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Numerical investigation of hydraulic instability of pump-turbines in fast pumpto-turbine transition

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15	Abstract: The fast pump-to-turbine transition of a pumped storage unit is highly responsive to electrical
16	power system regulation demands and is a critical process for the unit itself. The correct prediction of
17	this hydraulic transient process is crucial to avoid issues during machine operation. In this study, a three-
18	dimensional numerical model of the transition process of a pumped storage hydraulic system was
19	proposed and analyzed. Numerical simulations were performed to investigate the variations in external
20	parameters, pressure pulsations, and the evolution of internal flow patterns and vortical structures in the
21	pump turbine. The results showed that the stepwise variations in the flow rate, torque, and force were
22	synchronized with the movement of the guide vanes. Notably, during the pump braking operation and
23	runner speed reversal, a considerable time period with significant force oscillations and intense torque
24	fluctuations occurred, particularly in the vaneless region, where pressure fluctuations were notable.
25	Vortical structures that induce vibrations and intense pressure fluctuations were localized near the runner
26	inlet and at the leading and trailing edges of the guide and stay vanes. This study offers theoretical
27	insights that are crucial for enhancing the stability of the fast pump-to-turbine transitional process, thus
28	optimizing its control strategies.
29	

30 **Keywords:** Numerical investigation; Pumped storage; Pump-turbine; Fast pump-to-turbine; Hydraulic

31 instabilities

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3

33 Nomenclature

n _r	Runner rated speed (rad/s)
k	Turbulence kinetic energy (m^2/s^2)
ω	Specific dissipation rate (m ² /s ²)
t	Physical time (s)
<i>u</i> _j	Velocity component (m/s)
x_j	Cartesian coordinate component (m)
P_k	Turbulence production due to viscous forces (m^2/s^3)

v	Kinematic viscosity (m ² /s)
V_t	Eddy viscosity (m ² /s)
S	Invariant measure of the strain rate (s ⁻¹)
r	Grid refinement ratio
$h_{ m coarse}$	Size of the coarse grid
$h_{ m fine}$	Size of the fine grid
Q	Flow rate (m ³ /s)
η	Efficiency (%)
n_t	Runner speed (r/s)
Μ	Torque (N·m)
J	Rotational inertia (kg·m ²)
∇t	Time step (s)
γt	Angular velocity (r/s)
Н	Head (m)
$\overline{\omega}_{Ri}$	Reynolds-averaged rigid vorticity component (s ⁻¹)
$\overline{\omega}_{Si}$	Reynolds-averaged shear vorticity component (s ⁻¹)
$\overline{\omega}_i$	Reynolds-averaged vorticity component (s ⁻¹)
\overline{c}_i	Angular velocity component of the rotating system (rad/s)
\overline{u}_{ri}	Relative velocity component (m/s)
E_{v}	Evolution intensity of the enstrophy for the unit volume (s^{-3}/m^3)
\mathcal{Q}_R	Enstrophy of the rigid vorticity (s ⁻²)
Abbreviations	
ASME	American Society of Mechanical Engineers
ECFT	Enstrophy of the Coriolis Force Term
ERCT	Enstrophy of the Pseudo Lamb Vector Curl Term
ERDT	Enstrophy of the Rigid Vorticity Dilatation Term
ERST	Enstrophy of the Rigid Vorticity Stretching Term
ERRT	Enstrophy of the Reynolds Stress Term
ERVT	Enstrophy of the Viscous Term
GCI	Grid Convergence Index
GVO	Guide Vane Opening
RANS	Reynolds-Averaged Navier-Stokes
URANS	Unsteady Revnolds-Averaged Navier-Stokes
RSM	Reynolds Stress Model
SIMPLEC	Semi-Implicit Method for Pressure-Linked Equations-Consistent
SST	Shear Stress Transport
UDF	User Defined Function

1 Introduction

Widespread adoption of renewables is essential for achieving the environmental target of net-zero carbon emissions by 2050 [1]. In this context, hydropower has emerged as a highly promising solution for significantly boosting power generation capability and providing energy storage capacity. Pumped storage stations are among the most mature and efficient mechanical storage technologies [2]. Indeed, they fulfill several roles, such as peak shaving, valley filling, frequency and phase regulation, and standby capabilities, in the case of emergencies. Considering the prevailing trend towards the gradual integration of large-scale renewable energy into the power grid, pumped storage stations can play a pivotal role in counteracting the intermittency and instability associated with renewable energy sources, thus ensuring the safe and stable operation of modern power systems [3].

47 To align with the dynamic service demands of power systems, pumped storage stations often exhibit 48 distinctive features such as multipurpose operation, rapid mode transitions, frequent start-stop cycles, 49 and heightened pressure pulsations. Consequently, these characteristics increase complex hydraulic 50 transient issues in the water-conveyance system [4-7]. Several researchers have conducted extensive 51 numerical simulations and studies to explore various transition processes within pump turbines. Liu et 52 al. [8] performed numerical simulations of the transient power failure process of a prototype pump 53 turbine. The results showed that the reverse flow in the casing and vane, as well as the stall phenomenon 54 in the runner, led to a smooth change in the head when the flow rate was close to zero, whereas it became 55 evident when the flow rate increased. In particular, the stall phenomenon considerably influenced the 56 pressure distribution on the runner surface. Li et al. [9] performed numerical simulations of a prototype 57 pump turbine during a normal shutdown process. The results showed a concurrent decrease in the flow 58 rate and torque, along with a reduction in the rotational speed, owing to the negative torque applied to 59 the runner as the guide vanes close. They proposed an approach for predicting the transient characteristics 60 of a pump turbine during the shutdown process, and provided details on its internal fluid dynamic 61 behavior. Zhang et al. [10] developed a dynamic model for simulating pump-turbine operation in S-62 shaped regions under runaway conditions. They introduced dynamic relative parameters along with a 63 dynamic nonlinear model of the pump turbine, thus providing a new perspective for modeling transient 64 processes in pump turbines. Consequently, a good match between the dynamic model and theoretical 65 analysis was obtained, thus validating the proposed model. Regarding the load rejection in a pump-66 turbine, Mao et al. [11] explored the application of a co-adjusting inlet valve and guide vane method to 67 enhance the internal fluid stability. They combined a theoretical analysis with a numerical simulation, 68 and the results showed that the proposed method was beneficial to the operational stability of the entire 69 system. Jin et al. [12] simulated the start-up process of the turbine mode to pinpoint the precise location 70 of the flow energy dissipation within the pump turbine. The trailing edge of the guide vane had both high 71 flow energy dissipation and turbulence kinetic energy values, whereas the wall shear stress and axial and 72 radial forces had a significant effect on the fluid flow characteristics.

73 Using hydraulic braking to accelerate the transition from pump operation to turbine operation, the fast 74 pump-to-turbine transition process in pumped storage stations is one of the most intricate and hazardous 75 operating zones that involves complex hydraulic, mechanical, and electrical processes [13]. Modern 76 power systems require enhanced rapid response times from pumping to power generation, thus requiring 77 swift transitions. In contrast to the conventional transition mode involving electrical and mechanical 78 braking, the use of hydraulic braking to accelerate the pump shutdown introduces additional complexity 79 into the transition process. Current research on the transition processes of pumped-storage stations has 80 primarily focused on load-rejection, shutdown, startup, and runaway processes without providing 81 insights into the intricate hydraulic variations in the fast pump-to-turbine transition process [14-17]. To 82 the best of our knowledge, this hydraulic change has hampered the formulation of accurate and effective 83 control strategies for the transition process.

84 This study presents a three-dimensional numerical model of the entire flow system of a pumped storage 85 station operating during a fast pump-to-turbine transition process. The changes in runner speed, flow rate, 86 torque, and force characteristics under a specified control strategy were investigated. The study also 87 explored the duration to complete the transition and examined the pressure pulsations in the pump turbine 88 during the rapid transition. Furthermore, the evolution of the flow patterns and vortical structures was 89 analyzed. This study provides crucial references for reducing the time required for operational state 90 transitions and enhancing station responsiveness to the power grid. It also offers theoretical support for 91 improving stability and optimizing control strategies during the fast pump-to-turbine transition process. 92 The paper is structured as follows: Section 2 presents the numerical simulation model along with the 93 numerical settings and details of the numerical grid with its independence study. Section 3 describes the 94 fast pump-to-turbine transition process and its hydraulic transient mechanism from various perspectives, 95 including external parameter changes, pressure pulsations, flow patterns, and vortical structure evolution. 96 Finally, Section 4 reports the main research findings and future developments of this study.

97

98 2 Numerical considerations

99 2.1 Computational model and grid

The computational model of the entire flow system of the pumped storage station analyzed in this study is shown in Fig. 1, with a Guide Vane Opening (GVO) of 22° (rated operating conditions). The computational domain embedded an upstream penstock, a mixed-flow pump-turbine unit, and a downstream penstock. The computational domain of the pump-turbine unit was divided into the spiral casing, stay vanes (20 vanes), guide vanes (20 vanes), runner (9 vanes), and draft tube. The essential parameters of both the power station and the pump-turbine unit are listed in Table 1.



Parameter	Value
Upstream reservoir normal storage level (m)	716
Downstream reservoir normal storage level (m)	299
Type of pump-turbine	HLNA1518-LJ-412
Rated output power in turbine mode (MW)	306.1
Maximum input power in pump mode (MW)	≤325
Rated rotational speed n_r (rad/s)	44.88
Moment of inertia of the generator $(t \cdot m^2)$	6000
Moment of inertia of the pump-turbine $(t \cdot m^2)$	240
Elevation of guide vane centerline (m)	192

112 Hexahedrally structured grids were used in the upstream penstock, spiral casing domain, stay vane 113 domain, runner domain, draft tube domain, and downstream penstock to control the overall grid node 114 count. In the guide vane domain, a prism grid was generated using the dynamic mesh method to simulate 115 guide vane closure or opening. Two-dimensional refined quad and triangular meshes were generated in 116 the boundary layer regions and main-flow region of the guide vane bottom side. Subsequently, the twodimensional mesh on the bottom side of the guide vane was extruded to the top side of the guide vane 117 118 using a copper meshing operation in Gambit. Specifically, grid generation for the runner domain was performed using ANSYS-TurboGrid software. The grids for the remaining computational domains were 119 120 created using ANSYS-ICEM software. For the pump-rated operating conditions, the y + values of the 121 runner blades were maintained in close proximity to 30. Since these y+ values are compatible with the 122 Shear Stress Transport k- ω (SST k- ω) turbulence model, they were selected by examination of the 123 ANSYS software manual and other research studies in the scientific literature located in [18]. Because it 124 uses an autonomous wall function to consider near-wall flow features, this model is regarded as one of 125 the best for analyzing a number of issues related to the study of the transition process in hydraulic turbines. 126 This enables a good compromise between simulation timing and accuracy [19]. 127

128 **2.2 Governing equations and numerical settings**

ANSYS Fluent 2022R1 software was used to perform the numerical simulations. As previously mentioned, the SST k- ω turbulence model has been selected to close the control equations, providing a dual-equation eddy-viscosity model that integrates the advantages of the standard k- ε and k- ω turbulence models [20, 21]. It can be described as the k and ω equations:

$$\frac{\partial k}{\partial t} + \frac{\partial (u_j k)}{\partial x_j} = P_k - \beta^* k \omega + \frac{\partial}{\partial x_j} \left[\left(\nu + \sigma_k \nu_t \right) \frac{\partial k}{\partial x_j} \right]$$
(1)

$$\frac{\partial\omega}{\partial t} + \frac{\partial(u_j\omega)}{\partial x_j} = \gamma S^2 - \beta \omega^2 + \frac{\partial}{\partial x_j} \left[\left(v_t + \sigma_\omega v_t \right) \frac{\partial\omega}{\partial x_j} \right] + 2\left(1 - F_1 \right) \frac{\sigma_{\omega 2}}{\omega} \frac{\partial k \partial \omega}{\partial x_j \partial x_j}$$
(2)

133

134 where k denotes the turbulence kinetic energy, m^2/s^2 ; ω denotes the specific dissipation rate, m^2/s^2 ; t

denotes time, s; u_j (j=1, 2, 3) represents the velocity component, m/s; x_j (j=1, 2, 3) represents the Cartesian coordinate component, m: P_k denotes the turbulence production due to viscous forces, m²/s³; v denotes kinematic viscosity, m²/s; v_l denotes eddy viscosity, m²/s; S denotes the invariant measure of the strain rate, s⁻¹. All coefficients for this model are obtained by functions of $\varphi = \varphi_1 F_1 + \varphi_2 (1 - F_1)$. The constants are as follows: $\beta^* = 0.09$, $\beta_1 = 0.075$, $\beta_2 = 0.0828$, $\sigma_{k1} = 0.85$, $\sigma_{k2} = 1$, $\gamma_1 = 0.556$, $\gamma_2 = 0.44$, $\sigma_{\omega 1} = 0.5$, $\sigma_{\omega 2} = 0.856$.

141 For transient simulations using unsteady Reynolds-averaged Navier (URANS) equations, the initial 142 conditions were derived from steady-state simulations. Both steady-state and transient computations used 143 the Semi-Implicit Method for Pressure-Linked Equations-Consistent (SIMPLEC) algorithm for the 144 coupled solution of the velocity and pressure equations. Convective and diffusive terms were discretized 145 using a second-order upwind scheme, and time discretization employed a first-order implicit format. The 146 results of steady-state simulations act as the initial flow field for transient simulations, with a specified 147 transient simulation time step of $1.17 \cdot 10^{-3}$ s that corresponds to a 3° rotation of the runner at the rated 148 rotational speed. The convergence criterion for the residual of the Reynolds Stress Model (RSM) at each 149 time step is set to 10⁻⁵. Additionally, the gravitational acceleration was defined as $g = 9.81 \text{ m/s}^2$.

In the entire flow-domain system model, the downstream inlet exhibits a pressure inlet boundary condition, whereas the upstream outlet exhibits a pressure outlet boundary condition. The reservoir depths are calculated based on the actual water levels downstream and upstream, and the pressure values for the downstream inlet and upstream outlet are set to 426,330.76 Pa and 836,199.15 Pa, respectively.

155 **2.3 Grid independence study**

156 The Richardson extrapolation method was applied in this investigation to verify grid independence using 157the Grid Convergence Index (GCI) for a quantitative evaluation of the computational results [22-25]. The three grid partitioning strategies used for the convergence analysis are identical, involving a 158 159reduction in both grid quantity and scale. According to the recommendations of the American Society of 160 Mechanical Engineers (ASME), the refinement ratio r should exceed 1.3 for grid refinement [26], which is defined by Eq. (3), where h_{coarse} represents the coarse grid size and h_{fine} represents the fine grid size, 161 162 respectively. Consequently, the quantities of the three sets of grids are 20.43 million, 8.62 million, and 163 3.88 million, respectively.

$$r = \frac{h_{\text{coarse}}}{h_{\text{fine}}}, h = \left[\frac{1}{N}\sum_{i=1}^{N} \left(\Delta V_i\right)\right]^{1/3}$$
(3)

164

Table 2. Quantities of three sets of grids

Computational domain	Number of nodes	Number of nodes	Number of nodes
Computational domain	in S1 (10 ⁵)	in S2 (10 ⁵)	in S3 (10 ⁵)
Upstream penstock	15.66	6.46	2.34
Spiral casing	24.79	11.19	4.87
Stay vanes	29.41	12.41	5.72
Guide vanes	27.92	11.58	5.13
Runner	68.85	28.73	13.54

Draft tube	28.34	12.07	5.43
Downstream penstock	9.37	3.72	1.78
Sum	204.34	86.16	38.81

167 Numerical simulations of all grid sets were performed under the rated conditions of the pump mode, 168 considering the flow rate and efficiency as variables for the convergence analysis. Table 3 lists the results 169 of the grid independence study, showing convergence indices for the flow rate and efficiency of 2.53% 170 and 1.35%, respectively. These values confirmed that the grids satisfied the convergence criterion (less 171 than 3%) [27]. Considering both the simulation accuracy and computational costs, a computational grid 172 with 8.62 million elements was chosen for performing all numerical simulations.

173 174

Table 3. C	Frid inde	pendence	studv
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Parameter	$\varphi = Q \text{ (m}^{3/\text{s})}$	<i>φ</i> =η (%)
\mathbf{N}_1	20,428,557	
N_2	8,624,839	
N_3	3,880,253	
Mesh refinement factor r_{21}	1.33	
Mesh refinement factor r_{32}	1.31	
Numerical value φ_1	60.47	93.14
Numerical value φ_2	60.18	93.02
Numerical value φ_3	54.47	91.72
Apparent order p	11.1541	8.8932
Extrapolated value φ_{ext}^{21}	60.482	93.15
Relative error e_a^{21}	0.480%	0.129%
Extrapolated error e_{ext}^{21}	0.020%	0.011%
Grid convergence index GCI _{fine} ²¹	2.53%	1.35%

175

176 **2.4 Control method of varying the angular speed**

The fast pump-to-turbine transition process significantly reduces the transition time compared with the normal pump-to-turbine transition process, thus enhancing the responsiveness of the pumped-storage station to power system demands. The main distinction occurs in the normal transition when a spherical valve undergoes a "fully open \rightarrow fully closed \rightarrow fully open" process, using a combination of electrical and mechanical braking. In contrast, during the rapid transition, the spherical valve remained stationary, and a rapid shutdown was achieved through hydraulic braking.

183 The control process for the fast pump-to-turbine is as follows: Upon receiving the transition command, 184 the unit closes the guide vanes to reduce the pump input. Once the guide vanes were fully closed, the 185 electric motor circuit breaker was disconnected, the system was demagnetized, and the unit speed began 186 to decrease. When the speed decreased to approximately 50% of the rated speed, the guide vanes were 187 preopened for hydraulic braking. As the speed reversed and entered turbine mode, reaching 188 approximately 50% of the rated rotational speed, the guide vanes were further opened to the no-load 189 position, ultimately transitioning the unit to the turbine no-load operating condition. 190 The algorithmic implementation of the control strategy in this study is outlined as follows. The speed of

- 191 the runner changes with time according to the angular momentum equation shown in Eq. (4). During the
- 192 transient simulations, the torque acting on the runner blades was monitored at each time step, and the

193 runner speed for the next time step was predicted using Eq. (4).

$$n_{t} = \begin{cases} n_{r} \left(t \le 26s \right) \\ n_{t-\Delta t} + \frac{30}{\pi} \frac{M}{J} \times \Delta t \left(t > 26s \right) \end{cases}$$

$$\tag{4}$$

194

195 where M is the total torque of the runner, N·m; J is the total rotational inertia of runner and 196 generator, kg·m²; t is the time, s; Δt is the time step, s. 197 The motion of the guide vanes is described by Eq. (5) and shown in Fig. 3, where γ_t represents the angular 198 velocity of the guide vanes at time t in r/s. To simulate the motion process of the guide vanes, dynamic 199 mesh techniques were used in this study, with a combination of elastic smoothing and a 2.5D remeshing 200 algorithm. It is worth noting that, in practical computations, as the opening of the guide vanes approaches 201 0° , the grid quality in the guide vane region deteriorates sharply. To ensure smooth calculations, the 202 opening of the guide vanes was maintained at 0.5° between 26s - 45.36s.

r

$$\gamma_{t} = \begin{cases} 0(t \le 10s) \\ -0.0234528625(10s < t \le 26s) \\ 0(26s < t \le 45.36s) \\ 0.00872665(45.36s < t \le 50.36s) \\ 0(50.36s < t \le 80.88s) \\ 0.00872665(80.88s < t \le 87.88s,) \\ 0(t > 87.88s) \end{cases}$$
(5)

203

By implementing a user-defined function (UDF) program within ANSYS Fluent 2022R1, Eq. (4) dynamically modulates the runner's rotational speed, and Eq. (5) simulates the motion of guide vanes. The UDF allows the system to dynamically adjust the simulation parameters in real-time, thus aligning with the fast pump-to-turbine transition process control strategy. This approach accurately replicates the dynamic responses of an actual system runner and guide vanes.

209

210 **3 Results and Discussion**

3.1 Validation of the numerical results

Fig. 2 shows the experimental and numerical results of the pump-turbine performance in both pump and turbine modes at various guide vane openings. The numerical simulations revealed good agreement with the experimental results. In the pump mode, the maximum relative error percentage difference in the head and efficiency was always below 5%. Similarly, in the turbine mode, the relative error percentage difference in the head and efficiency was always below 4.61%.



Figure 2. Experimental vs. numerical results of the pump-turbine performance in both pump
 and turbine modes

3.2 Time histories of external parameters during the fast

222 pump-to-turbine transition process

223 The control strategy and variations in the rotational speed and flow rate during the fast pump-to-turbine 224 transition process of the pumped storage system are shown in Fig. 3. Initially, the unit operated under a 225 stable rated condition in the pump with the guide vane opening set at 22° (relative opening of 100%). 226 After 10 s, the unit received a transition command that started the gradual closure of the guide vanes and 227 the subsequent reduction in the flow rate. After 26 s, the guide vanes completed their closure (relative 228 opening of 2.3%) and the circuit breaker of the electric motor was disconnected, resulting in 229 demagnetization. Consequently, the runner's rotational speed decreases, leading to a continuous decrease 230 in the flow rate. At 35.2 s, the instantaneous flow rate drops to zero and marks the commencement of the 231 flow reversal in the turbine-mode direction. At 45.36 s, the guide vanes initiate a pre-opening to 3° 232 (relative opening of 13.6%) for hydraulic braking. At 66.7 s, the runner's instantaneous rotational speed 233 drops to zero and initiates a reverse rotation in the turbine mode. At 80.88 s, the guide vanes began to 234 open further to a no-load opening of 6.5° (a relative opening of 29.5%). After 119.5 s, the unit enters a 235 stable turbine no-load operating condition and completes the transition process at 109.5 s.



Figure 3. Control strategy and time histories of speed and flow rate

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From the initial moment until 35.2 s, the pump-turbine unit operated in pump mode. From 35.2 to 66.7

s, the pump-turbine unit undergoes pump braking that accelerates the pump shutdown under the influence of hydraulic braking. After 66.7 s, the pump-turbine unit started to operate in the turbine mode. The

of hydraulic braking. After 66.7 s, the pump-turbine unit started to operate in the turbine mode. The

schematic diagrams of the three operational modes of the pump-turbine are shown in Fig. 4.



243



Figure 4. Schematic diagram of different operating modes of pump-turbine

245

The torque and force variations during the fast pump-to-turbine transition process in the pumped-storage system are shown in Fig. 5. The torque and force experienced by the runner were primarily influenced by the combined effects of the unit flow rate and runner speed. During the pump shutdown, the torque decreased steadily. Upon entering the pump-braking operating conditions and activating the guide vanes for hydraulic braking, the torque on the blades increased and exhibited significant fluctuations. Once the unit has started its operation in the turbine mode and the guide vanes are opened to the no-load position, the blade torque consistently decreases to the null value.

Positive axial forces were present during the reversal of the runner's rotational direction. The combined action of the X- and Y-axis radial forces is reflected in the oscillation direction and the intensity of the unit in the horizontal plane (the definitions of the X- and Y-axis directions are shown in Fig. 1). It is evident that under the pump-braking operating condition, the unit experiences more pronounced oscillations when the guide vanes move.



Figure 5. Time histories of torque and force

261 **3.3 Pressure fluctuations during the fast pump-to-turbine**

262 transition process

263 To investigate the pressure fluctuations during the fast pump-to-turbine transition process in the pump-264 turbine system, eight monitoring points were strategically placed within the pump-to-turbine unit, as 265shown in Fig. 6. P-SC1 is located in the spiral casing region near the upstream penstock; P-SC2 is placed 266 in the spiral casing region near the stay vanes; P-SV1 is situated in the stay vane region; P-GV1 is 267 positioned in the guide vane region; P-VL1 is placed in the vaneless region between the guide vanes and 268 the runner; P-RN1 is located in the runner region; P-DT1 is situated in the draft tube region near the 269 interface between the runner and the draft tube; and P-DT2 is positioned within the elbow section of the 270 draft tube region on the inner side. Fig. 6 provides a graphical overview of the setup of the monitoring 271 points described thus far.



Figure 6. Schematic of monitoring points

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The pressure fluctuations at each monitoring point during the fast pump-to-turbine transition are shown in Fig. 7. Monitoring points P-SC1, P-SC2, P-SV1, and P-GV1 showed similar trends in temporal pressure variation. However, the magnitude of the pressure change increased from the spiral casing region to the guide vanes. Using the guide vanes as a boundary, during the fast pump-to-turbine transition process, when the guide vanes are closed, there is a rapid decrease in the pressure in the spiral casing and stay vane regions. The inertia of the water flow led to a sudden pressure increase in the runner and draft tube regions.

The pressure variations in the vaneless and runner regions exhibited similar trends. During pump braking, under the influence of vortical structures at different scales and intensities, there was a significant intensification of pressure fluctuations in both the vaneless and runner regions, with the most pronounced fluctuations observed in the former. The pressure changes in the draft tube region at the two monitoring points were nearly identical (P-DT1 and P-DT2 in Fig. 6), indicating that the pressure in the straight cone section of the draft tube did not exhibit significant variations. No evident phenomena, such as the formation of a tailrace eddy, caused intense pressure pulsations within the draft tube.



289 290

Figure 7. Time histories of pressure fluctuations at different monitoring points

291

3.4 Evolution of flow patterns and vortical structures during

293 the fast pump-to-turbine transition process

3.4.1 Selection of transition process characteristic moments

295To further explore the variations in the internal factors influencing the runner force during the fast pump-296to-turbine transition process, a detailed study and analysis of the evolution of the internal flow field of297the pump-turbine unit were conducted. Six representative moments were selected for an in-depth analysis298as follows:

 $t_1=10.0$ s: initial pump operating condition; $t_2=26.0$ s: onset of runner rotational speed decrease; $t_3=35.2$ s: zero flow rate within the pump-turbine unit; $t_4=66.7$ s: runner rotational speed reaches zero; $t_5=83.5$ s: runner rotational speed rises to 50% of the rated speed in turbine mode. Blade torque continuously decreases and leads to the onset of turbine no-load conditions; and $t_6=119.5$ s: stable turbine no-load operating conditions.

304 **3.4.2 Identification and distribution of vortical structures**

During the fast pump-to-turbine transition process in pumped storage, unstable flow structures are predominantly concentrated in the stay vanes, guide vanes, and runner regions. Employing the rigid vorticity-based vortex identification method, as well as the improvements made in this area by other scientists [28–32], has been shown to be reliable and may prevent significant shear mixing compared to other vortex identification methods [33-37]. With a rigid vorticity amplitude of 120 s⁻¹, as an isosurface colored by turbulent kinetic energy, the distribution of vortical structures at the six characteristic moments is shown in Fig. 8.



312

Figure 8. Vortical structures during the fast pump-to-turbine transition process (Rigid vorticity =120 s⁻¹, colored by turbulent kinetic energy)

315

316 In the initial pump operating condition, vortical structures were primarily distributed near the runner inlet 317 and leading edges of the guide and stay vanes under the impact of the water flow. The results indicate that the flow field at the leading edges of the guide vanes has high turbulent kinetic energy, which means that a high energy dissipation will be generated owing to the high flow rate and strong impact of the water flow on the leading edges of the guide vanes under this condition. Regardless of whether the unit is operating in pump or turbine mode, vortical structures in the runner domain consistently appear near the runner inlet.

323 In pump mode, vortical structures are likely to occur at the leading and trailing edges of both the guide 324 and stay vanes. In turbine mode, vortical structures in the guide and stay vane regions primarily manifest 325 at the trailing edges of the vanes. In addition, the turbulent kinetic energy dissipation in the turbine mode 326 was correspondingly reduced owing to the lower flow rate and velocity during the fast pump-to-turbine 327 process. Notably, the mesh used in this study is not sufficiently fine to resolve all the relevant vortical 328 structures because the computational resources and accuracy must be balanced when simulating the fast 329 pump-to-turbine transition process for more than 100 s. Besides, some of the vortical structures are dampened out by the SST k-w turbulence model. Therefore, not all the vortical structures were resolved, 330 331 as shown in Fig. 8.

332 **3.4.3 Analysis of vortex dynamics characteristics**

To further understand the hydraulic instability during the fast pump-to-turbine transition process, the Reynolds-averaged transport equation for the enstrophy of rigid vorticity [35] was adopted to analyze the generation and dissipation of vortical structures:

$$\Omega_R = \frac{\omega_R \cdot \omega_R}{2} \tag{6}$$

$$\frac{\partial \left(\overline{\omega}_{Ri} \cdot \overline{\omega}_{Ri} / 2\right)}{\partial t} + \overline{u}_{j} \frac{\partial \left(\overline{\omega}_{Ri} \cdot \overline{\omega}_{Ri} / 2\right)}{\partial x_{j}} \\
= \overline{\omega}_{Rj} \frac{\partial \overline{u}_{i}}{\partial x_{j}} \overline{\omega}_{Ri} + \overline{u}_{i} \frac{\partial \overline{\omega}_{Rj}}{\partial x_{j}} \overline{\omega}_{Ri} + \left[\left(\frac{\partial \overline{u}_{i} \overline{\omega}_{Sj}}{\partial x_{j}} - \frac{\partial \overline{u}_{j} \overline{\omega}_{Si}}{\partial x_{j}} \right) \overline{\omega}_{Ri} - \frac{\partial \overline{\omega}_{Si}}{\partial t} \overline{\omega}_{Ri} \right] + \nu \frac{\partial^{2} \overline{\omega}_{i}}{\partial x_{j} \partial x_{j}} \overline{\omega}_{Ri} + \nu_{\iota} \frac{\partial^{2} \overline{\omega}_{i}}{\partial x_{j} \partial x_{j}} \overline{\omega}_{Ri}$$

$$(7)$$

336

337 where Ω_R indicates the enstrophy of the rigid vorticity, s⁻²; $\overline{\omega}_{Ri}$ (*i*=1, 2, 3) indicates the Reynolds-338 averaged rigid vorticity component, s⁻¹; $\overline{\omega}_{Si}$ (i=1, 2, 3) indicates the Reynolds-averaged shear vorticity 339 component, s⁻¹; $\overline{\omega}_i$ (*i*=1, 2, 3) indicates the Reynolds-averaged vorticity component, s⁻¹. In Eq. (7), the 340 left-hand side is the total derivative of the enstrophy of the rigid vorticity, encompassing terms for both 341 the time and convective derivatives. On the right side, the initial term denotes the enstrophy of the rigid 342 vorticity stretching term (ERST), which represents the positive strain intensity of the vortex core line 343 embedding both stretching and compression deformations. The subsequent term, denoted by the 344 enstrophy of the rigid vorticity dilation term (ERDT), captures the shear strain intensity along the vortex 345 core line, accounting for torsional and bending deformations. This term arises from the effect of shear 346 effects on the vortical shape. The third term, known as the enstrophy of the pseudo-lamb vector curl term 347 (ERCT), involves both shear and temporal effects. The fourth term, designated as the enstrophy of the 348 viscous term (ERVT), reflects the inherent dynamic action due to shear processes, representing the 349 diffusion and dissipation strength of vortices induced by fluid viscosity. The fifth term, called the 350 enstrophy of the reynolds stress term (ERRT), captures the diffusion and dissipation strength of vortices 351 induced by turbulent fluctuations. In the context of fluid within a rotating reference frame, an additional

352 term $-2\left(\partial \overline{c}_{i}\overline{u}_{ri} / \partial x_{j} - \partial \overline{c}_{j}\overline{u}_{ri} / \partial x_{j}\right)\overline{\omega}_{Ri}$, namely the enstrophy of the coriolis force term (ECFT), is

introduced and accounts for the influence of inertial effects due to the rotation of vortex evolution, where \bar{c}_i represents the angular velocity component of the rotating system (a constant vector in this study), and \bar{u}_{ri} signifies the velocity component of the fluid relative to the rotating coordinate system.

356 To quantitatively analyze the distribution and intensity of the Reynolds-averaged enstrophy transport 357 terms for the rigid vorticity in these flow components along the spanwise and streamwise directions, the 358 stay vanes, guide vanes, and runner regions were divided into 20 volumes each along the spanwise 359 direction and 60 volumes in the streamwise direction (each region was divided into 20 volumes). The 360 spanwise volumes were numbered from 0 to 1, moving from the hub to the shroud. The streamwise 361 volumes were numbered from 0 to 1, moving from the inlet to the outlet of the stay vane region; from 1 362 to 2, moving from the inlet to the outlet of the guide vane region; and from 2 to 3, moving from the inlet 363 to the outlet of the runner region. The schematic of the volume division is shown in Fig. 9. The evolution 364 intensity of the Reynolds-averaged enstrophy for a unit volume $E_{\rm v}$ is defined by Eq. (7):

$$E_{v} = \frac{\int \frac{D\Omega_{R}}{Dt} dV_{i}}{V_{i}}$$
⁽⁷⁾

365



366

367

Figure 9. Schematic diagram of the volume division

368

369 The distribution and variation of the various transport terms of the Reynolds-averaged enstrophy for a 370 rigid vorticity along the spanwise direction are shown in Fig. 10. It is evident that ERCT plays a key role 371 in the evolution of vortices. From the initial pump operating condition at time t_1 to t_2 when the guide 372 vanes begin to close and the runner speed starts to decrease, and further to t₃ when the flow rate drops to 373 zero, the dissipative effect of the ERCT on the vortices gradually weakens. As the pump turbine 374 transitions into the turbine mode, the ERCT transforms into a vortex generation effect. However, with 375 the phased opening of the guide vanes, the vortex generation effect of the ERCT gradually diminished 376 until it entered the turbine no-load operating condition. Subsequently, the ECFT becomes the primary 377 driver in the evolution of the vortices owing to the small opening of the guide vanes and the unloaded 378 pump-turbine unit. This is primarily manifested by the rotation of the blade, which generates vortices 379 within the flow passage. Additionally, the intensity of the ERDT increased near the hub and shroud, 380 making the vortical structures prone to twisting deformation.





384

Figure 10. The distribution and intensity of E_v variation along the spanwise direction

386 The distribution and variation of the various transport terms of the Reynolds-averaged enstrophy for a 387 rigid vorticity along the streamwise direction are shown in Fig. 11. Consistent with the distribution and 388 variation in the spanwise direction, the ERCT remains the primary factor influencing the evolution of 389 vortices in the streamwise direction. In the turbine no-load operating condition, the ECFT replaces the 390 ERCT as the predominant factor governing vortex evolution in the streamwise direction. The significant 391 variation in the ECFT between the runner inlet and outlet intensified and led to the formation of vortical 392 structures at these locations (see Fig. 8). The magnitude of $E_{\rm v}$ is notably larger in the streamwise volume 393 range of 2 to 3, indicating that the transport terms of the Reynolds-averaged enstrophy for the rigid 394 vorticity within the runner domain exert a stronger influence on the evolution of vortical structures than 395 the stay and guide vane regions. Moreover, in pump mode, the ERDT affects the entire streamwise 396 direction within the runner domain, thus resulting in a twisting deformation of the vortical structures 397 along the streamwise direction.

In the stay and guide vane regions, vortical structures are prone to occur in the middle of the guide vane passages, specifically around a streamwise volume of 1.5. This result is consistent with the vortex identification results presented in Fig. 8. In the stay-vane region, vortical structures primarily appeared near the leading and trailing edges of the blades.



Figure 11. The distribution and intensity of E_v variation along the streamwise direction

406

407 **3.4.4 Evolution of flow patterns and vortical structures**

To provide further evidence of the flow pattern evolution and vortices in the fast pump-to-turbine process
in pumped storage, the distributions of pressure, velocity, rigid vorticity, and ERCT, which play primary
roles in the evolution of vortices, are depicted at six characteristic moments (see Fig. 12).
At the initial moment t₁, the pump-turbine unit operates in pump mode, pumping water from the runner

- 412 into the guide vanes. Owing to this impact, vortices appeared at the leading edge of the guide vanes. High
- 413 ERCT values were primarily distributed in regions near the leading edges of the guide and stay vanes as 414 well as on the suction side of the runner blades. The pressure distribution within the runner blades 415 increases uniformly from the runner to the spiral casing.
- 416 At t₂, the guide vanes are fully closed, and the runner speed starts to decrease. Owing to inertia, the runner
- 417 continues to rotate and pumps water out. However, obstruction due to closed guide vanes causes a rapid
- 418 pressure increase in the vaneless region, generating high-speed circulation. The flow in the stay vanes
- 419 and spiral casing domain started to become chaotic, vortical structures emerged near the runner blade
- 420 suction side, and the guide vanes were cleared.
- 421 At t_3 , the flow rate drops to zero, and both the pressure and high-speed circulation in the vaneless region
- 422 decrease owing to decreases in the speed and flow rate. The flow in the stay vanes and spiral casing
- 423 domain became increasingly chaotic, and regions with high ERCT values and vortical structures
- 424 appeared near the runner blade pressure side.





426

Figure 12 (a). Evolution of the flow patterns and vortical structures at time t_1 , t_2 , and t_3

427

From t_3 to t_4 , the pressure within the runner region decreased continuously as the speed continued to decrease. From t_4 to t_6 , the runner changed its rotational direction to the turbine mode, and the pumpturbine unit entered the turbine mode. As the guide vanes gradually opened, the pressure gradient within the flow domain gradually became more uniform, and the intensity of high-speed circulation in the vaneless region decreased.



433 434

Figure 12 (b). Evolution of the flow patterns and vortical structures at time t_4 , t_5 , and t_6

436 **4 Conclusions**

437 This paper reports a numerical three-dimensional model of the entire flow system of a pumped storage 438 station. Simulations were performed to investigate fast pump-to-turbine transition processes. This 439 research delved into the variations in external parameters, pressure fluctuations within the pump-turbine 440 unit, and the evolution of flow patterns and vortical structures during the fast pump-to-turbine transition 441 process. To the authors' opinion, this research considerably contributes to the understanding of the 442 hydraulic transient mechanisms during the fast pump-to-turbine transition process and provides 443 theoretical guidance for enhancing stability and optimizing control strategies during the transitional 444 process. In particular:

During the fast pump-to-turbine transition process, the opening and closing of the guide vanes
instantaneously impact the hydrodynamic performance within the pump-turbine unit. Variations in
the flow rate, torque, and axial force exhibited stepwise changes during the movement of the guide
vanes. Periods of significant oscillation and drastic fluctuations in torque are prone to occur during
pump braking and reversal of runner speed.

Under the influence of vortical structures at different scales and intensities, the pressure fluctuations
 within the vaneless region and runner exhibit significant variations during the transition process.
 Notably, the pressure fluctuations in the vaneless region were particularly pronounced. The temporal
 variations in pressure at the monitoring points in the spiral casing, stay vanes, and guide vane regions

- exhibit similar trends, with the magnitude of pressure variations increasing from the spiral casing to
 the guide vanes. The draft tube region did not generate a tailrace eddy, thereby avoiding intense
 pressure fluctuations.
- 3) During the fast pump-to-turbine transition process, the inertia of the water flow and the blocking effect of the guide vanes contributed to the occurrence of high-speed circulation in the vaneless region, which was the reason for the intense pressure fluctuations observed in the vaneless region. The internal flow was radically altered and became more complex when the guide vanes were closed, the flow rate approached zero, or the runner speed decreased to zero. Rapid changes in the guide vane opening, flow direction, and rotational direction were the main causes of hydraulic instability of the pump turbine.
- 464

Building on the foundation laid in this study, future investigations can delve deeper into the hydraulic transient mechanisms during the critical phases of the transition process. Furthermore, exploring the impact of various guide vane control strategies on this transition process and optimizing these control strategies represents another avenue for future research.

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