Managing Inventory Level and Bullwhip Effect in Multi Stage Supply Chains with Perishable Goods: A New Distributed Model Predictive Control Approach

Beatrice Ietto[1][a] and Valentina Orsini[2][b]
[1]*DIMA, UNIVPM, Ancona, Italy*
[2]*DII, UNIVPM, Ancona, Italy*

Keywords: Supply Chain Management, Inventory Control, Bullwhip Effect, Model Predictive Control.

Abstract: We consider the inventory control problem for multi stage Supply Chains (SC) whose dynamics is characterized by uncertainties on the perishability factor of stocked goods and on the customer forecast information. The control problem is to define a replenishment policy keeping the inventory level as close as possible to a desired value and mitigating the Bullwhip Effect (BE). The solution we propose is based on Distributed Robust Model Predictive Control (DRMPC) approach. This implies solving a constrained min-max optimization problem. To drastically reduce the numerical complexity of this problem, the control signal is parametrized using B-spline functions.

1 INTRODUCTION

MPC techniques for multi stage SC are usually implemented according to three different control architectures: centralized, decentralized and distributed. The first two are discussed in (Alessandri et al., 2011),(Fu et al., 2014),(Fu et al., 2016),(Mestan et al., 2016),(Perea-Lopez et al., 2003). The main limitations of centralized approach are: numerical complexity, computational cost, reluctance to share information. Decentralized approach does not have these drawbacks but causes a loss of performance because control agents decide control actions independently on each other. Thus, the interest has recently focused on Distributed MPC (DMPC) (Fu et al., 2019),(Fu et al., 2020),(Kohler et al., 2021).The above mentioned papers do not take into account the presence of deteriorating items in the inventory system. On the other hand, if the effect of perishable goods is not taken into consideration, a serious degradation of the supply chain system is observed. Centralized MPC of inventory level for perishable goods has been investigated in (Hipolito et al., 2022; Lejarza and Baldea, 2020). These latter papers assume an exactly known deteriorating factor. However, this simplifying assumption is not satisfied in the overwhelming part of practical cases (Chaudary et al., 2018).

[a] https://orcid.org/0000-0001-5617-8228
[b] https://orcid.org/0000-0003-4965-5262

Given the previous literature, the purpose of this paper is to propose a DRMPC approach for the optimal inventory control of a multi stage SC with deteriorating items. Extending previous results on single stage SC (Ietto and Orsini, 2022), our purpose is to define a DRMPC policy optimally conciliating the three following antagonist Control Requirements (CR) at each stage: CR1) maximize the satisfied demand issued by the neighboring downstream stage, CR2) minimize the on hand stock level, CR3) mitigate the BE.

The first step to face this problem is to define a suitable predictive information on the end-customer demand. In this paper we only assume that at any time instant $k \in Z^+$ and over a finite prediction horizon, the future end-customer demand entering the first stage of the SC is arbitrarily time varying inside a given compact set $\mathcal{D}_{1,k}$. Coherently with this assumption we conciliate CR1 and CR2 defining a desired inventory level that, for the first stage of the SC, is given by the upper bounding trajectory of $\mathcal{D}_{1,k}$. Then, the target inventory level of each other upward stage is iteratively defined on the basis of the predicted demand coming from the previous downstream stage. Satisfying CR3 is a problem of a paramount importance in the multi-stage SC management as testified by the impressive amount of relevant literature, (Dejonckeere et al., 2003),(Giard and Sali, 2013).

We face this problem simultaneously acting on two Fundamental Features (FF) of BE.

Ietto, B. and Orsini, V.
Managing Inventory Level and Bullwhip Effect in Multi Stage Supply Chains with Perishable Goods: A New Distributed Model Predictive Control Approach.
DOI: 10.5220/0011885300003396
In *Proceedings of the 12th International Conference on Operations Research and Enterprise Systems (ICORES 2023)*, pages 229-236
ISBN: 978-989-758-627-9; ISSN: 2184-4372

FF1) irregularity of stock replenishment orders, FF2) progressive upward amplification of the intervals over which the replenishment orders issued by each stage take values.

FF1 is addressed defining a replenishment policy parametrized in terms of smooth functions and defining a cost functional penalizing excessive differences between consecutive orders. As for FF2, we prove that, using our approach, the upward interval amplification is proportional to the perishability rate. The interesting corollary is that, in the case of non perishable goods, the values of orders issued by all stages may be contained in the same fixed amplitude interval. Coherently with the assumptions on the uncertainties and with the CR's , we develop a DRMPC approach based on a min-max optimization procedure: the control law is obtained minimizing the worst case of a quadratic cost functional, which is computed by maximizing with respect to all the possible perishability factor values. Another significant novelty of our approach is the parametrization of the replenishment policy as a polynomial B-spline function. The main reasons for this choice are: 1) polynomial B-splines are smooth functions 2) B-splines are admit a parsimonious parametric representation. given by a time varying, linear, convex combination of some parameters named "control points". These properties allow us to obtain a replenishment order with a smooth waveform and to transfer any hard constraint on the control law to its control points. This is very useful to deal with FF2 of BE. Property 2 also allows us to reformulate the constrained minimization of the cost functional with respect to the replenishment order signal as a Weighted Constrained Robust Least Square (WCRLS) estimation problem. that can be efficiently solved using interior point methods (Lobo et al., 1998).

2 PRELIMINARIES

2.1 B Splines Functions

A scalar, continuous time, B-spline curve is defined as a linear combination of polynomial basis functions and control points, (De-Boor, 1978):

$$s(t) = \mathbf{B}_d(t)\mathbf{c}, \quad t \in [\hat{t}_1, \hat{t}_{\ell+d+1}] \subseteq \mathbb{R}, \tag{1}$$

where $\mathbf{c} = [c_1, \cdots, c_\ell]^T$ and $\mathbf{B}_d(t) = [B_{1,d}(t), \cdots, B_{\ell,d}(t)]$. The c_i's are real numbers representing the control points of $s(t)$, the integer d is the degree of the B-spline, the $(\hat{t}_i)_{i=1}^{\ell+d+1}$ are the non decreasing knot points and the basis functions $B_{i,d}(t)$ are computed by the Cox-de Boor recursion formula.

Remark 1. Eq. (1) shows that, once the degree d and the knot points $\hat{t}_i$ have been fixed, the scalar B spline function $s(t)$, $t \in [\hat{t}_1, \hat{t}_{\ell+d+1}]$, is completely determined by the corresponding vector $\mathbf{c}$ of ℓ control points.

2.2 The RLS Problem

Consider a set of linear equations $Df \approx b$, with $D \in \mathbb{R}^{r\times m}$, $b \in \mathbb{R}^r$, $m > r$, subject to unknown bounded errors: $\|\delta D\| \le \beta$ and $\|\delta b\| \le \xi$ (where the matrix norm is the spectral norm). The RLS estimate $\hat{f} \in \mathbb{R}^m$ is the value of f minimizing

$$\min_f \max_{\|\delta D\|\le\beta,\, \|\delta b\|\le\xi} \|(D+\delta D)f - (b+\delta b)\|, \tag{2}$$

In ((Lobo et al., 1998), p. 206), it is shown that problem (2) is equivalent to minimizing the following sum of Euclidean norms

$$\min_f \|Df - b\| + \beta\|f\| + \xi \tag{3}$$

Possible linear constraints on f can be taken into account imposing

$$\underline{f} \le f \le \bar{f}. \tag{4}$$

3 THE SYSTEM MODEL

As shown in Fig. 1, we consider an SC network consisting of a cascade of stages (nodes) $\mathcal{S}_i$, $i = 1, \cdots, n$, characterized by counter-current order and material streams. Management decisions for each node are taken periodically at equally distributed time instants kT where $k \in Z^+$ and T is the review period. At the beginning of each review period $[kT, (k+1)T)$ the operations across the SC network are performed sequentially from $\mathcal{S}_1$ to $\mathcal{S}_n$. Inside each review period, each $\mathcal{S}_i$ executes five actions in the following order: receives delivery from supplier S_{i+1}, logs the demand of customer S_{i-1}, measures its on hand stock level, delivers the goods to meet demand and finally places an order according to a suitably defined replenishment policy. Accordingly, five variables are defined: $s_i(k)$, $d_i(k)$, $y_i(k)$, $h_i(k)$ and $u_i(k)$. They represent the shipment of goods from supplier S_{i+1}, the demand from S_{i-1}, the on hand stock level, the delivery to customer S_{i-1} and the replenishment order, respectively. Each node $\mathcal{S}_i$ is regulated by an agent $\mathcal{A}_i$ that solves a local RMPC problem based on the following assumptions:

- **A1)** At any time instant k, and limitedly to an M_1-steps prediction horizon $[k+1, k+M_1]$, the unknown future end-customer demand $d_1(k+j)$, $j = 1, \cdots, M_1$, fluctuates within a compact set $\mathcal{D}_{1,k}$ limited below and above by two known boundary trajectories: $d_1^-(k+j)$

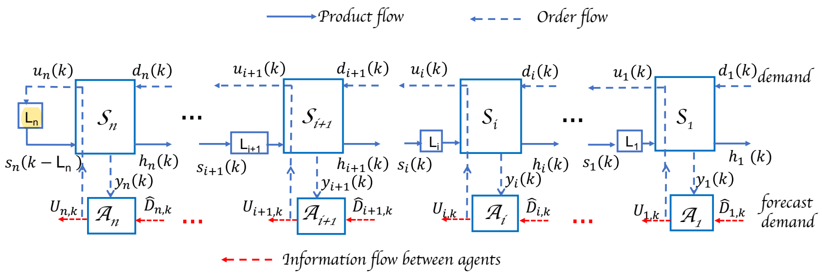

Figure 1: Distributed control scheme of the n-subsystems SC network.

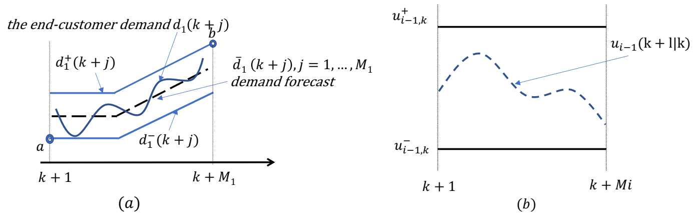

Figure 2: (a) Example of a set $\mathcal{D}_{1,k}$, (b) Example of a set $\mathcal{D}_{i,k}$, $i>1$.

and $d_1^+(k+j)$, $j=1,\cdots,M_1$. The minimum value of $d_1^-(k+j)$ and the maximum value of $d_1^+(k+j)$, $j=1,\cdots,M_1$, are denoted by $d_{1,k}^-$ and $d_{1,k}^+$ (points a and b of Figure 2.(a) respectively). The demand forecasting $D_{1,k} \triangleq [d_1(k+1|k),\cdots,d_1(k+M_1|k)]$ for agent $\mathcal{A}_1$ is assumed to coincide with the central trajectory of $\mathcal{D}_{1,k}$ namely $D_{1,k}=[\bar{d}_1(k+1),\cdots,\bar{d}_1(k+M_1)]$. Figure 2.(a) shows a typical example of an end-customer demand $d_1(k+j)$ and of a predicted end-customer demand $\bar{d}_1(k+j)$ over a fixed $\mathcal{D}_{1,k}$.

- **A2)** At any time instant k, the predicted demand $D_{i,k}=[d_i(k+1|k),\cdots,d_i(k+M_i|k)]$ for the other agents $\mathcal{A}_i$, $i=2,\cdots,n$, coincides with the predicted optimal control sequence (i.e. the optimal predicted replenishment policy $\mathcal{U}_{i-1,k} \triangleq [u_{i-1}(k+1|k),\cdots,u_{i-1}(k+N_{i-1}-1|k)]$ transmitted by $\mathcal{A}_{i-1}$ to $\mathcal{A}_i$ where $M_i=N_{i-1}-1$. Note that also $D_{i,k}$ belongs to a given compact set $\mathcal{D}_{i,k}$ limited by the imposed lower and upper values $u_{i-1,k}^-$ and $u_{i-1,k}^+$ respectively (as shown in Fig. 2.(b)). How to compute $\mathcal{U}_{i-1,k}$, $u_{i-1,k}^-$ and $u_{i-1,k}^+$ is explained in Section 4.
- **A3)** The goods shipped from supplier S_{i+1} arrive at customer S_i with a time delay $L_i=n_iT$, where $n_i\in Z^+$. Goods arrive at customer S_i new and deteriorate while kept in stock.
- **A4)** Inside each review period, the perishability rate of the goods stocked in S_i is $\alpha_i\in[\alpha_i^-,\alpha_i^+]\subset(0,1)$.
- **A5)** The operations of inventory replenishment and goods delivery are executed simultaneously at the beginning of each review period. Sales are not backordered.

The above assumptions imply that the stock level dynamics of the i-th node is described by the following uncertain equation

$$y_i(k+1)=\rho_i(y_i(k)+s_i(k-L_i)-h_i(k)) \quad (5)$$

where:
- $y_i(k)$ is the on hand stock level of $\mathcal{S}_i$, i.e. the amount of goods left in stock after satisfying the demand at the beginning of the $k-1$ review period;
- $s_i(k-L_i)$ is the goods delivered to the stage $\mathcal{S}_i$ with a time delay L_i;
- the sum $y_i(k)+s_i(k-L_i)$ represents the effective amount of goods available for sale at the beginning of k-th review period;
- $h_i(k)$ is the demand fulfilled by $\mathcal{S}_i$, $i=1,\cdots,n$

$$h_i(k)\triangleq\min\{d_i(k),y_i(k)+s_i(k-L_i)\} \quad (6)$$

where $d_1(k)$ is the end-customer demand, $d_i(k)=u_{i-1}(k)$, $i=2,\cdots,n$, is the demand issued by $\mathcal{S}_{i-1}$;
- $\rho_i\triangleq 1-\alpha_i\in[\rho_i^-,\rho_i^+]$ is the uncertain decay factor.

For future developments we now rewrite equation (5) assuming **A6)**: there exists a $\bar{k}\geq 0$ such that

$$y_i(k)+s_i(k-L_i)\geq d_i(k),\ \forall k\geq\bar{k},\ i=1,\cdots,n \quad (7)$$

By (6) and (7) we have $h_{i+1}(k)=d_{i+1}(k)$. As $d_{i+1}(k)=u_i(k)$ and $h_{i+1}(k)=s_i(k)$ (see Fig. 1) we also have $s_i(k-L_i)=u_i(k-L_i)$. Hence an equivalent expression of (5) is

$$y_i(k+1)=\rho_i(y_i(k)+u_i(k-L_i)-h_i(k)),\ \forall k\geq\bar{k} \quad (8)$$

Assumption **A6)** is justified because, at each stage, the control sequence is obtained minimizing the maximum weighted ℓ_2 norm of the distance between the on-hand stock level and the maximum demand.

4 PROBLEM SETUP

To simplify the derivation of the control strategy we refer to (8) in the ideal case $\bar{k} = 0$. Each $\mathcal{A}_i$ uses equation (8) and the predicted optimal control policy $\mathcal{U}_{i-1,k}$ communicated by $\mathcal{A}_{i-1}$ to predict the future inventory level of the local subsystem $\mathcal{S}_i$. This latter is in turn used to compute $\mathcal{U}_{i,k}$ minimizing the worst case of a local quadratic cost functional subject to hard constraints $u^-_{i,k}$ and $u^+_{i,k}$. Coordination between contiguous agents $\mathcal{A}_{i-1}$ and $\mathcal{A}_i$, is imposed by relating the respective constraints u^-_{i-1} and $u^+_{i-1,k}$ with $u^-_{i,k}$ and $u^+_{i,k}$. Each local RMPC requires each agent $\mathcal{A}_i$ to repeatedly solve a Min-Max Constrained Optimization Problem (MMCOP) over a future N_i steps control horizon, and, according to the receding horizon control, to only apply the first sample of the computed predicted optimal control sequence.

4.1 Local MMCOP for $\mathcal{A}_i$

The local MMCOP for any $\mathcal{A}_i$, $i = 1, \cdots, n$ is formally defined as follows $\forall k \in Z^+$

$$\min_{[u_i(k|k),\cdots,u_i(k+N_i-1|k)]} \max_{\rho_i \in [\rho_i^-, \rho_i^+]} J_{i,k}, \quad (9)$$

$$u^-_{i,k} \leq u_i(k+j|k) \leq u^+_{i,k}, \quad j = 0, \cdots, N_i - 1, \quad (10)$$

where:

$$J_{i,k} = \sum_{l=1}^{N_i} e_i^T(k+L_i+l) q_{i,l} e_i(k+L_i+l) + \sum_{l=1}^{N_i-1} \lambda_{i,l} \Delta u_i^2(k+l|k) \quad (11)$$

where $e_i(k+L_i+l)$ denotes the future values of the tracking error given by

$$e_i(k+L_i+l) \triangleq r_i(k+L_i+l) - y_i(k+L_i+l) \quad (12)$$

with

$$y_i(k+L_i+l) = \rho_i^{L_i+l} y_i(k) + \sum_{\ell=0}^{L_i-1} \rho_i^{L_i+l-\ell} u_i(k+\ell-L_i)$$
$$+ \sum_{\ell=0}^{l-1} \rho_i^{l-\ell} u_i(k+\ell|k) - \sum_{\ell=0}^{L_i+l-1} \rho_i^{L_i+l-\ell} h_i(k+\ell). \quad (13)$$

$$r_i(k+L_i+l) \triangleq \begin{cases} d_1^+(k+L_1+l) & i = 1 \\ u^+_{i-1,k} & i = 2, \cdots, n \end{cases} \quad (14)$$

and

$$\Delta u_i(k+l|k) \triangleq u_i(k+l|k) - u_i(k+l-1|k) \quad (15)$$

Remark 2. Some considerations on $J_{i,k}$ are now in order.

-1) By **A1)**, **A2)** and (14), it can be seen that $M_1 \geq N_1 + L_1$ and $M_i = N_{i-1} - 1 = N_i + L_i$, $i > 1$, namely $N_{i-1} = N_i + L_i + 1$.

-2) The time varying target inventory level $r_i(k)$ for $\mathcal{S}_i$ is defined as follows:

$$r_1(k) = d_1^+(k) \text{ and } r_i(k) = u^+_{i-1,k}, \; i \geq 2 \quad (16)$$

For each fixed k and over the corresponding prediction interval $[k, k+M_i]$, $i \geq 2$, definition (14) implies that the values of the target inventory level are frozen on $u^+_{i-1,k}$. Keeping the on hand stock level as near as possible to the possible maximum level of the demand forecasting maximizes the amount of fulfilled demand over each shifted prediction horizon and prevents unnecessarily larger stock levels.

-3) The term $\sum_{l=1}^{N_i-1} \lambda_{i,l} \Delta u_i^2(k+l|k)$ and the way the hard constraints (10) are defined allow us to deal with FF1 and FF2 respectively.

4.2 Determining $u^-_{i,k}$ and $u^+_{i,k}$

By CR1-CR3, the constraints on $u_i(k+j|k)$ imposed by (10) are determined on the basis of the two following criteria: 1) maximize the amount of demand satisfied by each stage $\mathcal{S}_i$, 2) limit the amplitude (defined as $A_{i,k}$) of the interval $[u^-_{i,k} \; u^+_{i,k}] \triangleq \mathcal{C}_{i,k}$. We estimate $u^-_{i,k}$ and $u^+_{i,k}$ with reference to two possible, limit situations compatible with (8). Consider the following scenario:

-$d_i(k+L_i+j)$, $j = 0, \cdots, N_i - 1$, is a constant signal with value $\tilde{d}_{i,k} \in [d^-_{i,k}, d^+_{i,k}] = [u^-_{i-1,k}, u^+_{i-1,k}]$. The two mentioned limit situations are $\tilde{d}_{i,k} = u^-_{i-1,k}$ and $\tilde{d}_{i,k} = u^+_{i-1,k}$.

- Each control horizon $H_{i,k}$ is long enough to allow $y_i(k+L_i+j)$, $j = 1, \cdots, N_i$, to practically attain the steady-state value $\tilde{y}_{i,k}$ under the forcing action of a constant $u_i(k+j) = \tilde{u}_{i,k}$, $j = 0, \cdots, N_i - 1$. The problem we now consider is: for a given constant demand $\tilde{d}_{i,k}$ it is required to find the interval $\mathcal{C}_{i,k}$ where the corresponding constant control input $\tilde{u}_{i,k}$ takes values, such that the resulting constant steady state state output $\tilde{y}_{i,k}$ satisfies $\tilde{y}_{i,k} \geq \tilde{d}_{i,k}$, $\forall \rho_i \in [\rho_i^-, \rho_i^+]$.

Some algebraic calculations (not reported for brevity) based on z-transform methods and on the final value theorem (Kuo, 2007) show that

$$\mathcal{C}_{1,k} \triangleq [u^-_{1,k}, u^+_{1,k}] = \frac{1}{\rho_1^-} [d^-_{1,k}, d^+_{1,k}] \quad (17)$$

$$\mathcal{C}_{i,k} \triangleq [u^-_{i,k}, u^+_{i,k}] = \frac{1}{\rho_i^-} \left[u^-_{i-1,k}, u^+_{i-1,k} \right], i = 2, \cdots, n \quad (18)$$

Recalling that $A_{i-1,k}$ denotes the amplitude of $\mathcal{C}_{i-1,k}$, from (17),(18) we have

$$A_{1,k} = \tfrac{1}{\rho_1^-}(d^+_{1,k} - d^-_{1,k}), A_{i,k} = \tfrac{1}{\rho_i^-} A_{i-1,k}, i = 2 \cdots n \quad (19)$$

To quantify the BE at node $\mathcal{S}_i$ according to FF2 we define the following measure:

$$\mathcal{B}_{i,k} = \frac{A_{i,k}}{A_{i-1,k}} \tag{20}$$

According to (19)-(20), the proposed DRMPC scheme implies $\mathcal{B}_{i,k} = 1/\rho_i^- \stackrel{\triangle}{=} \mathcal{B}_i > 1$.

The two salient conclusions are: 1) an estimate of the overall BE (corresponding to FF2) which propagates along the SC network, can be computed "a priori" and is given by $\mathcal{B} = 1/(\prod_{i=1}^n \rho_i^-)$, 2) our approach does not entail this kind of BE for $\rho_i^- \to 1$.

5 REFORMULATION OF THE MMCOP

We reformulate the local MMCOP as a WCRLS estimation to drastically reduce the numerical complexity of the algorithm solving the original MMCOP. The functional $J_{i,k}$, defined in (9), is minimized assuming that the control sequence $\mathcal{U}_{i,k}$, is given by the sampled version of a B-spline function. Adapting the notation in (1) to specify that it is relative to the i-th node and the k-th fixed time instant we have

$$u_i(j|k) \stackrel{\triangle}{=} \mathbf{B}_{i,d}(j)\mathbf{c}_{i,k}, \;\; j = k, \cdots, k+N_i-1, \tag{21}$$

with $\mathbf{B}_{i,d}(j) = [B_{i,1,d}(j), \cdots, B_{i,\ell,d}(j)]$ and $\mathbf{c}_{i,k} = [\mathbf{c}_{i,k,1}, \cdots, \mathbf{c}_{i,k,\ell}]^T$. The parameter vector $\mathbf{c}_{i,k}$, defining $u_i(j|k)$, is computed as the solution of the WCRLS estimation problem defined beneath.

As $\rho_i \in [\rho_i^- \rho_i^+]$, an equivalent representation of ρ_i is $\rho_i = \bar{\rho}_i + \delta\rho_i, \bar{\rho}_i = (\rho_i^- + \rho_i^+)/2$ where $\bar{\rho}_i$ is the nominal value and $\delta\rho_i$ is the perturbation with respect to $\bar{\rho}_i$ satisfying $|\delta\rho_i| \le (\rho_i^+ - \rho_i^-)/2$. It follows that

$$\rho_i^k = (\bar{\rho}_i + \delta\rho_i)^k = \bar{\rho}_i^k + \Delta\rho_{i,k} \tag{22}$$

where $\Delta\rho_{i,k} \stackrel{\triangle}{=} (\bar{\rho}_i + \delta\rho_i)^k - \bar{\rho}_i^k$ is the sum of all terms containing $\delta\rho_i$ in the explicit expression of $(\bar{\rho}_i + \delta\rho_i)^k$. Analogously, by **A1**), **A2**), **A6**) and (6), the future values $h_i(k+\ell)$ in (13) can be expressed as

$$h_i(k+\ell) = \bar{h}_i(k+\ell|k) + \delta h_i(k+\ell|k) \tag{23}$$

where $\bar{h}_i(k+\ell|k) = d_i(k+\ell|k)$. Exploiting (21)-(23) it can be shown that the future tracking error given by (12) can be expressed as

$$e_i(k+L_i+l|k) = (b_{i,k,l} + \delta b_{i,k,l}) - (D_{i,k,l} + \delta D_{i,k,l})\mathbf{c}_{i,k}$$

where

$$D_{i,k,l} \stackrel{\triangle}{=} \sum_{\ell=0}^{l-1} \bar{\rho}_i^{l-\ell} \mathbf{B}_{i,d}(k+\ell) \tag{24}$$

$$\delta D_{i,k,l} \stackrel{\triangle}{=} \sum_{\ell=0}^{l-1} \Delta\rho_{i,l-\ell} \mathbf{B}_{i,d}(k+\ell) \tag{25}$$

$$\begin{aligned} b_{i,k,l} &\stackrel{\triangle}{=} r_i(k+L_i+l) - \bar{\rho}_i^{L_i+l} y_i(k) \\ &- \sum_{\ell=0}^{L_i-1} \bar{\rho}_i^{L_i+l-\ell} u_i(k+\ell-L_i) \\ &+ \sum_{\ell=0}^{L_i+l-1} \bar{\rho}_i^{L_i+l-\ell} d_i(k+\ell|k) \end{aligned} \tag{26}$$

$$\begin{aligned} \delta b_{i,k,l} &\stackrel{\triangle}{=} -\Delta\rho_{i,L_i+l} y_i(k) - \sum_{\ell=0}^{L_i-1} \Delta\rho_{i,L_i+l-\ell} u_i(k+\ell-L_i) \\ &+ \sum_{\ell=0}^{L_i+l-1} \bar{\rho}_i^{L_i+l-\ell} \delta h_i(k+\ell|k) + \sum_{\ell=0}^{L_i+l-1} \Delta\rho_{i,L_i+l-\ell} h_i(k+\ell) \end{aligned}$$

Similarly $\Delta u_i(k+l|k) = b_{u_{i,k,l}} - D_{u_{i,k,l}} \mathbf{c}_{i,k}$ with $D_{u_{i,k,l}} = -(\mathbf{B}_{i,d}(k+l) - \mathbf{B}_{i,d}(k+l-1))$ and $b_{u_{i,k,l}} = 0$.
Defining the following extended error vector

$$\underline{e}_{i,k} = \begin{bmatrix} q_{i,1}^{1/2} e_i(k+L_i+1) \\ \vdots \\ q_{i,N_1-1}^{1/2} e_i(k+L_i+N_i-1) \\ \lambda_{i,1}^{1/2} \Delta u_i(k+1|k) \\ \vdots \\ \lambda_{i,N_i-1}^{1/2} \Delta u_i(k+N_i-1|k) \end{bmatrix}$$

and the corresponding extended vectors $\underline{b}_{i,k}$, $\underline{\delta b}_{i,k}$ and matrices $\underline{D}_{i,k}$, $\underline{\delta D}_{i,k}$ (not reported for brevity) allow us to reformulate the local MMCOP (9)-(11) as the following local WCRLS estimation problem:

$$\min_{\mathbf{c}_{i,k}} \max_{\|\underline{\delta D}_{i,k}\| \le \beta_{i,k} \|\underline{\delta b}_{i,k}\| \le \xi_{i,k}} \|\underline{e}_{i,k}\|^2 \tag{27}$$

where

$$\|\underline{e}_{i,k}\|^2 = \|(\underline{b}_{i,k} + \underline{\delta b}_{i,k}) - (\underline{D}_{i,k} + \underline{\delta D}_{i,k})\mathbf{c}_{i,k}\|^2 \tag{28}$$

$$\text{subject to } \; u_{i,k}^- \le \mathbf{c}_{i,k,r} \le u_{i,k}^+, r = 1, \cdots, \ell. \tag{29}$$

It is seen that (28)-(29) define a problem of the kind (2)-(4). Hence, according to Section 2.2, at any k the local WCRLS estimation problem (27)-(29) can be reformulated as

$$\min_{\mathbf{c}_{i,k}} \|\underline{b}_{i,k} - \underline{D}_{i,k} \mathbf{c}_{i,k}\| + \beta_{i,k} \|\mathbf{c}_{i,k}\| + \xi_{i,k} \tag{30}$$

where the components of $\mathbf{c}_{i,k}$ must satisfy (29).

Remark 3. Note that $\xi_{i,k}$ of (30) is independent of $\mathbf{c}_{i,k}$ so that it can be removed from the objective function. This implies that in (30) only the upper bound $\beta_{i,k}$ on $\|\underline{\delta D}_{i,k}\|$ needs to be determined at each k. Moreover the way the B-spline basis functions are defined by the Cox de Boor formula (De-Boor, 1978) implies that $\mathbf{B}_{i,d}(\tau) = \mathbf{B}_{i,d}(\tau + N_i)$, $\forall \tau \in H_{i,k}$, $k \in Z^+$. Hence, by (25), one has that $\beta_{i,k} \stackrel{\triangle}{=} \beta_i$, $\forall k = 0, 1, \cdots$ and moreover β_i is can be determined putting $\rho_i = \rho_i^+$.

Feasibility and stability properties of the proposed control strategy can be now formally stated in the following theorem (whose proof is omitted for brevity).
Theorem The proposed DRMPC strategy guarantees the feasibility of each local MMCOP, the positivity of all physical variables $u_i(k)$ and $y_i(k)$, $i = 1, \cdots, n$, and their uniform boundedness.

6 NUMERICAL RESULTS

In this section, we perform simulated experiments on the application of the proposed RDMPC to the management of an uncertain SC composed of $n = 3$ stages $\mathcal{S}_i$, $i = 1, \cdots, 3$, (retailer-distributor-factory). We assume that the equations describing the stock level dynamics of each $\mathcal{S}_i$ are characterized by the same perishability factor α_i, time delay L_i and initial state $y_i(0)$. The model parameters of each $\mathcal{S}_i$ are reported in Table 1. At each k, the unknown future end-customer demand $d_1(k)$, belongs to a known compact set $\mathcal{D}_{1,k}$, with $M_1 = 24$. Figure 3 shows the actual end-customer demand enclosed in the contiguous positioning of all the $\mathcal{D}_{1,k}$'s. The dashed red trajectory is the predicted end-customer demand $d_1(k+l|k)$. We use B-spline functions of degree $d = 3$ with $\ell = 6$ control points over each control horizon $H_{i,k}$. The other tuning parameters of the local MMCOP for any $\mathcal{A}_i$, $i = 1, 2, 3$ are reported in Table 2.

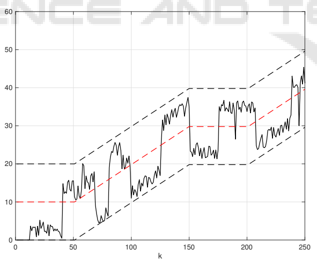

Figure 3: The end-customer demand $d_1(k)$ and the two boundary trajectories $d_1^+(k)$ and $d_1^-(k)$.

We evaluate the effectiveness of the proposed method by defining performance indicators that take into account the ability to satisfy the demand at each stage, to limit the inventory level and to reduce the BE. The first performance indicator, that we define, measures the normalized amount of Unsatisfied Demand at each stage and is given by $\mathcal{UD}_i \triangleq \frac{1}{\sum_{k=0}^{T_s} d_i(k)} \sum_{k=0}^{T_s} |d_i(k) - h_i(k)| \in [0\ 1]$, $i = 1, 2, 3$, where T_s is the length of the simulation. The second performance indicator is the total sum of the Inventory Stock in the SC after satisfying the demand at each $k = 0, \cdots, T_s$. In accordance with (5), it is given by $\mathcal{IS} \triangleq \sum_{i=1}^{n} \sum_{k=0}^{T_s} y_i(k)$. As for the BE, we define a second performance indicator according to FF1: $\mathcal{BE}_{\Delta u,i} = \sum_{k=0}^{T_s-1} |u_i(k+1) - u_i(k)|$, $\quad i = 1, \cdots, n$. It measures "a posteriori" the smoothness property of each replenishment order $u_i(k)$, $i = 1, \cdots, n$. The simulation has been performed choosing $\rho_i = 0.885$, $i = 1, 2, 3$ and it has been stopped at time $k = 200$ (namely $T_s = 200$). The generated orders $u_i(k)$, $i = 1, 2, 3$ are displayed on the left hand side of figure 4. This figure shows the ordering signal issued by each stage $\mathcal{S}_i$, $i = 1, 2, 3$ with the respective time-varying lower and upper bounds. The resulting inventory level $y_i(k)$ and the time varying desired inventory level $r_i(k)$ for each $\mathcal{S}_i$, $i = 1, 2, 3$ are reported on the left hand side of figure 5. The imposed and fulfilled demands $d_i(k)$ and $h_i(k)$ respectively at each $\mathcal{S}_i$ are displayed on the left hand side of figure 6.

A comparison has been performed with the DTCM proposed in (Ignaciuk, 2013) where equations (34),(35) have ben adapted to the case of an n-stage SC with $n = 3$, uncertain decay factors $\rho_i \in [0.86, 0.9]$ and known time delays $L_i = 4$, $i = 1, 2, 3$ obtaining

$$u_i(k) = \text{sat}[\omega_i(k)] = \text{sat}[y_{ref,i} - PIP_i(k)] \qquad (31)$$

where $PIP_i(k) = \rho_i^{L_i} y_i(k) + \sum_{j=0}^{k-1} \rho_i^{k-j} s_i(j) - \sum_{j=0}^{k-L_i-1} \rho_i^{k-j} s_i(j)$ and the saturation function $\text{sat}(\omega_i) \triangleq \{\omega_i$ if $\omega_i \in [0, u_{\max,i}]$; 0 if $\omega_i < 0$; $u_{\max,i}$ if $\omega_i > u_{max,i}\}$. According to (45),(46) in (Ignaciuk, 2013), $u_{\max,i}$ and $y_{ref,i}$ are inferiorly limited as: $u_{\max,i} > d_{\max,i}$ and $y_{ref,i} > d_{\max,i} \sum_{j=0}^{L_i} \rho_i^j$. The topology of the SC network shown in figure 1 is such that: $d_{\max,1} = \max_k d_1(k)$ and $d_{\max,i} = u_{max,i-1}$, $i = 2, 3$. The modified DTCM (31) has been applied choosing: $\rho_i = \bar{\rho}_i = 0.88$, $i = 1, 2, 3$, $d_{\max,1} = 40$, $u_{\max,1} = 45 > d_{\max,1}$, $u_{\max,2} = 50 > d_{\max,2} = 45$, $u_{\max,3} = 55 > d_{\max,3} = 50$, $y_{ref,1} = 160 > 157$, $y_{ref,2} = 180 > 177$ and $y_{ref,3} = 200 > 196$.
The orders $u_i(k)$ generated (with $\rho_i = \bar{\rho}_i = 0.88$) and the resulting on hand stock level $y_i(k)$ (generated with $\rho_i = 0.885$) are reported on the right hand side of figures 4 and 5 respectively.
The performance evaluation of both methods is performed on the basis of the performance indicators previously defined. The results are summarized in table 3. Both methods fully satisfy the end-customer demand $d_1(k)$ (as numerically quantified by the performance indicator $\mathcal{UD}_1$ reported in table 3) but the DRMPC approach requires a very smaller warehouse occupancy with respect to DTCM. This is visually evidenced by figure 5 and numerically quantified by $\mathcal{IS}$ (see table 3). The reduction of warehouse occupancy is a consequence of tracking a time varying inventory level $r_i(k)$ which is adapted

Table 1: Parameters of each node $\mathcal{S}_i$, $i = 1,2,3$.

time delay	*perishability factor*	*decay factor*	*initial state*
$L_i = 4$	$\alpha_i \in [\alpha_i^-, \alpha_i^+] = [0.1, 0.14]$	$\rho_i = 1 - \alpha_i \in [\rho_i^-, \rho_i^+] = [0.86, 0.9]$	$y_i(0) = 0$

Table 2: Tuning parameters of the local MMCOP for any $\mathcal{A}_i$, $i = 1,2,3$.

length of each $H_{i,k}$			*scalar weights in (11)*		*the scalars $\beta_{i,k} \triangleq \beta_i$ in (30)*		
$N_1 = M_1 - L_1$	$N_i \triangleq N_{i-1} - (L_i + 1)$ $i > 1$		$q_{i,l}$	$\lambda_{i,l}$			
$N_1 = 20$	$N_2 = 15$	$N_3 = 10$	$e^{-0.1\,(l-1)}$	$e^{-1\,(l-1)}$	$\beta_1 = 1.88$	$\beta_2 = 1.38$	$\beta_3 = 0.89$

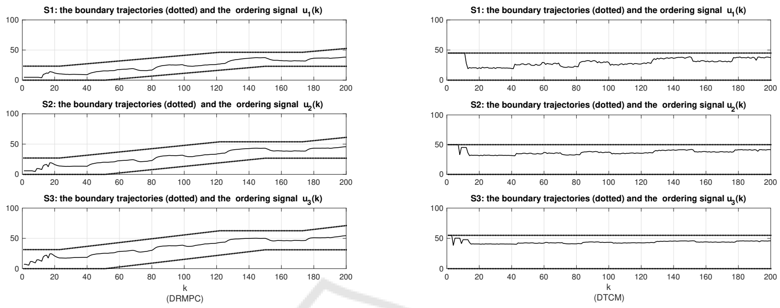

Figure 4: Comparison (DRMPC)-(DTCM): the ordering signal $u_i(k)$ issued by each $\mathcal{S}_i$.

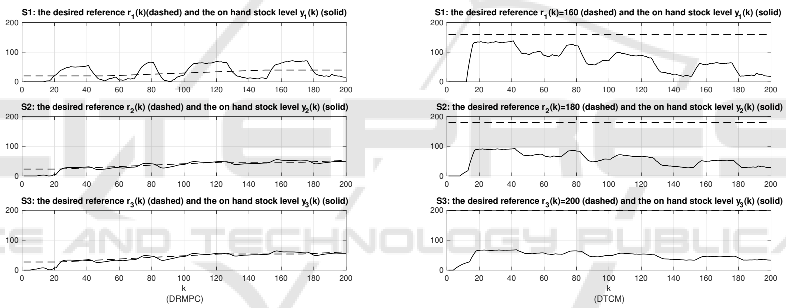

Figure 5: Comparison (DRMPC)-(DTCM): the desired inventory level $r_i(k)$ and the on hand stock level $y_i(k)$ of each $\mathcal{S}_i$.

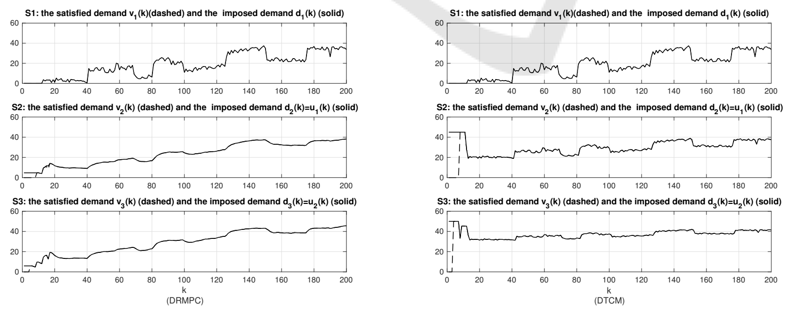

Figure 6: Comparison (DRMPC)-(DTCM): the imposed demand $d_i(k)$ and the fulfilled demand $h_i(k)$ at each $\mathcal{S}_i$.

Table 3: The performance evaluation of the DRMPC and DTCM strategies.

	$\mathcal{UD}_1$	$\mathcal{UD}_2$	$\mathcal{UD}_3$	$\mathcal{IS}$	$\mathcal{BE}_{\Delta u,1}$	$\mathcal{BE}_{\Delta u,2}$	$\mathcal{BE}_{\Delta u,3}$
DRMPC	0	0.0089	0.004	2.1894×10^4	74.3	92	106.8
DTCM	0	0.0526	0.0201	3.4895×10^4	232.4	152	108.7

at any k on the basis of the current demand $d_i(k)$. On the contrary DTCM defines a constant desired inventory level $y_{ref,i}$ for each $\mathcal{S}_i$, which is "a priori" computed using a conservative formula requiring the "a priori" knowledge of the maximum value $d_{\max,1}$ of the end-customer demand over an indefinitely long future time interval. Moreover, as $d_{\max,1}$ is never exactly known, it is often over-estimated.

The diagrams displayed in figure 4 and the entries of columns 5-7 of table 3 show that the DRMPC policy provides a smoother control signal with respect to the DTCM strategy. Moreover figure 4 evidences how the interval containing each replenishment order $u_i(k)$ is tighter in the DRMPC strategy. Our approach is able to limit the amplitude of such intervals and consequently to strictly control the FF2 of BE.

7 CONCLUSIONS

The main novelties we propose in this paper are: 1) the supply chain dynamics is characterized by perishable goods with uncertain decay factor, 2) the proposed DRMPC approach provides a B-splines parametrization of the replenishment order. The B-splines parametrization allows us to reformulate the min-max optimization problem implied by the DRMPC as a simpler WCRLS estimation problem. The method we propose also allows us to define a time-varying inventory level conciliating the opposite control requirements CR1 and CR2. CR3 is addressed penalizing the difference between control moves and also parametrizing the control moves as polynomial B-spline functions. The numerical test confirms the validity of the approach: it is actually able to reduce the inventory level without affecting customer service quality and without incurring an excessive control effort.

REFERENCES

Alessandri, A., Gaggero, M., and Tonelli, F. (2011). Min-max and predictive control for the management of distribution in supply chains. *IEEE Transactions on Control System Technology*, 19:1075–1089.

Chaudary, V., Kulshrestha, R., and Routroy, S. (2018). State of the art literature review on inventory models for perishable products. *Journal of Advances in Management Research*, 1:306–346.

De-Boor, C. (1978). *A practical guide to splines*. Springer Verlag, New York, 2nd edition.

Dejonckeere, J., Disney, S., Lambrecht, M., and Towill, D. (2003). Measuring and avoiding the bullwhip effect: a control theoretic approach. *European Journal of Operational Research*, 147:567–590.

Fu, D., Ionescu, C., Aghezzaf, E., and Kayser, R. D. (2014). Decentralized and centralized model predictive control to reduce the bullwhip effect in supply chain management. *Computers & Industrial Engineering*, 73:21–31.

Fu, D., Ionescu, C. M., Aghezzaf, E., and Kayser, R. D. (2016). A constrained epsac to inventory control for a benchmark supply chain system. *International Journal of Production Research*, 54:232–250.

Fu, D., Zhang, H., Dutta, A., and Chen, G. (2020). A cooperative distributed model predictive control approach to supply chain management. *IEEE Transactions on Systems Man and Cybernetics*, 50:4894–4904.

Fu, D., Zhang, H., Ionescu, C. M., Aghezzaf, E., and Kayser, R. D. (2019). A distributed model predictive control strategy for the bullwhip reducing inventory management policy. *IEEE Transactions on Industrial Informatics*, 15:932–941.

Giard, V. and Sali, M. (2013). The bullwhip effect in supply chains: a study of contingent and incomplete literature. *International Journal of Production Economics*, 51:3380–3893.

Hipolito, T., Nabais, J., Benitez, R., Botto, M., and Negenborn, R. (2022). A centralised model predictive control framework for logistics management of coordinated supply chain of perishable goods. *International Journal of Systems Science: Operation & Logistics*, 9:1–21.

Ietto, B. and Orsini, V. (2022). Resilient robust model predictive control of inventory systems for perishable good under uncertain forecast information. In *2022 International Conference on Cyber-physical Social Intelligence (Best paper finalists award)*. IEEE.

Ignaciuk, P. (2013). Discrete inventory control in systems with perishable goods- a time delay system perspective. *IET Control Theory and Applications*, 8:11–21.

Kohler, P., Muller, M., Pannek, J., and Allgower, F. (2021). Distributed economic model predictive control for cooperative supply chain management using customer forecast information. *IFAC Journal of Systems and Control*, 15:1–14.

Kuo, B. (2007). *Digital Control Systems*. Oxford University Press, Oxford, 2nd edition.

Lejarza, F. and Baldea, M. (2020). Closed-loop real-time supply chain management for perishable products. *IFAC PapersOnLine*, 53:11458–14463.

Lobo, M., Vandenberghe, L., Boyd, S., and Lébret, H. (1998). Second-order cone programming. *Linear Algebra and its Applications*, 284:193–218.

Mestan, E., Metin, M. T., and Arkun, Y. (2016). Optimization of operations in supply chain systems using hybrid systems approach and model predictive control. *Ind. Eng. Chem*, 45:6493–6503.

Perea-Lopez, E., Ydstie, B., and Grossmann, I. (2003). A model predictive control strategy for supply chain optimization. *Computers and Chemical Engineering*, 27:1201–1218.