How severe are the EBA macroeconomic scenarios for the Italian Economy? A joint probability approach

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How severe are the EBA macroeconomic scenarios for the Italian Economy? A joint probability approach

Manuel Bonucchi* [1] and Michele Catalano† [1,2]

[1]Prometeia Associazione, Piazza Trento e Trieste, 3, 40137 Bologna - Italy
[2]IIASA, Schlossplatz 1, Laxenburg, 2361, Austria

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Abstract

Measures of the severity of macroeconomic scenarios have been widely used in the literature, but a consistent methodology for their calculation has not been developed yet. Against this background, we provide a general method for calculating the joint probability of observing a macroeconomic scenario, which can be applied to various structural models. By doing so, we can attach probabilities to scenarios produced with multidimensional economic models to compare their severity and plausibility. We apply our methodology to the 2016 and 2018 EBA stress test scenarios and also provide reverse stress test applications. Our results show that for the Italian economy, the 2016 and 2018 EBA scenarios are unlikely, especially the 2016 one. The reverse stress tests allow us to identify the key variables that affect our probabilities.

JEL classification: C30, E30, E44, G10, G20, G28

Keywords: multiple simultaneous equation models; stress tests; financial instability; macroprudential; joint probability.

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†corresponding author: michele.catalano@prometeia.com

1 Introduction

Stress tests are procedures for assessing the potential impact on banks' balance sheets of certain risks, typically financial and macroeconomic shocks. Assessing such risks is a complex task requiring analyzing the interactions between the financial system and the real economy using macro-financial models to identify latent fragilities. These types of tools can also be used to perform reverse stress test analysis. Reverse stress tests, using the hierarchical structure of the models employed, are procedures that try to quantify how severe a scenario needs to be to bring a given target variable, i.e., the banks' capital ratio below or above a given threshold, for example, 6% Core Tier 1 ratio threshold. The severity of the scenario is usually assessed by looking at the marginal probability distribution of one variable, for example GDP, obtained by model simulations or calculating an aggregate index of a set of variables through a loss function aggregation. This approach is limited as it cannot capture the complexity of the macroeconomic channels affecting the banking system. In this perspective, our paper uses a probabilistic approach in order to obtain the unique expected value of the worst-case scenarios avoiding multiplicity or indeterminacy on exogenous macroeconomic shocks [(Grasserman et Al. 2015), (Pritsker 2013)].

In order to create sufficient hypothetical stress on financial institutions, scenarios should be severe enough but at the same time realistic. The scenario design process should be aided by analytical tools capable of generating unusual events, given the constraint of causal links between sectors of the economy. In other words, when designing stress test scenarios, an adequate level of severity must be ensured (i.e. with a significant impact on banks) but not an unlikely one (i.e. non-compatible with the economic structure as well as the history of a given country) (Breuer 2020).

A model-based scenario generation tool can weight empirical knowledge with the theoretical properties of the model to generate stress events even if it has not been experienced in the same combination in the data. When using an estimated structural macroeconomic model, for example, historical correlations play a minor role in defining a scenario compared to a statistical or pure time series model.

However, the literature still needs to clarify better how to quantitatively assess the severity and the plausibility of a scenario even if some work in literature addresses the issue quantitatively, as in (Breuer 2009).

Our paper provides a robust and unified framework for estimating i) the probability of observing a given scenario, and ii) the expected profile of exogenous variables for inverse stress testing exercises. Our approach can be applied to any type of dynamic structural model and for scenarios jointly defined by a set of variables over time. However, a number of caveats apply i) our quantitative results are general but model-dependent. This means that the probabilities of the scenarios, i.e., severity and plausibility are contingent on the model used. Moreover, our approach could be applied to any model for which a system of multipliers[1] and second moments are available. This also means that this

[1] The strategy of collecting multipliers approximates exactly the solution in the reduced form of the model.

method could be applied to any model for which the general PDF family can be inferred. For example, in the following, we will use a simultaneous macro-econometric equations model that allows us to get a sound multivariate density assumption.[2] (ii) At the current stage of the analysis, we provide a method of calculating the direct multidimensional scenario probability. We leave important issues, such as, non-standard error distributions of shocks that would lead to fat-tailed shock distributions for a future work.

(Borio et Al. 2014) argue that "[...] rather than being part of the solution, stress tests turned out to be part of the problem [...]". In particular, they identify two main shortcomings. First, the "model" is used to simulate financial distress. They are generally large linear autoregressive models with several dimensions and a high degree of complexity, such as the GVAR approach as in (Olli 2010). Despite its complexity, the GVAR lacks structural relationships, which prevents the computation of comprehensive probabilities (see Section 2). Moreover, the assumption of linearity represents a limit. Several approaches attempt to avoid the linearity assumption, as (Kanas and Philip 2018) who use a non-parametric quantile regression and find those macro variables non-linearly affect the upper tail distribution (90% and 95% quantiles) of non-performing loans (NPLs). They also find that default probabilities are highly affected by the macro variables selected.

We focus on the severity of the macroeconomic scenarios and do not discuss the specific effects of the macroeconomic shocks on the banking sector. (Barbieri et Al. 2021), using a Large Bayesian VAR model, which accounts for potential spillover between the macro-economy and the banking sector, propose a model-based approach to assess Italian banks' resiliency to adverse scenarios.

Our paper suggests a structural macro-econometric model-based perspective instead of an empirical one to exploit the information contained in the model. The reason for our choice is twofold. First, following this approach, there is a direct connection between the models used by policymakers that allow their reuse in stress-testing and judging scenario properties through joint probability. Second, the structural approach has a quantitative consequence that affects the joint probability of large-scale events such as macroeconomic scenarios involving many variables/sectors, as we will see shortly.

The second shortcoming of the existing models is the context in which the stress tests are run. The theoretical framework underlying macro stress testing allows one to assess macro variables' impact on banks' balance sheets. However, usually, the focus is on macroeconomic shocks and how they affect banks' solvency rather than on models that can capture the negative feedback loops between the banking sector and the rest of the economy. The methodology followed by EU regulators is to coordinate the design between the ESRB and the EBA by converting the macro risk identification into a set of shocks included in a suite of ECB models.[3] A multiplicity of models is used

[2]This assumption would be relaxed in the extensions of this paper.

[3]for a standard discussion see Henry, J., "Macro financial Scenarios for System-wide Stress Testing: Process and Challenges", in Quagliariello, M. (ed.), Europe's New Supervisory Toolkit: Data, Benchmarking and Stress Testing for Banks and their Regulators, Risk Books, London, 2015, and for a more up-to-date report on the debate, see Reforming bank stress testing in the EU: reflections in light of the EBA's discussion paper on the issue Javier Suarez

to calibrate shocks to variables such as foreign demand, consumer and business confidence, house prices, financial prices, and even bank balance sheet variables. These shocks are then used as inputs to Stress Test Elasticities (STEs), which are provided by national central banks and produce the final response of macroeconomic variables (real GDP, unemployment rate, and HICP) to the shocks generated. Stress Test elasticities are basically the elasticities of the main macroeconomic variables to different shocks and are mainly based on the forecast models of the national central banks of the European System of Central Banks.[4] It is worth to note that the STEs are similar in nature to the multipliers contained in our solution strategy and suggest how our approach can also be used in the context used by the ESRB, EBA and ECB regulators. Based on the scenarios provided, EBA suggests the use of a metric of severity to be applied in a framework including many variables. EBA chooses as an indicator the maximum decline in real GDP divided by the maximum historical decline in real GDP over a maximum of three years. In our framework, we suggest the use of joint probability as a comprehensive measure, including all the endogenous variables.

To overcome the aforementioned limitations, we propose a framework to associate a number of macroeconomic variables along a time period (a scenario) to a probability level (scalar). To this end, we exploit a reduced form of the Prometeia's Quarterly Macro-econometric Model (Tomasini et Al. 2018). The model includes the banking sector and its feedback to the other sectors of the economy. Importantly, our approach does not eliminate the structural relationships characterizing the original model.[5] The macro-economic model used here is only illustrative of the approach, as it could accept any type of dynamic model whether national or multinational. The approach is not bound by the literature on stress test scenarios but could be used by policymakers to obtain probabilities of alternative scenarios, including different monetary and fiscal policy assumptions. The approach not only provides scenario probabilities along with a large dimensional set of endogenous variables (unemployment rate, GDP, inflation rate, etc.) in order to communicate the probability and severity of the scenarios. The approach could also be used to obtain in a reverse engineering approach the conditional expected value of exogenous variables such as monetary policy rates or public investment growth rates consistent with the macroeconomic objectives. In this perspective, the plausibility of the policy variables associated with the target combinations could be judged by looking at the probability measure of the scenario.

Our structure works in two directions: from exogenous to endogenous variables to determine scenario probabilities and from endogenous to exogenous variables to perform reverse stress tests. In this way, we are able to combine the stressed profile of the endogenous variables with the most likely dynamics of the exogenous variables. The intuition of our approach is explained in detail in Section 2, is to track the evolution of the system and to collect all the available empirical moments, such as variances and covariances between variables in their contemporaneous, lagged, and leading versions. The goal is to maintain the structural properties of the model and allow an accurate and

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[4]for details, see Annex 3 of the "Macro financial Scenario for 2020 EU-wide banking sector stress test", ESRB, January 2020.

[5]In this class of models we could also consider Structural VAR models.

consistent estimation of the probability of a scenario, conditional on the joint multivariate distribution of the model's shocks. The joint use of modelled economy and estimated empirical moments would give us a full probabilistic model-based environment to assess the joint probability of a particular scenario.

We apply this methodology to assess the severity of the macroeconomic scenarios of the 2016 and 2018 EBA stress tests.

As shown in section 3, the joint probability approach delivers the possibility of a robust scenario severity determination by probability comparison. In our application, we obtain that the 2018 and 2016 scenarios have a non-zero joint probability mass, respectively 0.50% and 0.15%, suggesting a less severe profile for the 2018 EBA stress test exercise.[6] Our analysis resembles very closely the approaches found in literature as in [(Breuer 2018), (De Meo 2019)] looking for a formal and general approach for the design of macroeconomic stress testing. With respect to a similar approach, we prefer to stress the theoretical content embodied into a structural macroeconomic model. We will explain that this aspect will play a central role in probability determination. In fact, following a model-dependent approach, we will have benefits in terms of robust joint probability determination.

The methodology we propose, the probability determination and the reverse stress testing tasks, are useful not only in normal times but also during severe crises periods when it is fundamental to evaluate the conditions under which the banks have sufficient capital to continue to support economic activity. Thanks to our tool, the macroeconomic counterpart and the corresponding policy levers implied in the scenario can be easily determined. For example, the Bank of England 2021 stress test has evaluated heuristically adverse scenarios in the wake of the Covid-19 crisis for the UK, finding a cumulative GDP loss of 37% of 2019 UK GDP, UK residential and commercial property prices fall by around 33% and unemployment rises by 5.6 percentage points to peak at 11.9%.[7]

Our framework could strengthen the process of heuristic identification performed by Central Banks of the drivers or risk factors during stress test design.[8] Given a certain probability level (even really small) attached to a stress test scenario, central banks determine the minimum requirement for such drivers. We are able to determine them analytically in a straightforward way through the analytical reverse stress testing apparatus. Finally, it is worth to note that our framework is easily extendable to include internal bank models to be used with any general macroeconomic model to determine a unified joined probabilistic framework.

The rest of the paper is organized as follows. Section 2 discusses the methodology, while Section 3 discusses the empirical application. Section 4 concludes.

[6] The paper was written before the shock of the Coronavirus pandemic. We will update the model estimate in order to include the deep macroeconomic shocks of 2020 and 2021 in a companion paper in the future. A broad and straightforward consideration should be addressed: before updating the model, the probability of the shock would be close to zero, due to the severity of the shock, but after the structural and empirical model updates, the probability of such a macroeconomic scenario would be measurable due to our generalized framework.

[7] Financial Stability, Report December 2021.

[8] 2022 Stress Test Scenarios, February 2022. The Federal Reserve System, Washington.

2 The methodology

We begin by discussing the standard econometric techniques used in stress testing exercises. If the model used to analyze alternative scenarios is a time series model (VAR and its modifications), there is a limit to the number of observations and time periods that the model can include to yield a non-zero probability. The increase in the number of dimensions (N) and time periods (T) of the model leads to the joint probability for the random vector $Y_t \in R^N$, representing the simulated model's endogenous vector, converges to zero very quickly. The reason is the (un)desired Markov property of the model, according to which the joint probability of the simulated scenario, represented by the set of random vectors $Y_1, Y_2, \cdots, Y_t$, is given by the product of the conditional probabilities:

$$P(Y) = P(Y_1, Y_2, ..., Y_t) = P(Y_t|Y_{t-1}) \cdots P(Y_2|Y_1) \cdots P(Y_1) \tag{1}$$

We are interested in using probability measures as an assessment for plausibility and severity in the realm of macroeconomic models used to generate macroeconomic stress. Equation (1) suggests that a macro-economic model with null or poor structural properties would be useless in satisfying this goal. Alternatively, we could use a very simple statistical and structural model in order to exploit the non-Markovian property of the data generation process. The ingredients to build such a model are:

1. Use a structural model that provides a baseline and alternative stressed scenarios. In the following, we discuss the case of a linear model, but the methodology could be extended to nonlinear models as well.

2. Assume the joint distribution of the residuals.[9] We use a multivariate Gaussian distribution as a standard hypothesis.[10]

3. Perform a set of deterministic cumulative shocks on all the exogenous variables and then collect cumulative multipliers given the effect of the shocks on each endogenous variable.

4. For each given shock, compute Montecarlo simulations in order to get covariance matrices for the system for each time period of simulation and, given the multiplier matrices system (see below) and to determine the covariance matrix for the whole scenario.

5. Identify a stress scenario for each endogenous variable of the system, arbitrarily or methodologically based (Breuer 2020).

6. Define the tail of the marginal distribution of the endogenous variable representing the preferences of the macro-prudential policy maker.

[9] The assumption would affect the results as the analytical moments would depend on such a choice.

[10] The choice of the multivariate distribution for shocks is not independent of the model selected and the econometrical estimation strategy. For example, here, we are able to select multivariate Gaussian distribution given OLS estimation that would justify Gaussian error distributions.

7. Evaluate the probability of the scenario as the multivariate joint distribution of the endogenous variable scenario, given the tail specifications (step 6) and the covariance matrix (step 4).

Formally, let us define the deviation between the shocked scenario Y_s and the baseline scenario Y_0 as y_{st}. The same applies for the exogenous variable z_{st}. The endogenous variable $y_{it}, i = 1, \cdots, n$ and exogenous variable $z_{jt}, j = 1, \cdots, p$ are vertically stacked into $Y_t \in R^N, Z_t \in R^P$, respectively. The multiplier at time t is defined as $m_{ijt} = \frac{y_{it}}{z_{jt}}$ and $M_t \in R^{nxp}$, for $t = 1, \cdots, T$ is the respective vector. We build the compact model by collecting each M_t in a matrix as follows:

$$\boldsymbol{M} = \begin{bmatrix} M_t & 0 & \cdots & 0 \\ M_{t+1} & M_t & \cdots & 0 \\ \vdots & \ddots & \cdots & 0 \\ M_T & M_{T-1} & \cdots & M_t \end{bmatrix}.$$

Similarly, for the endogenous, exogenous and shock variables, respectively, we have:

$$\boldsymbol{Y} = \begin{bmatrix} Y_t \\ Y_{t+1} \\ \vdots \\ Y_T \end{bmatrix}, \boldsymbol{Z} = \begin{bmatrix} Z_t \\ Z_{t+1} \\ \vdots \\ Z_T \end{bmatrix}, \boldsymbol{E} = \begin{bmatrix} E_t \\ E_{t+1} \\ \vdots \\ E_T \end{bmatrix}. \tag{2}$$

Then, we can build the compact system of equations as follows:[11]

$$\boldsymbol{Y} = \boldsymbol{M}\boldsymbol{Z} + \boldsymbol{E}. \tag{6}$$

Assuming first and second moments of the errors and the exogenous variables $(\sigma^2_{e_i}, \sigma_{e_i e_j}, \sigma^2_{z_i}, \sigma_{z_i z_j})$ and their Gaussian multivariate distributions ($E_t \sim \mathcal{N}(0, \Sigma_{E,t})$, $Z_t \sim \mathcal{N}(0, \Sigma_{Z,t})$), we obtain:

[11]We can consider the resulting system as the simultaneous equations format (Canova 2007) of a dynamic simultaneous system. In fact, we can write a general macroeconomtric model as a recursive system of simultanoues equations:

$$\mathcal{A}_0 \mathcal{Y}_{t+1} = \mathcal{A}_1 \mathcal{Y}_t + \mathcal{B}\mathcal{Z}_t + \mathcal{E}_t. \tag{3}$$

where $\mathcal{A}_0$ includes simultanoues relationships among system variables. After simple algebra devoted to find the reduced form for the deviation from baseline representation of the above system, we get :

$$\mathcal{Y}_T = \Gamma_y^T \mathcal{Y}_t + \sum_{i=0}^{T} \Gamma_y^{i-1} [\Gamma_Z \mathcal{Z}_{t+i} + \Gamma_E \mathcal{E}_{t+i}]. \tag{4}$$

where $\Gamma_y = \mathcal{A}_0^{-1}\mathcal{A}_1$, $\Gamma_Z = \mathcal{A}_0^{-1}\mathcal{B}$ and $\Gamma_E = \mathcal{A}_0^{-1}$. We will record the shocks in the matrices M_i where not the only impact and delayed effects of exogenous shocks are included but also the dynamic effects included in $\Gamma^T \mathcal{Y}_t$. In this way we can define the multiplier

$$\frac{Y_{t+i}}{Z_t} = M_i. \tag{5}$$

$$\mathbf{\Xi} = \begin{bmatrix} \Xi_t & \Xi_{t,t+1} & \cdots & \Xi_{t,T} \\ \Xi_{t+1,t} & \Xi_{t+1,t+1} & \cdots & \Xi_{t+1,T} \\ \vdots & \ddots & \cdots & \vdots \\ \Xi_{T,t} & \Xi_{T,t+1} & \cdots & \Xi_T \end{bmatrix} \tag{7}$$

$$\Xi_t = \begin{bmatrix} \Sigma_{Z_t} & \Sigma_{Z_t,E_t} \\ \Sigma_{Z_t,E_t} & \Sigma_{E_t} \end{bmatrix}, \Sigma_Z = \begin{bmatrix} \sigma^2_{z_1} & \cdots & \sigma_{z_1 e_n} \\ \vdots & \ddots & \vdots \\ \sigma_{z_n z_1} & \cdots & \sigma^2_{z_n} \end{bmatrix}, \Sigma_E = \begin{bmatrix} \sigma^2_{e_1} & \cdots & \sigma_{e_1 e_n} \\ \vdots & \ddots & \vdots \\ \sigma_{e_n e_1} & \cdots & \sigma^2_{e_n} \end{bmatrix}. \tag{8}$$

With system (6) and covariance matrices (7) and (8) we arrive at the final multivariate joint distributions:

$$\boldsymbol{Y} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{M}'\mathbf{\Xi}\boldsymbol{M}). \tag{9}$$

Model 9 can replicate the original macro-econometric model and represents the general solution of the model as represented by a Montecarlo analysis. With model (9) we can measure the probability of the scenario $\boldsymbol{Y}_k$ as $P(\boldsymbol{Y}_k \in \bar{\boldsymbol{Y}})$. [12] The tail of the multivariate Gaussian distribution is compared with the policy maker's preference set, $\bar{\boldsymbol{Y}}$. For example, we can assume:

$$\bar{\boldsymbol{Y}}_a = \{y_{1t} > a_1, y_{2t} > a_2\}$$

$$\bar{\boldsymbol{Y}}_b = \{y_{1t} < a_1, y_{2t} < a_2\}$$

$$\bar{\boldsymbol{Y}}_c = \{y_{1t} < a_1, y_{2t} > a_2\}$$

i.e. several combinations of the informative set defined as: (i) a right tail ($>$), (ii) a left tail ($<$) and (iii) an internal interval ($<<$) set. In this way, it is possible to estimate a large combination of conditional probabilities to the preferences of the policy maker. For example, in stress test exercises, the probability along the GDP dimension is measured at the left tail ($\infty < GDP < a$) and that along with the unemployment rate (UR) at the right tail ($\infty > UR > a$), to then combine them jointly.

The main question is: why does not the probability converge to zero given the high dimension of the probabilistic system? The answer is simple: the system is not Markovian, that is, the joint probability has not a multiplicative form because the model is a structural one, and the whole history of the system is considered (i.e. the entire solution of the dynamic model is acknowledged). By doing so, we do not lose the data required by the structural relationships of the system, keeping the necessary information in the global covariance matrix.

[12] Actually, the joint-probability is a conditional probability given model elasticities and stochastic properties, formally: $P(\boldsymbol{Y}_k \in \bar{\boldsymbol{Y}}|\boldsymbol{M}, \boldsymbol{E})$. The multivariate Gaussian distribution is numerical evaluate as in Genz(1992,1993,2004,2009,2014). Packages in R https://cran.r-project.org/web/packages/mvtnorm/mvtnorm.pdf and python are available (scipy.stats.mvn).

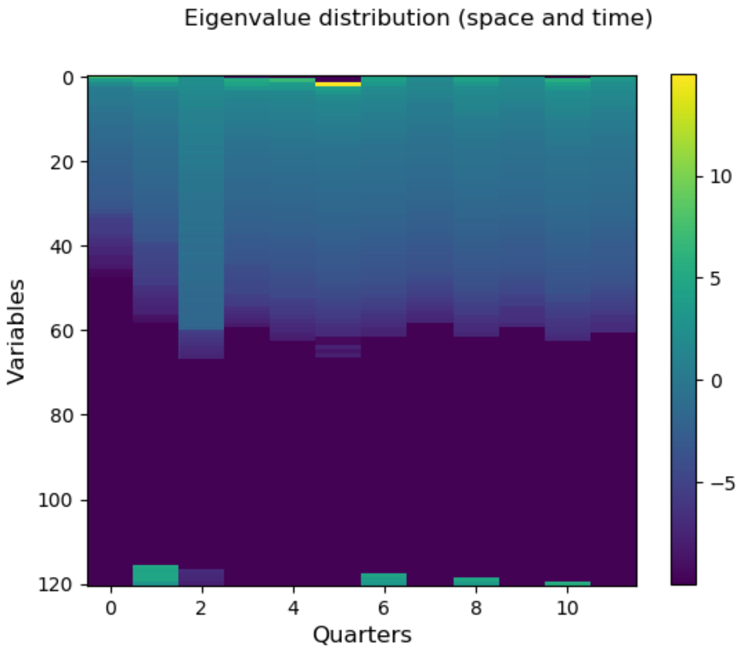


Figure 1: Matrix covariance matrix eigenvalue inspection (Log)
Source: Authors' own calculations

The presence of interdependence is equivalent to the presence in the system of a subset of common factors. Looking at the covariance matrix, we can understand why the probability does not converge to zero. The multivariate Gaussian distribution is:

$$P(x) = \int me^{-1/2x'\Sigma^{-1}x}dx = \int me^{-1/2\sum_i \frac{1}{\lambda_i}e_i^2 y_i}dy_i \tag{10}$$

where the right-hand side is obtained after an eigenvalue decomposition of the covariance matrix Σ and where λ_n are the eigenvalues and m is $\frac{1}{\sqrt{(2\pi)^k|\Sigma|}}$. If there are common factors, we should observe large eigenvalues of the covariance matrix that allow the probability to be non-zero. High values of the eigenvalues are a condition for a non-vanishing probability mass. In Figure 1, we show the distribution of eigenvalues by variables and periods. The structure of the model is generated when the first 40-60 variables trigger the rest of the system dynamics and are interrelated. Moreover, as the region of high eigenvalues expands, we can observe the role of the main components increasing over time. This means that the full explanatory power of macroeconomic variables reaches its maximum after a few quarters, increasing the importance of taking a multi-period assessment approach. Before proceeding with our applications in the present work, we must recognize that the task of assessing plausibility and severity, in our case, the joint probability measure has a number of limitations: first, the model used is always a strong approximation of strong interactions that can trigger complex events that are difficult to contextualize. Therefore, our work needs a number of extensions or applications to more complex, micro-founded, and dynamic models capable of triggering endogenous fluctuations in the economy and capable of originating fat-tailed distributions. Second, even with richer models, the likelihood remains a synthetic and optimistic reduction in uncertainty that must be used with caution. The risk is to provide statistical measures to events that cannot be measured because they are surrounded by unavoidable and incalculable deep uncertainty (Knight 1921) with dangerous consequences for economic policy.

Finally, our framework allows us to determine reverse stress testing analytically. We can solve for the exogenous variables vector $\boldsymbol{Z}$ from system 6 to get:

$$E(\boldsymbol{Z}_k|\boldsymbol{Y}_k) = (\boldsymbol{M'M})^{-1}\boldsymbol{M'Y}_k. \tag{11}$$

System (11) is the expected conditional exogenous profile given $\boldsymbol{Y}_k$, i.e. $E(\boldsymbol{Z}_k|\boldsymbol{Y}_k)$. It allows identifying the most likely path of the exogenous variable. If the original macro-econometric model is well identified, the uniqueness of the solution for the system (11) is ensured.

Given the properties of the general model (6) the inversion is possible and the uniqueness of the solution is always satisfied. The solution in system (11) is equivalent to the OLS or Maximum Likelihood methods for the multivariate regression model. The $\boldsymbol{Z}$ are the same as the linear regression coefficients and the $\boldsymbol{M}$ are the same for the $\boldsymbol{X}$ matrix observations. Therefore, under the easily satisfied condition of linear independence of exogenous shocks, the invertibility conditions are satisfied and the solution is unique[13]. The solution $\boldsymbol{Z}$ is the most likely solution of a set of potential solutions. As we use a linear model (corresponding to a linear approximation) this expected likelihood is maximum in a unique point (the whole scenario $\boldsymbol{Z}$).[14]

In this section, we provide a general model to calculate the probability of a macroeconomic scenario based on the properties of the macro-econometric model. The assumptions are simple but still too general. One of the extensions that are currently planned to be implemented will be the determination of heterogeneous multipliers depending on the phases of the business cycle in which is possible to perform stress tests. In section 4 we present the model that allows us to evaluate the probability of scenarios conditioned to the baseline, i.e. to consider a skewed distribution of shocks based on cyclical conditions and to avoid the assumption of homogeneous multipliers. Our preliminary inspection supports the evidence of skewed distributions of exogenous shocks.

3 Applications

This section shows an application of the methodology described in Section 2. For our analysis, we rely on Prometeia's Quarterly Macroeconometric Model (Tomasini et Al. 2018). The model is a large-scale estimated vector error correction equation system for the Italian economy. It includes several sectors,

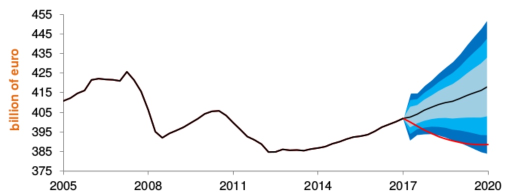
[13]However, in the case of low probability scenarios, in a reverse-engineering the confidence intervals around the expected value would be larger, signalling a decrease in general meaningfulness of such a unique solution.

[14]Furthermore, in a nonlinear model, a bimodal distribution with two expected values should be theoretically possible but always numerically rank-able.

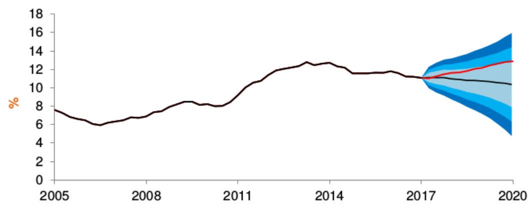
including firms, households and the credit sector at the macroeconomic level. As a practical application of our approach, we analyze a set of endogenous variables (GDP, unemployment rate and the BTP-Bund 10y spread) resulting from the inclusion of EBA shocks in the Prometeia's macro-econometric model as shown in Figure 2. In particular, we show the historical series and the forecasted fan chart resulting from our stochastic simulations.

Then, we compare the fan chart with the 2018 EBA scenario. We can observe a severe fall in GDP and an increase, albeit less severe, in the unemployment rate together with an increase in the BTP-Bund spread. If we calculate marginal probabilities, we get a value of 28.7% for the BTP-Bund spread, 35.2% for the unemployment rate and 6.8% for GDP. The corresponding probabilities for the 2016 EBA scenario the are 22.6%, 30.6% and 3.2%, respectively. However, if we include housing prices, we find a marginal probability of the variable close to zero, suggesting that its profile is implausible. It is just a visual inspection that cannot provide a systematic probabilistic assessment of the overall scenario design. To do this, we apply the methodology in the following as in section 2.

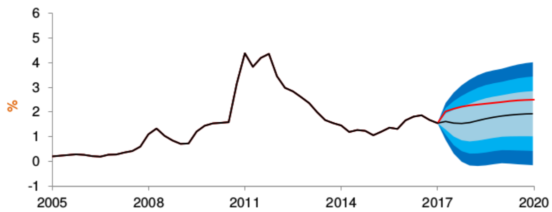
Figure 2: EBA 2018 endogenous variables



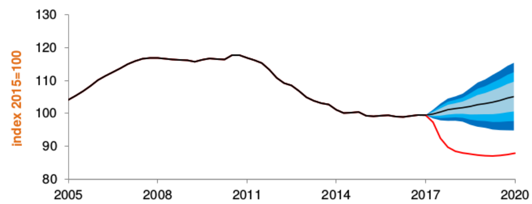
(a) GDP, Probability 3.1% (% dev. from baseline)



(b) Unemployment Rate Probability 30.3% (% dev. from baseline)



(c) SPREAD BTP-Bund, Probability 22.3% (abs dev. from baseline)



(d) Dwellings Price, Probability 0%: (% dev. from baseline)

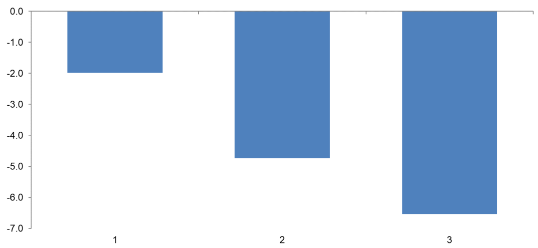
Source: Authors' own calculations on EBA data

We use the baseline forecast of Prometeia Associazione, and we perform a one standard deviation shock[15] on all the exogenous variables. Then, we collect the multipliers and the distribution of the estimated errors and exogenous variables. To perform the probability assessment, we use the reduced form (6) of the model, thereby preserving the structural relationships of the original model. This property is also achieved by calculating ex-post the dynamic multipliers.

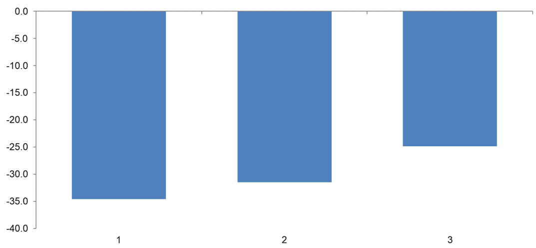
We obtain the joint distribution for the endogenous variables with equation (9). The dimension of the reduced system is given by $N = 121$ endogenous variables, including the main variables of the macro-econometric model (GDP, unemployment rate, inflation, consumption, aggregate wealth, etc.) and a set of bank interest rates and credit indicators. The number of exogenous variables is $P = 22$, including the exogenous shocks to the main endogenous and exogenous variables, represented in its vector error correction form model (VECM). The model is simulated for 12 quarters, and therefore, the global matrix of multipliers (M) has a dimension of 1452×276, while the covariance matrix $M\Sigma M'$ of 1452×1452.

Figure 3: EBA 2018 endogenous variables

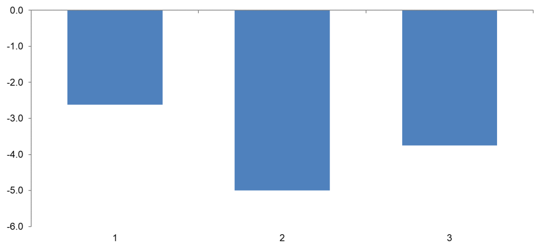
(a) GDP (% dev. from baseline)



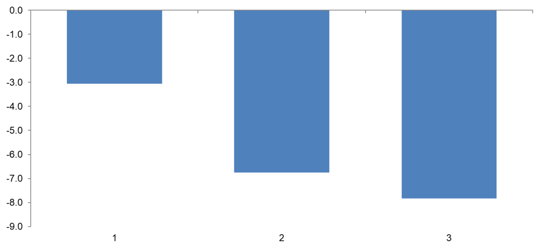
(b) FTSE MIB (% dev. from baseline)



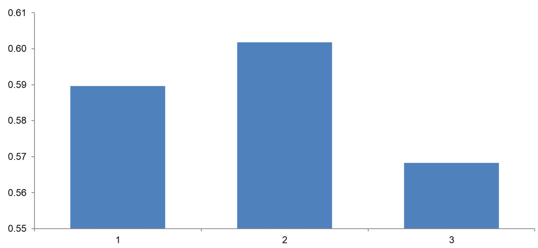
(c) US GDP (% dev. from baseline)



(d) EU GDP (% dev. from baseline)



(e) SPREAD BTP-Bund (abs dev. from baseline)



Source: Authors' own calculations on EBA data

[15] The shock size is not relevant as the macroeconomic model is linear.

In order to calculate the probability of the scenario, we collect data on the exogenous variables included in the EBA scenarios.

As a first step, we apply the probability to both the 2018 and 2016 EBA scenarios. We manipulate the data considering quarterly frequencies, and we make a hypothesis on all the variables not explicitly indicated by the EBA, however, that is necessary for the simulation of our model. We use the EBA variables given the profile conditioning the model to simulate the rest of the endogenous model.

Accordingly to the Prometeia's model, the profile of the main endogenous and exogenous variables for the 2018 EBA scenario are shown in Figure 3. A negative and persistent shock triggers the adverse shock on the Italian economy to the Euro area GDP (excluding Italy) (-7.8%), a less severe slowdown of the US economy (-3.8%) and a deterioration of asset prices (initially -30% but slowly recovering).

As for the whole set of endogenous variables, we initially set a preference vector for all variables on the left tail.

Formally it translates in $P(Y < Y_{EBA})$ where Y_{EBA} is a vector containing the EBA scenario opportunely managed. The probability is 0.201% and 0.069% for the 2018 and 2016 EBA scenarios, respectively, indicating that the latter is the most severe from a quantitative point of view.

In order to get a more precise probability for the scenarios, we select a subset of variables as indicated in Figure 4. In particular, we consider a vector with different preferences. We have an adverse preference on the left tail for the oil price, exchange rates and GDP. For example, we assume a loss if GDP is comprised between $-\infty$ and the EBA deviation from the baseline.[16] For the unemployment rate, and the BTP-Bund 10y spread we have an adverse preference for the right tail. In this case, the 2018 EBA scenario yields a probability of 0.503%, while the 2016 EBA scenario has a probability of 0.148%. Although the 2016 EBA scenario is still the most severe, the difference between the two scenarios is small.

[16] The case for oil price shows a possible subjectivity in preferences as recessions have been experienced for Italy with a negative shock in terms of oil price and not only in case of positive shocks. This affects the choice of the side (tail) of distribution.

Figure 4: Scenario preference configuration

Variable	Tail
Oil price	rhs
Exhange Rate €/$	rhs
GER Bund Rate	rhs
Ermergin Gdp	lhs
US Gdp	lhs
Euro Area Gdp	lhs
FTSE MIB	lhs
Italy GDP	lhs
Unemployment rate	rhs
Spread Btp-Bund 10y rhs	rhs
Euribor 3M	rhs

Source: Authors' own calculations

We perform a set of robustness checks. First of all, we change the severity of the scenario using a common factor for the deviations from the baseline. We use a simple ratio of the whole set of variables using respectively severity ratios: 0.5 and 4. We get a probability for EBA 2018 scenario as respectively of 1.140 and 0.054, signalling that the estimation procedure is robust and slightly non-uniform in moving severity. Moreover, we change the tail in oil-price and EU-US nominal exchange rate, leaving unchanged the remaining preference set. When setting both left-hands of the considered shocks, we observe an increase in probability for the EBA 2018 scenario of 0.85 per cent, meaning that the right-hand side is less likely for historic reasons.[17] We can also detect which variable causes a decrease in probabilities. When we include the price of dwellings (Figure 2d) we obtain a joint probability of 0%, as expected from preliminary inspection of the marginal distribution. Therefore, dwelling prices are the most implausible endogenous variables, as it is assumed to deviate from its baseline value to -20% persistently. From an empirical point of view, Italy has not experienced such shock in a 3-year time period. To better appreciate the scenario's plausibility, we perform a second application: the reverse stress test. Looking at the most-likely (maximum-likelihood) exogenous profile it is important per-se but it is also useful as a double check to look at which implied exogenous is at odds with a scenario and which channel to adjust to increasing the scenario's plausibility.

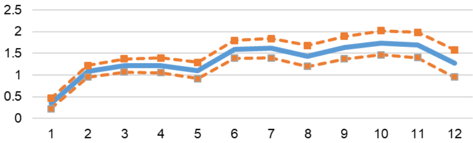
We use the 2018 EBA endogenous set of variables (Figure 3) and fit it to our model specification. We use system (11) to obtain the exogenous profile with the higher conditional expected value consistent with our model. The adverse additional shock profile of the exogenous variables is shown in Figure (5). They consist in an initial adverse shock to the oil price, the Euro area GDP and the US economy. The unemployment rate starts growing, while the ECB increases

[17]Setting just oil price to the left-hand, moves probability to 0.225 per cent, showing that a stronger shock in oil price reduces probability.

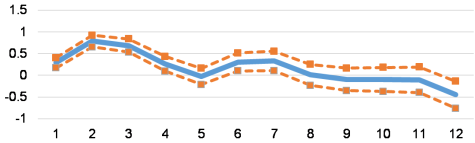
its policy rate. The BTP-Bund spread increases over the entire period, while neither for GDP nor for equity prices an additional shock is required. Overall, these results allow us to quantify the additional shocks needed to replicate the EBA scenario given the Prometeia macroeconometric model. They also help to identify how the EBA scenario can be improved to make it more plausible. From a political economy perspective, the approach could be useful in reading the associated movements of exogenous policy controls like policy rates. For example, the movements in the world equilibrium natural rate would suggest a response of monetary authorities around the world in cutting policy rates to sustain recovery given the stressed scenario profile as the EU-GDP would recover. It is worth to note that even if the model is national, the implied structural relationships are able to generate sound reactions. Finally, we recall that this application is only exemplificative and that using a more general macroeconomic model like a multi-country simultaneous model would yield useful outcomes.

Figure 5: Distribution of the sub-periods by quartile of selected macroeconomic indicators % dev from baseline

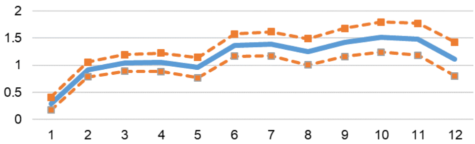
(a) 10y BTP-Bund Spread



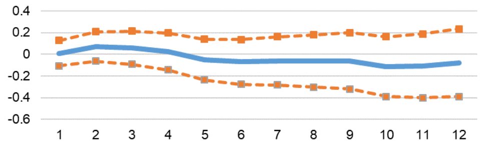
(b) BUND 10y rate



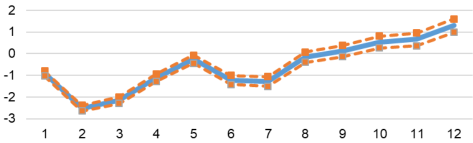
(c) Unemployment rate



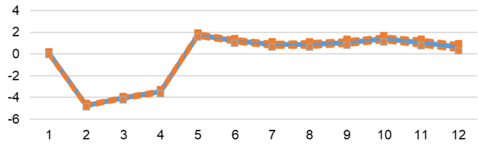
(d) Employment rate



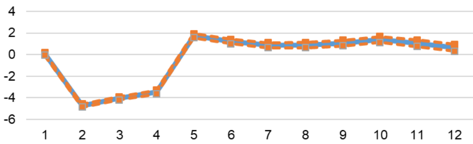
(e) Public expenditure



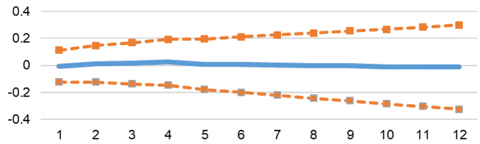
(f) Compensation of employees per capita



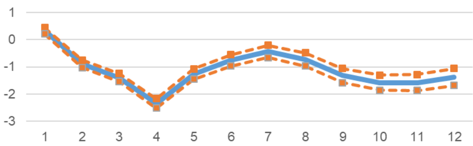
(g) Index of prices of residential dwellings



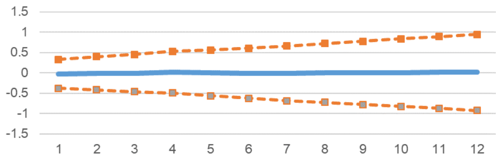
(h) FTSE-MIB



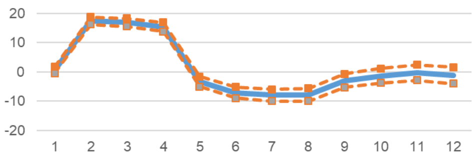
(i) EU GDP



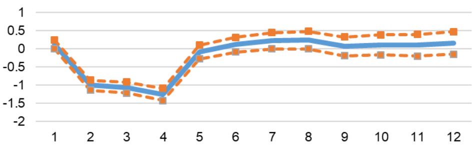
(j) GDP Italy



(k) Petroleum Brent: $ per barrel



(l) US GDP



Source: Prometeia Calculations based on EBA data

4 Extensions: Conditioning probability to the business cycle phase

The method presented in the previous section assumes a constant structure of the joint probability distribution. In some sense, it is unsatisfactory as it provides the same probability measure for a stress test scenario. It would be preferable to perform stress testing exercises conditional on the actual macroeconomic condition, with a more flexible model taking into account the current macroeconomic conditions. A stress test implemented during a recession could have a probability measure much higher than if performed during expansionary phases. We could consider several ways to complicate such a model and in this section, we show the simplest extension: a two-state multivariate distribution. The goal is to measure the joint probability of a macroeconomic scenario considering the positive and negative phases of the business cycle.

As the simplest example for an extension of the model in section 2 we provide the following two-state system:

$$\boldsymbol{Y} = \begin{cases} \boldsymbol{M}^{-}\boldsymbol{Z} + \boldsymbol{E}^{-} & \text{with probability } p \text{ (negative output gap)} \\ \boldsymbol{M}^{+}\boldsymbol{Z} + \boldsymbol{E}^{+} & \text{with probability } (1-p) \text{ (positive output gap)} \end{cases}$$

This model allows performing conditional stress tests into two possible states depending on the phase of the business cycle. If the output gap is negative (positive), we can assume a skewed distribution of shocks. The same hypothesis holds for the several endogenous variables with the heavier tail, depending on the particular variable of the model. At the same time we could expect a non-zero multiplier for exogenous shocks impacting during recessionary business cycle phases. Formally, it translates into $E(\boldsymbol{M}^{+}\boldsymbol{Z}) = \boldsymbol{\mu}_{z+} < 0$, $E(\boldsymbol{E})^{+} = \boldsymbol{\mu}_{e+} < 0$ and $E(\boldsymbol{M}^{-}\boldsymbol{Z}) = \boldsymbol{\mu}_{z-} > 0$, $E(\boldsymbol{E})^{-} = \boldsymbol{\mu}_{e-} > 0$ and a log-normal distribution for $Y^{-} \sim log(\mathcal{N})(\boldsymbol{\mu}_{z-} + \boldsymbol{\mu}_{e-}, \Sigma_{E,t})$ and $Y^{+} \sim log(\mathcal{N})(\boldsymbol{\mu}_{z+} + \boldsymbol{\mu}_{e+}, \Sigma_{E,t})$. Figure 6 depicts the whole picture of the model for positive and negative phases and the use of the prior distribution for the output-gap, weighting the two multiplier sub-systems.

As a first numerical example, we consider the hypothesis in which the conditional probability is obtained by keeping the estimated parameters of the model and the matrix of the multipliers, $\boldsymbol{M}^{-} = \boldsymbol{M}^{+} = \boldsymbol{M}$, constant between the two states.

What we assume is the heterogeneity of the shocks obtained bootstrapping the sample distribution of shocks in the two states, conditioned to the positive or negative output-gap, i.e., considering the distributional moments of E^{+} and E^{-}.

Figure 6: Conditional Probability distribution

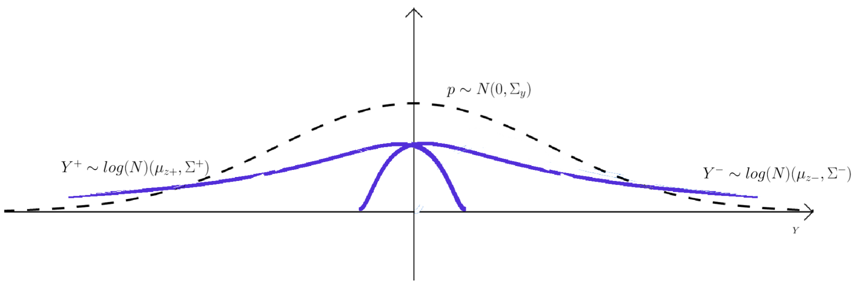
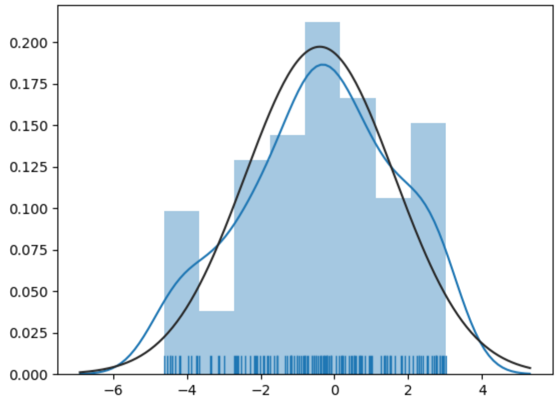


Figure 7: Output Gap distribution, Empirical (bars), Kernel (blue) and Gaussian (black)



Source: Authors' own calculations

First of all, we estimate the kernel distribution as in figure 7 in order to obtain the probability of a positive state of the macroeconomic cycle, namely an accelerating positive output gap for the Italian economy. Then, we can calculate the conditional probability of a particular scenario given the probabilities of the negative and positive states, $P(Y^+)$ and $P(Y^-)$.

We can observe in Figure 8 that the simulated endogenous distributions has a skewed shape and that it approximates a log-normal bi-variate PDF. On top of this distribution, we can calculate a transformation by shifting and logging the original empirical distribution considered for each variable and time period.[18]

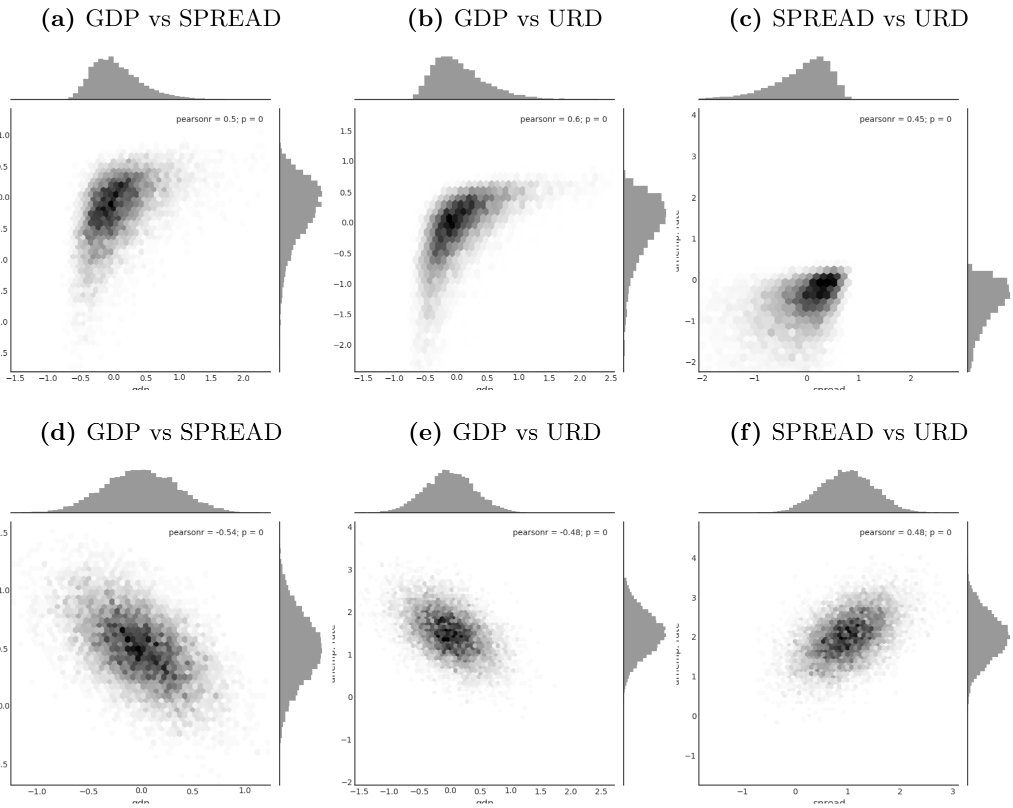
In this way, we can lead back to a multivariate normal distribution specific for each of the two states.

Once we have obtained our multivariate normal distribution, we can apply the same procedure presented in the previous sections. We apply the EBA 2018 scenario as in section 2 for each of the two states. In this way, we obtain a scenario probability in the positive output-gap state of 0.003 per cent and for the negative state of 8.4. This means that the 2018 EBA scenario is much more likely during recessionary states of the economy, as expected.

Finally, applying the conditional probability calculation recursively during the sampling period, we can obtain a dynamic probability of the 2018 scenario conditioned to the dynamics of output-gap, as shown in Figure 9. As expected, at the beginning of the Great Recession and Covid-19 crisis, the likelihood of the scenario designed for the 2018 stress-test exercise has increased.

[18]We fit a shifted log normal distribution per each variable belonging to the state of the economy, ie. $y^S_{it} \sim log\mathcal{N}(0, \sigma^2_{S,i,t}, \delta_{S,i,t}), S = +, -, i \in N, t \in T$, where $\delta_{S,i,t}$ is the location parameter.

Figure 8: Pair of bivariate simulated distributions. Positive state. Original (1st row) and transformed (2nd row) data. Transformation : $log(X) + \delta$

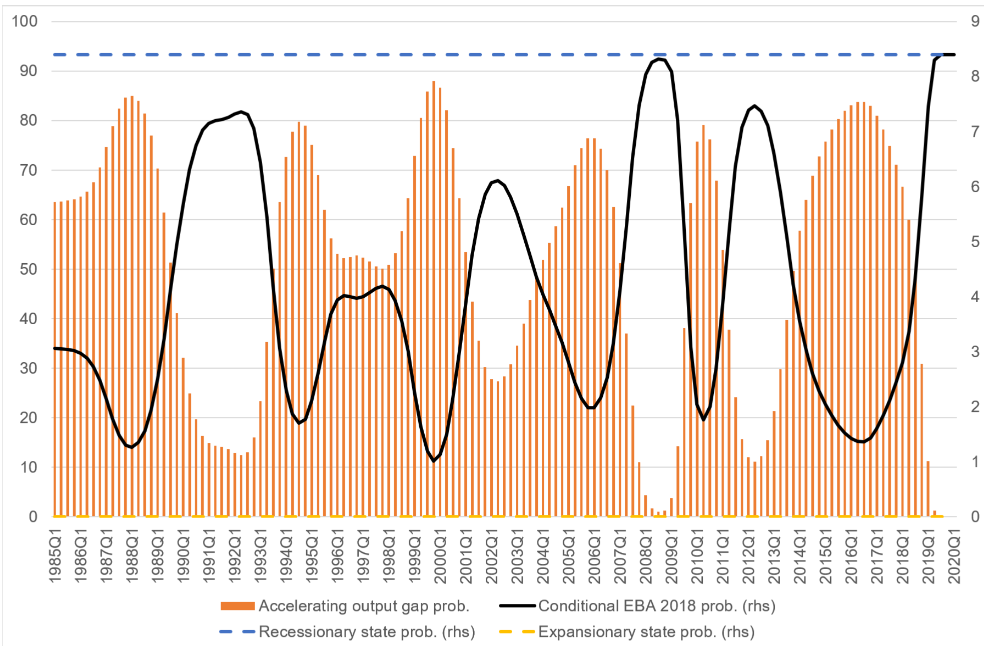


(a) GDP vs SPREAD **(b)** GDP vs URD **(c)** SPREAD vs URD

(d) GDP vs SPREAD **(e)** GDP vs URD **(f)** SPREAD vs URD

Source: Authors' own calculations

Figure 9: 2018 EBA Scenario, conditional probabilities over time



Source: Authors' own calculations

5 Conclusions

This article examines the severity of the EBA shocks concerning the macroeconomic scenario considered in past stress tests for Italian banks. To the best of our knowledge, no standard analytical method exists in the literature to measure the plausibility and severity of stress testing scenarios. We provide a very simple analytical model, which allows a precise computation of the probability of the scenario, a remarkable result. Even if the approach is model-dependent, it is suitable to be used with any dynamic model by directly exploiting its algebraic properties and with a simple and easily applicable procedure in any macroeconomic context.

This approach, if performed with reverse stress tests, is suitable for detecting exogenous variables that are not plausible. In this framework, the macroeconomic scenario designed by the EBA in 2016 is more severe than 2018 one. Compared to the latter, our methodology allows us to obtain an additional profile of exogenous variables from Prometeia's macro-econometric model, consistent with the profile of endogenous variables of the EBA.

This is achieved by inverting the fully reduced form, and considering opportunely the stochastic properties it is possible to get the maximum likelihood estimation of the exogenous variables associated with an observed endogenous variable profile. In this context, a much more severe shock on the BTP-Bund spread, unemployment rate, the Euro area, and US GDP is necessary to induce the EBA scenario, indicating that the latter potentially includes some implausible variables.

This means that some channels of stress diffusion may be underestimated, such as disposable income through the unemployment rate or financial market stress induced by rising interest rates relative to the values in the baseline scenario. These results should be taken with caution because implausible variables may regain plausibility by changing the base model used. This confirms the practical utility of the approach by allowing it to be applied with different models and then choosing the results based on the ranking of their empirical explanatory ability. Moreover, the paper presents only a first sound attempt to perform the approach. Here, we provide the first extension, including a switching regime probability estimation to the business cycle to get a more flexible estimation of scenario probability. Several possible extensions could be addressed in future research, like non-Gaussian error distribution to reach probability estimation in fat-tail regimes or extend to a second-order approximation of the model when using nonlinear macroeconomic models.

Finally, in our view, the approach is useful in application outside the stress-test framework, such as macroeconomic scenario assessment, particularly when assessing the implied probability of a given scenario and induced exogenous policy variables given a targeted variable profile. In this context, a given profile of targeted GDP growth rates could be assessed in its probability and hence implied monetary policy rates, for example, could be estimated consistently.

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