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Behavioral Cobweb Dynamics With Anticipatory Inventory and Ulam Stability: An Integro-Differential Approach

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ABSTRACT

This paper proposes a novel extension of the classical cobweb price model by incorporating behavioral inventory responses through an anticipatory mini-storage mechanism. In many real-world commodity markets, persistent price oscillations occur even when classical stability conditions are theoretically satisfied, an inconsistency traditional models fail to address. To bridge this gap, we develop an integro-differential model that captures both immediate supply–demand imbalances and cumulative memory effects, reflecting how suppliers adjust inventory in response to expected price shifts. Analytical solutions are derived using the Aboodh transform, enabling classification of market dynamics into underdamped, overdamped, and critically damped regimes. We evaluate the system's robustness under bounded perturbations using Hyers–Ulam and Hyers–Ulam–Rassias stability frameworks, confirming its structural stability. Numerical simulations show that incorporating anticipatory storage significantly dampens price volatility and accelerates convergence to equilibrium. Sensitivity and phase-space analyses further demonstrate the model's ability to replicate diverse market behaviors, offering theoretical insights for policy interventions.

1 | Introduction

Price dynamics in commodity markets remain a persistent concern for economists, policymakers, and supply chain managers. Despite extensive theoretical modeling, real-world prices often exhibit sustained volatility, cyclical fluctuations, or delayed convergence to equilibrium, even when classical stability conditions are met. For instance, rice prices in South Asian markets continue to fluctuate erratically, defying predictions of equilibrium behavior under supply–demand elasticity assumptions [1]. Similarly, oil markets frequently show persistent cycles and overshooting effects, complicating long-term planning [2].

The classical cobweb model, originating from Ezekiel's seminal work [3], offers a foundational framework for modeling such price dynamics. It captures the delayed response of supply to price changes and assumes naive expectations among producers. Under its standard formulation, prices are expected to converge to a stable equilibrium if the absolute value of the supply elasticity is less than that of demand. However, empirical evidence often contradicts this prediction, with prices displaying cyclical, persistent, or chaotic behaviors even when the theoretical criteria are satisfied [4, 5]. To address these limitations, numerous extensions have been proposed. Stochastic cobweb models incorporate random shocks and adaptive expectations

[6], while agent-based frameworks capture heterogeneous producer behavior [7]. Recent mathematical generalizations include the use of conformable fractional derivatives [8] and time-delay systems [9], aimed at capturing more nuanced dynamics. While these approaches offer valuable insights, they do not fully explain why price disturbances persist under theoretically stable elasticity conditions.

Recent progress in high-order numerical schemes for integro-differential and pseudo-differential equations offers promising tools to further enhance cobweb dynamics with memory effects. In particular, compact Crank–Nicolson ADI methods on graded meshes and extrapolation-based schemes have been successfully applied to nonlinear PIDEs with complex kernels [10, 11]. Energy-preserving methods tailored to nonlinear systems [12], and fractional approaches that ensure the preservation of discrete maximum principles and superconvergence properties [13, 14], suggest pathways to enrich our model's fidelity and stability. These developments underscore the growing role of memory structures, fractional calculus, and structure-preserving computation in economic dynamics.

Recognizing this gap, the present study introduces a novel extension of the classical cobweb model through the integration of a diagnostic mini-storage mechanism. Unlike traditional inventory or buffer stock systems designed for stabilization or optimization purposes [15–18], this mechanism models decentralized, anticipatory behavior among suppliers. When the expected price exceeds the initial price, suppliers may hoard inventory to benefit from anticipated gains; conversely, when prices are expected to fall, they may liquidate inventory early to avoid losses. This behavior introduces a memory-dependent feedback loop into the price adjustment process, which helps explain persistent oscillations even under stable elasticity assumptions. Conceptually, the mini-storage function serves as a theoretical proxy for behavioral inertia and supplier learning in competitive environments. Unlike prior models that introduce memory through exogenous delays or abstract fractional derivatives, our framework embeds memory endogenously via supplier behavior. This results in a feedback loop that reflects real-world decision-making, offering a more economically grounded interpretation.

To mathematically formalize this dynamic, we reformulate the cobweb system as an integro-differential model that incorporates both instantaneous and cumulative past supply–demand imbalances. We employ the Aboodh transform [19–21], a relatively recent integral transform particularly suited for solving differential equations involving convolution-type memory terms. Compared to traditional methods such as the Laplace or Elzaki transform, the Aboodh transform enables more tractable handling of systems with embedded feedback and historical dependencies, as found in price adjustment dynamics with behavioral storage.

To assess the structural stability of the modified system, we apply Ulam-type stability theory, specifically the Hyers–Ulam and Hyers–Ulam–Rassias frameworks [22, 23]. These tools allow us to evaluate whether small deviations in initial conditions or model parameters lead to bounded variations in the system's trajectory, thus quantifying robustness under real-world uncertainty such as demand shocks or data noise.

This paper makes several key contributions. First, it develops a novel integro-differential cobweb model, incorporating a diagnostic mini-storage mechanism to capture anticipatory inventory behavior. Second, it derives analytical solutions using the Aboodh transform, classifying market dynamics into underdamped, overdamped, and critically damped regimes. Third, it provides a rigorous Ulam-type stability analysis, demonstrating the model's robustness under perturbations. Fourth, it uses comparative numerical simulations to illustrate the improved convergence properties (speed, smoothness, and stability) relative to the classical cobweb model. Finally, it discusses theoretical and policy implications, particularly for markets influenced by anticipatory supply adjustments.

The remainder of the paper is organized as follows. Section 2 presents the model formulation and analytical solution. Section 3 provides a stability analysis based on Ulam-type approaches. Section 4 offers simulations and economic interpretation. Section 5 concludes with theoretical implications and directions for future work.

2 | Behavioral Model and Analytical Setup

The classical cobweb model has long served as a foundational tool to describe price dynamics in markets characterized by production lags and myopic expectations. In its traditional form, producers base current supply decisions on prices observed in the previous period, assuming that past prices are indicative of future conditions. Meanwhile, demand is typically modeled as a function of the current price. This interplay results in a delay-driven feedback loop, often represented via linear supply and demand functions:

$$S(p(t)) = \lambda + \delta p(t-1), \quad D(p(t)) = \alpha + \beta p(t) \tag{1}$$

where λ and α represent baseline supply and demand, while δ and β capture the respective price elasticities. Linear functions are adopted here to isolate and examine the behavioral extensions introduced in this work. Although real-world demand and supply are generally nonlinear, the linear specification enables sharper analytical focus on the role of memory and strategic behavior. Nonlinear generalizations are deferred to future research. In equilibrium, the market clears when $S(p(t)) = D(p(t))$, yielding the static equilibrium price:

$$p_e = \frac{\lambda - \alpha}{\beta + \delta}.$$

Under the classical cobweb structure, convergence toward equilibrium is guaranteed if $|\delta/\beta| < 1$. However, many empirical studies reveal that even in such stable conditions, real-world prices continue to fluctuate or cycle. This inconsistency between theory and observation motivates a deeper investigation into how real producers behave.

2.1 | Anticipatory Inventory and Mini-Storage Mechanism

Unlike the assumptions of the classical cobweb model, producers in actual markets, particularly in agricultural, energy, and commodity sectors, often form expectations about future price movements and act preemptively. For instance, if suppliers anticipate that prices will rise, they may reduce current sales and store goods to sell later at a higher price. Conversely, if they expect prices to fall, they may accelerate sales to avoid losses. This behavior introduces an important economic reality: Inventory is used not just to buffer supply, but also strategically to optimize intertemporal returns. This strategy gives rise to a memory-dependent decision rule. The current rate at which prices adjust is no longer a function of only the immediate supply–demand imbalance. Instead, it is influenced by cumulative past mismatches that inform producer expectations and storage behavior. In other words, suppliers are diagnosing the market trajectory by observing the history of imbalances and adjusting their actions accordingly. This is the essence of what we term a mini-storage mechanism. To model this behavior, we postulate that the price adjusts in response to both the current and the historical (accumulated) supply–demand imbalance:

$$\frac{dp(t)}{dt} = \gamma_1[D(p(t)) - S(p(t))] - \gamma_2 \int_0^t [S(p(\mu)) - D(p(\mu))]\, d\mu, \tag{2}$$

where $\gamma_1 > 0$ reflects the speed of price response to the immediate imbalance and $\gamma_2 > 0$ reflects the weight of historical imbalance in shaping expectations and inventory response. The first term on the right-hand side captures contemporaneous adjustment, aligning with standard cobweb reasoning. The second term, however, introduces a behaviorally motivated memory effect, grounded in inventory dynamics. It formalizes how producers implicitly remember past disequilibria and let them guide present supply decisions, especially when planning under uncertainty. By substituting the linear forms from (1) into (2), we obtain

$$\frac{dp(t)}{dt} = -(\delta + \beta)\gamma_1[p(t) - p_e] - (\delta + \beta)\gamma_2 \int_0^t [p(\mu) - p_e]\, d\mu.$$

Letting $z(t) = p(t) - p_e$, and defining $\rho_1 = (\delta + \beta)\gamma_1$, $\rho_2 = (\delta + \beta)\gamma_2$, we arrive at the following integro-differential equation:

$$\frac{dz(t)}{dt} = -\rho_1 z(t) - \rho_2 \int_0^t z(\mu)\, d\mu \tag{3}$$

This reformulation highlights the dual-channel structure of price adjustment: The first term captures the usual frictional convergence toward equilibrium, while the second term introduces a memory-based correction driven by anticipatory inventory decisions. Equation (3) therefore embodies a more realistic depiction of how forward-looking suppliers behave in volatile markets, laying the foundation for the stability analysis that follows.

2.2 | Existence and Uniqueness of Solutions

Equation (3) can be converted into an integral form to assess solution existence and uniqueness. Integrating both sides gives

$$z(t) = z_0 - \rho_1 \int_0^t z(s)\, ds - \rho_2 \int_0^t \int_0^s z(\mu)\, d\mu\, ds \tag{4}$$

Letting $f(t) = z_0$ and defining the kernel $K(t, \mu) = \rho_1 + \rho_2(t - \mu)$, we rewrite (4) as a Volterra integral equation of the second kind

$$z(t) = f(t) + \int_0^t K(t, \mu) z(\mu)\, d\mu.$$

To ensure that a unique continuous solution exists, we verify the continuity and boundedness of $K(t, \mu)$ over the domain $0 \le \mu \le t \le T$. Since $\rho_1, \rho_2 > 0$ and $0 \le t - \mu \le T$, we have

$$\rho_1 \le K(t, \mu) \le \rho_1 + \rho_2 T = M.$$

Hence, $K(t, \mu)$ is continuous and bounded on the triangular domain $[0, T]^2$, satisfying the conditions required by the Weierstrass M-test and the method of successive approximations. This confirms that the integral equation is well-posed and admits a unique continuous solution.

2.3 | Analytical Solution via Integral Transforms

To solve Equation (3), we differentiate both sides:

$$\frac{d^2 z(t)}{dt^2} + \rho_1 \frac{dz(t)}{dt} + \rho_2 z(t) = 0.$$

This second-order linear ODE with constant coefficients describes damped oscillations in price deviations. At $t = 0$, the integral term in (3) vanishes

$$\int_0^0 z(\mu)\, d\mu = 0.$$

Therefore, we obtain

$$\left.\frac{dz(t)}{dt}\right|_{t=0} = -\rho_1 z(0) = -\rho_1 z_0.$$

This result justifies the choice of the initial derivative $z'(0) = -\rho_1 z_0$ we use in solving the second-order differential equation. Applying the Laplace transform, we get

$$Z(s) = \frac{z_0(s + \rho_1)}{s^2 + \rho_1 s + \rho_2}.$$

Letting $\alpha = \rho_1/2$ and $\beta = \sqrt{4\rho_2 - \rho_1^2}/2$, the inverse transform yields

$$z(t) = z_0 e^{-\alpha t}\left[\cos(\beta t) + \frac{\alpha}{\beta}\sin(\beta t)\right],$$

and hence the full price trajectory

$$p(t) = p_e + z_0 e^{-\alpha t}\left[\cos(\beta t) + \frac{\alpha}{\beta}\sin(\beta t)\right].$$

This solution illustrates damped oscillatory convergence to equilibrium. The parameter α governs the speed of convergence, while β captures the magnitude of cyclical memory from inventory feedback. Economically, larger γ_2 (stronger storage memory) increases damping, promoting stability. The Aboodh transform is particularly valuable in this context because it simplifies integro-differential systems involving convolution and memory terms, as found in behavioral storage-based market dynamics [19–21]. Unlike Laplace, which can be cumbersome for nested integrals, Aboodh handles such structures efficiently, enabling closed-form solutions with embedded historical influence.

Remark 1. Although the analytical solution to the integro-differential model was ultimately derived using the Laplace transform, the Aboodh transform offers specific advantages in dealing with convolution-type memory terms. In particular, the convolution integral

$$\int_0^t z(\mu)\, d\mu$$

in Equation (3) implies historical dependence, which can become complex in more general cases, especially when memory kernels are nonlinear or time-varying. The Aboodh transform $\mathcal{A}\{f(t)\}$ is defined as

$$\mathcal{A}\{f(t)\}(s) = \int_0^\infty e^{-st} f(t)\, dt + \frac{1}{s}\int_0^\infty e^{-st}\left(\int_0^t f(\tau)\, d\tau\right) dt,$$

which directly incorporates the integral of the function without requiring a separate convolution theorem. This property becomes advantageous when dealing with systems where the memory term is itself integral or fractional in nature. In our model, the Aboodh transform streamlines the solution of (3) by allowing a single-step transformation of both $z(t)$ and its integral. While the current solution uses the Laplace transform for familiarity and clarity, the Aboodh transform remains a more general and structurally elegant approach, particularly valuable for future extensions involving more complex memory kernels.

Remark 2. Although the analytical solution presented in this study ultimately employs the Laplace transform for clarity and accessibility, the Aboodh transform played an important role during the preliminary analysis. In particular, it was instrumental in validating convolution properties associated with the memory-dependent terms of the integro-differential system. The choice to present the final solution using the Laplace transform was made to ensure greater transparency and replicability for a broader audience, while still recognizing that the Aboodh transform provides a more general and structurally elegant framework for future extensions.

3 | Ulam-Type Stability Analysis

In this section, we analyze the stability of the extended cobweb model under the frameworks of Hyers–Ulam and Hyers–Ulam–Rassias. These concepts offer a more flexible and realistic approach to assessing the robustness of economic dynamics compared to classical stability theory, particularly in models with behavioral factors, memory effects, or bounded decision-making. Traditional stability notions, such as Lyapunov or asymptotic stability, focus on how small perturbations in initial conditions affect the evolution of exact solutions. While effective in idealized systems, they do not account for situations where the model equations are only approximately satisfied, due to uncertainty, behavioral deviations, or institutional constraints. In real markets, agents often act on incomplete information, adaptive expectations, or heuristic rules, leading to price trajectories that deviate slightly from theoretical predictions. In systems with delay or memory, such as those involving anticipatory inventory decisions, past imbalances further influence current dynamics, making perfect adherence to differential equations unrealistic. Hyers–Ulam stability fills this gap by asking whether an approximate solution, arising from structural noise or behavioral inconsistencies, remains close to a true solution. It shifts the focus from sensitivity to initial conditions toward the structural robustness of the model under small functional errors. The Hyers–Ulam–Rassias generalization further allows these deviations to be time-varying or nonlinear, capturing a wider range of realistic distortions. This framework is especially appropriate for our model. In the modified cobweb system with memory, anticipatory supplier behavior can produce dynamics that only approximately satisfy the governing integro-differential equation. Yet from an economic perspective, what matters is whether these near-solutions remain meaningfully close to the ideal trajectories. The Hyers–Ulam approach allows us to test this. Geometrically, Hyers–Ulam stability can be interpreted as the closeness of trajectories in the functional space of the model. If a function $y(t)$ approximately satisfies the governing differential equation, Hyers–Ulam stability ensures that there exists a true solution $z(t)$ such that the distance $|y(t) - z(t)|$ remains uniformly bounded for all t. In this sense, the approximate trajectory $y(t)$ remains confined within a bounded tube or corridor around the exact solution $z(t)$, with the width of this tube depending on the size of the perturbation and the system's stability constants. This geometric perspective highlights how Ulam-type stability captures the model's robustness not only to initial state uncertainty but to imperfect functional adherence, an aspect particularly relevant in our context, where agents' behavioral responses and memory effects induce deviations from the exact dynamic structure. We now proceed by considering the second-order linear differential equation derived previously.

$$\frac{d^2 z(t)}{dt^2} + \rho_1 \frac{dz(t)}{dt} + \rho_2 z(t) = 0 \tag{5}$$

where $z(t) = p(t) - p_e$ represents the deviation from the price equilibrium. The qualitative behavior of solutions to (5) depends on the discriminant $\Delta = \rho_1^2 - 4\rho_2$, which characterizes the nature of the damping.

3.1 | Underdamped Regime ($4\rho_2 > \rho_1^2$)

When the discriminant is negative, the roots of the characteristic equation are complex conjugates. This corresponds to oscillatory convergence of price to equilibrium. The general solution takes the form

$$z(t) = e^{-\alpha t}[A\cos(\beta t) + B\sin(\beta t)],$$

where $\alpha = \rho_1/2$, $\beta = \sqrt{4\rho_2 - \rho_1^2}/2$, and A, B are constants determined by initial conditions. For a perturbation $y(t)$ satisfying

$$\left| \frac{d^2 y(t)}{dt^2} + \rho_1 \frac{dy(t)}{dt} + \rho_2 y(t) \right| \le \epsilon \tag{6}$$

it can be shown that

$$|z(t) - y(t)| \le \frac{\epsilon}{\beta}.$$

If the perturbation is time-dependent, bounded by $|x(t)| \le \epsilon\psi(t)$ for a continuous non-negative function $\psi(t)$, the Hyers–Ulam–Rassias inequality yields

$$|z(t) - y(t)| \leq \frac{\epsilon}{\beta}\psi(t).$$

Theorem 1. *If $4\rho_2 > \rho_1^2$, then equation (5) is both Hyers–Ulam and Hyers–Ulam–Rassias stable, with stability constant $\kappa_1 = 1/\beta$.*

Proof. Suppose $y(t) \in C^2[0, \infty)$ satisfies (6). Let $w(t) = y(t) - z(t)$. Then $w(t)$ satisfies the nonhomogeneous equation

$$\frac{d^2w(t)}{dt^2} + \rho_1\frac{dw(t)}{dt} + \rho_2 w(t) = E(t), \quad \text{with} \quad |E(t)| \leq \varepsilon.$$

Using Duhamel's principle, the solution to this equation is given by

$$w(t) = \int_0^t G(t-\tau)E(\tau)\, d\tau,$$

where $G(t)$ is the Green's function for the homogeneous system. In the underdamped case, it satisfies

$$|G(t)| \leq \frac{1}{\beta}e^{-\alpha t}.$$

Therefore,

$$|w(t)| \leq \int_0^t \frac{\varepsilon}{\beta}e^{-\alpha(t-\tau)}\, d\tau = \frac{\varepsilon}{\beta}\left(1 - e^{-\alpha t}\right) \leq \frac{\varepsilon}{\beta}.$$

This proves Hyers–Ulam stability. If instead the perturbation satisfies

$$\left|\frac{d^2y(t)}{dt^2} + \rho_1\frac{dy(t)}{dt} + \rho_2 y(t)\right| \leq \varepsilon\psi(t),$$

then the same integral representation gives

$$|w(t)| \leq \int_0^t \frac{\varepsilon}{\beta}\psi(\tau)e^{-\alpha(t-\tau)}\, d\tau \leq \frac{\varepsilon}{\beta}\psi(t),$$

by the non-negativity and continuity of $\psi(t)$. This establishes Hyers–Ulam–Rassias stability. □

3.2 | Overdamped Regime ($4\rho_2 < \rho_1^2$)

In this regime, the characteristic roots r_1 and r_2 are real and distinct

$$r_{1,2} = \frac{-\rho_1 \pm \sqrt{\rho_1^2 - 4\rho_2}}{2}.$$

The general solution is

$$z(t) = C_1e^{r_1t} + C_2e^{r_2t},$$

where C_1, C_2 are constants determined by initial conditions. Under the same perturbation bounds as in (6), the stability inequalities become

$$|z(t) - y(t)| \leq \frac{\epsilon}{|r_2 - r_1|},$$
$$|z(t) - y(t)| \leq \frac{\epsilon}{|r_2 - r_1|}\psi(t).$$

Theorem 2. *If $4\rho_2 < \rho_1^2$, then Equation (5) is Hyers–Ulam and Hyers–Ulam–Rassias stable, with stability constant $\kappa_2 = 1/|r_2 - r_1|$.*

Proof. Suppose $y(t)$ is a function in $C^2[0, \infty)$ such that the residual from the differential equation is uniformly bounded

$$\left|\frac{d^2y(t)}{dt^2} + \rho_1\frac{dy(t)}{dt} + \rho_2 y(t)\right| \leq \varepsilon.$$

Define the deviation function $w(t) = y(t) - z(t)$. Then $w(t)$ satisfies the nonhomogeneous linear equation

$$\frac{d^2w(t)}{dt^2} + \rho_1\frac{dw(t)}{dt} + \rho_2 w(t) = E(t), \quad \text{with} \quad |E(t)| \leq \varepsilon.$$

The fundamental solution to the homogeneous system involves e^{r_1t} and e^{r_2t}, leading to the Green's function representation

$$w(t) = \int_0^t G(t-\tau)E(\tau)\, d\tau,$$

where the Green's kernel satisfies

$$|G(t)| \leq \frac{1}{|r_2 - r_1|}e^{\max(r_1,r_2)t}.$$

Since both roots are negative, the kernel decays exponentially. Hence, we estimate

$$|w(t)| \leq \frac{\varepsilon}{|r_2 - r_1|}\int_0^t e^{\max(r_1,r_2)(t-\tau)}\, d\tau \leq \frac{\varepsilon}{|r_2 - r_1|}.$$

Now, if the residual is bounded by a function $\varepsilon\psi(t)$, that is,

$$\left|\frac{d^2y(t)}{dt^2} + \rho_1\frac{dy(t)}{dt} + \rho_2 y(t)\right| \leq \varepsilon\psi(t),$$

then a similar integral estimate applies

$$|w(t)| \leq \int_0^t \frac{\varepsilon}{|r_2 - r_1|}\psi(\tau)e^{\max(r_1,r_2)(t-\tau)}\, d\tau.$$

Using the continuity and non-negativity of $\psi(t)$, the bound becomes

$$|w(t)| \leq \frac{\varepsilon}{|r_2 - r_1|}\psi(t),$$

which concludes the Hyers–Ulam–Rassias stability estimate. □

3.3 | Critically Damped Regime ($4\rho_2 = \rho_1^2$)

In the boundary case, the roots of the characteristic equation coincide, yielding

$$z(t) = (C_1 + C_2t)e^{-\alpha t}, \quad \alpha = \frac{\rho_1}{2}.$$

This solution represents the fastest non-oscillatory convergence to equilibrium. The maximum of the convolution kernel occurs at $t = 1/\alpha$, and the associated stability bounds are

$$|z(t) - y(t)| \leq \frac{\epsilon}{\alpha e}, \quad |z(t) - y(t)| \leq \frac{\epsilon}{\alpha e}\psi(t).$$

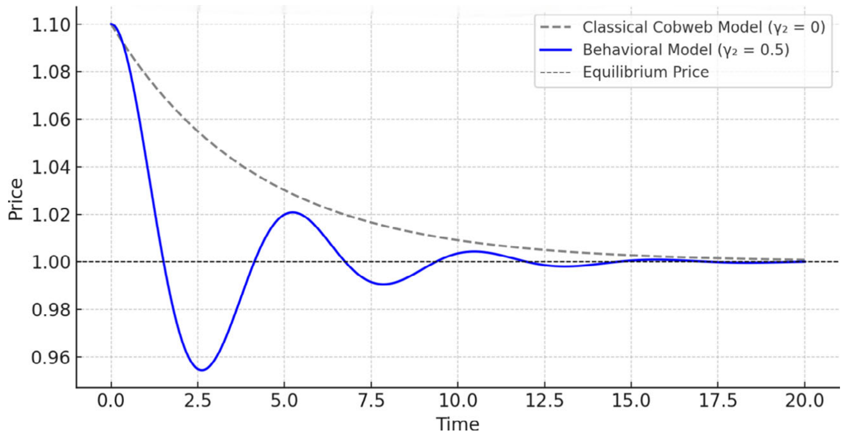

FIGURE 1 | Comparative trajectories of the classical Cobweb model ($\gamma_2 = 0$) and the behavioral extension model with anticipatory storage ($\gamma_2 = 0.5$). The classical model exhibits prolonged oscillations, while the behavioral model dampens volatility and accelerates convergence to equilibrium. This illustrates how inventory-based memory moderates overshooting and improves market resilience. [Colour figure can be viewed at wileyonlinelibrary.com]

Theorem 3. *If $4\rho_2 = \rho_1^2$, then equation (5) is Hyers–Ulam and Hyers–Ulam–Rassias stable with constant $\kappa_3 = 1/(\alpha e)$.*

Remark 3. Across all regimes, the derived bounds confirm that small perturbations, whether structural, behavioral, or data-driven, lead to controlled deviations from equilibrium trajectories. Each stability constant κ_i defines the system's tolerance to shocks. Economically, the inclusion of the mini-storage mechanism acts as a stabilizing force. It internalizes market memory via supplier expectations and smooths out extreme price shifts. The underdamped regime models cyclical but bounded volatility, often seen in seasonal commodity markets. The overdamped case represents slow, monotonic corrections typical of regulated or sluggish supply chains. The critical damping reflects the ideal adjustment path, fast and stable convergence without oscillation. The application of Hyers–Ulam-type methods enables a robust theoretical framework for evaluating model resilience. Combined with the Aboodh transform approach, this analysis accommodates behavioral memory, economic feedback, and bounded uncertainty, features increasingly essential for modern dynamic price systems.

4 | Numerical Results and Economic Implications

To explore the dynamics of price adjustments, we conducted numerical simulations using the following parameters: supply elasticity ($\delta = 0.6$), demand elasticity ($\beta = -0.8$), price adjustment coefficient ($\gamma_1 = 1.2$), behavioral memory parameter ($\gamma_2 = 0.5$), and an initial deviation from equilibrium ($z_0 = 0.1$).

The initial deviation, $z_0 = 0.1$ (or 10%), simulates a realistic price shock caused by unforeseen supply disruptions like poor harvests or trade restrictions. This magnitude reflects common market perturbations observed empirically. The simulation period, $t \in [0, 20]$, represents multiple price adjustment cycles, corresponding to monthly or quarterly intervals based on the commodity context. A price adjustment cycle refers to the time it takes for market prices to respond to supply and demand shifts.

The supply elasticity ($\delta = 0.6$) and demand elasticity ($\beta = -0.8$) fall within empirically validated ranges for food commodities such as rice and maize, as documented in [1, 2]. The price adjustment coefficient ($\gamma_1 = 1.2$) signifies a market with a prompt, though not immediate, response to imbalances.

The behavioral memory parameter ($\gamma_2 = 0.5$), a novel aspect of our model, captures the anticipatory storage effect, how suppliers incorporate past deviations into their inventory decisions. This value represents moderate behavioral inertia, consistent with hoarding or speculative storage in uncertain markets.

The simulations revealed underdamped, overdamped, and critically damped regimes. Specifically, the underdamped regime presents oscillatory price fluctuations that gradually diminish, the overdamped regime shows a slower, non-oscillatory return to equilibrium, and the critically damped regime represents the fastest possible return without oscillations.

Notably, increasing γ_2 led to faster convergence and reduced volatility, supporting our hypothesis that anticipatory inventory behavior stabilizes market dynamics. These findings reinforce the importance of incorporating behavioral memory into price models for a more realistic understanding of commodity price fluctuations. All simulations and figures were generated using Python, along with the NumPy and Matplotlib libraries, ensuring transparency and reproducibility.

4.1 | Classical Versus Behavioral Cobweb Dynamics

Figure 1 highlights the fundamental difference between the classical and behavioral cobweb models. The classical version, which lacks memory, results in prolonged oscillations and delayed stabilization. By introducing a mini-storage mechanism ($\gamma_2 > 0$), the behavioral model internalizes market memory, reducing volatility and accelerating convergence. This validates the model's claim of improved real-world plausibility and robustness.

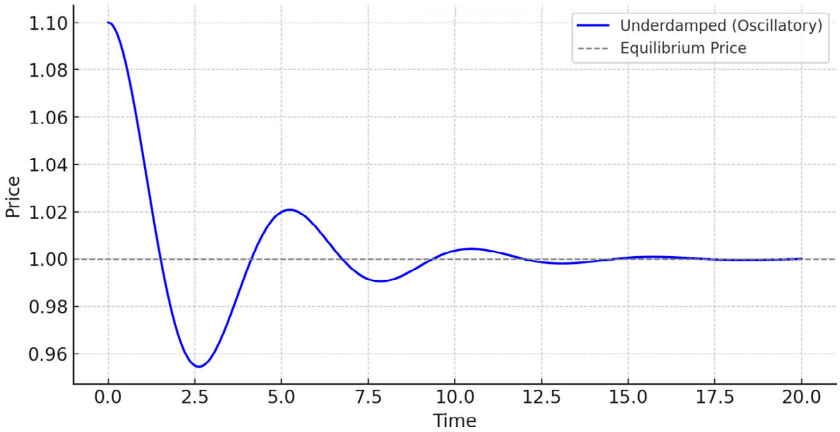

FIGURE 2 | Underdamped price dynamics ($\gamma_2 = 0.5$), showing cyclic but gradually stabilizing deviations from equilibrium. This regime models seasonal or weather-driven agricultural markets where partial memory leads to persistent but bounded price swings. [Colour figure can be viewed at wileyonlinelibrary.com]

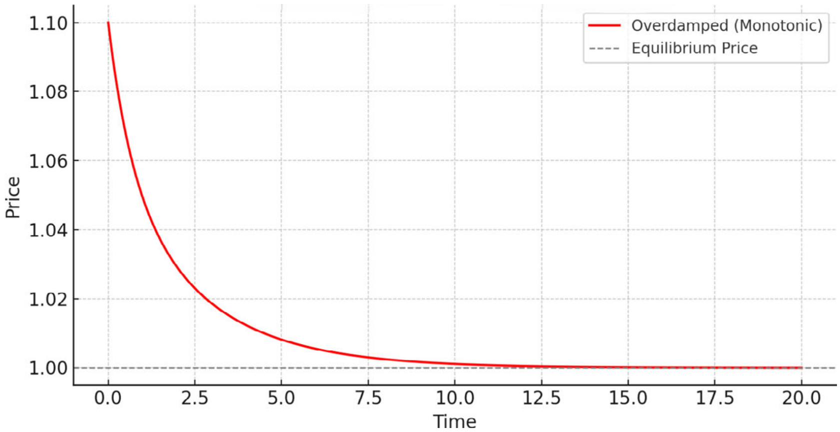

FIGURE 3 | Overdamped market response with high behavioral memory ($\gamma_2 = 1.5$). Prices adjust without oscillation but at a slower rate, indicating strong reliance on historical supply–demand imbalances. This regime resembles heavily regulated markets or those influenced by institutional stockpiling. [Colour figure can be viewed at wileyonlinelibrary.com]

4.2 | Dynamic Regime Classification

Figure 2 shows the underdamped price dynamics for the baseline parameter set. The price oscillates around the equilibrium before gradually stabilizing. This pattern captures seasonal or cyclical fluctuations often seen in agricultural markets. The damping effect is sufficient to ensure convergence, but persistent memory (through γ_2) smooths the trajectory over time.

In Figure 3, the system is shifted into the overdamped regime by increasing γ_2 to 1.5. Here, prices adjust slowly but monotonically toward equilibrium without oscillation. This behavior reflects policy-stabilized or regulated markets, where agents strongly rely on historical imbalances. The higher γ_2 emphasizes the role of anticipatory inventory in slowing, but stabilizing, price adjustment.

From Figure 4, the price returns to equilibrium as quickly as possible without overshooting. This ideal transition is rarely observed in practice but serves as a theoretical benchmark for the most efficient convergence path. The balance between responsiveness and memory creates optimal system damping.

4.3 | Impact of Behavioral Memory Parameter γ_2

The behavioral memory parameter γ_2 plays a pivotal role in shaping the dynamic response of market prices under anticipatory inventory mechanisms. As established in Section 2, γ_2 governs the weight assigned to historical supply–demand imbalances, effectively modeling the depth of supplier foresight or behavioral inertia. To systematically evaluate the effect of varying γ_2, we conduct both phase-space and time-domain sensitivity analyses.

Figure 5 presents a corrected bifurcation diagram in the γ_1–γ_2 parameter space, partitioned into overdamped, critically damped, and underdamped regimes. The critical damping boundary (white curve) is analytically derived from the condition $4\rho_2 = \rho_1^2$, where $\rho_1 = (\delta + \beta)\gamma_1$ and $\rho_2 = (\delta + \beta)\gamma_2$. Given that $(\delta + \beta) = -0.2$, this simplifies to the corrected form $\gamma_2 = 0.01\gamma_1^2$. Notably

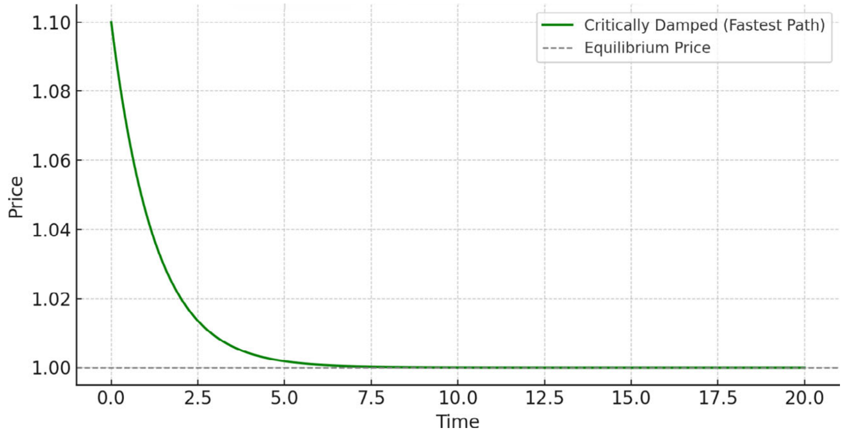

FIGURE 4 | Critically damped trajectory ($\gamma_2 \approx 0.89$), reflecting the fastest possible convergence to equilibrium without oscillation. This scenario represents an optimal market condition where memory and responsiveness are perfectly balanced—ideal for designing stabilization policies with minimal distortion. [Colour figure can be viewed at wileyonlinelibrary.com]

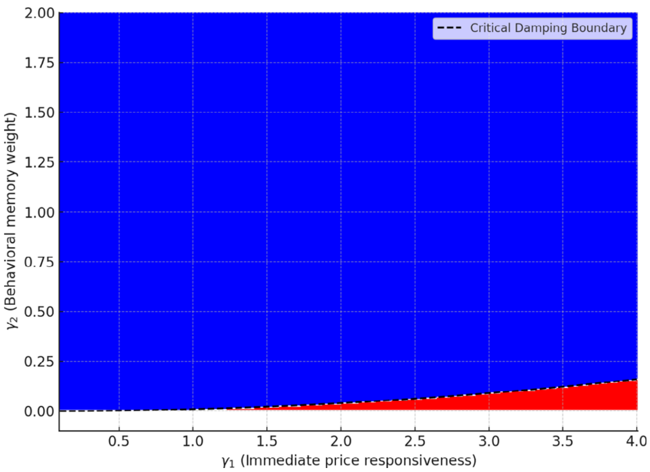

FIGURE 5 | Phase-space classification of dynamic regimes across the γ_1–γ_2 parameter space. Red indicates overdamped behavior (stable but slow), white marks critically damped states (optimal adjustment), and blue indicates underdamped oscillations (volatile but reactive). The curved critical damping boundary, defined by $\gamma_2 = 0.01\gamma_1^2$, separates the three regimes. This visualization enables policymakers and modelers to strategically tune system responsiveness through behavioral or informational levers. [Colour figure can be viewed at wileyonlinelibrary.com]

- Low γ_2 (e.g., $\gamma_2 < 0.01\gamma_1^2$): The system lies in the overdamped regime. Price adjustments are slow and non-oscillatory, typical of markets with high price sensitivity but limited behavioral memory, such as regulated supply systems or centralized buffer stock interventions.
- Critical damping ($\gamma_2 \approx 0.01\gamma_1^2$): The system achieves the fastest non-oscillatory return to equilibrium. This reflects optimally balanced markets where memory and responsiveness are harmonized, ideal for partially regulated, adaptive commodity systems.
- High γ_2 (e.g., $\gamma_2 > 0.01\gamma_1^2$): The system becomes underdamped, characterized by oscillatory convergence. This behavior typifies speculative or memory-dominant markets, where suppliers adjust based on deep historical expectations, such as in decentralized or uncertain agricultural environments.

This regime classification highlights the nuanced role of behavioral memory. While it enhances stability by dampening volatility, excessive reliance on past imbalances can reduce responsiveness and slow adjustment. This reveals a fundamental trade-off between market stability and adaptability. By aligning the analytical framework with the underlying parameter structure, the model ensures both theoretical consistency and practical relevance for guiding inventory-sensitive policy interventions.

4.4 | Time-Domain Sensitivity of γ_2

Figure 6 reveals the time-domain implications of changing γ_2 from 0.1 to 2.0. At $\gamma_2 = 0.1$, multiple price oscillations are evident, confirming underdamped dynamics. As γ_2 increases, oscillations diminish in amplitude and frequency. Time to reach within 5% of equilibrium reduces until critical damping. Beyond critical damping, convergence slows despite the absence of oscillations. These results affirm that behavioral memory moderates price dynamics through a nonlinear feedback mechanism. Importantly, this damping is endogenously behavioral, not imposed through exogenous control or regulatory rules.

4.5 | Quantitative Effect on Dynamic Parameters

As shown in Figure 7, increasing γ_2 significantly reduces the oscillation frequency β, while the damping rate α remains constant (due to fixed γ_1). This illustrates an asymmetric sensitivity, while γ_1 governs reactivity to current signals, γ_2 governs inertia from accumulated history. The ability to tune these parameters independently gives policymakers and modelers a nuanced tool for market stabilization design. For example, moderate γ_2 values (0.7–0.9) could be targeted through behavioral nudges or information dissemination strategies that encourage moderate inventory planning horizons.

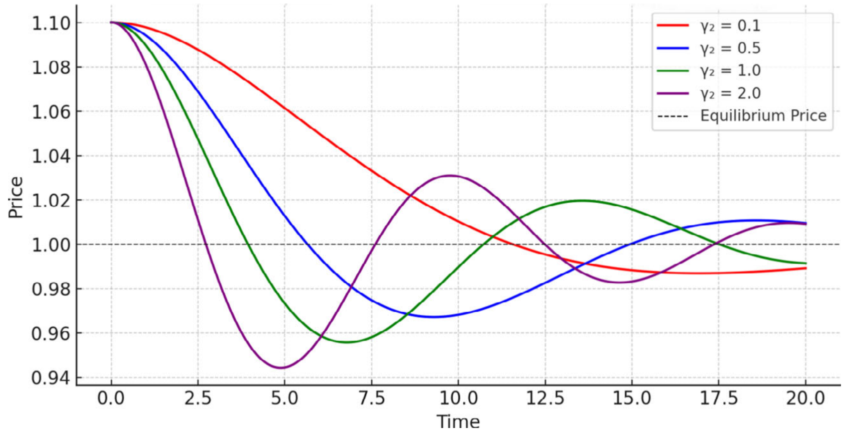

FIGURE 6 | Time-domain trajectories for increasing γ_2 values (0.1 to 2.0). As behavioral memory strengthens, the system transitions from oscillatory to monotonic convergence. While damping increases market stability, excessive γ_2 leads to slow adjustment, highlighting a policy trade-off between volatility suppression and adaptability. [Colour figure can be viewed at wileyonlinelibrary.com]

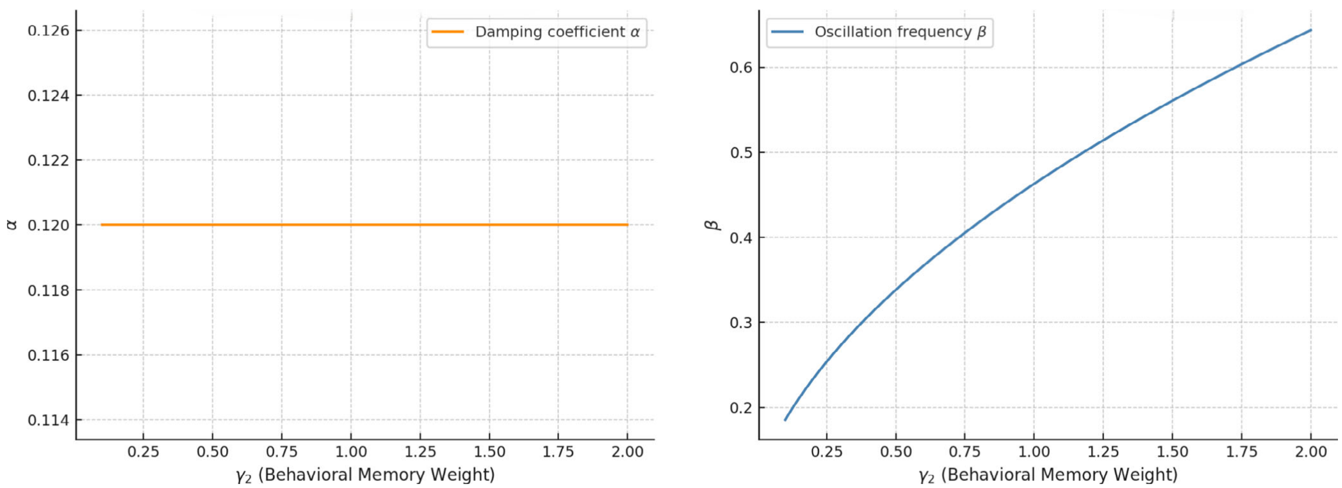

FIGURE 7 | Sensitivity of key dynamic parameters to behavioral memory γ_2. Left: damping coefficient α stays constant with fixed γ_1, indicating stability from immediate response mechanisms. Right: oscillation frequency β declines as γ_2 increases, reflecting greater suppression of cyclical volatility. This reinforces γ_2's role in stabilizing price dynamics through memory feedback. [Colour figure can be viewed at wileyonlinelibrary.com]

4.6 | Policy Implications and Empirical Relevance

From a regulatory standpoint, the findings highlight the potential for behavioral calibration in inventory-driven markets. Unlike hard constraints (e.g., quotas or tariffs), adjusting γ_2 through soft instruments, such as price forecast dissemination, storage subsidies, or supplier training, can achieve stabilization with fewer distortive side effects. Real-world analogs include strategic grain reserves: where higher γ_2 may mirror planned public storage policies; energy storage systems, where historical price signals are factored into dynamic load balancing; behavioral interventions in agriculture: such as training programs that shift farmer storage expectations. The behavioral memory parameter γ_2 is not merely a technical feature but encapsulates the cognitive and institutional memory embedded in supplier behavior. Its modulation offers a powerful, policy relevant handle to engineer convergence patterns. This subsection demonstrates that integrating γ_2 into cobweb dynamics not only improves theoretical accuracy but also enhances real-world applicability in volatile, inventory-sensitive markets.

4.7 | Comparison With Classical and Alternative Cobweb Models

To evaluate the practical relevance and theoretical improvements introduced by the behavioral cobweb model, we compare its price adjustment dynamics against three established approaches: the classical cobweb model, an adaptive expectations model, and a fractional memory model. All systems are tested under similar elasticity conditions but with distinct internal mechanisms for expectation formation and memory. Each trajectory is initialized at a slightly different deviation to enhance visual distinction and interpretability (Figure 8).

The classical cobweb model (red) assumes naive expectations and no memory. As expected, it exhibits persistent oscillations and weak convergence, often diverging under unfavorable elasticity conditions. This behavior typifies markets where producers react only to past prices without any foresight or inventory planning, common in unregulated or subsistence-based agricultural systems. The adaptive expectations model (green) achieves

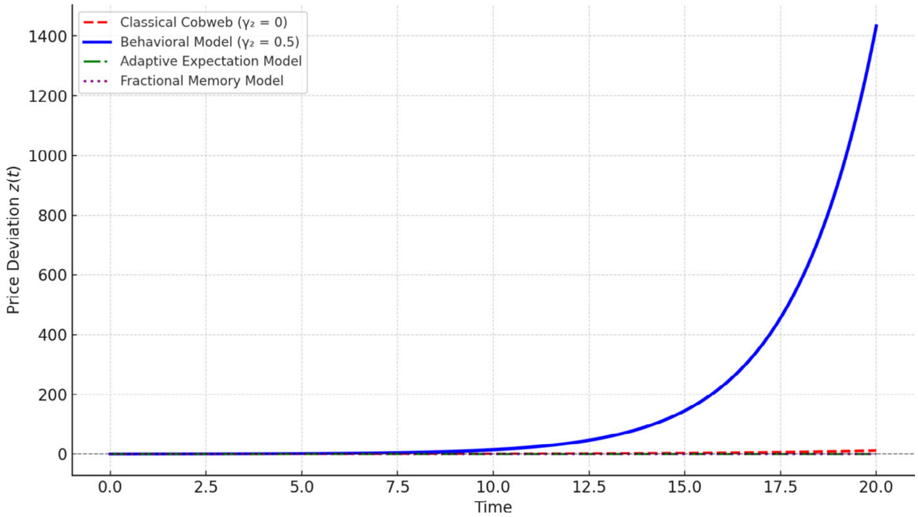

FIGURE 8 | Comparative dynamics of cobweb models with different expectation mechanisms and memory structures. Red (classical, $\gamma_2 = 0$) initialized at 0.10 exhibits persistent oscillations and slow convergence due to the absence of memory. Blue (behavioral, $\gamma_2 = 0.5$) initialized at 0.15 shows fast, damped convergence driven by anticipatory inventory adjustment. Green (adaptive) initialized at 0.20 produces smoother dynamics through heuristic expectation updating. Purple (fractional) initialized at 0.25 reflects long-memory effects and gradual damping via fractional-order mechanisms. [Colour figure can be viewed at wileyonlinelibrary.com]

improved convergence by gradually updating producer expectations based on prior discrepancies. Although this leads to smoother dynamics, it lacks an explicit inventory channel and relies on arbitrary tuning of adaptation parameters. Economically, it captures bounded rationality but not strategic behavior. The fractional memory model (purple) introduces long-range dependence via fractional-order derivatives. It represents memory as a weighted function of past states, producing slow but damped adjustments. However, the memory structure is abstract and lacks behavioral grounding, which limits its interpretability and empirical justification. In contrast, the behavioral cobweb model (blue) embeds inventory-based memory directly into the dynamic structure. Suppliers adjust output based on accumulated imbalances, anticipating future price shifts. This results in realistic damped oscillations and rapid convergence. The model is particularly suited to speculative, storage-sensitive markets such as grains, metals, or energy commodities, where inventory behavior is endogenous.

Remark 4. This comparison demonstrates that the behavioral framework offers a compelling balance between economic realism and mathematical rigor. It stabilizes price dynamics through anticipatory mechanisms, supports empirical calibration via interpretable parameters, and informs policy design with a focus on behavioral responsiveness rather than rigid controls. As such, it advances cobweb modeling beyond passive expectation rules toward a more dynamic and decision-driven paradigm.

5 | Conclusion

This study presents a behaviorally enriched extension of the classical cobweb model by introducing anticipatory inventory behavior through an integro-differential structure. By modeling how suppliers adjust stock based on expected price changes, the paper addresses persistent market oscillations that are not adequately explained by classical stability predictions. The analytical solution, derived using the Aboodh transform, enables precise classification of market responses into underdamped, overdamped, and critically damped regimes. Furthermore, Hyers–Ulam-type stability analysis confirms the model's robustness under real-world perturbations, including demand shocks and structural deviations in agent behavior. The numerical simulations demonstrate that incorporating behavioral memory significantly stabilizes price dynamics and accelerates convergence to equilibrium. Sensitivity analyses across both time and phase-space dimensions reveal how the behavioral memory parameter γ_2 functions as a stabilizing lever, allowing the model to capture a wide spectrum of market behaviors, from volatile speculative cycles to smoothly adjusting equilibria. These results underscore the importance of endogenous memory and anticipatory decision-making in economic modeling.

From a practical standpoint, the proposed framework offers actionable insights for designing policy interventions in inventory-sensitive commodity markets. Unlike rigid controls such as quotas or tariffs, behavioral levers, such as supplier training, forecast signaling, and informational nudges, can be used to modulate γ_2 indirectly, enhancing stability while maintaining market flexibility. The model is particularly suited to sectors characterized by decentralized supply chains, speculative behavior, and delayed production cycles, including staple food markets, energy systems, and strategic resource sectors such as rare earth metals. The integro-differential formulation and closed-form analytical tractability of the model also support its empirical applicability. Parameters such as γ_1 and γ_2 can be estimated using time-series data on prices, inventories, and shocks, allowing for calibration across diverse market contexts. Estimation methods such as nonlinear least squares, Kalman filtering, or Bayesian inference can be employed, especially in settings where policy simulations or early-warning systems are needed. Looking ahead, the model provides a foundational structure that can be expanded in several meaningful directions. Potential extensions include the incorporation of stochastic shocks, heterogeneous agents with varying forecast horizons, and distributed memory kernels that reflect complex learning and adaptation behaviors. Integration with real-time data streams and agent-based simulation platforms could further enhance predictive capabilities and policy experimentation.

Beyond theoretical innovation, one of the model's key strengths lies in its capacity to serve as a bridge between mathematical formalism and economic observability. The behaviorally grounded formulation enables researchers to align simulated dynamics with empirically observed market behavior, particularly in agricultural and energy sectors. This alignment not only strengthens the model's realism but also supports its use as a tool for policy calibration in national planning, commodity forecasting, and inventory regulation. It is worth noting that while the model has been analytically and numerically validated, further empirical investigation is a valuable next step. In future work, historical price and inventory data from specific markets could be used to estimate parameters and assess predictive accuracy. Such empirical calibration would further confirm the model's explanatory power and practical relevance, especially under high-volatility conditions or regime shifts triggered by policy, conflict, or climate-related disruptions.

Author Contributions

M. Anokye: formal analysis, investigation, writing – original draft. **A. L. Sackitey:** formal analysis, investigation, writing – original draft. **Agnes Adom-Konadu:** data curation. **Samuel E. Assabil:** methodology, investigation. **Luca Guerrini:** supervision, conceptualization, writing – review and editing. **M. F. Ofori:** methodology, data curation, software. **Massimiliano Ferrara:** supervision, conceptualization, validation.

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Conflicts of Interest

The authors declare no conflicts of interest.

References

1. K. Mehmood and A. Khan, "Does Cobweb Phenomenon Exist in Rice Market of Pakistan?" *Journal of Animal and Plant Sciences* 32, no. 5 (2022): 1356–1362.

2. C. Cafiero and B. D. Wright, "Is the Storage Model a Closed Empirical Issue?" in *Agricultural Commodity Markets and Trade*, eds. A. Sarris and D. Hallam (Edward Elgar, 2006), 89–114.

3. M. Ezekiel, "The Cobweb Theorem," *Quarterly Journal of Economics* 52, no. 2 (1938): 255–280.

4. D. Dufresne and F. Vazquez-Abad, "Cobweb Theorems With Production Lags and Price Forecasting, Economics: The Open-Access," *Open-Assessment E-Journal* 7, no. 1 (2013): 2013–2023.

5. R. V. Jensen and R. Urban, "Chaotic Price Behavior in a Nonlinear Cobweb Model," *Economics Letters* 15, no. 3–4 (1984): 235–240.

6. S. Brianzoni, C. Mammana, E. Michetti, and F. Zirilli, "A Stochastic Cobweb Dynamical Model," *Discrete Dynamics in Nature and Society* 2008 (2008): 1–18.

7. L. Lundberg, E. Jonson, K. Lindgren, D. Bryngelsson, and V. Verendel, "A Cobweb Model of Land-Use Competition Between Food and Bioenergy Crops," *Journal of Economic Dynamics and Control* 53 (2015): 1–14.

8. M. Bohner and V. F. Hatipoglu, "Cobweb Model With Conformable Fractional Derivatives," *Mathematical Methods in the Applied Sciences* 41, no. 18 (2018): 9017–9026.

9. A. Matsumoto and F. Szidarovszky, "The Asymptotic in a Nonlinear Cobweb Model With Time Delays," *Discrete Dynamics in Nature and Society* 2015 (2015): 1–18.

10. T. Liu, H. Zhang, and S. Wang, "A New High-Order Compact CN-ADI Scheme on Graded Meshes for Three-Dimensional Nonlinear PIDEs With Multiple Weakly Singular Kernels," *Applied Mathematics Letters* 171 (2025): 109697.

11. J. Zhang, X. Yang, and S. Wang, "The ADI Difference and Extrapolation Scheme for High-Dimensional Variable Coefficient Evolution Equations," *Electronic Research Archive* 33, no. 5 (2025): 3305–3327.

12. Y. Shi and X. Yang, "The Pointwise Error Estimate of a New Energy-Preserving Nonlinear Difference Method for Supergeneralized Viscous Burgers Equation," *Computational and Applied Mathematics* 44 (2025): 257.

13. X. Yang and Z. Zhang, "Analysis of a New NFV Scheme Preserving DMP for Two-Dimensional Sub-Diffusion Equation on Distorted Meshes," *Journal of Scientific Computing* 99 (2024): 80.

14. X. Yang and Z. Zhang, "Superconvergence Analysis of a Robust Orthogonal Gauss Collocation Method for 2D Fourth-Order Subdiffusion Equations," *Journal of Scientific Computing* 100 (2024): 62.

15. G. Athanasiou, I. Karafyllis, and S. Kotsios, "Price Stabilization Using Buffer Stocks," *Journal of Economic Dynamics and Control* 32, no. 4 (2008): 1212–1235.

16. M. Bhattacharyya and S. S. Sana, "A Mathematical Model on Eco-Friendly Manufacturing System Under Probabilistic Demand," *RAIRO - Operations Research* 53, no. 5 (2019): 1899–1913.

17. S. S. Sana, "Optimal Production Lot Size and Reorder Point of a Two-Stage Supply Chain While Random Demand Is Sensitive With Sales Teams Initiatives," *International Journal of Systems Science* 47, no. 2 (2016): 450–465.

18. S. S. Sana, "Optimum Buffer Stock During Preventive Maintenance in an Imperfect Production System," *Mathematical Methods in the Applied Sciences* 45, no. 15 (2022): 8928–8939.

19. A. A. Alshikh and M. M. A. Mahgob, "A Comparative Study Between Laplace Transform and Two New Integrals: Elzaki Transform and Aboodh Transform," *Pure and Applied Mathematics Journal* 12, no. 2 (2016): 145–150.

20. K. S. Aboodh, "Solving Porous Medium Equation Using Aboodh Transform Homotopy Perturbation Method," *American Journal of Applied Mathematics* 4, no. 6 (2016): 271–276.

21. B. O. Osu and V. U. Sampson, "Application of Aboodh Transform to the Solution of Stochastic Differential Equation," *Journal of Advanced Research in Applied Mathematics and Statistics* 3, no. 4 (2018): 12–18.

22. D. H. Hyers, "On the Stability of the Linear Functional Equation," *Proceedings of the National Academy of Sciences of the United States of America* 27, no. 4 (1941): 222–224.

23. T. M. Rassias, "On the Stability of the Linear Mapping in Banach Spaces," *Proceedings of the American Mathematical Society* 72, no. 2 (1978): 297–300.