International historical evidence on money growth and inflation: The role of high inflation episodes

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Abstract

How long is the long run in the relationship between money growth and inflation? How important are high inflation episodes for the unit slope finding in the quantity theory of money? To answer these questions, we study the relationship between excess money growth and inflation over time and across frequencies using annual data from 1870 to 2013 for 16 developed countries. Wavelet-based exploratory analysis shows the existence of a close stable relationship between excess money growth and inflation only over long time horizons, i.e. periods greater than 16-24 years, with money growth mostly leading. When we investigate the sensitivity of the unit slope finding to inflation episodes using a 'time-frequency-based' panel data approach, we find that low-frequency regression coefficients estimated over variable-length subsamples are largely affected by high inflation episodes occurring in the 1910s, the 1940s and the 1970s. Taken together, our results suggest that inflationary upsurges affect regression coefficients, but not the closeness of the long-run relationship. This reconciles the validity of the Quantity Theory of Money with the current disinterest of monetary policy making in money growth.

JEL codes: C22, E40, E50, N10.
Key words: Quantity theory of money, time-frequency analysis, low frequency relationships, high inflation episodes.

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1 Introduction

Since Friedman (1956), the literature has analyzed whether changes of the growth rate of money supply lead to equal changes of the growth rate of the price level. Later on, following Lucas (1980) contribution, there has been a proliferation of empirical studies on the quantity theory of money (henceforth QTM) using both time-series data for individual countries and international cross-country datasets (e.g. Whiteman, 1984; McCallum, 1984; Geweke, 1986; Stock and Watson, 1988; Dwyer and Hafer, 1988, 1999; Barro, 1990; King and Watson, 1992; Christiano and Fitzgerald, 2003).

The results of previous research investigating QTM, however, are dependent on some critical issues. To begin with, there is no consensus, across studies, on the exact definition of the frequency span over which the QTM is supposed to hold. If the frequency range is well defined in its lower limit, generally identified with the upper range of business cycle fluctuations (8 or 10 years), the definition of its upper limit is more indeterminate, spanning from 20 to 50 years and even longer (e.g. Blanchard, 1997; Rotemberg, 1999; and Comin and Gertler, 2006). This lack of consensus has led to studies on the money growth-inflation relationship using different pre-determined frequency bands, such as 8 to 20 years, 20 to 40 years (Christiano and Fitzgerald, 2003), 8 to 40 years (Haug and Dewald, 2004), and even periods beyond 30 years (Benati, 2009).

Secondly, one needs to work with datasets that provide sufficient volatility in the growth rate of money to measure the effects of money growth on the inflation rate (Fisher and Seater,1993). However, that comes with a drawback: the inclusion of high growth money and prices rates may break down the one-to-one long-run relationship between these two variables (e.g., Barro, 1993; McCandless and Weber, 1995; Lucas, 1996). Thirdly, many contributions that study high inflation episodes focus on countries (typically developing countries) with a history of both high monetary growth and inflation rates. In contrast, by including pre-WWII data, we can assess the impact of high-inflation episodes within a general history of price and monetary stability.[1] That is, we concur with Christiano and Fitzgerald (2003, p. 22) that "much can be learned by incorporating data from the first half of the [20th] century into the analysis of inflation and monetary policy".

The aim of this contribution is twofold: to shed light on the length of the long-run relationship between excess money growth and inflation, and to investigate the sensitivity of the estimated coefficients in the money growth inflation relationship to the presence of high money and inflation episodes in the estimation sample. In contrast with previous contributions (e.g., Neumann and Greiber, 2004; Rua, 2012; and Mandler and Scharnagl, 2014) , we use the new historical macro database by Jorda et al. (2017) that covers 16 developed countries and spans 140 years (1870-2013). This international dataset is well suited for investigating both issues, as it contains several high inflation and monetary growth episodes experienced by a group of low inflation countries in the

[1] Few studies use long historical data going back to the beginning of the 20th century: for the US, see Christiano and Fitzgerald (2003) and for a sample of industrial countries, see Dewald (2003), Haug and Dewald (2004), and, more recently, Sargent and Surico (2011).

1910s, the 1940s, and the 1970s (Dewald, 2003). Methodologically, we exploit the wavelet transform, both in the continuous and the discrete framework, to capture the intrinsic time-frequency dependence of the excess money growth - inflation relationship.

Our results, in the continuous framework, show that the time horizon of the long-run relationship is quite long: strong stable co-movements are evident for the Anglo-Saxon countries over 24-year time horizons, and for the rest of the countries over 16-year time horizons, with a widespread tendency of the long-run relationship to shift towards lower frequencies in the second half of the post-WWII period. We also document a stable coincident time lag relationship, especially at lower frequencies, and two significant changes in the slope of the coefficient of inflation with respect to excess money growth; the first is an increase in coincidence with high inflation episodes, the second is a decrease, especially after the 1990s, that is common to almost all countries. Finally, after decomposing the variables using the Maximal Overlap Discrete Wavelet Transform (MODWT), we estimated parametrically the relationship between inflation and excess money growth using separate panel datasets, each composed by data at different frequency ranges (e.g. Gallegati et al., 2016). Upward and downward shifts in the low-frequency regression coefficients are clearly detected in both the pre- and post-WWII periods, which are critically dependent on high-inflation episodes. Our headline finding is that a proper verification of the QTM and understanding of the current conduct of monetary policy requires testing contemporaneously for time and frequency variations. The evidence that inflationary upsurges affect regression coefficients, but not the closeness of the long-run relationship between excess money growth and inflation, may reconcile the validity of the quantity theory of money with the current disinterest in money growth rates in the current low inflation policymaking environment.

The paper is divided in four sections. Section 2 briefly introduces the continuous and discrete wavelet transforms. Section 3 applies several CWT bivariate tools to detect the time horizon of the long-run relationship in the QTM. Section 4 tests the QTM unit slope hypothesis applying a 'time-frequency-based' panel regression approach to variable-length sub-samples in the pre- and post-WWII periods. Section 5 offers the main conclusions of the paper.

2 Continuous and discrete wavelet transforms

Wavelet methods, by simultaneously extracting the time and frequency content of a signal, are well suited to investigate questions such as the QTM.[2] The wavelet transform uses a set of local basis functions that are dilated, or compressed, through a scale or dilation factor and shifted along the signal through a translation or location parameter. This property is particularly useful when dealing with complex, non-stationary signals, such as historical time series, since their secular movements

[2] A brief technical introduction to wavelet analysis is provided in the Appendix. Introductory surveys for economists are provided in Gencay et al. (2002), Crowley (2007), and Aguiar-Conraria and Soares (2014).

are likely to exhibit structural changes due to shocks such as wars and crisis periods.[3] Moreover, by using relatively short rolling windows at shortest time scales and relative long windows at longest time scales, the wavelet filter,[4] in contrast to band-pass filtering methods, allows researchers not to be committed to any class of models. There are two types of wavelet transforms: the continuous wavelet transform (CWT) and the discrete wavelet transform (DWT). The CWT operates with smooth continuous functions and decompose signals at all scales. It is a highly redundant transform that produces information in a two-dimensional format, where each wavelet coefficient is represented by a pair of data, location or time and scale (Gencay et al., 2002). DWT, instead, uses only a limited discrete number of translated and dilated versions of the wavelet basis to decompose the original signal, with locations and scales normally based on a dyadic arrangement.

A full description of the wavelet methodology is beyond the scope of this introduction. Aguiar-Conraria and Soares (2014) have written an excellent survey of the CWT both in a bivariate and in a multivariate set up. Recent contributions in economics and finance exploiting the CWT include Crowley and Mayes (2008), Rua (2012), Gallegati and Ramsey (2014), Aguiar-Conraria et al. (2018, 2020), Verona (2020). In what follows we briefly describe those CWT tools employed in the paper. We start with the wavelet coherence, which measures the local correlation between two variables. It is defined as the modulus of the wavelet cross spectrum normalized by the wavelet spectra of each signal, W_x and W_y:

$$R_{xy} = \frac{|S(W_{xy})|}{\left[S(|W_x|^2)S\left(|W_y|^2\right)\right]^{1/2}}, \quad (1)$$

where S is a smoothing operator (Torrence and Webster, 1999). Practically, plotting a color map of the wavelet coherence, one can easily identify the strength of the correlation between x and y both in time and frequency, with darker colors associated with higher correlation.

Next, the wavelet phase difference provides information about the sign and the lead-lag relationships between x and y. It is defined as:

$$\phi_{xy} = Arctan\left(\frac{\mathrm{I}\left(S(W_{xy})\right)}{\mathrm{R}\left(S(W_{xy})\right)}\right), \quad (2)$$

where I and R are the imaginary and the real part of the complex wavelet coherency, respectively. For a chosen frequency band: $\varphi_{x,y}=0$ indicates that x and y move together at the specified frequency; $\varphi_{x,y}\in(-\pi/2, \pi/2)$ indicates that x and y are in-phase; $\varphi_{x,y}\in(-\pi, -\pi/2)$ or $\varphi_{x,y}\in(\pi/2\pi)$ indicate instead are in an anti-phase relationship; $\varphi_{x,y} \in (0, \pi/2)$ or $\varphi_{x,y} \in(-\pi, -\pi/2)$ indicate that x leads y; and $\varphi_{x,y} \in (-\pi/2, 0)$ or $\varphi_{x,y} \in (\pi/2\pi)$ indicate that y leads x. Finally, following Mandler and Scharnagl (2014),

[3] This is evident, for example, in the new dataset recently assembled by Schularick and Taylor (2012) where two very different patterns in the long-run trends of money and credit aggregates relative to GDP are evident with a trend break occurring around 1950.
[4] By decomposing a signal into a set of time scale components, each associated to a specific frequency band and with a resolution matched to its scale, the wavelet transform, in contrast to frequency domain methods, attains an optimal trade-off between time and frequency resolution levels (Lau and Weng 1995, Mallat 1998).

the wavelet gain is defined as:

$$G_{yx} = \frac{|S_{yx}|}{S_{xx}} = R_{yx}\frac{\sigma_y}{\sigma_x}, \tag{3}$$

which can be interpreted as the modulus of the regression coefficient of y over x both in time and frequency. Since the average wavelet gain in each frequency band is obtained by computing the absolute value of the mean of the corresponding complex gains, its interpretation must be complemented with that of the phase difference to glean information about the sign and the timing (leading or lagging) of the estimated relationship.

The application of the DWT, based on arbitrary mother and father wavelets $\psi(.)$ and $\varphi(.)$,[5] allows to separate a function f(t) into multiresolution components, which can be re-written in terms of collections of coefficients at given scales as:

$$f(t) \approx S_J + D_J + D_{J-1} + \ldots D_2 + D_1 \, . \tag{4}$$

S_J contains the "smooth" component of the signal, and D_j , j = 1, 2, .., J, the detail signal components at ever increasing levels of detail. S_J provides the large-scale road map, D_1 shows the pot holes.[6] In practical applications, instead of the DWT, the MODWT is used (e.g. Gencay et al. 2010, Gallegati et al., 2011, Scott et al. 2012, Michis, 2015). The MODWT is a compromise between the CWT, with continuous variations in scale, and the DWT, where the power of the transform is highly localized. The MODWT is highly redundant so that the transformations at each scale are not orthogonal, but the offsetting gain is that applying the transform leaves the phase invariant, a very useful property in analyzing transformations, and the transform is not restricted to restrictions imposed by the dyadic expansion used by the DWT. Therefore, because of the practical limitations of the DWT, wavelet analysis is generally performed by applying the MODWT, a non-orthogonal variant of the classical discrete wavelet transform that, unlike the DWT: is translation invariant, as shifts in the signal do not change the pattern of coefficients; can be applied to datasets of length not divisible by 2^J; and returns at each scale a number of coefficients equal to the length of the original series.

3 How long is the QTM long run?

How long is the long-run relationship between excess money growth and inflation? To answer this question, we draw from the Jordà, Schularick, and Taylor international dataset (Jordà et al., 2017), which consists of annual observations for a sample of developed countries over the 1870-2013 period. Specifically, for 16 countries we use data for: real GDP, by multiplying real GDP per capita (PPP)[7] by

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5 The mother (father) wavelet is good at representing the high(low)-frequency parts of a signal.

6 When the number of observations N is divisible by 2^J, the number of coefficients at each specified component is

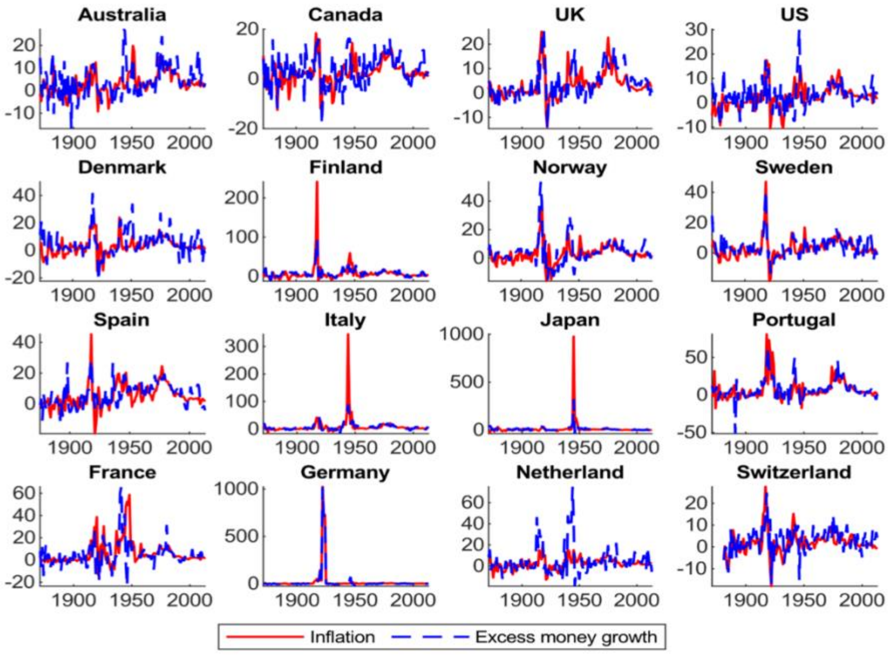
$$N = N/2^J + N/2^J + N/2^{J-1} + \ldots N/4 + N/2,$$

that is, there are $N/2^J$ coefficients $s_{J,k}$, $N/2^J$ coefficients $d_{J,k}$, $N/2^{J-1}$ coefficients $d_{J-1,k}$... and N/2 coefficients $d_{1,k}$.

7 Real GDP data are from The Maddison-Project, http://www.ggdc.net/maddison/maddisonproject/home.htm, 2013

population; broad money (nominal, local currency); and consumer prices (index, 1990=100). These variables are then transformed to their annual growth rates. We define excess money growth as the difference between nominal money growth and real output growth, a measure that purges the money growth-inflation relationship from the effects of shifts in trend output (Assenmacher-Wesche and Gerlach, 2007; Teles et al., 2016).[8] Figure 1 shows the rate of inflation and the excess money growth for each of the 16 countries over the entire period. It is worth noticing the triple-digit hyperinflation episodes of Finland in 1919, Germany in 1923-25, Italy in 1944, and Japan in 1945.

Figure 1 – Inflation and excess money growth, percentage change, 1870-2013.



Note: The solid red line represents inflation while the dashed blue line is excess money growth both in percentage change. Excess money growth is defined as the difference between nominal money growth and real gdp growth.

We exploit the wavelet methods described in the previous section to answer the first of our two critical questions: at what time horizon does a significant link occur between excess money growth and the rate of inflation? Figures 2 to 5 show wavelet coherence and phase difference plots[9] between the rate of inflation and excess money growth over the period 1871-2013.[10] Time is plotted on the

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version.

[8] Seven countries have missing data for monetary aggregates: Denmark (1946-50), France (1914-20), Germany (1923-25 and 1939-48), Netherlands (1942-45), Spain (1871-74 and 1936-41), Switzerland (1871-80) and Sweden (1871-80). Four countries have missing data for prices: Finland (1919), Germany (1923-25), Italy (1944) and Japan (1945). We replace missing values-using an interpolation technique. These decisions do not affect the ability of wavelet coherence analysis to determine the length of the long-run relationship between inflation and money growth.

[9] The analysis has been performed using the ASToolbox2018 package developed by Aguira-Conraria and Soares (2018).

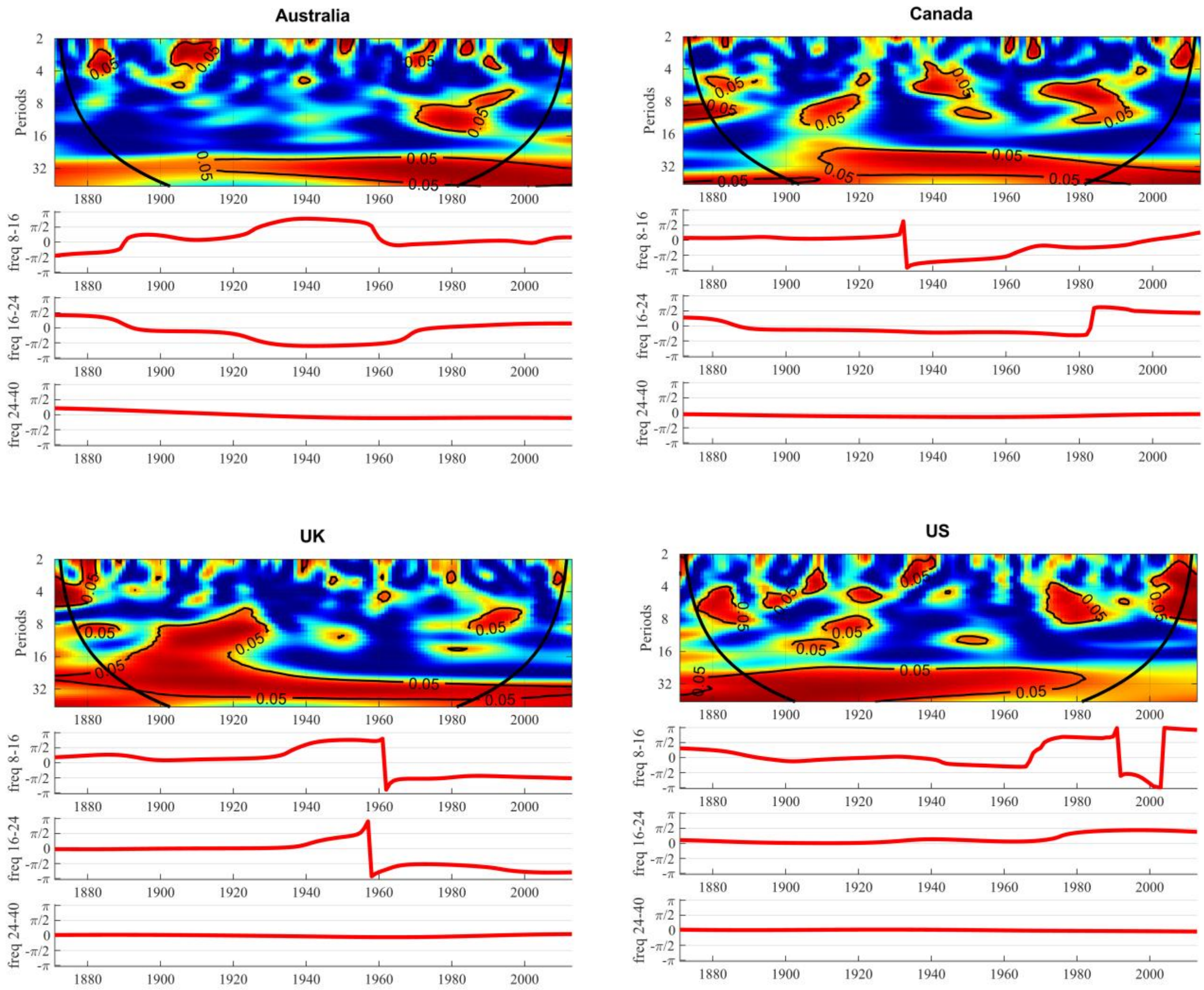
[10] The countries of our sample are divided in four groups: the Anglo-Saxon group consisting of Australia, Canada, the UK and the US; the Northern group consisting of Denmark, Finland, Norway, and Sweden; the Euro-Mediterranean group consisting of Italy, Spain and Portugal plus Japan; and the Euro-core group consisting of France, Germany, and the Netherlands plus Switzerland.

horizontal axis, periods, with the corresponding scales expressed in years, on the vertical axis. By reading horizontally across the graph at a given value of the wavelet scale, one sees how the power of the projection varies over time; reading down vertically at a given point in time, one sees how the power varies with the wavelet scale (Ramsey et al., 1995). The power of the projection is color coded and ranges from dark blue (low power) to dark red (high power): regions with warmer colors and thus high power correspond to wavelet transform coefficients of large modulus. The five percent significance level against the null hypothesis of red noise is estimated with Monte Carlo methods and it is shown in the figures with a thick black contour. The cone of influence[11] is marked by a black thin line: values outside the cone of influence should be interpreted very carefully, as they result from a significant contribution of zero padding at the beginning and the end of the time series. The lower half of Figures 2 to 5 shows the wavelet phase-difference plots for selected frequency bands to determine the average dynamic lead–lag relationship of the two variables in the sample. The area above (below) zero indicates that inflation leads (lags) excess money growth, while the zero line indicates no time lags. We present the results of the phase difference for those frequency bands that, in the wavelet coherency plots, are mostly red and statistically significant, that is for cycles of period 8-16, 16-24 and 24-40 years.[12]

[11] The wavelet transform, like other types of transforms, suffers from a distortion at the boundaries because the finiteness of the time series impacts on the wavelet transform coefficients at the beginning and at the end of the series. The affected region is called the "cone of influence", an area where results are to be interpreted carefully (Percival and Walden, 2000). Since the effective support of the wavelet at scale s is proportional to s, these edge effects increase with s so that the number of wavelet coefficients affected by the boundary conditions tends to increase as the wavelet scale increases.

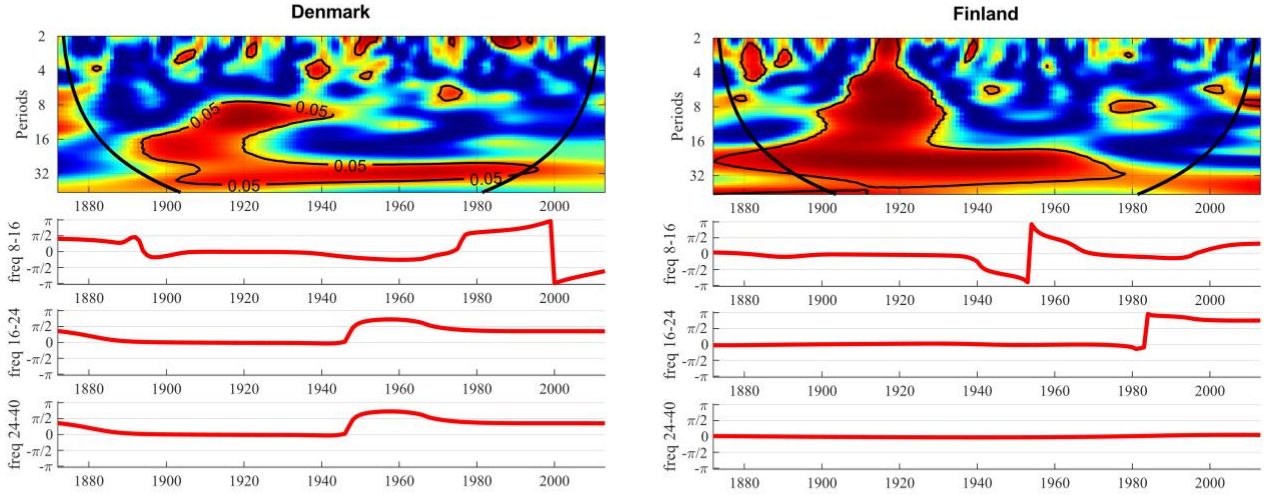
[12] The findings on the phase-difference at higher frequencies, 2-4 and 4-8 years, not reported here for brevity, display an unstable dynamics with the average phase relationship shifting direction frequently.

Figure 2 – Wavelet coherence and phase difference between inflation and excess money growth for the Anglo-Saxon countries.

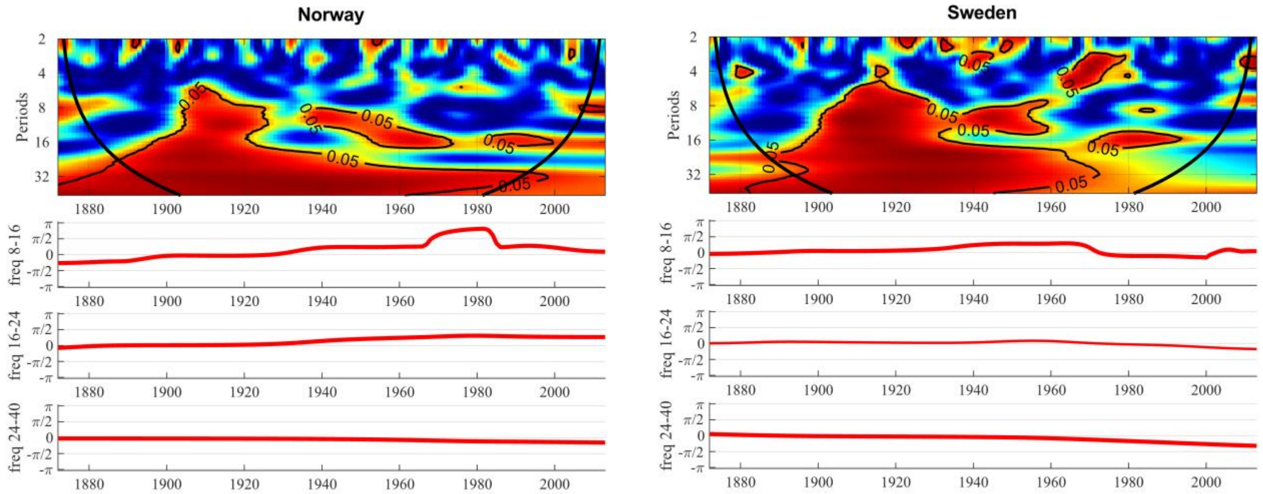


In the charts, time is recorded on the horizontal axis and the scale of the wavelet transform on the vertical axis, with frequency converted to time units (years) to facilitate interpretation. The color code for power ranges from blue (low coherence) to red (high coherence). A 95% confidence intervals for the null hypothesis that coherency is zero are plotted as contours in black in the figure. The cone of influence is marked by a black line. Phase difference: frequency is recorded on the y-axis while time on the x-axis.

Figure 3 – Wavelet coherence and phase difference between inflation and excess money growth

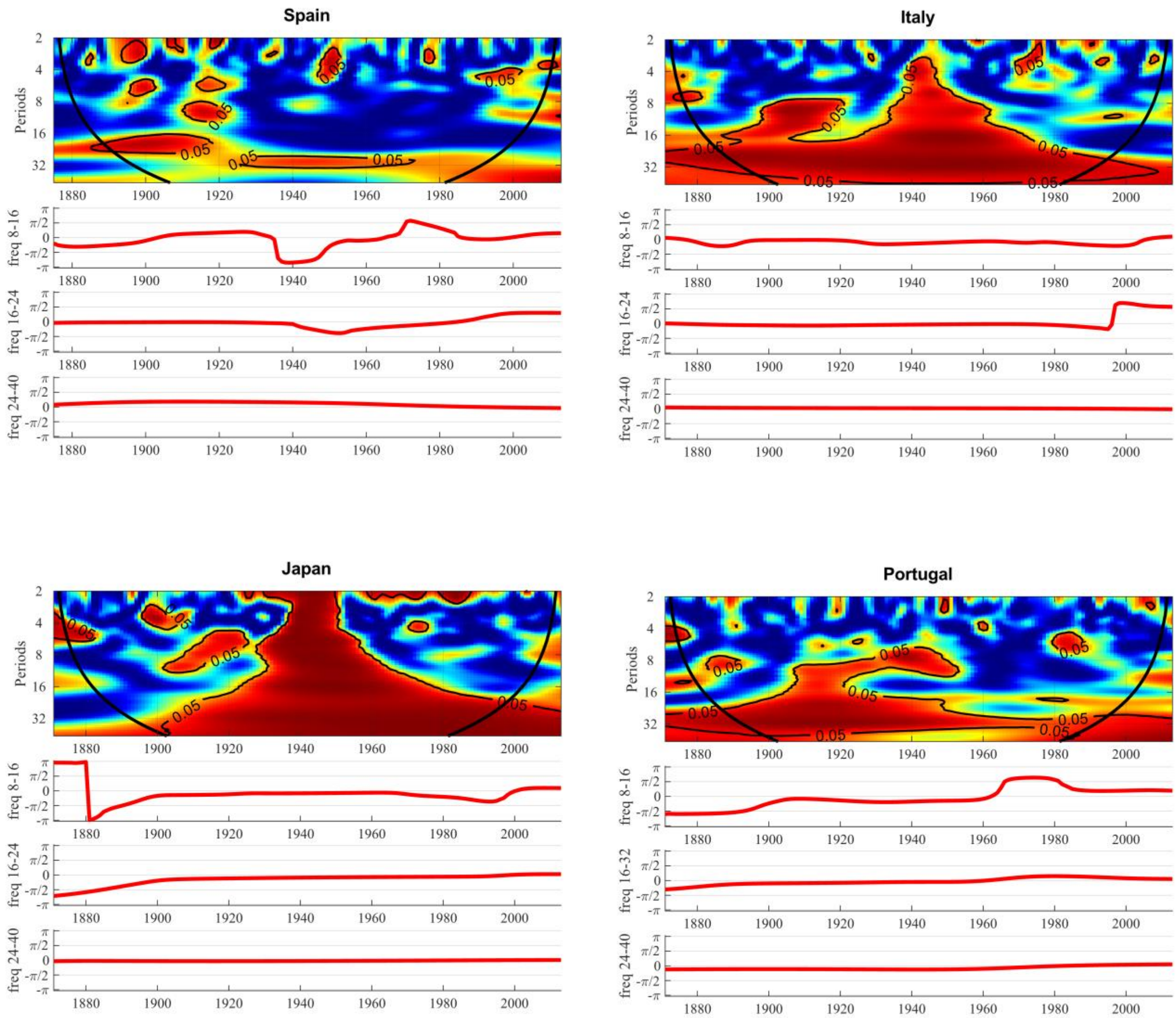


for the Scandinavian countries.



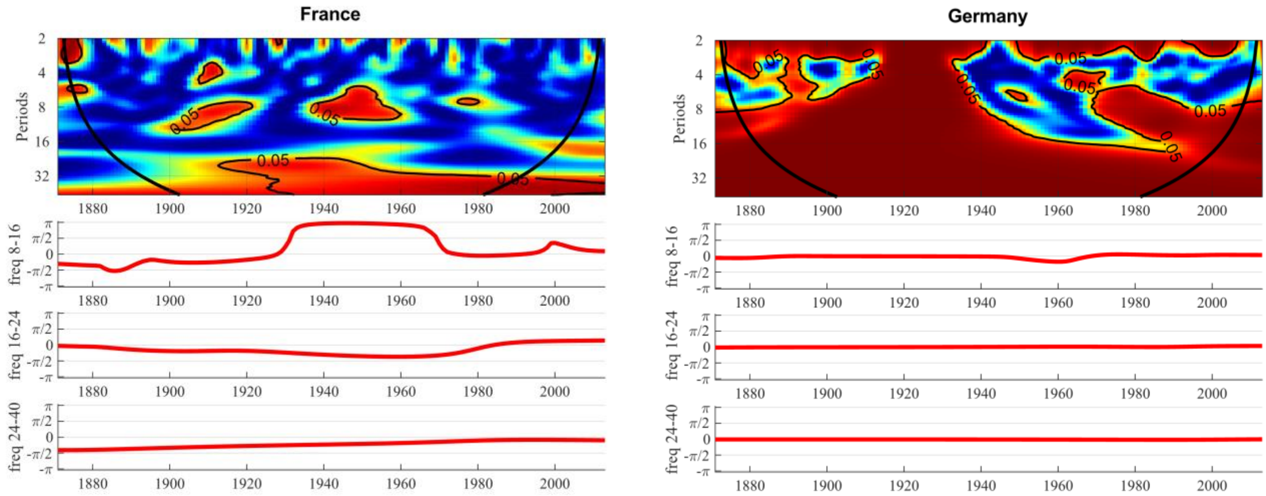
In the charts, time is recorded on the horizontal axis and the scale of the wavelet transform on the vertical axis, with frequency converted to time units (years) to facilitate interpretation. The color code for power ranges from blue (low coherence) to red (high coherence). A 95% confidence intervals for the null hypothesis that coherency is zero are plotted as contours in black in the figure. The cone of influence is marked by a black line. Phase difference: frequency is recorded on the y-axis while time on the x-axis.

Figure 4 - Wavelet coherence and phase difference between inflation and excess money growth for the Southern European countries and Japan.

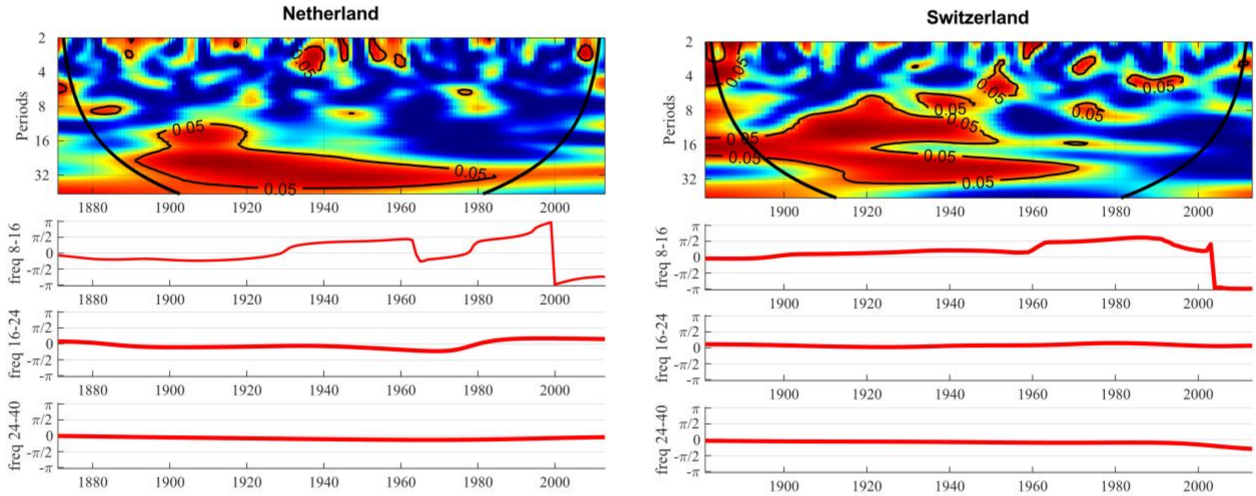


In the charts, time is recorded on the horizontal axis and the scale of the wavelet transform on the vertical axis, with frequency converted to time units (years) to facilitate interpretation. The color code for power ranges from blue (low coherence) to red (high coherence). A 95% confidence intervals for the null hypothesis that coherency is zero are plotted as contours in black in the figure. The cone of influence is marked by a black line. Phase difference: frequency is recorded on the y-axis while time on the x-axis.

Figure 5 - Wavelet coherence and phase difference between inflation and excess money growth



for the continental European countries.



In the charts, time is recorded on the horizontal axis and the scale of the wavelet transform on the vertical axis, with frequency converted to time units (years) to facilitate interpretation. The color code for power ranges from blue (low coherence) to red (high coherence). A 95% confidence intervals for the null hypothesis that coherency is zero are plotted as contours in black in the figure. The cone of influence is marked by a black line. Phase difference: frequency is recorded on the y-axis while time on the x-axis.

Two main findings are evident from the visual inspection of the upper panels in Figures 2 to 5. The first is that for Finland, Germany, Italy and Japan there is a high coherence region at shorter-time scales coincident with the hyperinflationary episodes previously mentioned, clear evidence of a strong, stable and significant medium to long-run relationship between excess money growth and inflation, with excess money growth mostly leading inflation. Notwithstanding this common pattern of strong long-term co-movements throughout the sample period, some differences emerge. The Anglo-Saxon countries exhibit strong stable co-movements at frequencies corresponding to periods greater than 24 years (Great Britain, makes partial exception by also displays significant co-movements at frequencies greater than eight years until the end of WWI). By contrast, for all other countries in the sample such strong and stable co-movements extend to frequencies corresponding to 16 years, with co-movements going back to frequencies corresponding to 8 years between late 19th and mid-20th centuries for Norway, Sweden, Italy, Netherlands, Germany and Switzerland. Specific patterns are displayed by Japan, Spain and Switzerland. In the first case the relationship is limited to the longest run until the early 20th century; while in Spain co-movements shift from 16 to 32 years starting from the early 20th century; in Switzerland, the relationship first stretches towards higher frequencies (periods of 8 years and shorter), then gradually shifts to lower frequencies (periods between 16 and 32 years) until the late 1960s, and finally disappearing at the end of the 20th century. Finally, for Germany, from the 1970s, there is clear evidence of a high coherence region in the 4-16 frequency range that tend to progressively shifts towards the lower (8-16 years) frequency range, throughout the sample.

The phase differences are stable and consistently located around zero at the 24-40 year frequency range, where coherences are mostly significant throughout the sample for almost all countries in the sample. In this range, the positive co-movement occurs with excess money growth leading inflation. More variable over time are the phase differences in the 16-24 year frequency band; when the coherence is statistically significant, the two variables display a positive co-movement. Even more variable are the phase differences in the 8-16 year frequency band; it is worth noting that, even at this relatively high frequency range, the phase difference tends to move close to zero in coincidence with high-coherence regions. In sum, the QTM relationship is not only strong, but stable in the sense that it holds consistently from the medium to the long run. However, since the 1970s a shift of the relationship towards lower frequencies is a common feature to most countries in the sample.[13] Interestingly, at high coherence regions, irrespective of the frequency band, the phase-differences indicate a positive co-movement between inflation and excess money growth, with money generally leading.

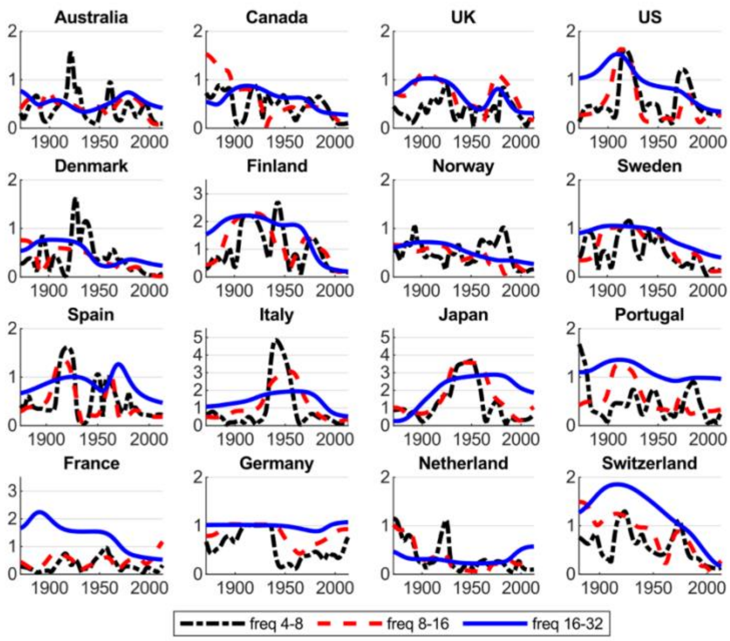
What policy lessons can we draw from the evidence in Figures 2-5? First, the stability over time and the validity across countries corroborates of Friedman's (1963) statement that "inflation is always and everywhere a monetary phenomenon" in the medium to long run. Second, the observed strong correlation in the medium to long-term between excess money growth and inflation justifies the policy of monitoring and targeting monetary aggregates with the aim of stabilizing medium-to-long-run prices. This is the significance of the monetary pillar in the European Central Bank’s monetary policy strategy: the intent there is to capture the effects of money growth at the lower frequencies.

Our evidence is also supportive of Svensson's (1999, p. 215) criticism that "this long-run correlation is irrelevant at the horizon relevant for monetary policy" and provides a strong rationale for the inflation targeting policy framework adopted by most central banks during the last 25 years. Similarly, Woodford (2008, p. 1561) notes that there is no compelling reason "for assigning an important role to tracking the growth of monetary aggregates when making decisions about monetary policy." On this basis, the decision of the European Central Bank to assign a prominent role to money in the conduct of the Euro-area monetary policy stands out as an exception that can be rationalized as a remaining umbilical cord of the Deutsche Bundesbank.[14] However, if monetary policy is partly concerned with pinning down steady-state (trend) inflation, long-run relationships remain of interest.

4 QTM and high inflation episodes

How important are high inflation episodes for the unit slope finding in the QTM? To check for the sensitivity of the QTM regression coefficient over time and across scales, we estimate the wavelet gain. Figure 6 shows the wavelet gain at 4-8 (black dotted lines), 8-16 (red dotted lines) and 16-32 (blue solid lines) frequency ranges.

Figure 6 – Wavelet gain at 4-8, 8-16 and 16-32 frequency ranges.



The solid blue line is the wavelet gain at a frequency range between 16 and 32 years, the red dashed line is the wavelet gain at a frequency range between 8 and 16 years, while the black dotted line is the wavelet gain at a frequency range between 4 and 8 years.

According to the QTM, inflation and excess money growth move in a one-to-one step, Thus, an estimated gain of one would corroborate the QTM. At the longest frequency range, 16-32 years, there is evidence of three distinct patterns. In countries with hyperinflation episodes, plus Portugal and Switzerland, the average wavelet gain is consistently above unity and displays a hump-shaped pattern with a reduction of its value in the last part of the sample. This decline of the wavelet gain, especially after the 1980s, denotes a weakening of the relationship; this is common to all countries and holds at any frequency range.[13] The hump-shaped pattern is also evident in a subset of countries - France, Spain, Sweden, UK and the US - and displays an average wavelet gain close to unity, or slightly above, in about one half of the sample. A hump-shaped pattern also applies to the remaining countries, with an average wavelet gain consistently below unity.

The taxonomy of the countries into three groups is confirmed also at the frequency range corresponding to the medium term, 8-16 years. The main differences with respect to the lower frequency band are the presence of a second peak located in the late 1970s-early 1908s period, and of values of the estimated gain at peaks that are generally higher and better aligned with high-inflation episodes than those at a lower frequency band. At the frequency band corresponding to lower business cycle frequencies, 4-8 years, the gain exhibits stronger time-variation, but with patterns close to those displayed at lower frequency bands. In general, the critical finding is that high-inflation episodes tend to affect the estimated values of the QTM slope coefficient.

While the wavelet gain is a powerful tool for investigating the changes in the slope of the excess money growth-inflation relationship at different frequency ranges, computing the confidence interval of such estimates remains an open issue (see Aguiar-Conraria et al, 2018). In what follows, we refine the analysis with the objective of obtaining results comparable to those reached by standard econometric methods, especially in terms of statistical significance of the estimated coefficient. We perform a parametric analysis of the QTM relationship on a scale-by-scale basis. First, we decompose the inflation rate and excess money growth variables for each country using the MODWT. We apply the LA(8) Daubechies (1992) wavelet filter (with reflecting boundary conditions) for a number of levels J=4. A J=4 level decomposition produces four wavelet detail vectors, $D_1,\ldots,D_4$, each associated with a specific frequency range (2-4, 4-8, 8-16 and 16-32 years, respectively), and one wavelet smooth vector, S_4, capturing fluctuations greater than 32 years. By sequentially adding the detail level components D_4, D_3, D_2 to the lower "smooth" component S_4 we get three additional levels of approximation, S_3, S_2 and S_1. The higher the index, the smoother is the function: S_1 captures fluctuations greater than 4 years, S_2 greater than 8 years, and S_3 greater than 16 years. Table 1 presents the frequency domain interpretation in terms of periods for each detail and approximation level when annual data are used.

Table 1: Frequency interpretation of detail and approximation levels

Detail level, D_j	**Years**	**Approximation level, S_J**	**Years**
D_1	*2-4*		
D_2	*4-8*	S_1	*from 4 to ∞*
D_3	*8-16*	S_2	*from 8 to ∞*
D_4	*16-32*	S_3	*from 16 to ∞*

[13] The weakening of the relationship, with the reduction of the regression coefficient in post-WWII estimates in the 1990s and subsequently, is shown in Mandler and Scharnagl (2014) for Germany based on CWT analysis.

		S_4	*from 32 to ∞*

Following Assenwacher-Wesche and Gerlach (2008), we estimate the inflation equation using the low frequency components of inflation and excess money growth corresponding to periodicities greater than 4, 8 and 16 years (S_1, S_2, S_3,). In addition, we also estimate equations at different frequency ranges, that is D_1, D_2, D_3, D_4. As in Gallegati et al. (2016) the (approximation and detail level) components of inflation and excess money growth of each country are first stacked into separate panel datasets, one for each approximation and detail level component. Then the inflation equation is estimated on a "scale-by-scale" basis using

$$\pi[S_j]_{it} = \alpha_i + a_j\, emg[S_j]_{it-1} + e_{j,it}$$

and

$$\pi[D_j]_{it} = \alpha_i + b_j\, emg[D_j]_{it-1} + e_{j,it},$$

where $[S_j]_{it}$, and $[D_j]_{it}$ represent the j-level approximation and detail components of inflation rate, π, and excess money growth, *emg*, for country *i* at time t, with J=1,2,….,4, and α_i individual effects with i=1,…,N.[14]

Table 2: Full sample approximation and detail levels panel regressions: 1871-2013

$\pi[S_j]_{it} = \alpha_i + a_j\, EMG[S_j]_{it-1} + e_{j,it}$ and $\pi[D_j]_{it} = \alpha_i + b_j\, EMG[D_j]_{it-1} + e_{j,it}$

Approx. and detail level	**S_3 > 16 yrs**	**S_2 > 8 yrs**	**S_1 > 4 yrs**	**D_4 16-32 yrs**	**D_3 8-16 yrs**	**D_2 4-8 yrs**	**D_1 2-4 yrs**
12 countries							
a_{it}, b_{it}	**0.822**	**0.700**	**0.621**	**0.560**	***0.264***	*0.102*	*0.022*
	(0.116)	*(0.122)*	*(0.121)*	*(0.111)*	*(0.118)*	*(0.093)*	*(0.060)*
R^2	0.747	0.594	0.484	*0.590*	0.099	0.016	0.001
plus Finland							
a_{it}, b_{it}	**0.911**	**0.798**	**0.783**	**0.685**	**0.382**	*0.548*	*0.150*
	(0.141)	*(0.153)*	*(0.195)*	*(0.178)*	*(0.165)*	*(0.397)*	*(0.137)*
R^2	0.714	0.583	0.493	*0.539*	0.175	0.177	0.025
plus Italy							
a_{it}, b_{it}	**0.901**	**0.794**	**0.770**	**0.874**	**0.735**	*0.656*	*0.218*
	(0.127)	*(0.139)*	*(0.177)*	*(0.235)*	*(0.362)*	*(0.350)*	*(0.148)*
R^2	0.721	0.592	0.496	*0.604*	0.258	0.179	0.026
plus Japan							
a_{it}, b_{it}	**0.855**	**0.748**	**0.719**	**1.184**	**1.569**	**1.370**	**1.676**
	(0.123)	*(0.132)*	*(0.167)*	*(0.313)*	*(0.617)*	*(0.620)*	*(0.548)*
R^2	0.691	0.553	0.454	*0.593*	0.496	0.261	0.562

Note: HAC (heteroskedastic consistent) robust standard errors in parenthesis. 5% significance level in bold. The 12 countries included in the initial estimation sample are: Australia, Canada, Denmark, France, Great Britain, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland and the US. a_{it} and b_{it} are parameter estimates of approximation and detail level regressions, respectively.

[14] With cross-sectional units such as this group of developed countries, the individual effects can be treated as fixed constant parameters rather than to be drawn from a distribution as in the random effect model.

Table 2 presents the panel regression results for the inflation equation estimated over the entire 1871-2013 period using different subsets of countries. In particular, the first row excludes those countries that experienced a triple-digit increases of inflation and money growth during war years.[15] The following rows show the results when these countries are included, one at a time.

The estimates of Table 3 resemble those reported by Assenmacher-Wesche and Gerlach (2008). The smoother is the series, the larger are the regression slopes of filtered data, and the explanatory power of the regressions tends to rise monotonically when higher frequency components are progressively excluded. Interestingly, the results point to instability of the regression coefficients across different panels, with included countries experiencing hyper-inflation episodes determining a significant increase of the values of estimated coefficients at both the approximation and detail levels, without a corresponding increase of the explanatory power. This is consistent with Fisher and Seater's (1993) argument that we shouldn't expect to find strong evidence of an effect of money growth on inflation unless the sample of data includes episodes with sufficient variation in money growth.

Beyond hyper-inflation episodes at the individual country level, the inflation-excess money growth relationship is likely to be affected by episodes of inflation acceleration at the global level. Dewald (2003) reports that sustained inflationary trends, measured by ten-year inflation rate averages, have occurred in almost every country during wartime periods in the 1910s and the 1940s, as well as after the oil supply shock of the early 1970s; and these high rates of inflation were generally accompanied by equally high rates of money growth. According to Benati (2009) such infrequent inflationary upsurges are responsible for the one-to-one correlation between the long-term components of inflation and money growth.

To see whether high-inflation episodes create a potential bias in favor of the unit-slope hypothesis, we estimate the QTM relationship for different approximation and detail level components using moving variable-length sub-samples windows for the pre- and post-WWII periods.[16] In particular, in the pre-WWII period we use sub-sample windows with a fixed starting point, 1871, while the ending point is allowed to move forward 5 years from 1905 to 1940. In contrast, in the post-WWII period we assemble sub-sample windows with a fixed endpoint in 2013, while the starting point is allowed to move forward 5 years from 1955 to 1990. We expect that adding or deleting observations pertaining to inflationary periods will lead to significant upward or downward changes in parameter estimates. Moreover, we apply the MODWT separately to pre- and post-WWII periods data. The aim of this procedure is to verify the impact of high-inflation episodes on the unit

[15] During war periods several countries suffered hyperinflation. Triple-digit inflation rates were experienced by Finland in 1918, and by several countries in the mid-1940s including France, Germany, Italy (peak in 1944) and Japan (from 1946 to 1949).

[16] This procedure handles the inevitable trade-off between identification of time variation in the data and estimation of long-run relationship which requires a comparatively long window.

slope result in the QTM, while avoiding the distortionary effects stemming from the presence of hyperinflationary periods at the country level.

Table 3: *Approximation (upper) and detail (lower) levels panel regressions Pre-WWII estimates (1870-)*

$$\pi[S_j]_{it} = \alpha_i + a_j\, emg[S_j]_{it-1} + e_{j,it}$$

	-1905	**-1910**	**-1915**	**-1920**	**-1925**	**-1930**	**-1935**	**-1940**
$a_{J[S3]}$	0.406	**0.460**	**0.763**	**1.003**	**1.013**	**0.985**	**0.981**	***0.899***
> 16 yrs	(0.212)	(0.211)	(0.224)	(0.179)	(0.164)	(0.163)	(0.159)	*(0.157)*
R^2	0.347	0.345	0.509	0.706	0.734	0.730	0.728	0.693
$a_{J[S2]}$	0.267	0.291	**0.530**	**0.926**	**0.930**	**0.923**	**0.918**	**0.850**
> 8 yrs	(0.206)	(0.205)	(0.209)	(0.194)	(0.178)	(0.173)	(0.169)	*(0.161)*
R^2	0.153	0.160	0.326	0.631	0.645	0.639	0.637	0.603
$a_{J[S1]}$	0.170	0.169	0.216	**0.939**	**0.940**	**0.934**	**0.926**	**0.868**
> 4 yrs	(0.180)	(0.176)	(0.147)	(0.272)	(0.247)	(0.239)	(0.232)	*(0.220)*
R^2	0.073	0.068	0.093	0.537	0.546	0.545	0.542	0.515

$$\pi[D_j]_{it} = \alpha_i + b_j\, emg[D_j]_{it-1} + e_{j,it}$$

	-1900	**-1905**	**-1910**	**-1915**	**-1920**	**-1925**	**-1930**	**-1935**
$b_{J[D4]}$	0.465	0.540	**0.819**	**0.828**	**0.885**	**0.882**	**0.826**	**0.801**
16-32 yrs	(0.363)	(0.348)	(0.332)	(0.268)	(0.227)	(0.215)	(0.195)	(0.183)
R^2	*0.185*	*0.241*	*0.440*	*0.484*	*0.580*	*0.584*	*0.596*	*0.594*
$b_{J[D3]}$	**0.447**	**0.466**	**0.635**	**0.799**	**0.815**	**0.781**	**0.737**	**0.689**
8-16 yrs	*(0.086)*	*(0.072)*	*(0.132)*	*(0.184)*	*(0.171)*	*(0.156)*	*(0.151)*	*(0.153)*
R^2	*0.184*	*0.197*	*0.319*	*0.448*	*0.507*	*0.510*	*0.484*	*0.439*
$b_{J[D2]}$	*-0.004*	*0.001*	*0.010*	*0.408*	*0.844*	*0.819*	*0.806*	*0.763*
4-8 yrs	*(0.144)*	*(0.136)*	*(0.127)*	*(0.312)*	*(0.492)*	*(0.445)*	*(0.429)*	*(0.418)*
R^2	*0.001*	*0.001*	*0.001*	*0.129*	*0.320*	*0.299*	*0.294*	*0.274*

Note: HAC (heteroskedastic consistent) robust standard errors in parenthesis. 5% significance level in bold. All countries, except Germany, are countries are included in the estimation sample.

Tables 3 and 4 report the results for the inflation equation estimated over moving (subsample) windows with fixed starting-points before WWII and fixed endpoints after WWII, with approximation and detail levels regressions in the upper and lower panel of the tables, respectively. The results point to an instability of the coefficient of the excess money growth across sub-samples, with upward and downward shifts in the estimated values occurring both before and after WWII.

Table 3 shows that between the sample ending in 1910 and that ending in 1920—namely, after the sharp increase in the rate of inflation during the 1910s--a *dramatic* upward shift occurs at all approximation levels S_1 S_2 and S_3, with the excess money growth parameter reaching unity values at lower frequencies, that is beyond 16 years. Thereafter, the estimated coefficients and the explanatory power of the regressions remain remarkably stable around these high points until the end of the pre-WWII period. [17] The same pattern, upward shift followed by stability, is also matched by the explanatory power of the regressions.

Table 4 shows the estimation results for the approximation and detail levels in the post-WWII period. There are strong similarities between the pre- and post-WWII estimates. The similarities show up in the estimated coefficients and the explanatory power of the regressions increasing monotonically for the approximation components, when higher frequency components are progressively excluded; and for the detail components, when moving from higher to lower frequency bands. Similar, although opposite in sign, is the shift of the estimated coefficients in the post-WWII period after the end of the inflationary period of the 1970s. When we exclude observations from the great inflation of the 1970s, we observe a rapid decline in the values of the estimated coefficients in the S_1-S_3 regressions. Indeed, from the early-1980s to the early-1990s, the estimated coefficient values of excess money growth fell to about one-third of their initial values. Estimated coefficients corresponding to the sub-samples, including the 1970s, reach values between 0.89 and 0.92 at the S_3 approximation level, between 0.77 and 0.80 at the S_2 level, and between 0.65 and 0.69 at the S_{12} level. Comparable is also the stability pattern of the estimated coefficients and explanatory power of the regressions across sub-samples, including the high-inflation 1970s. If there is a difference between the pre- and post-WWII periods, it has to do with the deterioration of the estimation results after the war. [18]

Table 4 - *Approximation (upper) and detail (lower) levels panel regressions - Post-WWII estimates (-2013)*

$\pi[S_J]_{it} = \alpha_i + b_J\, emg[S_J]_{it-1} + e_{J,it}$

	1955-	**1960-**	**1965-**	**1970-**	**1975-**	**1980-**	**1985-**	**1990-**
$a_{J[S3]}$	**0.895**	**0.920**	**0.919**	**0.923**	**0.919**	**0.817**	**0.555**	**0.260**
> 16 yrs	(0.051)	(0.042)	(0.039)	(0.039)	(0.048)	(0,062)	(0.131)	(0.108)
R^2	0.801	0.837	0.853	0.873	0.879	0.832	0.643	0.489
$a_{J[S2]}$	**0.772**	**0.798**	**0.802**	**0.806**	**0.793**	**0.638**	**0.382**	**0.249**
> 8 yrs	(0.055)	(0.048)	(0.047)	(0.044)	(0.049)	(0.064)	(0.103)	(0.079)
R^2	0.703	0.729	0.739	0.758	0.758	0.695	0.515	0.468
$a_{J[S1]}$	**0.652**	**0.680**	**0.685**	**0.689**	**0.691**	**0.562**	**0.285**	**0.191**
> 4 yrs	*(0.072)*	(0.065)	(0.063)	(0.060)	(0.067)	(0.077)	(0.082)	(0.063)
R^2	0.579	0.615	0.628	0.646	0.683	0.604	0.441	0.376

$\pi[D_J]_{it} = \alpha_i + b_J\, emg[D_J]_{it-1} + e_{J,it}$

[17] Significant upward shifts of the estimated coefficients are also evident at the detail levels D_4 and D_3. The coefficients of excess money growth increase from lower to higher detail levels, with the higher detail level, D_4, providing the highest estimated coefficient values, i.e. about 0.88 in sub-samples ending in early-to-mid 1920s. The estimated coefficients at the detail levels D_3 to D_4 are generally significant at the 5% level, whereas those at the D_2 level are never significant.

[18] For instance, the magnitude of the estimated coefficients is comparatively higher in the pre-WWII period with values for all approximation levels greater than .90.

	1955-	**1960-**	**1965-**	**1970-**	**1975-**	**1980-**	**1985-**	**1990-**
$b_{J[D3]}$	***0.626***	***0.592***	***0.571***	***0.556***	***0.517***	***0.444***	***0.432***	**0.347**
16-32 yrs	*(0.101)*	(0.070)	(0.071)	(0.063)	(0.068)	(0.063)	(0.070)	(0.063)
R^2	0.419	0.464	0.474	0.507	0.508	0.487	0.489	0.483
$b_{J[D2]}$	***0.207***	***0.243***	***0.218***	***0.185***	***0.156***	***0.139***	***0.149***	***0.112***
8-16 yrs	*(0.061)*	(0.068)	(0.067)	(0.069)	(0.063)	(0.057)	(0.061)	(0.064)
R^2	0.085	0.120	0.106	0.082	0.073	0.089	0.122	0.096
$b_{J[D1]}$	***0.067***	***0.067***	***0.064***	***0.069***	***0.117***	***0.064***	***0.047***	***0.039***
4-8 yrs	*(0.040)*	(0.039)	(0.038)	(0.035)	(0.038)	(0.029)	(0.031)	(0.031)
R^2	0.013	0.013	0.012	0.014	0.047	0.023	0.018	0.015

Note: HAC (heteroskedastic consistent) robust standard errors in parenthesis. 5% significance level in bold. All 16 countries are included in the estimation sample.

The key finding stemming from this “scale-by-scale" panel regression analysis on variable-length sub-samples is that the excess money growth coefficient has been subject to upward and downward shifts associated, respectively, with the start of the inflationary upsurges around the time of WWI and the end of the great inflation of the early 1980s. In sub-samples that include these inflationary surges the estimated coefficients are boosted towards one and exhibit remarkable stability. Outside these periods, the values of estimated coefficients are much lower than unity.

These findings are consistent with those reported by Sargent and Surico (2011) who examine US data for the 1900-2005 period and show substantial deviations from unit slopes, except for the 1900-28 and 1960-1983 years (and to a lesser extent between 1955 and 1975). Our results are also comparable with those of Sargent and Surico (2011) and Teles et al. (2016) on the weakening of the long-run link between money growth and inflation in low inflation countries during the post-WWII years, especially after the Great Inflation period and the adoption of inflation targeting.

Finally, the evidence on changing values of regression coefficients suggests an alternative interpretation of Fisher and Seater's (1993) comment that one can't identify the effects of a permanent change in money growth on inflation unless money growth and inflation are nonstationary. This non-stationarity, instead of reflecting money growth and inflation exhibiting permanent movements in the data, could result from the difficulty to distinguish a stationary process with a structural break from a nonstationary process, as it is evidenced in the literature on unit root test with structural breaks (Perron, 1989).

5 Conclusions

Our exercise confirms the problems, risks, but also the benefits of using historical macroeconomic datasets to investigate the observed changes in the low-frequency relationships between money growth and inflation over time. We draw several conclusions from our empirical analysis of the QTM relationship using time-frequency methods. First, there is clear evidence of a stable and significant medium- to long-term relationship between excess money growth and inflation in the sample of developed countries analyzed in this study. This finding confirms Friedman's (1963) statement that "inflation is always and everywhere a monetary phenomenon" in the medium to long run. Second, the strong medium to long-term correlation between excess money growth and inflation supports the policy of monitoring and targeting monetary aggregates in stabilizing medium-to-long-run prices; for example, this is the intent of the monetary pillar in the European Central Bank's monetary policy strategy. Third, the "scale-based" panel data evidence suggests a strong proportional relationship between excess money growth and inflation, but only when the sample includes sufficient experimental variation in money growth, as in the pre-WWI period and during the Great Inflation period. The otherwise low estimated values of the excess money growth coefficient indicates that the one-to-one relationship is associated with large fluctuations in the low frequency component of inflation and excess money growth during periods of inflationary upsurges (Benati, 2009).

Finally, the evidence that inflationary upsurges affect regression coefficients, but not the closeness of the long-run relationship, may reconcile the validity of the QTM with the current disinterest in money growth rates in the conduct of monetary policy, at least at current low rates of inflation.

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