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Resilient and robust management policy for multi-stage supply chains with perishable goods and inaccurate forecast information: A distributed model predictive control approach

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[Correction added on 6 September 2024, after first online publication: The copyright line was changed.]

Abstract

An efficient supply chain (SC) management requires that decisions are taken to minimize the effects of parametric uncertainties and unpredictable external disturbances. In this article, we consider this problem with reference to a multi-stage SC (MSSC) whose dynamics is characterized by the following elements of complexity: perishable goods with uncertain perishability rate, an uncertain future customer demand that is only known to fluctuate inside a given compact set. The problem we face is to define a resilient and robust Replenishment Policy (RP) such that at any stage the following requirements are satisfied: the fulfilled demand is maximized, overstocking is avoided, the bullwhip effect (BE) is mitigated. These objectives should be pursued despite the mentioned uncertainties and unexpected customer demand behaviors violating the bounds of the compact set. Robustness is here intended with respect to uncertainty on the perishability rate, and resiliency as the ability to quickly react to the mentioned unforeseen customer demands. We propose a method based on a distributed resilient robust model predictive control (DRRMPC) approach. Each local robust MPC (RMPC) involves solving a Min-Max constrained optimization problem (MMCOP). To drastically reduce the numerical complexity of each MMCOP, we parametrize its solution by means of B-spline functions.

KEYWORDS

distributed resilient robust model predictive control, min-max optimization, optimal inventory management, supply chain

1 | INTRODUCTION

Efficient SC management requires maximizing the satisfied customer demand without incurring excessive inventory levels. The antagonism of these requirements calls for a control policy based on an optimality criterion. In this regard, the importance of MPC is widely recognized and documented.[1,2] This is due to the natural ability in handling physical constraints and to the receding horizon nature of the control law.[3] The first feature allows limiting the inventory level and the replenishment orders, the second one allows determining appropriate on line corrections to the actual control action according to the incoming observations. This second property is particularly useful in SC design where only a time-varying forecast of customer demand is available over a limited time interval.

For large scale systems, the research on MPC techniques was directed toward the application of three different control architectures[4]: centralized, decentralized and distributed. The first two are applied to MSSCs in References 5–10. The main limitations of centralized approach are the numerical complexity of the optimization problem, the costs associated to communication channels, the barriers to information sharing among supply chain partners. The decentralized approach does not have these drawbacks but causes a loss of performance in the supply chain management because control agents decide control actions independently on each other. This motivated the recent interest toward distributed model predictive control (DMPC) strategies, see for example, References 11–15. They make a compromise between centralized and decentralized architectures because some information exchange among local agents is allowed.

The above papers do not face the problem raised by the presence of deteriorating items. On the other hand management of perishable products is quite important because it has a direct impact on factors determining profitability like: price, sales, inventory level, deterioration cost, product availability.

MPC of inventory level in the case of perishable goods has been investigated in References 16 and 17 for single-echelon SCs and in References 18–20, for multi-echelon SCs. The importance of this topic also motivated many alternative control methods outside the MPC framework: for example, single-stage SCs the optimal inventory control problem with perishable goods has been considered in References 21–26 from different points of view. A Smith predictor based approach has been proposed in References 27–29. The case of an MSSC with perishable goods has been considered in Reference 30 using genetic algorithms.

The common factor of all the above mentioned approaches (MPC and non-MPC) dealing with perishable goods is the assumption of an exactly known decaying factor. However, this simplifying assumption is not satisfied in the overwhelming part of practical cases due to unstable and variable storage conditions.[31]

MPC of single stage SC's with perishable good and uncertain deterioration rate has been considered in References 32–34.

Besides the robustness requirement, resilience is another very important property required to SCs. Resilience is usually defined as the ability of an SC to quickly react to unexpected events. With reference to the purpose of this article we define the resilience property as the capacity of promptly restore operational normality in response to sudden unpredicted changes of demand patterns.

Based on the foregoing considerations and cited literature, the purpose of this article is to propose a DRRMPC strategy for the optimal inventory control problem of an MSSC working in the following operating conditions:

1. perishable goods with an interval type uncertainty on the perishability rate;
2. an uncertain future customer demand that may show unforeseen behaviors with respect to some "a priori" assumptions on its uncertainty.

We aim at obtaining a resilient and robust RP conciliating the following antagonist control requirements (CR) at each stage:

(CR1) the satisfied demand coming from the neighboring downstream stage should be maximized;

(CR2) overstocking should be avoided;

(CR3) the replenishment orders issued by each stage should take values inside intervals with a predetermined amplitude that is slowly increasing in the upward direction;

(CR4) the RP should not incur sharp and frequent order quantity changes.

The first step to satisfy CR1 is defining a suitable predictive information on the end customer demand. According to the interval prediction approach (see e.g., References 35 and 36 and references therein), we only assume that at any time instant $k \in Z^+$ and over an M_1-steps prediction horizon, the future end customer demand entering the first stage of the MSSC is arbitrarily time varying inside a given compact set $D_{1,k}$. This assumption is general enough to include all situations of a practical interest and is independent of the sources of uncertainty on the future customer demand.

Owing to its intuitive nature, the interval prediction approach has been attracting a lot of interest as an alternative to point forecast based on time series analysis. Its advantageousness is particularly evident when the data sequence shows large randomness and volatility: in this case, point forecast methods based on time series analysis are not able to capture several statistical phenomena underlying the nature of the demand generation process.[37] Interval prediction prescinds from this information and can better capture the uncertain trend of the data sequence and is a more effective reference for formulating robust control strategies.

CR1 and CR2 are conciliated defining a time-varying target inventory level for the first stage given by the upper bounding trajectory of $D_{1,k}$. Going in the upwards direction, we iteratively define suitable time-varying target inventory levels and keep the actual inventory of any stage as close as possible to the respective desired trajectory (see Section 4.1 for details).

Meeting CR3 and CR4 is necessary to counteract the BE, that is, the continuous upwards amplification of amplitude and frequency of small changes in the end customer demand. This is a problem of a paramount importance in MSSC management as testified by the impressive amount of relevant literature. See for example, References 38 and 39, and references therein.

As for CR3, we limit the progressive upstream amplification of demand imposing suitable hard constraints on the optimal robust RP solution of the local RMPC problem defined at each stage of the MSSC. In this regard our approach reveals an important fact: the presence of deteriorating items contributes to the progressive amplification proportionally to the perishability rate (see Section 4.2). The interesting corollary is that, in the case of nonperishable goods, our approach allows us to contain the values of orders issued by all stages in the same fixed amplitude interval.

As for CR4, we define an RP through a parametrized solution of the optimization problem in terms of smooth functions and define a cost functional penalizing excessive differences between consecutive orders.

To address the problem described so far we propose a DRRMPC based on a set of MMCOPs: the control law (i.e., the replenishment order) of any stage is obtained through a receding horizon implementation of a predicted control sequence minimizing the worst case of a local quadratic cost functional on the basis of the information coming from the downstream stage. The worst case is computed as the maximum with respect to all the possible perishability rate values belonging to a known compact set.

We introduce a coordination requirement between contiguous agents: the constraints on the replenishment orders issued by each agent are related to the analogous constraints relative to the neighboring downstream agent. This allows us to strictly control the BE (see Section 4.2). Resilience w.r.t. anomalous end customer demands is achieved by introducing an agile adjustment mechanism of $D_{1,k}$ (see Section 4.3).

Another significant novelty of our approach is the use of polynomial B-spline functions to parametrize the solution of each MMCOP. The main reasons for this choice are: (1) polynomial B-splines are smooth functions that can be used as universal approximators of curves which exhibit different shapes over different time-intervals; (2) B-splines admit a parsimonious parametric representation given by a time varying, linear, convex combination of some parameters named "control points."[40]

Property 1 allows us to obtain a predicted replenishment order signal with a smooth waveform. Property 2 allow us to transfer any hard constraint on the predicted control sequence to its control points and to reformulate the MMCOP as a Constrained Robust LS estimation problem with only constraints on the unknowns (the control points defining the admissible B-spline function). The Constrained Robust LS problem can be efficiently solved using interior point methods.[41] Finally, as shown in the theorem of Section 5, Property 2 allows us to rigorously prove both stability and feasibility of the MMCOP without any further assumption.

As for the stability of MPC, two main approaches exist in the literature: finite, sufficiently large, prediction horizon with terminal constraints (see e.g., References 42 and 43), infinite prediction horizon (see e.g., References 44 and 45). On the contrary, we achieve stability and feasibility regardless of the prediction horizon length (see Theorem 1, Section 5). This is very important because allows us to limit the future knowledge on the end customer demand to prediction horizons whose length is inferiorly limited by considerations only involving the architecture of the MSSC (see Section 4.1).

We also remark that although, stability and feasibility are fundamental issues of MPC approach, most MPC techniques for SCs do not explicitly address these topics.

A preliminary version of this contribution was presented at Reference 46. Here we provide more theoretical and implementation insights answering many issues not addressed in the previous version:

1. a more general description of the MSSC dynamics: for example, here we remove the assumption that the demand is fully satisfied starting from a specific time instant;
2. a different formulation of the cost functional where large deviations between two consecutive control actions are penalized as to meet CR4 (see (16));
3. definition of an RP endowed with an agile adjustment mechanism to achieve resilience w.r.t. unexpected patterns of the end customer demand
4. much more extensive numerical results: centralized and decentralized implementation of our approach and comparison.

TABLE 1 Acronyms.

SC	Supply chain
MSSC	Multi-stage SC
RP	Replenishment policy
BE	Bullwhip effect
MPC	Model predictive control
RMPC	Robust MPC
DRMPC	Distributed RMPC
DRRMPC	Distributed resilient RMPC
CR	Control requirement
MMCOP	Min-max constrained optimization problem
LS	Least squares
UD	Unsatisfied demand
IL	Inventory level
IO	Issued orders
WG	Wasted goods
MNLCS	Modified non-linear control strategy

The article is structured as follows. Brief mathematical preliminaries on B-splines and Robust LS are provided in Section 2. The plant model is described in Section 3. The control problem is stated in Section 4 and reformulated as a Constrained Robust LS estimation problem in Section 5. The numerical results of Section 6 include a comparison with Reference 29 and with the decentralized and centralized versions of our approach. Concluding remarks are reported in Section 7. The Appendix explains the mathematical derivation of decentralized and centralized architectures. Acronyms and nomenclature are reported in Tables 1 and 2, respectively.

2 | MATHEMATICAL BACKGROUND

2.1 | B-splines

A scalar, continuous time, B-spline curve $b_s(t)$ is defined as a linear combination of B-splines basis functions and control points[40]:

$$b_s(t) = \sum_{i=1}^{\ell} c_i B_{i,d}(t), \qquad t \in [\hat{t}_1, \hat{t}_{\ell+d+1}] \subseteq R, \tag{1}$$

where: the c_i's are real numbers representing the control points of $b_s(t)$, the integer d is the degree of the B-spline, the knot sequence $(\hat{t}_i)_{i=1}^{\ell+d+1}$ is a nondecreasing sequence of time instants and the $B_{i,d}(t)$ are the uniformly bounded B-spline basis functions that can be computed by the Cox-de Boor recursion formula

$$B_{i,d}(t) = \frac{t-\hat{t}_i}{\hat{t}_{i+d}-\hat{t}_i} B_{i,d-1}(t) + \frac{\hat{t}_{i+1+d}-t}{\hat{t}_{i+1+d}-\hat{t}_{i+1}} B_{i+1,d-1}(t), \qquad d \geq 1, \tag{2}$$

with $B_{i,0}(t) = 1$ if $\hat{t}_i \leq t < \hat{t}_{i+1}$, otherwise 0.

In (2) we use the convention that "fractions with zero denominator have value zero."[47]

An equivalent representation of $b_s(t)$ in (1) is

$$b_s(t) = \mathbf{B}_d(t)\mathbf{c}, \qquad t \in [\hat{t}_1, \hat{t}_{\ell+d+1}] \subseteq R, \tag{3}$$

TABLE 2 Parameters/sets/variables of the MSSC network.

$\mathcal{S}_i$	The *i*th node
$\mathcal{A}_i$	The *i*th agent for $\mathcal{S}_i$
$d_1(k)$	The actual customer demand
N_i	The length of control horizon $H_{i,k}$ *for* $\mathcal{A}_i$
M_i	The length of prediction horizon $P_{i,k}$ *for* $\mathcal{A}_i$
$\mathcal{D}_{1,k}$	The compact set containing the future customer demand over $[k, k+M_1]$
$\mathcal{D}_{i,k}, i = 2, \dots, n$	The compact set containing the forecasted demand for $\mathcal{A}_i$
$\hat{\mathcal{D}}_1(k)$	The predicted customer demand for $\mathcal{A}_1$
$U_{i,k}, i = 1, \dots, n$	The predicted optimal RP determined by $\mathcal{A}_i$
$\hat{\mathcal{D}}_i(k) = U_{i-1,k}, i = 2, \dots, n$	The forecasted demand for $\mathcal{A}_i$
$u_i(k), i = 1, \dots, n$	The order issued by S_i
$u^-_{i,k}$ and $u^+_{i,k}$	The lower and upper bounds on $U_{i,k}$
$d_i(k) = u_{i-1}(k), i = 2, \dots, n$	The demand issued by S_{i-1} *for* S_i
$s_i(k - L_i)$	The goods delivered to $\mathcal{S}_i$ *from* $\mathcal{S}_{i+1}$ *with time delay* L_i
$h_i(k)$	The amount of demand issued by S_{i-1} *for* S_i *and delivered to* S_{i-1} *from* S_i
$y_i(k)$	The on hand inventory level of S_i
α_i, ρ_i	The perishability rate and the decay factor of S_i

where $\mathbf{c} \triangleq [c_1, \dots, c_\ell]^T$ and $\mathbf{B}_d(t) \triangleq \left[B_{1,d}(t), \dots, B_{\ell,d}(t)\right]$.

Convex hull property. Any value assumed by $b_s(t)$, $\forall t \in [\hat{t}_j, \hat{t}_{j+1}]$, $j > d$, lies in the convex hull of its $d+1$ control points $c_{j-d}, \dots, c_j$.

Smoothness property. Suppose that $\hat{t}_i < \hat{t}_{i+1} = \dots = \hat{t}_{i+m} < \hat{t}_{i+m+1}$, with $1 \le m \le d+1$ then the B-spline function $b_s(t)$ has continuous derivative up to order $d - m$ at knot $\hat{t}_{i+1}$. This property implies that the spline smoothness can be changed using multiple knot points. It is common choice to set $m = d+1$ multiple knot points for the initial and the last knot points and to evenly distribute the other ones. In this way (1) assumes the first and the final control points as initial and final values.

Remark 1. From (3) it is apparent that, once the degree d and the knot points $\hat{t}_i$ have been fixed, the scalar B spline function $b_s(t)$, $t \in [\hat{t}_1, \hat{t}_{\ell+d+1}]$, is completely determined by the corresponding vector $\mathbf{c}$ of ℓ control points.

2.2 | The robust LS problem

Consider the overdetermined set of linear equations $Df \approx b$, where $D \in R^{r\times m}$ is the design matrix and $b \in R^r$ is the observations vector. Both D and b are subject to unknown but bounded errors[41]: $\|\delta D\| \le \beta$ and $\|\delta b\| \le \xi$ (where the matrix norm is the spectral norm). The robust least squares estimate $\hat{f} \in R^m$ is the value of f minimizing

$$\min_f \max_{\|\delta D\|\le\beta,\ \|\delta b\|\le\xi} \|(D+\delta D)f - (b+\delta b)\|, \tag{4}$$

Using norm properties, it can be shown that

$$\max_{\|\delta D\|\le\beta,\ \|\delta b\|\le\xi} \|(D+\delta D)f - (b+\delta b)\| = \|D\,f - b\| + \beta\|f\| + \xi. \tag{5}$$

Hence (4) is equivalent to minimize the following sum of euclidean norms

$$\min_f \|Df - b\| + \beta\|f\| + \xi, \tag{6}$$

The constrained robust LS problem also requires that f satisfies the following conditions

$$\underline{f} \le f \le \overline{f}. \tag{7}$$

Remark 2. Note that the term $\|\delta b\|$ in (4) only appears in (6) through its norm upper bound ξ, which is independent of f. Hence ξ can be removed from the objective function without affecting the value of f solving the minimization problem. As shown in Section 5, this allows us to solve the MMCOP implied by the RMPC algorithm even in the case of uncertain future customer demand.

3 | THE SYSTEM MODEL

As shown in Figure 1, we consider an MSSC network consisting of a cascade of stages (nodes) $\mathcal{S}_i$, $i = 1, \dots, n$, characterized by counter-current order and material streams. Orders are propagated upstream from $\mathcal{S}_1$ to $\mathcal{S}_n$ and the products are shipped along the opposite direction.

Management decisions for each node are taken periodically at equally distributed time instants kT where $k \in Z^+$ and T is the review period. At the beginning of each review period $[kT, (k+1)T)$ the operations across the SC network are performed sequentially from $\mathcal{S}_1$ to $\mathcal{S}_n$.

Inside each review period, each $\mathcal{S}_i$ executes five actions in the following order: receives delivery from supplier S_{i+1}, logs the demand of customer S_{i-1}, measures its on hand stock level, delivers the goods to meet demand and finally places an order according to a suitably defined RP. Accordingly, five variables are defined: $s_i(k)$, $d_i(k)$, $y_i(k)$, $h_i(k)$ and $u_i(k)$. They represent the shipment of goods from supplier S_{i+1}, the demand from S_{i-1}, the on hand stock level, the delivery to customer S_{i-1} and the replenishment order, respectively.

Each node $\mathcal{S}_i$ is regulated by an agent $\mathcal{A}_i$ that solves a local RMPC problem based on the following assumptions:

- **(A1)** The end customer demand $d_1(k)$, $k \in Z^+$, is uniformly bounded. Moreover, at any time instant k, and limitedly to an M_1-steps prediction horizon $[k, k+M_1] \triangleq P_{1,k}$, the end-customer demand $d_1(k+j)$, $j = 0, \dots, M_1$, fluctuates within a given compact set $\mathcal{D}_{1,k}$ limited below and above by two boundary trajectories: $d_1^-(k+j)$ and $d_1^+(k+j)$, $j = 0, \dots, M_1$. The forecasted demand $\hat{\mathcal{D}}_{1,k} = [d_1(k+1|k), \dots, d_1(k+M_1|k)]$ for agent $\mathcal{A}_1$ coincides with the central trajectory of $\mathcal{D}_{1,k}$. Figure 2A shows a typical example of an end-customer demand $d_1(k+j)$ and of a predicted end-customer demand $d_1(k+j|k)$ over a fixed $\mathcal{D}_{1,k}$. The set D_1 containing the whole customer demand is given by the consecutive contiguous overlapping of all the sets $D_{1,k}$, $k \in Z^+$. Figure 2B shows an example of a partial overlapping for $k = 0, 1, 2$.

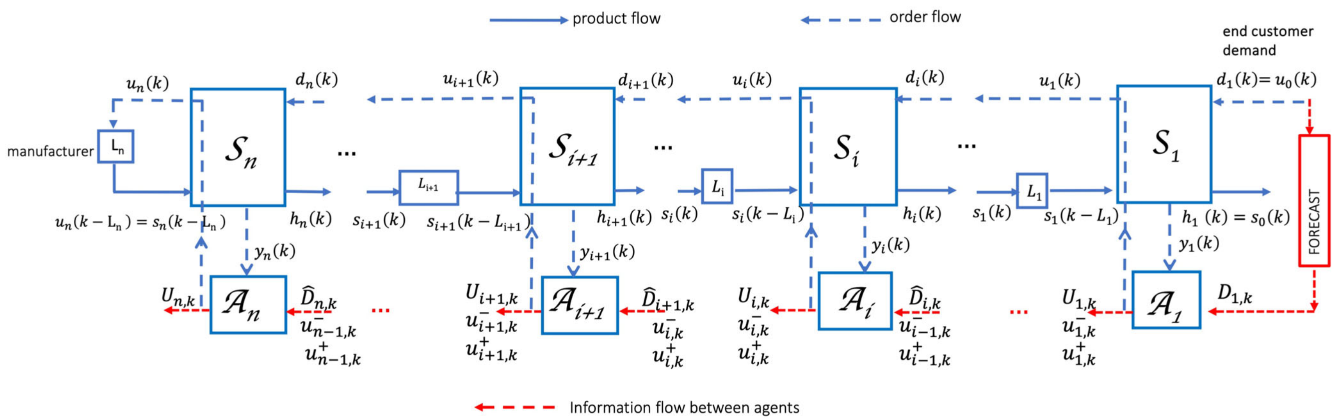

FIGURE 1 Distributed control scheme of the MSSC network.

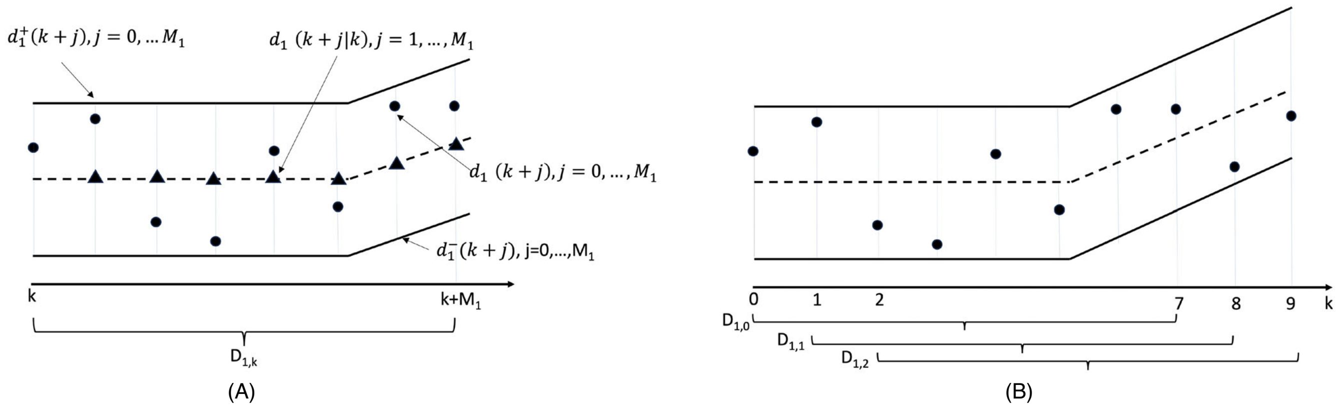

FIGURE 2 (A) Example of a set $\mathcal{D}_{1,k}$ with known time varying boundaries trajectories: $d_1^-(k+j)$ and $d_1^+(k+j), j=0,\ \dots\ ,M_1$. The sequence of bullets denotes a possible trajectory of the actual end-customer demand $d_1(k+j), j=0,\ \dots\ ,M_1$. The sequence of triangles denotes the predicted end customer demand $d_1(k+j|k), j=1,\ \dots\ ,M_1$. (B) An example of a partial overlapping of sets $\mathcal{D}_{1,k}$ for $k=0,1,2$, with $M_1=7$.

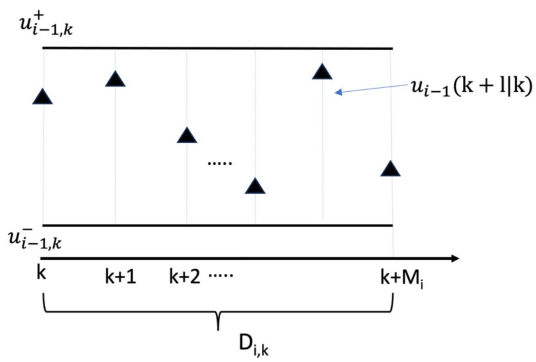

FIGURE 3 Example of a set $\mathcal{D}_{i,k}$, $i>1$, limited by the known constant boundaries trajectories given by the lower and upper values $u_{i-1,k}^-$ and $u_{i-1,k}^+$ respectively. The sequence of triangles denotes the predicted demand $d_i(k+l|k)=u_{i-1}(k+l|k), l=1,\ \dots\ ,M_i$ with $M_i=N_{i-1}-1$, for agent $\mathcal{A}_i$, $i>1$.

- **(A2)** At any time instant k, the predicted demand $\hat{\mathcal{D}}_{i,k}=[d_i(k+1|k),\ \dots\ ,d_i(k+M_i|k)]$ for the other agents $\mathcal{A}_i$, $i=2,\ \dots\ ,n$, coincides with the predicted optimal control sequence (i.e., the optimal predicted RP) $U_{i-1,k}\triangleq[u_{i-1}(k+1|k),\ \dots\ ,u_{i-1}(k+N_{i-1}-1|k)]$ transmitted by $\mathcal{A}_{i-1}$ to $\mathcal{A}_i$ (so that $M_i=N_{i-1}-1$). Note that also $\hat{\mathcal{D}}_{i,k}$ belongs to a given compact set $\mathcal{D}_{i,k}$ limited by the imposed lower and upper values $u_{i-1,k}^-$ and $u_{i-1,k}^+$ respectively (as shown in Figure 3).
  How to compute $U_{i-1,k}$, $u_{i-1,k}^-$ and $u_{i-1,k}^+$ is explained in Sections 4.1 and 4.2.
- **(A3)** Any goods shipped from supplier S_{i+1} arrive at customer S_i with a time delay $L_i=n_iT$, where $n_i\in Z^+$. The goods arrive at customer S_i new and deteriorate while kept in stock.
- **(A4)** Inside each review period, the uncertain perishability rate of the goods stocked in S_i is $\alpha_i\in[\alpha_i^-,\alpha_i^+]\subset(0,1)$. Hence the perishable goods are subject to a decay equal to $\rho_i\triangleq 1-\alpha_i\in[\rho_i^-,\rho_i^+]\subset(0,1)$. For example, if inside each review period the uncertain percentage of stock that must be discarded due to deterioration can take values inside the interval [3%, 5%], then the percentage of still available goods belongs to the range [95%, 97%]. Hence: $\alpha_i\in[0.03,0.05]$ and $\rho_i\in[0.95,0.97]$
- **(A5)** the operations of inventory replenishment and goods delivery are executed simultaneously at the beginning of each review period. Sales are not backordered.

The above assumptions imply that the stock level dynamics of the ith node is described by the following uncertain equation

$$y_i(k+1)=\rho_i(y_i(k)+s_i(k-L_i)-h_i(k)), \tag{8}$$

where:

- $y_i(k+1)$ is the on hand nondeteriorated stock level of $\mathcal{S}_i$ available at the beginning of the $(k+1)$th period;
- $s_i(k-L_i)$ is the goods delivered to the stage $\mathcal{S}_i$ with a time delay L_i. For $i=n$ we have $s_n(k-L_n) \equiv u_n(k-L_n)$;
- The sum $y_i(k)+s_i(k-L_i)$ represents the effective amount of goods available for sale at the beginning of kth review period;
- $h_i(k)$ is the demand fulfilled by $\mathcal{S}_i$

$$h_i(k) \triangleq \min\{d_i(k), y_i(k)+s_i(k-L_i)\} \in [0,\ d_i(k)], \quad i=1, \dots, n; \tag{9}$$

where: $d_1(k)$ is the end-customer demand and $d_i(k)=u_{i-1}(k)$, $i=2, \dots, n$ is the demand issued by $\mathcal{S}_{i-1}$ for $\mathcal{S}_i$.

For future developments we now rewrite equation (8) in a more convenient form where $u_i(k)$ is made explicit.

Figure 1 shows that $d_{i+1}(k)=u_i(k)$ and $h_{i+1}(k)=s_i(k)$ for $i=0, \dots, n-1$, moreover by (9): $s_i(k)=h_{i+1}(k) \in [0,\ d_{i+1}(k)]=[0,\ u_i(k)]$. Therefore

$$s_i(k)=u_i(k)-z_i(k), \quad i=0, \dots, n-1, \tag{10}$$

for some $z_i(k) \in [0\ \ u_i(k)]$, that represents the difference between the ordered and the amount of goods dispatched to the ith stage (where $\mathcal{S}_0$ is the client). For $\mathcal{S}_n$ we have $z_n(k)=0$ because $s_n(k) \equiv u_n(k)$. Moreover Figure 1 and (10) imply

$$z_i(k)=u_i(k)-s_i(k)=d_{i+1}(k)-h_{i+1}(k), \quad i=0, \dots, n-1. \tag{11}$$

By (11) it follows that $z_i(k)=0$ iff S_{i+1} fully satisfies the demand coming from S_i. By (10) and (11), Equation (8) can be rewritten as

$$y_i(k+1)=\rho_i(y_i(k)+u_i(k-L_i)-z_i(k-L_i)-u_{i-1}(k)+z_{i-1}(k)), \quad i=1, \dots, n \tag{12}$$

In Section 4.1 we use (12) to formally derive the prediction equation of the on hand stock level $y_i(k)$ over the prediction horizon. The apparent difficulty due to the unknown terms is dealt with in Section 5.

4 | PROBLEM SETUP

This section describes the procedure to define the resilient robust RP for the MSSC described in Section 3. Resilience and robustness are here considered with respect to unforeseen customer demand outliers and to uncertainty on the decay factor respectively.

Each $\mathcal{A}_i$ exploits the information carried by Equation (8) and by the predicted optimal control policy $U_{i-1,k}$ coming from $\mathcal{A}_{i-1}$. This information is used to predict the future inventory level of the local subsystem $\mathcal{S}_i$ which in turn is used to compute $U_{i,k}$. This last step is performed minimizing the worst case of a local quadratic cost functional subject to hard constraints $u^-_{i,k}$ and $u^+_{i,k}$. The worst case is computed as the maximum with respect to all the possible values of the uncertain decay factor $\rho_i \in [\rho^-_i, \rho^+_i]$.

Coordination between contiguous agents $\mathcal{A}_i$ and $\mathcal{A}_{i-1}$, is imposed by relating the respective constraints $u^-_{i,k}$ and $u^+_{i,k}$ with $u^-_{i-1,k}$ and $u^+_{i-1,k}$, $i=1, \dots, n$, with the aim of guaranteeing the satisfaction of CR3 (see Section 4.2).

The proposed DRMPC requires each agent $\mathcal{A}_i$ to repeatedly solve an MMCOP over a future N_i steps control horizon $H_{i,k} \triangleq [k, k+N_i-1]$, (for some $N_i < M_i$), and, according to the receding horizon control, to only apply the first sample of the computed optimal control sequence $U_{i,k}=[u_i(k|k), \dots, u_i(k+N_i-1|k)]$, $k \in Z^+$.

The bounds $u^-_{i,k}$ and $u^+_{i,k}$ on $U_{i,k}$, are computed at the beginning of each $H_{i,k}$ $i=1, \dots, n$, $k \in Z^+$, before solving the local MMCOP.

The counterpart of this powerful approach is the numerical complexity of the algorithm.[48] As explained in Section 5, this drawback is drastically reduced through a Constrained Robust LS formulation of the MMCOP.

4.1 The local MMCOP

At each time instant $k \in Z^+$ and on the basis of CR1)-CR4), the local MMCOP for any $\mathcal{A}_i, i = 1, \dots, n$, is formally defined as follows

$$\min_{U_{i,k}} \max_{\rho_i \in [\rho_i^-, \rho_i^+]} \sum_{l=1}^{N_i} \left[e_i^T(k+L_i+l|k) q_{i,l}(k) e_i(k+L_i+l|k)\right] + \lambda_i(k)(\Delta u_i(k|k))^2, \tag{13}$$

$$\text{subject to:} \qquad u_{i,k}^- \leq u_i(k+j|k) \leq u_{i,k}^+ < \infty, \;\; j = 0, \dots, N_i - 1, \tag{14}$$

$$e_i(k+L_i+l|k) \triangleq r_i(k+L_i+l|k) - y_i(k+L_i+l|k), \tag{15}$$

$$\Delta u_i(k|k) \triangleq u_i(k|k) - u_i(k-1), \tag{16}$$

By (12) we have

$$\begin{aligned} y_i(k+L_i+l|k) = \rho_i^{L_i+l} y_i(k) &+ \sum_{\ell=0}^{L_i-1} \rho_i^{L_i+l-\ell} u_i(k+\ell-L_i) + \sum_{\ell=0}^{l-1} \rho_i^{l-\ell} u_i(k+\ell|k) \\ &- \sum_{\ell=0}^{L_i-1} \rho_i^{L_i+l-\ell} z_i(k+\ell-L_i) - \sum_{\ell=0}^{l-1} \rho_i^{l-\ell} z_i(k+\ell|k) \\ &- \sum_{\ell=0}^{L_i+l-1} \rho_i^{L_i+l-\ell} u_{i-1}(k+\ell|k) + \sum_{\ell=0}^{L_i+l-1} \rho_i^{L_i+l-\ell} z_{i-1}(k+\ell|k). \end{aligned} \tag{17}$$

Definition of the above predicted quantities implies

$$[k+L_i+1, k+L_i+N_i] \subseteq [k, k+M_i] \triangleq P_{i,k}, \;\; i = 1, \dots, n. \tag{18}$$

Inside each $P_{i,k}$, the predicted $r_i(k+L_i+l|k), l = 1, \dots, N_i$ is defined as

$$r_i(k+L_i+l|k) \triangleq \begin{cases} d_1^+(k+L_1+l) & i = 1, \\ u_{i-1,k}^+ & i = 2, \dots, n. \end{cases} \tag{19}$$

Remark 3. Some considerations on (13) are now in order.

1. By **A1)** and (19), the number M_1 of future steps over which the upper bounding trajectory $d_1^+(k+j), j = 0, \dots, M_1$ must be known is inferiorly limited as

$$M_1 \geq N_1 + L_1. \tag{20}$$

2. By (18) and recalling that the demand forecasting for $\mathcal{A}_i$ is $\hat{D}_{i,k} = U_{i-1,k}$ (see Figure 3), it is easily seen that $M_i = N_{i-1} - 1 = N_i + L_i, i > 1$, namely

$$N_{i-1} = N_i + L_i + 1. \tag{21}$$

By (21), each $\mathcal{A}_{i-1}$ computes N_{i-1} on the basis of the information transmitted by $\mathcal{A}_i$. This iterative procedure starts from an "a priori" value N_n chosen by $\mathcal{A}_n$.
3. By (19), the actual time-varying target inventory level $r_i(k), k \in Z^+, i = 1, \dots, n$, is given by:

$$r_1(k) = d_1^+(k) \;\; \text{and} \;\; r_i(k) = u_{i-1,k}^+, i = 2, \dots, n, \tag{22}$$

where $d_1^+(k)$ is the "a priori" known upper limit of the actual end-customer demand entering $\mathcal{A}_1$ inside $P_{1,k}$. For $i = 2, \dots, n$, the situation is different because an upper boundary trajectory of the actual future demand that each $\mathcal{A}_{i-1}$, will forward to $\mathcal{A}_i$, $i = 2, \dots, n$ is not "a priori" known (due to the receding control horizon philosophy of MPC). Hence, inside each $P_{i,k}$, $i = 2, \dots, n$, the predicted reference level $r_i(k + L_i + l|k)$ is frozen on the "a priori" computed maximum value $u^+_{i-1,k}$ of the predicted demand $U_{i-1,k}$ that each $\mathcal{A}_{i-1}$, forwards to $\mathcal{A}_i$, $i = 2, \dots, n$ (see Figure 3).

4. The hard constraints (14), computed as explained in Section 4.2, need to guarantee the internal stability of the MSSC. Moreover, forcing the control effort to fluctuate within a predefined amplitude range allows us to contain the BE. The term $\lambda_i(k)\Delta u_i^2(k|k)$ has been introduced in order to meet CR4.
5. The terms $q_{i,l}(k)$, $l = 1, \dots, N_i$, and $\lambda_i(k)$, are positive coefficients weighting two conflicting objectives: small tracking error and small difference between two consecutive control moves. According to the guidelines proposed in Reference 49, these coefficients should be inversely proportional to the square of the interval where the relative physical variables are allowed to vary. We choose

$$q_{i,l}(k) = \frac{1}{\bar{e}_i(k + L_i + l)^2}\zeta_i^{l-1}, \tag{23}$$

$$\lambda_i(k) = \frac{1}{\overline{\Delta} u_i(k)^2}, \tag{24}$$

where $\bar{e}_i(k + L_i + l)$ is the maximum tolerable tracking error defined as a fixed percentage $\varepsilon_{e,i}$ of $r_i(k + L_i + l)$, $0 < \zeta_i < 1$ is a forgetting factor progressively decreasing the weight of future observations, $\overline{\Delta} u_i(k)$ is the maximum tolerable variation between two consecutive values of the control effort defined as a fixed percentage $\varepsilon_{u,i}$ of $u_i(k-1)$.

4.2 | Determining the hard constraints on the predicted control sequence $U_{i,k}$

The constraints (14) on $U_{i,k}$ are determined on the basis of the following criteria:

(1) maximize the amount of demand satisfied by $\mathcal{S}_i$ (CR1),
(2) limit the amplitude $A_{i,k}$ of $[u^-_{i,k} \;\; u^+_{i,k}] \triangleq C_{i,k}$, to contain the BE (CR3).

We assume:

(A6) The agent $\mathcal{A}_i$ derives the minimum $A_{i,k}$ of each $C_{i,k}$ so that $\mathcal{S}_i$ guarantees the full satisfaction of the demand coming from $\mathcal{S}_{i-1}$ that is, to guarantee $h_i(k) = d_i(k), i = 1, \dots, n$. Analogously, each $\mathcal{A}_i$ also assumes that its demand will be fully satisfied by $\mathcal{A}_{i+1}$.

In accordance with the foregoing considerations, (11) gives $z_i(k) = 0, i = 0, \dots, n-1$. As also $z_n(k) = 0$, (12) becomes

$$y_i(k+1) = \rho_i(y_i(k) + u_i(k - L_i) - u_{i-1}(k)), \quad i = 1, \dots, n, \quad u_0(k) = d_1(k). \tag{25}$$

Owing to the uncertainty on the future values of $u_{i-1}(k)$ and on the decay factor ρ_i, we compute $u^-_{i,k}$ and $u^+_{i,k}$ with reference to two possible, limit situations compatible with (25). Consider the following hypothetic scenario:

- the demand $u_{i-1}(k)$, entering S_i is a constant signal with value $\tilde{u}_{i-1,k} \in [u^-_{i-1,k}, u^+_{i-1,k}]$. The two mentioned limit situations are $\tilde{u}_{i-1,k} = u^-_{i-1,k}$ and $\tilde{u}_{i-1,k} = u^+_{i-1,k}$;
- each control horizon $H_{i,k}$ is long enough to allow the output (the on hand stock level), to practically attain the steady-state value $\tilde{y}_{i,k}$ under the forcing action of a constant signal $\tilde{u}_{i,k}$.

Note that the existence of an output steady-state response is assured by the asymptotic stability of (25) (consequence of $\rho_i < 1$). The problem we now consider is: for a given $\tilde{u}_{i-1,k} \in [u^-_{i-1,k}, u^+_{i-1,k}]$ it is required to find the corresponding constant control input $\tilde{u}_{i,k}$ over each $H_{i,k}$, such that S_i fully satisfies the demand coming from S_{i-1}, namely $\tilde{y}_{i,k} \geq \tilde{u}_{i-1,k}$, $\forall \rho_i \in [\rho^-_i, \rho^+_i]$.

Using classical z-transform methods and applying the final value theorem[50] we have

$$\tilde{y}_{i,k} = [W_{u_i y_i}(z)]_{z=1}\tilde{u}_{i,k} - [W_{u_{i-1} y_i}(z)]_{z=1}\tilde{u}_{i-1,k}, \tag{26}$$

where $W_{u_i y_i}(z) = \frac{\rho_i}{z^{L_i}(z-\rho_i)}$ is the transfer function between the $\mathcal{Z}$ transforms of $u_i(k)$ and $y_i(k)$, $k \in Z^+$, and $W_{u_{i-1} y_i}(z) = \frac{\rho_i}{(z-\rho_i)}$ is the transfer function between the $\mathcal{Z}$ transforms of $u_{i-1}(k)$ and $y_i(k)$.

If ρ_i were exactly known, then, choosing $\tilde{u}_{i,k} = \frac{\tilde{u}_{i-1,k}}{\rho_i}$, equation (26) would readily imply $\tilde{y}_{i,k} = \tilde{u}_{i-1,k}, \forall \tilde{u}_{i-1,k} \in [u^-_{i-1,k}, u^+_{i-1,k}]$. As ρ_i is uncertain, the minimum $\tilde{u}_{i,k}$ that guarantees $\tilde{y}_{i,k} \geq \tilde{u}_{i-1,k}, \forall \rho_i \in [\rho^-_i, \rho^+_i]$ is $\tilde{u}_{i,k} = \frac{\tilde{u}_{i-1,k}}{\rho^-_i}$.

In conclusion, over each $H_{i,k}$ we choose $u^-_{i,k}$ according to the limit scenario 1: $\tilde{u}_{i-1,k} = u^-_{i-1,k}$ and $u^+_{i,k}$ according to the limit scenario 2: $\tilde{u}_{i-1,k} = u^+_{i-1,k}$. Hence, from (14), we obtain

$$u_i(k+j|k) \in C_{i,k} = [u^-_{i,k} \;\; u^+_{i,k}] \triangleq \frac{1}{\rho^-_i}\left[u^-_{i-1,k} \;\; u^+_{i-1,k}\right], \quad j = 0, 1, \ldots, N_i - 1, \tag{27}$$

with $C_{1,k} \triangleq [u^-_{1,k} \;\; u^+_{1,k}] \triangleq \frac{1}{\rho^-_1}\left[u^-_{0,k} \;\; u^+_{0,k}\right] = \frac{1}{\rho^-_1}[d^-_{1,k} \;\; d^+_{1,k}]$. The limits $d^-_{1,k}$ and $d^+_{1,k}$ are the minimum and maximum values respectively of the end customer demand over the time interval $[k + L_1 + 1, k + M_1] \subseteq P_{1,k}$, namely over the subset of the prediction interval $P_{1,k}$ where the corresponding predicted tracking error (15) is defined. Recalling that $C_{i-1,k} \triangleq [u^-_{i-1,k} \;\; u^+_{i-1,k}]$ and $A_{i-1,k}$ denotes the amplitude of $C_{i-1,k}$, from (27) we derive

$$A_{i,k} = \frac{1}{\rho^-_i}A_{i-1,k}, \quad \text{with} \quad A_{1,k} \triangleq \frac{1}{\rho^-_1}(d^+_{1,k} - d^-_{1,k}). \tag{28}$$

To quantify the BE at node $\mathcal{S}_i$ according to CR3 we introduce the following measure:

$$\mathcal{B}_{i,k} = \frac{A_{i,k}}{A_{i-1,k}}, \tag{29}$$

where $\mathcal{B}_{i,k} > 1$ ($\mathcal{B}_{i,k} < 1$) indicates the amplification (attenuation) of the amplitude of range $C_{i,k}$ with respect to that of range $C_{i-1,k}$. According to (27), the proposed DRRMPC scheme implies

$$\mathcal{B}_{i,k} = 1/\rho^-_i \triangleq \mathcal{B}_i > 1. \tag{30}$$

The two salient conclusions are:

(1) an estimate of the overall BE (corresponding to CR3) which propagates along the SC network can be computed "a priori"

$$\mathcal{B} = \frac{1}{\prod_{i=1}^{n}\rho^-_i}, \tag{31}$$

(2) by (30) it is evident that the interval amplification disappears for $\rho^-_i \to 1$.

4.3 | The resilient robust RP

To satisfy the further requirement of resilience with respect to unpredicted anomalous patterns of the end customer demand, we introduce the following agile adjustment mechanism of the set $D_{1,k}$. Assume that, for some $\bar{k}$, an unforeseen value $d_1(\bar{k})$ violating A1 is detected (namely $d_1(\bar{k}) \notin D_{1,\bar{k}}$ as shown in Figure 4 (where $\bar{k} = 3$), then we assume that $d_1(\bar{k}+j), j = 0, 1, \cdots M_1$, belongs to a new set $D_{1,\bar{k}}$ centered on $d_1(\bar{k})$ and given by the parallel translation of the current $D_{1,k}$. The amount of translation is $d_1(\bar{k}) - d_1(\bar{k}|\bar{k})$ (upward translation if $d_1(\bar{k}) > d_1(\bar{k}|\bar{k})$, downward translation if $d_1(\bar{k}) < d_1(\bar{k}|\bar{k})$). In accordance with the new $D_{1,\bar{k}}$, also new $d^-_1(\bar{k}+\ell)$, and $d^+_1(\bar{k}+\ell)$, $d_1(\bar{k}+\ell|\bar{k})$, $\ell = 0, \ldots, M_1$, are defined. As a consequence, also the desired inventory level is updated to the new $d^+_1(\bar{k}+L_i+i)$, $i = 1, \ldots, N_i$.

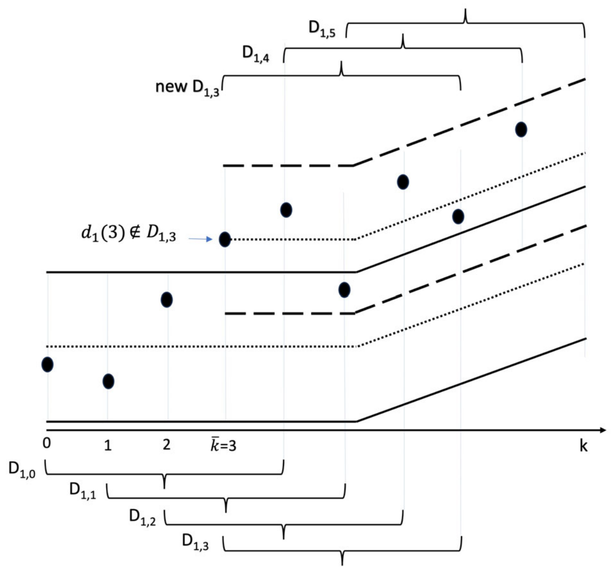

FIGURE 4 An example of violation of A1: $d_1(\bar{k}) \notin D_{1,\bar{k}}$ with $\bar{k} = 3$. The sequence of new sets $D_{1,k}$, $k \geq 3$ (delimited by the dashed trajectories) are obtained by an upward parallel translation of the previously defined $D_{1,k}$ falsified by the measured $d_1(\bar{k})$.

Owing to the receding horizon nature of the MPC, the RP issued by S_1 is promptly redesigned according to this new situation. As a consequence, also the RPs of all the upstream echelons are automatically updated. The remarkable advantage of this procedure is the possibility of fulfilling A1 without resorting to a very conservative estimate of $D_{1,k}$, $k \in Z^+$ that, in turn, would imply overordering and overstocking. This is all the more harmful the higher the perishability rate and the number of SC stages.

Remark 4. We remark the difficulty of obtaining an analogous agile flexibility using demand forecasting methods based on time series analysis: even using adaptive identification algorithms, the intrinsic inertia of ARMA models slows down the process of adjusting the parameter estimates according to the incoming measures of customer demand.

5 | REFORMULATION OF THE LOCAL MMCOP

In this section, we reformulate the local MMCOP as a Constrained Robust LS estimation problem. The purpose is to drastically reduce the numerical complexity of the algorithm solving the MMCOP.

For any fixed k, the predicted optimal control sequence $U_{i,k} = [u_i(k|k), \dots, u_i(k + N_i - 1|k)]$, solving the MMCOP (13)-(14), is given by the sampled version (with sampling period coinciding with the review period T) of a B-spline function. Adapting the notation in (3) to specify that it is relative to the ith node and the kth fixed time instant we have

$$u_i(j|k) \triangleq \mathbf{B}_{i,d}(j)\mathbf{c}_{i,k}, \quad j \in [\hat{k}_1,\ \hat{k}_{\ell+d+1}] \tag{32}$$

with

1. $\mathbf{B}_{i,d}(j) \triangleq [B_{i,1,d}(j), \dots, B_{i,\ell,d}(j)]$,
2. $\mathbf{c}_{i,k} \triangleq [\mathbf{c}_{i,k,1}, \dots, \mathbf{c}_{i,k,\ell}]^T$,
3. $\hat{k}_1 = \cdots = \hat{k}_{d+1} = k$ and $\hat{k}_{\ell+1} = \cdots = \hat{k}_{\ell+d+1} = k + N_i - 1$,
4. the remaining $\ell - d - 1$ knot points are evenly distributed over $(k, k + N_i - 1)$.

Remark 5. Point 3 and the smoothness property of B splines (recalled in Section 2.1) imply that the first sample $u_i(k|k)$ of the B spline $u_i(j|k)$ coincides with the first control point $c_{i,k,1}$ of the vector $c_{i,k}$.

The parameter vector $\mathbf{c}_{i,k}$ defining $u_i(j|k), j = k, \ldots, k + N_i - 1$, is computed as the solution of the Constrained Robust LS estimation problem defined beneath.

As $\rho_i \in [\rho_i^- \rho_i^+]$, an equivalent representation of ρ_i is

$$\rho_i = \overline{\rho}_i + \delta\rho_i, \qquad \overline{\rho}_i \triangleq (\rho_i^- + \rho_i^+)/2, \tag{33}$$

where $\overline{\rho}_i$ is the nominal value and $\delta\rho_i$ is the perturbation with respect to $\overline{\rho}_i$ satisfying $|\delta\rho_i| \leq (\rho_i^+ - \rho_i^-)/2$.

From (33) it follows that

$$\rho_i^k = (\overline{\rho}_i + \delta\rho_i)^k = \overline{\rho}_i^k + \Delta\rho_{i,k}, \tag{34}$$

where $\Delta\rho_{i,k} \triangleq (\overline{\rho}_i + \delta\rho_i)^k - \overline{\rho}_i^k$ is the sum of all terms containing $\delta\rho_i$ in the explicit expression of $(\overline{\rho}_i + \delta\rho_i)^k$. Exploiting (34) one has that the term $\rho_i^{L_i+l} y_i(k)$ of (17) can be rewritten as

$$\rho_i^{L_i+l} y_i(k) = (\overline{\rho}_i^{L_i+l} + \Delta\rho_{i,L_i+l}) y_i(k). \tag{35}$$

Analogously, the following terms of (17) can be rewritten as

$$\sum_{\ell=0}^{L_i-1} \rho_i^{L_i+l-\ell} u_i(k+\ell-L_i) = \sum_{\ell=0}^{L_i-1} (\overline{\rho}_i^{L_i+l-\ell} + \Delta\rho_{i,L_i+l-\ell}) u_i(k+\ell-L_i), \tag{36}$$

$$\sum_{\ell=0}^{L_i-1} \rho_i^{L_i+l-\ell} z_i(k+\ell-L_i) = \sum_{\ell=0}^{L_i-1} (\overline{\rho}_i^{L_i+l-\ell} + \Delta\rho_{i,L_i+l-\ell}) z_i(k+\ell-L_i), \tag{37}$$

$$\sum_{\ell=0}^{l-1} \rho_i^{l-\ell} u_i(k+\ell|k) = \sum_{\ell=0}^{l-1} (\overline{\rho}_i^{l-\ell} + \Delta\rho_{i,l-\ell}) \mathbf{B}_{i,d}(k+\ell) \mathbf{c}_{i,k}, \tag{38}$$

$$\sum_{\ell=0}^{L_i+l-1} \rho_i^{L_i+l-\ell} z_{i-1}(k+\ell|k) = (\overline{\rho}_i^{L_i+l} + \Delta\rho_{i,L_i+l}) z_{i-1}(k|k) + \sum_{\ell=1}^{L_i+l-1} \rho_i^{L_i+l-\ell} z_{i-1}(k+\ell|k), \tag{39}$$

and

$$\sum_{\ell=0}^{L_i+l-1} \rho_i^{L_i+l-\ell} u_{i-1}(k+\ell|k) = \sum_{\ell=0}^{L_i+l-1} (\overline{\rho}_i^{L_i+l-\ell} + \Delta\rho_{i,L_i+l-\ell}) u_{i-1}(k+\ell|k). \tag{40}$$

By (35)–(40), an equivalent representation of the predicted tracking error given by (15) is

$$e_i(k+L_i+l|k) = (b_{i,k,l} + \delta b_{i,k,l}) - (D_{i,k,l} + \delta D_{i,k,l}) \mathbf{c}_{i,k}, \tag{41}$$

where

$$\begin{aligned} b_{i,k,l} \triangleq\ & r_i(k+L_i+l|k) - \overline{\rho}_i^{L_i+l} y_i(k) - \sum_{\ell=0}^{L_i-1} \overline{\rho}_i^{L_i+l-\ell} u_i(k+\ell-L_i) \\ & + \sum_{\ell=0}^{L_i-1} \overline{\rho}_i^{L_i+l-\ell} z_i(k+\ell-L_i) + \sum_{\ell=0}^{L_i+l-1} \overline{\rho}_i^{L_i+l-\ell} u_{i-1}(k+\ell|k) - \overline{\rho}_i^{L_i+l} z_{i-1}(k|k), \end{aligned} \tag{42}$$

$$\begin{aligned} \delta b_{i,k,l} \triangleq\ & -\Delta\rho_{i,L_i+l} y_i(k) - \sum_{\ell=0}^{L_i-1} \Delta\rho_{i,L_i+l-\ell} u_i(k+\ell-L_i) + \sum_{\ell=0}^{L_i-1} \Delta\rho_{i,L_i+l-\ell} z_i(k+\ell-L_i) \\ & + \sum_{\ell=0}^{L_i+l-1} \Delta\rho_{i,L_i+l-\ell} u_{i-1}(k+\ell|k) + \sum_{\ell=0}^{l-1} \rho_i^{l-\ell} z_i(k+\ell|k) \\ & - \Delta\rho_{i,L_i+l} z_{i-1}(k|k) - \sum_{\ell=1}^{L_i+l-1} \rho_i^{L_i+l-\ell} z_{i-1}(k+\ell|k), \end{aligned} \tag{43}$$

$$D_{i,k,l} \triangleq \sum_{\ell=0}^{l-1} \bar{\rho}_i^{l-\ell} \mathbf{B}_{i,d}(k+\ell), \tag{44}$$

$$\delta D_{i,k,l} \triangleq \sum_{\ell=0}^{l-1} \Delta \rho_{i,l-\ell} \mathbf{B}_{i,d}(k+\ell). \tag{45}$$

Similarly, recalling that $u_i(k|k) = c_{i,k,1}$ (see Remark 5), also the term $\Delta u_i(k|k) = u_i(k|k) - u_i(k-1)$ in (13) can be rewritten in the following way:

$$\Delta u_i(k|k) = b_{u_{i,k}} - D_{u_{i,k}} \mathbf{c}_{i,k},$$

where $b_{u_{i,k}} = -u_i(k-1)$ and $D_{u_{i,k}} = -\begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}$.

Define the following vectors and matrices

$$\underline{e}_{i,k} = \begin{bmatrix} q_{i,1}^{1/2}(k) e_i(k+L_i+1|k) \\ \vdots \\ q_{i,N_i}^{1/2}(k) e_i(k+L_i+N_i-1|k) \\ \lambda_i^{1/2}(k) \Delta u_i(k|k) \end{bmatrix}, \underline{D}_{i,k} = \begin{bmatrix} q_{i,1}^{1/2}(k) D_{i,k,1} \\ \vdots \\ q_{i,N_i}^{1/2}(k) D_{i,k,N_i-1} \\ \lambda_i^{1/2}(k) D_{u_{i,k}} \end{bmatrix} \tag{46}$$

$$\underline{b}_{i,k} = \begin{bmatrix} q_{i,1}^{1/2}(k) b_{i,k,1} \\ \vdots \\ q_{i,N_i}^{1/2}(k) b_{i,k,N_i} \\ \lambda_i^{1/2}(k) b_{u_{i,k}} \end{bmatrix}, \underline{\delta b}_{i,k} = \begin{bmatrix} q_{i,1}^{1/2}(k) \delta b_{i,k,1} \\ \vdots \\ q_{i,N_i}^{1/2}(k) \delta b_{i,k,N_i} \\ 0 \end{bmatrix} \tag{47}$$

$$\underline{\delta D}_{i,k} = \begin{bmatrix} q_{i,1}^{1/2}(k) \delta D_{i,k,1} \\ \vdots \\ q_{i,N_i}^{1/2}(k) \delta D_{i,k,N_i} \\ 0 \end{bmatrix}. \tag{48}$$

Exploiting the above defined vectors and matrices, we reformulate the local MMCOP (13)-(14) as the following local Constrained Robust LS estimation problem:

$$\min_{\mathbf{c}_{i,k}} \max_{\|\underline{\delta D}_{i,k}\| \le \beta_{i,k} \; \|\underline{\delta b}_{i,k}\| \le \xi_{i,k}} \|(\underline{b}_{i,k} + \delta b_{i,k}) - (\underline{D}_{i,k} + \underline{\delta D}_{i,k}) \mathbf{c}_{i,k}\|^2 \tag{49}$$

$$\text{subject to } \; u_{i,k}^- \le \mathbf{c}_{i,k,r} \le u_{i,k}^+, r = 1, \dots, \ell. \tag{50}$$

Constraints (50) derive from (32) and the convex hull property of B splines.

By (5) and recalling that

$$\arg\min_x \left(\sum_i \|f_i(x)\| \right) = \arg\min_x \left(\sum_i \|f_i(x)\| \right)^2$$

it is seen that (49), (50) define a problem of the kind (4), (7). Hence, according to Section 2.2, at any k the local Constrained Robust LS estimation problem (49)–(50) can be reformulated as

$$\min_{\mathbf{c}_{i,k}} \|\underline{b}_{i,k} - \underline{D}_{i,k} \, \mathbf{c}_{i,k}\| + \beta_{i,k} \|\mathbf{c}_{i,k}\| + \xi_{i,k}, \tag{51}$$

where the components of $\mathbf{c}_{i,k}$ must satisfy (50).

As for the numerical calculation of $\beta_{i,k}$ and $\xi_{i,k}$, the following considerations hold:

1. Equations (42)–(48) evidence that the uncertain future samples $z_i(k+\ell|k)$ and $z_{i-1}(k+\ell|k)$ are only contained in the terms $\delta b_{i,k,l}$ given by (43) and collected in $\underline{\delta b}_{i,k}$ given by (47). Recalling Remark 2, it follows that the unpredictability affecting these samples does not affect the value of the $\mathbf{c}_{i,k}$ solving (51) because $\xi_{i,k}$ (the upper bound on $\|\underline{\delta b}_{i,k}\|$) of (51) is independent of $\mathbf{c}_{i,k}$. The two important consequences are (1) the local MMCOP defined in Section 4.1 can be solved though the uncertainty on $z_i(k+\ell|k)$ and $z_{i-1}(k+\ell|k)$, (2) in (51) only the upper bound $\beta_{i,k}$ on $\|\underline{\delta D}_{i,k}\|$ needs to be determined at each k.
2. The way the B-spline basis functions are defined by the Cox de Boor formula (2) implies that $\mathbf{B}_{i,d}(\tau)=\mathbf{B}_{i,d}(\tau+N_i), \forall \tau \in H_{i,k}$, $k \in Z^+$. Hence, by (45) one has that $\delta D_{i,k,l}$ is independent of k and by (48) the scalars $\beta_{i,k}$, $k \in Z^+$, are determined putting $\rho_i=\rho_i^+$.

The proposed DRRMPC strategy is based on the solution of a sets of MMCOPs whose feasibility and stability properties are stated in the following theorem.

Theorem 1. *The proposed DRRMPC strategy guarantees the feasibility of each local MMCOP and the uniform boundedness of all physical variables $u_i(k)$ and $y_i(k)$, independently of the lengths M_i, $i=1,\dots,n$ of the prediction intervals.*

Proof. The feasibility of the MMCOP solved by each local $\mathcal{A}_i$, $i=1,\dots,n$, is a consequence of parametrizing $u_i(j|k)$ as in (32). In fact the ℓ-components vector $\mathbf{c}_{i,k}$ solves the equivalent Constrained Robust LS estimation problem and, simultaneously, satisfies (50). The uniform boundedness of $u_i(k)$ derives from (27), and the uniform boundedness of $y_i(k)$ derives from:

1. the internal stability of each $\mathcal{S}_i$ due to $\rho_i<1$, $i=1,\dots,n$,
2. the assumed uniform boundedness of the customer demand,
3. the uniform boundedness of $u_i(k)$ and $h_i(k)$ (this latter consequence of (9)), $i=1,\dots,n$.

■

Remark 6. Owing to the uncertainty on the future customer demand and on the decay factor, large values of M_i would lead to unreliable predictions and, as a consequence, to a poor control performance. Theorem 1 overcomes this inconvenience: the lengths M_i are only imposed by the constraints (20) and (21) due to the structure of the MSSC and to the coordination between consecutive agents. This avoids unnecessarily longer prediction intervals.

5.1 | A schematic step by step summary of the whole procedure

In brief, the method to define the resilient robust RP can be schematically summarized in the sequential execution of the following steps:

1. set $k=0$, (k is the time instant),
2. set $i=1$, ($i=1,\dots,n$, denotes the ith stage),
3. define the tube $D_{1,k}$ containing the foreseen end-customer demand $d_1(k+j|k), j=1,\dots,M_1$, and assume the central trajectory as the most probable future demand. If necessary apply the procedure of Section 4.3,
4. if $i>1$ define the tube $D_{i,k}$ (as explained in A2, Section 3) and assume as predicted demand $\hat{D}_{i,k}$ the predicted optimal control policy $U_{i-1,k}$ coming from $\mathcal{A}_{i-1}$ (see Figure (3)),
5. compute the bounds $u^-_{i,k}$ and $u^+_{i,k}$ limiting the predicted control sequence $U_{i,k}=[u_i(k|k),\dots,u_i(k+N_i-1|k)]$,
6. define the degree d and number ℓ of control points of the B-spline in (32) parametrizing $U_{i,k}$,
7. solve the minimization problem (51) with respect to $\mathbf{c}_{i,k}$ and compute $U_{i,k}$ through (32),
8. if $i=n$ place $u_n(k|k)$ to the manufacturer else place $u_i(k|k)$ to the $(i+1)$ th stage and communicate $U_{i,k}$, $u^-_{i,k}$ and $u^+_{i,k}$ to $\mathcal{A}_{i+1}$,
9. put $i=i+1$,
10. if $i\le n$ go to Step 4 else put $k=k+1$ and go to Step 2.

6 | NUMERICAL RESULTS

In this simulation we consider an MSSC composed of $n = 3$ stages $\mathcal{S}_i$, $i = 1, \dots, 3$. We apply the DRRMPC to define a resilient robust RP based on the adjustment mechanism described in Section 4.3 and compare the results obtained with those ones of an RP based on a conservative estimate of $D_{1,k}$, $k \in Z^+$, without any adjustment mechanism. Then we carry out a detailed comparison between the DRRMPC and the nonlinear control policy proposed in Reference 28. The method described in Reference 28 accounts for the effects of time delay and perishable goods and all details for its reproducibility are provided. Finally we propose a comparison with the decentralized and centralized implementations of our approach. Unlike Figures 2–4, all the diagrams reported in this section are plotted with continuous curves.

The numerical simulations have been implemented in MATLAB R2018 (9.5.0) on a MacBook Pro (retina) 2,2 GHz Intel Core i7 quad-core, 16GB 1600 MHz DDR3. The program is available on request.

6.1 | Performance indices

We define four performance indices. The first one measures the normalized amount of Unsatisfied Demand (UD) at each stage and is defined as

$$UD_i \triangleq \frac{1}{\sum_{k=0}^{N_s} d_i(k)} \sum_{k=0}^{N_s} |d_i(k) - h_i(k)| \in [0\ 1], \quad i = 1, 2, 3,$$

where N_s is the length of the simulation. The smaller UD_i, the greater the demand satisfied by the ith stage.

The second performance indicator is the total Inventory Level (IL) in each stage measured as the amount of goods left in stock after satisfying the demand at each $k = 0, 1 \dots, N_s$. In accordance with (8), it is given by

$$IL_i \triangleq \sum_{k=0}^{N_s} y_i(k), \quad i = 1, 2, 3.$$

The third performance indicator measures the total amount of Issued Orders (IO) by each stage:

$$IO_i \triangleq \sum_{k=0}^{N_s} u_i(k), \quad i = 1, 2, 3.$$

The fourth performance indicator measures the actual amount of Wasted Goods (WG) in each stage due to perishability. In accordance with (8), it is computed as

$$WG_i \triangleq \sum_{k=0}^{N_s} \frac{1 - \rho_i}{\rho_i} y_i(k), \quad i = 1, 2, 3.$$

6.2 | Initialization

To simplify notation (but without any loss of generality) we assume that the equations describing the stock level dynamics of each node $\mathcal{S}_i$, $i = 1, 2, 3$, are characterized by the same time delay L_i, perishability rate α_i, decay factor $\rho_i = 1 - \alpha_i$ and initial stock $y_i(0) = 0$. The model parameters of each $\mathcal{S}_i$ are reported in Table 3.

TABLE 3 Parameters of each stage $\mathcal{S}_i$, $i = 1, 2, 3$.

Time delay	Perishability rate	Decay factor	Initial state
$L_i = 3$	$\alpha_i \in [\alpha_i^-, \alpha_i^+] = [0.1,\ 0.14]$	$\rho_i \in [\rho_i^-, \rho_i^+] = [0.86,\ 0.9]$	$y_i(0) = 0$

TABLE 4 Tuning parameters of the local MMCOP for any $\mathcal{A}_i$, $i = 1, 2, 3$.

$\varepsilon_{e,i}$ *in* (23)	$\varepsilon_{u,i}$ *in* (24)	ζ_i *in* (23)
$\varepsilon_{e,i} = \varepsilon_e$	$\varepsilon_{u,i} = \varepsilon_u$	$\zeta_i = \zeta$
0.004	0.001	0.368

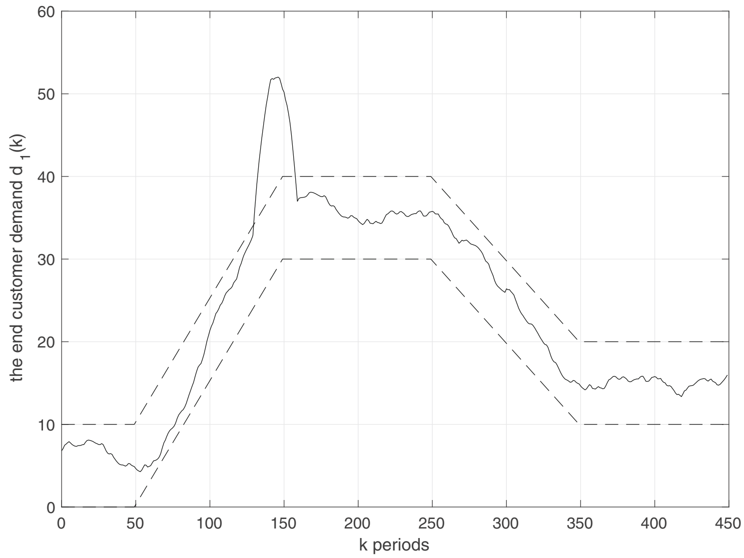

FIGURE 5 The actual end customer demand $d_1(k)$ (solid line) with an unexpected behavior. The dashed lines represent the consecutive contiguous overlapping of all the “a priori” given sets $D_{1,k}$’s.

At each $k \in Z^+$, the local MMCOP for any agent $\mathcal{A}_i$, $i = 1, 2, 3$ is solved parametrizing $U_{i,k} = [u_i(k|k), \ldots, u_i(k + N_i - 1|k)]$ as a B spline function of degree $d = 3$ with $\ell = 8$ control points. The length N_i of each control horizon $H_{i,k}$, $i = 1, 2, 3$ is computed according to (21) starting from $N_3 = 16$. This gives $N_2 = 20, N_1 = 24$ and hence by (20) we derive $M_1 \geq 27$. The other tuning parameters are: the percentages $\varepsilon_{e,i}$ in (23), $\varepsilon_{u,i}$ in (24) and the forgetting factor ζ_i in (23) are given in Table 4.

6.2.1 | The end customer demand

According to A1, at any $k \in Z^+$, the future end customer demand is known to belong to a given compact set $D_{1,k}$. We assume $M_1 = 27$. Figure 5 shows the actual demand $d_1(k)$ characterized by an unexpected behavior: at $k = \overline{k} = 130$, the actual $d_1(\overline{k})$ is not confined inside the consecutive contiguous overlapping of all the "a priori" given sets $D_{1,k}$, $k \in [130 - M_1,\ 158] = [103,\ 158]$. To achieve resilience we apply the adjustment mechanism of Section 4.3. Figure 6 shows the new compact set D_1 given by the consecutive contiguous positioning of all redetermined sets $D_{1,k}$’s so as to enclose the whole actual end customer demand.

With reference to the same profile of customer demand and refraining from applying the proposed adjustment mechanism, the "a priori" fulfillment of A1 at any $k \in Z^+$ can be assured by a conservative estimate of the all sets $D_{1,k}$, $k \in Z^+$ enclosing the whole $d_1(k)$, $k \in Z^+$. An example is shown in Figure 7.

In the next section we present the results obtained applying the robust resilient RP endowed with the adjustment mechanism (we refer to this simulation with Sim1) and compare them with those obtained through the robust RP not endowed with the resilience property (we refer to this simulation with Sim2).

6.3 | Simulation results

The dynamic Equation (8) of each $\mathcal{S}_i$ has been implemented assuming $\rho_i = 0.885$, $i = 1, 2, 3$. Both Sim1 and Sim2 have been stopped at time $k = N_s = 400$. The generated orders $u_i(k)$, $i = 1, 2, 3$ in both simulations are displayed in Figures 8 and

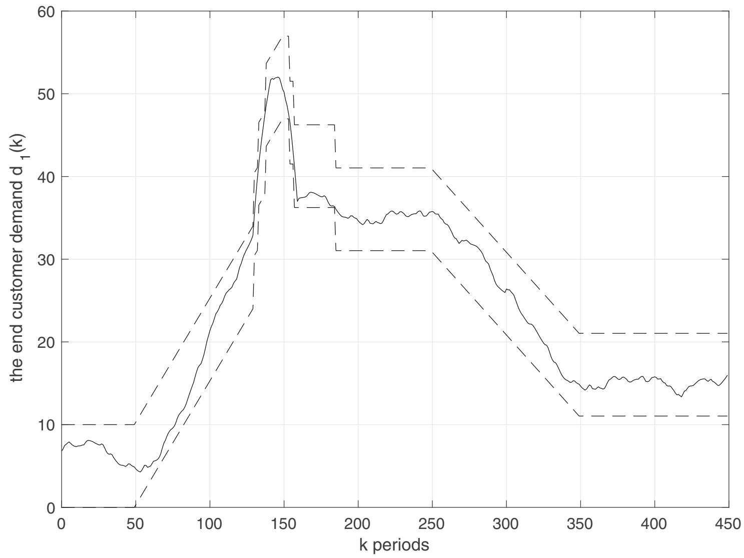

FIGURE 6 Robust Resilient RP (Sim1): the actual end customer demand $d_1(k)$ (solid line) enclosed in the new compact set D_1 delimited by the new upper $d_1^+(k)$ and the new lower d_1^- boundaries (dashed lines).

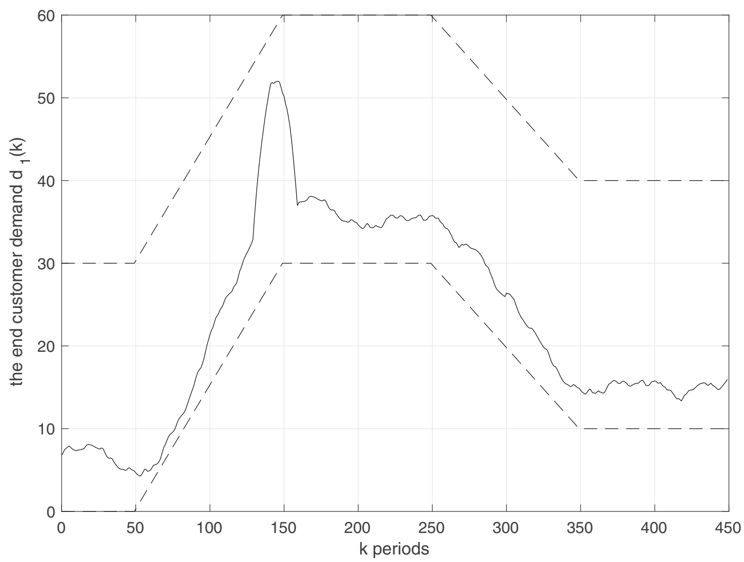

FIGURE 7 Robust RP (Sim2): the actual end customer demand $d_1(k)$ (solid line) enclosed in the new compact set D_1 delimited by the upper $d_1^+(k)$ and the lower $d_1^-(k)$ boundaries (dashed lines) computed in a conservative way.

9 respectively. These figures show the ordering signal issued by each stage $\mathcal{S}_i$, $i = 1, 2, 3$ with the respective time-varying lower and upper bounds. It is seen that the amplitude $A_{i,k}$ of the the corresponding intervals $C_{i,k}$ grows as in (28). The resulting inventory level $y_i(k)$ and the time varying desired inventory level $r_i(k)$ for each $\mathcal{S}_i$, $i = 1, 2, 3$ in both simulations are reported in Figures 10 and 11 respectively. Analogously, the imposed and fulfilled demands $d_i(k)$ and $h_i(k)$ at each $\mathcal{S}_i$, $i = 1, 2, 3$ are displayed in Figures 12 and 13 respectively.

The performance evaluation in both simulations is performed on the basis of the indicators defined in Section 6.1. The results are summarized in Table 5. It evidences that the amount of unsatisfied demand is comparable, but the resilient RP yields much smaller values of: warehouse occupancy, wasted goods, issued orders. This means a significant decrease in warehouse costs and a reduction in profit losses.

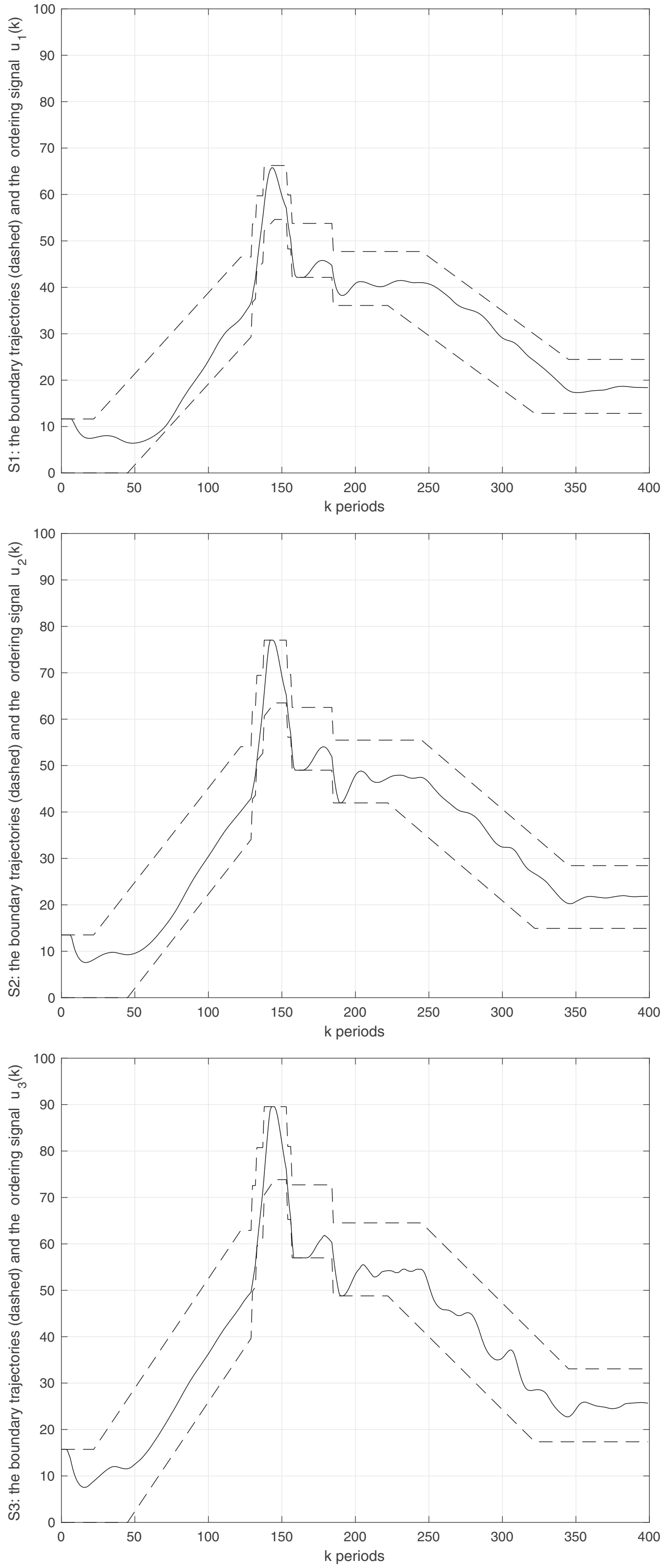

FIGURE 8 Robust Resilient RP (Sim1): The ordering signal $u_i(k)$ (solid line) issued by each $\mathcal{S}_i$, $i = 1, 2, 3$, with the respective boundary trajectories (dashed line) computed by (27).

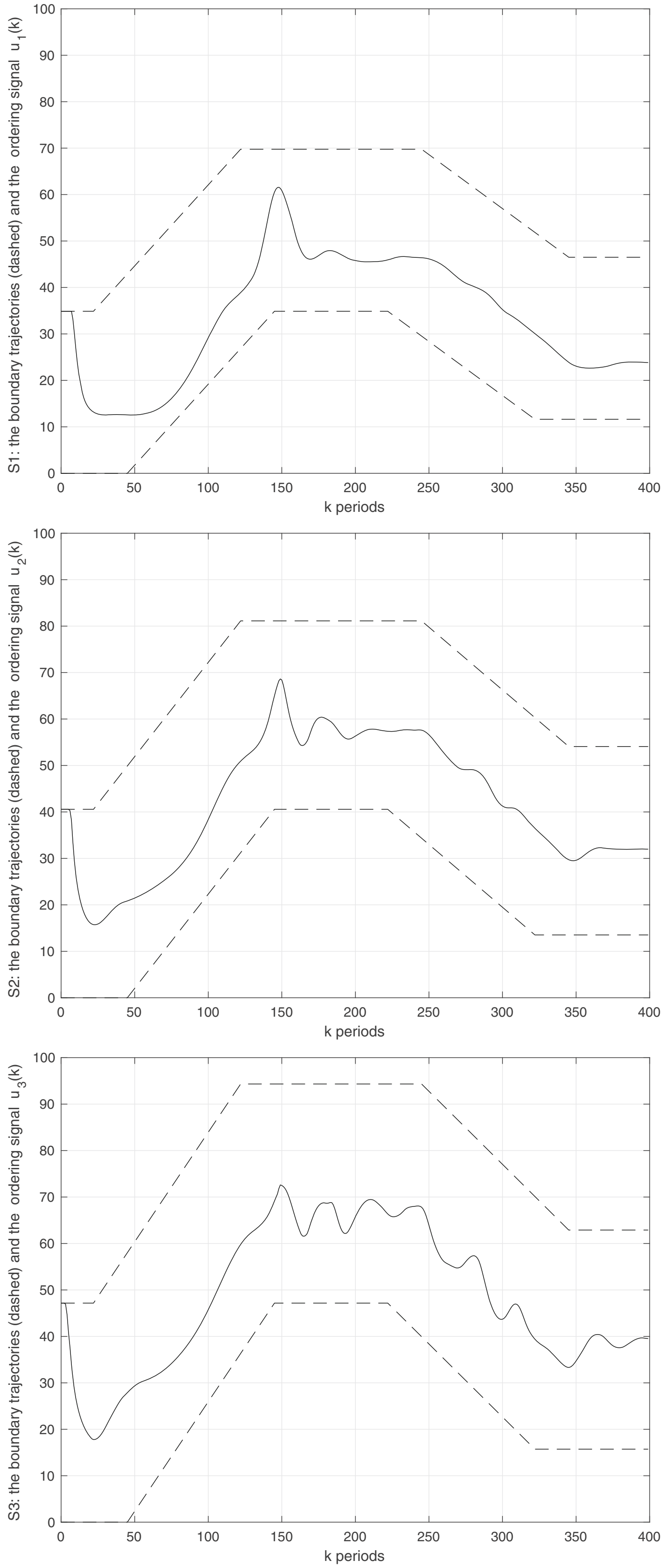

FIGURE 9 Robust RP (Sim2): The ordering signal $u_i(k) = u_i(k|k)$ (solid line) issued by each S_i, $i = 1, 2, 3$, with the respective boundary trajectories $u^-_{i,k}$ and $u^+_{i,k}$ (dashed line) computed by (27).

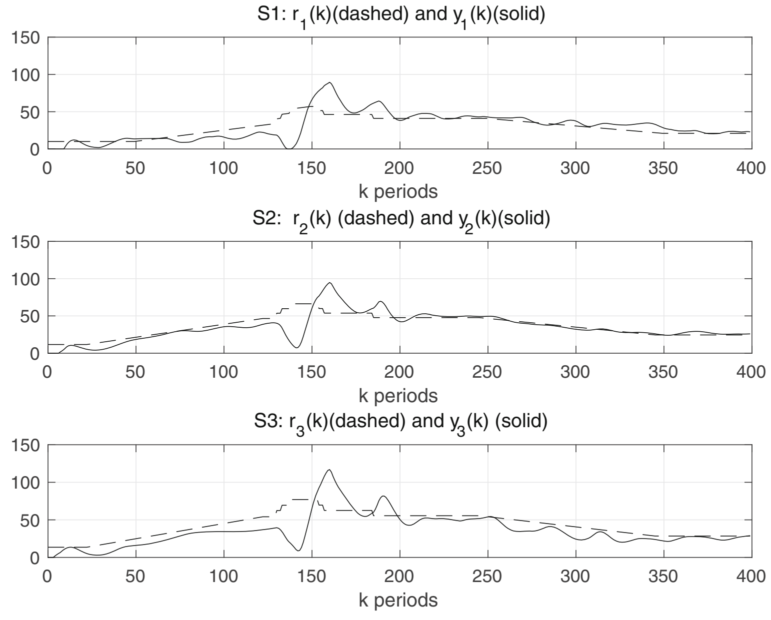

FIGURE 10 Robust Resilient RP (Sim1): the desired time varying inventory level $r_i(k)$ (dashed line) and the on hand stock level $y_i(k)$ (solid line) of each node $\mathcal{S}_i$, $i = 1, 2, 3$.

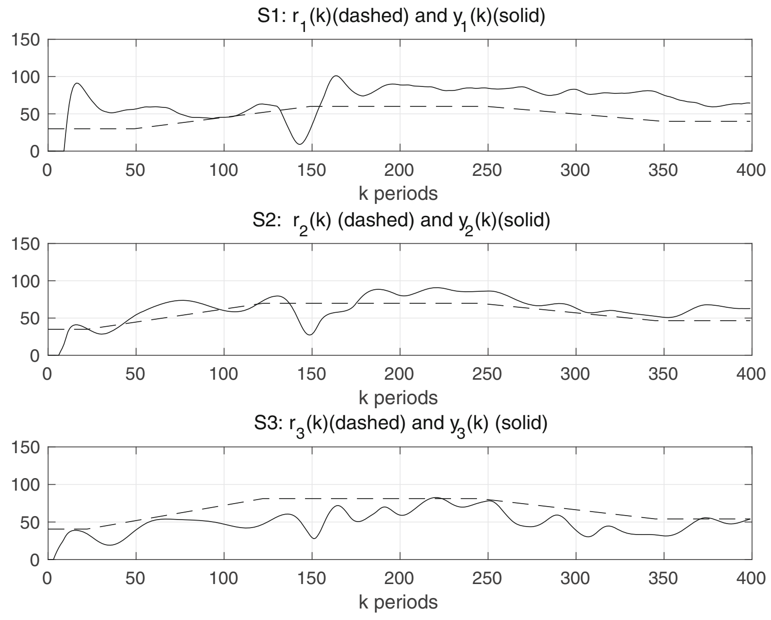

FIGURE 11 Robust RP (Sim2): the desired time varying inventory level $r_i(k)$ (dashed line) and the on hand stock level $y_i(k)$ (solid line) of each node $\mathcal{S}_i$, $i = 1, 2, 3$.

6.3.1 | Comparison with the nonlinear control strategy

Equations (34)–(36) in Reference 28 have been rewritten in the case of an uncertain n stage SC with $n = 3$, decay factors $\rho_i \in [0.86,\ 0.9]$ and known time delays $L_i = 3$, $i = 1, 2, 3$ obtaining

$$u_i(k) = \text{sat}[\omega_i(k)], \tag{52}$$

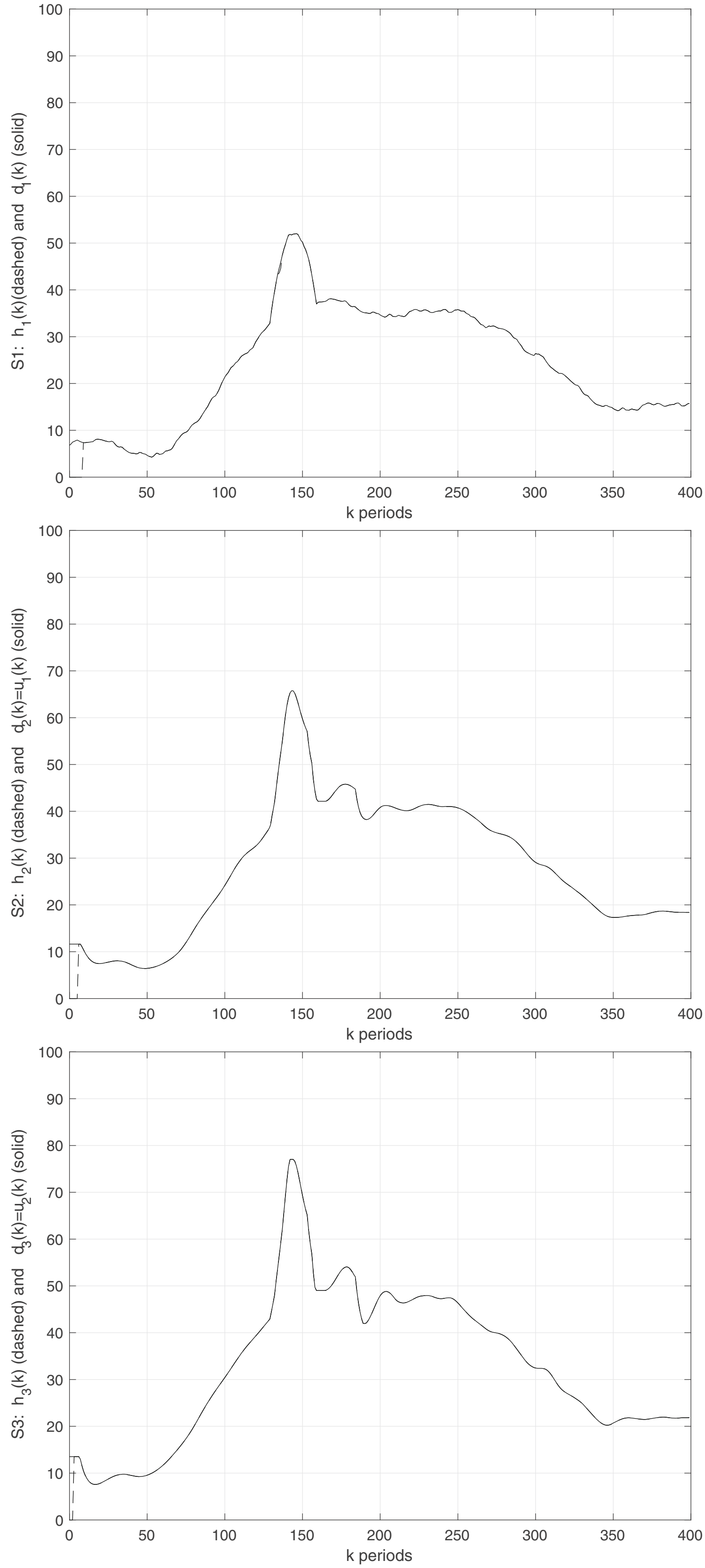

FIGURE 12 (Robust Resilient RP (Sim1): the imposed demand $d_i(k)$ (solid line) and the fulfilled demand $h_i(k)$ (dashed line) at each $\mathcal{S}_i$, $i = 1, 2, 3$.

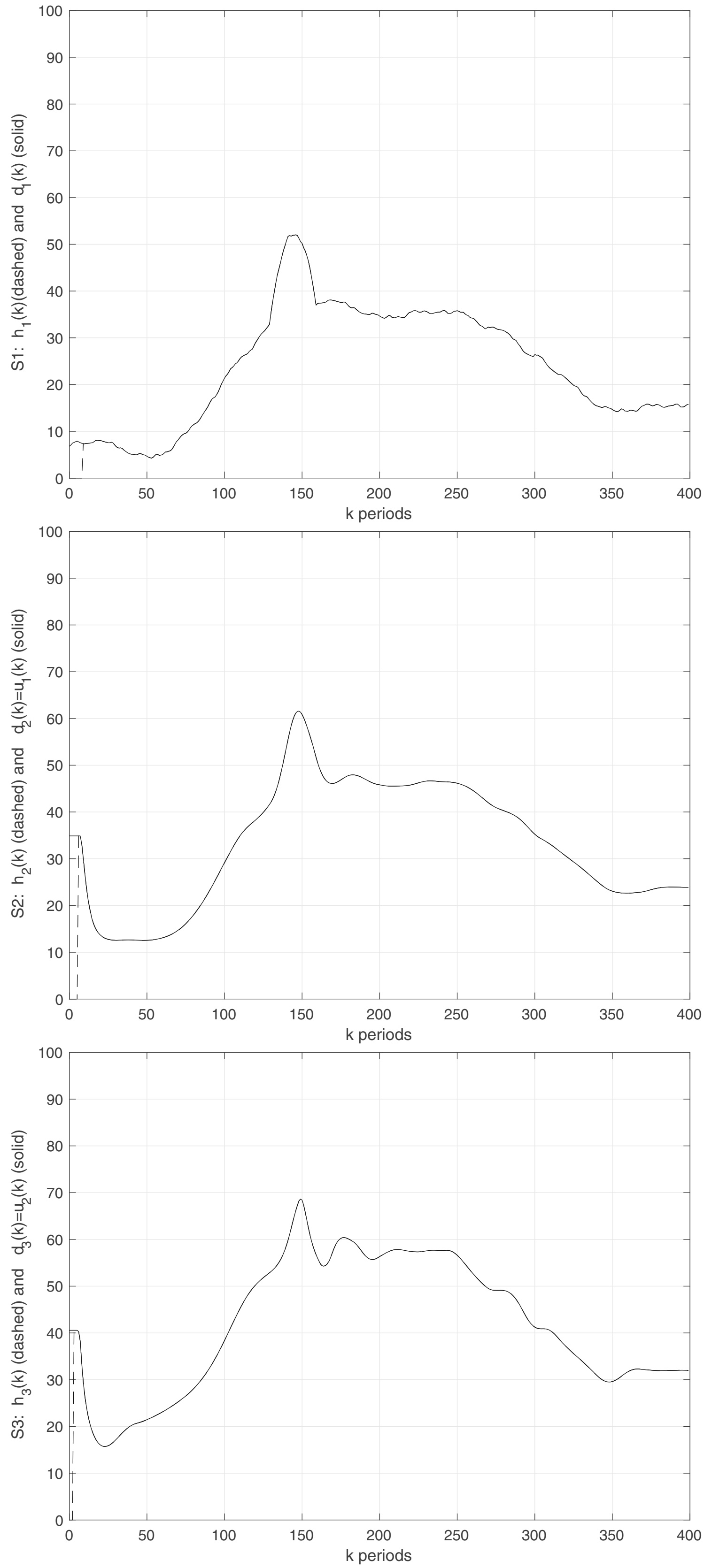

FIGURE 13 Robust RP (Sim2): the imposed demand $d_i(k)$ (solid line) and the fulfilled demand $h_i(k)$ (dashed line) at each $\mathcal{S}_i$, $i = 1, 2, 3$.

TABLE 5 Quantitative comparison between the robust RP equipped with the adjustment mechanism (Sim1) and that not equipped (Sim2).

	UD_1	UD_2	UD_3	IL_1	IL_2	IL_3	WG_1	WG_2	WG_3	IO_1	IO_2	IO_3
Robust resilient RP (Sim1)	0.007	0.0062	0.0031	1.19e04	1.39e04	1.41e04	1.54e03	1.80e03	1.84e03	1.12e04	1.31e04	1.50e04
Robust RP (Sim2)	0.007	0.0156	0.0073	2.68e04	2.52e04	1.96e04	3.48e03	3.28e03	2.55e03	1.33e04	1.67e04	1.93e04

TABLE 6 The performance analysis of MNLCS.

	UD_1	UD_2	UD_3	IL_1	IL_2	IL_3	WG_1	WG_2	WG_3	IO_1	IO_2	IO_3
MNCLS	0.007	0.0229	0.0097	4.47e04	2.86e04	1.85e04	5.81e03	3.72e03	2.40e03	1.60e04	1.97e04	2.22e04

where

$$\omega_i(k) = y_{ref,i} - \rho_i^{L_i} y_i(k) + \sum_{j=0}^{k-1} \rho_i^{k-j} s_i(j) - \sum_{j=0}^{k-L_i-1} \rho_i^{k-j} s_i(j), \tag{53}$$

and the saturation function

$$\mathrm{sat}[\omega_i(k)] = \begin{cases} \omega_i(k) & \text{if } \omega_i(k) \in [0, u_{\max,i}], \\ 0 & \text{if } \omega_i(k) < 0, \\ u_{\max,i} & \text{if } \omega_i(k) > u_{\max,i}. \end{cases} \tag{54}$$

According to (45), (46) in Reference 28 and taking into account that $\rho_i \in [\rho_i^-, \rho_i^+]$, $u_{\max,i}$ and $y_{ref,i}$ are inferiorly limited as:

$$u_{\max,i} > d_{\max,i} \quad \text{and} \quad y_{ref,i} > d_{\max,i} \sum_{j=0}^{L_i} \rho_i^{+j}. \tag{55}$$

The topology of the SC network shown in Figure 1 is such that:

$$d_{\max,1} = \max_k d_1^+(k) \quad d_{\max,2} = u_{\max,1} \quad d_{\max,3} = u_{\max,2}. \tag{56}$$

According to (55), (56) we fix: $u_{\max,1} = 61 > d_{\max,1} = 60$, $u_{\max,2} = 62 > d_{\max,2} = 61$, $u_{\max,3} = 63 > d_{\max,3} = 62$, $y_{ref,1} = 210 > 209.7$, $y_{ref,2} = 214 > 213.2$ and $y_{ref,3} = 217 > 216.6$.

We refer to the control strategy (52)–(54) as modified non-linear control strategy (MNLCS). The MNLCS has been applied putting $\rho_i = \overline{\rho}_i = 0.88$, while the model equation (8) has been implemented assuming $\rho_i = 0.885$, $i = 1, 2, 3$. The generated orders $u_i(k)$ and the resulting on hand stock level $y_i(k)$ are displayed in Figures 14 and 15 respectively. The imposed and fulfilled demands $d_i(k)$ and $h_i(k)$ respectively at each $\mathcal{S}_i$ are given in Figure 16.

The MNLCS has been evaluated with the same three quantitative indicators defined in Section 6.1. The results are summarized in Table 6. A comparison with those relating to Sim1 (see row 2 Table 5) we note that the amount of unsatisfied demand is comparable, but the RRDMPC requires a very smaller warehouse occupancy with respect to the nonlinear control strategy (52)–(54). The remarkable reduction of warehouse occupancy is a consequence of tracking a time varying inventory level which is updated at any k on the basis of the current value of the demand. On the contrary MNLCS defines a constant desired inventory level $y_{ref,i}$ for each $\mathcal{S}_i$, which is "a priori" computed using a conservative formula requiring the "a priori" knowledge of the maximum value $d_{\max,1}$ of the end-customer demand over an indefinitely long future time interval. Moreover, as $d_{\max,1}$ is never exactly known, it is often over-estimated.

In addition, the reduced waste implied by the RRDMPC (compare the WG_i, $i = 1, 2, 3$ in Tables 5 and 6) leads to a lower loss of profit. Finally comparing Figures 8 and 14 we note that the interval containing each replenishment order $u_i(k)$ is tighter in the RRDMPC strategy. Our approach is able to limit the amplitude of such intervals and consequently to strictly control the BE.

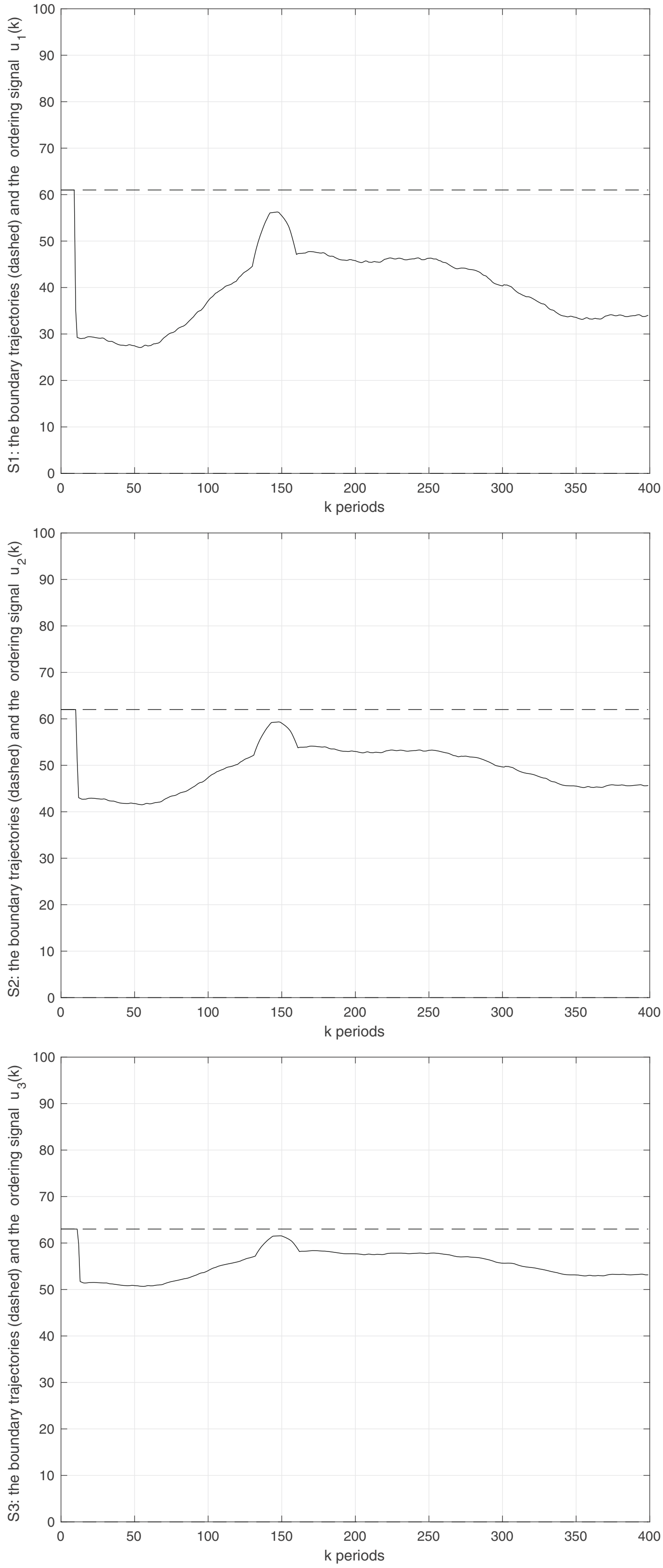

FIGURE 14 (MNLCS): the ordering signal $u_i(k)$ (solid line) issued by each S_i, $i = 1, 2, 3$ with the respective constant lower $u_{\min,i} = 0$, $i = 1, 2, 3$, and upper $u_{\max,i}$ bounds ($u_{\max,1} = 61$, $u_{\max,2} = 62$, $u_{\max,3} = 63$) according to (55)–(56).

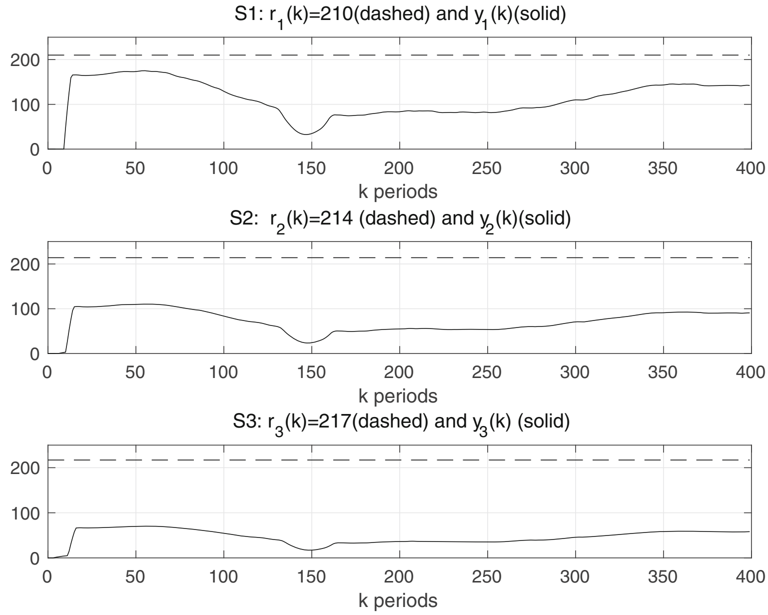

FIGURE 15 MNLCS: The desired constant inventory level $r_i(k)$ and the on hand stock level $y_i(k)$ of each $\mathcal{S}_i$.

TABLE 7 Performance indicators of the centralized and decentralized implementation of the RRMPC.

	UD_1	UD_2	UD_3	IL_1	IL_2	IL_3	WG_1	WG_2	WG_3	IO_1	IO_2	IO_3
Centralized	0.007	0.0062	0.0031	1.19e04	1.33e04	1.39e04	1.54e03	1.71e03	1.81e03	1.12e04	1.30e04	1.49e04
Decentralized	0.0075	0.0062	0.0032	1.19e04	1.60e04	2.18e04	1.54e03	2.07e03	2.83e03	1.12e04	1.33e04	1.53e04

Remark 7. Figure 14 evidence that the MLNCS does not imply any significant increase in the upstream direction of the interval containing the replenishment order $u_i(k)$. This is a direct consequence of (55) and (56): allowing $u_1(k)$ to range over a very large interval $\forall k \in Z^+$ prevents subsequents upstream increases of the interval amplitude, but leads to the very serious inconvenience of huge over-ordering and overstocking. Compare the numerical values of IL_i, WG_i, IO_i reported on Table 5 with those ones reported on Table 6.

6.3.2 | Comparison with the decentralized and centralized architectures

To contain the length of this article to a manageable size, we only report (on Table 7) the values of the same performance indicators used to evaluate the distributed architecture.

Calculations relative to these schemes and their architectures are reported in the Appendix.

Comparing Table 5 (Robust Resilient RP (Sim1)) and Table 7, we observe that the amount of unsatisfied demand is comparable in all three schemes, somewhat larger values are obtained with the decentralized architecture. The centralized scheme yields slightly smaller values of: warehouse occupancy, wasted goods, issued orders. However, these advantages are counteracted by the drawbacks mentioned in the introduction. Above all, the reluctance of agents to share information must be kept in mind.

7 | CONCLUDING REMARKS

The main feature of our contribution is to consider an MSSC whose dynamics is affected by several elements of uncertainties including possible demand-side shocks. The challenge we faced in this involved context is to define a resilient, robust RP conciliating the usual opposite control requirements that can be briefly summarized as: matching supply with demand

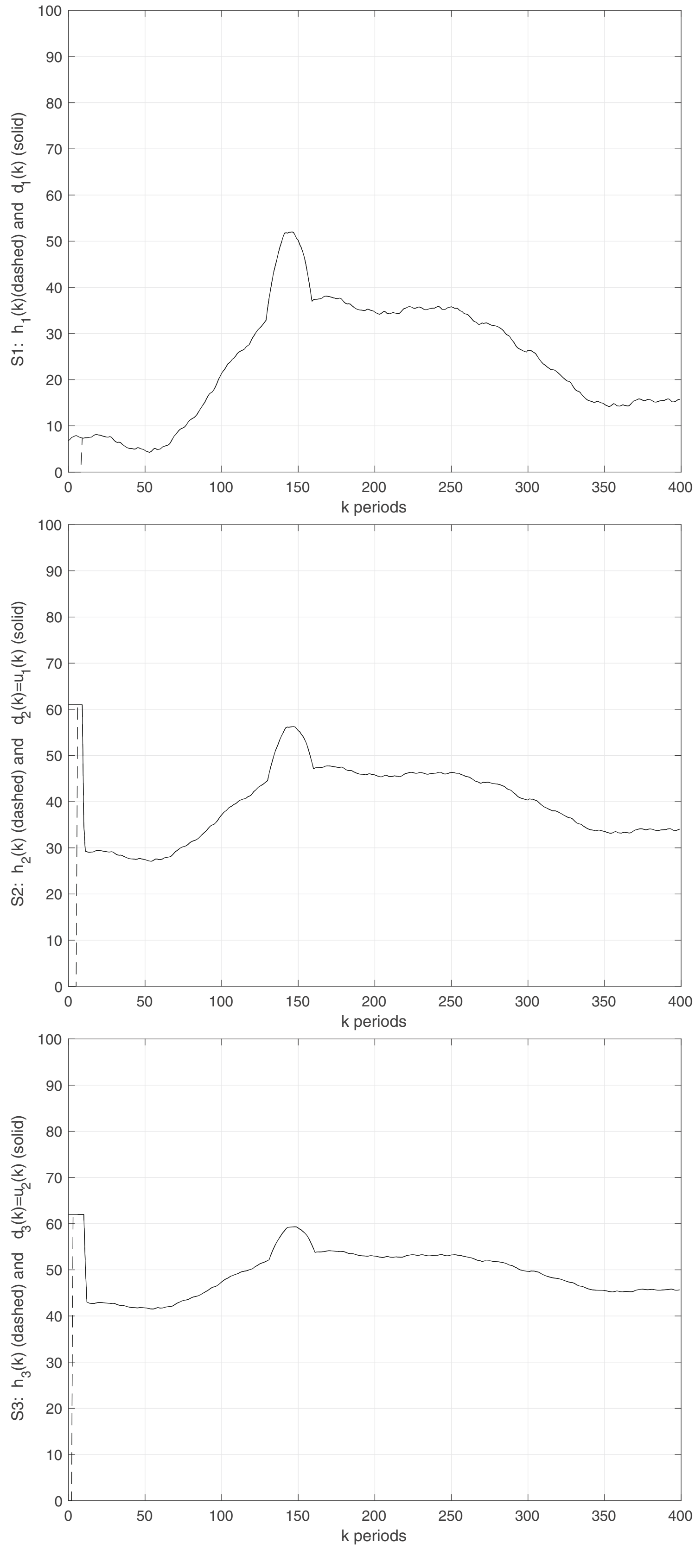

FIGURE 16 MNLCS: The imposed demand $d_i(k)$ and the fulfilled demand $h_i(k)$ at each $\mathcal{S}_i$, $i = 1, 2, 3$.

in a cost and project time effective manner. The new DRRMPC we proposed to deal with this problem is based on the solution of a set of MMCOPs. The B-splines parametrization allowed us to reformulate the conceptually and numerically demanding MMCOP as a simpler Constrained Robust LS estimation problem. Mitigation of the BE has been obtained imposing suitable constraints to the RP and penalizing sharp changes of control moves.

We also provided a simple, rational method with a low computational cost to achieve resiliency w.r.t. unpredicted patterns of the end customer demand. The reported numerical simulations show fully acceptable results from the point of view of the considered criteria of performance evaluation. A rigorous proof of feasibility and stability of the RMPC strategy has been also provided.

The theoretical foundations of our approach also provides the following ManageriaI Implications (MI):

(MI1) the robust approach to customer demand forecasting avoids the use of complicated parameter estimation methods and cumbersome numerical procedures. An experience based analysis of historical data and seasonal trends more easily provides information on upper and lower limits of the predicted demand rather then on its actual value. This, in turn, greatly facilitates the choice of the time-varying target inventory defined in any stage of the SC.

(MI2) by Theorem 1, the manager of each stage is "a priori" sure that at any k the MMCOP is feasible and that all the physical variables are uniformly bounded.

(MI3) the limits (27) on $u_i(k)$ clarify a fundamental aspect of the BE: the amplitude of the interval over which the issued restocking orders take values. Definition (29) and inequality (30) show that this amplitude is increasing in the upward direction proportionally to $1/\rho_i^-$. Therefore, to limit this undesired amplification, each warehouse should be organized so as to prevent large deterioration rate $\alpha_i = 1 - \rho_i$.

It is our intention to extend the proposed approach to the case of multi stage SC models characterized by further elements of complexity for example, time-varying perishability rate with large uncertainty, inaccurate information on the actual inventory level.

AUTHOR CONTRIBUTIONS

The authors have equally contributed to the development both of the theoretical and numerical aspects.

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CONFLICT OF INTEREST STATEMENT

The authors declare no potential conflict of interests.

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DATA AVAILABILITY STATEMENT

Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

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APPENDIX. DECENTRALIZED AND CENTRALIZED ARCHITECTURES

The architectures of the decentralized and centralized implementation of our approach are reported in Figures A1 and A2 respectively. In accordance with the DRRMPC, the following calculations are derived considering an MSSC composed of $n = 3$ stages $\mathcal{S}_i$, $i = 1, \dots, 3$, where each $\mathcal{S}_i$ is characterized by the same lead time (i.e., $L_i = L = 3$ $i = 1, 2, 3$). Unlike the DRRMPC, both the decentralized and centralized control schemes do not need to impose any constraints of the kind (21) on the lengths N_i of the control horizons $H_{i,k}$. Hence here we assume $N_i = N = 16$, $i = 1, 2, 3$ with $M_1 = N + L = 19$ (length of the end customer demand prediction interval).

- Decentralized architecture.

  As there is no information sharing between two consecutive agents, each $\mathcal{A}_i$, $i = 2, \dots, n$, has to solve its own local MMCOP by exploiting an empirical knowledge (based on historical series) of the compact set $D_{i,k}$ containing the predicted demand $\hat{D}_{i,k} = d_i(k + l|k) = u_{i-1}(k + l|k)$ issued by the downstream agent $\mathcal{A}_{i-1}$. This lack of communication with $\mathcal{A}_{i-1}$, causes $\mathcal{A}_i$, $i = 2, \dots, n$, to fix $\hat{\mathcal{D}}_{i,k}$, $\mathcal{D}_{i,k}$ and its boundary trajectories in a conservative way.

  Similarly to the distributed scheme we assume: $\hat{\mathcal{D}}_{i,k}$ coincides with the central trajectory of the compact set $D_{i,k}$ that is delimited by the lower and upper values $\gamma_i u^-_{i-1,k}$ and $\gamma_i u^+_{i-1,k}$ respectively, with $\gamma_i > 1$.

  Hence we derive the following bounds $u^-_{i,k}$ and $u^+_{i,k}$ on the optimal predicted control sequence $u_i(k + l|k)$, $i = 1, 2, 3$, $l = 0, \dots, N - 1$:

$$u^-_{1,k} \triangleq \frac{d^-_{1,k}}{\rho^-_1} \le u_1(k+l|k) \le \frac{d^+_{1,k}}{\rho^-_1} \triangleq u^+_{1,k},$$
$$u^-_{2,k} \triangleq \frac{\gamma_2 u^-_{1,k}}{\rho^-_2} \le u_2(k+l|k) \le \frac{\gamma_2 u^+_{1,k}}{\rho^-_2} \triangleq u^+_{2,k},$$
$$u^-_{3,k} \triangleq \frac{\gamma_3 u^-_{2,k}}{\rho^-_3} \le u_3(k+l|k) \le \frac{\gamma_3 u^+_{2,k}}{\rho^-_3} \triangleq u^+_{3,k}$$

  with $\gamma_3 > \gamma_2 > 1$.

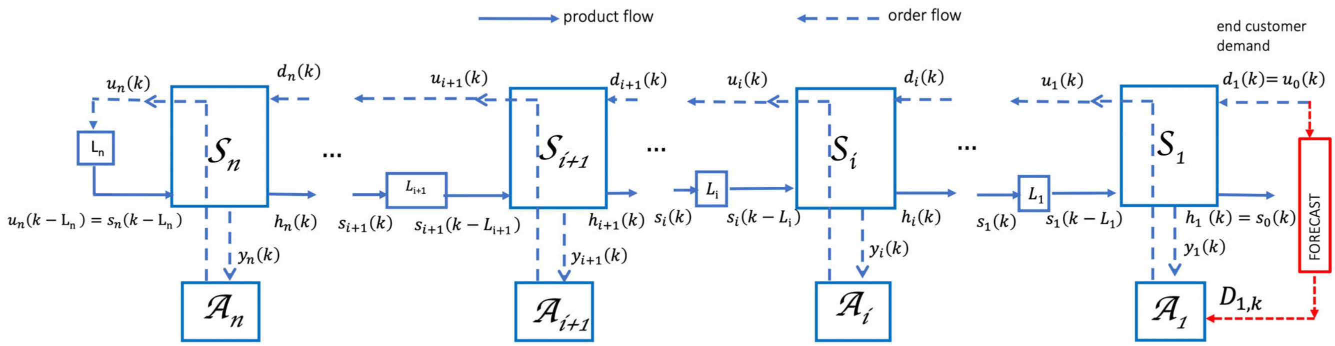

FIGURE A1 Decentralized control scheme for the MSSC network.

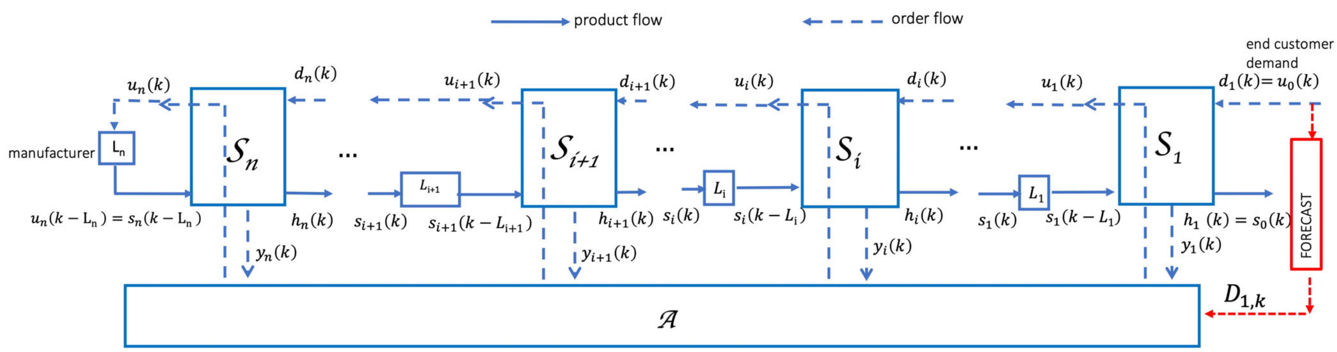

FIGURE A2 Centralized control scheme for the MSSC network.

As a consequence the predicted target inventory levels $r_i(k+L+l|k)$, $i=1,\ldots,3$, $l=1,\ldots,N$, are defined in the following way:

$$r_1(k+L+l|k)=d^+(k+L+l),\quad r_2(k+L+l|k)=\gamma_2 u^+_{1,k},\quad r_3(k+L+l|k)=\gamma_3 u^+_{2,k}.$$

- Centralized architecture.

As shown in Figure A2 there exists only one agent $\mathcal{A}$ that at each $k \in Z^+$ simultaneously computes all the replenishment orders $u_i(k)$, $i=1,\ldots,n$ by solving a unique MMCOP of larger dimensions.

In accordance with (8), the MMCOP in Section 4.1 is defined on the basis of the following augmented model

$$\underline{y}(k+1)=\begin{bmatrix}\rho_1&0&0\\0&\rho_2&0\\0&0&\rho_3\end{bmatrix}\underline{y}(k)+\begin{bmatrix}\rho_1&0&0\\0&\rho_2&0\\0&0&\rho_3\end{bmatrix}\underline{s}(k-L)-\begin{bmatrix}\rho_1&0&0\\0&\rho_2&0\\0&0&\rho_3\end{bmatrix}\underline{h}(k),$$

where

$$\underline{y}(k)\triangleq\begin{bmatrix}y_1(k)\\y_2(k)\\y_3(k)\end{bmatrix},\quad \underline{s}(k)\triangleq\underline{u}(k)-\underline{z}(k)\triangleq\begin{bmatrix}u_1(k)\\u_2(k)\\u_3(k)\end{bmatrix}-\begin{bmatrix}z_1(k)\\z_2(k)\\0\end{bmatrix},\quad \underline{h}(k)\triangleq\begin{bmatrix}s_0(k)\\s_1(k)\\s_2(k)\end{bmatrix}\triangleq\begin{bmatrix}u_0(k)-z_0(k)\\u_1(k)-z_1(k)\\u_2(k)-z_2(k)\end{bmatrix}=\begin{bmatrix}d_1(k)-z_0(k)\\d_2(k)-z_1(k)\\d_3(k)-z_2(k)\end{bmatrix}.$$

At each k the MMCOP is solved with respect to the optimal predicted $\underline{u}(k+i|k)$, $i=0,\ldots,N-1$ exploiting the information on the compact set $\mathcal{D}_{1,k}$ (see Figure 2A) containing the future values of the end customer demand over $[k+1,\ k+M_1]$.

Recalling that $d^+_{1,k} = \max_{j=1,\dots,M_1} d^+_1(k+j)$ and $d^-_{1,k} = \min_{j=1,\dots,M_1} d^-_1(k+j)$, the predicted target inventory level $\underline{r}(k+L+i|k)$, $i = 1, \dots, N$, the forecasted demand $\underline{d}(k+\ell|k)$, $\ell = 1, \dots, L+N$ and the constraints on the predicted $\underline{u}(k+i|k)$, $i = 0, \dots, N-1$, are defined in the following way

$$\underline{r}(k+L+i|k) = \begin{bmatrix} d^+_1(k+L+i) \\ d^+(k+L+i)/\rho^-_1 \\ d^+(k+L+i)/(\rho^-_1\rho^-_2) \end{bmatrix}, \quad \underline{d}(k+\ell|k) \triangleq \begin{bmatrix} \overline{d}_1(k+\ell) \\ \overline{d}_1(k+\ell)/(\rho^-_1) \\ \overline{d}_1(k+\ell)/(\rho^-_1\rho^-_2) \end{bmatrix}$$

$$\begin{bmatrix} d^-_{1,k}/(\rho^-_1) \\ d^-_{1,k}/(\rho^-_1\rho^-_2) \\ d^-_{1,k}/(\rho^-_1\rho^-_2\rho^-_3) \end{bmatrix} \leq \begin{bmatrix} u_1(k+i|k) \\ u_2(k+i|k) \\ u_3(k+i|k) \end{bmatrix} \leq \begin{bmatrix} d^+_{1,k}/(\rho^-_1) \\ d^+_{1,k}/(\rho^-_1\rho^-_2) \\ d^+_{1,k}/(\rho^-_1\rho^-_2\rho^-_3) \end{bmatrix},$$

where $\overline{d}_1(k+\ell)$ is the central trajectory of $\mathcal{D}_{1,k}$.