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ESSAYS ON ICT DIFFUSION

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ABSTRACT

This thesis introduces a class of epidemic diffusion models specifically tailored to the description and analysis of ICT technologies, by defining a dynamic potential market that depends on the network size of the number of individuals who have already adopted. Compared to traditional "stand alone" products, ICT technologies have peculiar characteristics and different adoption behaviours that can be explained by network effects and externalities. After an overview of the state of the art of the literature on the diffusion of innovations and on networks (chapter 1), the theoretical work is presented (chapter 2). Here, we carry out a systematic functional study leading to the construction of a class of new models, to their parameterization and analysis in comparative statics, and finally simulation. The basic Bass model, which assumes a fixed potential market, is taken as a reference for comparisons, beside being the backbone of our class of models. From the simulations, it emerges that our class of models is able to describe the network effects (and externalities) and their role in shaping the diffusion of such technologies. In chapter 3, we test the capability of this class of models to explain empirically, with market data, the historical ICT diffusion paths, trying to derive useful implications for the policy-maker (for example, in the realm of contemporary digital agendas). This chapter features the NLS econometric estimation of the previous models, taking as a reference the real diffusion paths of broadband technologies in selected EU countries: in particular, we focus on the ITU time series of fixed broadband subscriptions of the "five big" European countries. The econometric estimates confirm that our class of models provides an original analytical and empirical instrument for capturing and stylizing the network phenomena that dominate the diffusion paths of the main telecommunications and media markets, such as

fixed broadband. As such, it also enables a series of future policy evaluation exercises.

INTRODUCTION

The study of the diffusion of innovations appears to be of crucial importance, since innovative products and technologies have become part of the everyday life of people, society and economy.

The literature about innovation diffusion is vast, and it spills over many conventional disciplinary boundaries.

The diffusion of innovation is a phenomenon of essentially social nature. However, it has an interdisciplinary nature that combines and integrates concepts and theories from different disciplines such as mathematics, biology, statistics, economics and marketing.

The formal representation of aggregated diffusion models has historically been borrowed from biology epidemic models, holding the hypothesis that an innovation spreads in a social system through the mechanism of communication, like an infection infects people.

The most famous evolution of the logistics equation is the Bass model, introduced in the field of marketing and then become a reference point for research activities on diffusion processes due to its parsimony and its remarkable predictive capacity.

The main objective of a diffusion model is to describe the pattern of spread of an innovation among potential adopters in terms of a mathematical function of time. The cumulative curve of adoption of an innovation over time typically features a sigmoid shape, more or less symmetrical or regular.

As a matter of fact, the bulk of the diffusion of innovations literature has analyzed “stand alone” products or services, for which the sigmoid pattern provided a convincing analogy. However, the digital age and its continuous transformations relentlessly provide many new instances of diffusion cases, and in these cases adaptations of theoretical frameworks used in the earlier techno-economic paradigms are in need. In fact, many types of Information and Communication Technologies (henceforth ICT) may describe developments that significantly delay or anticipate the growth phase of the typical ‘stand-alone’ S-curve, because of the activation of events and behaviours specific to these digital industries. Many products or services of this market segment contain features based on the fact that the utility of these products cannot be regarded as a constant value. In the case of ICT, the utility is a dependable variable resulting in a distinct diffusion behaviour which can be explained by a concept called network effect. A technology exhibits network effects (or network externalities), for the individual consumer, when the value of the product depends on the number of adopters who use the same product, or on the number of the compatible complementary goods available (Liebowitz and Margolis, 1994). The utility of technologies showing network effects rises with this value leading to an interdependency of users. This interdependency is based on two properties of technologies with network externalities called “chilling effect” and “bandwagon effect”.

Now, our goal is to capture network effects and externals using macro - diffusive models. Even the most widely used and the most popular model in the literature, the Bass model, despite being able to measure such effects, cannot completely capture the phenomenon because of the considerable limitations of its basic assumptions.

This thesis introduces a class of diffusion models specifically tailored to the description and analysis of ICT by setting a dynamic market potential that depends on the network size of the number of individuals who have already adopted.

In the first chapter of this thesis we conduct an extensive review of the literature on innovations diffusion and network externalities, in particular referring to ICT innovations.

In the second chapter, we build a class of diffusion models that capture network effects, by extending the standard structure of the mixed influence model (in particular the Bass (1969)-type model), through the change from static to dynamic of the parameter measuring the market potential. In addition, two more models are also introduced that seek to capture the effect of network externalities by introducing a time-dependent parameter. We carry out the simulations for each model and then we compare our class of models with the standard Bass.

In the third chapter, we focus on the determination of comparisons between different countries and the analysis of the diffusive paths of selected ICT technologies. The determination of the comparisons between some European countries has been possible thanks to some indicators that allowed the assessment of any digital divide. The morphological analysis of diffusion paths then allowed to compare the different characteristics of the countries considered and to verify the adequacy of the class of models we have built. This analysis focuses on curve fitting and on empirical estimation with real market data. In addition, we describe the estimation methods useful for aggregate models describing the strengths and weaknesses of each one. In particular, we deepen the "Nonlinear Least Square" which is the most suitable method to estimate the parameters of the Bass model and the built models.

Finally, we expose the conclusions, the positive aspects and the difficulties encountered.

CHAPTER 1

LITERATURE REVIEW

1.1 INFORMATION COMMUNICATION TECHNOLOGY

1.1.1 Information Communication Technology (ICT) and the constituent components

This paragraph provides the basics knowledge on ICT, to understand what we mean when we talk about ICT and then to analyze what are the constituent elements of ICT, namely e-access, infrastructure and e-content.

In the last few years we talk of ICT every day. If, on the one hand, some of the essential elements of ICT can be identified, on the other hand, it is not easy to provide a unique definition of ICT, since we are talking about "fluid fields" and sectors where there is no general and shared definition.

Many times, rather than a definition of ICT, it is preferred to define the areas where ICT operates. For example, Dutch National Institute for Statistics (CBS) draws a distinction between ICT operating environments: a first field linked to more industrial aspects, and a second field linked to the services sector. This definition follows the most general one performed by Organization for Economic Cooperation and Development (OECD), which operates a classification related to the sectors where ICT operates, that is:

- the manufacturing sector, for example the manufacture of office machines or computers and computer systems or the manufacture of radio receivers for the recording and reproduction of sound or images and related products;

- the sector of goods referring services, such as those relating distribution and wholesale of telecommunications equipment, electrical apparatus, computers, etc.;
- the sector referring intangible services, such as radio and telecommunications activities, software and hardware consultancy, database activities, telematics or robotic services, etc.;
- the sector linked to the content industry, such as publishing books, sound media, movie projections, etc.;

Although this distinction appears limited, since it is essentially linked to industrial production, in recent years the aspect of the use of ICT as a tool to produce information, new knowledge and new content has acquired more strategic importance.

Always in the attempt to provide a definition, other agents, both institutional and non-governmental, have adopted different methodologies, ranging from financial related approaches to new economy sectors. These attempts, rather than providing epistemological clarifications, were in fact purely technical operations designed to provide methodological bases for their respective operating environments. If we want clarify some basic concepts, we can say that ICT includes different components, such as computer technology, telecommunications, electronics and media. Examples are PCs, Internet, mobile telephony, cable TV, electronic payment systems, etc. In this sense, ICT has become increasingly tied to the Information Technology (IT) component with Communication Technology (CT). CT and IT have evolved over the years to come to the digital shapes that have progressively shaded their boundaries. In particular, with the advent of Internet technologies, information has lost that characteristic of processing on "stand alone" machines to become a shared component with other machines in a network (both the LAN and the global network are represented by Internet).

The constituent elements of ICT are:

- electronic infrastructures
- electronic content
- electronic access.

Electronic infrastructure is the hardware backbone of ICT. It is the physical part of the system that is made up of servers, PCs or mobile phones and also from infrastructure such as cables, antennas, fiber optics, etc.

Electronic content, on the other hand, is represented by information produced, stored, distributed or received through websites, electronic publications or databases. Each of these technologies has its own typology of content and, hence, a possible user target. For example, mobile telephony has its main objective in voice communication, also recalling the additional information possibilities offered by operators via SMS as well as through the network of videophone.

Electronic access is the ability provided to each organization, company, entity and individuals to access and benefit from the opportunities that new technologies can offer. It is obvious that the development of each technology depends mainly on its ability to access it. Van der Meer and Van Winden (2003) attribute access to two dimensions: ownership and management; knowledge and skills in the use of technology. The use of the Internet, for example, can provide many benefits, both in the social and economic sphere. Then access to the network would allow access to a large number of databases and information. It is precisely in this sense that access to ICT, and in particular to the network, is a cause for debate, especially for the role that the state should play in promoting access. Finally, very interesting is the aspect of interaction between the three components, which seem to strengthen each other. For example, in many cases there is a strong link between access and infrastructure, and this is particularly true in network systems, such as the

Internet and telephony. In this regard, Shapiro and Varian (1999) show that the utility of network technology is a quadratic function of the number of users.

1.1.2 BROADBAND TECHNOLOGY

The innovative services generated by Internet bring great social and economic value, in terms of quality of life and productivity. Internet potentially spreads knowledge and culture to all, offering essential services and new opportunities in areas such as work, education, health, social relations and relations with institutions. The evolution of telecommunications networks towards increasingly greater capacity, such as broadband and ultra-broadband, is the necessary condition for the development and diffusion of innovative services, with increasing levels of integration, multimedia and interactivity. In fact, telecommunications networks represent the basic infrastructure to allow the exchange of information and content between all the subjects involved in the Information Society: citizens, companies, institutions. The impact of the availability of advanced infrastructures on innovative processes can be outlined in different ways for the different actors of the Information Society:

- for citizens (individuals and households), the development of communication systems, which multiply the exchange and flow of content and information, generally increases the predisposition for the adoption of innovative technologies and services, expanding the sphere of possibilities and opportunities;
- for companies, the value is twofold, in terms both of process innovation and of product. On the one hand, advanced infrastructures allow better interaction between the various company structures (even if they are distributed locally) and between these and the external environment (customers, suppliers, partners), with direct repercussions on effectiveness and efficiency of business processes. Furthermore, the availability of a new "intangible" distribution channel (telecommunications networks) makes it

possible to expand the reference territorial market, creating new opportunities for development. On the other hand, through the new telecommunications networks, it is possible to create new products or services, which can represent an important factor for companies to differentiate and diversify their activities, thus intervening directly on product innovation;

- for institutions, services enabled by advanced infrastructures have a direct impact on internal intra- and inter-administrative processes, as well as on the quality of relations with citizens and businesses. Moreover, the triggering of an innovative process in the Institutions, based on network technologies, can activate a virtuous circle for the affirmation of innovative products and services, destined not only to the public sector, but susceptible of diffusion to a wider number of users.

The term broadband defines a set of technologies that allow to increase the speed of communication in general, and access to the Internet in particular, exploiting infrastructures and / or innovative technologies compared to traditional ones (enabled by analogical or digital telephone lines) and offering the opportunity to use high interactivity services.

The European Union defines broadband according to a non-technical definition, but at a performance level, that is as a set of networks and services that allow interactivity at a comfortable speed for the user. Although there is no precise definition, broadband refers to the set of platforms consisting of optical fiber, ADSL, wireless, HiperLAN, WiMAX, satellite, UMTS, HSDPA and LTE. Currently the most used sources in the literature are those ITU and OECD that define broadband access networks capable of ensuring the download speed of at least 256 Kbs.

In fact, the most obvious difference between broadband and ultra-fast broadband consists of the maximum speed that can be reached by the link, even if a performance boundary has not been universally chosen. We can reasonably assume that the boundary is roughly equal to 30 Mbps of download speed, but in any case the real ultra-fast broadband will allow symmetrical speeds of the order of 100 Mbps.

To allow these speeds, optical fibers must be used instead of traditional copper cables. These optical networks are the infrastructural basis for the construction of the NGAN (Next Generation Access Network) telecommunications networks.

Technological evolution, both in fixed networks and in mobile networks, has created several generations of broadband over the years. In particular, for fixed networks:

- the first generation, with speeds up to 8 Mbps in download (ADSL threshold);
- the second generation that goes up to 20 Mbps in download (ADSL2 + threshold);
- the third generation that exceeds this threshold and reaches 100 Mbps and more in download / upload (through the use of VDSL and optical fiber technologies up to the last user, in the case of FTTH solutions).

1.1.2.1 Fixed Broadband

The xDSL technology family has been designed to allow the use of telephone networks consisting of copper cable (pair) laid by all operators in the world. XDSL technology has been used for over 15 years. Technological improvements in transmission systems have gradually made it possible to increase the amount of data transmitted on the pairs that connect users to the telephone exchange of the traditional network.

This technology has now become mature and reliable and its evolution will still be able to guarantee evolutionary improvements, waiting for an ever-increasing introduction of the optical fiber in the access network. The main constraints on the use of xDSL technologies are:

- equipping the telephone network exchanges with new equipment;
- problems deriving from the conformation of the existing copper network (for example, the performance decrease with increasing length of the pairs);
- possible interferences among users of pairs located in the same bundle.

The lack of coverage in some telephone exchanges defines a gap called "digital divide", between users connected to service-enabled telephone exchanges (that is, where xDSL access systems have been installed) and users connected to telephone exchanges that are not enabled for the service. This gap, depending on the types of services enabled in the telephone exchange, can be considered for different technological generations (for example, considering the gap between users with second generation ADSL or ADSL2+).

Same people cannot use the xDSL service for different reasons:

- connections to telephone exchanges not enabled for service;

- excessive length of the pairs;
- telephone network devices that do not allow a connection without interruption.

The above mentioned problems lead to digital divide between users who have access to broadband services and users unable to use them. The extension of the performances with new technologies (for example VDSL) require sections in a very short length pair (0.5-1 km) and a series of interventions and investments for the modification of the current access infrastructure, with the introduction of optical fiber sections and a radical change in network architectures.

1.1.2.2 Fixed Broadband FTTH, FTTC and FTTB

The optical fiber is a very important infrastructural component, thanks to its transmission capacity and greater protection against noise and interference compared to copper. Since the beginning of the 1990s, mainly in major cities or for important business users, various access technologies based on fiber-optic links up to users have developed. Thanks to the ability to transmit huge amounts of information (millions of Mbps on a single fiber), this type of connection is used to provide the user with very high access speeds, far beyond those possible with xDSL technologies. For fiber optic accesses, the most significant operational constraints are due to the high investments required to build the new infrastructure. While the laying of fiber optics in the private sector (in a building or campus, equipped with cavities or other forms of channelling) is relatively easy, laying in public areas requires particularly onerous civil works (excavations, poses, pits, piling). This means that this technology is limited to the most densely populated and economically most developed areas. Even in these areas, however, the economic returns are long-term.

A further slowdown in the deployment of fully fiber-optic infrastructure (FTTH, Fiber To The Home) is due to the greater complexity of fiber termination, which makes the provision of the traditional telephone service more complex (lack of tele-power supply, the need of specific termination devices).

To keep costs down, migration to a fiber-optic network can also go through mixed copper and fiber architectures. The fiber can reach the proximity of buildings (FTTC, Fiber To The Cabinet) or the same buildings (FTTB, Fiber To The Building), but the final sections of the connection remain in copper.

This architecture makes it possible to provide traditional services according to the classical scheme, but also to provide ultra-fast broadband services (up to 50-100 Mbps). The mixed copper fiber architectures can therefore be seen as an intermediate phase of the path that will lead to the creation of an access network entirely in fiber. These solutions may allow the extension, in a first phase, of ultra-fast broadband coverage even in areas where an FTTH architecture would not be economically viable. In a second phase, compatibly with an adequate development of potential demand, it is possible to evaluate the opportunity to make further infrastructural investments.

FTTH access architectures are particularly interesting, from the point of view of the potential offered for the support of ultra-fast broadband services. The different possible variants of FTTH differ mainly according to the optical technology (Ethernet) and to the architecture of the passive optical network (Point-Multipoint or Point-to-Point). The diffusion of the optical fiber, inside the access network, forms the basis for the construction of the new generation access network. Besides, the need to have high speed links throughout the territory is also common for the creation of broadband wireless networks, both free from physical and mobile sites.

1.1.2.3 Mobile broadband

In recent years mobile networks have seen a rapid evolution, starting from UMTS technologies up to HSDPA (High Speed Download Pack Access) evolutions. They have brought the nominal band available to the user at a level comparable to that of fixed access in ADSL technology. A further recent evolution of mobile network technologies to bring the band available to the user at 70-100 Mbps is the LTE (Long Term Evolution) and LTE Advanced networks.

However, the performance of a mobile network (and in general, of any wireless network) is influenced by the intensity of the radio signal between the antennas of the network and the user. It varies both for the position of the user (the distance from the base station, the use inside the buildings, obstacles to transmission), and for temporary changes in transmission characteristics (atmospheric phenomena, disturbances, temporary reflections of the signal, speed of user movement, etc.). These causes can substantially change the available transmission speed in a hardly controllable manner.

Another not insignificant aspect is related to the need to share the radio resources of the single cell with the other users who are using it at the same time. The use of new transmission techniques can increase the availability of resources of the single cell, but the increase in the number of cells is still required to maintain the nominal performance of the technology over a certain number of concurrent users.

The evolution of the performance of mobile networks goes through two types of factors: on the one hand, the gradual adoption of new technologies that can induce, even in this case, a phenomenon of "generational" digital divide between covered areas and not covered areas by new technologies; on the other hand, there is a greater need for cell connectivity, both to ensure increasing capillarity, and because the increase in the bandwidth supplied must necessarily correspond to an increase in capacity of the connection to the network.

1.2.2.4 Evolutionary scenarios of broadband technologies

The spread of broadband and ultra-fast access networks improves the use of high bandwidth services (in particular, video content such as IPTV) and decreases the latency times of the ultra-fast broadband. In fact, the evolution of broadband performance has led to the development of a series of peer-to-peer services (such as file exchange), which have rapidly spread.

The evolution of access technologies will propose new scenarios of convergence and use of the network. A range of advanced services, enabled by increased speed of connections, is revolutionizing the way to do business of the companies, but also managing the daily activities of individuals. Cloud computing, a new model of on demand access to IT resources (applications, hardware resources, platforms, etc.), has already been made available today by the evolution of the broadband access network. This latter type of service, which makes IT resources accessible on the web, will become increasingly important, as the performance of the networks will increasingly support the new model of use. There is broad consensus on the crucial impact and benefits of a widespread coverage of ultra-fast broadband connectivity for economies and society: ultra-fast broadband connectivity favours efficiency and economic growth and creates the conditions so that economies can remain competitive and make it possible to benefit from the typical network externalities. The European Commission indicates the affirmation of the Information and Knowledge Society as a necessary condition to favour the economic and social development of the member countries. In this context, the availability of broadband services is considered the enabling condition. Broadband connectivity, in fact, plays a central role in the development, adoption and use of ICT technologies in the economy and in society.

The strategic importance of broadband derives from the ability to accelerate the contribution of ICT technologies to the growth and innovation in all economic sectors, as well as to social and territorial cohesion. The increase in the diffusion of digital technologies and the investments of the telematic infrastructures have a high multiplicative factor in terms of development, resulting in a real enabling factor for the growth of a country. According to a recent study by the European Commission "The socio-economic impact of bandwidth", the adoption of broadband has a significant impact on the economic growth and benefits on employment.

We can therefore conclude by saying that in the current economic phase, the investments for the development of broadband and ultra-fast broadband take on a strategic value.

After briefly discussing the ICT and broadband technologies, which are the subject of empirical estimates in this work, we focus on the literature review of diffusive theory and network externalities.

1.2 DIFFUSION THEORY

1.2.1 Introduction to the theory of the diffusion of innovation

The diffusion of innovation is an integral part of the concept of technological progress. Technological change is understood as a multidimensional process and consists of three phases (Schumpeter, 1942; Davies, 1979):

- the *invention*, that consists of conceiving a new idea;
- the *innovation*, that concerns the invention translated into economic activities through the application and verification in the market;
- the *diffusion*, that takes place when innovation is used over time by more users.

The literature on innovation diffusion is vast, and it spills over many conventional disciplinary boundaries.

The diffusion of an innovation has been defined by Everett Rogers as the process by which that innovation is communicated through certain channels over time among the members of a social system (Rogers, 2003). This definition is chosen as the basis for the typical analytical framework employed in the literature, featuring four elements:

- an *innovation*
- one or *more communication channels*
- *time*
- a *social system.*

Innovation means any idea, practice, object that is perceived as new by members of a system. For example, a commercial product, a new technology or a new social trend. The concept of the new is not an absolute concept, but it must be considered by individuals or units that are seen as companies, institutions or countries. In addition, two types of innovations can be distinguished: in the first group we find all the innovations that are essentially the improvement of others or the addition of an attribute and are called incremental innovations; in the second group, we find radical innovations that are completely new to the market and that satisfy in a completely different way the consumer needs. However, innovations cannot be considered all the same, because it would be too much of a simplification of reality and this, among other things, would not explain the different developmental speeds of the various products. Some characteristics that diversify the innovations are also outlined, and these are seen as the result of individual perception; the sum of such perceptions gives rise to the collective behaviour. Rogers (1962) delineates five different characteristics of innovations. Each of them is a bit empirically interrelated with the other four, but they are conceptually different. These features are:

1) *Relative advantage* concerns the degree to which an innovation is considered an improvement over the existing one. This should be considered in terms of economy, social prestige, convenience and satisfaction.
2) *Compatibility*, if an innovation is compatible with the existing technologies, or even with the values, experiences and needs of potential adopters;
3) *Complexity* is the degree to which an innovation is perceived as relatively difficult to understand and to use;
4) *Trialability* the degree to which a product can be tried before being bought; similarly, it can be defined as the degree in which the innovation allows "the learning by using" before the actual purchase. In some cases there are the possibility to buy the basic

version or a module of the product, before expanding and completing it;

5) *Observability* is how the innovation is visible to consumers, who can then see how it works or see the results before the adoption.

Briefly, the more an innovation benefits, the more is compatible with the surrounding environment, testable and observable and less complex, the greater is the speed of adoption.

The *social system*, in this context, is constituted by individuals, groups of individuals, organizations that share certain features and they are considered potential users of innovation. Therefore, members of a system can be consumers of certain types of product, but also companies and organizations.

Time is the time span between the awareness of the existence of the new product and its possible adoption. Time is often considered as a discriminating factor between the types of consumers. According to Rogers (1962) there are five types of consumers:

- *innovators*
- *early buyers*
- *early majority*
- *late majority*
- *laggards*.

This heterogeneity of individual behavior is called Innovativeness by the author.

Communication is the process by which the agents create and share information with one another in order to reach mutual understanding. A particular importance in this context is played by the concept of influence: consumers are influenced either "externally" or "internally".

The external influence is the "official" information, id est, the information carried by the mass media (through advertising) and distributed from business to consumer. Mass media channels are often the most rapid and efficient means to inform an audience of potential adopters about the existence of the innovation, to make them aware of it. However, mass media channels are deemed to lead to changes only in weakly held attitudes.

The internal influence means interpersonal communication channels and in particular through “word of mouth”. This propagation mechanism is very efficient, and cannot directly controlled by the companies. This type of communication channel is more effective in persuading an individual to accept a new idea, especially if the interpersonal channel links more individuals who are similar in socioeconomic status, education or other important ways.

In recent years the role of internal influence has greatly increased due to the introduction of new channels, catering for word of mouth (mobile telephony, social networks, broadband communications, etc.). The same occurred also for the external influence; just look at the huge advertising expenditures for promotions through digital media, including Internet channels.

Internet is a multifaceted means of communication and, depending on its use, it can become a bearer of external or internal influence, positive or negative, with respect to any product. In launching a new product, special attention should therefore be devoted to create a potential word of mouth by using targeted strategies. In particular, to take the advantage of word of mouth, business communication should be more direct to the market segment that has more propensity to disseminate information (opinion leaders). This way you can increase the penetration of the new product into the market by limiting advertising costs.

All the treated models have the concept of influence and information that it occurs at all stages of the adoption process.

Despite the many types of diffusion processes, there is a recurring regularity in academic and practitioners' analyses: if we draw a graph of the cumulative adoption of an innovation over time, the resulting curve has almost always a sigmoid shape. Thus, a diffusion model allows the prediction of the shape of the diffusion process and enables a theoretical explanation of the dynamics of the same process in dependence of certain characteristics of the social system and the used communication channels.

The diffusion process of a product can be thought as the flow of adoptions due to potential consumers through two market segments. If, for simplicity, the consumer can only make one adoption, in this case adoption or consumer is equivalent. The market can be distinguished in:

- Residual market potential $\boldsymbol{m_R(t)}$, adopters who can be considered potential adopters at the time t;
- Actual market $\boldsymbol{N(t)}$, actual consumers at the time $t_0 \leq t$.

The sum $\boldsymbol{m(t) = m_R(t) + N(t)}$, or, in the hypothesis above, the number of adoptions made before the withdrawal of the product from the market defines the total market (market potential).

The total market is not an abstract amount. It is, by definition, the number of consumers who will plausibly adopt the product before withdrawing from the market. It follows that the residual market potential is composed of consumers who have not adopted the innovation but they are expected to adopt it in the future. For example, the total market of a new household appliance cannot simply be the number of households in the market on which it is launched. It should be the expected number of households that, by family composition, income, willingness to buy, etc., are likely to be really interested in purchasing within a period of time equal or less than the expected time of stay on the household appliance market.

It is important the distinction between sales and adoptions.

Many misunderstandings, partly terminological, may arise on this point. If the modelled product belongs to the category of durable consumer goods (which are purchased only once and last over time), the distinction is useless since, as seen above, the quantity of products sold (the adoptions) coincides with the number of buyers (consumers). In the case of goods, subject to repeated purchases, adoptions do not coincide with the number of consumers.

Our statistical units will then be adopters if they also coincide with sales or, in the opposite case, directly the consumers. In the first case, that is, whether the object of study is a good subject to repeated purchases, it can be understood by market potential the number of expected unit of product to be sold in the future. It will then be necessary to examine carefully the model hypotheses as they may prove themselves inadequate or completely wrong. From a statistical and mathematical point of view, the distinction may not be so significant. Everything depends on modelling choices. If only the adoption data is detected, the information on the buyer would not be available, and therefore the possibility of controlling the repetition of purchases by a single consumer would be no longer available.

Although we limit our review to the marketing literature that focuses on the dissemination of new products, we must not forget that there is another equally important literature that studies the diffusion processes, namely the economic one. In this regard, we recommend the literature review by Stoneman and Battisti (2010) that analyse both the demand side and the supply side of the diffusion process at different levels of aggregation, from the worldwide to the interfirm or household level. They discuss the theoretical foundations of explored diffusion models as well as econometric models, data availability and diffusion policy.

1.2.2 Some classifications of the diffusion models

There are several classifications of the diffusion models. We take into consideration two types of classification. The first is described by Roberts and Lattin (2000) who divides the diffusion models into three categories:

- *aggregate level diffusion models*
- *individual level diffusion models*
- *intermediate level diffusion models*

Aggregate level diffusion models they are aimed at the understanding of overall market development and its response to managerial and environmental variables without a direct microeconomic derivation of the individual's adoption decision. Originally proposed as a way of explaining and forecasting the aggregate sales profile of a new consumer durable, these models have extended to include the effects of the marketing mix, environmental factors such as competitions and dynamic market potential, and a variety of other consumer phenomena such as repeat purchase, awareness, etc. Their ability to fit macrolevel data and to provide an understanding of the drivers of adoptions over time is well established (Bass, Krishnan and Jain, 1994).

Individual level diffusion models start from classical utility and attitude models from economics and psychology and attempt to represent changes in expected utility over time. Discrete choice theory then provides a method to transform these utilities to probabilities of purchase and thus expected market shares. Individual level models consider that the different individuals of the population adopt at different times. These models consist of three components:

- a *utility function*;
- an *updating process* by which that utility function changes over time;

- *a choice decision* based on the utility.

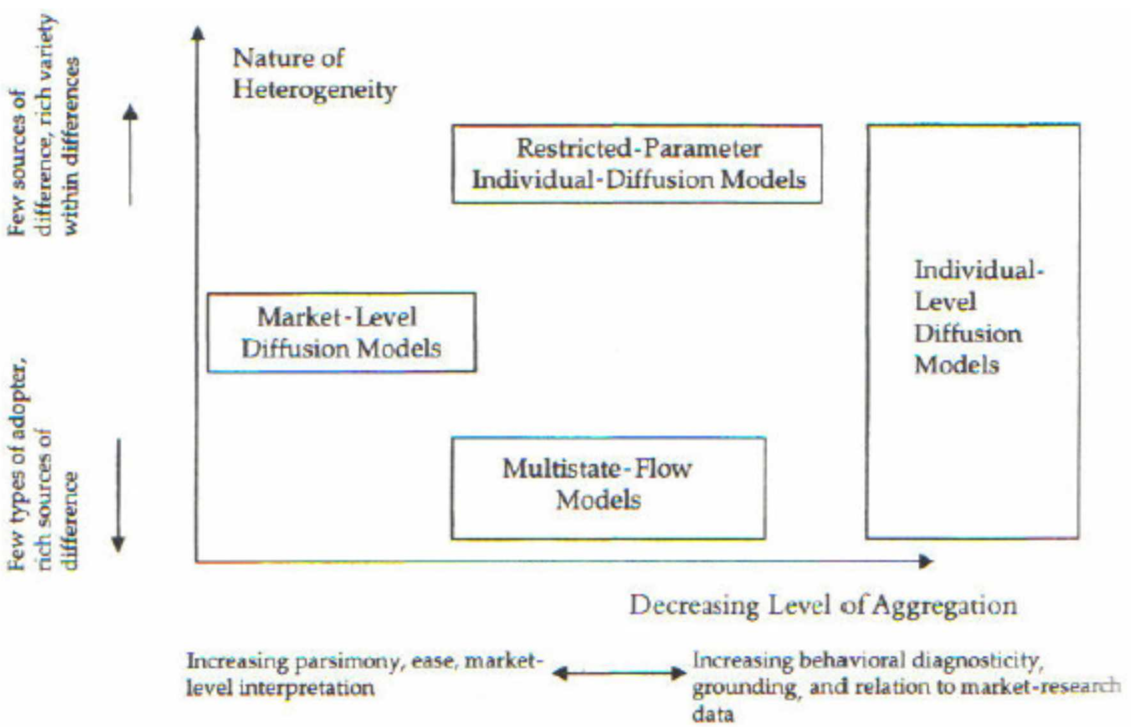
Intermediate level diffusion models are placed between these two extremes described above. We can distinguish two types of intermediate level models:

- *multistate flow models*
- *restricted-parameter individual models*.

Multistate-flow models segment the market into a number of behavioural stages and then observe the flows between them. These models achieve parsimony by restricting consumer heterogeneity to a small number of groups although the differences between these groups can be quite richly specified.

Restricted-parameter individual models retain the richness and theoretical rigour of individual level models, but agents are heterogeneous only with respect to a small number of parameters. The figure 1 show the different models of the sales of a new product.

Figure 1. Different models of the sales of a new product



Source: Roberts and Lattin (2000)

Another useful and important classification divides the diffusion models in the following categories (Geroski, 2000):

- *aggregate models or epidemic models*
- *microeconomic models or probit models*
- *evolutionary models*

Epidemic models are the most common diffusion models. These models are based on the premise that the cause limiting the diffusion speed is the lack of available information about the innovation. The hypothesis underlying these models is that at any time all agents involved could benefit from adoption. *Probit models* are the leading alternate models of the epidemic models. These models analyse individual adoption decisions behind the hypothesis that the different agents (consumers and firms), with different objectives and abilities, will probably want to adopt innovation at different times. In this type of models, the agents are heterogeneous.

Evolutionary models share with probit models the presumption that adopters are heterogeneous. These models analyse the effect that selection mechanisms have on innovation adoption choices in a context of uncertainty and limited information. In these models, the original innovation changes during the process of diffusion as learning by different types of agents creates feedback effects that enhance the original innovation. Also, central elements of explanation of innovation diffusion are information contagion, path dependent processes, increasing returns and technology choice under uncertainty (Nelson and Winter, 1982).

The first two diffusion categories discussed represent *inter-firm diffusion*. Another component of the diffusion that we neglect is the so-called *intra-firm diffusion*, which measures the time with which companies that adopt a new technology completely convert their productive apparatus to the latter. For a deeper understanding of the intra-firm diffusion we recommend Stoneman (1983), Stoneman and Battisti (2010) and Battisti (2008).

1.2.3 Macro level diffusion models

Macro level diffusion models were the first to be used in social sciences and are still the most common. We define "macro-level" diffusion models the models that describe the aggregate (country-wide, region-wide) adoption path of a new technology.

The underlying hypothesis of these models is that at any moment all concerned agents could benefit from adoption.

The only cause of the slowness of dissemination is the lack of information about the existence or the actual utility of innovation: at every moment, not all potential adopters know that innovation exists and it is available or, more likely, not everyone is convinced that innovation is really superior to old technologies. Each agent therefore considers it risky to abandon the old technologies and adopt the new ones. To spread innovation, it is necessary and sufficient to disseminate information about the product: once the effective utility of the innovation is proven, the agent will be willing to adopt it. We can have different models depending on the assumptions about the origin and the way of information transmission.

Diffusion models focus on the development of the product life cycle (Kotler, 1971). The product life cycle theory is an empirical generalization that recognizes distinct phases in product sales, from when they are born and put on the market, to when they become obsolete. These phases reflect the behaviour of consumers towards the good that is being studied. The canonical form of the product life cycle is an S-shaped curve, represented in the Cartesian plane with the time on the abscissa and the sales volume on the ordinate. The life cycle of a product is usually divided into four phases:

1) *introduction* of the product into the market,
2) *growth*,

3) *maturity*,
4) *decline*.

They are preceded by the product development phase. The *development phase* begins when the company starts designing a new product idea, which can be a radical innovation in that market or the improvement of an existing product.

The *introduction* is the time of launching the product that sees a slow increase in sales. The company seeks to build the market as quickly as possible by investing in distribution and promotion activities to form its market share.

The product enters the *growth phase* when it is accepted by the market and the sales grow so much to create profit. Initial adopters continue to buy the product and other consumers decide to follow their example. *Maturity* is when we try to keep the reached market share. As the product has been accepted by the majority of potential customers, we strive to boost brand loyalty and repurchase by defending the product from the competition. At this stage, the increase in sales volume slows down and in general this stage is more difficult than the previous one.

The *decline* is the time of the decline in sales and profits. It may be slow or fast, but investments are reduced and the decision is made whether the product should be removed from the market or not.

High quality products are therefore characterized by a more or less long life cycle, whose canonical form follows this trend.

It is true that diffusion models, like any other model, are simplifications of reality. However, they constitute a wide range of useful tools, in both the academic and business context.

Technological innovation in the theoretical framework of the neoclassical economics is interpreted by a very fragile and static scheme. This structure is in trouble in having to analyse the phenomenon of the diffusion of innovation that technology history has shown to be very dynamic. Neoclassical hypotheses consider scientific knowledge freely and equally accessible to all entrepreneurs (this clearly discloses the diffusion of immediate and general innovation). This setting does not match the reality. The invention, and therefore innovation, is not freely and completely accessible to the enterprise, just consider the example of a patent-protected invention, or when it is the result of internal R & D activity to another firm. In such cases it is not readily available for other companies. The neoclassical scheme is such as to neutralize the dynamic nature of technological innovation and its diffusion. These neoclassical setting limits have led some economists to analyse the phenomenon of the spread of innovation with evolutionary concepts borrowed from biology. Indeed, the formal representation of macro diffusion models has historically been borrowed from biology epidemic models, holding the hypothesis that an innovation spreads in a social system through the mechanism of communication, like an infection infects people (Geroski, 2000). Therefore, epidemiology concepts were used to compare the spread of information with the transmission of diseases from infected individuals to other uninfected ones. An important contribution to the analysis of the process of technological diffusion comes from the studies of Griliches, who demonstrated that certain types of diffusion processes can be adequately described in terms of logistic development (Griliches, 1957). Griliches, with the spread of hybrid corn in the USA (first noticeable empirical study on the diffusion of an innovation) shows that economic factors (expected profits and economies of scale) are crucial determinants of diffusion. The author showed that within 25 years the share of hybrid seeds in corn production had grown rapidly, but the process had taken off at different speeds in different states. He pointed out that economic incentives can explain the different rates of diffusion. After the 1950's, a certain consensus emerged on the fact that the spread of technological innovation over time is satisfactorily represented by the logistics curve. The logistic function is a S-shaped symmetric curve that in mathematical terms expresses the form of a phenomenon that passes from one point of equilibrium to another one through a continuous transition path. Frequently empirical research on the processes of diffusion of innovations shows an element of asymmetry. In such cases, the use of the logistic function is unsuitable and a more adherent model is the one using S-shaped asymmetric curves such as that produced by Gompertz or the log-normal cumulative distribution function or the resulting curve from the Bass (1969) model.

In the following, we sketch the basics of this modelling tradition, in order to better underline our expected theoretical contribution.

A diffusion function *y* captures the diffusion pattern of a new product during its life cycle. Given the fact that this pattern is time dependent, we denote a diffusion function by *y (t)*. The cumulative diffusion function is usually modelled as the solution of a differential equation $\frac{dy}{dt} = f(y,t)$, where the function *f* determines the shape of the diffusion curve. (Ruiz–Conde, Leeflang and Wieringa, 2006). By making the standard assumptions that are used in the diffusion of innovations theory (Mahajan and Schoeman, 1977; Kalish and Sen, 1986; Mahajan, Muller and Bass, 1990), we arrive at a mathematical expression for the fundamental diffusion model. The first assumption is that the rate of diffusion or the number of adopters at any given point in time is directly proportional to the number of remaining potential adopters at that moment. Mathematically, this can be represented as:

$$\boldsymbol{n(t)} = \frac{\boldsymbol{dN(t)}}{\boldsymbol{dt}} = \boldsymbol{g(t)}\ [\boldsymbol{m} - \boldsymbol{N(t)}] \tag{1.1}$$

where: (1.2)

$$\boldsymbol{N(t)} = \int_{t_0}^{t} \boldsymbol{n(t)dt}$$

n (t) is the number of adopters at time *t*, *N(t)* is the cumulative number of adopters at time *t*, and *m* is the market potential (the carrying capacity or the ceiling of the social system). The function *g(t)* is known as the rate of adoption or individual probability of adoption, namely as the probability that a potential adopter adopts at time *t*. The second assumption is that *g(t)* depends on time through a linear function of *N(t)* (Mahajan and Peterson; 1985):

(1.3)

$$\boldsymbol{g(t)} = (\boldsymbol{p} + \boldsymbol{q}\, \boldsymbol{N(t)})$$

Substituting this equation in:

(1.4)

$$\boldsymbol{n(t)} = \boldsymbol{g(t)}\, [\boldsymbol{m} - \boldsymbol{N(t)}]$$

so, we get the fundamental diffusion model:

(1.5)

$$\boldsymbol{n(t)} = \big(\boldsymbol{p} + \boldsymbol{q}\, \boldsymbol{N(t)}\big)\, [\boldsymbol{m} - \boldsymbol{N(t)}]$$

The specific value of *g(t)* depends on the characteristics of the diffusion process such as the degree of innovation, the properties of the communication channels, and the social system properties. In addition, *g(t)* can be interpreted as the probability, for a potential adopter, of an adoption at time t. Communication channels can play different roles during the adoption process.

Depending on the importance of each source of influence, different versions can be derived from the fundamental diffusion model. When $q = 0$, the model only considers external influence, when $p = 0$, it only considers internal influence. When $p \neq 0$ and $q \neq 0$, the resulting model is called a mixed influence diffusion model (Mahajan and Peterson, 1985; Ruiz – Conde, Leeflang and Wieringa, 2006). So, there are three specific types of innovation spread patterns:

- *The external influence model*, where g(t) is a constant p
- *The internal influence model*, where g(t) is $q\ N(t)$
- *The mixed influence model*, where g(t) is $p + q\ N(t)$

1.2.4 The external influence model

The external influence model based on the assumption that the rate of diffusion only depends on the number of potential adopters at time *t*.

External influence directly affects the innovative behaviour of members of the social system through external sources. External sources of information are not dependent on the current level of dissemination, which is not dependent on the number of adopters. These sources include innovation providers, various media (television, radio, more or less specialized newspapers) and public promotion agencies for innovation. These subjects deliver amount of information that is not strictly dependent on the user's experience and uniformly reaches all potential adopters.

The hypotheses underlying the model are:

- the population of potential adopters remains constant over time and all members of it can adopt the innovation;
- the diffusion process derives from a constant influence factor that does not depend on the number of members.

The model can be represented by the following equation:

$$\frac{dN(t)}{dt} = n(t) = p\,(m - N(t)) \tag{1.6}$$

where *n (t)* is the number of adopters at time *t*, *N(t)* is the cumulative number of adopters at time *t*, *m* is the market potential and *p* is the constant of external influence.

Adoptions at a given time *t* are directly proportional to the residual market $(m - N(t))$ with constant proportionality *p* scalar parameter. The residual market assumes the character of saturation effect, while the parameter p is the diffusion coefficient (external influence parameter).

The maximum value for *n(t)* is reached when it is launched on the product market and it decreases in relation to the value of parameter p.

The equation represents a first order differential equation and it can be resolved analytically. We add to the system the initial condition by imposing that adoptions are void at the time of launching the product on the market.

(1.7)

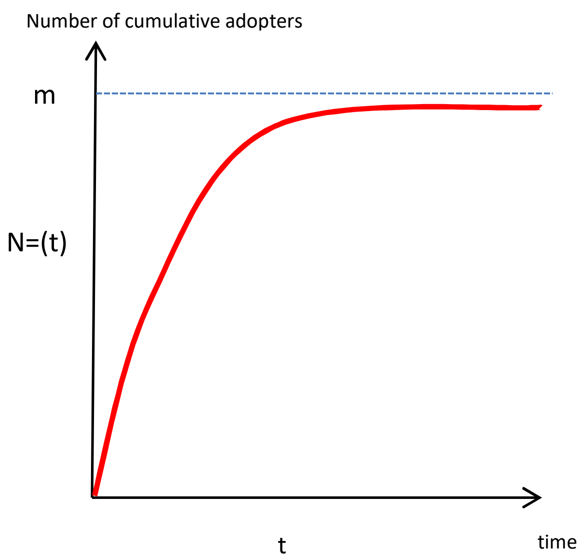
$$\begin{cases} \dfrac{dN(t)}{dt} = n(t) = p\,(m - N(t)) \\ N(0) = 0 \end{cases}$$

A unique solution is obtained that corresponds to the modified exponential function:

(1.8)

$$N(t) = m\,(1 - e^{-p\,t})$$

Figure 2. Modified exponential function



Source: our elaboration

The instantaneous adoptions (not cumulative adoptions) are:

$$\frac{dN(t)}{dt} = n(t) = m\,p\,e^{-p\,t} \tag{1.9}$$

The function *N(t)* does not have maximum points, it is tightly growing and it has always the second-order derivative.

The parameter can be interpreted as a measure of the influence of mass media on the diffusion of the product.

The interpretation of p is reinforced by the fact that this model has proved to be valid in explaining the adoption of products that in the introduction phase do not encounter great resistance from consumers. It is well represented by the market's response to fashion items with limited market presence, for which the launch is crucial.

Its biggest limit is the inability to incorporate the influences that exert the first customers on the rest of the potential market.

Pioneering works in the use of the diffusion model of external influence are those of Fourt and Woodlock (1960), Coleman, Katz and Menzel (1966) and Hamblin, Jacobsen and Miller (1973). Fourt and Woodlock (1960) demonstrate that sales predictions for certain consumer products require the application of a modified exponential curve.

1.2.5 The internal influence model

The internal influence model based on the assumption that the rate of diffusion depends both on the number of potential adopters at time t and the level of diffusion reached *N(t)*.

The model is based on the existence of communication between members of the social system through social interaction, represented in the model by the product of previous and potential adopters. A mechanism similar to epidemiological contagion is established and it is no coincidence that these patterns originate from them and are called epidemic models. In this case it is the imitation mechanism that is similar to the mechanism of contagion of an infection occurring in biology.

In this type of model, the probability of adoption increases directly proportional with the increase in the number of adopters in the social system: as the greater the number of previous adopters, the more information there will be in the market on the characteristics, advantages and previous adopters' experience of the innovation, which would reduce the risk aversion of potential adopters and encourage the decision to adopt the product. This assumption is consistent with the assumption of diffusion driven by word of mouth, which acts from the inside of the potential market. There is also the possibility of a negative interaction, but most authors consider only the positive component of interpersonal communication.

The hypotheses underlying the model are:

- the population of potential adopters remains constant over time and all members of it can adopt the innovation;
- the diffusion process derives from a constant influence factor that does not depend on the number of members;

- all adopters are imitators and they only adopt after getting in touch with other adopters who use the product;
- the rate of diffusion depends both on the maximum number of potential adopters at time t that still have not adopted on the number of previous adopters *N(t)*.

The model can be represented by the following equation:

$$\frac{dN(t)}{dt} = n(t) = q\, N(t)\, (m - N(t)) \tag{1.10}$$

This equation represents a diffusion model of pure imitation and the parameter q is defined as a parameter of internal influence or an index of potential adopters' imitation of previous adopters. Gray (1973) denominates q as the parameter of diffusion through interaction.

The above mentioned equation is a first order differential equation (Bernoulli type) and through its integration can be solved:

$$\begin{cases} \dfrac{dN(t)}{dt} = n(t) = p\,(m - N(t)) \\ N(0) > 0 \end{cases} \tag{1.11}$$

So, we get the cumulative number of adopters:

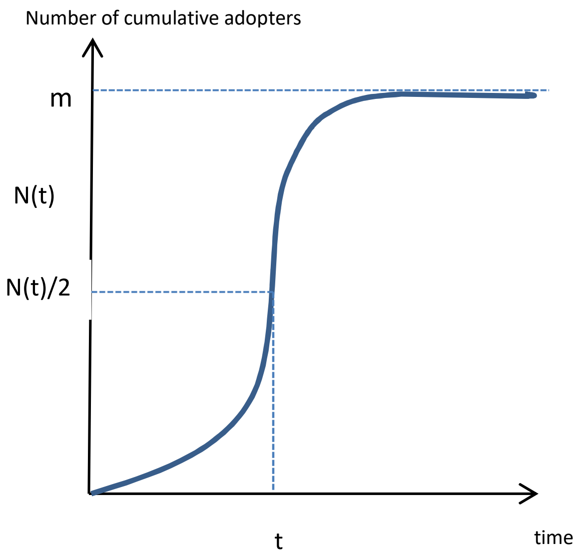
$$N(t) = \frac{m}{1 + e^{-(a+b)\,t}} \tag{1.12}$$

where: $a = ln\frac{m-N_0}{N_0}$ and $b = m\,p$

It can only be applied after you know the first sales data. It is necessary to assume that the model is valid only after sales have begun. This is however consistent with the underlying hypothesis, since word of mouth can only occur if there is a certain number of information diffusers that are consumers and adopters themselves.

This equation is a logistic function where m is the market potential, namely the carrying capacity or the saturation level. The concept of logistic function implies that the cumulative growth of a product in a market over a period of time presents a characteristic S-shaped curve, which is symmetrical respect to the inflection point. In the inflection point, the cumulative adoptions are exactly half of the potential market.

Figure 3. Logistic function



Source: our elaboration

This means that the propensity to adopt increases until the half of the total market, and then decrease and tend to 0 for $n \to \infty$.

At the beginning of market development, we notice that the diffusion coefficient is very small. When the actual market increases, it increases the interaction of adopters who have already adopted with the potential adopters, thus accelerating adoption decisions for new consumers. Overcoming a certain level, the interaction decreases as the market potential decreases.

The instantaneous adoptions (not cumulative) are:

$$\frac{dN(t)}{dt} = n(t) = \frac{m\,c\,e^{-(a+b)\,t}}{(1+e^{-(a+b)\,t})^2} \tag{1.13}$$

The logistic model was formulated for the first time by Verhulst in 1838 and was originally used in natural sciences for describing growth processes, like the spread of a disease. Fisher and Pry (1971) and Meade and Islam (1998) demonstrated the usefulness of the logistic equation in representing the diffusion of basic technologies. Among the best known there is Mansfield's work (1961) in the field of technology substitution studies of industrial innovations.

1.2.6 The mixed influence model

The internal influence model considers that both types of influence are present in the decision to adopt an innovation. This model exceeds the capabilities of the other two because it incorporates both forms of communication that can influence consumer behaviour. The main assumptions established for the previous models are valid in the mixed or generalized context. This is the most general specification of the fundamental diffusion model.

The model can be represented by the following equation:

$$n(t) = \frac{dN(t)}{dt} = (\, p + q\, N(t))[m - N(t)] \tag{1.14}$$

Integrating this first order differential equation, we get the following numbers of cumulative adopters:

$$N(t) = \frac{m - \frac{p\ (m - N_0)\ e^{[-p + (q\,m)\ (t - t_0)]}}{(p + q\,m)}}{1 + \frac{q\ (m - N_0)\ e^{[-p + (q\,m)\ (t - t_0)]}}{(p + q\,m)}} \tag{1.15}$$

Now, we describe the most famous and most widely used mixed influence model, namely the Bass model.

1.2.7 The Bass model

The Bass (1969) model is the most parsimonious mixed influence diffusion model suggested in the marketing literature (Parker, 1994) and inspired several hundreds of contributions (Mahajan, Muller and Wind, 2000). Mahajan, Muller and Bass (1990) provide a good overview of the Bass model, its extensions, and directions for further research.

The theoretical justification that Bass explains in his article published in 1969 is based on the division of adopters into two categories:

- *Innovators*
- *Imitators*

Innovators are the first to adopt without affecting the influence of other individuals. Imitators, on the contrary, mainly adopt innovation after undergoing the influence of those who have already adopted. Innovators and imitators do not stand out for the period of purchase. Their difference is in the different communicative channel that has influenced adoption and both are present at all periods. The importance of innovators is larger in the period immediately after the launch and decreases over time.

Bass saw that Rogers' work on the spread of innovations in social systems due to word of mouth could be the basis of a new mathematical theory of how new products diffuse among potential adopters. The Bass model assumes that sales of a new product are primarily driven by word of mouth from satisfied customers. Innovation is first adopted by a small group of innovators who, afterwards, influence the other consumers through the interpersonal communication.

The mathematical structure of the Bass model (Bass, 1969) is derived from a hazard function corresponding to the conditional probability that an adoption will occur at time t given that it has not occurred yet. This probability is a linear function of the number of previous adopters:

$$\frac{\boldsymbol{f(t)}}{[\boldsymbol{1-F(t)}]} = \boldsymbol{p + q\, F(t)} \tag{1.16}$$

where the variable t denotes the time of adoption of a new product by an individual (adopter), $f(t)$ is the density function of adoption at time t, $F(t)$ the cumulative distribution function, and p and q are the parameters of innovation and imitation, respectively.

An adoption is a first-time purchase of a product (including services) or the first-time uses of an innovation.

In the above equation, t represents time from product launch and it is assumed to be non-negative.

From the first order differential equation with the initial condition $F(0)=0$, it could be find the solution of cumulative distribution function $F(t)$, cumulative adoptions $N(t)$, and noncumulative adoptions.

$$\begin{cases} \dfrac{f(t)}{[1 - F(t)]} = p + q\, F(t) \\ F(0) = 0 \end{cases} \tag{1.17}$$

The cumulative distribution function is:

$$F(t) = \frac{1 - e^{-(p+q)\,t}}{1 + \frac{q}{p} e^{-(p+q)\,t}} \tag{1.18}$$

The proportion of adoptions *F(t)* provided by equation describes the dynamics of the diffusion process, in terms of adoption parameters, *p* and *q*.

We also can refer to the absolute scale representation, that is to the number of adoptions, *N(t)*, just multiplying *F(t)* by the market potential *m*, acting as a scale parameter:

(1.19)

$$N(t) = m\frac{1 - e^{-(p+q)\,t}}{1 + \frac{q}{p}e^{-(p+q)\,t}}$$

Previous equations indicate cumulative adoptions at time t, but if we are more interested on instantaneous adoptions we will use the correspondent first order derivative, that is the density function:

(1.20)

$$\frac{dF(t)}{dt} = f(t) = \frac{(p+q)^2}{p}\frac{1 - e^{-(p+q)\,t}}{\left(1 + \frac{q}{p}e^{-(p+q)\,t}\right)^2}$$

or the corresponding absolute version:

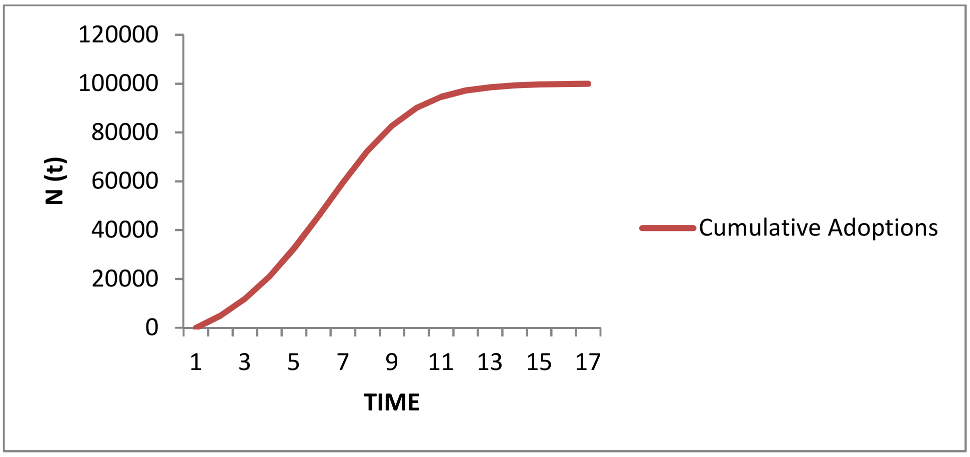
(1.21)

$$\frac{dN(t)}{dt} = n(t) = m\,\frac{(p+q)^2}{p}\frac{1 - e^{-(p+q)\,t}}{\left(1 + \frac{q}{p}e^{-(p+q)\,t}\right)^2}$$

Thus, we can define *m* as the market potential of adopters, *n(t)* as the density function of the number of adopters at time *t*, with $n(t) = m\,f(t)$, and *N(t)* the cumulative number of adopters up to time *t* ($N(t) = m\,F(t)$) , and we can write the *Bass* model expressed in the form of the fundamental model (1.14):

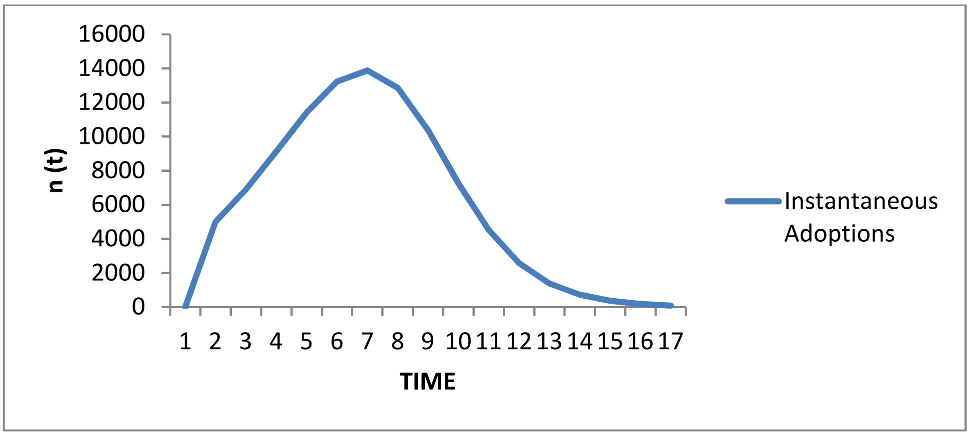
$$n(t) = \left(p + q\,\frac{N(t)}{m(t)}\right)[m(t) - N(t)]$$

Figure 4. Cumulative adoptions of Bass Model with p=0.05, q=0.45 and m=100000



Source: our elaboration

Figure 5. Instantaneous adoptions of Bass Model with p=0.05, q=0.45 and m=100000



Source: our elaboration

This model has three parameters: the parameter of innovation or external influence *(p)*, the parameter of imitation or internal influence *(q)* and the market potential *(m)*. Parameter *q* reflects the influence of those users who have already adopted the innovation (word of mouth communication from previous adopters), while p captures the influence that is independent from the number of adopters (external communication) (Bass, 1969).

Instantaneous adoptions show the presence of a peak, corresponding to the point of maximum growth of the diffusion process. After this period which represents the maturity phase of the life cycle of innovation, the product enters the decline phase and the diffusion process tends to decrease. We can calculate the peak of adoptions t^*, deriving the equation of instantaneous adoptions and equalizing the result to 0. We get:

$$t^* = \frac{ln\,\frac{q}{p}}{p+q} \tag{1.22}$$

The corresponding values of cumulative adoptions and instantaneous adoptions are:

$$N(t^*) = \frac{m\,(q-p)}{2q} \tag{1.23}$$

$$n(t^*) = \frac{m\,(q+p)^2}{4q} \tag{1.24}$$

The maximum of the instantaneous adoption is no longer fixed as in the logistic model, but is a function of the p and q parameters. This fact represents a huge step forward in terms of flexibility. In fact, this mathematical property translates into the ability of the model to adapt to fairly different adoptions trends, providing adequate economic interpretations.

This model is, therefore, able to examine both the phenomenon of word-of-mouth and the effect of mass media on the diffusion of products.

The Bass model has been successfully applied in the explanation of diffusion processes for a large number of innovations: durable consumer products, industrial processes, medical equipment and telecommunication systems. Its applications do not stop at the economic-productive environment. In literature there are cases of social phenomena, such as the spread of contraceptive pill in Thailand, metropolitan violence and federal laws in the United States.

1.2.8 Assumptions: the limitations of the fundamental diffusion model

The fundamental diffusion model in its three meanings is based on different assumptions which on the one hand limit and reduce the reality of the diffusive phenomenon, on the other they allow the model to obtain analytical solutions.

These are the different assumptions:

1. the adoption process is a binary process;
2. the population is homogeneous;
3. market potential of the new technology remains constant over time;
4. the parameters of external and internal influence remain constant;
5. there is only one adoption by an adopter;
6. the geographical borders of the social system do not change over the diffusion;
7. diffusion of a new technology is independent of all other innovations;
8. the characteristics of an innovation and its perception do not change;
9. there are no supply restrictions;
10. the diffusion of a product is not influenced by marketing strategies.

Among these, we particularly examine the assumption that the potential market does not change over time, because we relax this restriction.

Now, we briefly discuss each of the assumptions on which the fundamental model is based.

Assumption number 1: the adoption process is a binary process.

The fundamental diffusion model assume that potential adopters have only two options: adopt or reject innovation. As a result of this assumption, the adoption process is treated as a discrete behaviour with respect to continuous behaviour. In addition, the fundamental diffusion model does not take into account stages in the adoption process. Various authors have studied models that relax this assumption. Examples are the works of Dodson and Muller (1978), Mahajan and Muller (1982), Sharif and Ramanathan (1982), Mahajan, Muller and Kerin (1984), Kalish (1985), Bayus (1987) and Jain, Mahajan and Muller (1991) extending the basic dissemination model by increasing the number of phases in the adoption process and creating multinomial, polynomial, and multistage diffusion models (Ruiz Conde, 2005).

Assumption number 2: 2. the population is homogeneous

The fundamental diffusion model assumes that the population of potential adopters is homogeneous. One way to relax this restriction is by multistage diffusion models. One possibility to relax this assumption is though multi-stage diffusion models. Another way is to introduce a parameter that permits heterogeneity of individuals with respect to their susceptibility to an innovation. Given the importance of this assumption, we briefly describe some models.

Roberts and Urban (1988) assume that individual consumers choose brands that provide them with the highest expected risk and update their previous brand convictions with the arrival of new information. This update occurs in two ways:

1) word-of-mouth communications (positive or negative reviews) can change estimated average levels of the trademark attribute;

2) uncertainty may decrease due to the availability of new information. The authors take the single purchase risk as a multinomial logit model. They apply the model to the pre-launch planning of a new car in which they collect measurements of average values, perceived attribute levels, uncertainty and probability of purchase by respondents and aggregate the probability of purchase on consumers to obtain market share expected.

Chatterjee and Eliashberg (1990) develop a micro level diffusion model that incorporates heterogeneity in the population with respect to initial perceptions, preference characteristics (degree of risk aversion and price sensitivity), and responsiveness to information about the innovation. Consumers update their performance expectations based on the information they receive. Consumers are, therefore, heterogeneous in the cumulative information they need for adoption. The authors derive a diffusion curve by aggregating the expected individual adoption behaviour with respect to the potential adopter population. They obtain individual level parameters for price, risk and uncertainty through a survey among respondents.

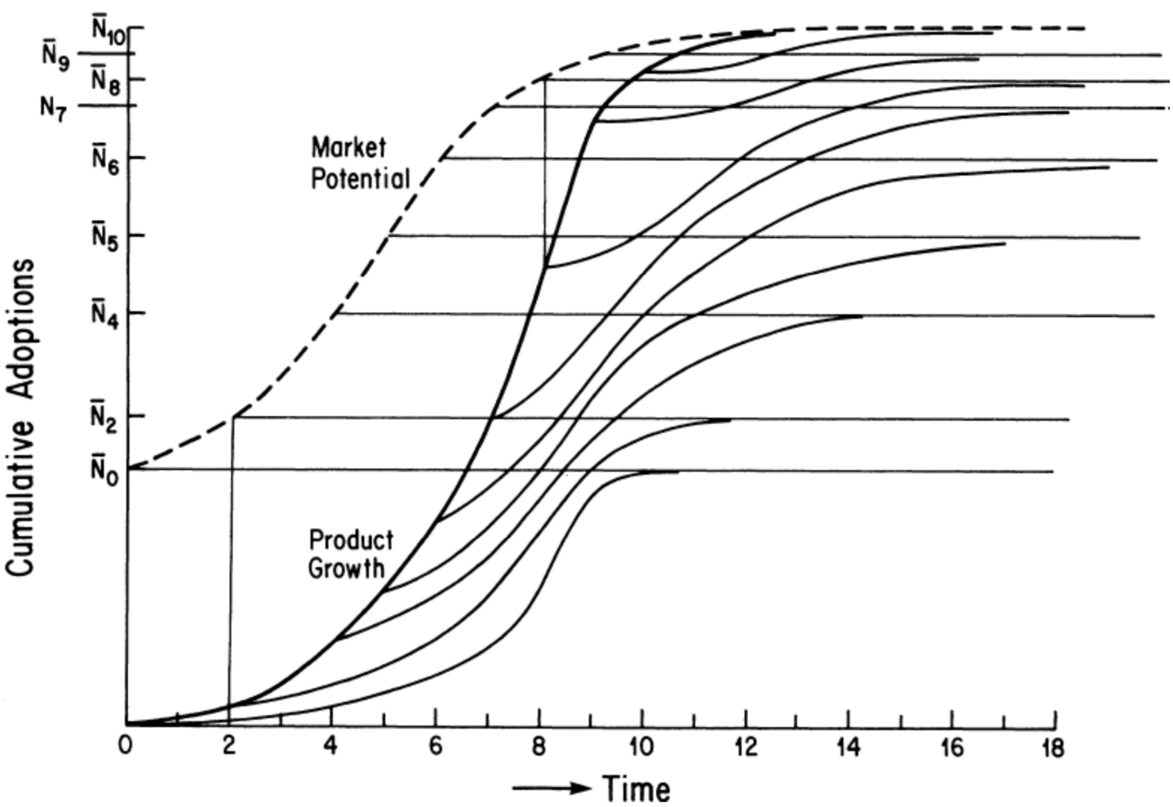
Karshenas and Stoneman (1993) and Stoneman (2002) describe "rank", "stock" or "order" models. In the models that consider the "rank" effects, the actors adopt as soon as the usefulness of the innovation exceeds a critical level or threshold. If the utility systematically increases over time and the thresholds follow a bell distribution, the cumulative number of adopters, id est the diffusion curve, will be in the sigmoidal shape. In models considering "stock" effects, the assumption is that the marginal benefit from adoption decreases with the number of prior adopters (Karshenas and Stoneman, 1993; Stoneman, 2002). Over time, cost of acquisition falls, increasing the number of adopters. As more firms adopt the new technology, the costs of production fall, increasing output. In the models that incorporate the "order" effects the hypothesis is that there are advantages of the first move in the use of a new technology. The returns to the company from the new technology depend on its position, with the higher-order companies obtaining higher profits than lower-level companies. Each company, considering how to move down the order affects its return, generates the path of diffusion. For any given acquisition cost, only some companies will find it useful to adopt in a specific point of the order. Karshenas and Stoneman (1993) determine the effects of rank, stock, order, and epidemic effects on the diffusion of CNC machine. They estimate a risk model and discover that the rank and endogenous learning effects play an important role in the diffusion process.

Assumption number 3: market potential of the new technology remains constant over time.

The fundamental diffusion model assumes that the market potential of a new technology is determined at the time of introduction and remains unchanged over its entire life. Theoretically, there is no rationale for a static potential adopter population because there are many exogenous factors (such as economic, social or technological conditions) and endogenous factors (such as product improvements or changes to distribution channels) that could affect it. Sharif and Ramanathan (1981) present good reasons for considering the case in which the size of the potential market changes over time. They point out that if the demand for the output generated by innovations grows over time, the number of potential adopters will also grow. They demonstrate that technological innovations are a particularly strong motivation for the entry of new companies. Moreover, they show that improvements on innovations widen their practical applications and increase the number of potential users over time. The precision of a dynamic diffusion model depends, to a large extent, on identifying the variables that affect $m(t)$ and on determining the way in which they affect: $m(t)=f(V(t))$, where $V(t)$ is a vector of all the potentially relevant exogenous and endogenous variables that affect *M(t)*, and *f(V(t))* is the functional shape of this influence. A sensible selection of these variables can help explain and clarify the reasons why the diffusion process of one innovation is much faster or slower than that of another. (Ruiz Conde, 2005).

The figure below shows the diffusion curve of an innovation when *M(t)* grows over time. As can be seen, when we consider a dynamic mixed influence diffusion model the ceiling of the cumulative number of adoptions, *M(t)*, is dynamic and grows over time; and the difference between the cumulative number of adoptions and the product growth curve decreases over time until the two curves finally meet.

Figure 6. Diffusion process with a dynamic potential market



Source: Mahajan, Peterson, Jain and Malhotra (1979)

Various authors have relaxed this assumption. We mention some works, especially the first ones who ventured into choosing to make the dynamic market potential. We deepen this assumption more than the others because it is the one we have relaxed for our models.

Dodson and Muller (1978) built two models that extend the mixed influence model. The movement from unawareness of the product to awareness is a function of a firm's advertising expenditure.

Mahajan and Peterson (1978) consider the mixed influence diffusion model and assume a market potential as a function of exogenous and endogenous variables such as socioeconomic conditions, population changes and government actions.

Sharif and Ramanathan (1981) represented the market potential as a function of population growth through three models with various specifications for market potential. The authors provided three applications and their results show the superiority of the proposed models, in comparison with existing models, in terms of forecasting accuracy.

Kamakura and Balasubramanian (1988) extend the models of external, internal and mixed influence diffusion by considering a dynamic market potential and incorporating price explicitly in the model. They assume two specifications for the dynamic market potential. They test their models on six consumer durables and the results show that price does not affect the market potential. This result is important and strengthens our model that does not directly introduce the price variable.

Among the most current authors, the work of Guseo-Guidolin is interesting. The Guseo-Guidolin model, arises from the need to define procedures for estimating the potential market in order to be able to quantify it in a more reliable way than possible with the Bass model. It is reasonable to consider that the market potential for an innovation can be influenced by the communication related to the new product: in fact, without knowing it, one cannot be a potential buyer. They observe that the variability of the market potential is particularly evident in the first phase of the spread, called the incubation period, where the success of an innovation is still uncertain. The incubation period is the time that passes from the product development (process that ends when the product is technologically ready) to the mass diffusion of the same. During this phase, the authors argue that marketing and management activities play a crucial role in stimulating the take-off of the product. Choen and Levinthal (1990) define the concept of "absorptive capacity" as readiness, receptivity and the ability to recognize the value of new information and to exploit it. This capacity is greater the more the prior knowledge on the subject is rooted. The intuition that led to the formulation of the Guseo-Guidolin model (2009) consists in considering the market potential as a function of this knowledge and therefore a direct measurement of the absorptive capacity, related to the purchase of the product, present in society. In fact, the Guseo-Guidolin model is the expression of a co-evolution of processes, whose advantage lies in producing a double amount of information compared to the use of the standard Bass model, requiring only aggregated data from inputs sale.

Price and the number of households are the variables mostly used to influence the market potential. Price has received the most attention due to its critical role in influencing the demand for a product (Kalish and Sen, 1986).

Assumption number 4: the parameters of external and internal influence remain constant.

All three types of fundamental diffusion model assume the parameters constant during the diffusion phenomenon.

Some authors relax this assumption by introducing the parameters of external and internal influence as functions of factors that affect the diffusion process: $p(t)=f(V(t))$ and $q(t)=f(V(t))$, where $V(t)$ is a vector of potentially important factors in the diffusion, $f(V(t))$ is the functional shape of this influence, $p(t)$ is a function that expresses external influence (time-varying external influence parameter) and $q(t)$ is a function that expresses internal influence (time-varying internal influence parameter). Easingwood, Mahajan and Muller (1981) developed the Non Symmetric Responding Logistic model (NSRL). Their model assumes that the word of mouth effect on potential adopters is flexible and may increase, decrease or remain constant over time.

Assumption number 5: there is only one adoption by an adopter.

The fundamental diffusion model captures only the first-time consumers. replacement or multiple adoptions. But, for a great many product innovations, the increase in the number of adopters may consist both first-time consumers and repeat consumers. In fact, this restriction of only one adoption per adopter can be valid for some consumer durables. Several authors have relaxed this assumption. Kamakura and Balasubramanian (1987) included product replacements in the Bass model to assess long-term sales for consumer durable products and they demonstrated the incorporation of replacement purchases into a diffusion setting even when replacement data was not specifically available. Norton and Bass (1987) assume that adopters continue to buy and that the average repeat buying rate over the population of adopters is constant. While these models throw light on how to capture replacement demand and multiple purchases, they do not give insights on what drives these processes.

Assumption number 6: the geographical borders of the social system do not change over the diffusion.

The fundamental diffusion model assumes that the innovation is confined to a certain geographical. Spatial diffusion models focus on the way innovation diffuse over space rather than over time as the prior models do. Casetti and Semple (1969), Haynes, Mahajan and White (1977) and Mahajan and Peterson (1979) extend the mixed influence diffusion model by integrating space and time dimensions of the diffusion of an innovation. Mahajan and Peterson (1979) introduce the notion of the "neighborhood effect" in technological substitution models in the marketing literature, that is, the further a region is from the "innovative region," the later substitution will occur.

Assumption number 7: diffusion of a new technology is independent of all other innovations.

The fundamental diffusion model assumes that the adoption of a new technology does not complement, substitute, or enhance the adoption of any other innovation. In reality, however, an innovation does not exist in isolation. Other innovations are present in the marketplace and may have an influence positive or negative on its diffusion. Consideration of simultaneous diffusion of multiple innovations is especially critical if the diffusion of one innovation is contingent upon the diffusion of another innovation (for example compact disc software and compact disc hardware) or if the diffusion of one innovation complements the diffusion of another innovation. Mahajan and Peterson (1978) extend the mixed influence diffusion model by developing four classes of multi-product growth models:

- independent products
- complementary products

- contingent products
- substitute products.

Furthermore, the fundamental diffusion model does not consider possible competition between companies or brands. Chatterjee, Eliashberg and Rao (2000) offer a good critical review of diffusion models incorporating competition.

Assumption number 8: the characteristics of a product and its perception do not change and do not influence diffusion patterns.

The fundamental diffusion model does not consider explicitly the impact of product features on diffusion patterns. These characteristics do not change in the life cycle of this innovation. This assumption is not adequate for many products, especially for those which are subject to continuous modifications and improvements.

Kalish and Lilien (1986a) analyse the impact of product characteristics on diffusion patterns. They consider the changing consumer perceptions of the product features as the product is accepted over time. The authors define the coefficient of imitation as changing over time due to changes in the product features.

Assumption number 9: there are no supply restrictions.

The fundamental diffusion model is a demand model. If the demand for a product cannot be met because of supply restrictions, such as the unavailability of the product due to limitations on production capacity, the excess unmet demand is likely to generate a waiting line of potential adopters (Mahajan, Muller and Bass, 1993).

Jain, Mahajan and Muller (1991) makes a contribution to the diffusion modelling literature by suggesting a parsimonious formulation that integrates both the supply and the demand sides of the diffusion process.

Assumption number 10: the diffusion of a product is not influenced by marketing strategies.

The fundamental diffusion model, implicitly captures the impact of marketing variables through the parameters p and q. These models are parsimonious and do not provide an understanding of the specific effect of these variables.

Many authors have relaxed this assumption by introducing marketing mix variables. We mention some: Horsy and Simon (1983) (by introducing the advertising), Robinson and Lakhani (1975) (by introducing the price) and Jones and Ritz (1987) (by introducing the distribution). In this thesis we do not focus on this aspect, but we recommend for those who want to deepen this topic, a good literature review of Ruiz – Conde, Leeflang and Wieringa (2006).

It is important to understand these assumptions. In building our class of models, let's relax one or more assumptions.

1.3 NETWORK EXTERNALITIES

In the original article of Bass and in following studies of the diffusion literature, the internal parameter q was interpreted simply as the influence of word-of-mouth between individuals. Lately, several authors have revised this interpretation to identify and discuss other types of social interactions. Among the market factors that drive the diffusion of innovations and involve the direct level of interpersonal communication there are the network externalities, the social signals and the social networks (Peres, Muller and Mahajan, 2010).

Peres, Muller and Mahajan revisit and extend the definition of diffusion of innovation according to them. Innovation diffusion is the process of the market penetration of new products and services, which is driven by social influences. Such influences include all of the interdependencies among consumers that affect various market players with or without their explicit knowledge (Peres, Muller and Mahajan, 2010).

Technologies such as ICT tend to exhibit different durations in the introduction phase, with varying speeds of adoption, before that the so-called "critical mass" is reached. In some ICT (like mobile phones, compared to fixed line communications), when the critical mass is reached, the diffusion process may even experiment explosive growth. Hence, the path to the critical mass is important in the study of ICT technologies, and the attainment of this threshold is much related to networks effects (see *infra*).

Technologies that have strong network externalities (see *infra*) generally have a long life on the market and are rapidly growing after having passed a critical dimension. This is the result of positive feedbacks that is the key feature of network industries: when a network user's base grows, more and more users will find it profitable to join that network; the value of membership to a network depends on the number of other users who have already joined it. Positive feedback makes strong growth even stronger and weaker growth is even weaker.

Main authors having identified positive feedbacks and network effects were Brian Arthur and Paul David. Arthur (1994) explores the adoption dynamics in a context in which increasing returns naturally arise: agents who choose between technologies competing for adoption. Modern and complex technologies often show increasing returns with adoption, as the more are adopted, the more experience is gained and the more are improved. When two or more technologies, with increasing returns, compete for a market of potential adopters, some “insignificant events” can provide one of them with an initial advantage in adoption. This technology can improve more than others and therefore it could attract a larger percentage of potential adopters. Therefore, a technology that, by chance, obtains an initial advantage in adoption, can eventually “affect the market” of potential adopters, excluding other technologies. Of course, based on different “insignificant events”[1] a different technology could have levels of adoption and improvements sufficient to get to dominate. Competition between technologies can provide different potential outcomes. It is known that adoption problems with increasing returns tend to exhibit multiple equilibria, and therefore it is not surprising that they present more results. Considering the possibility of “random events” occurring during the adoption, Arthur examines how these events influence the “selection” of the result, that is how some sets of random historical events could accumulate to guide the process towards a result of sharing the market; other events lead to other results. Arthur also discusses how the two properties of increasing return - unpredictability and potential

[1] For example, unexpected successes in the execution of prototypes, whims of early developers, political circumstances

inefficiency - are born; how increasing returns act to amplify random events during adoptions, so that ex-ante knowledge of adoption preferences and technological possibilities may not be sufficient to predict the "market outcome"; and how increasing returns could drive the adoption process in developing a technology that has a long-term potential. A dynamic approach could also indicate two new properties: inflexibility, since once a result (a dominant technology) begins to emerge, it becomes progressively more "locked"; and non-ergodicity, since the historical "small events" are not mediated, so that can decide the result. Arthur's work compares the dynamics of the "market shares" of technologies in conditions of increasing, decreasing and constant returns. The author pays particular attention to how returns affect predictability, efficiency, flexibility and ergodicity, and to circumstances where the economy is blocked from "historical events" to the monopoly of a lower technology.

David's previous ideas on the path-dependence of economic events on history were similar to those of Arthur and found a mathematical foundation in the dynamics of positive feedbacks, increasing returns and possible lock-in effects. He understood that he could use the mathematical foundations of Arthur's work to assert that the history of past events was not only a topic of cultural interest, but contributes to determine the states of economic equilibrium, influencing the choices of the actors. David researched on the history of typewriter keyboards. The result, his 1985 paper "Clio and the Economics of QWERTY", became a classic instantly. In the hands of David, the history of the QWERTY keyboard proved to be a particularly effective tool to demonstrate the importance of positive feedbacks. In this object of common use it appears in fact quite clear how the original decisions on the layout of the keys of the first mechanical typewriters could influence the way in which we still write to the personal computer, forcing us to use a keyboard that today seems inefficient and inadequate. Although some better alternatives have been proposed for some time, at that point the collective cost of switching to a different standard of keyboard would have been too high: we therefore remained anchored to this inefficient choice, because of the 'accidents' happened in the past. Hence, today's choices are conditioned by the past, that is, they depend on the "path" initially followed. The obvious conclusion is that the QWERTY keyboard is not at all the best.

All this really appears to be something accidental in the history: a standard is able to establish itself on competitors and become universal even though it is not technical superior. The economic principles that explain the accidents in terms of dependence on the path are called by David "QWERTYnomics", and they recall the essential concepts of the so-called "network economics". David explains the QWERTY domain based on three key factors: indirect network effects (technical interrelatedness), economies of scale and switching costs (quasi-irreversibility of investments). The terms used by David (1985) are different from those that were established in the following decade, but the basic concepts are the same: the technical interrelation between hardware (the typewriter) and software (the "mental" programs of typists) corresponds to the technical compatibility underlying the indirect network effects in hardware / software systems; the economies of scale that David calls "system" correspond to the sum of economies on the demand and supply side; the "quasi-irreversibility of investments" refers to the sunk costs related to specific investments, such as those on training on the QWERTY keyboard, which cannot be recovered when moving to a different standard: they are therefore switching costs. David also observes that the typewriter hardware requires, for optimal use, the storage of sequences and appropriate procedures (software) by typists, thus generating indirect network effects: the greater availability of typewriters QWERTY determines indirectly a wider market of expert typists, with "system" economies of scale, ie both on the demand side and on the supply side. As QWERTY became dominant, even the producers who had adopted competing standards tended to adopt it: the technical conversion of non-QWERTY production was relatively inexpensive, especially when compared to the retraining of the majority of typists who were already using "Universal". Even more so when the most efficient DSK standard emerged: QWERTY was now so widespread as to make the transition too expensive. Therefore, the course of past events can influence subsequent choices and history matters.

Always David (1990), he used the history of the electric motor as an analogy for computers. The author observed that the contribution made by the electric motor to productivity was initially slow, because it took time to improve both the quality of the instrument and the ways to use it. He then argued that computers, which at that time did not generate a major impact on productivity, would follow a similar path and that there would be increases in productivity as the diffusion process developed. And his prediction again proved to be correct.

Network effects describe a situation where the utility that an individual derives from the adoption or use of a good depends, in a positive or negative way, on the number of other individuals who adopt the same good and are thus connected to each other a network of relationships. In other words, when the size of the network of adopters influences the utility that good brings to the individual. The consequence is that the adoption of an asset by an individual indirectly causes an increase (decrease) in the benefit to other individuals who bought it. Focusing on the case of positive network effects, we can state that these effects tend to become significant at achieving a certain numbers of users (installed base) defined in critical mass literature. The critical mass is the minimum number of users of the network making it convenient become part of the same network. It is only by overcoming the critical mass that the benefits considered become significant and potential users will begin to buy a good that has got positive network externalities (Economides e Himmelberg 1995).

A technology exhibits network effects (or network externalities), for the individual consumer, when the value of the product depends on the number of adopters who use the same product, or on the number of the compatible complementary goods available (Liebowitz and Margolis, 1994). This could apply to mobile phones or the broadband access to the Internet, that represent original extensions after the old style consumer products studied before in the literature (like consumer electronics goods such as DVD recorders and TV sets).

Network externalities exist when the utility of a product to a consumer increases as more consumers adopt the new product (Rohlfs, 2001). A positive network externalities is defined as increasing of consumers' utility of an innovation when the total number of users of the same product increases. These network externalities are considered to be:

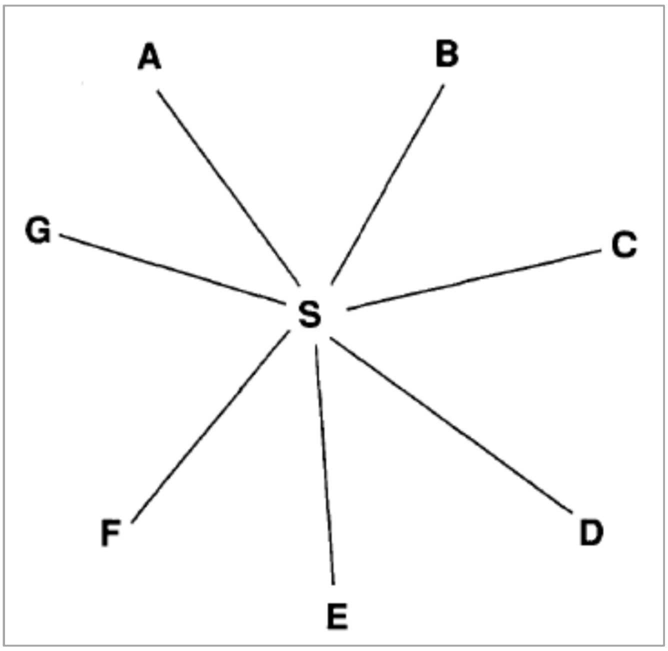
- *direct* if utility is directly affected by the number of other users of the same product, as in the case of telecommunication products and services such as phone, internet and e-mail (Peres, Muller and Mahajan, 2010). This, for example, can be motivated by the fact that he (or she) can interact with more people. By analogy, it can be assimilated to a sort of economies of scale from the demand side. The presence of direct network externalities also benefits network product manufacturers because, overcame the critical mass, they may have increased scale returns in the production. Direct externalities also emerge in the absence of a physical network.

- *indirect* if the utility increases with the number of users of another, complementary good, content or updates. For example, in the PC market if more consumers use the same type of hardware, software producers will be required to produce a wider variety, compatible with that type of hardware. On the other hand, an increase in the variety of software compatible with that ' hardware, will increase demand and reduce the price of the latter if there are economies of scale in hardware production.]

Now let's see how a network is structured and we can better understand the difference between the two types of network externalities.

A network is typically composed of nodes connected to each other by means of connections through which streams of energy (electricity), information (sounds, voices, images, data) and materials (water, goods, passengers) are transmitted. Let us consider the following figure (fig. 6), which describes a star topology telephone network (Economides, 1996): S is the central switch (central exchange unit) that communicates the different nodes A, B, C, etc. (which can be imagined as the location of consumers making calls); AS, BS, etc., are called calls and represent the connection of users to the central unit; ASBs, BSAs, ASCs, etc., are compound goods id est phone calls.

Figure 7: star network



Source: Economides (1996)

Network services consist of several components and each component can have several substitutes. For example, each asset ASB consists of AS and BS that are complements (and which can be thought of as "switch accesses") and have ASC, ASD, etc. as substitutes. The network of fig. 6 is the simplest representation of a two-way network, such as telecommunications, railways and roads. In the specific case of telecommunication networks, a direct network externality is recorded. Observing the figure 6 we can say that in such networks, such as the telephone or the Internet, the connection AB is different from the BA, it can take place either from A to B and from B to A. This type of network is like we first defined a Two-Way Network. The two authors, in accordance with Metcalfe's law, observe that *n (n - 1)* connections are possible in a network like this, where n represents the number of users in the network. When a new user joins a network generates *2n* new potential connections, providing positive direct network externalities. Summarizing the characteristics:

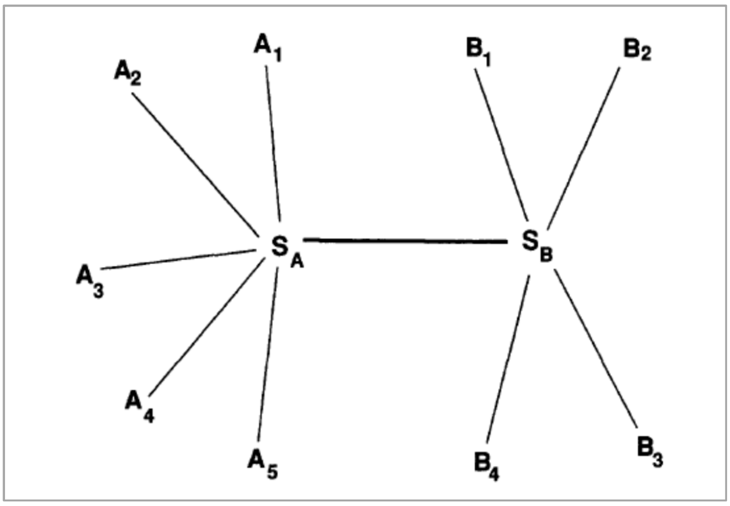
- all the components (AS, BS) are complementary to others so that they can form several composite goods such as ASB, BSC;
- the components are complementary to the others even if they are similar (they are all phone calls);
- the composite goods ASB and BSA are different, but similar, because they are travelled in both directions (A calls B and B calls A);
- the consumers are identified with the components (A is a user getting a phone);
- the composite goods that share a component (for example ASB and BSC) are not necessarily close substitutes;
- the components must not necessarily be compatible so they can always be combined; the producers intentionally choose the compatibility by joining to particular technical standards;
- as already said, direct network externalities are created so that adding a new spoke (component) to the previous *n*, it creates *2n* new composite goods.

Therefore, two-ways networks are characterized by direct externalites because a user's utility is positively conditioned by the use of the network by another user; id est entering of a new consumer into network has a positive and direct impact on the utility of old consumers. The demand for a network good is a function of both the price of the product and the expected size of the network. It is therefore necessary to determine the relevant size of the network. For the two-way networks the problem is whether users of a service, provided by a company, can contact the users of the service provided by another company. If the two services are compatible, the relevant network consists of the total number of subscribers. If the services are incompatible, the relevant network is measured by the users of the single service.

If we consider more complex representations of networks, some complementarity between the components is missing. Observing the figure below SASB is called a gateway linking two central nodes SA and SB; A1SAA2 and B1SBB2 are, as before, composite goods, but components such as A1SA and B1SB become complementary and can be connected only in the presence of the SASB gateway to form the A1SASBB1 good. With the following scheme (fig. 7) you can find two types of externals:

- *local network externalities*;
- *long distance network externalities*.

Figure 8. Local and distance network



Source: Economides (1996)

This second more complex representation of a two-way network can also be used to represent a one-way network where only long-distance composite good, as A1SASBB1, makes sense. In a typical one-way network there are two types of components that are combined to form a composite goods. Examples are PCs, consisting of hardware and software, movies available on DVDs and DVD players, credit cards and POS systems. In this case we can refer to the joint use of two complementary goods and for example we hypothesize that A represents a good like that of credit cards and B represents POS systems. A and B are, therefore, two complementary goods whose only the composite use can generate utility, so that A becomes equal to B. An increase on demand of the good A will necessarily lead to an increase on demand of the good B and vice versa, producing indirect network externalities: If more credit cards are offered on the market for this type of support, the number of POS systems will also increase on demand and vice versa. Summarizing, the features of a one-way network are:

- there is no reciprocity, id est goods and services A1SASBB1 and B1SBSAA1 are equal;
- consumers are generally not identified with components since they buy components;
- composite goods that share a component are generally substitutes;
- also a one-way network exhibits network externalities, but these are indirect: as we have just mentioned earlier, if there are *n* type A components and *m* component type B, we have *nm* composite goods; if we add a new type A component, we create new composite good; if we add a new type B component, we create n new composite good, so a user, who asks for new components increases the demand for components A and B and therefore the number of compound goods available on the market available to all other consumers.

The distinction between a one-way and a two-way network must be sought in the different modes of accessing users to the network. In a one-way network generally access is controlled by a monopoly company, with a number of good and upstream and downstream service companies. An example is the electricity supply service where, because of the high fixed costs needed to build a network, it is not possible to have more than one (one service provider). Instead, two-ways networks may have multiple structures connected to each other. For example, if a user wants to subscribe to a single phone operator to communicate with subscribers to other operators, it is necessary for the various operators to agree to each other to ensure signal transmitting in both directions.

We will primarily deal with the first type of network externality, and thus we will not deepen again about the indirect network externalities. Before discussing about critical mass, it is necessary to talk of the sources of network externalities. Such origins must be sought in agents 'expectations, agents' coordination, component complementarity and agents' compatibility and in switching costs of incompatible technologies. Also in this section the reference literature will be Economides (1996).

Consumer expectations, when choosing products or services in a network market, have a decisive impact on the sales of these products or their complements, because the utility of each consumer is linked to the number of other consumers who will buy the same product. So, the value of the good depends also on consumers' expectations about the future extent of the network, so the demand is not only a function of the price of the good, but also of the actual size of the network. The good demand curve for the good network might not have the usual negative slope, but have a growing trajectory: the marginal consumer of the good can gain more utility than the intramarginal consumer because a larger network attributes a value to the good that makes up for the reduction in value due to the marginal unit purchase. On the other hand, producers will try to influence consumer expectations to maximize their profit, especially when consumers have imperfect information about the size of the installed base.

Coordination is a demand side question: as their utility functions are interdependent, each user must anticipate which technology will be adopted by other users. For consumers, changing choices owing to the introduction of a new technology could be very expensive (even if the new technology is more efficient than the old one) if the products were incompatible. In addition, they may have different preferences regarding the technology to be adopted. Two types of inefficiency can emerge: excessive inertia and *excess momentum* (Farrel and Saloner, 1985). a situation of *excessive inertia* emerges when each user is afraid to move alone and prefers not to adopt new products; In this case, companies are not stimulated to invest in innovation. Instead, an excessive momentum emerges when each user is afraid to remain the only user of the old technology and switch to the new one; In this case, companies invest too much in innovation.

Compatibility is an issue related to the supply side and needs to be defined by linking two or more systems. Two-way networks are incompatible if users in a network cannot communicate with users of the other network; two one-way networks are incompatible if components of a system cannot be combined with the components of the other system. In a market characterized by network externality, you might think there is a natural tendency towards standardization, that is, the use of a single standard by all consumers. Really, there is a trade-off among the benefits of standardization and the benefits of a greater variety of marketable systems attributable to the difference in tastes of consumers (who might prefer products with different characteristics) and the problem of the inefficiency of the only standard. But compatibility remains an important strategic choice for companies. For two-way networks, compatibility prevents duplication of facilities for the provision of services, resulting in reduced costs. For one-way networks, compatibility leads to a reduction in costs due to the use of economies of scale.

Switching costs are barriers that prevent consumers from switching from one network to another due to the adoption of incompatible technologies. When a consumer decides to switch to a new standard, he must consider two types of costs: a private cost linked to the investment in the original technology, and a social cost linked to the comparison of the benefits expected from joining the new network and the benefits of the old network.

As previously anticipated to trigger network externalities, it is necessary to reach a critical mass of adopters beyond which, for the individual user, not belonging to the network, becomes a significant disadvantage. This is because the marginal user's advantage over a certain threshold is much greater than that of the first users. This will push a new mass of users to choose to join the network, thus generating positive feedback that will determine network externalities.

But what is the “critical mass”? The term “critical mass” is usually used in physics to indicate the amount of radioactive material needed to create a nuclear fission. However, it is often used metaphorically in social studies, to refer to the amounts of participants (or individual actions) necessary for the occurrence of collective action (Oliver, Marwell and Teixeira, 1985). The phenomenon has been extensively studied by Oliver, Marwell and Teixeira (1985), observing that critical mass plays a crucial role in the production of different types of collective action and in particular examines the case of the supply of a public good. According to the authors, collective actions often result from the actions of a group (critical mass) belonging to the community that behaves differently from other members of the community. Therefore, they assume that the heterogeneity of community members can play a fundamental role, that is: the higher the degree of heterogeneity, the higher the probability of generating a critical mass that is triggered by mass actions. Sometimes such a mass generates benefits for all members of the community, despite of the fact that the other members do not pose any beneficial action. On the other hand, however, it takes on the initial costs coming from its actions aimed at achieving common benefits, and subsequently generating collective actions that will have the greatest benefits for the community. In the case examined by researchers, there are two particular functions of public goods production: the first one is referred to the decreasing marginal benefits of individual contributions, while the second is referred to the increasing marginal benefits of individual contributions. In the first case, the first taxpayers have a strong impact on the ability to obtain public goods. In the second case, however, the marginal benefit of new taxpayers grows as the number of taxpayers increases. The critical mass concept that interests us in this work is more closely related to the second typology of production function. We refer to networks where new users have marginal benefits more and more as the number increases. This could mean that the community group most concerned with network development plays a key role in the initial phase. This group could take on the initial costs of network development because it is more interested in getting the potential benefits. The increasing network size such draws attention of the less interested people, who will be attracted by the strong marginal benefits they could obtain.

The size of the network is also influenced by the price of products and consumer expectations. Some information about good future sales are imperfect, consumers' expectations can lead to a sub-optimal network amplitude. For example, if all consumers think nobody would buy the good, then the network would have no size, even if everyone would benefit from being there; if they all expect to buy the product, the network would be broad. Thus, the demand for network goods will be a function not only of the price but also for the consumers' expectations on the future size of the network. The construction of the demand function is essential for the determination of critical mass, that is, the minimum size of the network that can be sustained in equilibrium, due to the costs and the structure of the market. The interpretation of the critical mass is related to the paradox of "chicken and egg": if consumers expect the network to be small, they will not join it. Conversely, if no consumer joint it, the expected size of the network will be small. The critical mass is observable in different market structures. Economides and Himmelberg (1995) compare perfect competition, monopoly and oligopoly with compatible goods and describe under what conditions the critical mass exists. They demonstrate that the market structure does not influence the existence and size of critical mass.

The effects of network externalities on diffusion are seen especially after the flex point, when the critical mass is activated. Several authors suggest that these network effects should lead to a faster diffusion of technology thanks to the bandwagon effect (Economides and Himmelberg, 1995; Rolfhs, 2001; Shapiro and Varian, 1999; Goldenberg, Libai and Muller, 2010, Guseo and Guidolin, 2009 and 2010).

Shapiro and Varian (1999) attribute network externalities to positive feedback and they suggest that “if a technology is on a roll … positive feedback translates into rapid growth: success feeds on itself.”

However, networks, for example through adverse expectations - can also create the opposite effect of slowing down diffusion dynamics - in particular this is frequently the case of many ICT that do not succeed in the market. Potential users and consumers don’t have sufficient knowledge to evaluate the advantage implied by a new service or product compared to investment’s risks. Indeed, many ICT experience difficulties right in the first part of their life cycle, because a limited knowledge about their features, or even about their existence, prevents users to adopt them. This phenomenon is called “chilling effect” (Goldenberg, Libai and Muller, 2010). Goldenberg, Libai, and Muller (2010) use an agent-based model to demonstrate that network externalities have this type of effect on new product diffusion, id est. they slow down new product adoption since many consumers wait before enough people have adopted. They perform their simulations using theoretical Moore lattices as the underlying social network structure of the consumers (Mukherjee, 2014).

Our modelling work remains exclusively aggregated, and we follow the line of Katz and Shapiro when they summarize the evidence that consumer utility can be a function of the size of the network (Katz and Shapiro, 1986, 1994).

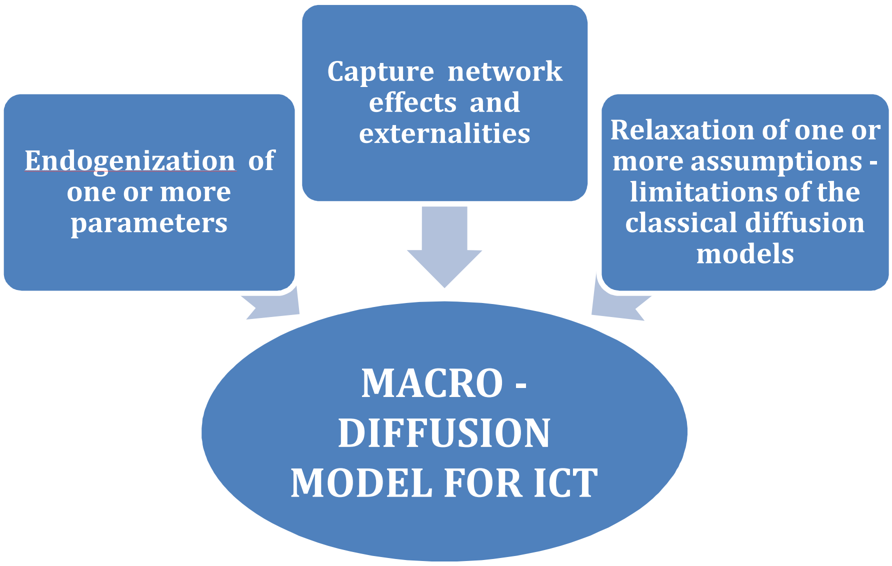
CHAPTER 2

BUILDING A MACRO DIFFUSION MODEL

2.1 CLASS OF MACRO – LEVEL DIFFUSION MODELS WITH NETWORK EFFECTS

We build a class of models (to start with, the three illustrated below) with the market potential variable that capture the network effects by extending the standard structure of the mixed influence model (in particular the standard Bass (1969)-type model) through the endogenization of this key parameter.

Figure 9. Synthesis of my work



Source: our elaboration

One of the characterizing assumptions of the Bass model relates to the size of the market potential *m*, whose value is fixed at the time of introducing the new technology and remains constant along the whole diffusion process: this assumption is the least acceptable from an economic point of view, face to the last generation of ICT. To take into account the impact of the accessibility of new technologies on a growing number of users, several authors have proposed dynamic models. In fact, the issue of a variable market potential is not new to the diffusion literature (Mesak and Darat, 2002; Jain and Rao, 1990; Guseo and Guidolin, 2009 and 2010), but from the point of view of macro-diffusion, it has not been adequately addressed. In all models the reasoning behind is that the larger the size of the network of adopters of a technology, the greater the value of this innovation to potential adopters, and, therefore, the higher the probability of its adoption.

The Bass model is great for explaining the diffusion phenomenon and it incorporates the count of network externalities, but it fails to capture it adequately because the assumptions on the diffusion parameters are very restrictive. In addition, for the first phase of the life cycle of technologies that show network effects, the number of potential adopters tends to be overestimated, and hence the diffusion rate calculated initially results higher than what actually happens: these products, in fact, are characterized by delays, due to limited knowledge of their features and benefits in terms of utility. The model extension here goes through by changing the market potential, whose determination changes from static to dynamic:

$$\boldsymbol{n(t)} = \left(\boldsymbol{p} + \boldsymbol{q}\, \frac{\boldsymbol{N(t)}}{\boldsymbol{m(t)}} \right) [\boldsymbol{m(t)} - \boldsymbol{N(t)}] \tag{2.1}$$

Let's point out that the socio-economic system of reference, in a context of macro-diffusion modeling, is a country. This choice is also consistent with the availability of aggregated data.

In the country of reference, only a portion of its population considers useful to adopt an innovation at any time. This is because not all of them contemplate the actual utility or simply because they are not interested. So, let's not consider the fraction of the population that do not want to adopt, but just the portion that think is useful enough to buy or adopt a certain technology. This concept is important to reinforce the very meaning of the market potential.

The market potential variable will depend on the size of the network of users and consumers of the technology in question. Its growth will be positively influenced by the network of adopters, as the higher the value of the technology for the same potential adopters, the greater the probability will be the adoption of that technology.

Naturally, the influences generated by the network are not necessarily positive, they can also be negative. To simplify the model, we take into account only the network effects generated by positive word of mouth, while we neglect the negative effects.

A further clarification is necessary: we take into consideration only the ICT technologies that spread with the increase of the network and therefore do not consider the so-called snob products, such as the IPhone. The diffusion of these particular products follows a different trend: in fact, the potentials who want to adopt this type of technology are guided by a search for an exclusive product rather than by the utility induced by other users who own this product.

The network will consist simply of those who have previously adopted, $N(t)$. So, a fraction of the population and consequently the market potential will depend on the network size. The greater the cumulative adopters $N(t)$, the higher will be the value of $m(t)$.

The portion of the population that considers useful and profitable to adopt the technology is nothing other than the market potential in the country of reference:

$$\boldsymbol{m(t) = Z(t)\, Pop(t)} \tag{2.2}$$

where $Z(t)$ is a functional form bounded between 0 and 1 $(0 < Z(t) \leq 1)$.

The variable $Z(t)$ can never assume the value 0, because there can be no diffusion without a fraction of the population that is interested in adopting the product and consequently the total market potential cannot be null. The maximum value 1 is restrictive in the case in which one wishes to maintain the assumption of only one permitted adoption. But for some technologies such as mobile telephony, also a multiple adoption and not just one could be considered. Just think of the possibility of having more than one sim. For these particular technologies we can also relax the assumption of a single allowed adoption and therefore the variable $Z(t)$ can also assume values greater than 1.

Depending on the shape assumed by the variable $Z(t)$, we can have different models that represent all extended versions of the Bass model.
We have developed three different forms of $Z(t)$:

$$\boldsymbol{Z(t) = \beta + \gamma \frac{N(t)}{Pop(t)}} \tag{2.3}$$

$$\boldsymbol{Z(t) = \left(1 - \frac{\alpha}{1 + \frac{N(t)}{Pop(t)}}\right)} \tag{2.4}$$

$$\boldsymbol{Z(t) = \left(\beta + \frac{\alpha}{1 + e^{-\frac{N(t)}{pop(t)}}}\right)} \tag{2.5}$$

As a result, their respective markets potential are:

MPDL (2.6)

$$\boldsymbol{m(t) = Pop(t)\,\beta + \gamma\,N(t)}$$

MPD (2.7)

$$\boldsymbol{m(t) = \left(1 - \frac{\alpha}{1 + \frac{N(t)}{Pop(t)}}\right) Pop(t)}$$

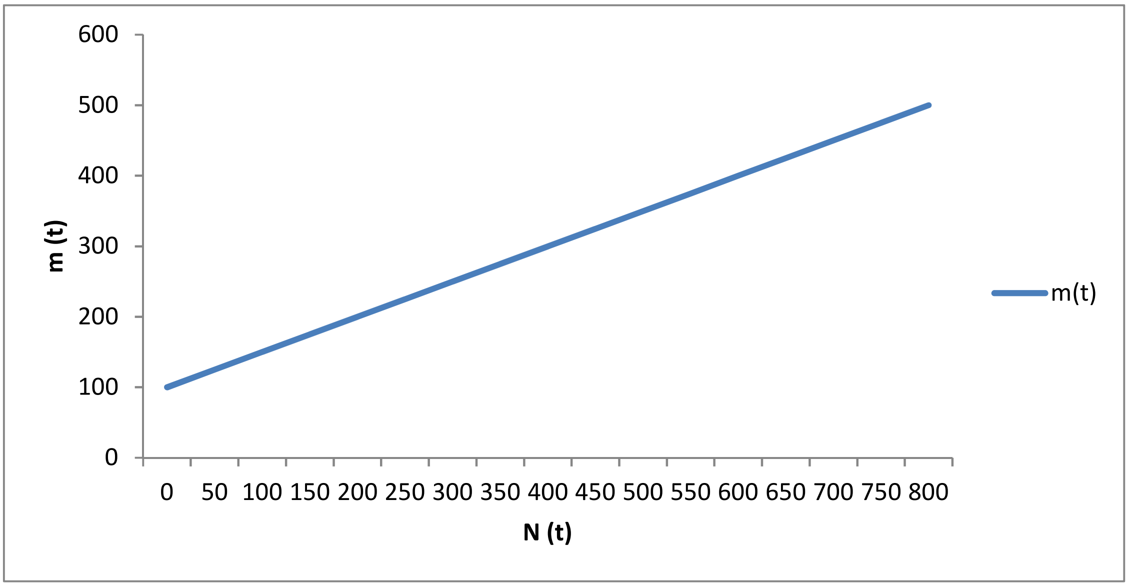
MPDT (2.8)

$$\boldsymbol{m(t) = \left(\beta + \frac{\alpha}{1 + e^{-\frac{N(t)}{pop(t)}}}\right) Pop(t)}$$

where *pop(t)* is the population of the reference country, β is the parameter that indicates the fraction of the initial adopters that does not depend on the size of the network; α is the parameter that indicates the fraction of the later adoptions that are connected to the initial fraction of adopters β, (with the sum $\alpha + \beta = 1$); γ represents the network effect that describes the intensity and the power of the network, and *N(t)* is the same network.

The three different types of functional forms present different characteristics related to the relationship between the potential market and the cumulative of adoptions, id est the network. The first model having market potential described by expression (2.6) is called MPDL, market potential dynamic linear. The relationship between the market potential and the size of the network is linear. The graph below (graph 1) shows the linearity of the relationship.

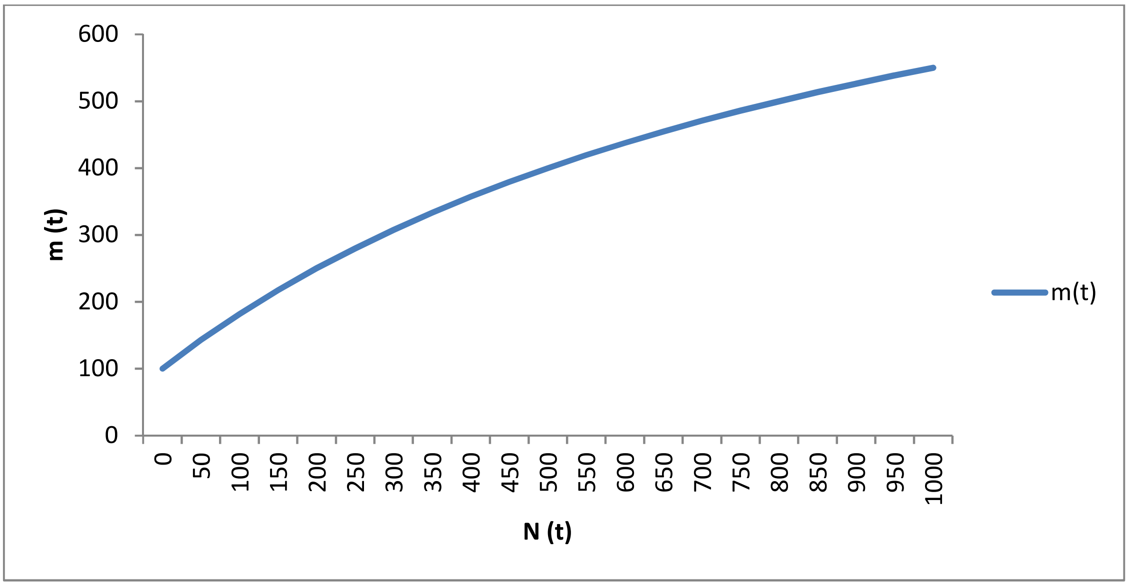
Graph 1. Relation Market potential – Network Size of MPDL model.



Legend: Relation Market potential – Network Size of MPDL model.

It is important to note how the network parameter intervenes in strengthening the positive dependence between the market potential and the network of the adopters. The second model with market potential described by expression (2.7) is called MPD, market potential dynamic.

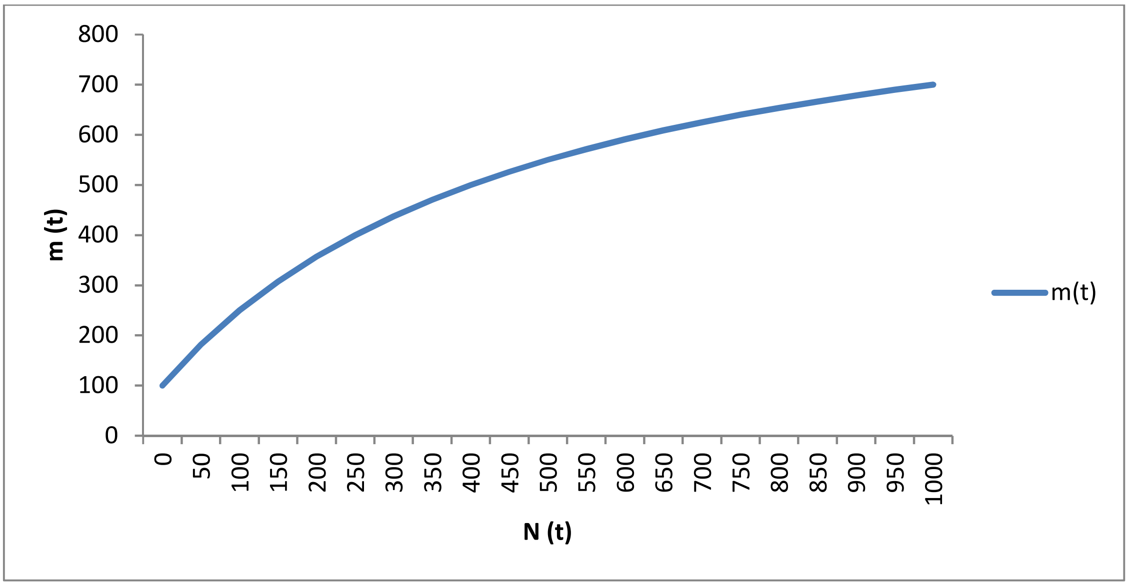
Graph 2. Relation Market potential – Network Size of MPD model.



Legend: Relation Market potential – Network Size of MPD model.

In this case the relationship between the market potential and the size of the network presents a non-linear trend (as shown in the graph 2) that can be further amplified by a parameter (γ) that describes the strength of this network (graph 3). To distinguish it from the previous case, we add to the name an n that means network.

Graph 3. Relation Market potential – Network Size of MPDn model.

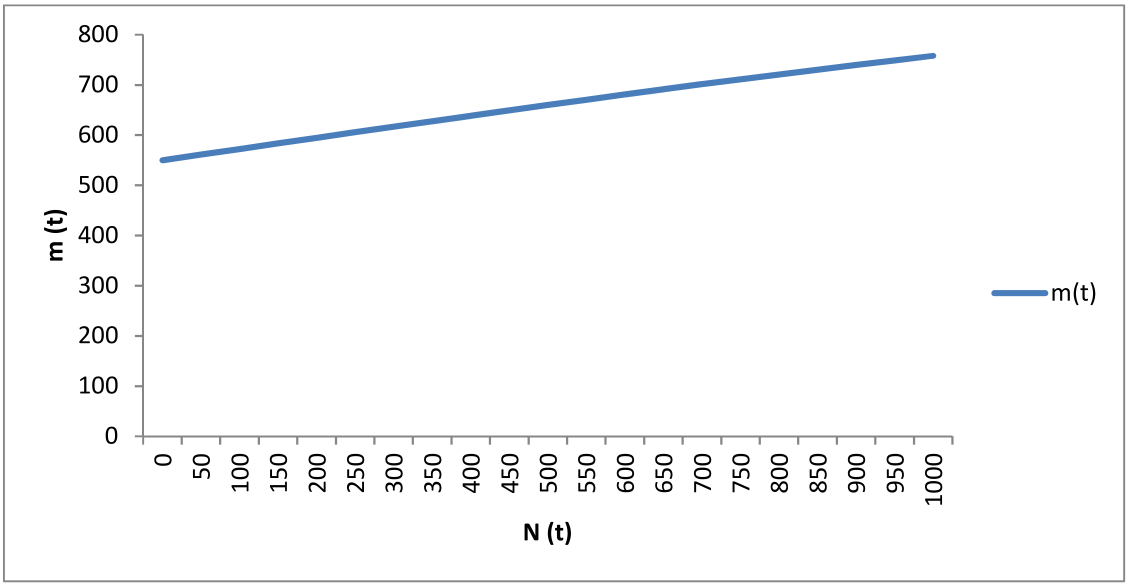


Legend: Relation Market potential – Network Size of MPDn model.

The size of the network is particularly important at the beginning during which the number of potential adopters increases more than in the subsequent phases.

The third model is called MPDT (2.8), market potential dynamic technology. The market potential in relation to the cumulative *N(t)* tends to take a slightly non-linear trend.

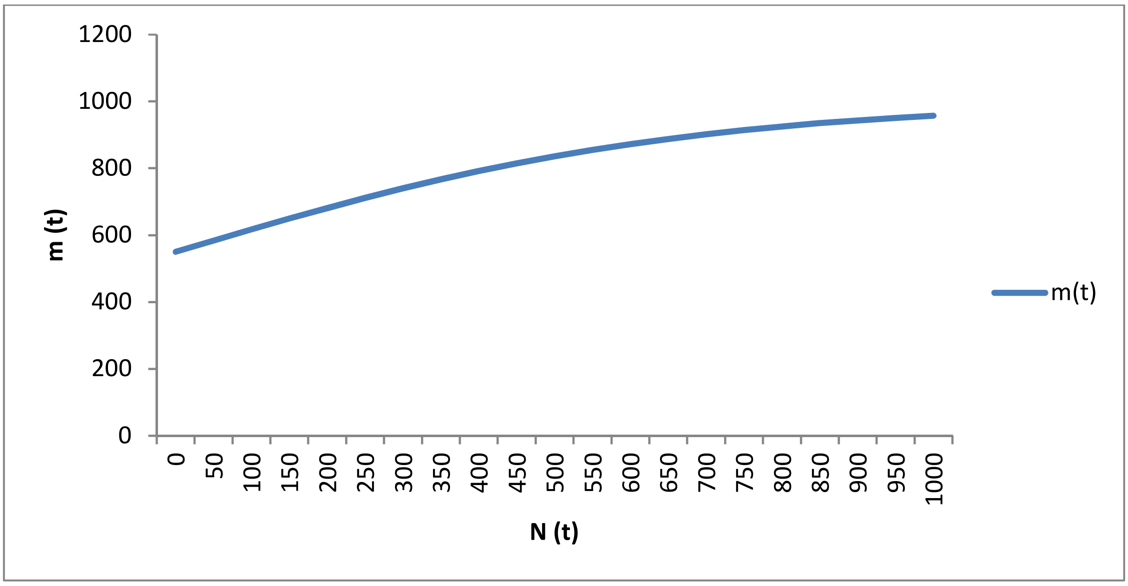
Graph 4. Relation Market potential – Network Size of MPDT model.



Legend: Relation Market potential – Network Size of MPDT model.

In the initial phase the value of the market potential is higher than the previous models. Also in this case we can introduce a network parameter (γ) to amplify this relation, in particular in the presence of strong network effects ($\gamma > 1$) as we can see from the graph below (graph 5).

Graph 5. Relation Market potential – Network Size of MPDTn model.



Legend: Relation Market potential – Network Size of MPDTn model.

This last model can be indicated for the technologies that tend to anticipate the S diffusion curve, products that spread quickly and reach soon the saturation phase.

The population can be considered, depending on the type of technology, the demographic population or the sum of households and companies. Let us suppose that *pop(t)* is constant, neglecting significant demographic shocks. The parameters β and α will be comprised each between 0 and 1 ($0 < \beta < 1$). Naturally, β, being the fraction of the population during the initial phase, reasonably assumes values lower than 0.5. The parameter γ represents the power of the network and will have values $0 \leq \gamma \leq 1$, with $\gamma = 0$ identifying no network effects. Under this last condition, a constant potential market is restored. The greater will be the market potential. In addition, greater network power generates at higher rate of diffusion and a high adoption rate.

Various combinations of the β, α and γ parameters give rise to different diffusion curves. We calculate the market potential *m(t)* e and replace it in the following Bass model equation (2.1):

$$\boldsymbol{n(t)} = \left(\boldsymbol{p} + \boldsymbol{q}\,\frac{\boldsymbol{N(t)}}{\boldsymbol{m(t)}} \right) [\boldsymbol{m(t)} - \boldsymbol{N(t)}] \tag{2.1}$$

But we cannot get an analytical solution. We can, however, solve numerically the corresponding difference equation and trace the different diffusion profiles through simulations; in a second step of the research, we will subsequently make the estimation of the various parameters directly from the discrete models.

Thus, the instantaneous adopters can be calculated by the difference equation:

$$\boldsymbol{n(t+1) = \left(p + q\,\frac{N(t)}{m(t)}\right)[m(t) - N(t)]} \tag{2.9}$$

Various combinations of α, β and γ parameters give rise to different diffusion curves. When γ and β values are low and α values are high, the market potential is stable for different periods of the diffusion process.

Technologies that exhibit network externalities are characterized by a low fraction of initial adopters and powerful network externalities. Adoption is slower in initial periods, but increases relatively quickly when a certain threshold for adopters is reached. Sigmoidal curves may present asymmetries or deviations from the regular logistic pattern.

The proposed model class does not include the presence of the price as independent variable. At first glance, it might seem a considerable simplification, but there are both theoretical (higher complexity and lower analytical tractability of the model) and practical reasons. Concerning the latter, we need to point out that in some ICT sectors the cost conditions (based on high fixed infrastructural capex expenditures) do not enable easy pricing decisions, and there is not a reference market price. Hence, it would be very difficult to get meaningful market-representative time-series for ICT and utility prices, in the following econometric analysis. This is particularly valid for fixed broadband, that will be the main object of our empirical applications.

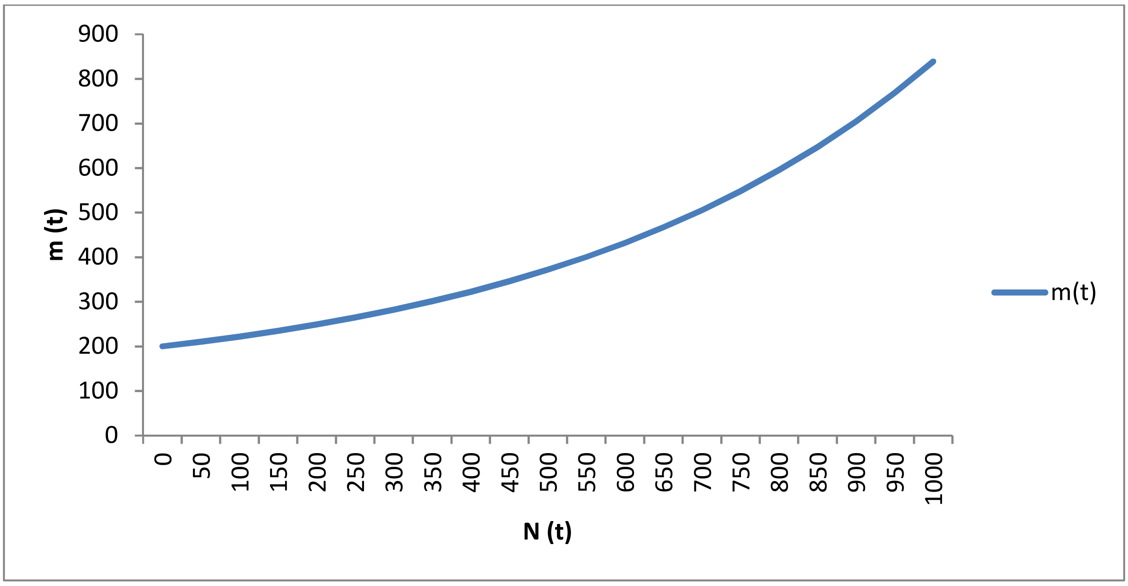
2.1.2 Further model with dynamic market potential

To the class of models just described, we add a model that presents a variable market potential depending on the size of the network constituted by the number of agents that have already adopted. In this case, we have also taken into consideration the initial potential market which also becomes crucial because it will have a dual role, id est it can also influence the number of potential adopters interacting as a network and with the network that is created. The formulation of the functional form is completed by the introduction of a network parameter that measures the intensity and strength of the network itself. Moreover, the relation of the potential market, considering the size of the network, assumes a nonlinear trend guaranteed by the exponential form of market penetration. The following expression is called MPDII, market potential dynamic initial influenced:

$$\boldsymbol{m(t)} = \boldsymbol{m_0}\left(\mathbf{1} + \boldsymbol{e}^{\gamma\frac{\boldsymbol{N(t)}}{\boldsymbol{pop(t)}}}\right) \tag{2.10}$$

As we can see from the equation, the initial potential market is important for determining a large number of potential total adopters. A high number allows a higher probability of generating a greater network which can therefore positively influence the number of potential adopters. The exponential form of market penetration confers a nonlinear trend of diffusion typical of most ICT products. The network parameter can assume values greater than 0 and can exceed the value 1 when there are strong network effects. This assumption allows you to have different curves depending on the strength of the network. In fact, when the value of γ is very high, the market potential curve in relation to the size of the network can have a considerable convexity compared to low levels of γ between 0 and 0.5. In the figure below (graph 6), we can see an example of the relationship between the potential market and the size of the network with strong network effects ($\gamma > 1$).

Graphs 6. Relation Market potential – Size Network of MPDII model.



2.1.3 Model with exogenous market potential

After discussing about class of models with dynamic market potential, we introduce a model with a constant potential market that depends on a network parameter and follows the functional form of the MPD. Again, we have an extension of the Bass model. In contrast to the MPD model, this model does not present an endogenous market potential. In fact, the potential market is exogenous because it depends on other parameters (such as the network parameter that is introduced) and does not depend on variables over time.

The functional form of the market potential is:

MPDR (3.11)

$$\boldsymbol{m_R} = \left(\mathbf{1} - \frac{\boldsymbol{\alpha}}{\mathbf{1} + \boldsymbol{R}}\right) \boldsymbol{Pop}$$

where *pop* is the population of the reference country, α is the parameter that indicates the fraction of the later adoptions and R is the network parameter that measure the size of network.

Like the MPD model, α is connected to the initial fraction of adopters β (with the sum $\alpha + \beta = 1$).

As with previous models, the higher α, the greater is the market potential.

As the network increases, the value of the market potential is higher and therefore the probability of adopting the technology is higher.

The model, with this form of the potential market, is a differential equation with an analytical solution.

We now proceed to simulate the models so far analytically presented.

2.2 MODEL SIMULATIONS AND RESULTS

To test the dynamics of the class of the models, in this first step of the research project, we carry out a series of simulations. In a second step, the class of our models will be empirically estimated with real world market data (in primis, substituted into *n(t)* and *N(t)*, so that the parameters will be directly computed during the estimation process, instead of being imputed like we do in this first phase[2].

For each model of our class of models, we consider variations in the parameters, and trace the corresponding dynamics of the cumulative and instantaneous adoptions (the latter meant as annually measured).

The four models have clear peculiarities that differentiate them, and can potentially describe distinctive trends in the adoption process of various technologies, over time.

Now, let's look at the results of a first bunch of simulations, done with some parameters formulated with reference to market trends. Specifically, we introduce the parameters q and p, whose attributed range of variation has been drawn from the magnitude registered in previous studies on broadband technology, while we choose the parameters α, β and γ to describe the dynamics of adoptions, taking mainly into account their economic meaning and definition (see table 1)[3]. For example, concerning β, it expresses the initial fraction of adopters, so that it cannot reasonably surpass the 0.5 threshold, and is typically well below this value (we initially set it to 0.3, in

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[2] We have preliminary estimated the three models with the Matlab package, on telecom data coming from OECD and ITU sources. Results are very encouraging.

[3] This fact also depends on the novelty of our functional forms and parameterizations, that do not easily find previous benchmarks in the available literature.

Table 1): in fact, during the first stages of the life cycle, pioneer adopters (having a low risk aversion and superior knowledge and skills) are a small minority of the overall potential population. The population is set to 1.420.000 units for the first two models, 3.150.000 for the third model and 1.300.00 for the latest model. We preferred to choose these numbers to compare our class with the Bass model (all models reach saturation at 1 million). After simulating a particular case for each of the models, we compare their respective dynamics with that of the Bass model (with *m*=1 million, *p*=0,04 and *q*=0,5).

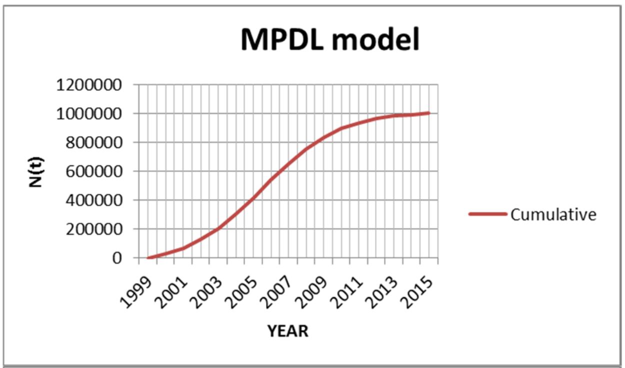
We first start with the MPDL model, whose acronym describe its main characteristics: "Market Potential Dynamic Linear".

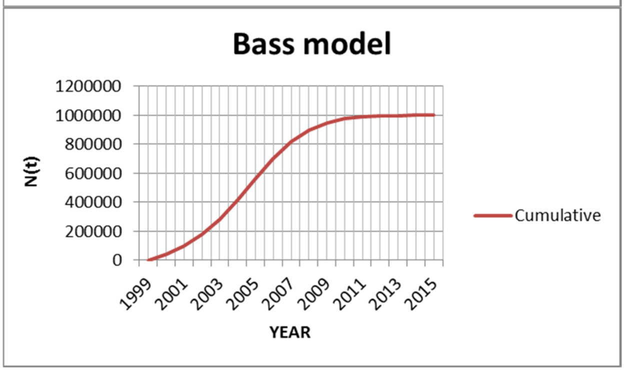
Table 1. MPDL model

0,3	**β**
1420000	***Pop***
0,04	***p***
0,5	***q***
0,5	**γ**

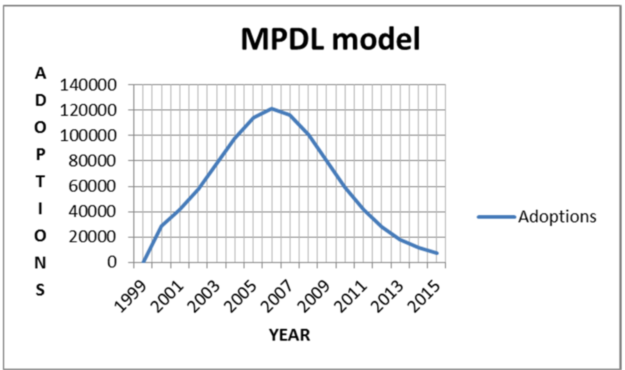
Legend: parameters set for simulating the MPDL model, with $m(t)=Pop*\beta+\gamma*N(t)$

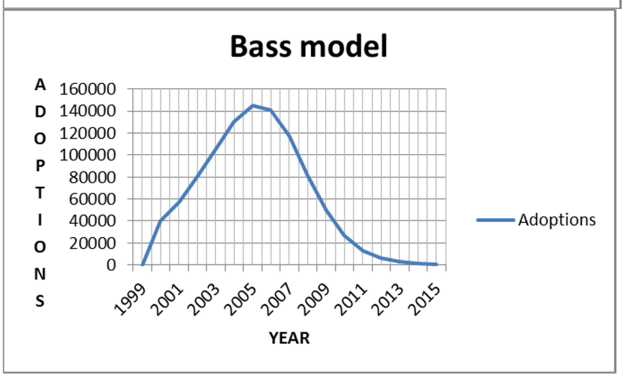
Graphs 7. Comparative simulations on model MPDL and Bass





Legend: our simulations of model MPDL



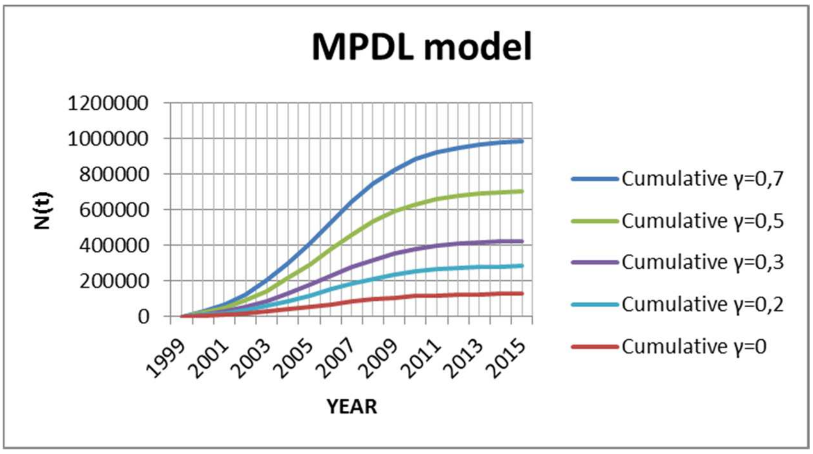


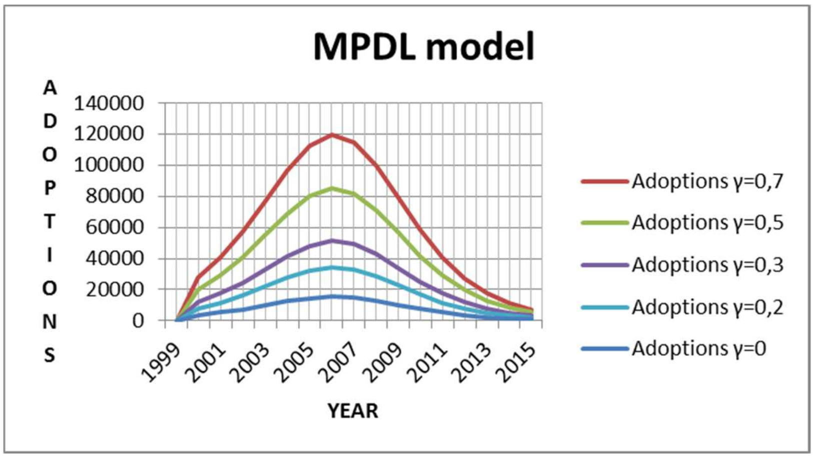
Legend: our simulations of model MPDL

We note from the MPDL charts (Graph 7) how the pattern dynamics can feature the intended chilling effect and the inertia of the diffusive phenomenon, that happens when a diffusion process is struggling to take off, also due to a small number of initial adopters. We started imputing an "average" value for the γ parameter (γ=0.5), and in fact this choice had direct implications on the generation of the resulting bandwagon effect (see *infra*).
On overall, the lower charts of Graph 7 (those relating to annual adoptions) let to appreciate that Bass arrives before to the respective saturation point: in fact, in 2015, the Bass' right tail is around the zero level, while the MPDL's one is still much ticker.

Now, we focus on the role of the γ parameter on the MPDL model: let's look at its different values.

Graphs 8. Parameter sensitiveness of the MPDL model.





Legend: sensitiveness analysis on simulations of the model MPDL

An increase in the intensity and strength of the network effect expressed by γ causes, as expectable, a higher rate of diffusion (an increase in the number of both instantaneous and cumulative adoptions) and a clearer bandwagon effect. The contrary happens when the parameter expressing the power of the network decreases. Hence, in the further step of model estimation, the MPDL model will provide a first natural test-bed framework to test the role of network effects, and their dynamic impact on the market potential, for a series of telecom technologies for which we have an informed guess on their crucial role.

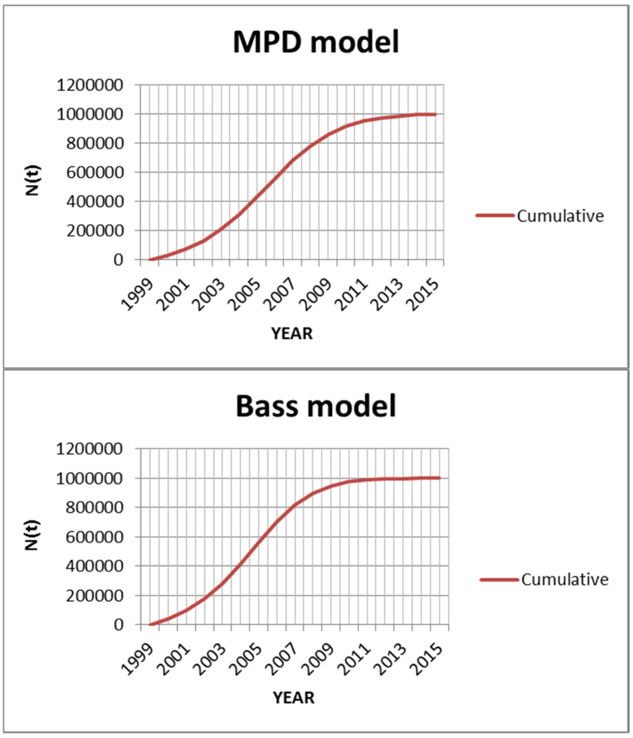
Let's now move to the second model of our class, model MPD (Market Potential Dynamic). Table 2 presents the parameter's initial values, set for the first simulation exercise (presented in Graph 3). For the analysis of the parameters sensitiveness of the MPDL model and the next models, the population is set to 1 million.

Table 2. MPD Model

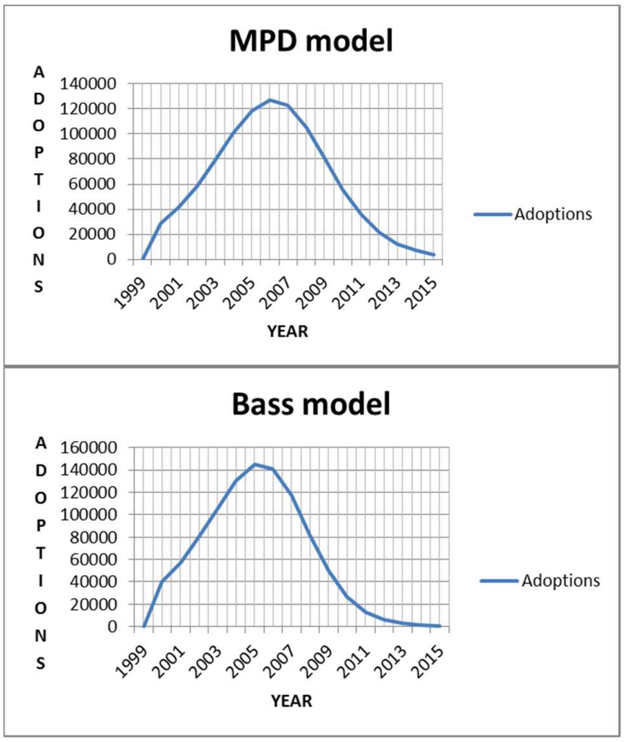
0,5	***α***
1420000	***Pop***
0,04	***p***
0,5	***q***

Legend: parameters set for simulating the MPDL model, with: m(t)=(1-α/(1+(N(t))/Pop))*Pop

Graphs 9. Comparative simulations on model MPD and Bass



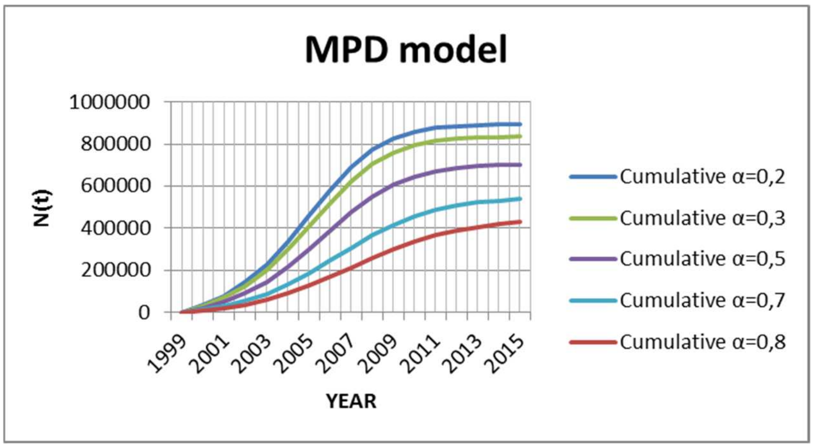
Legend: our simulations of model MPDL

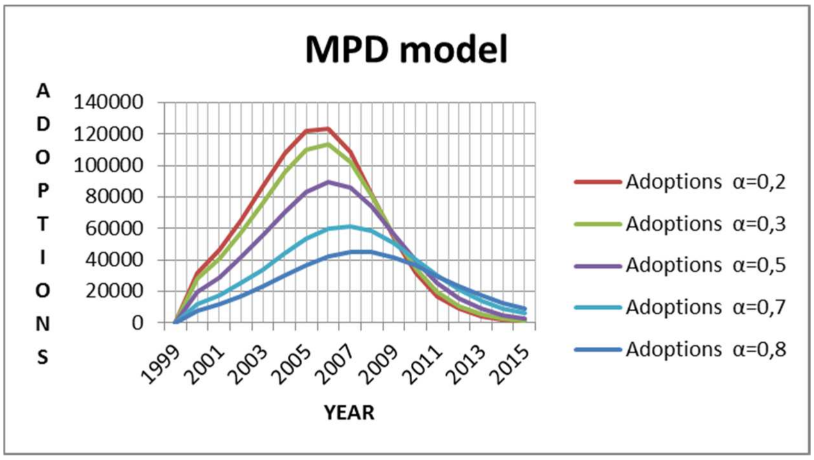


Legend: our simulations of model MPDL

Even in this case, we notice from the charts of Graph 9 that there is a delay in the diffusion phenomenon in the early stages of the diffusion process, for the MPD model. The number of adoptions in early life cycle is lower and requires a greater number of initial adopters. In this case, there is a greater bandwagon effect than the previous case when the critical mass is reached. We can see this by changing the α parameter as in the underlying charts. In particular, augmenting the parameter α, due to the constraints on α and β, the initial number of adopters decreases, so that the critical mass and the network effect are lower. At the same time, since $m(t)$ is a function positively growing with $N(t)$, augmenting $N(t)$ leads to an increase of the network effect (here implicitly present, although not directly reflected in a specific parameter), that ultimately leads to a higher market potential. The charts of Graph 10 present the sensitiveness analysis on the MPD model, with respect to alternatives values for parameter α. In short, increasing α induces a smaller speed of diffusion, a longer duration and a lower saturation level.

Graphs 10. Parameter sensitiveness of the MPD model.





Legend: sensitiveness analysis on simulations of the model MPD

Now, we present the parameter initial choice and the plots for the third model of our class: MPDT.

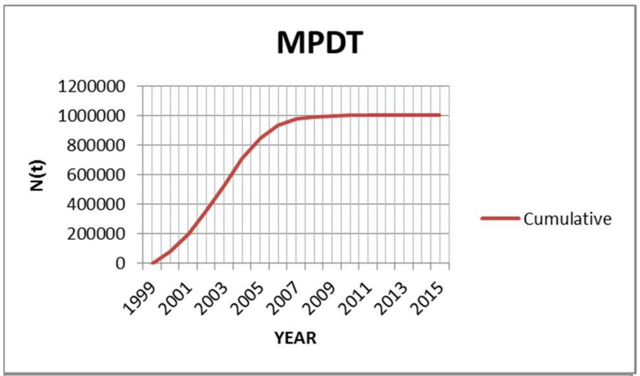
Table 3. MPDT Model

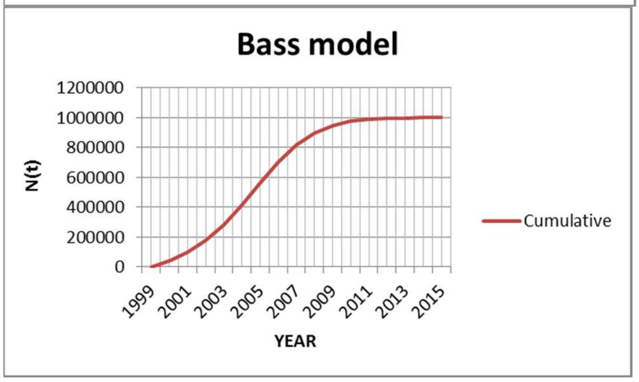
0,7	***α***
3150000	***Pop***
0,04	***p***
0,5	***q***
0,3	**β**

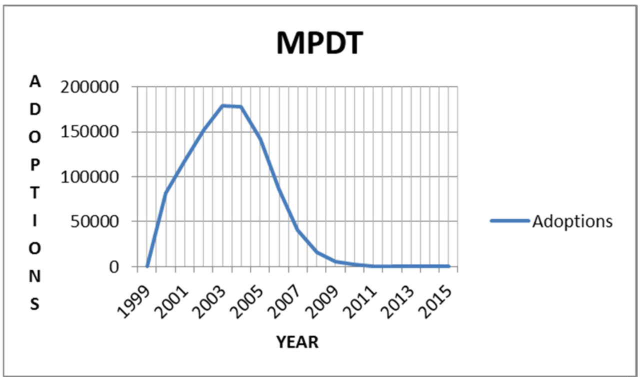
Legend: parameters set for simulating the MPDL model, with: m(t)=(β+α/(1+e^-(N(t))/Pop))*Pop

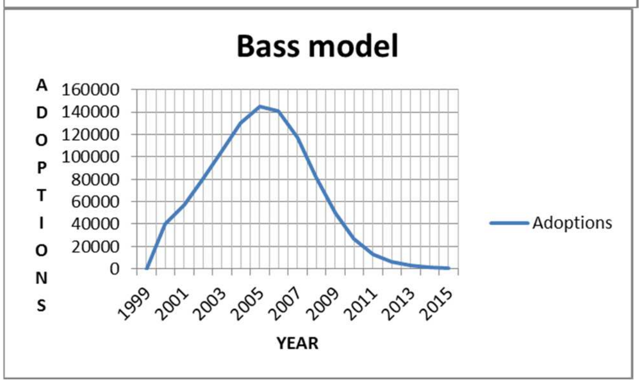
This third model is well fit to represent the bandwagon effect, and well describe those processes that have an explosive diffusion path, and frequently reach saturation relatively soon. This is the typical case of low-entry technologies and consumer goods that are aggressively priced and promoted, or that of products whose life cycle is short and features rapid substitution of obsolete versions. In Graph 5, at intermediate levels of the relevant parameter (α =*5*), it shows a sigmoid curve on overall rather steep, that leads to rapid saturation. Also with alternative parameter values (Graph 6), the cumulative shape remains asymmetric, with a predominance of the concave part. In other words, the model is immune from the chilling effect, no matter the initial adopters value. In fact, from the sensitiveness analysis of the parameter, we see two main regularities. The first is that, augmenting α, the diffusion is pushed down; in other words, for the constraint on the parameters α and β (with $\alpha + \beta = 1$), this means that the fraction of initial adopters is progressively reduced and this has an obvious impact on the speed of the diffusion process and its absolute size: ceteris paribus, in this type of diffusion processes, it is better to have more pioneers than followers, holding constant the same population. This is justified by the strong dynamics effects materialised during the early stages. Finally, also in this case, the market potential registers the effect of the network size that, despite not being explicitly parametrized, pushes up the *N(t)* and the *m(t)*.

Graphs 11. Comparative simulations on model MPDT and Bass



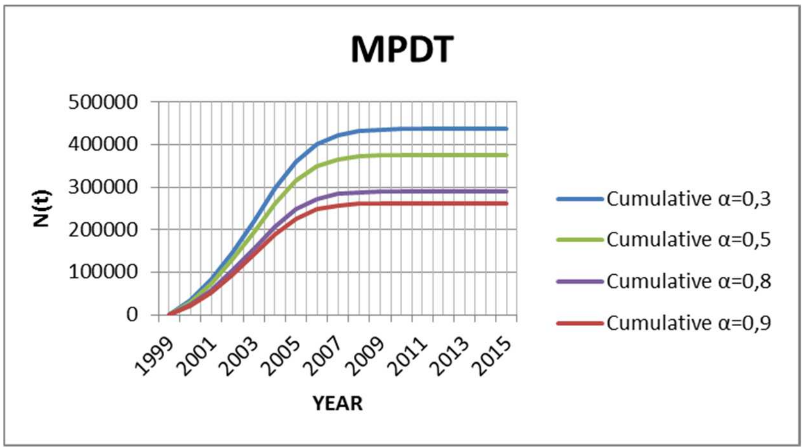


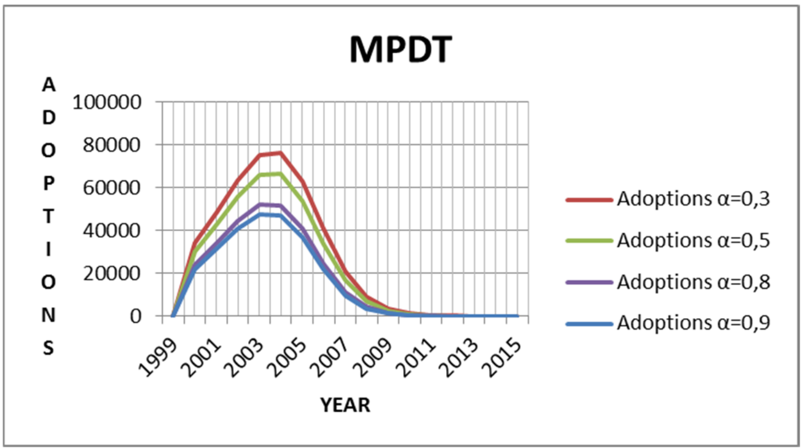




Legend: our simulations of model MPDT

Graphs 12. Parameter sensitiveness of the MPDT model.





Legend: sensitiveness analysis on simulations of the model MPDT

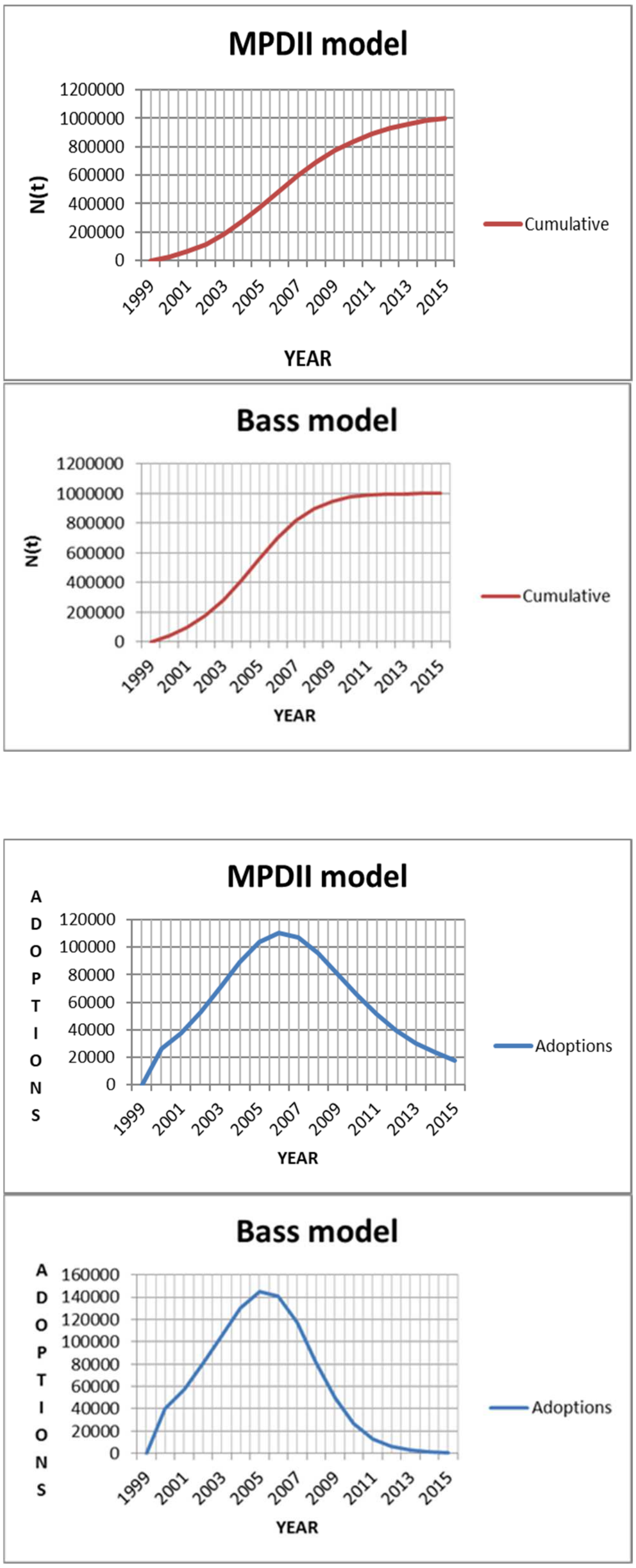
The last model to be analysed is the MPDL model. As for the other models, we trace the profile of instantaneous adoptions and cumulative adoptions. Following the sensitiveness analysis of the initial market potential and the network parameter.

Table 4. MPDII Model

0,3	**β**
1300000	***Pop***
0,04	***p***
0,5	***q***
1	**γ**

Legend: parameters set for simulating the MPDL model, with: m(t)=(Pop β)*(1+e^(γ*(N(t)/Pop)))

Graphs 13. Comparative simulations on model MPDII and Bass



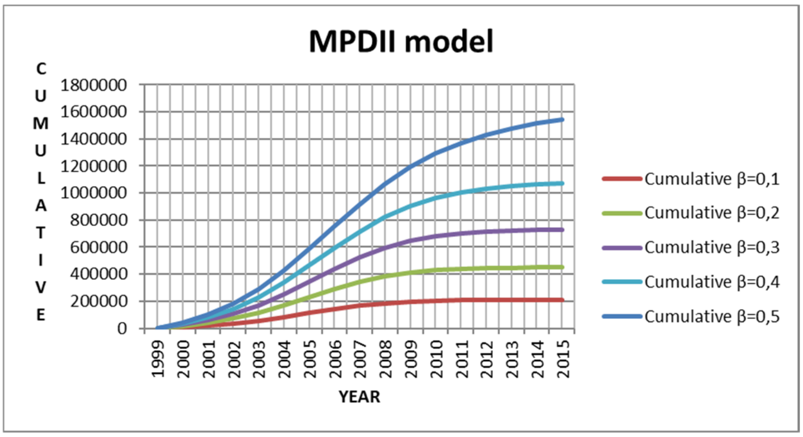
Legend: our simulations of model MPDII

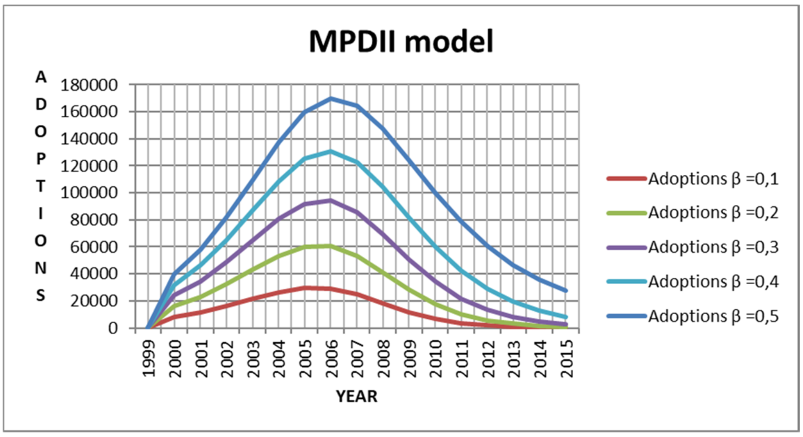
We notice from the MPDII charts (Graph 13) how the pattern dynamics can feature the intended chilling effect and the inertia of the diffusive phenomenon, that happens when a diffusion process is struggling to take off, also due to a small number of initial adopters. Among the four models of our class, this model is the one that registers more time to reach and a higher chilling effect. We started imputing a value for the γ parameter equal to 1 (γ=1), and in fact this choice had direct implications on the generation of the resulting bandwagon effect.

On overall, the lower charts of Graph 13, relating to instantaneous adoptions, let understand that the Bass model arrives before to the respective saturation point: in fact, in 2015, the Bass' right tail is around the zero level, while the MPDII's one is still much ticker. In this case, the diffusion speed difference is even more evident.

Now let's analyse the sensitiveness of the parameters. Let's start with the initial market potential. To help our analysis, we take this parameter as an initial fraction of the population by measuring it with β. We note immediately that as b increases, the number of both instantaneous and cumulative adopters increases significantly (graphs 14).

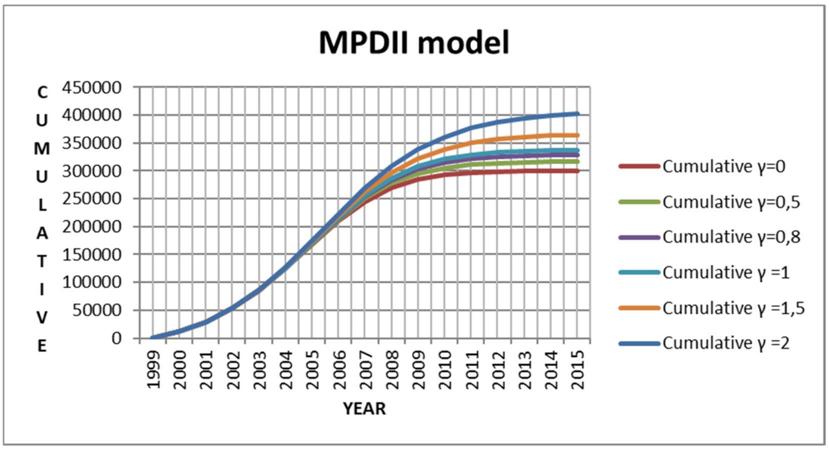
Graphs 14. Parameter sensitiveness of the MPDII model.

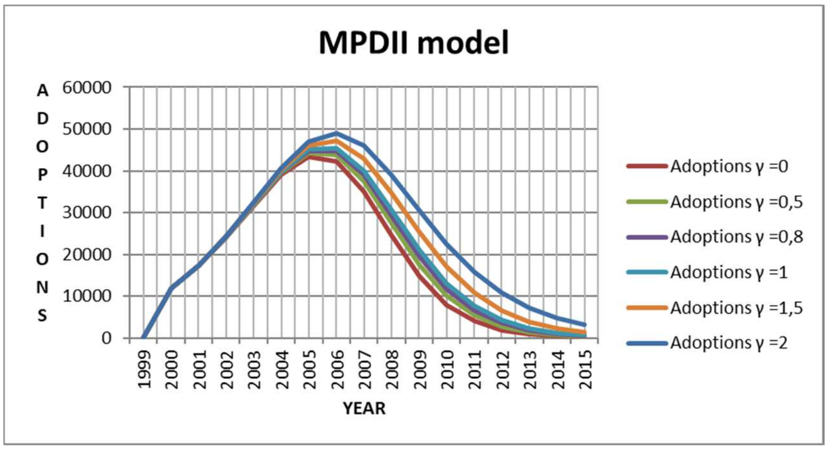




Legend: sensitiveness analysis on simulations of the model MPDII

Graph 15. Parameter sensitiveness of the MPD model.





Legend: sensitiveness analysis on simulations of the model MPDII

Moving on to analyse how both annual and cumulative adoptions behave as the network parameter γ changes. It is evident that this parameter acts on scales by increasing the level of both curves (graphs 15), in particular after the inflection point. We can see how the number of adopters increases with this parameter.

CHAPTER 3

REAL DATA, ESTIMATION METHODS AND EMPIRICAL APPLICATIONS

3.1 REAL DATA AND FEATURES OF THE TIME SERIES

The second step in our research concerns the determination of comparisons between different countries and the analysis of the diffusive paths of selected ICT technologies. The determination of the comparisons between some European countries has been possible thanks to some indicators that allowed the assessment of any digital divide. The morphological analysis of diffusion paths then allowed to compare the different characteristics of the countries considered and to verify the adequacy of the class of models we have built. This analysis focuses on curve fitting and on empirical estimation with real market data.

But are we sure that the data available depict strong regularities in the sigmoidal forms? The answer is provided by the same historical series that we now analyse to understand its peculiar characteristics and move to more sophisticated empirical methods.

Therefore, this section describes the available data, the problems inherent to these types of surveys and the main features of the time series.

We have taken into consideration the European countries, in particular the so - called "Big Five" countries in Europe (France, Italy, Germany, Spain and the UK) that are most directly comparable in terms of socio - demographic characteristics, being the latter of fundamental importance to evaluate the diffusion processes of an ICT technology.

The main technology analysed is fixed broadband. The source of the data used is the International Telecommunication Union (ITU). The data used are historical series of annual subscriptions to broadband services from 2000 to 2016 for Italy, Germany, Spain and the UK and from 1998 to 2016 for France, that was the European leader and early adopter of this technology.

International comparisons on a complex theme such as broadband diffusion move from hypotheses and statistical conventions that need to be explicitly expressed, to appreciate the real cognitive power provided by relevant data. It is then necessary to first highlight how broadband is a technologically fluid concept whose definition characters are subject to fast qualitative and quantitative evolution, so that all available statistics, including the most accurate and recent, have unavoidable conceptual and methodological inadequacies (for a methodological treatment, see Matteucci, 2013). Their main intrinsic limitation is that the capacity and transmission speeds used in broadband definition become technologically obsolete over time; on the other hand, their frequent adaptation could compromise the continuity of the time series. All this provokes an evident diachronic bias in the comparative detection of broadband diffusion processes that causes a growing downward flattening of cases in the distribution of countries, a growing overestimation of the performance of the laggard countries and underestimates the real variability of the international situation. The main quality indicator used for broadband Internet speed is the commercial data capacity per unit of time, measured in bits and expressed in its multiple Kbs and Mbs (kilobits per second and megabits per second). Currently the most widely used sources in the literature are those ITU and OECD, that define broadband access networks those that can provide download speeds of at least 256 Kbs.

Let's start with the static comparative broadband analysis using an indicator that is the fixed broadband diffusion rate, that is, the number of technology subscriptions in relation to the population. The table below (table 4) shows the absolute diffusion figures, the normalized rate of subscriptions per 100 inhabitants, and the relative country ranking. In general, the discrepancies between OECD and ITU data are minimal, also due to the homogeneity of the broadband definition adopted and the source used. Also in the case of table 5, the differences between the alternative series are negligible.

The situation measured by a simple indicator as the diffusion rate shows a striking digital divide for Italy, which is also overtaken by Spain despite having about 15 million inhabitants less. France is at the top of the standings: UK and Germany follow with inferior but similar penetration levels. The same situation occurred in previous years. Comparing the number of absolute cumulated subscriptions, we can see how Italy's delay is evident with respect to a country similar in terms of population, such as France. In Italy there is only 56% of the broadband subscriptions of France, in practice a substantial delay that can be attributed to several reasons. Matteucci (2013, 2015) describes the various issues and some reasons that have led to a substantial digital divide for Italy, also considering a regional perspective.

Table 5: Fixed broadband subscriptions at 2016 - "Big five" countries in Europe.

Country	Broadband subscriptions	Diffusion rate (for 100 inhabitants)	Ranking
France	27.664.000	42.83	1
UK	25.153.203	38.74	2
Germany	31.377.178	38.17	3
Spain	13.941.138	30.00	4
Italy	15.563.278	25.71	5

Legend: cumulative subscriptions ITU data and diffusion rate of fixed broadband.

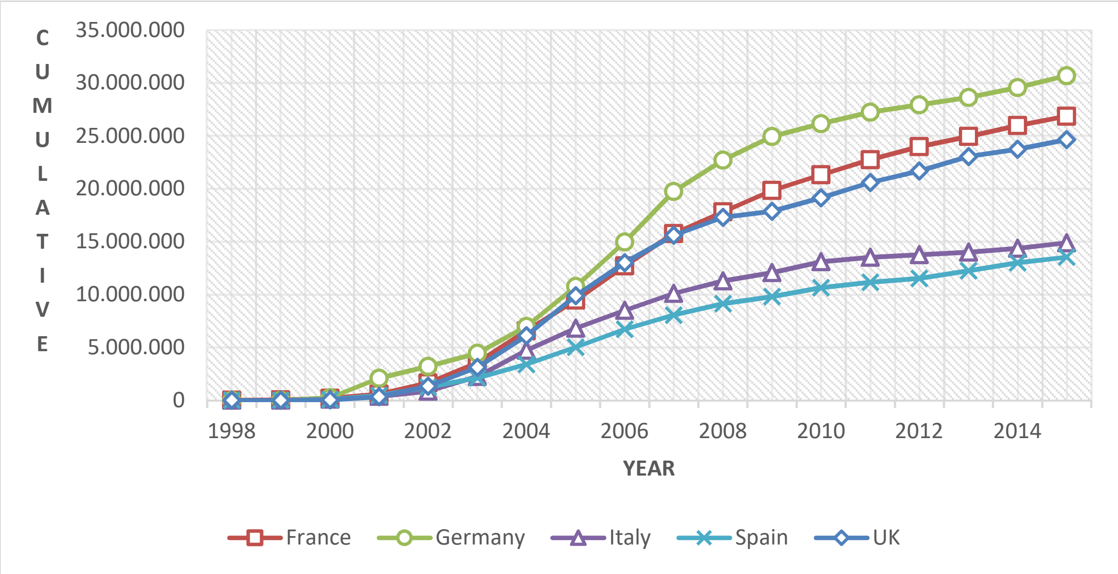
Source: our elaboration on ITU data (2017)[4].

Now let's proceed with the analysis of the basic features of the historical series at our disposal. As anticipated, our analysis focuses on the historical ITU series of fixed broadband subscriptions. We have the time series of the cumulative subscriptions and the annual subscriptions for the period 2000-2016 for several European countries, including Italy, Germany, Spain and the UK. For France, the year of the first survey is 1998. Observing in figure 10 the case of the five European countries that we analyse, we immediately notice a marked and broad similarity of the respective diffusion processes with the sigmoidal curve, although with some specific characteristics: in fact, the potential inflection points are different even if contained in a narrow range (2004 - 2008). We note that the countries of France, Germany and the UK continue to show sustained growth even in recent years. On the contrary, Italy and Spain gradually accumulate an increasing delay in the period from 2004 onwards. with the last few years that signal a noticeable and evident gap with the other group of countries examined. Furthermore, over the last two years (2015 - 2016), all countries

[4] ITU data (2017), Fixed broadband series 2000-2016, extracted from World Telecommunication/ICT Indicators database 2017 (21th edition), available on www.itu.int/ITU-D/Statistics.

have grown more than we would have expected and this fact can be seen even better in Figure 10.

Figure 10. Diffusion of fixed broadband of "Big five" countries in Europe



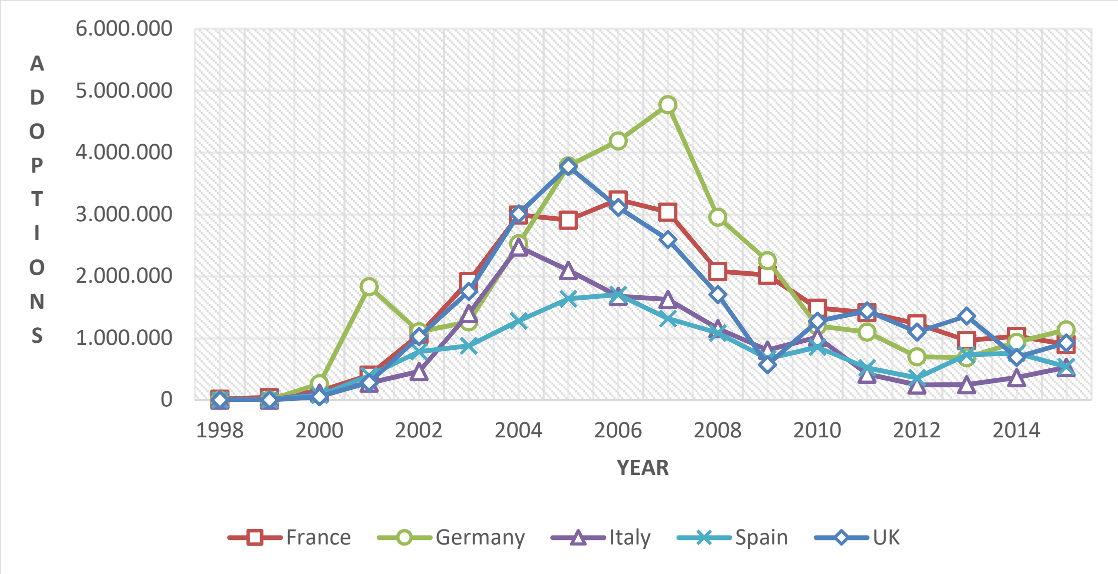
Legend: cumulative subscriptions of fixed broadband, time series 1998-2016.

Source: our elaboration on ITU data (2017).

Figure 11 represents the trend of the annual subscriptions, id est the speed of diffusion in the countries taken into consideration. One aspect on which we can focus our attention is the period when the maximum subscription peak is reached. Italy is paradoxically the first country to reach the peak of subscriptions, a clear follower country for broadband: the year of the peak appears to be 2004. However, it should be noted that the follower character does not depend on the early peak but on a set of more informative considerations, including first and foremost the fact that the estimated level of saturation is the lowest of all the countries considered here. Furthermore, other elements show that the behaviour in big cities, already covered, explains the rapid start. After the coverage of the larger cities, the speed of the diffusion process decreases because the rest of the small and medium urban centres is poorly covered. UK follows the peak in 2005, France and Spain in 2006 and finally Germany in 2007. There are also some peculiarities, always observing figure 10: Germany has a small peak in subscriptions in 2001 to return to lower levels in the next two years and then follow an upward trend again before reaching its highest peak in 2007.

Another noticeable peculiarity is the return to growth in the last two years for almost all countries. This could mean that the saturation phase turned out to be slower than expected, and that in many countries the diffusion of this technology has not yet reached the phase of maturity and many potential users are preparing to adopt the product. The last fact opens up new hypotheses that can be put forward. For example, starting with the less likely, broadband could feature an intrinsically anomalous diffusion process; or there could be noises or definition breaks in the series. Turning to more economically-grounded raisons, there could be a reactivation of the adoption process following the introduction of new improved generations of the same product (this could be the case of next generation broadband networks (NGN), that respond to the same minimum definition of performance). Lastly, a new diffusion momentum could be triggered by the availability of new downstream services (e-Government, etc.), and/or new classes of users may be involved in the usage of this technology, spurring additional network effects and adoption value.

Figure 11 Speed diffusion of fixed broadband of "Big five" countries in Europe



Legend: annual subscriptions of fixed broadband, time series 1998-2016.

Source: our elaboration on ITU data (2017).

3.2 ESTIMATION METHOD OF DIFFUSION MODELS

This section discusses the most widely used methodologies for the estimation of diffusion patterns and the major problems associated with it.

The currently used methodologies for estimating diffusion models include:

- Ordinary Least Squares (OLS)
- Maximum Likelihood (MLE)
- Non-linear Least Squares (NLS)
- Algebraic Estimation (AE)

There are other methods used in literatures, such as Bayesian methods and stochastic differential equations, but we will not discuss them.

There are several books and papers taken into consideration to drawing up this presentation. Really useful they were Seber and Wild (2003), Bates and Watts (1988), Boswijk and Franses (2003), Satoh (2001) Mahajan and Sharma (1986), Schmittlein and Mahajan (1982), Srinivasan and Mason (1986) and Satoh (2001).

Choosing the method to be used depends of course on the functional shape of the model, the richness of available data, and the model hypotheses. Since the models discussed above and our models are nonlinear, our attention will be focused mainly on the method of the non-linear least squares for regression models.

3.2.1 ORDINARY LEAST SQUARES

In his original work Bass (1969) observes that the differential equation $n(t) = \left(p + q\,\frac{N(t)}{m}\right)[m - N(t)]$ can be developed in such a way that the first adoptions are a quadratic function of the previous cumulated adoptions:

$$n(t) = p\,m + (q-p)\,N(t) - \left[\frac{q}{m} N^2(t)\right] \tag{3.1}$$

So, he suggested estimating model parameters using the Ordinary Least Squares (OLS) applied to the regressive model of instant sales based on a discrete formalization:

$$N(t) - N(t-1) = \beta_1 + \beta_2\,N(t-1) + \beta_3\,N^2(t-1) + \varepsilon(t) \tag{3.2}$$

where β_1, β_2, β_3 represent the parameters and $\varepsilon(t)$ is a residual error normally distributed, with $\varepsilon \sim N(0, \Sigma)$, $\Sigma = (\delta^2 \boldsymbol{I})$. We remember that $\beta_1 = pm$, $\beta_2 = (q-p)$ and $\beta_3 = \frac{q}{m}$. You can use the estimates of β_1, β_2, β_3 to solve for the estimators $\hat{p}$, $\hat{q}$ and $\widehat{m}$:

$$\hat{p} = \frac{-\beta_2 + \sqrt{\hat{\beta}_2^2 - 4\hat{\beta}_1\hat{\beta}_3}}{2}$$

$$\hat{q} = \frac{\beta_2 + \sqrt{\hat{\beta}_2^2 - 4\hat{\beta}_1\hat{\beta}_3}}{2}$$

$$\widehat{m} = \frac{-\beta_2 - \sqrt{\hat{\beta}_2^2 - 4\hat{\beta}_1\hat{\beta}_3}}{2\,\widehat{\beta_3}}$$

(3.3)

This estimation method is simple but at the same time presents some problems: the first problem is that OLS estimates often provide estimates for *p*, *q* and *m* with high multicollinearity. In fact, $N(t\text{-}1)$ e $N^2(t\text{-}1)$ are typically intrinsically correlated and, using OLS in the presence of strong multicollinearity, estimates of β_1, β_2 are characterized by very high standard errors and very low significance levels (Satoh, 2001). It is not possible to resort to the remedies proposed by econometric literature to eliminate multicollinearity, for example, by removing a coefficient to regression, because it would lose meaning in this context. Another major problem highlighted by numerous empirical studies is the tendency to obtain parameters with negative sign which in the case of this category of models are unreasonable and symptomatic of a general structural weakness of the model. So, with a few data points, the OLS estimates are often unstable due to the issues mentioned.

In addition, the discretization of the Bass model made to be able to use OLS regression introduces a time interval bias (Boswijk and Franses, 2005). For these reasons, the parameters of the Bass models are typically estimated with the Nonlinear Least Squares (NLS), as suggested by Srinivasan and Mason (1986).

3.2.2 MAXIMUM LIKELIHOOD (MLE)

The MLE estimation method tries to solve problems related to OLS estimates. MLE was introduced by Schmittlein e Mahajan (1982) and, as a starting point, it uses the distribution of adoption times instead of the discrete version of Bass:

$$F(t) = \frac{\beta_3 \left(1 - e^{-\beta_2 t}\right)}{1 + \beta_1 e^{-\beta_2 t}} \tag{3.4}$$

where $F(t)$ is the unconditional probability for adoption by time t, $\beta_1 = \frac{q}{p}$, β_3 is the probability of an eventual adoption and $\beta_2 = q + p$.

Thus, the likelihood function of the diffusion phenomenon can be written as follows:

$$L(\beta_1, \beta_2, \beta_3, x_k) = (1 - F(t_T - 1))^{X_T} \prod_{k=1}^{T-1} (F(t_k) - F(t_k - 1))^{X_k} \tag{3.5}$$

where x_k indicates the number of individuals that adopt in the interval $k = (t_k, t_k - 1)$. The MLE estimators for the parameters β_1, β_2, β_3 are then obtained by the authors using the Hooke-Jeeves accelerated search model, as there are no explicit formulas for parameters that maximize the likelihood function. Once β_1, β_2, β_3 estimates are obtained, it is possible to obtain the value of p, q and m through the following equations:

(3.6)

$$\hat{p} = \frac{\beta_2}{1 + \beta_1}$$

$$\hat{q} = \frac{\beta_2 \, \beta_1}{1 + \beta_1}$$

$$\hat{m} = \beta_3\, m$$

The MLE methodology overcomes several defects in the OLS estimation method. Thanks to this method we get the most stable estimates that show the correct sign, id est p, q and m are all non-negative. In addition, we can have approximated standard errors of the parameter estimates and this improves the stability of the estimates themselves. However, the procedure is heavily based on regularity conditions and therefore it is not completely fit when we have small sample size. In particular, the hypothesis that individual adoption times are independent seems very questionable, as the imitative phenomenon is a major driver of the diffusion process.

3.2.3 NONLINEAR LEAST SQUARES

Srinivasan and Mason (1986) proposed the Nonlinear Least Squares method (NLS) to overcome the issues that OLS estimates often present. Additionally, it is an approach that tries to solve some defects of the MLE method that in fact underestimates standard errors as MLE only considers sampling errors, but no other sources of error (such as the effects of excluded marketing variables). Using the autoregressive formulation:

$$N(t) = m\frac{1 - e^{-(p+q)\,t}}{1 + \frac{q}{p}e^{-(p+q)\,t}} + \varepsilon(t) \tag{3.7}$$

or

$$\frac{dN(t)}{dt} = n(t) = m\,\frac{(p+q)^2}{p}\frac{1 - e^{-(p+q)\,t}}{\left(1 + \frac{q}{p}e^{-(p+q)\,t}\right)^2} + \varepsilon(t) \tag{3.8}$$

where $\varepsilon \sim N(0, \Sigma)$, $\Sigma = (\sigma^2 \boldsymbol{I})$, it is possible to obtain reliable estimates of the parameters p, q and m with appropriate confidence intervals.

So, the parameters p, q and m and the respective standard errors can be directly estimated by NLS. The method is able to generate more reliable standard errors and overcomes the time aggregation bias of the OLS methodology. Hence NLS is the most commonly used method for the estimation of the Bass parameters (Satoh, 2001).

The first formulation is the autoregressive form of the cumulative equation, the second is the instantaneous autoregressive equation. Theoretically it should be possible to estimate the parameters using both approaches because, from a deterministic point of view, the two models are absolutely equivalent. The difference is identified in the various specifications regarding the distribution of residuals. But it is improper to use the instantaneous approach as the available data are basically cumulative, whatever the time reference period.

A rather neglected problem in literature that characterizes reputable models with the NLS method is the difficulty of evaluating the origin for t. Generally, there are ordered time series available and it is customary to associate the first value given to the value $t = 1$. It identifies t_c, that is the first value of adoptions ($N(1)$) corresponding to adoptions from time 0 to time 1. In many cases this choice can reveal wrong because the innovation launch could be previous to the surveys.

We hypothesize that t_c is time interval between the launch of innovation and the moment when it starts the detection of adoptions. If there is a censorship between the time 0 and the time t_c, the observed quantities are:

$$Y(t) = N(t) - N(t_c)\,, \quad t = t_c + 1, t_c + 2, \dots , t_c + T \tag{3.9}$$

with unknown $N(t_c)$. Thus, the autoregressive formulation becomes:

$$Y(t) = m\frac{1 - e^{-(p+q)\,t}}{1 + \frac{q}{p}e^{-(p+q)\,t}} - N(t_c) + \varepsilon(t) \tag{3.10}$$

Even $N(t_c)$ can be expressed in regressive form but it is not directly observable:

$$N(t_c) = m\frac{1 - e^{-(p+q)\,t_c}}{1 + \frac{q}{p}e^{-(p+q)\,t_c}} + \varepsilon(t_c) \tag{3.11}$$

Finally, the autoregressive formulation can be written as follows:

$$Y(t) = m\frac{1 - e^{-(p+q)\,t}}{1 + \frac{q}{p}e^{-(p+q)\,t}} - m\frac{1 - e^{-(p+q)\,t_c}}{1 + \frac{q}{p}e^{-(p+q)\,t_c}} + \varepsilon(t) \tag{3.12}$$

where $t = t_c + 1, t_c + 2, \dots, t_c + T$.

There are two typical cases depending on whether the actual launch date, and consequently t_c, is known with certainty or not:

1) The factor t_c is a constant assigned. It is generally possible to estimate the parameters, even if special arrangements are needed.
2) t_c is a further parameter to estimate. In this case, the estimation problem is not always solvable because the model is likely to be unidentifiable and therefore cannot be estimated.

Fortunately, with regard to the diffusion of technological innovations, the second case, characterized by uncertainty in relation to the launch date, is rare.

NLS estimation requires starting values for the model parameters. The criteria for choosing initial values are not rigidly coded. This is done by global or local linearization of the reference model, then applying OLS. Or appropriate transformations are applied. One approach suggested in the literature for the choice in the field of diffusion processes consists, in the case of Bass model, to use as starting values the parameter estimates obtained through the contribution of the linearized model by analogy.

Now, let's go to a more detailed description of the non-linear regression and the numerical estimation methods typically used in this context.

Suppose to have a certain numbers of observations n (x_i,, y_i) and a model nonlinear in the parameters having fixed regressors expressible in the form:

$$y_i = f(\boldsymbol{x}_i, \boldsymbol{\theta}) + \varepsilon_i \ , \qquad i = 1,2,\dots,n \tag{3.13}$$

where $f(\boldsymbol{x}_i, \boldsymbol{\theta})$ is an non-linear function relating $E(y_i)$ to the independent variables x_i , x_i is a $k \times 1$ vector of independent variables (fixed), θ_i is a $l \times 1$ vector of parameters and ε_i is iid variable with mean 0 and variance σ^2 . The true value of θ, indicated by θ^*, belongs to Θ, an appropriate subset of R.

The estimate according to the least squares of θ^*, we indicate it with $\hat{\theta}$, for construction minimizes $SSR\ (\theta)$, namely the sum of squared residuals between the observations y_i and the model $f(x_i, \theta)$:

$$SSR(\theta) = \sum_{i=1}^{n} [y_i - f(x_i, \theta)]^2 \tag{3.14}$$

Unlike the case of linear minimum squares, it can have multiple relative and absolute minimum.

It can be demonstrated that under appropriate conditions of regularity, assuming that the errors ε_i are iid with constant variance σ^2 and mean 0, $\hat{\theta}$ and $s = \frac{SSR(\hat{\theta})}{(n-l)}$ are consistent estimates of θ^* and σ^2 respectively. With additional regularity conditions, $\hat{\theta}$ is asymptotically normal for $n \to \infty$. If we also assume that ε_i have normal distribution, then $\hat{\theta}$ is also a maximum likelihood estimator.

If $f(\boldsymbol{x}_i, \boldsymbol{\theta})$ can be differentiated in θ and $\hat{\theta}$ is into Θ, $\hat{\theta}$ will satisfy the system of normal equations:

(3.15)

$$\frac{\partial SSR(\theta)}{\partial \theta_r}\bigg|\,\hat{\theta} = 0$$

Let's express $y_i = f(\boldsymbol{x}_i, \boldsymbol{\theta}) + \varepsilon_i$ in the following form so it does not make the notation too heavy: $y = \boldsymbol{f}(\,\boldsymbol{\theta}) + \boldsymbol{\varepsilon}$

Also for greater simplicity, we use as notation:

$$f(\theta) = [f_1(\theta), f_2(\theta), \dots, f_n(\theta)]' \tag{3.16}$$

Be $\underset{.}{F}(\theta) = \frac{\delta f(\theta)}{\delta \theta'} = \frac{\delta f_i(\theta)}{\delta \theta_r}$, $\quad i = 1,2, \dots, n$ and $r = 1,2, \dots, l$

(3.17)

the Jacobian matrix $n \ x \ l$ of the first partial derivatives of the function f respect to $\hat{\theta}$. Turning to the vector notation, the sum of squared residuals *SSR(θ)* becomes:

(3.18)

$$SSR(\theta) = [y - f(\theta)]' \, [y - f(\theta)] = \varepsilon'\varepsilon = \|y - f(\theta)\|^2$$

that is the square of the norm of $\varepsilon = y - f(\,\theta)$.

Deriving the last equation and equating that expression to zero, we get:

(3.19)

$$\hat{\underset{.}{F}}'[y - f(\,\theta)] = \hat{\underset{.}{F}}'\hat{\varepsilon} = 0$$

These expressions are the normal equations for the nonlinear model, whose solutions lead to estimates, under the orthogonality hypothesis. Particular attention must be paid to the minimization of $SSR(\theta)$, because, unlike the linear regression, it is possible that there are several relative minima. This is one of the main problems to be solved in order to obtain a reliable estimate of $\hat{\theta}$. Most nonlinear models do not have analytic solution and it is necessary to use iterative estimation methods such as the algorithms of Gauss-Newton and Levenberg - Marquardt.

3.2.4 ITERATIVE ESTIMATION METHODS

Since most nonlinear models we consider do not have an analytical solution, it is necessary to introduce iterative estimation methods. We will focus on two in particular:

- *the Gauss - Newton method*
- *the Levenberg - Marquardt method.*

Both algorithms are present on Matlab software, the software we used to run estimates of our class of models.

The *Gauss-Newton algorithm* is obtained starting from a first-order Taylor series approximation of $f(\theta)$ in a neighborhood of θ_a, where θ_a is a vector of parameters considered to be a good approximation, as a starting point, of the estimate of $\hat{\theta}$. Then:

$$f(\theta) \approx f(\theta_a) + \frac{\partial f(\theta_a)}{\partial \theta'} (\theta - \theta_a) \tag{3.20}$$

where $\frac{\partial f(\theta_a)}{\partial \theta'} = F_{.}(\theta_a)$, using the notation of the previous subparagraph. For simplicity, we denote $F_{\cdot a} = F_{.}(\theta_a)$. Using the approximation (3.20) in (3.18) we obtain the approximated sum of squared residuals in linear terms:

$$\begin{gathered} SSR(\theta) = [y - f(\theta)]' \, [y - f(\theta)] \\ \approx [y - f(\theta_a) - F_{\cdot a} \, (\theta - \theta_a)]' \, [y - f(\theta_a) - F_{\cdot a} \, (\theta - \theta_a)] \\ \approx [Z_a - F_{\cdot a} \; \beta]' \, [Z_a - F_{\cdot a} \; \beta] \end{gathered} \tag{3.21}$$

where $Z_a = y - f(\theta_a)$ and $\beta = (\theta - \theta_a)$.

Consequently, the normal equations for the non-linear model were re-expressed as a linear model in parameter β. The minimum of β can be obtained for:

$$\hat{\beta} = (F_{\cdot a}{}' F_{\cdot a})^{-1} F_{\cdot a}{}' Z_a \tag{3.22}$$

We then arrive at the formulation of the Gauss-Newton algorithm:

$$\theta_{a+1} = \theta_a + \delta_a \tag{3.23}$$

where $\delta_a = \hat{\beta} = \theta_{a+1} - \theta_a = (F_{\cdot a}{}' F_{\cdot a})^{-1} F_{\cdot a}{}' \left(y - f(\theta_a)\right)$

When the minimum sum of squared residuals *SSR(θ)* is reached, the iterative mechanism stops and this situation occurs when the value of δ_a is zero.
Therefore, in correspondence with an effective minimum solution, the distance measured by δ_a will be zero, thus stopping the sequential update. Observe that the value of δ_a is null if $F_{\cdot a}{}' \left(y - f(\theta_a)\right) = 0$, which verifies the condition of orthogonality (3.19) previously exposed.
This method is the basis for the most used nonlinear least squares algorithms.

The Levenberg-Marquardt algorithm introduces a substantial modification to the Gauss - Newton algorithm (3.22), eliminating any sources of singularity due to the matrix $(F_{\cdot a}{}' F_{\cdot a})$.
The new upgrade step introduces an appropriate diagonal matrix of full rank:

$$\delta_a = \hat{\beta} = (F_{\cdot a}{}' F_{\cdot a} + \eta_a D_a)^{-1} F_{\cdot a}{}' \left(y - f(\theta_a)\right) \tag{3.24}$$

where D_a is a full rank diagonal matrix with positive elements.

It should be noted that in any case at the solution the equation of orthogonality (3.19) is satisfied.

The parameter η_a is modified according to the value of the deviance $SSR(\theta)$. If the deviance is reduced with respect to the previous step, the value of η will also be reduced to the next step, bringing the method closer to that of Gauss-Newton (where $\eta=0$); if, on the other hand, the value of the deviance increases, the value of η will also be increased and this leads in the direction of the criterion of the maximum descent.

The Levenberg-Marquardt algorithm is extremely powerful as it solves the convergence problems of the Gauss-Newton algorithm, encountered when $F_{\cdot a}{}' F_{\cdot a}$ is badly conditioned; this is done at the cost of the approximation of the Hessian of $f(\theta)$ to a very coarse identity matrix. In its various implementations, we can consider it the main algorithm for the estimation of nonlinear least squares.

3.2.5 ALGEBRAIC ESTIMATION

The Algebraic Estimation method (AE) (Mahajan and Sharma, 1986) has the virtue of simplicity in obtaining reasonably good estimates of the Bass parameters. This approach is usually inferior to NLS estimates in terms of fit and forecasting accuracy, but can be used to provide reasonable starting values for the NLS procedure. The reasoning of Mahajan and Sharma (1986) is that the sophisticated NLS and MLE methodologies depend heavily on search algorithms and thus are sensitive to the quality of starting values. Poor starting values might lead to slow down convergence or even failure to reach the global optimum (Jukic, 2011).

Mahajan and Sharma claim that AE is relatively simple and this procedure require the knowledge of the point of inflection., which could be obtained using data, experience of analogous products, or even expert opinions. They argue that knowledge of the inflection point t^* provides knowledge about the cumulative fraction of adopters F^* at time t^* and the noncumulative fraction of adopters f^* at time t^*. Given t^*, F^* and f^*, the algebraic estimation procedure consists of solving a system of simultaneous equations:

$$t^* = \frac{ln\,\frac{q}{p}}{p+q}$$

$$F^* = \frac{N^*}{m} = \frac{(q-p)}{2q} \qquad (3.25)$$

$$f^* = \frac{n^*}{m}\frac{m\,(q+p)^2}{4q}$$

The values of t^*, N^*(cumulative adopters) and n^* (noncumulative adopters) can be used to solve for p, q, and m by reformulating the system:

$$p = \frac{(m - 2\,N^*)\,n^*}{(m - N^*)^2} \tag{3.26}$$

$$q = \frac{m\,n^*}{(m - N^*)^2} \tag{3.27}$$

$$t^* = \frac{m - N^*}{2\,n^*} \ln\left(\frac{m}{m - 2\,N^*}\right) \tag{3.28}$$

The last equation can be used to find m either numerically or by trial and error, and once m is known, p and q can be obtained from 3.26 and 3.27 equations (Kijek and Kijek, 2010).

The algebraic estimation procedure is not to be regarded an alternative to NLS estimation, not the least because it cannot produce standard errors for the parameter estimates. However, given its simplicity it is a useful source of starting parameter values for NLS estimation.

3.3 EMPIRICAL APPLICATIONS ON REAL MARKET DATA

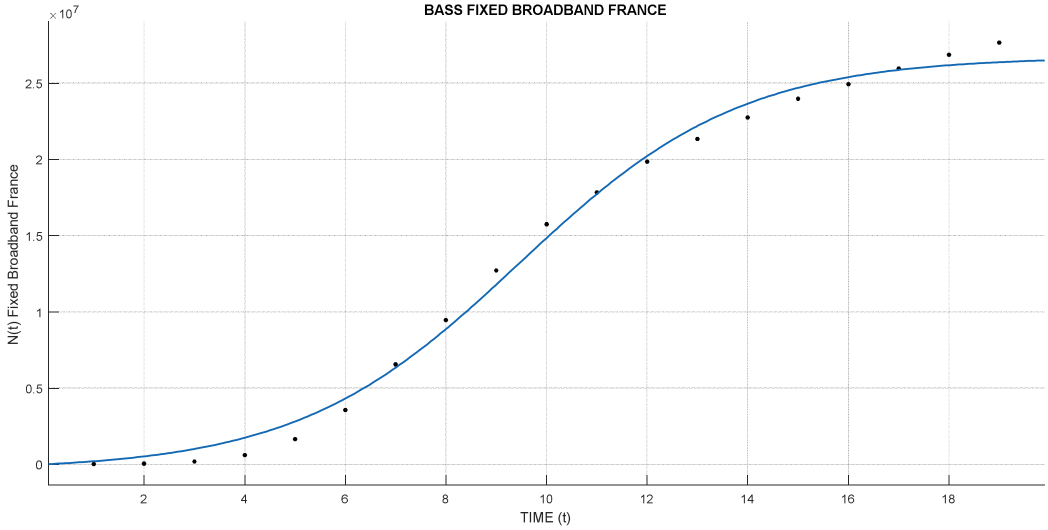
After discussing the characteristics of the data and the time series available and describing the main estimation methodologies, we now focus on the morphological shape of diffusion paths to compare the different characteristics of the countries considered, to verify the adequacy of the class of models we have built and, possibly, to analyse policy implications for Governments' digital agenda. This analysis focuses on curve fitting and on empirical estimation of diffusion curves with real market data. We carry out both to get more information. Both have the same algorithm, but the use of the two methodologies is different. With the curve fitting, we use a specific toolbox that in addition to tracing the curve that best fits the data, but also returns some measures regarding the goodness of fit. However, the limit is that the toolbox is a pre-made package that lacks some useful and important measures. With the estimation of the parameters, we can obtain the estimates with the corresponding standard errors and the various significance tests in addition to the goodness of the fit. Thus, by running both, we can get a more complete view.
We proceed by performing the curve fitting and the estimation of the parameters of each model starting from the Bass model and then we move to our class of models. As we perform the parametric analysis on the coefficients, we take into consideration the original time series without smoothing the data, because the smoothing would invalidate the hypothesis of normality of the errors underlying the parametric fitting methods. Moreover, the annual frequency of the observations guarantees a reliable observation of the process, also taking into account the fact that broadband subscriptions

may register some seasonality, and better account for the unavoidable noise and incidence of the series.

We carry out the fitting using the "Curve Fitting Toolbox" of the Matlab software. Two nonlinear regression algorithms are built into Matlab: “lsqcurvefit” and “nlinfit”. The former implements the Gauss-Newton algorithm and the latter implements the Levenberg-Marquardt algorithm. Each algorithm has its advantages and disadvantages. Experience shows the Levenberg-Marquardt algorithm to be more reliable (the Gauss-Newton method uses complex numbers that make the process more complex and occasionally fails), however it often finds results local to the initial guess. So, the choice of the methodology for the different types of models is as follows: the algorithm used is Levenberg–Marquardt, inserting each model as "Custom Equation".

In detail, concerning the “Custom Equation”, for the Bass model and the models with analytical solution we have inserted the formula of the analytical solution of cumulated form, while for the models without analytical solution we have introduced the corresponding instantaneous adoption equation, using the corresponding diffusion data. We have reported here the curve fitting graphs of a country, France, for the main models developed in the previous chapter. We have excluded only the MPDT which is not useful for this specific technology. The main reason is that they present a very rapid trend suitable for technologies that usually reach soon the saturation phase. In the appendix one can find the fitting graphs of Italy, another of the five countries considered (see Graphs A1-A4 in the Appendix).

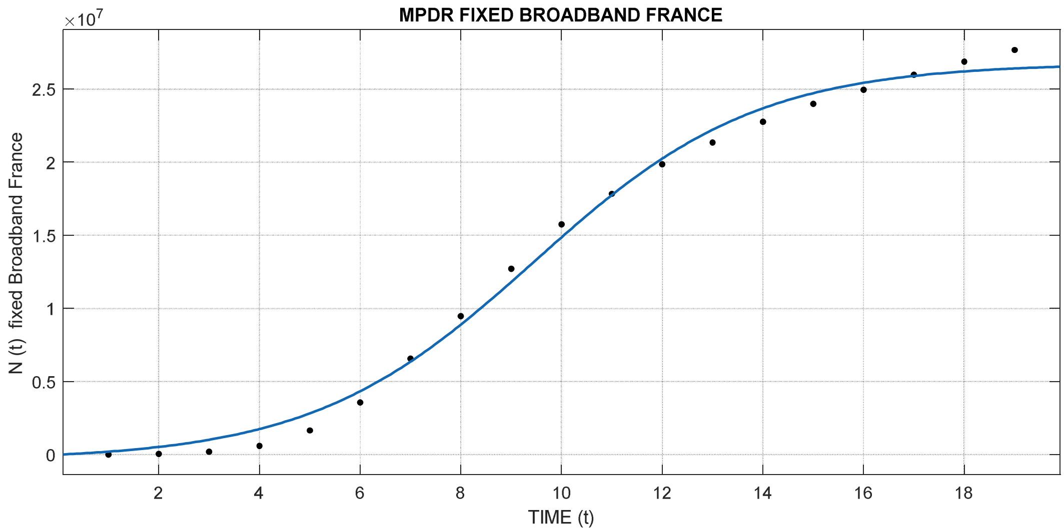
Graph 16. Curve fitting of fixed broadband France – Bass model – Years 1998-2016



Legend: curve fitting of fixed broadband France – Bass Model, years 1998-2016

Goodness of fit Bass model:
SSE: 1.08e+13
R-square: 0.9946
Adjusted R-square: 0.9939
RMSE: 8.217e+05

Graph 17. Curve fitting of fixed broadband France – MPDR model - 1998-2016



Legend: curve fitting of fixed broadband France - MPDR Model, years 1998-2016

Goodness of fit MPDR model:

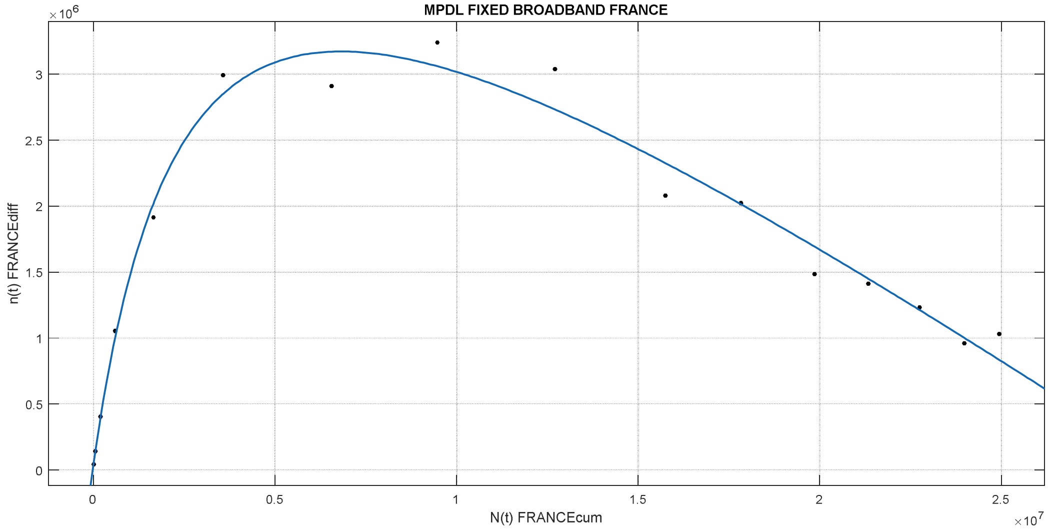
SSE: 1.08e+13

R-square: 0.9946

Adjusted R-square: 0.993

RMSE: 8.18e+05

Graph 18. Curve fitting of fixed broadband France – MPDL model - 1998-2016



Legend: curve fitting of fixed broadband France - MPDL Model, years 1998-2016

Goodness of fit MPDL model:

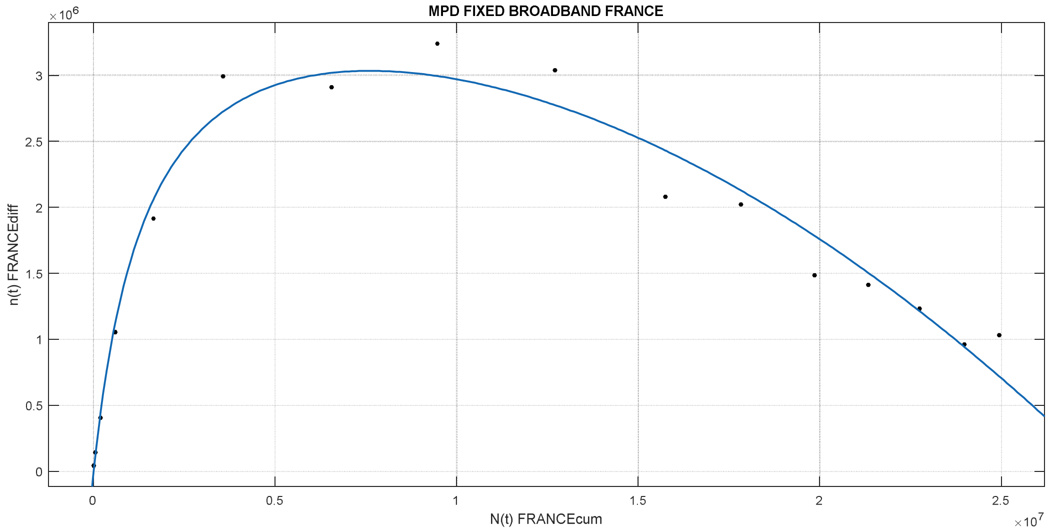
SSE: 3.738e+11

R-square: 0.9768

Adjusted R-square: 0.971

RMSE: 1.765e+05

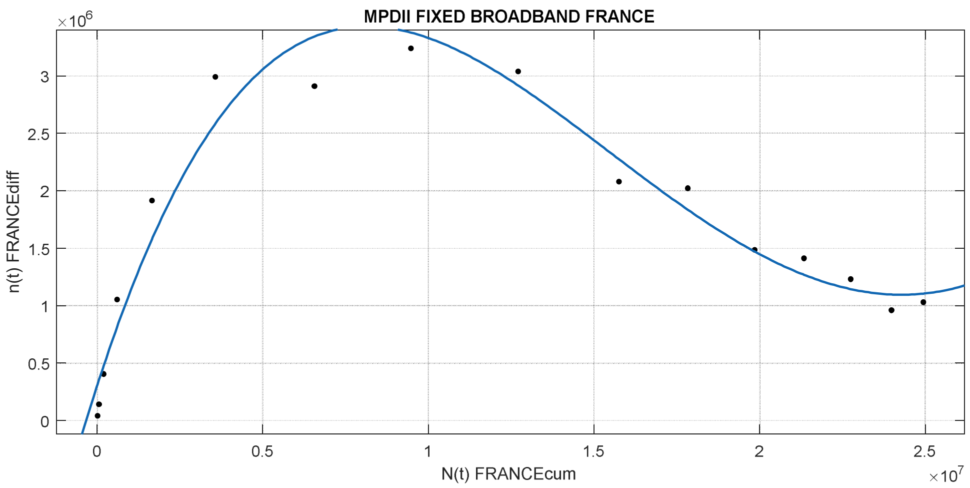
Graph 19. Curve fitting of fixed broadband France – MPD model - 1998-2016



Legend: curve fitting of fixed broadband France - MPD Model, years 1998-2016

Goodness of MPD model:
SSE: 5.716e+11
R-square: 0.9645
Adjusted R-square: 0.9557
RMSE: 2.182e+05

Graphs 20. Curve fitting of fixed broadband France – MPDL model - 1998-2016



Legend: curve fitting of fixed broadband France - MPDII Model, years 1998-2016

Goodness of fit MPDII model: SSE: 8.069e+11 R-square: 0.9499 Adjusted R-square: 0.9317 RMSE: 2.708e+05

R-square (and correspondingly, the adjusted one) in Bass is higher than in other models: however, this cannot be used to imply that the Bass is better suited to the data than the other models, since the models compared are functionally different in terms of variables.

Root Mean Square Error (RMSE) can be used to evaluate a model's goodness of fit to the historical pattern of diffusion. RMSE measures the differences between the forecasted values by the models and the values actually observed. As a result, as the tables shows, it was found that the MPDL model had the lowest RMSE value among the three models and this time the MPDL is the model that best fits the French data. Once the different parameters have been estimated, it would be convenient to compare the different countries to each model.

Now we present the estimates of the different models. As Mahajan and Srinivasan (1986) remark, the use of OLS in the Bass model requires a two steps estimation; as a result, exact standard errors for the parameters of the model cannot be obtained. Moreover, making the endogenous factor in the Bass model a function of economic variables yields nonlinearities and thus the modified Bass models cannot be estimated by OLS (Zettelmeyer and Stoneman, 1993). So, we use the NLS procedure.

For the Bass model and for the MPDR model, we use NLS proposed by Mahajan and Srinivasan (1986). The formulations used are the following:

$$N(t) = m \frac{1 - e^{-(p+q)\,t}}{1 + \frac{q}{p} e^{-(p+q)\,t}} + \varepsilon(t)$$

(3.30)

$$N(t) = \frac{\beta\, pop + R\, pop}{1 + R} \; \frac{1 - e^{-(p+q)\,t}}{1 + \frac{q}{p} e^{-(p+q)\,t}} + \varepsilon(t)$$

(3.31)

For equations without analytical solution, we carry out the estimation directly from the discrete models.[5]

Table 6 presents the results of our estimates, concerning the Bass model:

Table 6: Parameter estimates of Fixed Broadband - Bass model – 1998-2000

GERMANY							
Parameter	Estimate	Std Errors	t value	Pr(>|t|)	R^2	a- R^2	RMSE
p	0.0138	0.0014	9.591	0.000***	0.998	0.998	5.51e+05
q	0.5133	0.0196	26.209	0.000***			
m	2.97e+07	1.02e-09	2.91e+16	0.000***			
ITALY							
Parameter	Estimate	Std Errors	t value	Pr(>|t|)	R^2	a- R^2	RMSE
p	0.0165	0.0026	6.401	0.000***	0.995	0.995	3.99e+05
q	0.5305	0.0319	16.648	0.000***			
m	1.43e+07	3.74e-09	3.82e+15	0.000***			
SPAIN							
Parameter	Estimate	Std Errors	t value	Pr(>|t|)	R^2	a- R^2	RMSE
p	0.0248	0.0033	7.604	0.000***	0.993	0.992	4.31e+05
q	0.3464	0.0245	14.146	0.000***			
m	1.37e+07	2.02e-09	6.76e+15	0.000***			
UK							
Parameter	Estimate	Std Errors	t value	Pr(>|t|)	R^2	a- R^2	RMSE
p	0.0206	0.0042	4.949	0.000***	0.987	0.986	1.05e+06
q	0.4099	0.0382	10.716	0.000***			
m	2.42e+07	2.09e-09	1.16e+16	0.000***			
FRANCE							
Parameter	Estimate	Std Errors	t value	Pr(>|t|)	R^2	a- R^2	RMSE
p	0.0062	0.0012	5.763	0.000***	0.995	0.994	7.97e+05
q	0.4463	0.0230	19.404	0.000***			
m	2.67e+07	9.68e-10	2.76e+16	0.000***			

Legend: Parameter estimated through NLS regression with the Bass model on the series 1998-2000.

Source: our elaboration on ITU data (2017).

[5] We carry out the estimates by Matlab sotware. As for the numerical algorithm, we used the Levenberg - Marquardt method.

As we can see from the estimates, in the different countries the R-squared is very high, and all the estimated parameters are significant. While the first result can be somehow more expectable[6], the second deserves more attention and suggests comparative analysis, having also in mind the opportunity to derive policy implication for the national and European digital agendas. Regarding the parameter q, concerning the speed of diffusion, it is higher for Italy, followed by Germany, France, UK and finally by Spain. The fact that Italy presents the fastest diffusion process can first comes as a surprise, and seems to be counterintuitive due to Italy's backwardness in broadband. In reality, this situation is coherent with the broader picture: if we look at the market potential, we immediately notice that Italy's absolute value is remarkably low (together with Spain) and this is even more evident when weighted by the population residing in Italy (see Table 7). If compared to a country similar in terms of population size, like France, we can observe a considerable delay and a consequent digital divide for this technology: France has nearly the double than the broadband penetration ratio of Italy.

Table 7. Market potential of broadband subscriptions for 100 inhabitants - "big five" Europe – 2016

Country	Market Potential	Market Potential/Population	Ranking
France	26.734.000	41.18	1
UK	24.281.000	37.53	2
Germany	29.749.000	36.19	3
Spain	13.674.000	29.42	4
Italy	14.333.000	23.68	5

Legend: Market Potential estimated through NLS regression with Bass model on the series 1998-2000.
Source: our elaboration on ITU data (2017).

[6] On one side, a high R-squared is typical of the NLS method. Hence, we focus on deviations from the norm.

Furthermore, the previous fact is also coherent with what already observed in figure 11: Italy is the country that first reaches the maximum subscription peak, and this obviously comes at a relatively lower rate of diffusion than the other countries of comparison. Then, as a further robustness check, calculating the maximum peak with the formulas presented in the previous section, it is confirmed that Italy reaches this peak before (id est, 2004). Then come UK (2005), Spain and France in 2006 and finally Germany in 2007.

$$t^* = \frac{ln\,\frac{q}{p}}{p+q} \tag{3.32}$$

$$N(t^*) = \frac{m\,(q-p)}{2q} \tag{3.33}$$

The parameter *p*, concerning the effect of external influence, records Spain as the country with the highest value. Follow UK, Italy, Germany and finally France. Returning to the estimated market potential, we note that the value of all countries is underestimated and this is also highlighted with the curve fitting. This empirical evidence has already been discussed frequently in the literature. Due to the small number of explanatory coefficients, from a computational perspective, the Bass Model is less sensitive to the initial guess for the market size.

Let us now compare our results with the previous literature, for which evidences are rather limited, when not imperfect.

The results of the *q* parameter seem to be in line with a previous work by Matteucci (2013) which analysed the morphology of the diffusion paths of broadband in the same countries using the logistic and the Gompertz models as reference in a time span between 1997 and 2009 with OECD data. This research uncovers a similar delay of the countries Spain and Italy - especially the latter. Coherently, the study calculates a similarly low value of Italy's absolute and relative market potential, with a peak that is reached before and at a significantly lower value than the other countries considered. Moreover, the author highlighted how the Gompertz model, of an asymmetrical nature with respect to the logistic model, is better suited to fit the broadband data. This reinforces the idea that network externalities can influence the different aspects of the trend of data that have an asymmetrical sigmoidal pattern.

Our results on Bass can be also compared with those of the research of Kijek and Kijek (2010) for the diffusion of broadband in the period 2000-2009. The authors study the diffusion of this technology in 29 OECD countries using the logistic model, the Bass model and a dynamic model introduced by Sharif and Ramanathan. Also in this work, Italy for the q parameter is at the top right after UK and the ranking continues as our results. For the p it is always Spain with the highest value. For the market potential, Italy is always the last in the group, even exceeded in absolute terms by Spain. In this case, these estimates are not underestimated.

Another work to mention is the study by Turk and Trkman (2012) which estimates the parameters of the Bass model for 20 European countries to analyse the spread of broadband. In this case there are some differences probably attributable to the data used. The authors use as diffusion variable the number subscriptions normalised per 100 inhabitants, when it is usually preferred to use the absolute number of subscriptions, due to the contentious informative power of the population numeraire, in this specific case[7]. Italy and Spain have high p but low m. This is in line with our work, but the q has

[7] Basically, this choice is contentious especially for fixed broadband, which is not a personal service, being used by the members of the family or firm employees. These average users vary in number across countries.

high values for Germany and the UK and not for Italy which has a lower value than even Spain. France has a very high potential market value.

Now we take care of the next model, called MPDR. MPDR has an explicit analytical solution, and to study the country behaviours we use the NLS estimation, with the Levenberg - Marquardt algorithm. The analytical solution expressed in the form of the cumulative adoptions is:

$$N(t) = \frac{\beta\, pop + R\, pop}{1 + R} \frac{1 - e^{-(p+q)\,t}}{1 + \frac{q}{p} e^{-(p+q)\,t}} \tag{3.34}$$

Basically, this is a model featuring an exogenous market potential containing a network parameter. Looking at the estimation output (table 8) we note that the values are very similar to the estimated parameters of the Bass model and even the ranking for each parameter follows the broad pattern of the most famous benchmark model (in particular for the parameters p and q). It is interesting to observe the ranking of the R parameter that follows the same trend of the potential market relative to the population. In fact, being an exogenous measure of the number of adopters who have already adopted with respect to the population, it cannot but follow this trend. It is therefore not a coincidence that France is first, while Italy is the last also surpassed by Spain. Also β, which is the initial fraction of the population that adopts the technology, follows the same trend of the market potential related to the population with France first, followed by UK, Germany, Spain and finally Italy. Also in this case, as for the Bass model, all the parameters are significant (always at 99%). On the other hand, we have changed the market potential exogenously and therefore we did not expect big differences. We have really high R-squared and a good fit with the data. The goodness of fit is reduced if we estimate the parameters through the discrete formulation (not presented here).

Table 8. Parameter estimates of Fixed Broadband - MPDR model – 1998-2000

GERMANY							
Parameter	Estimate	Std Errors	t value	Pr(>|t|)	R^2	a- R^2	RMSE
p	0.0148	0.0021	7.0071	0.000***	0.997	0.996	7.17e+05
q	0.4911	0.0301	15.862	0.000***			
β	0.0764	0.0050	15.254	0.000***			
R	0.4607	0.0032	145.44	0.000***			
ITALY							
Parameter	Estimate	Std Errors	t value	Pr(>|t|)	R^2	a- R^2	RMSE
p	0.0180	0.0037	4.8605	0.000***	0.992	0.991	5.15e+05
q	0.4978	0.0478	10.405	0.000***			
β	0.0069	0.0036	1.9335	0.000***			
R	0.3093	0.0026	117.46	0.000***			
SPAIN							
Parameter	Estimate	Std Errors	t value	Pr(>|t|)	R^2	a- R^2	RMSE
p	0.02478	0.0039	6.3338	0.000***	0.993	0.991	4.47e+05
q	0.3464	0.0396	8.745	0.000***			
β	0.0247	0.0074	3.3493	0.000***			
R	0.3820	0.0052	73.51	0.000***			
UK							
Parameter	Estimate	Std Errors	t value	Pr(>|t|)	R^2	a- R^2	RMSE
p	0.0203	0.0050	4.085	0.001**	0.987	0.985	1.11e+06
q	0.4138	0.0571	7.2432	0.000***			
β	0.0296	0.0120	2.4701	0.000***			
R	0.5536	0.0075	73.997	0.000***			
FRANCE							
Parameter	Estimate	Std Errors	t value	Pr(>|t|)	R^2	a- R^2	RMSE
p	0.0062	0.0013	4.839	0.000***	0.995	0.994	7.39e+05
q	0.4463	0.0324	13.79	0.000***			
β	0.1089	0.0091	11.985	0.000***			
R	0.5149	0.0053	96.317	0.000***			

Legend: Parameter estimated through NLS regression with the MPDR model on the series 1998-2000.

Source: our elaboration on ITU data (2017).

Now, we can begin to analyse the estimates of our class of models without analytical solution. Not having a benchmark, we will try to analyse the characteristics of each model by comparing the different parameters and the corresponding ranking with the basic model of Bass, which still constitutes the backbone of the various models. Let's start from the MPDL (see table 9).

The estimated parameter q has the same trend in the ranking of estimated q of the Bass model. Let us remember that although the models are different, since the original model of Bass has undergone an extension due to an endogenization of the market potential parameter, and may present different numerical values, they express the same effect, id est the measure of internal influence or word of mouth.

There is only one exception: Germany. Germany with the introduction of the network parameter, suffers an effect of non-identification that does not allow a very precise estimate. In fact, by removing the network parameter γ and estimating again, we note that Germany returns to follow the same trends as before in the parameters.

Note that the values of q are higher than those of Bass: a possible explanation could be provided by the fact that once the network effects are triggered, the part of the population that is infected becomes bigger and bigger and this intensity is captured by this parameter.

Also for this model the parameter p represents the external influence. With the exception of Germany, even in this case the ranking reflects the same positions, with Spain always first and France always last. The initial population B (explained with the parameter β) follows the usual trend of the diffusion rate, with the exception of Germany, id est first France, then follow UK, Spain and Italy. The low value of the fraction of population of the first adopters explains the initial situation of Italy's delay towards the other countries taken into consideration. This model has a more easily identifiable functional form than the rest of our class of models.

An important test to be done is to estimate, for all five countries, the value of the parameters without the explicit introduction of the network parameter γ. This verification is essential to better compare it with the Bass reference mode. In this case, all five countries have the non-significant parameter p as opposed to the models above. R-squared values are high but do not reach the Bass model and MPDR model levels. But, if we compare them with the values of the discrete Bass, the values are very similar. (results not displayed here).

Below you will find some graphs (21-25) representing the actual adaptability of the model to the data, with both the simulated curve (from the estimated parameters) and the real data plots. The exercise was done both for cumulative adoptions and for instantaneous adoptions. We will do the same for the remaining models.

Tab. 9: Parameter estimates of Fixed Broadband - MPDL model – 1998-2000

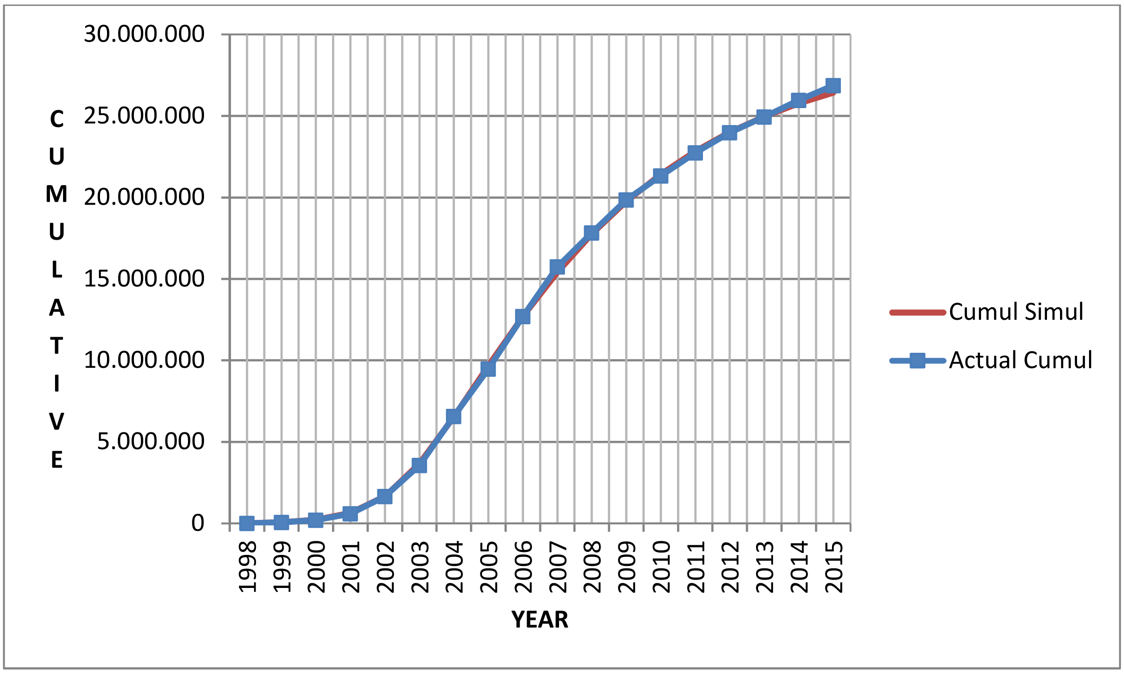
GERMANY							
Parameter	Estimate	Std Errors	t value	Pr(>|t|)	R^2	a- R^2	RMSE
p	0.0320	0.0191	1.673	0.1224	0.827	0.796	6.17e+05
q	0.5739	0.0753	7.627	0.000***			
γ	0.2384	0.0204	11.706	0.000***			
B	2.23e+07	1.04e-09	2.14e+16	0.000***			
ITALY							
Parameter	Estimate	Std Errors	t value	Pr(>|t|)	R^2	a- R^2	RMSE
p	0.0197	0.0848	0.2327	0.8199	0.943	0.934	1.93e+05
q	2.6818	0.1902	14.097	0.000***			
γ	0.90192	0.0025	356.18	0.000***			
B	1.46e+06	3.05e-07	4.79e+12	0.000***			
SPAIN							
Parameter	Estimate	Std Errors	t value	Pr(>|t|)	R^2	a- R^2	RMSE
p	0.0837	0.0497	1.682	0.1183	0.864	0.842	1.87e+05
q	1.1589	0.12374	9.366	0.000***			
γ	0.8326	0.0078	107.6	0.000***			
B	1.35e+06	4.36e-08	5.39e+13	0.000***			
UK							
Parameter	Estimate	Std Errors	t value	Pr(>|t|)	R^2	a- R^2	RMSE
p	0.0780	0.1558	0.501	0.6263	0.845	0.816	4.5e+05
q	2.4099	0.3153	7.643	0.000***			
γ	0.9143	0.0045	201.47	0.000***			
B	2.18e+06	3.08e-07	7.07e+12	0.000***			
FRANCE							
Parameter	Estimate	Std Errors	t value	Pr(>|t|)	R^2	a- R^2	RMSE
p	0.0121	0.0343	0.3519	0.7306	0.977	0.973	1.7e+05
q	1.9891	0.0809	24.581	0.000***			
γ	0.9097	0.0021	440.16	0.000***			
B	2.68e+06	4.89e-08	5.48e+13	0.000***			

Legend: Parameter estimated through NLS regression with the MPDL model on the series 1998-2000.

Source: our elaboration on ITU data (2017).

As we can see, the MPDL model fits well with broadband data for France. Even the peak is actually well estimated at around 2006. Also for other countries the MPDL has good data adaptability capabilities, but the model MPDL seems to fit France better. In fact, for this country is found highest value of R-squared and the lowest RMSE.

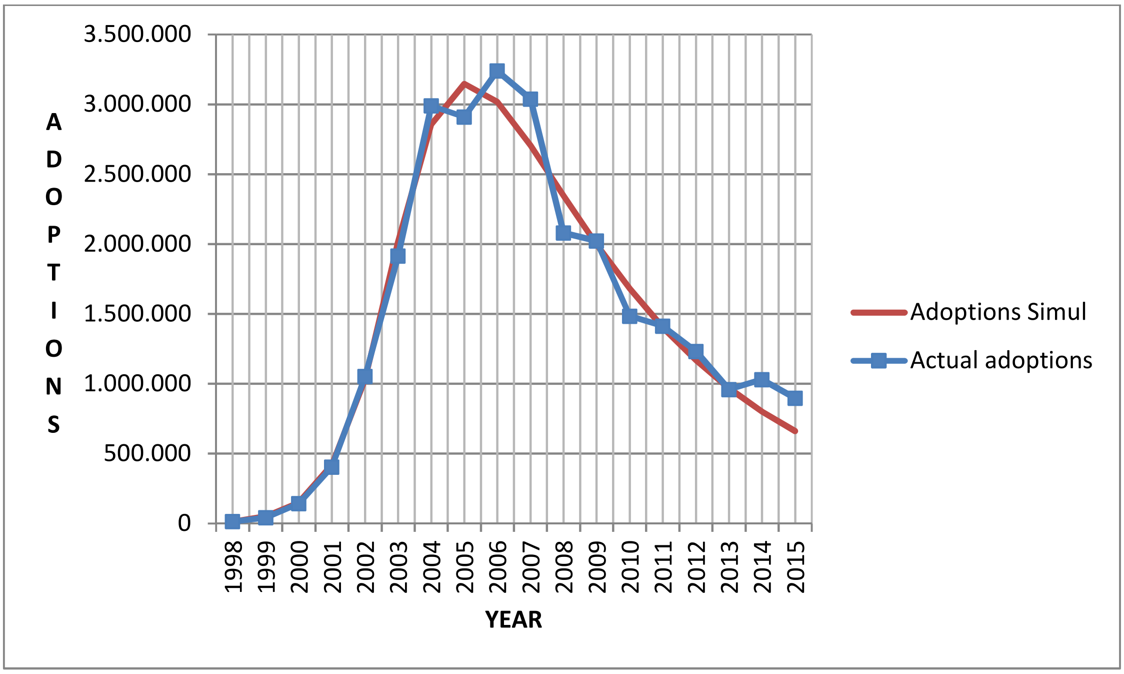
Graph 21. Cumulative series, MPDL France, fixed broadband, actual vs. simulated



Legend: Cumulative series, MPDL France, fixed broadband, actual vs. simulated.

Source: our elaboration on ITU data

Graph 22. Instantaneous series, MPDL France, fixed broadband, actual vs. simulated



Legend: Instantaneous series, MPDL France, fixed broadband, actual vs. simulated.

Source: our elaboration on ITU data

The next model is the MPD model (see Table 10). This model also reflects the same trends as the Bass reference model. The estimated parameter q has the same trend in the ranking of the estimated q in the Bass model. Also in this case, let us remember that although the models are different, since the original model of Bass has undergone an extension due to an endogenization of the potential market parameter, and may present different numerical values, they express the same effect, id est the measure of internal influence or word of mouth. The external influence parameter p presents an exception: Italy. In this case, Italy gains the top position, followed by Spain, which always ranks first for this type of parameter. The other positions remain unchanged with France always in the last position.

Not having explicitly introduced the network parameter, Germany has no problems with the estimated parameters. In this case, we have the fraction of late adopters α. The ranking follows as we expected the inverse path of that of the parameter of the fraction of the first adopters; Italy in first place, followed by Spain, UK, Germany and finally France. Moreover, thanks to the assumption $\beta = 1 - \alpha$, we can calculate the fraction of the initial adopters. Also, this ranking follows that of the diffusion rate and the market potential in relation to the population, with France taking the first position followed by UK, Germany, Spain and Italy.

Table 10. Parameter estimates of Fixed Broadband - MPD model – 1998-2000

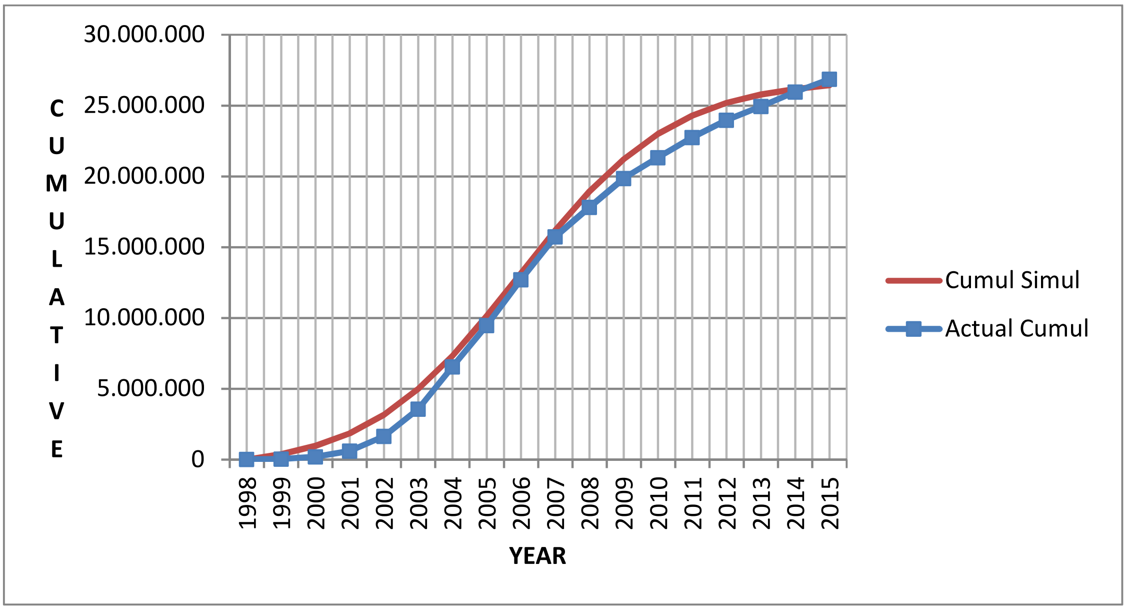
GERMANY							
Parameter	Estimate	Std Errors	t value	Pr(>|t|)	R^2	a- R^2	RMSE
p	0.0427	0.0623	0.686	0.5057	0.789	0.754	6.94e+05
q	0.9919	0.1806	5.491	0.000***			
α	0.9143	0.0067	137.33	0.000***			
ITALY							
Parameter	Estimate	Std Errors	t value	Pr(>|t|)	R^2	a- R^2	RMSE
p	0.1227	0.1098	1.117	0.2878	0.874	0.851	2.86e+05
q	1.5812	0.2514	6.288	0.000***			
α	0.9676	0.0018	542.39	0.000***			
SPAIN							
Parameter	Estimate	Std Errors	t value	Pr(>|t|)	R^2	a- R^2	RMSE
p	0.1604	0.067873	2.3634	0.0376**	0.803	0.767	2.09e+05
q	0.8716	0.17892	4.8712	0.000***			
α	0.9503	0.0041	229.8	0.000***			
UK							
Parameter	Estimate	Std Errors	t value	Pr(>|t|)	R^2	a- R^2	RMSE
p	0.1085	0.0633	1.714	0.1123	0.715	0.667	6.33e+05
q	0.9380	0.1998	4.694	0.000***			
α	0.9171	0.0079	115.48	0.000***			
FRANCE							
Parameter	Estimate	Std Errors	t value	Pr(>|t|)	R^2	a- R^2	RMSE
p	0.0611	0.0272	2.245	0.0428**	0.874	0.855	3.95e+05
q	0.8081	0.0978	8.27	0.000***			
α	0.8923	0.0068	132.14	0.000***			

Legend: Parameter estimated through NLS regression with the MPD model on the series 1998-2000.

Source: our elaboration on ITU data (2017).

With regard to the significance of the parameters, the parameters are all significant except for the parameter *p* for Germany and Italy.

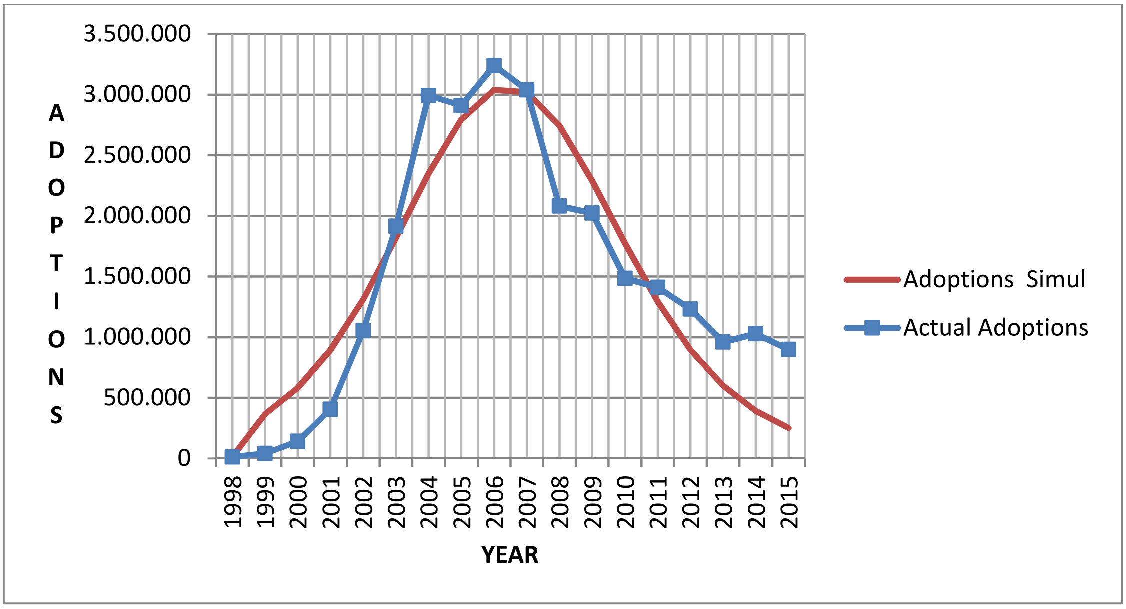
Graph 23. Cumulative series, MPD France, fixed broadband, actual vs. simulated



Legend: Cumulative series, MPD France, fixed broadband, actual vs. simulated.

Source: our elaboration on ITU data

Graph 24. Instantaneous series, MPD France, fixed broadband, actual vs. simulated



Legend: Instantaneous series, MPD France, fixed broadband, actual vs. simulated.

Source: our elaboration on ITU data

Also for this model we have seen how the model fits the data of the different countries. In this case, MPD fits well with the French data, but not like the previous model and the MPDII that we will see later.

Now, we present the latest model that concludes the analysis of the estimates of our class of models. The model in question is the MPDII model. The version that we present now is without the network parameter. This model is different from the others because it presents an initial market potential which can in turn influence the size of the network[8]. Table 11 show the estimates of the MPDII model.

Table 11. Parameter estimates of Fixed Broadband - MPDII model – 1998-2000

GERMANY							
Parameter	Estimate	Std Errors	t value	Pr(>|t|)	R^2	a- R^2	RMSE
p	0.0318	0.0175	1.7679	0.105	0.831	0.8	6.1e+05
q	0.5562	0.0751	7.405	0.000***			
M_0	1.20e+07	2.45e+05	49.136	0.000***			
ITALY							
Parameter	Estimate	Std Errors	t value	Pr(>|t|)	R^2	a- R^2	RMSE
p	0.0461	0.0179	2.58	0.024**	0.876	0.839	2.97e+05
q	0.5071	0.0877	5.785	0.000***			
M_0	6.23e+06	1.81e+05	34.499	0.000***			
SPAIN							
Parameter	Estimate	Std Errors	t value	Pr(>|t|)	R^2	a- R^2	RMSE
p	0.0550	0.0142	3.877	0.002***	0.907	0.879	1.51e+05
q	0.3639	0.0663	5.487	0.000***			
M_0	5.58e+06	1.97e+05	28.338	0.000***			
UK							
Parameter	Estimate	Std Errors	t value	Pr(>|t|)	R^2	a- R^2	RMSE
p	0.0473	0.0202	2.343	0.037**	0.913	0.887	3.53e+05
q	0.4600	0.1015	4.532	0.000***			
M_0	9.57e+06	4.05e+05	23.618	0.000***			
FRANCE							
Parameter	Estimate	Std Errors	t value	Pr(>|t|)	R^2	a- R^2	RMSE
p	0.0313	0.0115	2.715	0.018**	0.95	0.937	2.59e+05
q	0.4679	0.0609	7.689	0.000***			
M_0	1.046e+07	2.92e+05	35.858	0.000***			

Legend: Parameter estimated through NLS regression with the MPDII model on the series 1998-2000.

Source: our elaboration on ITU data (2017).

[8] Let's not forget that the functional form of MPDII is: $\boldsymbol{m(t)} = \boldsymbol{m_0}\left(\mathbf{1} + \boldsymbol{e}^{\gamma\frac{N(t)}{pop(t)}}\right)$

The parameter *q* immediately presents a surprise: Germany presents itself in first place. Then, follow Italy, France and Spain, id est the usual ranking. The parameter *p* follows the usual ranking, with Spain, Italy, UK, Germany and France.

Also for this model we can present an index to compare the market potential with respect to the population. In this case, however, this is an initial market potential. As can be seen from the table 12, we find ourselves in the same situation as the market potential of Bass and the fractions of the population that adopt early. Indeed, France maintains the top of ranking, followed by the UK, Germany, Spain and Italy.

Table 12: Initial Market Potential of broadband subscriptions for 100 inhabitants - "big five" Europe.

Country	Market Potential M_0	Market Potential M_0/Population	Ranking
France	10.465.000	16.12	1
UK	9.569.000	14.81	2
Germany	12.031.000	14.64	3
Spain	5.578.000	12.00	4
Italy	6.230.000	10.00	5

Legend: Initial Market Potential estimated by NLS regression with MPDII model on the series 1998-2000.
Source: our elaboration on ITU data (2017).

All parameters for each country are significant, except for the parameter *p* for Germany.

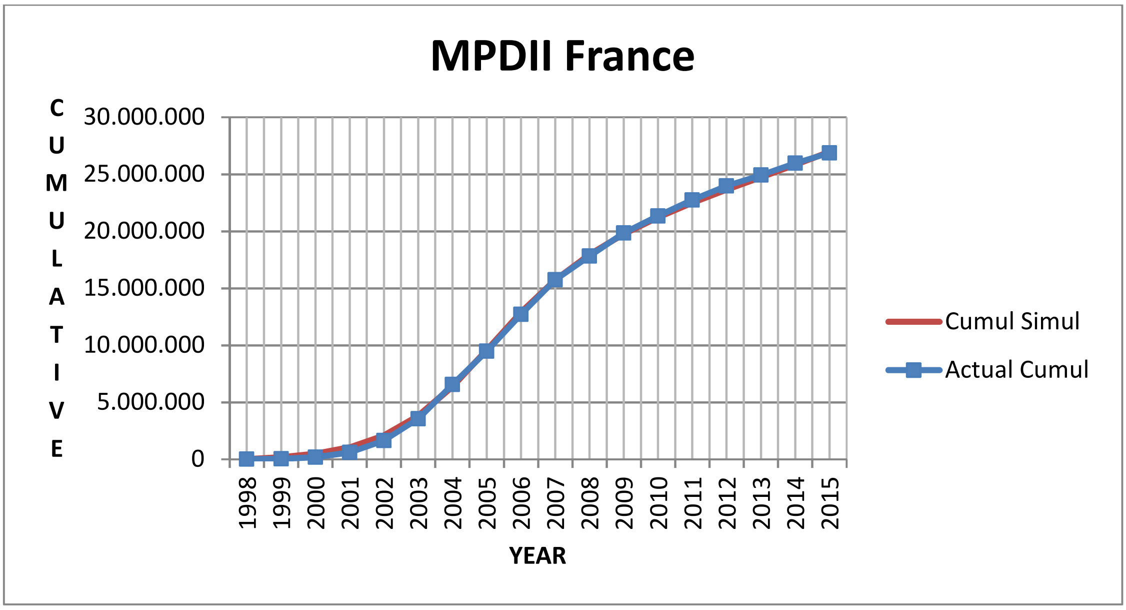
Regarding the version with the network parameter, the same discussion follows for the MPDL, remembering that this time Germany too follows an exceptional trend that does not occur when the parameter that measures the intensity of the network is not explicitly introduced. The estimated parameter

q has the same trend in the ranking of estimated q of Bass model. Also for this model the parameters are significant except for the p parameter for Germany and Italy.

Recall that the increase in the network effect intensity measured by the network parameter increases the convexity [9]of the market potential that causes a subsequent increase in the latest data that can explain the increases of the instantaneous subscriptions of the last two years. This peculiarity can be seen in the curve fitting and in the following graphs, in particular for instantaneous adoptions.

Also for this last model we see how the model fits the data of the various countries. Like MPDL, MPDII has excellent adaptability to French data as can be seen from the graphs 25 and 26 (version with explicit network parameter).

Graph 25. Cumulative series, MPDII France, fixed broadband, actual vs. simulated

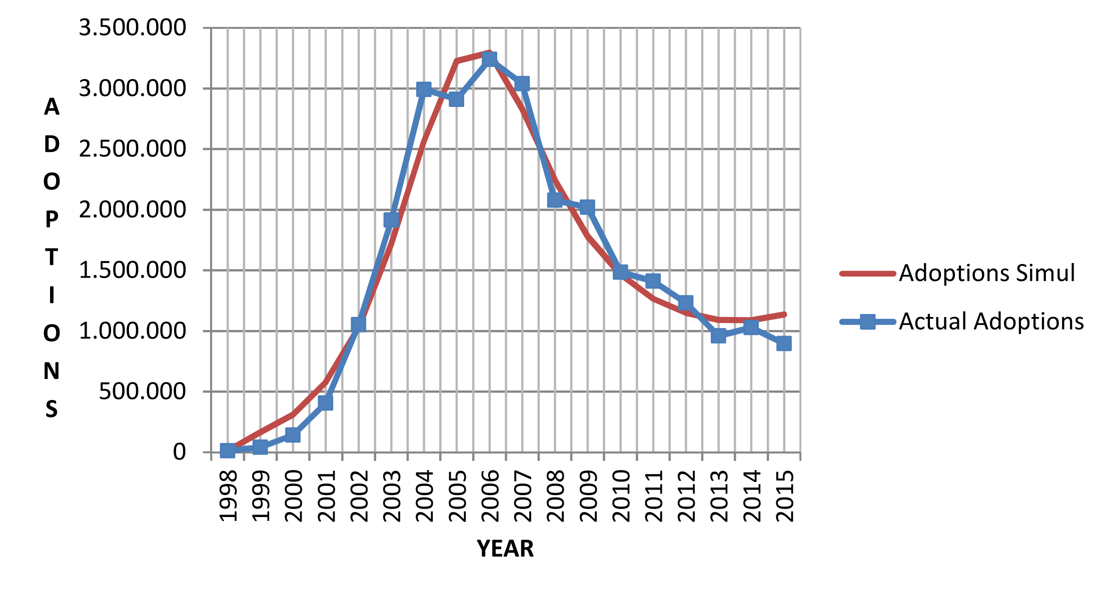


Legend: Cumulative series, MPDII France, fixed broadband, actual vs. simulated.

Source: our elaboration on ITU data

[9] The convexity of the potential market with respect to the size of the network.

Graph 26. Instantaneous series, MPDII France, fixed broadband, actual vs. simulated



Legend: Instantaneous series, MPDII France, fixed broadband, actual vs. simulated.

Source: our elaboration on ITU data

It seems necessary to make a summary of all the models with the weaknesses and the coherences found. Table 9 can help us providing a general framework

Table 13: Summary of the estimated parameters and ranking- "big five" Europe.

Model	Country / Parameter	GER	ITA	SPA	UK	FRA
BASS	***p***	0,0138	0,0165	0,0248	0,0206	0,0062
	Rank p	4	3	1	2	5
	q	0,5133	0,5305	0,3464	0,4099	0,4463
	Rank q	2	1	5	4	3
	m	29.749.000	14.333.000	13.674.000	24.281.000	26.734.000
	m/pop	0,36	0,24	0,29	0,37	0,41
	Rank m/pop	3	5	4	2	1

Model	Country / Parameter	GER	ITA	SPA	UK	FRA
MPDL	***p***	0,032	0,0197	0,0837	0,078	0,0121
	Rank p	3	4	1	2	5
	q	0,5736	2,6818	1,1589	2,4099	1,9891
	Rank q	5	1	4	2	3
	β	0,1982	0,0241	0,0366	0,0367	0,0413
	Rank β	1	5	4	3	2
	γ	0,2384	0,9019	0,8326	0,9143	0,9097
	Rank γ	5	3	4	1	2

Model	Country / Parameter	GER	ITA	SPA	UK	FRA
MPD	***p***	0,0635	0,1537	0,1312	0,1198	0,0523
	Rank p	4	1	2	3	5
	q	0,6402	0,7801	0,4414	0,4712	0,6101
	Rank q	2	1	5	4	3
	α	0,8598	0,9388	0,9057	0,8523	0,8389
	Rank α	2	5	4	3	1

Model	Country / Parameter	GER	ITA	SPA	UK	FRA
MPDII	***p***	0,0318	0,0461	0,055	0,0472	0,0312
	Rank p	4	3	1	2	5
	q	0,5562	0,5071	0,364	0,46	0,4679
	Rank q	1	2	5	4	3
	Mo	10.675.000	3.237.000	2.925.000	4.824.000	5.778.000
	Mo/pop	0,13	0,05	0,06	0,07	0,09
	Rank Mo/pop	1	5	4	3	2

Model	Country / Parameter	GER	ITA	SPA	UK	FRA
MPDIIn	***p***	0,0293	0,0501	0,0532	0,0468	0,0262
	Rank p	4	2	1	3	5
	q	0,614	1,223	0,7993	1,1694	0,9141
	Rank q	5	1	4	2	3
	Mo	12.031.000	6.230.000	5.578.000	9.569.000	10.465.000
	Mo/pop	0,146	0,10	0,12	0,148	0,16
	Rank Mo/pop	3	5	4	2	1

Model	Country / Parameter	GER	ITA	SPA	UK	FRA
MPDR	***p***	0,0148	0,018	0,0247	0,0203	0,0062
	Rank p	4	5	1	2	3
	q	0,4911	0,4978	0,3464	0,4138	0,4463
	Rank q	2	1	5	4	3
	β	0,0764	0,0069	0,0247	0,0296	0,1089
	Rank β	2	5	4	3	1
	R	0,4607	0,3093	0,3820	0,5536	0,5149
	Rank R	3	5	4	1	2

Model	Country / Parameter	GER	ITA	SPA	UK	FRA
MPDn	***p***	0,047	0,1746	0,1598	0,155	0,0799
	Rank p	5	2	1	3	4
	q	0,5473	0,8293	0,4494	0,495	0,6947
	Rank q	3	1	5	4	2
	α	0,8096	0,948	0,9122	0,8885	0,9019
	Rank α	1	5	4	2	3
	γ	0,6515	1,0492	1,0305	1,1535	1,2622
	Rank γ	5	3	4	2	1

Legend: Parameters estimated by NLS regression with all models on the series 1998-2000.

Source: our elaboration on ITU data (2017).

CONCLUSIONS

The fast pace of technological change is increasing the interest in studying the mechanisms of new products diffusion. Many types of these technologies, often belonging to the ICT sectors, have peculiar features and adoption behaviours, compared to typical “stand alone” products. As a matter of fact, ICT may feature developments that significantly delay or anticipate the growth phase of the traditional “stand alone” sigmoidal curve. In the case of ICT, this distinct diffusion behaviour can be explained by the network externalities.

This work introduces a class of models that describes the adoption of new products and services with a dynamic market potential that depends on the size of the network, in various ways (chapter 2). The simulations of each model and the comparisons with the standard Bass model, which assumes a fixed market potential, are given.

The various models proposed here are able to capture the effect of network externalities. The simulation study shows that the performances of our models are promising both for fitting and for parameter estimation.

Thus, summarizing the main features that emerged from the simulations:

- MPDL, MPD and MPDII initially describe a slower trend for the initial periods;
- MPDL is more useful to describe the slowest diffusions with linear trend;
- MPD is more suitable to describe faster diffusion processes, especially in the presence of strong network externalities;
- MPDII is more suitable to describe slowest diffusion phenomena with nonlinear trend;

- MPDT is more useful to describe diffusion processes with strong network externalities, when the diffusion quickly ends. This model describes very well the bandwagon effect, but it does not show the presence of the chilling effect. MPDT is suitable for technologies that undergo rapid replacement of obsolete versions;
- MPDR, which is the only model of the class that has the exogenous market potential, follows the trend of the Bass model, but with an initial period characterized by a slight delay.

In the second step of the research we verified the ability of this class of models to explain empirically, with real market data, the presence and effects of network externalities (chapter 3). In particular, this stage concerns the determination of comparisons between different countries and the analysis of the diffusion paths of selected ICT technologies.

The main technology analysed is fixed broadband. Our analysis focuses on the ITU time series of fixed broadband subscriptions of the “big five” countries in Europe. Comparisons between these European countries rely on some diffusion indicators that also allow to study the presence of the broadband digital divide.

The morphological analysis of the historical diffusion paths, while enabling the exam of the different characteristics of the countries considered, also verifies the heuristic potential of the class of models we have built. This analysis focuses on curve fitting and on empirical estimations with real market data.

Despite being not analytically solvable in their highly complex cumulative equation, our models can be fruitfully estimated, thanks to the usage of the newest releases of specific software packages. In particular, after a challenging training practice, we uncovered some operative routines to arrange our equations in a way amenable to empirical estimation with the software Matlab. Specifically, for the curve fitting and for the estimation of the parameters, we used NLS and the iterative method Levenberg – Marquardt.

We started with the Bass model, which stands as the main comparison benchmark in the literature. Despite being a model intensively used in the marketing literature, nevertheless it can be fruitfully employed also for conducting empirical studies more oriented to industry and innovation studies. Our study provides a convincing instance, since we start from the Bass model to conduct morphological analyses of country-level adoption paths. A main heuristic ambition is to examine phenomena such as the size of the national digital divide, the overall market potential for digital innovation and the dynamic behaviour of agents (delay or anticipation, structural break, and others).

When analysing the fixed broadband diffusion with the Bass model, and looking at the parameter estimates, a broad regularity and clear ranking of the different EU countries considered emerges beside letting scope for some country-specific characteristics:

- France is the country with the highest potential market (compared to the population), while Italy has the lowest one, relatively to the five countries considered;
- Italy reaches the peak of subscriptions before the other countries, at relatively lower levels of cumulative adoptions;
- on overall, the estimates confirm the presence of higher digital divides in Spain and Italy;
- Italy has high values of the parameter q, an evident sign that soon reaches the saturation phase.

Hence, these morphological results broadly confirm the findings of the stylised market facts stemming from earlier literature – also of those studies employing other diffusion models (logistic, etc.), and/or shorter time series. Among the broad regularities, we find that the diffusion processes were broadly similar across the Big-five states for their morphology and parameter behaviours, although with specificity (for example, market leads and retards). This broad similarity is a typical result found when the chosen countries share common paths of socio-economic development and homogeneous institutions and policies (like in EU with the digital agendas). Moreover, the morphological analysis can benchmark the country experiences and give the policy-maker some insights on the innovation policy: for example, we can use the estimated parameters to evaluate the digital agenda effectiveness and compatibility with the country digital needs. In turn, although the macro-models are not the best analytical tool to investigate the individual-level diffusion drivers, their results can be fruitfully coupled with the micro-level studies to detect the country-variant factors and specificities.

Indeed, the Bass model is great for explaining the diffusion phenomenon, but it does not explicitly capture the dynamics of network externalities, mainly because the assumptions on the diffusion parameters are very restrictive. Our class of models instead focused on these dynamics.

Then, we passed to analyse the estimates of our class of models not having analytical solution. Lacking a literature benchmark, we examine the characteristics of each model by comparing – in what possible - the different parameters and the corresponding rankings of the countries when modelled with the basic Bass, which can be considered as the backbone of the various models here developed.

All models fit well with the used historical data, although with minor specificities. In particular, the models which are better suited to the historical data are MPDL and MPDII. The parameter estimates follow the situation encountered with the Bass model and are quite consistent among all the countries, with the exception of Germany for the models that explicitly have a network parameter.

This consistency is important and conveys confidence on the verification of the adequacy of these models. Moreover, given the ability to explicitly capture network externalities, this class of models can be a valuable tool to support the Bass model to explain the diffusion phenomena of ICT technologies.

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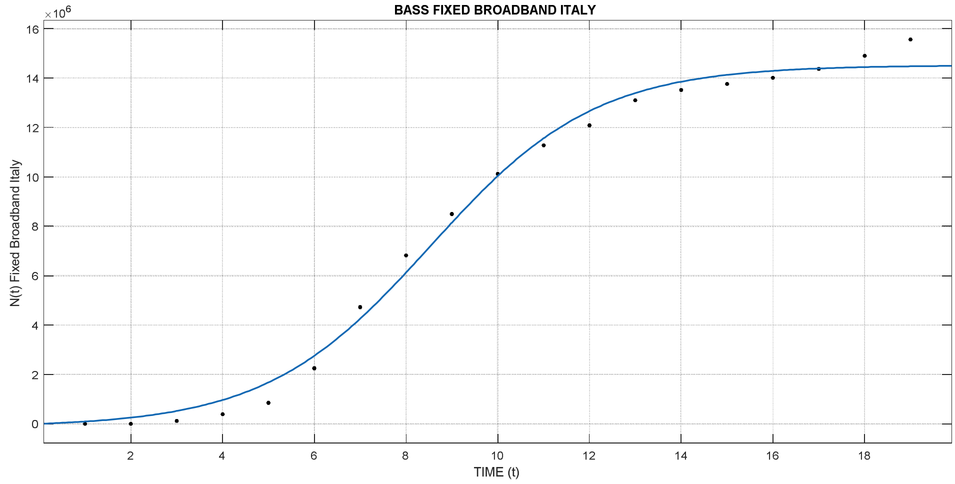
APPENDIXES

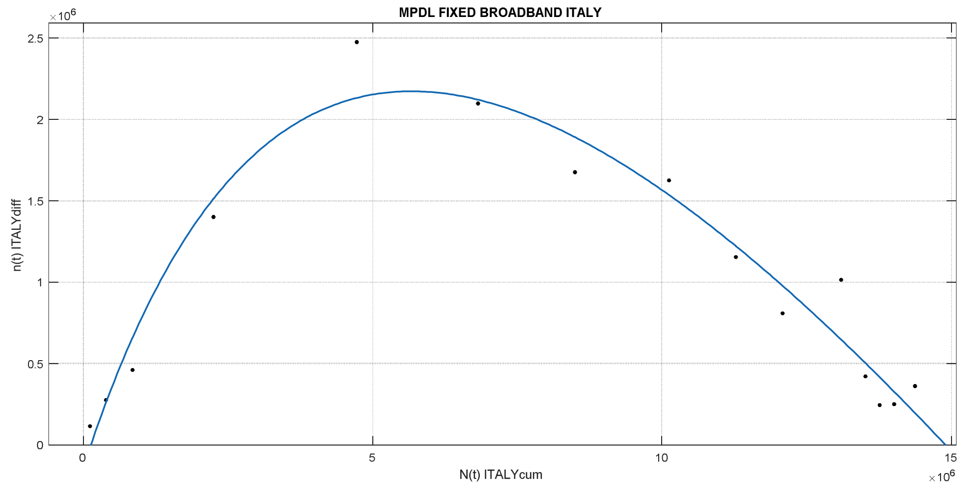
Graph A1. Curve fitting Italy – BASS Model



Legend: curve fitting of fixed broadband Italy - Bass Model, years 2000-2016

Goodness of fit: SSE: 4.556e+12 R-square: 0.993 Adjusted R-square: 0.9921 RMSE: 5.336e+05

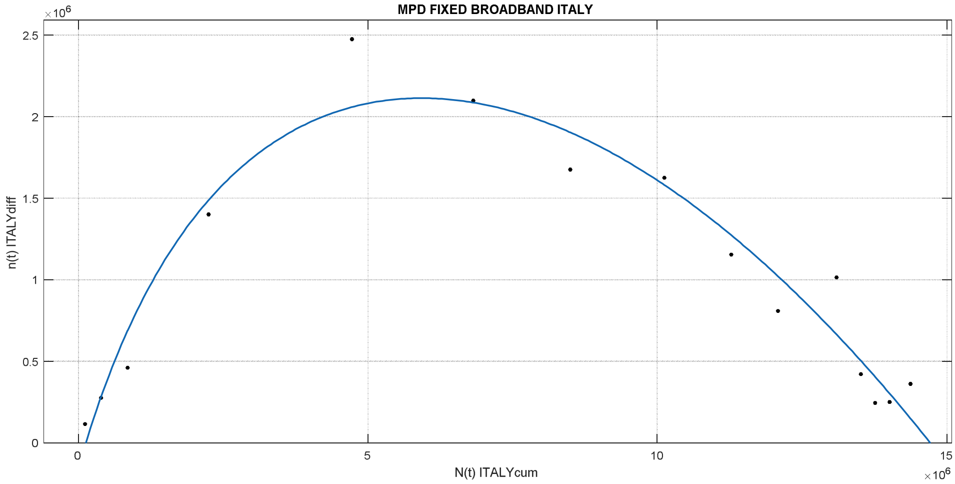
Graph A2. Curve fitting Italy – MPDL Model



Legend: curve fitting of fixed broadband Italy - MPDL Model, years 2000-2016

Goodness of fit: SSE: 4.79e+11 R-square: 0.9393 Adjusted R-square: 0.9228 RMSE: 2.087e+05

Graph A3. Curve fitting Italy – MPD Model



Legend: curve fitting of fixed broadband Italy - MPD Model, years 2000-2016

Goodness of fit:

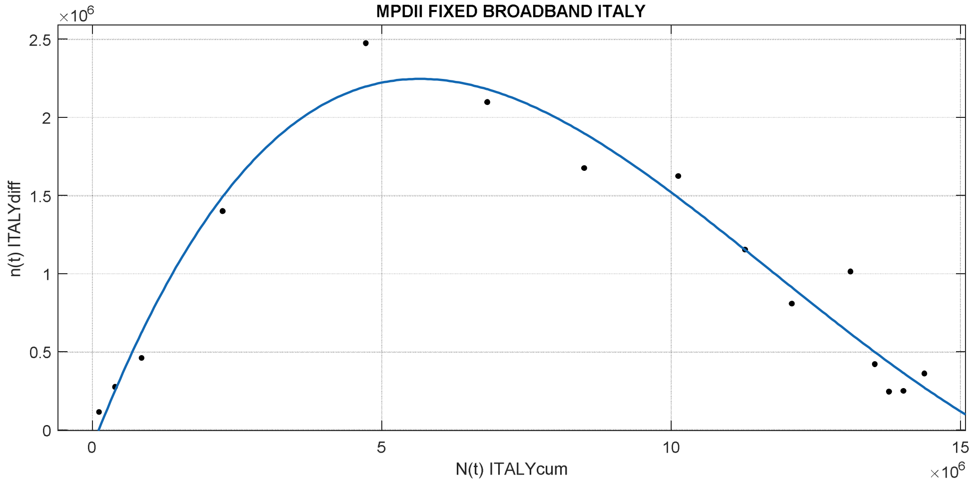
SSE: 5.653e+11

R-square: 0.9284

Adjusted R-square: 0.9089

RMSE: 2.267e+05

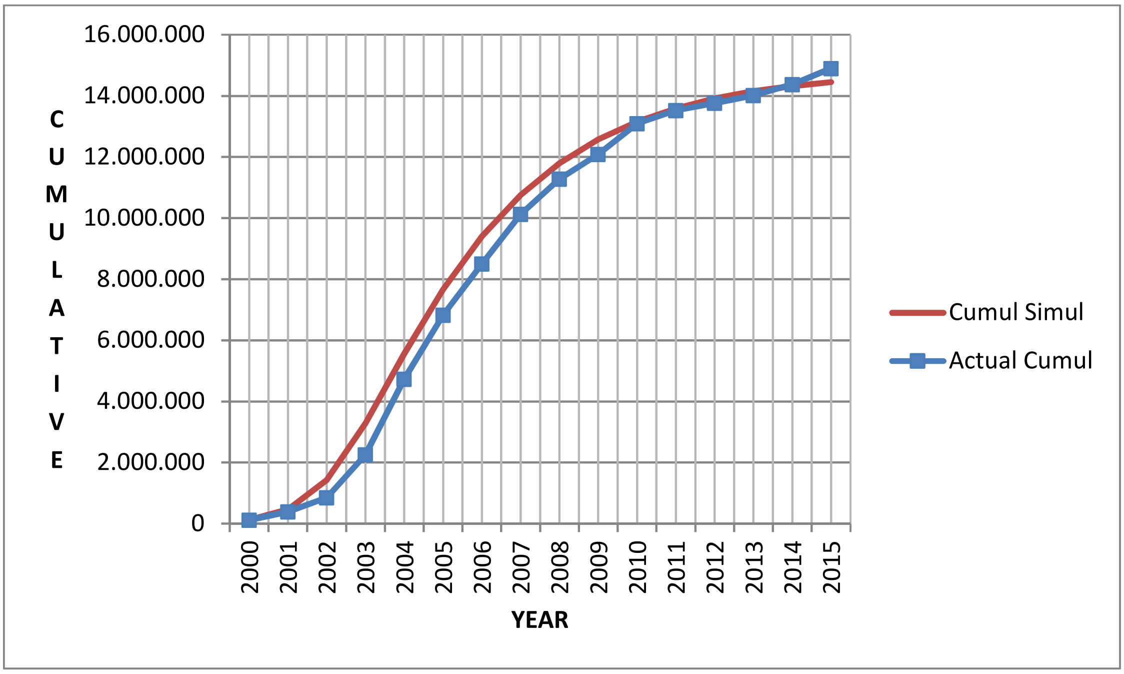
Graph A4. Curve fitting of fixed broadband Italy – MPDII Model



Legend: curve fitting of fixed broadband Italy - MPDII Model, years 2000-2016

Goodness of fit: SSE: 4.306e+11 R-square: 0.9455 Adjusted R-square: 0.9237 RMSE: 2.087e+05

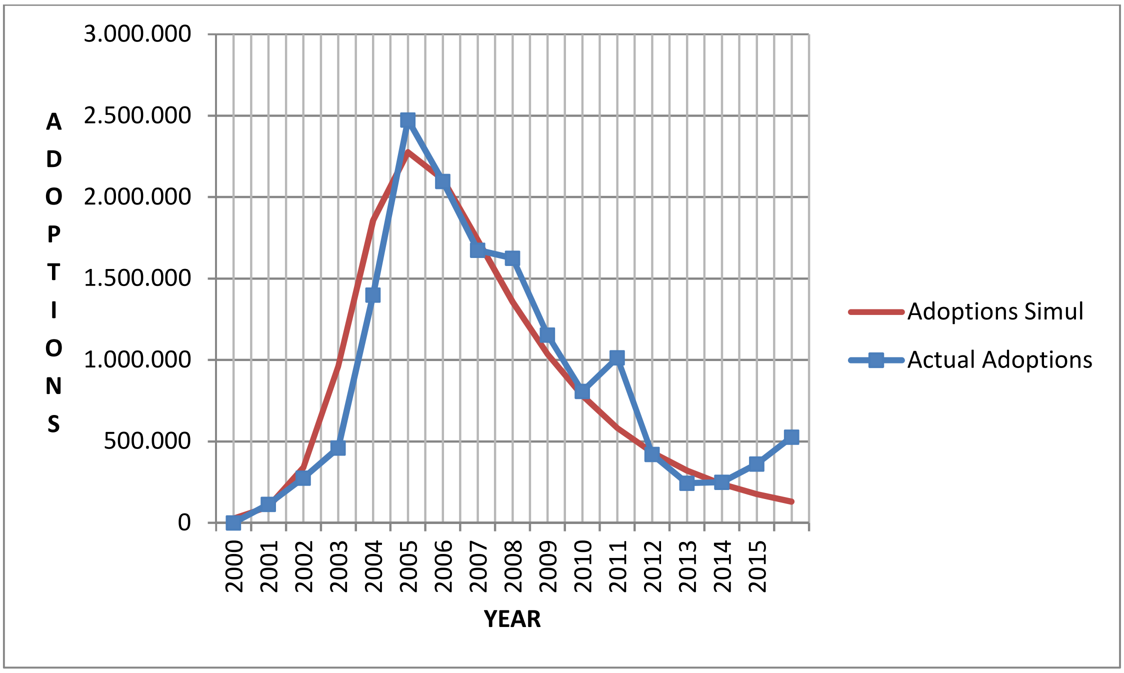
Graph A5. Cumulative series, MPDL Italy, fixed broadband, actual vs. simulated.



Legend: Cumulative series, MPDL Italy, fixed broadband, actual vs. simulated.

Source: our elaboration on ITU data

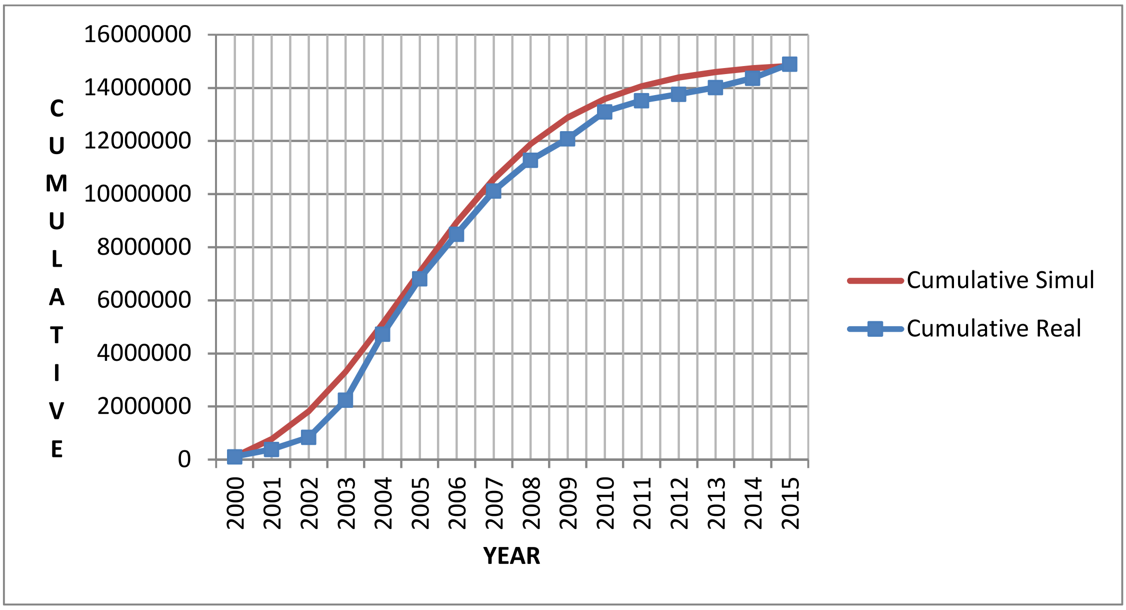
Graph A6. Instantaneous series, MPDL Italy, fixed broadband, actual vs. simulated.



Legend: Instantaneous series, MPDL Italy, fixed broadband, actual vs. simulated.

Source: our elaboration on ITU data

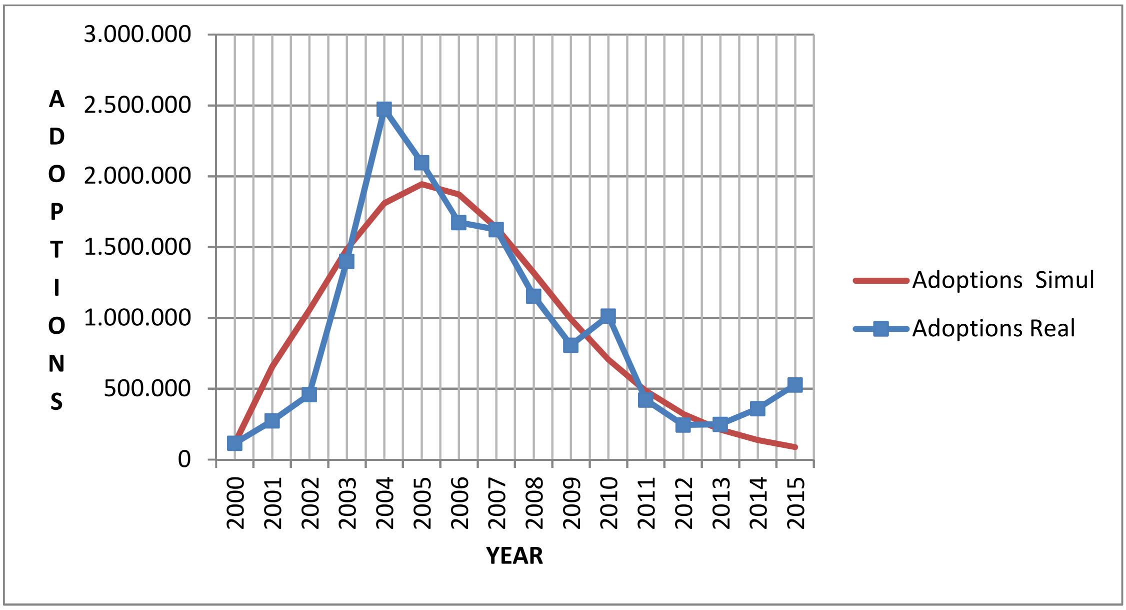
Graph A7. Cumulative series, MPD Italy, fixed broadband, actual vs. simulated.



Legend: Cumulative series, MPD Italy, fixed broadband, actual vs. simulated.

Source: our elaboration on ITU data

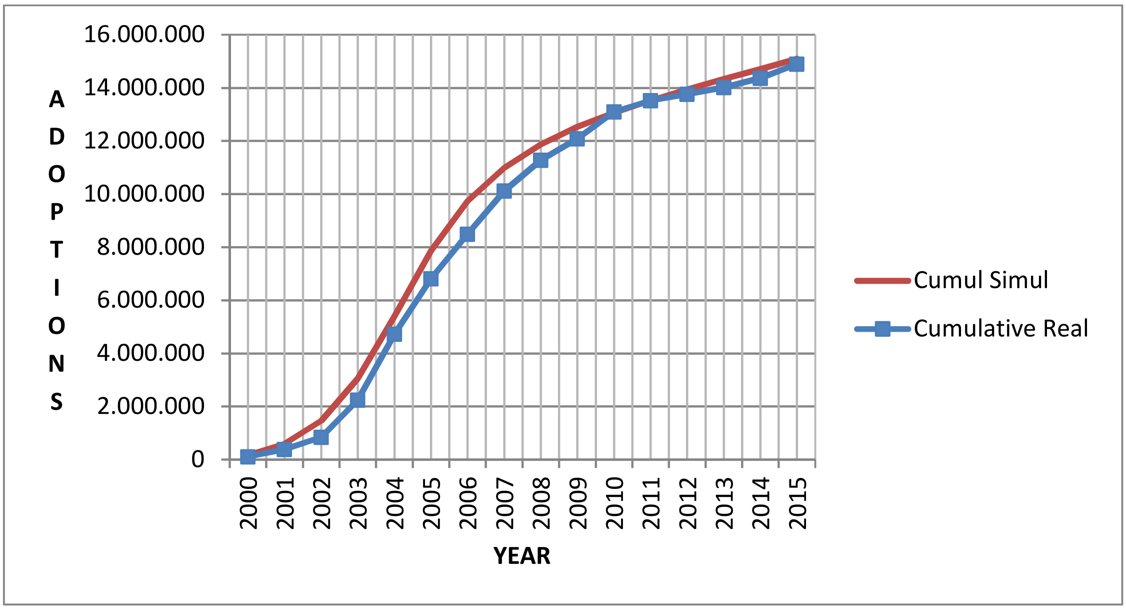
Graph A8. Instantaneous series, MPD Italy, fixed broadband, actual vs. simulated.



Legend: Instantaneous series, MPD Italy, fixed broadband, actual vs. simulated.

Source: our elaboration on ITU data

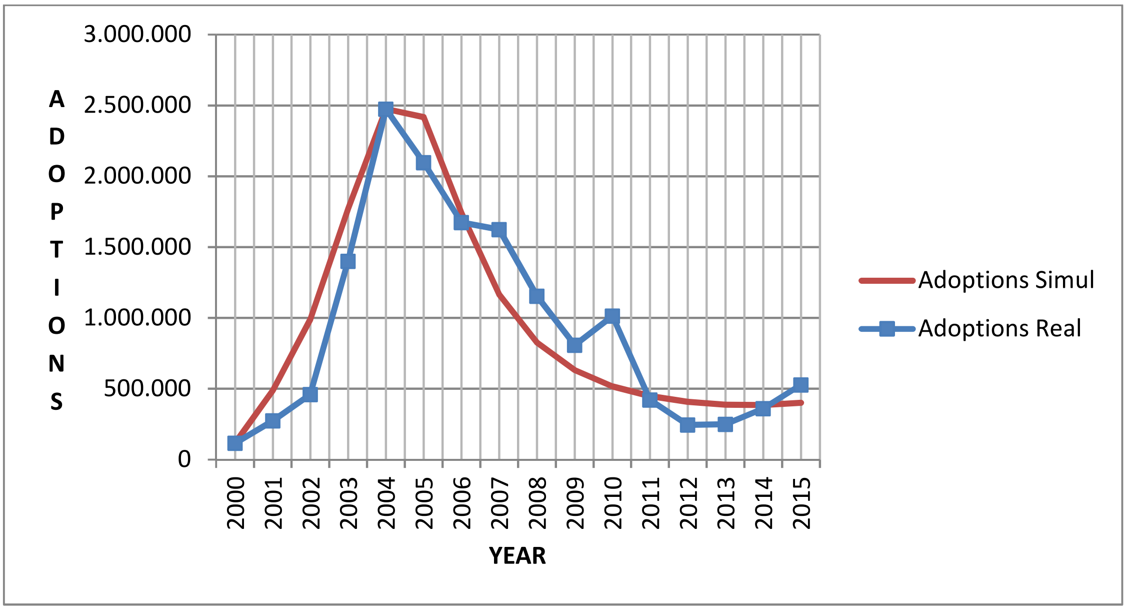
Graph A7. Cumulative series, MPDII Italy, fixed broadband, actual vs. simulated.



Legend: Cumulative series, MPDII Italy, fixed broadband, actual vs. simulated.

Source: our elaboration on ITU data

Graph A10. Instantaneous series, MPDII Italy, fixed broadband, actual vs. simulated.



Legend: Instantaneous series, MPDII Italy, fixed broadband, actual vs. simulated.

Source: our elaboration on ITU data

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