_{w1} f_w \left(\frac{\tilde{v}}{\tilde{d}} \right)^2 \right] + \frac{C_{b2}}{\sigma} \frac{\partial \tilde{v}}{\partial x_i} \frac{\partial \tilde{v}}{\partial x_i}. \quad (6)$$

The quantity $\gamma_{BC} \in [0, 1]$ is the intermittency function. The turbulence model is enabled if $\gamma_{BC} = 1$ and disabled if $\gamma_{BC} = 0$. This means that a

zero intermittency function corresponds to the solution of the Navier-Stokes (NS) equations. The intermittency function can be defined in several ways, however here we consider just the variant associated to the SA-BCM of [Cakmakcioglu et al. \(2020\)](#). Its implementation in our DG solver has been thoroughly validated on several boundary layer flows of the ERCOFTAC T3 series and an Eppler 387 airfoil ([Crivellini et al., 2023](#)). In the SA-BCM model one has

$$\gamma_{BC} = 1 - \exp\left(-\sqrt{T_1/\chi_1} - \sqrt{\chi_2 T_2}\right),$$

$$T_1 = \max\left\{\frac{Re_\omega}{2.193} - 1, 0\right\}, \quad T_2 = \max\left\{\frac{v_t}{v}, 0\right\}, \quad (7)$$

$$Re_\omega = \frac{d_w^2}{\nu} \Omega, \quad Re_{\theta c} = 803.73 (Tu_\infty + 0.6067)^{-1.027},$$

where $\chi_1 = 0.002$, $\chi_2 = 1000$ and the free-stream turbulent intensity Tu_∞ is manually tuned.

The SA-BCM model is used in conjunction with a correction of the coefficient C_{w2} for transitional flows at low-Reynolds number which was proposed by [Spalart and Garbaruk \(2020\)](#) and reads as

$$C_{w2,LRe} = C_{w4} + \frac{C_{w5}}{(\chi/40 + 1)^2}, \quad (8)$$

with $C_{w4} = 0.21$ and $C_{w5} = 1.5$.

Compared to a “standard” implementation of SA-BCM within IDDES, the changes proposed in this work are essentially three:

- (1) Unlike in pure RANS simulations, where γ_{BC} acts only on the production term, both production and dissipation are multiplied by the intermittency function due to the importance of keeping the eddy viscosity balance unaltered in LES mode, as underlined in [Yalçın et al. \(2021\)](#).
- (2) The IDDES shielding function in Eq. (5) is modified using the following novel expression

$$\tilde{f}_d^* = \max\{\tilde{f}_d, 1 - \gamma_{BC}\}. \quad (9)$$

in order to avert switching to LES in case of incipient transition phenomena. The reason is that transitional DES-like models can face issues when the turbulent transition takes place outside of the shielded area. Indeed, in that case the solver switches from a laminar solution directly to LES, with a negligible production of eddy viscosity. Although in principle one could think of capturing transition entirely in LES mode using dedicated subgrid-scale models, it would certainly require a much higher spatial resolution; this runs counter to the primary purpose of hybrid turbulence models of minimizing the computational cost of scale-resolving simulations. As a consequence, it is preferable to capture transition in RANS mode exploiting the correlations available from the literature, which are unlikely to perform equally well in the LES branch of DES-like models. With Eq. (9), we make sure that as long as $\gamma_{BC} < 1$ the shielding is active, regardless of $\tilde{f}_d$, thus forcing the sequence NS $\rightarrow$ RANS $\rightarrow$ LES without bypassing the RANS phase. Previous publications already tried to achieve this objective using their own strategies, e.g. fixing a limit for the intermittency function below which RANS is enforced ([Xiao et al., 2019](#)) or introducing a blending function between transitional RANS and DES ([Kim and Kwon, 2021](#)), however we decided to act directly on the definition of the shielding function. Note that the same concept can be straightforwardly applied to DDES by modifying the shielding function as $f_d^* = \min\{f_d, \gamma_{BC}\}$.

- (3) The model constant χ_2 is raised from 50 to 1000. This change is introduced to manually calibrate the correct transition onset location in case of natural transition, where SA-BCM does not work properly, as will be demonstrated in Sections 3.1, 3.2. The justification for such an increment comes from [Cakmakcioglu](#)

[et al. \(2020\)](#), where different values up to 10000 were tested for χ_2 on a Schubauer & Klebanoff test case, showing minimal alteration of the RANS skin friction coefficient. Although we actually observed that $\chi_2 = 1000$ can already alter pure RANS solutions, the suggestion is to consider this modification specifically for DES-like models where the eddy viscosity levels are much lower. In Section 3, we will show that the increased model constant yields a slight suppression of the LSB when IDDES operates in fully RANS mode, while it works well for massively separated flows ensuring a full propagation of the intermittency function up to the near-wall region that $\chi_2 = 50$ may fail to guarantee ([Yalçın, 2021](#)). We are aware that this modification is not sufficiently general and therefore we will also propose possible improvements.

It is important to observe that modifications (1) and (2) are not limited to SA-BCM and actually can be readily applied even to non-algebraic models. Multiplying both the production and dissipation terms by γ_{BC} and the modified shielding function of Eq. (9) can indeed benefit all transitional DES-like models based on an intermittency function, regardless of whether additional transport equations are used for its calculation.

2.2. Spatial and temporal discretization

The simulations are carried out by means of a high-order modal DG discretization method. The geometrical flexibility of such scheme allows for accurate solutions on unstructured grids with elements of arbitrary shape, possibly with curved edges and/or faces.

To obtain the weak formulation of Eq. (1), the governing equations are scalarly multiplied by an arbitrary smooth test function Φ and integrated over the domain Λ

$$\int_{\Lambda} \Phi^T \mathbf{P} \frac{\partial \mathbf{w}}{\partial t} d\mathbf{x} - \int_{\Lambda} \left(\frac{\partial \Phi}{\partial x_i} \right)^T (\mathbf{F}_{c,i} + \mathbf{F}_{v,i}) d\mathbf{x} + \int_{\partial\Lambda} \Phi^T (\mathbf{F}_{c,i} + \mathbf{F}_{v,i}) n_i d\sigma = \int_{\Lambda} \Phi^T \mathbf{s} d\mathbf{x}, \quad (10)$$

where n_i is the unit normal vector pointing outward from the boundary $\partial\Lambda$. In the DG method, the solution of Eq. (10) is approximated as $\mathbf{w}_h = \sum_{l=1}^{N_{dof}} \phi_l(x_i) \mathbf{W}_l(t)$, where $\mathbf{W}_l(t)$ are the unknown degrees of freedom (DOFs) and ϕ_l is a modal basis of polynomial functions of k th degree, which in our framework are orthonormal, hierarchical and defined in the physical space. Since we make use of Pascal polynomial basis functions, we have $N_{dof} = \prod_{m=1}^d (k+m)/m$ for a d -dimensional problem. As far as the physical space $\Lambda \subset \mathbb{R}^d$ is concerned, the discretization is obtained through a triangulation $\mathcal{K}_h = \{K\}$ composed of non-overlapping elements K , with the set of mesh faces identified by $\mathcal{F}_h = \{F\}$. Eventually, the DG discretized governing equations can be written as

$$\sum_{K \in \mathcal{K}_h} \int_K \phi_l \mathbf{P} \frac{d\mathbf{w}_h}{dt} d\mathbf{x} - \sum_{K \in \mathcal{K}_h} \int_K \frac{\partial \phi_l}{\partial x_i} (\mathbf{F}_{c,i} + \mathbf{F}_{v,i}) d\mathbf{x} + \sum_{F \in \mathcal{F}_h} \int_F \llbracket \phi_l \rrbracket (\hat{\mathbf{F}}_{c,i} + \hat{\mathbf{F}}_{v,i}) n_i d\sigma = \sum_{K \in \mathcal{K}_h} \int_K \phi_l \mathbf{s}_h d\mathbf{x}, \quad (11)$$

where the jump operator $\llbracket \cdot \rrbracket = \cdot^+ - \cdot^-$ is introduced to account for the discontinuities of functions at each mesh interface, while $\hat{\mathbf{F}}_{c,i}$ and $\hat{\mathbf{F}}_{v,i}$ indicate the convective and viscous numerical fluxes that replace their respective physical fluxes in the normal direction. More specifically, $\hat{\mathbf{F}}_{c,i}$ exploits a Godunov flux computed from the Riemann solver based on the artificial compressibility concept ([Bassi et al., 2006, 2007](#)), whereas $\hat{\mathbf{F}}_{v,i}$ follows the BR2 scheme of [Bassi et al. \(2005\)](#) with interface jump penalization. Being the solution vector a polynomial, the overall scheme is $(k+1)$ th order accurate for all unknowns except pressure, which is k th order accurate in this incompressible formulation.

The compact form of Eq. (11) reads as

$$\mathbf{M} \frac{d\mathbf{W}}{dt} + \mathbf{R}(\mathbf{W}) = \mathbf{0}, \quad (12)$$

where $\mathbf{W}$ is the global vector of unknowns, $\mathbf{M}$ is the global block diagonal mass matrix with null entries in correspondence of the DOFs associated to pressure and $\mathbf{R}$ is the residual vector. Eq. (12) constitutes a system of nonlinear differential algebraic equations (DAEs) which is integrated by means of the ROS3Pw scheme of Rang and Angermann (2005), which is a strongly A-stable linearly implicit (Rosenbrock type) Runge–Kutta method that at time step n is characterized by

$$\begin{aligned} \mathbf{W}^{n+1} &= \mathbf{W}^n + \sum_{j=1}^s m_j \mathbf{Y}_j \\ \left(\frac{\mathbf{M}}{\gamma_{ii} \Delta t} - \frac{\partial \mathbf{R}}{\partial \mathbf{W}}(\mathbf{W}^n) \right) \mathbf{Y}_i &= \mathbf{R} \left(\mathbf{W}^n + \sum_{j=1}^{i-1} a_{ij} \mathbf{Y}_j \right) \\ &+ \mathbf{M} \sum_{j=1}^{i-1} \frac{c_{ij}}{\Delta t} \mathbf{Y}_j \quad i = 1, \dots, s \end{aligned} \quad (13)$$

where $s = 3$ is the number of internal stages and $m_j, \gamma_{ii}, a_{ij}, c_{ij}$ are the real coefficients of the scheme. The advantage of Rosenbrock schemes is owed to linearization process which eliminates the nonlinear solution step, moreover there is the possibility to update the Jacobian matrix $\partial \mathbf{R} / \partial \mathbf{W}$ just once per time step rather than every stage. The linear system resulting from Eq. (13) is solved with the matrix-based Generalized Minimal RESidual (GMRES) algorithm whose convergence is sped up via the additive-Schwarz preconditioner.

3. Results

The performance of the transitional hybrid turbulence model is checked by means of several numerical experiments, all selected to highlight different modeling features. The first problem involves a flow around an Eppler 387 profile at various angles of attack with $Re_c = 300\,000$. In particular: (i) the configuration at 0° is ideal to assess the model in conditions of localized separation; (ii) at 8° it is possible to analyze how the model acts in case of natural transition without separation; (iii) at 14° allows to investigate how a localized separation at the leading edge may interact with a massive separation at the trailing edge; (iv) in the configuration at 16° we evaluate the behavior of the model in stall conditions. Then, we pass to a flow impinging on a sphere in the supercritical regime ($Re_D = 420\,000$), which is reserved for studying the suitability of the model to simulate a boundary layer undergoing natural transition followed by turbulent separation. The last test case is a flow past a NACA0012 foil at $Re_c = 130\,000$ and angle of incidence 11.5° . This is a very delicate case due to the establishment of a low-frequency cycle involving the formation, enlargement and burst of an LSB at the leading edge, that allows to test the ability of the model to capture subtle separation mechanisms. We employ this configuration also to analyze the behavior of the hybrid model in a situation where the separation point undergoes a continuous shift, with a view to using it in dynamic simulations.

The baseline configuration of the transitional hybrid turbulence model incorporates all three changes listed in Section 2.1.1; in the rest of the paper this layout will be denoted by IDDES_{TRA}($\tilde{f}_d^*$). The free-stream values for the eddy viscosity and the turbulence intensity supplied to the RANS background models for the various cases are collected in Table 1. Regarding $\tilde{\nu}_\infty / \nu$ for the baseline SA model, the value we use is in line with what is typically adopted in literature. In the transitional version, instead, we employ a value sufficiently low to make the solution insensitive. Indeed, it has been shown that below a certain threshold the flow field is barely affected by the inflow condition, at least in case of separation-induced transition (Crivellini et al., 2023). The only exception in this sense is the sphere, where the value is calibrated in such a way to have the transition onset at the location indicated by the experiments. As will be explained later, this need is dictated by the fact that in natural transition SA-BCM does not work as expected, since it is not T_1 that triggers transition but rather T_2 ; consequently, the eddy viscosity field becomes the sole factor in

Table 1

Free-stream parameters of the RANS branch in the hybrid turbulence models.

Case	SA	SA-BCM	
	$\tilde{\nu}_\infty / \nu$	$\tilde{\nu}_\infty / \nu$	Tu_∞ (%)
Eppler 387	3	0.2	0.1
Sphere	3	0.001	0.05
NACA0012	3	0.2	0.2

Table 2

Eppler 387 — Spatial resolution for the different DG polynomial orders. N_e = total number of grid cells, $\Delta_{y,w}^+$ = first cell height (normalized by $k+1$), N_{DOF} = total number of spatial DOFs.

N_e	$k = 3$		$k = 4$		$k = 5$	
	$\Delta_{y,w}^+$	N_{DOF}	$\Delta_{y,w}^+$	N_{DOF}	$\Delta_{y,w}^+$	N_{DOF}
7644	2.1	152 880	1.7	267 540	1.4	428 064

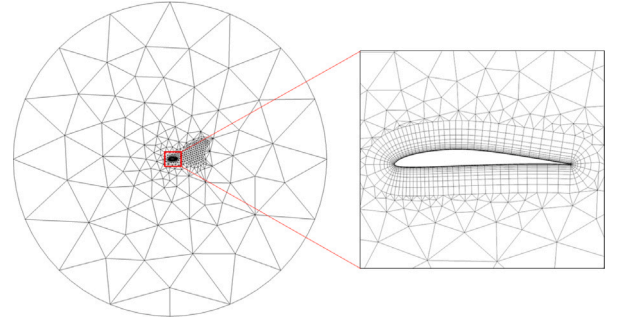


Fig. 1. Eppler 387 — Grid and relative close up view of the inflated layers at the $x - y$ middle plane.

establishing the onset of turbulence and hence $\tilde{\nu}_\infty / \nu$ (or alternatively χ_2) needs to be adjusted on a case-by-case basis. As far as Tu_∞ is concerned, for the airfoils we select values aligned with the reference experiments and for the sphere we use have a value which is compatible with an incoming laminar boundary layer.

3.1. Eppler 387

The flow around the Eppler 387 foil, characterized by $Re_c = 300\,000$, is simulated using a circular domain of diameter $25c$, where c is the chord length, centered at the leading edge (LE) of the foil and extruded up to $0.2c$ in the spanwise direction. The grid used for the simulations, depicted in Fig. 1 and described in quantitative terms in Table 2, is composed by a mix of hexahedrons and prisms with second order edges to follow the curvature of the profile. The near wall region is discretized with 12 inflated layers with growth rate 1.4. The study involves several angles of attack, specifically $0^\circ, 8^\circ, 14^\circ, 16^\circ$. The averages are performed over time windows of up to $50c/U_\infty$ in cases at higher angle of attack, where the flow unsteadiness is more pronounced.

The mean lift and drag coefficients for the various case studies are listed in Table 3, which also includes the experimental values of McGhee (1988) and Emmerson and Verstraete (2020) as a reference. We premise that in these specific test cases transitional IDDES with the original shielding function has been assessed but the solutions are practically identical to those obtained with $\tilde{f}_d^*$, therefore its results are never reported. The reason for the lack of a significant difference can be found both in the rather coarse spatial discretization and flow dynamics, which do not yield critical situations where regions affected by transition are not already protected by $\tilde{f}_d$.

For the first two angles of attack we show the results only at $k = 3$, since in the absence of a massive separation the required spatial resolution is similar to that needed for a pure RANS. In these

Table 3
Eppler 387 — Mean lift and drag coefficient.

α	Model	C_L	C_D
0°	SA-BCM steady $k = 3$	0.375	0.008
	IDDES $k = 3$	0.361	0.013
	IDDES _{TRA} ($\chi_2 = 50, \tilde{f}_d^*$) $k = 3$	0.375	0.009
	IDDES _{TRA} ($\tilde{f}_d^*$) $k = 3$	0.38	0.010
	Exp, McGhee (1988)	0.352	0.009
	Exp ^a , Emmerson and Verstraete (2020)	0.35	0.008
8°	SA-BCM steady $k = 3$	1.205	0.019
	IDDES $k = 3$	1.159	0.024
	IDDES _{TRA} ($\chi_2 = 50, \tilde{f}_d^*$) $k = 3$	1.235	0.019
	IDDES _{TRA} ($\tilde{f}_d^*$) $k = 3$	1.195	0.020
	Exp, McGhee (1988)	1.180	0.017
	Exp ^a , Emmerson and Verstraete (2020)	1.14	0.022
14°	SA-BCM steady $k = 3$	1.334	0.087
	SA-BCM steady $k = 4$	1.364	0.082
	IDDES $k = 3$	1.404	0.067
	IDDES _{TRA} ($\tilde{f}_d^*$) $k = 3$	1.318	0.083
	IDDES _{TRA} ($\tilde{f}_d^*$) $k = 4$	1.277	0.087
	IDDES _{TRA} ($\tilde{f}_d^*$) $k = 5$	1.224	0.106
	Exp, McGhee (1988)	1.179	–
	Exp ^a , Emmerson and Verstraete (2020)	1.18	0.12
	SA-BCM steady $k = 3$	1.168	0.153
16°	SA-BCM steady $k = 4$	1.190	0.148
	IDDES $k = 3$	1.427	0.119
	IDDES _{TRA} ($\tilde{f}_d^*$) $k = 3$	1.326	0.261
	IDDES _{TRA} ($\tilde{f}_d^*$) $k = 4$	1.303	0.266
	IDDES _{TRA} ($\tilde{f}_d^*$) $k = 4, N_{DOF} = 395k$	1.139	0.314
	IDDES _{TRA} ($\tilde{f}_d^*$) $k = 5$	1.169	0.300
	Exp, McGhee (1988)	0.99	–
	Exp ^a , Emmerson and Verstraete (2020)	1.09	0.25

^a Identifies digitized data.

specific configurations, in particular, we already achieve convergence at $k = 2$ (not shown for the sake of compactness). Starting from the case $\alpha = 0^\circ$, it can be noted from the plots of Fig. 2 how the curves associated to the transitional hybrid models overlap that of the transitional RANS. This represents a desirable property since without a massive separation it is not recommended to switch to LES, which would inevitably lead to a dissipative deficit considering the typical under-resolution of DES-oriented grids. Actually, the only transitional hybrid version manifesting a visible deviation, both in terms of pressure and skin friction coefficient, is IDDES_{TRA}($\tilde{f}_d^*$) with $\chi_2 = 1000$, which slightly suppresses the LSB. This is probably due to the fact that under such attached flow conditions, with consequent high levels of eddy viscosity, that value turns out to be too large and not function well with the other modeling parameters fine-tuned for a RANS-like solution. The result is indeed an early transition that shrinks the LSB. Conversely, the variant with $\chi_2 = 50$, despite keeping the same mean profiles as SA-BCM, promotes some instabilities downstream the bubble as depicted from Fig. 2(c). Therefore, even though $\chi_2 = 1000$ may be too large, $\chi_2 = 50$ is also not ideal. However, the incorporation of the transition RANS model into the hybrid framework still represents a major advantage. This is evident from the fact that the fully-turbulent IDDES model fails to detect the LSB and, instead, predicts a flow that is completely attached across the entire suction surface. Note that the oscillations in the C_f curves are just related to the discontinuous nature of the DG discretization which, combined with coarse spacings, may lead to visible jumps in the solution derivative at the interface between mesh elements.

Moving to the case at $\alpha = 8^\circ$, the plots of Fig. 3 support the observations already made for the version with $\chi_2 = 50$ at zero incidence. As a matter of fact, although IDDES_{TRA} ($\chi_2 = 50, \tilde{f}_d^*$) detects transition, the mechanism resembles more of a separation-induced process than a natural one, as noticeable from the distribution of the skin friction coefficient. The boundary layer detachment in turn generates quite prominent vortical structures (see Fig. 3(c)). In the model using $\chi_2 =$

1000, instead, transition is correctly predicted at $x/c \approx 0.2$ in agreement with the experiments (McGhee, 1988) and the solution remains in RANS mode. The unsatisfactory results related to $\chi_2 = 50$ can be better understood looking at Figs. 4, 5, which depict the solutions obtained at different χ_2 for IDDES_{TRA}($\tilde{f}_d^*$) at $k = 3$. In particular, it can be observed how χ_2 has a strong influence on the onset of the laminar-to-turbulent evolution. This comes from the fact that in natural transition SA-BCM does not work as expected, as will be also shown in detail in Section 3.2. Indeed, in unforced transition the activation of γ_{BC} is entrusted to T_2 rather than T_1 , with the location of the triggering point that becomes almost entirely contingent upon the values of $\tilde{v}_\infty/\nu$ and χ_2 (see Eq. (7)). This is clearly displayed by Fig. 5, where the activation γ_{BC} moves progressively upstream with the increment of χ_2 . In particular, only for $\chi_2 = 50$ the activation of T_2 is preceded by that of T_1 which takes place at $x/c > 0.6$, much later than the transition point observed in the experiments. The extreme sensitivity to χ_2 observed in the case of natural transition is obviously undesirable. To address this issue and derive a generally valid implementation, however, one should act on T_1 rather than T_2 , in such a way to make the SA-BCM model function as intended in the original design. Among the possible remedies, one could employ more sophisticated correlations for $Re_{\theta c}$ or replace Tu_∞ with an alternative local formulation. Such improvements, nonetheless, require an in-depth analysis and are therefore reserved for future work. In any case, the replacement of SA with SA-BCM always results in an improvement. In fact, IDDES generates a fully turbulent flow already starting from the LE, thereby completely ignoring the transition of the boundary layer.

In the last two angles of attack IDDES_{TRA} ($\chi_2 = 50, \tilde{f}_d^*$) is not included because its results are very similar to those generated by $\chi_2 = 1000$. The reason is that in case of massive separation a small loss of eddy viscosity owed to the partial penetration of the intermittency function up to the wall does not substantially alter the solution. At $\alpha = 14^\circ$ (see Fig. 6), although the difference between the turbulence models is rather thin, it is still possible to observe how the fully-turbulent IDDES fails to reproduce the LSB at the LE, resulting in the least accurate mean pressure profile. The most interesting observation, however, is linked to the different behavior of SA-BCM compared to IDDES_{TRA}($\tilde{f}_d^*$). The latter, in particular, converges towards the experimental data as the polynomial order is increased, improving the LSB assessment at the leading edge (treated in RANS mode, as intended), the turbulent reattachment and even the massively separated region at the trailing edge. These improvements are reflected in the mean lift and drag coefficients of Table 3.

A similar consideration can be made for the case at $\alpha = 16^\circ$ (see Fig. 7). At this angle of attack, the raise of the polynomial basis functions up to fifth order allows the transitional hybrid model to accurately predict the stall on the whole suction surface, which pure RANS is unable to do. Due to the notable variation in the solution from $k = 4$ and $k = 5$, results obtained on a refined mesh at $k = 4$ (395 325 DOFs) are included as evidence of spatial convergence. In addition, note that the fully-turbulent IDDES, which intuitively should behave similarly to the transitional version because of the complete stall conditions, actually gives much worse predictions highly underestimating the separated region, where it behaves more like SA-BCM than IDDES_{TRA}. This last case is also useful to show that $\chi_2 = 1000$, despite contributing to a general increase of eddy viscosity, does not spoil the simulation of fully detached flows.

3.2. Sphere

The flow around a sphere of diameter D is simulated at $Re_D = 420\,000$, i.e. in the supercritical regime, by means of a box of size $18D \times 9D \times 9D$, where the first dimension refers to the streamwise direction (x) and the last two (y, z) to the transverse directions. The center of the sphere coincides with the origin of the frame of reference,

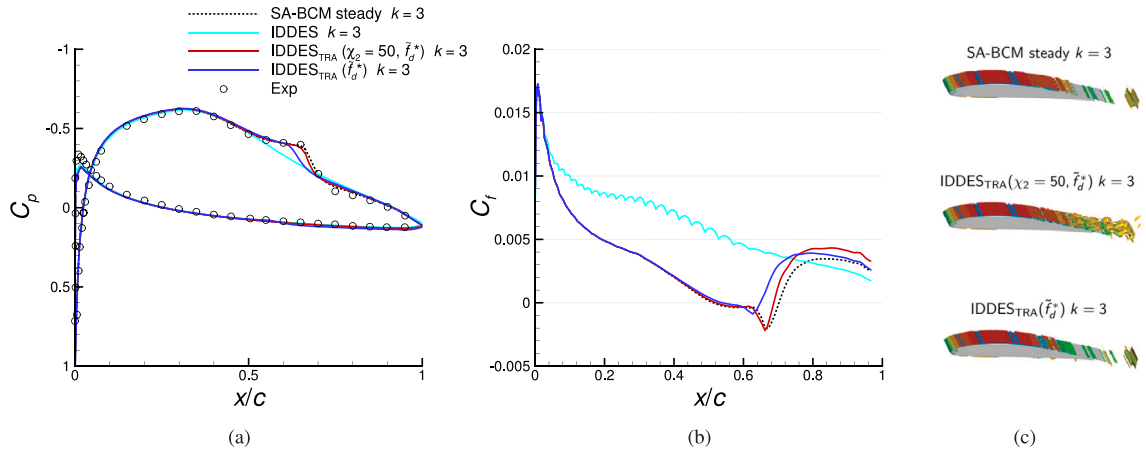


Fig. 2. Eppler 387 — $\alpha = 0^\circ$ — Mean profiles of (a) pressure and (b) skin friction coefficient. (c) Isosurfaces of $\lambda_2 = -1$ colored with the streamwise velocity. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
Source: Experimental data from McGhee (1988).

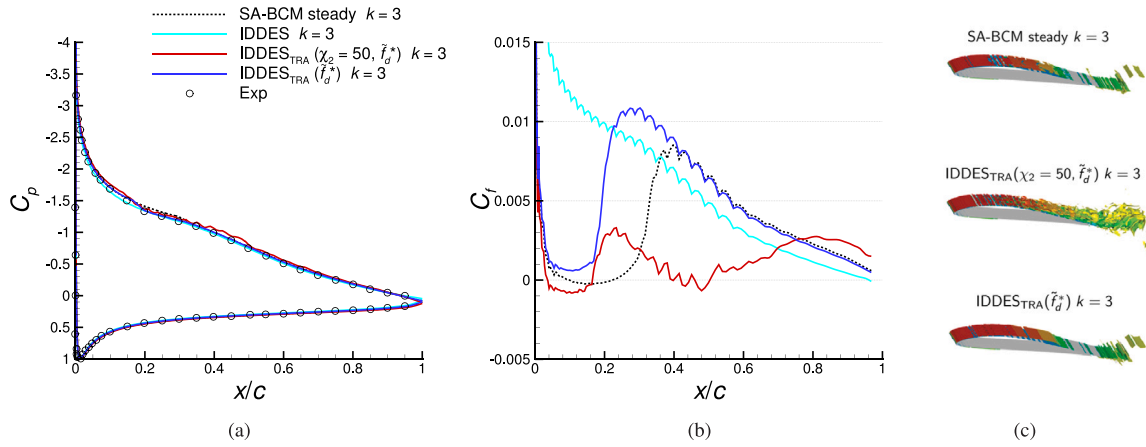


Fig. 3. Eppler 387 — $\alpha = 8^\circ$ — Mean profiles of (a) pressure and (b) skin friction coefficient. (c) Isosurfaces of $\lambda_2 = -1$ colored with the streamwise velocity. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
Source: Experimental data from McGhee (1988).

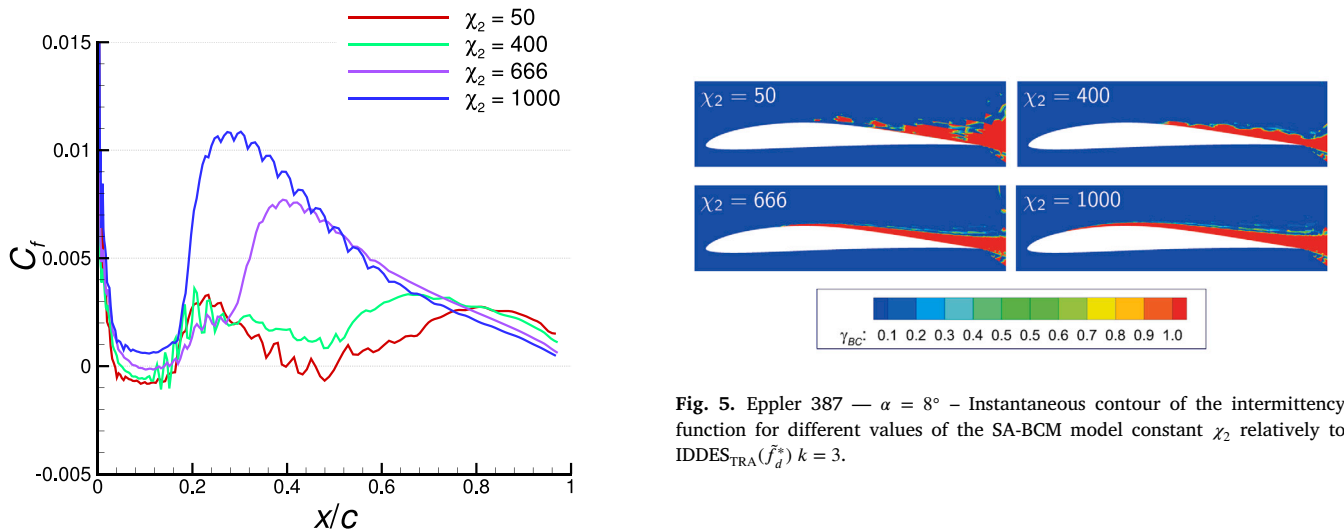


Fig. 4. Eppler 387 — $\alpha = 8^\circ$ — Mean profiles of skin friction coefficient for different values of the SA-BCM model constant χ_2 relatively to IDDES_{TRA} ($\tilde{f}_d^*$) $k=3$.

Fig. 5. Eppler 387 — $\alpha = 8^\circ$ — Instantaneous contour of the intermittency function for different values of the SA-BCM model constant χ_2 relatively to IDDES_{TRA} ($\tilde{f}_d^*$) $k=3$.

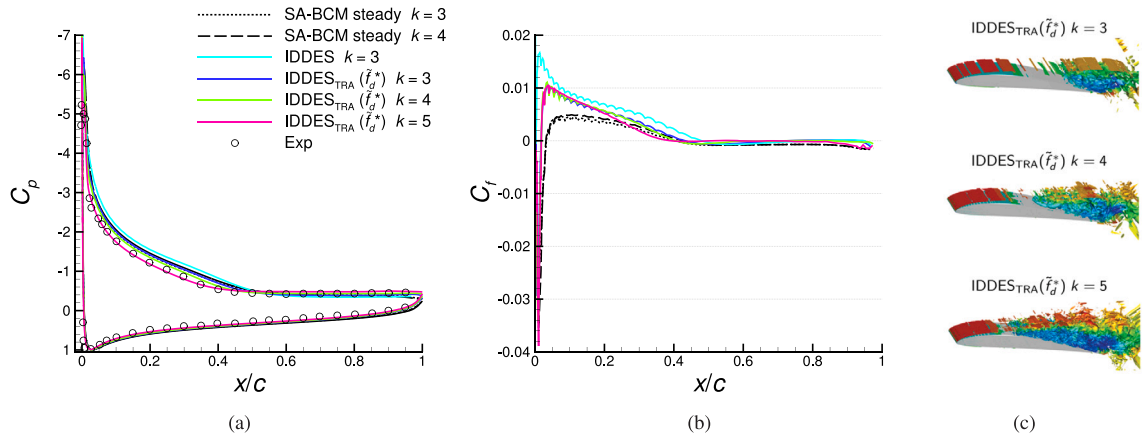


Fig. 6. Eppler 387 — $\alpha = 14^\circ$ — Mean profiles of (a) pressure and (b) skin friction coefficient. (c) Isosurfaces of $\lambda_2 = -1$ colored with the streamwise velocity. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Source: Experimental data from McGhee (1988).

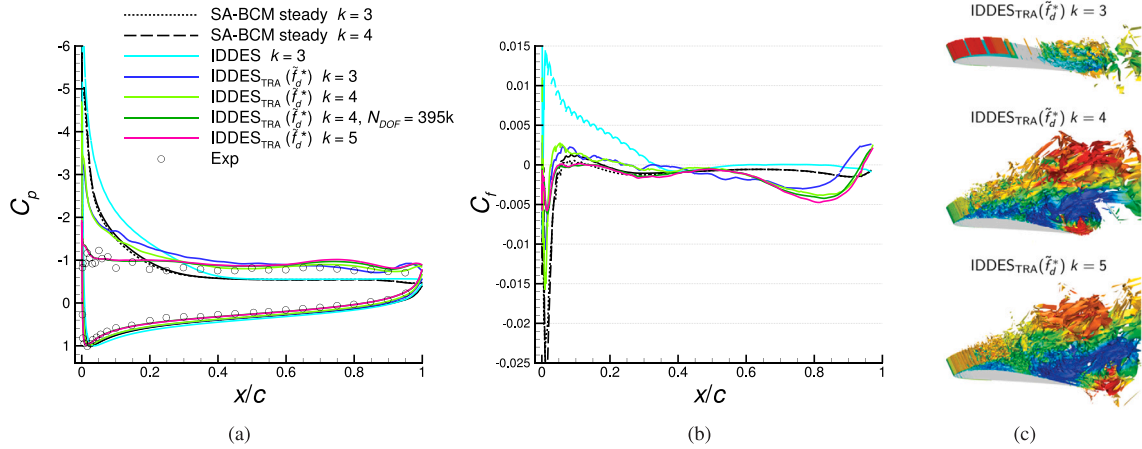


Fig. 7. Eppler 387 — $\alpha = 16^\circ$ — Mean profiles of (a) pressure and (b) skin friction coefficient. (c) Isosurfaces of $\lambda_2 = -1$ colored with the streamwise velocity. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Source: Experimental data from McGhee (1988).

Table 4

Sphere — Spatial resolution for the different DG polynomial orders. N_e = total number of grid cells, $\Delta_{y,w}^+$ = first cell height (normalized by $k+1$), N_{DOF} = total number of spatial DOFs.

N_e	$k=3$		$k=4$	
	$\Delta_{y,w}^+$	N_{DOF}	$\Delta_{y,w}^+$	N_{DOF}
53 360	1.0	1 067 200	0.8	1 867 600

with the domain extending $4.75D$ upstream and $13.25D$ downstream in the x -direction. The domain is discretized using the structured mesh showed in Fig. 8, whose spatial characteristics are summed up in Table 4. The elements have second order edges to better capture the curvature of the sphere. In the wall proximity there are 10 inflated layers with growth rate 1.4. The time span for the average is at least $200D/U_\infty$, to be sure to encompass a few vortex shedding cycles.

In this test case the turbulence model is particularly crucial, in fact the correct detection of the start and development of the natural transition process of the boundary layer is fundamental also to accurately predict the separation point and, consequently, the topology of the wake. In Fig. 9 are plotted the mean pressure and skin friction as a function of the elevation angle ϕ , where $\phi = 0^\circ$ corresponds to the stagnation point, for the various modeling configurations examined.

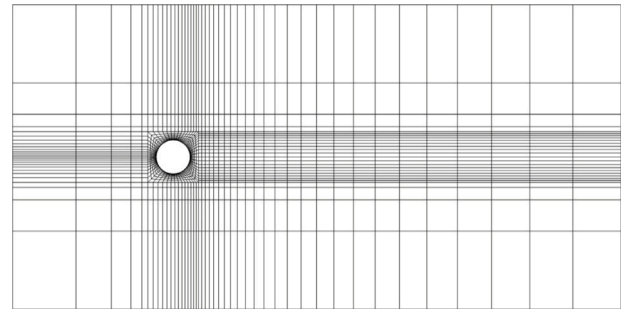


Fig. 8. Sphere — Cross section of the computational grid at the $x - y$ middle plane.

From the C_p graph one can notice how all hybrid models seem to be characterized by a similar behavior, reaching a plateau approximately at the same elevation angle. The unsteady transitional RANS is the only one giving a pretty different solution due to the asymmetry of the flow detachment, as shown in the mean velocity contour of Fig. 10. Looking at the C_f graph, instead, it is possible to appreciate substantial differences between the hybrid closures. In particular, the model with

Table 5

Sphere — Mean drag coefficient and location of the transition (ϕ_T) and separation (ϕ_S) point for the modeling setups explored.

	C_D	ϕ_T [deg]	ϕ_S [deg]
SA-BCM $k = 3$	0.375	123	130
IDDES _{TRA} ($\chi_2 = 50$) $k = 3$	0.146	99	99
IDDES _{TRA} $k = 3$	0.137	102	126
IDDES _{TRA} $k = 4$	0.145	79	115
IDDES _{TRA} ($\tilde{f}_d^*$) $k = 3$	0.095	97	124
IDDES _{TRA} ($\tilde{f}_d^*$) $k = 4$	0.083	96	118
Exp, Achenbach (1972)	0.085	98	119

$\chi_2 = 50$ actually produces a separation-induced transition, with the flow detachment being anticipated down to $\phi \approx 105^\circ$. This is most likely due to the fact that χ_2 is so low that it delays the activation of the intermittency function to the point of triggering transition only once separation has already occurred. Indeed, as anticipated in Section 3.1, in our framework SA-BCM does not work as expected for natural transition mechanisms and delegates the responsibility of activating γ_{BC} to T_2 rather than T_1 . In all IDDES_{TRA} models with $\chi_2 = 1000$, on the other hand, the flow follows a natural transition process that precedes turbulent separation, which correctly occurs around $\phi \approx 120^\circ$. Although in this case the correct prediction of the transition and separation point using $\chi_2 = 1000$ is certainly due, in part, to calibration with the inflow turbulent parameters, the effectiveness of the rest of the modeling structure is generally valid. The key aspect, indeed, is that while in the version with the original shielding function passing from $k = 3$ to $k = 4$ induces a substantial shift of the transition point upstream, this does not occur when using $\tilde{f}_d^*$. The anticipation of the laminar-to-turbulent evolution of the boundary layer leads to early thickening (at $\phi = 95^\circ$, for $k = 4$, IDDES_{TRA} predicts a boundary layer thickness of $\delta^+ = 1240$ whereas IDDES_{TRA}($\tilde{f}_d^*$) is characterized by $\delta^+ = 600$) and causes C_f to reach a higher peak. This can be interpreted as a successful attempt by $\tilde{f}_d^*$ to prevent a troublesome switch from pure Navier–Stokes directly LES in critical circumstances, here owed to the significant mesh refinement. In any case, it can be noted that the raise of the polynomial order leads to a flattening of the skin friction and pressure coefficient post-separation in both variants. The fact that C_p reaches a plateau at a higher level is supposedly the main reason for the decrease in the mean drag coefficient, which approaches the experimental value as is clear from Table 5. This is perfectly consistent with the fact that pressure distribution, especially the recirculation region within the wake, is primary responsible for generating drag in bluff bodies.

The observations made on the transition location for the various modeling configurations are also confirmed by the intermittency function contours in Fig. 11. Particularly, one can see the late activation of γ_{BC} for the transitional RANS and the postponement until separation for the model with $\chi_2 = 50$. Moreover, Fig. 11 clearly shows ϕ_T being push backward with the standard f_d from $k = 3$ to $k = 4$, while staying roughly the same for $\tilde{f}_d^*$.

3.3. NACA0012

The flow around the NACA0012 foil, characterized by $Re_c = 130\,000$ and $\alpha = 11.5^\circ$, is simulated by means of a computational domain exactly equal to that described in Section 3.1. The grid, depicted in Fig. 12 and with the main properties summarized in Table 6, is composed by elements with second order edges to follow the curvature of the profile. The near wall region is discretized with 13 inflated layers with growth rate 1.4. The averages are performed for about $100c/U_\infty$, with U_∞ being the inflow velocity.

The case at hand poses challenges from a modeling perspective due to the establishment of a periodic low-frequency cycle composed by the following phases: (i) formation of an LSB at the LE with a length of $0.2c$; (ii) enlargement of the LSB until its burst; (iii) completely

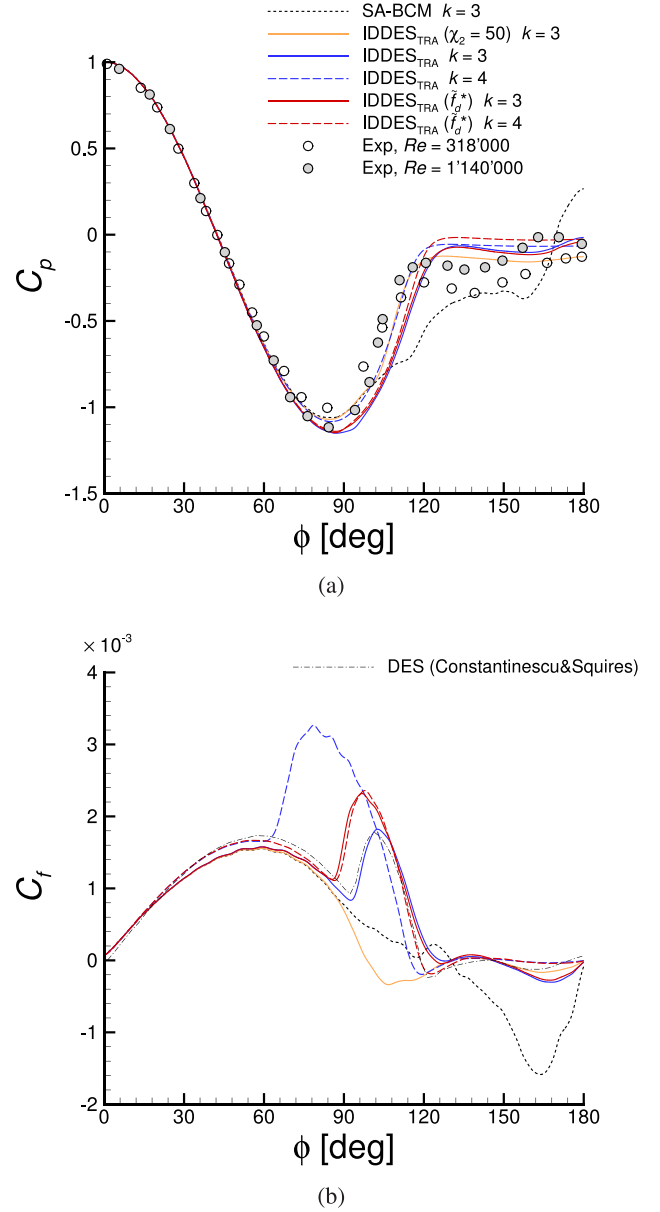


Fig. 9. Sphere — Mean profiles of (a) pressure and (b) skin friction coefficient. Source: Experimental data from Achenbach (1972) and DES curve from Constantinescu and Squires (2004).

Table 6

NACA0012 — Spatial resolution for the different DG polynomial orders. N_e = total number of grid cells, $\Delta_{y,w}^+$ = first cell height (normalized by $k+1$), N_{DOF} = total number of spatial DOFs.

N_e	$k = 3$		$k = 4$		$k = 5$	
	$\Delta_{y,w}^+$	N_{DOF}	$\Delta_{y,w}^+$	N_{DOF}	$\Delta_{y,w}^+$	N_{DOF}
9650	1.0	193 000	0.8	337 750	0.7	540 400

separated flow; (iv) flow reattachment with the LSB which shrinks back to $0.2c$; In this situation it is necessary to find the right balance of eddy viscosity, indeed excessively low values would maintain a persistent stall condition, whereas too high values would not allow to capture the LSB growth to the point of bursting. To do this, the turbulence model must be able to follow the movement of the separation point, which travels continuously back and forth during the low-frequency cycle.

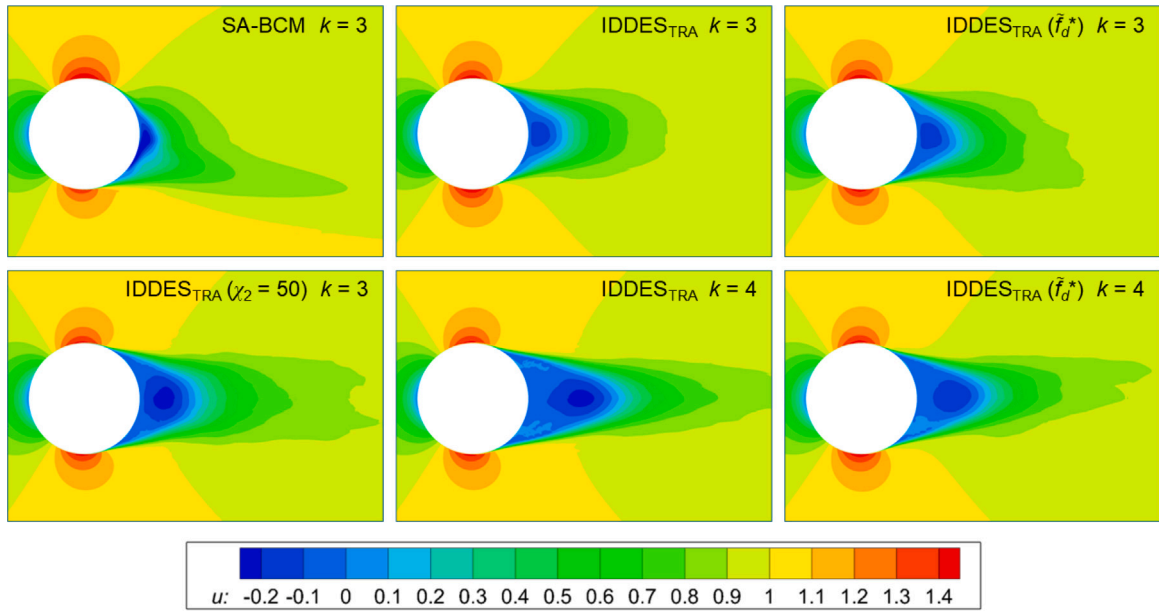


Fig. 10. Sphere — Mean streamwise velocity contours.

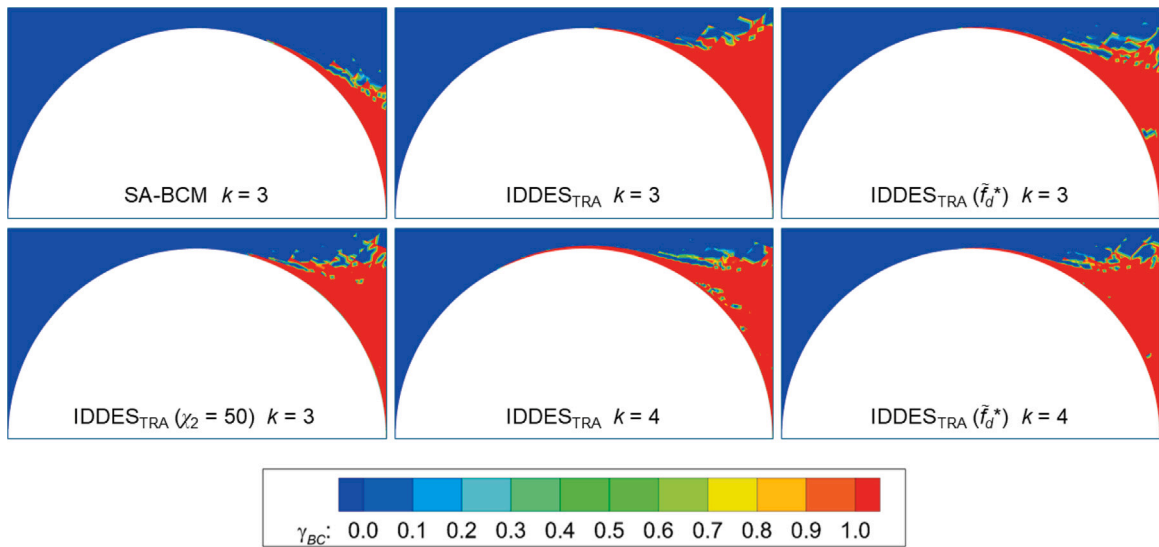
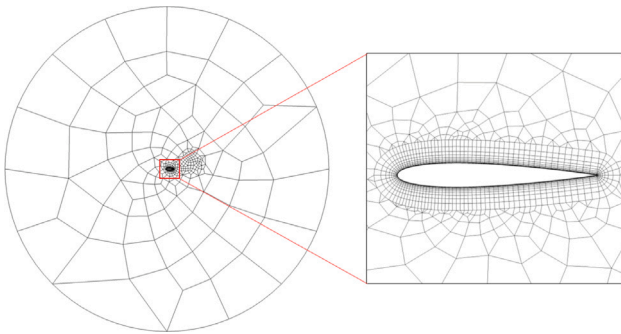


Fig. 11. Sphere — Instantaneous intermittency function contours.

Fig. 12. NACA0012 — $\alpha = 11.5^\circ$ — Grid and relative close up view of the inflated layers at the $x-y$ middle plane.

As depicted in Fig. 14, here the fully-turbulent IDDES represents the modeling approach with an overabundant eddy viscosity and in fact the flow does not develop a proper LSB but only exhibits a mild separation that starts from $x/c \approx 0.2$. Meanwhile, the steady transitional RANS delays the activation of the intermittency function enough to cause a separation over almost the entire suction surface, which worsens at $k = 4$ due to the further decrease in eddy viscosity. The transitional hybrid models at $k = 3$ are also unable to simulate the whole LSB evolution, overestimating the totally-separated flow phase at the expense of the state where there is only a small LSB upfront. However, passing to $k = 4$ they start to accurately capture each phase of the cycle, as can be seen from the time signal of the lift coefficient in Fig. 15. At first this may seem counterintuitive, since the decrease in eddy viscosity owed by a finer discretization could potentially cause an even greater overestimation of the stall phase. Nevertheless, in a case where it is the LES branch that governs the growth and the burst of the bubble, one has to consider that a higher resolution actually ensures a better assessment of the physical mechanisms promoting the reattachment.

Since the solutions undergo a significant jump passing from $k = 3$ to $k = 4$, we decided to include (just for the version with $\tilde{f}_d^*$) the results obtained with $k = 5$, as proof of convergence behavior; the similar trend between the solutions at $k = 4$ and $k = 5$ can be also appreciated in Fig. 15(b). Regarding the two transitional hybrid variants, one may notice that once again there is an advantage in using the modified shielding function over the original one. Indeed, at $k = 3$ and even more at $k = 4$ the average solution of $\text{IDDES}_{\text{TRA}}(\tilde{f}_d^*)$ gets closer to the experimental data. This is once again associated with a more consistent division of the laminar, RANS and LES regions provided by the modified shielding function. Fig. 13 displays exactly how $\tilde{f}_d^*$ preserves the desired progression $\text{NS} \rightarrow \text{RANS} \rightarrow \text{LES}$ at the location of incipient transition. It is worth to highlight that, unlike in the Eppler 387 airfoil, here the solver is globally characterized by a WMLES behavior, even when the flow is attached over most of the suction surface. The hybrid model is in fact not capable of “cleaning” the turbulent content developed during the stall phase and therefore never returns to a purely RANS operating mode. Focusing on the time signal of the lift and drag coefficient in Fig. 15, we can note that transitional hybrid models at $k = 4$ identify both the phase of attached flow with just a small LSB upfront, characterized by $C_L \approx 1$, and the phase of totally separated flow, where $C_L \approx 0.5$. Both variants successfully reproduce the frequency associated with the periodic evolution of the LSB, which was reported by Rinoie and Takemura (2004) and AlMutairi et al. (2017) to correspond to a Strouhal number of 0.008 and 0.0083, respectively. Specifically, $\text{IDDES}_{\text{TRA}}$ returns a value of 0.0077, while $\text{IDDES}_{\text{TRA}}(\tilde{f}_d^*)$ yields 0.0097. Considering the limited number of snapshots used for the statistical analysis ($\Delta t = 4 \times 10^{-3}$, with data sampled every 100 time steps), these results can be regarded as being in good agreement with the reference value. This is in stark contrast with the time signal of the same variants at $k = 3$, which show a lift coefficient fluctuating chaotically around $C_L \approx 0.6$ without a specific frequency. Three instants are selected within their time signals to take snapshots and give also a visualization of the instantaneous flow fields. As already described, it is possible to notice how $\text{IDDES}_{\text{TRA}}$ at lower polynomial order consistently predicts a widespread flow detachment past the separation point with both $\tilde{f}_d$ and $\tilde{f}_d^*$. Oppositely, at higher polynomial order it correctly identifies the completely separated flow in snapshot (I) which then leads to the progressive re-establishment of the LSB at the fore portion of the foil in snapshots (II) and (III). We remark that, despite the large presence of three-dimensional structures even in snapshot (III), on average the LES portion of IDDES is not working in an ILES fashion but as an actual hybrid model, as is clear from Fig. 16. In particular, Fig. 16(a) displays the contour of the average equivalent Smagorinsky constant, defined as

$$\tilde{C}_S = \sqrt{\frac{\nu_t}{\Delta^2 \Omega}}, \quad (14)$$

where Ω is the vorticity magnitude and $\Delta = V^{1/3}$, which is similar to the value assumed by $\tilde{\Delta}_\omega$ in the LES region for this specific grid. This constant should provide a measure of how much IDDES is working close to a Smagorinsky subgrid-scale model. The area where $\tilde{C}_S \approx 0.5$ identifies a region that, on average, is dominated by the RANS mode, additionally the vast majority of the LES portion is characterized by values around 0.1–0.2, which are consistent with a standard Smagorinsky model. Fig. 16(b) supports this finding as, in the near-wall region, the modeled Reynolds stress dominates over the resolved part at $x/c = 0.2$ and still contributes $\approx 10\%$ to the total Reynolds stress at $x/c = 0.4$. Furthermore, one can note that at $x/c = 0.4$ the equivalent Smagorinsky constant reaches a local plateau at 0.14, signaling an operating mode which resembles an explicit LES. The quality of the solution related to $\text{IDDES}_{\text{TRA}}$ and $\text{IDDES}_{\text{TRA}}(\tilde{f}_d^*)$ at $k = 4$ can also be observed from Table 7, where we note that the mean lift coefficients fall in between the values of the very refined LES of AlMutairi et al. (2017) and the experimental results of Rinoie and Takemura (2004), while the mean drag coefficients are practically identical to the LES

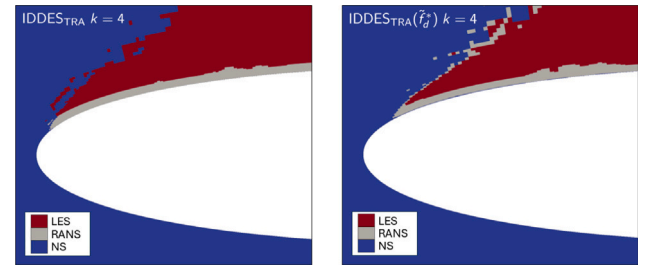


Fig. 13. NACA0012 — Comparison between the operating modes of $\text{IDDES}_{\text{TRA}}$ with the original (left) and modified (right) shielding function. The NS portion identifies all regions where $\gamma_{BC} < 0.1$, whereas the RANS/LES interface is set at $\tilde{f}_d$ (or $\tilde{f}_d^* = 0.5$).

Table 7

NACA0012 — $\alpha = 11.5^\circ$ – Mean lift and drag coefficient.

	C_L	C_D
SA-BCM steady $k = 3$	0.619	0.128
SA-BCM steady $k = 4$	0.593	0.134
IDDES $k = 3$	0.713	0.090
$\text{IDDES}_{\text{TRA}}$ $k = 3$	0.615	0.157
$\text{IDDES}_{\text{TRA}}$ $k = 4$	0.726	0.129
$\text{IDDES}_{\text{TRA}}(\tilde{f}_d^*)$ $k = 3$	0.635	0.155
$\text{IDDES}_{\text{TRA}}(\tilde{f}_d^*)$ $k = 4$	0.748	0.127
$\text{IDDES}_{\text{TRA}}(\tilde{f}_d^*)$ $k = 5$	0.756	0.119
LES, AlMutairi et al. (2017)	0.714	0.127
Exp, Rinoie and Takemura (2004)	0.880	—

in AlMutairi et al. (2017); this agreement is particularly noteworthy given that the present approach employs approximately two orders of magnitude fewer DOFs and a time step size roughly 50 times larger. In contrast, the same versions at $k = 3$ lead to a marked underestimation of C_L and a concurrent overestimation of C_D due to the persistent globally separated flow owed to an insufficient resolution in the scale-resolving portion of the domain. In a similar fashion, Table 7 shows how steady transitional RANS has problems in assessing the mean lift contribution which is much lower than the reference data because of the extensive recirculation region. Oppositely, the fully-turbulent IDDES highly undervalues the mean drag coefficient in virtue of its mildly detached solution.

4. Conclusions

The work focuses on the development of a hybrid RANS/LES model of IDDES-type with transitional capabilities within a high-order modal DG solver. The detection of the laminar-to-turbulent evolution is enabled by replacing the fully-turbulent SA model with an algebraically modified formulation, referred to as SA-BCM. To ensure an effective integration of the transition model, originally formulated for pure RANS, within a hybrid framework, three changes are proposed: (1) The first adjustment prevents the corruption of the eddy-viscosity balance in LES mode; (2) the second mitigates the risk of an undesired switch from laminar regions directly to LES for particularly subtle transition processes; (3) the third acts on the intermittency function activation in natural transition. We remark that these modifications are recommended only when SA-BCM is coupled with DES-like models. This is especially true for modification (3), where the eddy viscosity levels generated in pure RANS would surely spoil the original calibration. The test cases examined, including the flow over an Eppler 387 airfoil at various angles of attack and the flow past a sphere in the post-drag crisis regime, demonstrate the effectiveness of the proposed modifications and the ability to deal with localized and massive separation or even a combination of the two. Actually, the last revision exhibits

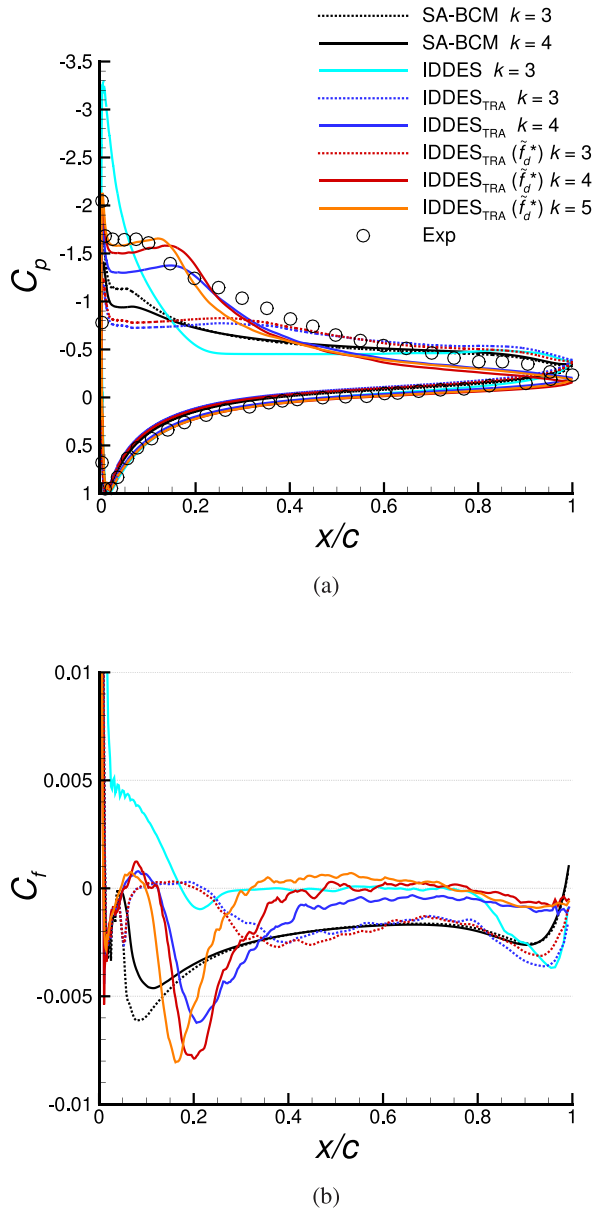


Fig. 14. NACA0012 — $\alpha = 11.5^\circ$ — Mean profiles of the (a) pressure and (b) skin friction coefficient.

Source: Experimental data from [Rinoie and Takemura \(2004\)](#).

some drawbacks, i.e.: (i) in the absence of massive separation, there is a risk of slightly suppressing the LSB; (ii) the natural transition location is highly dependent on χ_2 and so the chosen value is not always correct but needs to be either manually adjusted or balanced with the proper free-stream eddy viscosity (e.g. sphere). On the other hand, this modification proves effective for massively separated flows, where it also has the nice property of ensuring complete near-wall penetration of γ_{BC} . Although one could consider calibrating χ_2 on a case-by-case basis to ensure correct behavior, this does not constitute a general solution, highlighting the need for more refined strategies that will be explored in future studies. The final case, involving the flow around a NACA0012 airfoil, further illustrates the suitability of the transitional IDDES in scenarios with a non-fixed transition and separation point, which should also allow for the study of dynamic problems. Although

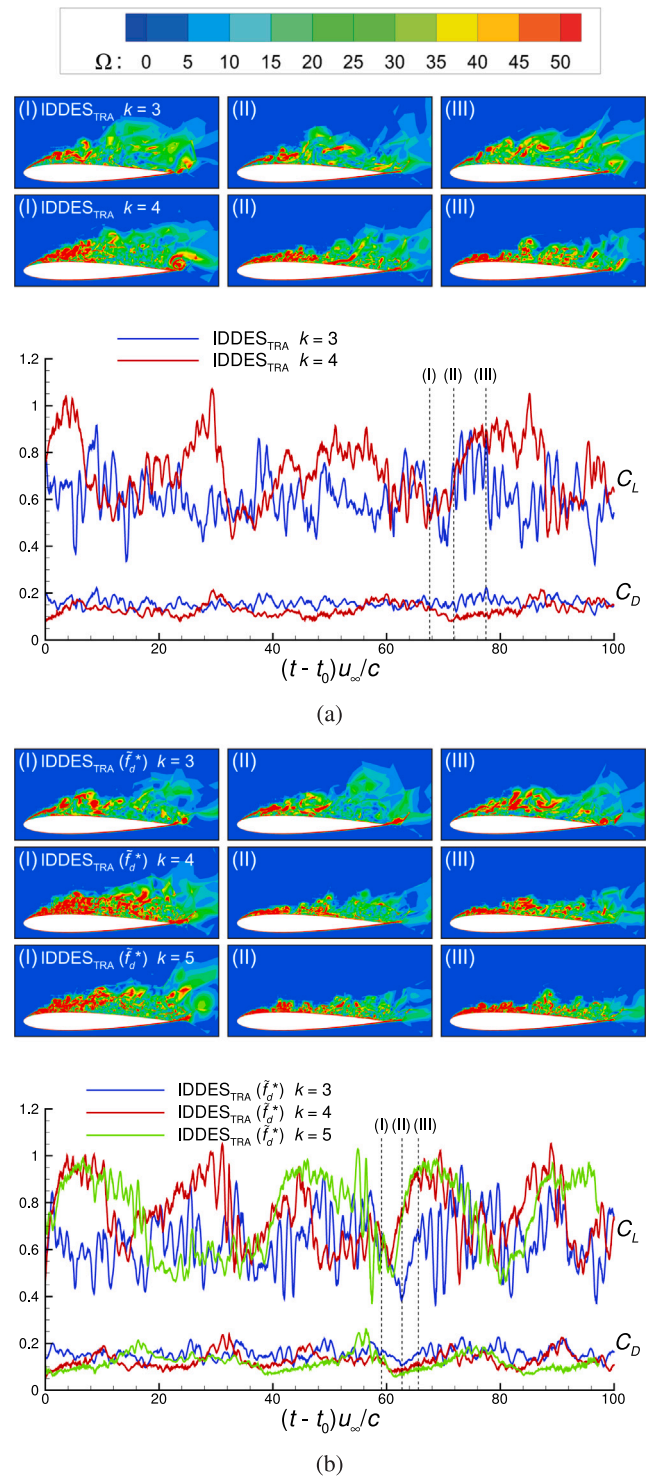


Fig. 15. NACA0012 — $\alpha = 11.5^\circ$ — Time signal of lift and drag coefficient together with contour plots of the vorticity magnitude field. Time is initialized at the start of the sampling window used for the averages.

in the current form SA-BCM exhibits some limitations in predicting natural transition, the generality of our proposals allows for an extension to more sophisticated transition models that may eventually supersede the current algebraic formulation.

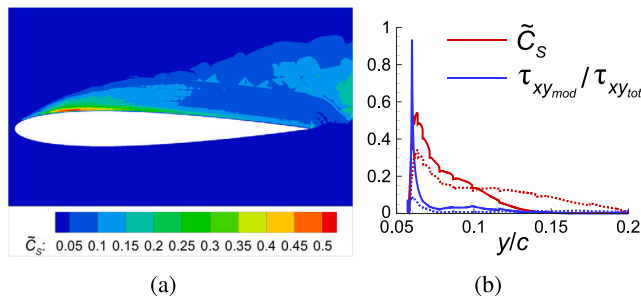


Fig. 16. NACA0012 — $\alpha = 11.5^\circ$ — (a) Contour of the average equivalent Smagorinsky constant for IDDES_{TRA}($\tilde{f}_d^*$) $k = 4$. (b) Mean profiles of the equivalent Smagorinsky constant and the ratio between modeled and total Reynolds stress at $x/c = 0.2$ (solid) and $x/c = 0.4$ (dashed).

CRedit authorship contribution statement

Luca Alberti: Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Emanuele Carnevali:** Writing – review & editing, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation. **Andrea Crivellini:** Writing – review & editing, Validation, Supervision, Software, Investigation, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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