Going through the Looking Glass with Mr. Phillips: A Time-Frequency Analysis on the (in)stability of the UK Phillips Curve

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Abstract

The debate on the inflation-unemployment relationship has focused almost exclusively on the distinction between the "short-run" and "long-run" Phillips curves, while virtually ignoring the "middle" horizons. Using a historical perspective, we show that the UK wage Phillips curve is essentially a medium-run phenomenon. At the frequency range beyond business cycle frequencies, that is 8 to 16 years, there is significant evidence of a negative and stable relationship between money wage inflation and unemployment.

Keywords: Phillips Curve, frequency bands, wavelet, medium run.

JEL classification codes: E00, E30, E31, E32.

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1 Introduction

Sixty-one years after A.W. Phillips (1958) published the "The relation between unemployment and the rate of change money wage rates in the United Kingdom, 1861-1957", the empirical validity of its Curve (PC for short) remains a hotly debated issue both in the world of academic research and policy applications. The very different opinions, "dead" or "alive", "flatter" or "steeper", about the status of the PC suggest that there is still much to be done despite the vastness of the contributions in the field; a sample of significant papers includes, among others, Samuelson and Solow (1960), Phelps (1967), Friedman (1968), Lucas (1972, 1973), Sargent and Wallace (1975), Gordon (1982, 2011), King and Watson (1994), Gali and Gertler (1999), Staiger et al. (1997a,b), Haldane and Quah (1999), Mankiw (2001), Mavroeidis et al. (2014), Galí (2011) and Hall and Sargent (2018).

In the last decade, the pattern of unemployment and inflation has raised doubts on the quality of the PC as a framework capable to explain satisfactorily how the disequilibrium in the goods or labor market impacts the inflation rate, leading to conclusions that the PC has been flattening and inward shifting (Cunliffe, 2017). Del Negro et al. (2020) indicate several explanations for this transformation of the PC. The first is related to the possible measurement error in both inflation and economic slackness. The second is how to deal with structural changes in the labor market that might have weakened the link between (wage) inflation and real economic activity; The third is the underlying monetary policy paradigm that emerged in the early 80s, in response to strong inflationary pressure, and flattened the aggregate demand curve and hence weakened the link between inflation and output. A crucial point emerging from many of these contributions is the focus on business cycle frequency dynamics, that is the time span generally defined as the interval between 2 and 8 years. Exploring what happens to the slope of the PC when we include lower frequencies, the so-called medium run (Borio, 2014), is the key point of the present paper.

The usefulness of frequency analysis applied to the PC was first raised by Desai (1975) who pointed out that Phillips' original averaging procedure, was not only crucial to his results, but also of general statistical and economic interest. The short-run trade-off between inflation and unemployment was not Phillips' original motivation. In fact, he dismissed price inflation as a contributing factor to his empirical model and argued that an excess demand for labor would have induced firms to bid up money wages independently of inflation. This assumption and the restriction that changes in unemployment were essentially set to zero, by averaging observations over the selected time intervals, suggest that Phillips was interested in a longer-run relationship.[1] Desai (1975) and, recently, Gallegati et al. (2021) show that Phillips' unorthodox data transformation is crucial for assessing the frequency resolution of Phillips' findings, meaning that his averaging procedure identifies a lower frequency relationship between wage inflation and unemployment.

Another novelty of our contribution is that it focuses on the UK using a sample that covers a long time span from 1857 to 2016. Much of the debate is centered on the US, evidence is more scarce for other countries. In this regard, Haldane and Quah (1999, Figure 3) produce a conventionally sloped price inflation-unemployment scatter plot from 50 years of post-WWII monthly data. The authors find that the UK PC is virtually vertical from 1948 to 1980 and then progressively flattens. Cunliffe (2017, Chart 6) fits trend lines through scatter plots of the UK money wage inflation-unemployment rate for the

[1] Phillips' argument goes as follows: "For suppose that productivity is increasing steadily at a rate of, say, 2 percent per annum and that aggregate demand is increasingly similarly so that unemployment is remaining constant at say, 2 percent. Assume that with this level of unemployment and without any cost of living adjustment wage rates rise by, say, 3 percent per annum as a result of employers' competitive for labor and that import prices and the prices of other factor services are also rising by 3 percent per annum. Then retail prices will be rising on average at the rate of 1 percent per annum…Under these conditions, the introduction of cost of living adjustments in wage rates will have no effect, for employers will merely be giving under the name of the cost of living adjustment part of the wage increases which they would, in any case, have given as a result of their competitive bidding for labor." (p.284).

periods 1971-1997, 1998-2012, and 2013-2017 and concludes that the PC relationship has continuously shifted inward as well as flattened. Empirical evidence suggests that the flattening of the PC in the UK follows the same pattern found elsewhere.

It is a central result of our paper that lower-frequency trends, defined as the cyclical components in the 8-to-16 years' frequency band, are statistically significant and economically important in the wage PC. In our investigation, we will revisit the UK PC using both static and forward-looking specifications with both Phillips' original data and Bank of England historical data covering the 1861-2016 period (Ryland et al., 2010). To do that, we employ wavelet methods (Crowley, 2007; Aguiar-Conraria and Soares, 2014) to analyze data both in the time and frequency domain. Our results suggest that the UK wage PC is stable at the medium-run frequencies even when it disappears at the business cycle frequencies.

Relatively few contributions have estimated the PC in the frequency domain using spectral analysis (King and Watson, 1994; Iacobucci, 2005; and Reinbold and Wen, 2020). Their conclusion is that the PC has a negative slope at the business cycle frequencies, but not at the higher frequencies (less than 18 months). Reinbold and Wen (2020), instead, find evidence that the relationship between unemployment and inflation is still negative and statistically significant at the medium run frequencies (between 6 to 50 quarters).

The wavelet methodology offers several appealing features: they can deal with stationary and non-stationary data, and can perform multi-resolution decompositions. Gallegati et al. (2011) were the first to estimate the wage PC for the US using wavelet tools, and found strong evidence of a negative relationship especially at the "major" NBER-dated business cycle frequency,16 to 32 quarters.[2] Other recent contributions have relied on continuous wavelet tools to investigate the shape of the PC. Mutascu (2019) supports the idea that the US PC is unstable over time with episodes of strong negative correlation both in the short and medium term. Aguiar-Conraria et al. (2019) apply the continuous wavelet transform to investigate the unemployment and price relationship in the context of the NKPC for the US and find evidence of a short-term correlation but no support for the medium-run hypothesis.

Our paper delivers four key results. First, over the period of direct interest to Phillips, 1861-1913, the negative relationship between wage inflation and unemployment is mainly present beyond the typical business-cycle frequency range. Second, this medium-run relationship survives until present days using a long stretch of historical annual data. Third, our exercise suggests that, while at business-cycle frequencies there is a steady decline of the PC slope, at the medium-run frequencies the slope is still negative and statistically significant.

The structure of the paper is as follows. Section 2 presents the mathematical details of the wavelet methodology. Section 3 applies the wavelet techniques to the original Phillips'data. Section 4 discusses the New Keynesian wage PC (NKWPC), our measure of the unemployment gap, empirical findings of annual data estimation over the longest available time, and a robustness exercise with post-WWII quarterly data. Section 5 offers concluding comments.

2 Wavelet methodology

Wavelet methodology was first applied to analyze seismic signals in the early 1980s (Morlet et al., 1982). Then, applications moved to the areas of signal processing and data compression (Daubechies, 1988). Ramsey et al. (1995) and Ramsey and Zhang (1996) were the first to employ the wavelet tool in economics and finance.

[2]According to the NBER, the business cycle frequencies 2-8 years can be further decomposed in fluctuations with periodicities between 2 and for years as "minor" business cycles and "major" the ones between 4 and 8 years. For a more detailed description see Flor and Klarl (2017) and Igan et al. (2009).

There are three important advantages of the wavelet transform. First, it is a local rather than a global transformation and consequently can handle both stationary and non-stationary time series.[3] Second, the multiresolution nature of the wavelet decomposition analysis attains an optimal trade-off between time and frequency resolution levels. It provides a flexible time-scale window that narrows on small-scale features and widens on large-scale features, thus displaying good time resolution (and poor frequency resolution) for short-lived high-frequency phenomena and good frequency resolution (and poor time resolution) for long-lasting low-frequency phenomena. Third, it provides a systematic way of performing band-pass filtering without being committed to any particular models.[4]

Wavelet multiresolution decomposition analysis may be performed using two types of wavelet transforms, the continuous (CWT) and the discrete wavelet transform (DWT). The former computes wavelet transform coefficients at all times and scales, while the latter provides a discretized version using only a limited number of translated and dilated versions of the wavelet basis function to decompose the original signal. In our investigation, we will use both transforms.

Regarding the CWT, most of the mathematical concepts are taken from the survey by Aguiar-Conraria and Soares (2014) and their later article (Aguiar-Conraria et al., 2018). Recent contributions using the CWT methodology include: Aguiar-Conraria et al. (2020), Faria and Verona (2020) and Fratianni et al. (2021). Most of the results are obtained with the partial wavelet coherence, the partial phase difference, and the partial wavelet gain. For the sake of simplicity, we start our discussion with the bivariate specification, even though it is not employed in our tests.

The wavelet power spectrum identifies, in the time-frequency plane, the location of variance of a time series. The complex wavelet coherence is defined as the modulus of the wavelet cross spectrum (W_{xy}) normalized by the squared root of the wavelet spectra of each signal (W_x and W_y),

$$\varrho_{yx} = \frac{|S(W_{xy})|}{(S(|W_x)|^2)S(|W_y)|^2)^{1/2}}, \tag{1}$$

where S is a smoothing operator (Torrence and Webster, 1999). The wavelet coherence is defined as the absolute value of the complex wavelet coherence:

$$R_{yx} = |\varrho_{yx}| = \frac{|S_{yx}|}{\sigma_x \sigma_y}, \tag{2}$$

where $|S_{yx}| = |S(W_{xy})|$, $\sigma_x = \sqrt{S(|W_x)|^2}$, and $\sigma_y = \sqrt{S(|W_y)|^2}$. In practical application, the wavelet coherence obtained from the CWT is shown in a graph where time is recorded on the x-axis and frequencies along the y-axis. Areas with the highest (lowest) values of local "correlation-type" appear with hottest (coldest) colors.[5]

We define the global wavelet coherence as in Schulte et al. (2016, 2018):

$$G\varrho_{yx} = \frac{|S(W_{xy})|}{[(\sum_{n=1}^{N} S(|W_x)|^2)(\sum_{n=1}^{N} S(|W_y)|^2)]^{1/2}}. \tag{3}$$

This is computed by taking the average of the wavelet coherence along the time dimension. This measure yields the statistical significance and the importance of each frequency band in the sample; the computation of the confidence interval of estimates such as the wavelet gain remains an open issue

[3] This property may be particularly useful when dealing with complex, non-stationary signals like historical time series since their secular movements are likely to exhibit structural changes due to shocks such as wars, economic and or financial crises.

[4] This contrasts with Baxter and King (1999) and Christiano and Fitzgerald (2003) who fit optimal band-bass filters to specific models.

[5] The coherence is a "correlation-type" measure in the sense that it is measured on a [0,1] scale, but it cannot be strictly qualified as a standard "correlation". In the text, for the sake of simplicity, we will continue to define coherences as "correlation".

(Aguiar-Conraria et al., 2018)

The wavelet phase difference yields information about the sign and the lead-lag structure of the relationship between x and y.

$$\Phi_{yx} = arctan\left(\frac{\Im(S_{yx})}{\Re(S_{yx})}\right), \tag{4}$$

where $\Im$ and $\Re$ are the imaginary and the real part of the cross spectrum (S_{yx}) between signals y and x. We glean this information as follow:

- if $\Phi_{yx} \in (\pi, \frac{\pi}{2})$ y and x are in anti-phase, then x leads y,
- if $\Phi_{yx} \in (\frac{\pi}{2}, 0)$ y and x are in phase, then y leads x,
- if $\Phi_{yx} \in (0, -\frac{\pi}{2})$ y and x are in phase, then x leads y,
- if $\Phi_{yx} \in (-\frac{\pi}{2}, -\pi)$ y and x are in anti-phase, then y leads x.

Finally, we define the wavelet gain as in Mandler and Scharnagl (2014):

$$G_{yx} = R_{yx}\frac{\sigma_y}{\sigma_x}. \tag{5}$$

The interpretation of the wavelet gain is straightforward: it is the equivalent to the coefficient β when we regress y on x at each time and frequency.

All these test measures can be extended to a multivariate setting at a cost of a more cumbersome notation.[6] For our purpose, the partial wavelet coherence ($\varrho_{yx|z}$) captures the local correlation between y and x after controlling for z. The same interpretation holds for the partial phase-difference ($\Phi_{yx|z}$) and the partial wavelet gain ($G_{yx}|z$).

The CWT is a highly redundant transform. The DWT, Unlike the CWT, uses only a limited number of translated and dilated versions of the mother wavelet to decompose the original signal, so that the information contained in the signal can be summarized in a number of wavelet coefficients equal to the number of observations. A complete overview of the DWT and its application in economics can be found in Crowley (2007). To implement the discrete wavelet transform on sampled signals we need to discretize the transform over scale and over time through the dilation and location parameters. Indeed, the key difference between the CWT and the DWT is that the DWT uses only a limited number of translated and dilated versions of the mother wavelet to decompose the original signal. Mathematically:

$$y(t) \approx S_J + D_J + D_{J-1} + ...D_2 + D_1, \tag{6}$$

where S_j is the long-run smooth component and D_j represents the detail components, each associated with a particular frequency range. With annual data, since scale 2^{j1} corresponds to frequencies in the interval $f[1/2^{j+1}, 1/2^j]$, a 3-level decomposition produces one smooth vector S_3 and three detail vectors, where wavelet detail coefficients at scales 1 to 3 are associated to 2−4, 4−8, 8−16 year intervals, and the smooth component S_3 captures oscillations with a period longer than 16 years, corresponding to the low-frequency components of the time series. At quarterly frequency, the 3 to 4 scales are associated with the traditional business cycle frequencies, while scale 5 is associated with the frequencies between 32 (8 years) and 64 (16 years) quarters, that is the medium run. In sum, scale D_3 at annual frequency and D_5 at quarterly data correspond to our definition of the medium run.

In practical applications, the maximal overlap discrete wavelet transform (henceforth MODWT) is typically employed instead of the DWT. The MODWT can be seen as a compromise between the CWT,

[6] The interested reader can refer to Aguiar-Conraria and Soares (2014) for the mathematical details of the multivariate extension.

with continuous variations in scale, and DWT where the power of the transform is highly localized. Examples of MODWT applications are Gençay et al. (2010), Gallegati et al. (2011), Michis (2015), and Crowley and Hudgins (2021).[7]

3 Phillips' original results seen through the wavelet methodology

Our empirical strategy is to go back to the origin and examine the negative relationship between money wage inflation and the unemployment rate occurring at different frequency bands. Phillips' procedure can be interpreted as a simple moving average with non-overlapping variable-width windows (Gallegati et al., 2021). Since a moving average removes high-frequency fluctuations of the time series, Phillips' averaging procedure is somewhat equivalent to applying a low-pass filter to the data. In brief, Phillips performed a crude form of wavelet analysis (Gallegati et al., 2011, Footnote 2). The more popular replication of the PC, done by Lipsey (1960), relied on familiar aggregated time-scale estimation techniques. It "became the standard method of analysis in economics by 1960...[and] inspired a booming industry in Phillips curve estimation" (Wulwick, 1996, Pag. 410).

The source of Phillips' original dataset is Wulwick (1996). Appendix A provides a description of the variables and their sources used in our exercise. We begin our analysis by investigating Phillips' original specification with the money wage inflation being a nonlinear function of the level of unemployment after controlling for the growth rate of imported prices,

$$\pi_t^w = \kappa U_t + \gamma \pi_t^{imp}. \tag{7}$$

Figure 1 shows the partial wavelet coherence in the top panel. The partial phase differences (red lines) and the partial wavelet gains (blue lines) are shown in the middle and lower panels, respectively, with the left panels reporting the results for the business-cycle frequencies (2-8 years) and the right panels their medium-run counterpart (8-16 years).

The partial wavelet coherence plot shows that the highest local correlation between wage inflation and the unemployment rate occurs in the period 1861-1885, with its most significant portion concentrated at frequencies between 8 and 16 years with different levels of significance. This time span was largely included in the original sample (1861-1913) Phillips used to estimate his curve (Phillips, 1958, Figure 1).

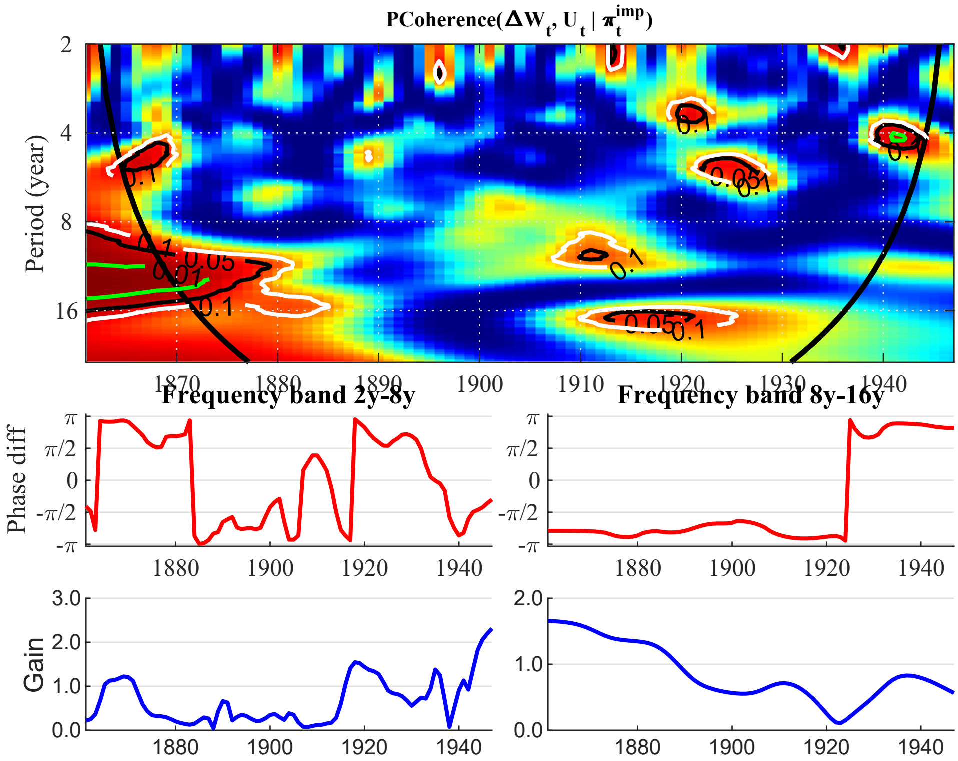
The partial phase difference reveals an important difference between business-cycle and medium-run frequencies in terms of the relationship stability. At business-cycle frequencies, both the sign and the timing of the relationship change, except in the period from the start of the sample up to 1885, when the relationship is negative and wages are leading unemployment. On the contrary, in the medium run, the relationship is always negative, with wage inflation shifting from leading to lagging unemployment after the 1920s.

The partial wavelet gains confirm the differences in results between the business-cycle and medium-run frequencies. In the former, the effect of the unemployment rate is negligible between 1880 and 1915, with peaks occurring in coincidence of price increases. By contrast, in the medium run, the impact of the unemployment rate tends to steadily decline from the start of the sample, when the value of the wavelet gain is around 1.8, to 1920, when the wavelet gain is close to zero.[8]

[7] All our analyses used two different Matlab's toolboxes: the one described in Aguiar-Conraria and Soares (2014), and the one developed by Grinsted et al. (2004).

[8] A problem in the interpretation of the phase difference may arise because when the lead becomes longer the phase shift of the relationship automatically switches to a lag, although that might not necessarily be the case.

Figure 1: Partial wavelet coherence, annual data, original Phillips



Note: In the partial wavelet coherence plot time is recorded on the horizontal axis and the scale of the wavelet transform on the vertical axis at yearly frequencies. The color coder ranges from blue (low coherence) to red (high coherence). The 99%, 95%, and 90 % confidence intervals for the null hypothesis that coherence being zero are plotted as contour lines in green, black, and white respectively. The cone of influence is marked by black lines.

To summarize, our evidence suggests that the PC relationship is sensitive to both time and frequency, specific to the period 1861-1885, and limited to the frequency range of 8 to 16 years.

4 The New Keynesian wage Phillips Curve

Our findings obtained from the application of wavelet techniques to Phillips' original dataset raise a critical question. Will these findings hold when we extend the sample to the post-1960 period and use a microfounded forward-looking specification of the Phillips curve? To answer our query, we apply the CWT tools to the specification of the NKWPC examined by Coibion and Gorodnichenko (2015) and Aguiar-Conraria et al. (2019). There is a wide consensus in the new Keynesian literature that this specification can be considered the benchmark supply equation of modern macroeconomics (Gordon, 2011).

$$\pi_t^w = \kappa U_t^{gap} + \beta E_t\{\pi_{t+1}\}, \tag{8}$$

where β is the coefficient of the expected future inflation path, $E_t\{\pi_{t+1}\}$, while U_t^{gap} is the unemployment gap defined as:

$$U_t^{gap} = U_t - U_N. \tag{9}$$

The estimation of equation (8) depends on obtaining estimates of the expected future inflation. We follow Ramsey et al. (2010) to create a measure of inflation expectations by shifting one period ahead the long-term component of the inflation rate. Similarly, we approximate the natural rate of unemployment with the trend component of the unemployment rate. The trend component of both variables is obtained by applying the MODWT to the raw series of the inflation and unemployment rate and, based on the multiresolution decomposition shown in equation (6), they corresponds to fluctuations greater than 16 years. Next, we define long-term components - in a three-scale level decomposition applied to annual data - as fluctuations greater than 16 years, a time horizon consistent with our concept of medium run. Thus, the S3 smooth component is our proxy of the expected future inflation and of the trend rate of the unemployment rate.

4.1 The NKWPC estimation results: annual data from 1861 to 2016

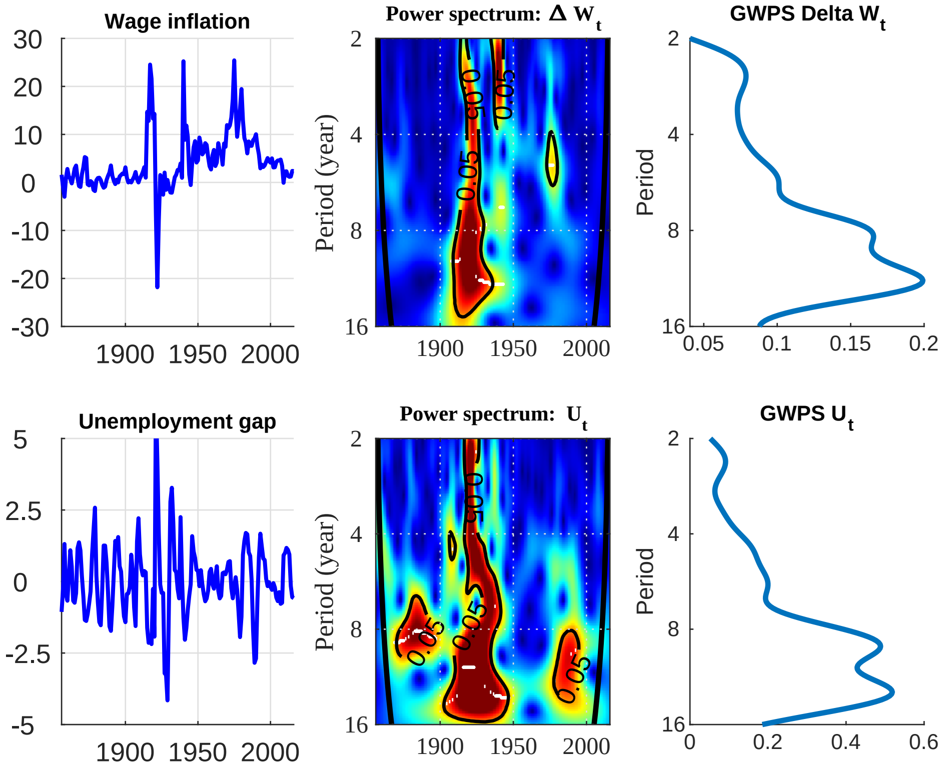
We proceed as follows. To maximize the length of the historical period being studied, we use for our analysis data from *A Millennium of macroeconomic data for the UK. The Bank of England's collection of historical macroeconomic and financial statistics* (Ryland et al., 2010).

Figure 2 shows the raw series (left panels), their wavelet local power spectra (middle panels), and their global wavelet spectra (right panels). The wavelet power spectrum is interpreted as the energy distribution of a time series and detects areas, in the time-frequency plane, where the variance is concentrated. Both wage inflation and the unemployment gap display evidence of a high power region in the first half of the 20th century across all frequencies. For the unemployment gap, these high power regions are also evident in the 2nd half of the 19th century and in the last quarter of the 20th century at scales corresponding to intervals between 8 and 16 years. The global wavelet spectrum identifies periodicities that are dominant in a time series. For both wage inflation and the unemployment gap, a well-defined peak is detected at the medium-run frequencies (8-16 years).[9]

The partial wavelet coherence between wage inflation and the unemployment gap with respect to expected inflation is reported in Figure 3. There is clear evidence of three statistically significant regions: at scales corresponding to the medium run located at the start of the sample (as in Phillips' dataset), in the first half of the 20th century, and in the last three decades of the same century. There is sparse evidence of statistically significant regions at the business cycle frequencies: two of them located in the first part of the sample, one in the first decade of the 20th century, another around the 1920s, and the

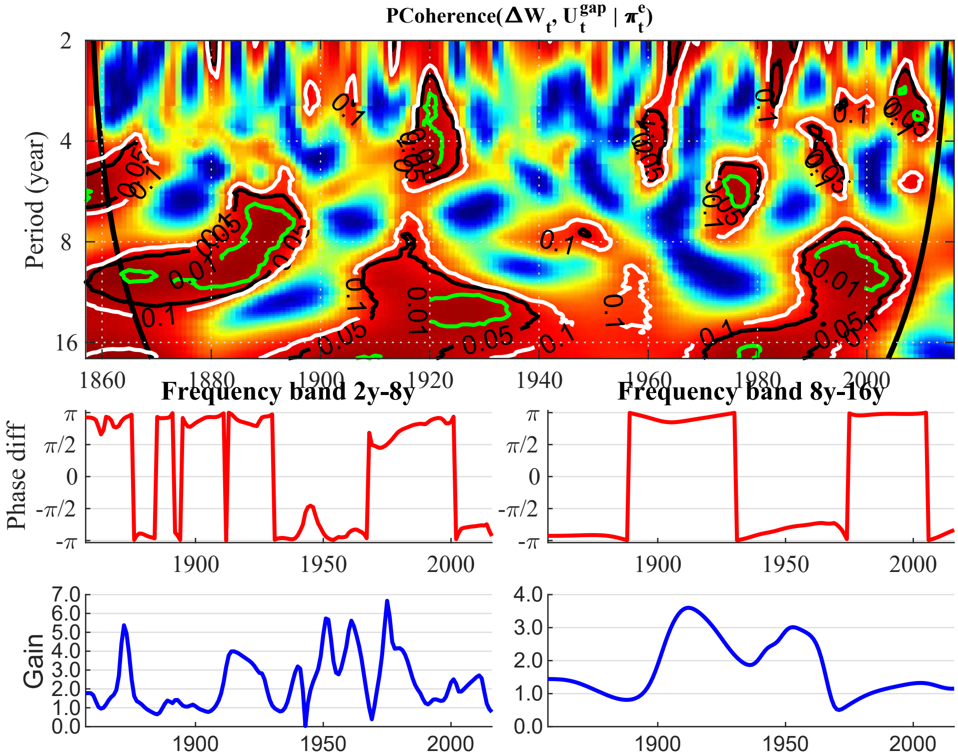
[9]For expected inflation such peak is located at longer frequencies.

Figure 2: Observations, wavelet local power spectrum, and global wavelet power spectrum (annual data 1857-2016)



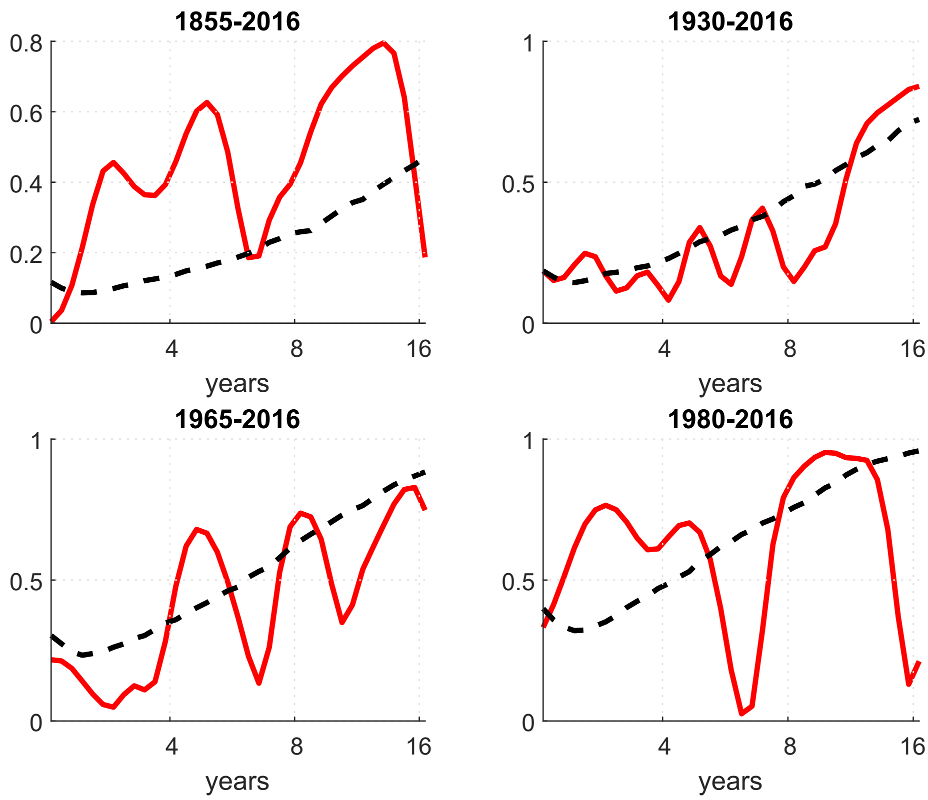
Note: In the wavelet local power spectrum time is recorded on the horizontal axis and the scale of the wavelet transform is recorded on the vertical axis at yearly frequencies. The color code ranges from blue (low power spectrum) to red (high power spectrum). The 95% confidence intervals for the null hypothesis that the power spectrum is zero is plotted as black contours. The cone of influence is marked by black lines. White lines on the power spectrum are associated with frequencies that locally maximize the variance.

Figure 3: Partial wavelet coherence, annual data



Note: Time is recorded on the horizontal axis and the scale of the wavelet transform is recorded on the vertical axis at yearly frequencies. The color code ranges from blue (low coherence) to red (high coherence). The 99%, 95%, and 90 % confidence intervals for the null hypothesis that coherence is zero are plotted as contours in green, black and white respectively. The cone of influence is marked by black lines.

Figure 4: Global wavelet coherence, annual data



Note: the red solid line represents the global wavelet coherence, while the black dashed line indicates the 90 % confidence interval. Periods (years) are recorded on the x-axis.

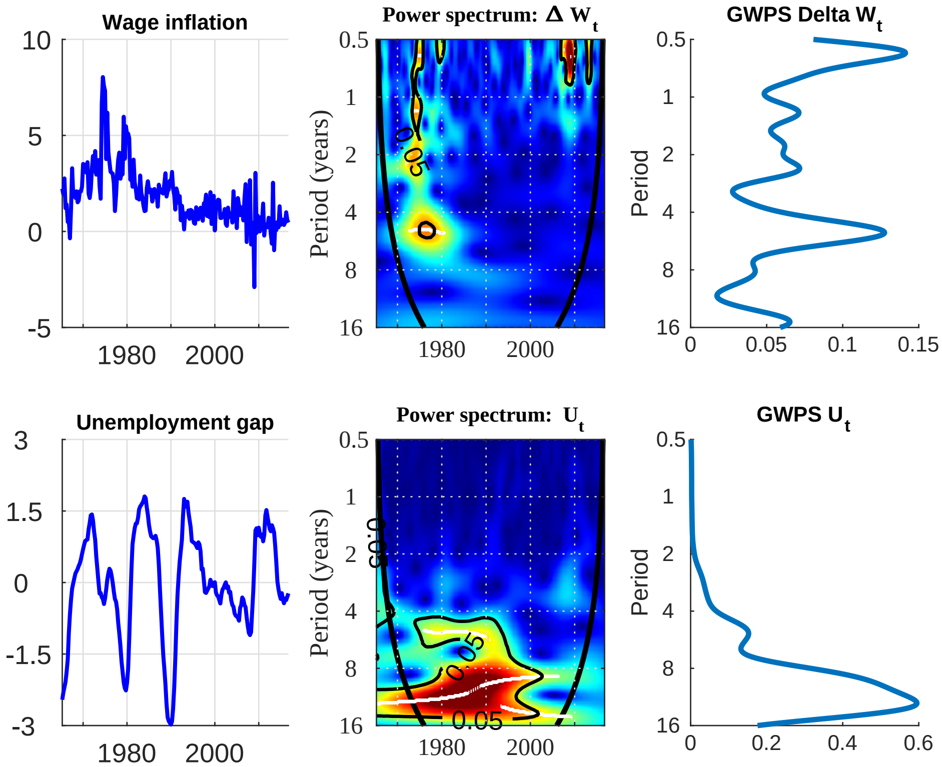
other significant regions located between the 1960s and early 21st century.

The partial phase difference and the wavelet gain, shown in Figure 3, partially confirm the results obtained with the sample overlapping Phillips' original dataset: for instance, in the medium run, there is a the shift in the wage inflation from leading to lagging and the decline of the wavelet gain ending in 1900 instead of 1920. In general, the two variables are out of phase with the leading-lagging relationship shifting in 1900, 1940, 1970, and early 2000s. These shifts coincide with the inversion of the evolution of the wavelet gain, especially when the gain rises after a declining phase. At business-cycle frequencies, the two variables are always out of phase, with wage changes mostly leading unemployment gap, except in the 1930-1970 years, when the timing is opposite. As to the wavelet gain, we find their highest values during high-inflation episodes (interwar period, aftermath of WWII, and mid-1970s) with the value steadily declining after the 1970s and falling below one after 2000.

As mentioned in the methodological section, it is not always straightforward to evaluate the level of relationship significance at different time scales. To improve on that, we take advantage of the global wavelet coherence (Schulte et al., 2016, 2018), which can be interpreted as the time-averaged version of the coherence. The global coherence provides further insights on the importance of different time scales across the whole sample. For example, if the global wavelet coherence (red line) is above the significance levels (dashed black line), the inference is that, over the whole sample, the frequency band is significantly different from zero. Results are presented in Figure 4.

Across the whole sample (1855-2016), the medium-run component appears to be significant, mainly driven by the 1980-2016 subperiod. To sum up, the investigation of annual data offers several clues on the importance of the medium-run frequencies in shaping the PC.

Figure 5: Observations, power spectrum, and global wavelet power spectrum, quarterly data



Note: Time is recorded on the horizontal axis and the scale of the wavelet transform is recorded on the vertical axis with frequencies converted to time units (years) to facilitate interpretation. The color code ranges from blue (low power spectrum) to red (high power spectrum). The 95% confidence intervals for the null hypothesis that power spectrum is zero is plotted as black contours. The cone of influence is marked by black lines. White lines on the power spectrum are associated with frequencies that locally maximize the variance.

4.2 Robustness analysis: quarterly data from 1965:1 to 2016:4

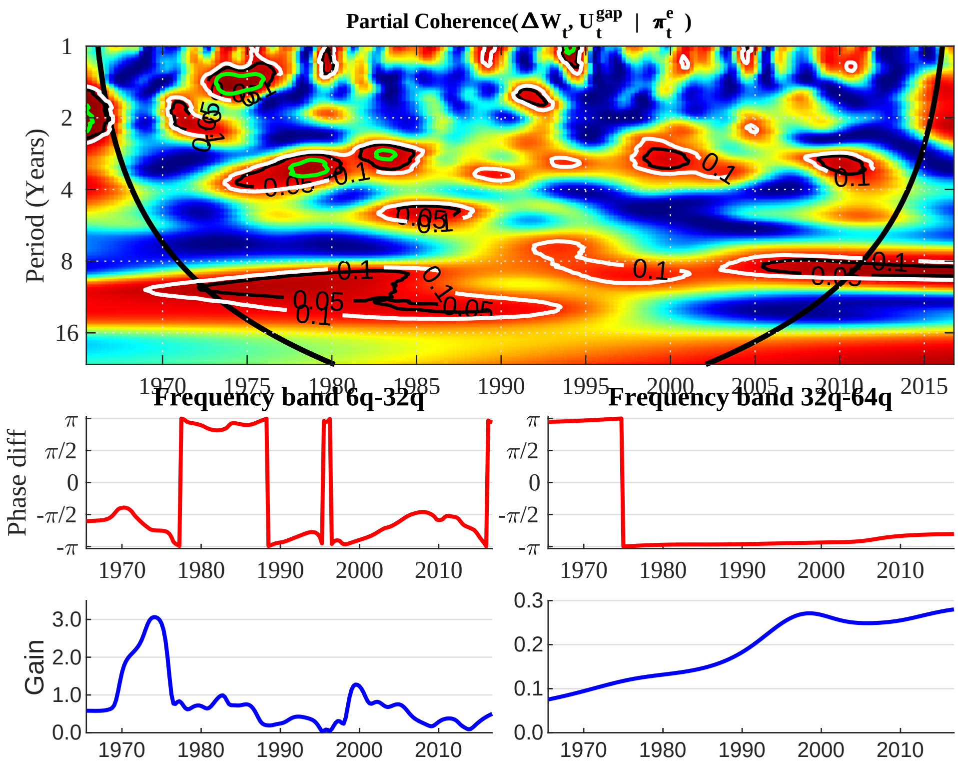
The analysis of the previous subsection may suffer from some limitations. First, with annual data, we can only explore relationships between variables at frequencies higher than the business cycle. However, according to the partial wavelet coherence plot in Figure 3, the very short run may be a time horizon also of interest for the PC, especially in the 1970s when several exogenous shocks hit the economy. To extend our analysis to shorter-term frequencies, i.e. smaller than 2 years, the wavelet transform must be applied to data collected more frequently. Therefore, in this section, we move from annual to quarterly data, from 1965:1 to 2016:4 (Ryland et al., 2010). Moreover, the choice of quarterly post-war data offers the additional advantage of exploiting professional forecasters' expectations as a measure of expected future inflation, thus testing the robustness of the findings obtained in the previous subsection.

We begin our analysis, mirroring the structure of the annual data section, by inspecting the power spectrum and the global power spectrum of wage inflation, the unemployment gap, and inflation expectation. The results are reported in Figure 5.

The wavelet local power spectra are similar for wage inflation and inflation expectations, with the variability mostly concentrated in the pre-mid 1980s years at business cycle frequencies and beyond. For the unemployment gap, the variability is mostly concentrated at frequencies corresponding to periods greater than 4 years, but, unlike other variables, this variability spreads throughout the sample.

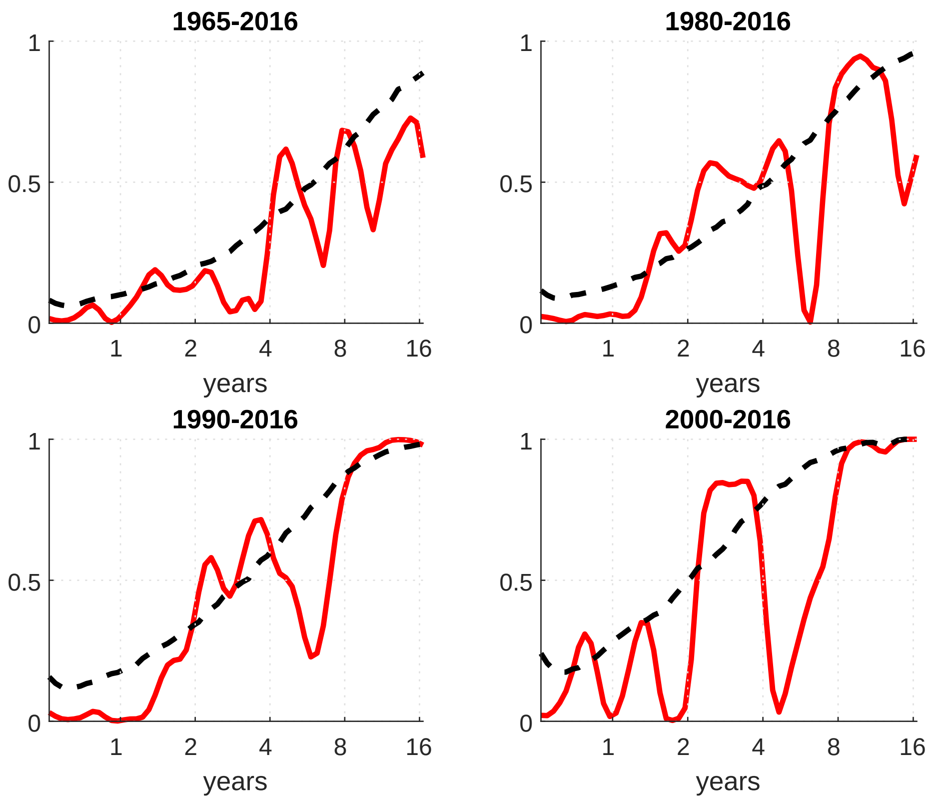
The partial wavelet coherence plot detects the presence of statistically significant regions at different frequencies. The widest areas are identified at frequencies corresponding to the 8-16 year range, one from 1970 to the mid-1990s, and the other from the early 200s onwards. Two other high coherence regions are

Figure 6: Partial wavelet coherence, quarterly data



Note: Time is recorded on the horizontal axis and the scale of the wavelet transform is recorded on the vertical axis with frequencies converted to time units (years) to facilitate interpretation. The color code ranges from blue (low coherence) to red (high coherence). The 99%, 95%, and 90 % confidence intervals for the null hypothesis that coherence is zero are plotted as contours in green, black and white respectively. The cone of influence is marked by black lines.

Figure 7: Global wavelet coherence, quarterly data



Note: The red solid line represents the global wavelet coherence while the black dashed line indicates the 90 % confidence interval. Time (years) is recorded on the x-axis.

detected between early 1970s and mid-1980s at frequencies higher than the typical business cycle's (2-4 years) and even shorter (1-2 years) in the 1970s. In sum, our evidence shows the dominant role of the lower frequencies in the PC that correlate with the short-term effects of the oil shocks of the 1970s (see Figure 6).

The partial phase difference confirms that the relationship between wage inflation and the unemployment gap is negative at all frequencies. At business cycle frequencies, wage inflation leads unemployment gap, with the exception of the late 1970s-1980s, when the unemployment gap is leading wage inflation. The medium-run relationship is remarkably stable with a unique phase shift occurring in the mid-1970s when wage inflation switched from lagging to leading the unemployment gap.

The partial wavelet gains show another stylized fact. At the business cycle frequencies, after the highest spike in the mid-1970s, the slope of the PC is steadily declining over time (except for a small spike in the early 2000s). On the contrary, in the medium run the slope is slightly, but persistently, increasing throughout the sample.

Like for annual data, we explore the strength of our medium-run relationship by computing the global wavelet coherence. Figure 7 shows that the medium run emerges once we omit the 1970s. This finding is in line with the yearly investigation. Starting with 1980, the medium run component is always present and statistically significant.

5 Concluding remarks

Going back to Phillips' original insight and using the maximum time span, we explore the negative relationship between money wage inflation and the unemployment rate at different frequencies and over time. Our main findings are that the wage PC is essentially a medium-run phenomenon which occur in the 8-16-years frequency band, and that the PC effect is evident at the business-cycle frequency (16 to 32 quarters), but is unstable over different time intervals.

That the PC relationship is primarily stable in the medium run is not surprising. Macroeconomic textbooks that refer to medium-run equilibrium in the labor market (e.g. Blanchard, 1997). Crowley and Hughes-Hallett (2018) imply the existence of embedded lower frequencies in real market variables. Although we do not have a comprehensive theory of why intermediate time scales may capture the "effective" time horizon of labor market variables, a few explanations are associated with this evidence. For one, labor is a "quasi-fixed factor" of production rather than the variable input described in conventional microeconomics texts (Oi, 1962, 1983). Hiring, firing, and training costs – reflecting in part the power of the unions – are responsible for this degree of fixity of labor. For the sluggish response of wages to changes in aggregate demand we refer to ongoing works in the literature on the source of wage and price stickiness and staggered wage contracts. The earlier promise that such contracts could explain nominal stickiness and retain sufficient flexibility to allow a credible monetary disinflation to be carried out with little or no economic loss of unemployment or output has not panned out. The alternative explanation is that monetary policy, the big driver in the growth of nominal magnitudes, suffers from very imperfect credibility.[10]

Finally, although based on a reduced form of both the PC and the NKWPC, our analysis suggests that the New Keynesian Model, the cornerstone of modern macroeconomics and business cycle analysis, may be re-examined on the basis of the evidence that variables move at different frequencies. For example, with regard to the estimates of the "deep" parameters, several studies, e.g. Sala (2015), Caraiani (2015), and

[10]The medium-run perspective as an explanation of the changing pattern of wages, unemployment, and inflation may be also consistent with Lipsey's 2016 evolutionary hypothesis that the economy evolves along a non-stationary path driven by waves of endogenously generated technological change (Freeman and Louçã, 2001 and Perez, 2009) rather than fluctuating around some static long-run equilibrium. Indeed, under this approach, also exogenous forces may have lasting effects on the evolution of the economy because of the path-dependency of endogenous technological change.

Gallegati et al. (2019), report that parameter estimates of DSGE models are highly sensitive to different subsets of frequencies used in estimation, an indication that models "cannot match all frequencies with one set of parameters" (Sala, 2015, p. 2019). The evidence that the Phillips Curve is predominantly a medium-run phenomenon has implications for the specification of structural macroeconomic models, as evidenced by Solow (2000) in his reflections on the time-horizon consistency problem of macroeconomic modelling.[11] Our paper indicates that the Phillips Curve may be a good candidate for this frequency-dependence. Clearly, additional research is required on the subject.

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[11] "At short term scales, I think, something sort of Keynesian is a good approximation, and surely better than anything straight neoclassical. At very long scales, the interesting questions are best studied in a neoclassical framework..... At the five to ten years time scale, we have to piece things together as best as we can, and look for an hybrid model that will do the job". (Solow, 2000, p. 156)

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Appendix A: Data

- The original database used by Phillips for the period 1860-1947 can be found in Wulwick (1996) Table 1 at page 427. We use:
  - Wage inflation is calculated as *Money-Wage inflation* (column 2).
  - Unemployment rate is the *unemployment* (column 3).
- We employ the *A Millennium of macroeconomic data for the UK. The Bank of England's collection of historical macroeconomic and financial statistics* based on the work by Ryland et al. (2010).
  - Annual data for the period 1861-2015 are calculated as:
    * Wage inflation calculated as the log difference of *Composite Average Weekly Earnings series* taken from worksheet A.47.
    * Unemployment rate is the *Unemployment rate* taken from the worksheet A.50.
    * Inflation expectations are calculated using Ramsey et al. (2010)' s methodology on the consumer price index taken from worksheet A.47.
  - Quarterly data for the period 1965:1-2016:4 are calculated as:
    * Wage inflation calculated as the log difference of the *Spliced Average Weekly Earnings series, 1919-2015* taken from worksheet Q.1
    * Unemployment rate is the *Monthly unemployment rate* taken from worksheet M.1 and transformed into quarterly frequency.
    * Inflation expectations are the professional forecasters' expectation taken from worksheet Q.7.