



UNIVERSITÀ POLITECNICA DELLE MARCHE
Repository ISTITUZIONALE

Multiscale and Multiphysics Modeling of Imperfect Interfaces

This is the peer reviewed version of the following article:

Original

Multiscale and Multiphysics Modeling of Imperfect Interfaces / Lebon, F., Rizzoni, R., Serpilli, M., Raffa, M.L., Dumont, S.. - 231:(2025), pp. 229-243. [10.1007/978-3-031-84379-2_18]

Availability:

This version is available at: 11566/364429 since: 2026-10-14T07:09:49Z

Publisher:

Springer

Published

DOI:10.1007/978-3-031-84379-2_18

Terms of use:

The terms and conditions for the reuse of this version of the manuscript are specified in the publishing policy. The use of copyrighted works requires the consent of the rights' holder (author or publisher). Works made available under a Creative Commons license or a Publisher's custom-made license can be used according to the terms and conditions contained therein. See editor's website for further information and terms and conditions.

This item was downloaded from IRIS Università Politecnica delle Marche (<https://iris.univpm.it>). When citing, please refer to the published version.

Publisher copyright:

Springer (book chapter) - Postprint/Author's accepted Manuscript

This version of the book chapter has been accepted for publication, after peer review (when applicable) and is subject to Springer Nature's AM terms of use <https://www.springernature.com/gp/open-research/policies/accepted-manuscript-terms>, but is not the Version of Record and does not reflect post-acceptance improvements, or any corrections. The Version of Record is available online at: 10.1007/978-3-031-84379-2_18.

(Article begins on next page)

Multiscale and Multiphysics modelling of Imperfect Interfaces

Frédéric Lebon¹, Raffaella Rizzoni², Michele Serpilli³, Maria Letizia Raffa⁴,
and Serge Dumont⁵

¹ Aix Marseille University, CNRS, Centrale Marseille, LMA, 4 impasse Nikola Tesla
13453 Marseille Cedex 13, France

² Department of Engineering, University of Ferrara, Ferrara, Via Saragat 1, 44122
Ferrara, Italy

³ Department of Civil and Building Engineering, and Architecture, Università
Politecnica delle Marche, via Brecce Bianche 10, 60131 Ancona, Italy

⁴ Laboratoire QUARTZ, ISAE-Supméca, 3 rue Fernand Hainaut 93400
Saint-Ouen-sur-Seine, France

⁵ Laboratoire de Modélisation Pluridisciplinaire et Simulations (LAMPS), Université
de Perpignan, 66860 Perpignan, France

Abstract. Interphases between solids are crucial for understanding the behavior of composite artificial and natural structures. Due to the thinness of these interphases, numerical modeling using finite element analysis may be challenging. Asymptotic techniques, modeling the interphase as an interface, are viable alternatives. In this paper, we review recent researches on imperfect interface laws modeling the multiphysic and damaging behavior of thin adhesive layers joining two adherents. Two main cases are considered: a linear multiphysics framework with a soft limit interface model, and the behavior of damaged structural elastic adhesives. The latter, considered as a particular case of the multiphysic setting, combines asymptotic theory and micromechanical homogenization to describe ductile and brittle damage in adhesives. A novel example of a composite block with an adhesive characterized by elastic soft damaging behavior under combined shear and tensile loads is provided.

Keywords: Asymptotic analysis, Imperfect Interfaces, Multiphysics materials, Damage.

1 Introduction

Interphases between solids play an essential role in the behaviour of structures, whether artificial or natural. It is very important to predict their behaviour and their ageing under various loads (mechanical, thermal, environmental, etc.). The difficulty in modelling is essentially due to the very small thickness of the interphases compared with those of the structures. This thinness makes numerical techniques as **volumic** finite element very difficult to use in this context, especially for coupled or non-linear problems. **Interface finite elements have been proposed over the past decades to solve these kind of problems [9, 17, 3, 22, 6,**

11, 5, 10, 14]. Most of these models use a constitutive law of the cohesive type (CZM, cohesive zone model) [1, 2]. The main limitation of CZMs is that the parameters are often not related to the physics of the thin layer being modeled. A now classical methodology for overcoming this difficulty is to use asymptotic techniques, where the interphase is modelled as an interface [16, 7]. After defining a small parameter ε , which will tend to zero, associated with the thickness and the constitutive coefficients of the interphase, different limit problems are characterized. In this paper, we briefly review the derivation of general imperfect interface laws for a composite, constituted by two solids, separated by a thin adhesive layer (Sec. 2). Two cases are essentially outlined. In the first one, a linear multiphysics framework is considered for the soft interface model, in which the constitutive coefficients depend linearly on ε . Notably, higher order terms are taken into account into the asymptotic analysis [23]. The second case is concerned by the behaviour of soft damaged structural adhesives. The model is obtained combining asymptotic theory and micromechanical homogenization for a micro-cracked material. The interface model with damage evolution is able to describe both ductile and brittle damage behaviour of structural adhesives [20]. To illustrate the implementation of the proposed models, a composite block under combined shear and traction loads is presented as an original example in Sec. 3. The block is obtained by bonding together two identical adherents with an adhesive characterized by an elastic soft damaging behavior. The micromechanical homogenization approach proposed by Kachanov and Sevostianov [12, 13] is applied to link the local microstructure and the damage parameter. Two different cases of soft and hard devices are considered. Conclusions are given in Sec. 4.

2 A review of imperfect interface models

2.1 Generalities on the asymptotic expansions method

In the sequel, the Einstein's summation convention is used. Latin indices range in the set $\{1, 2, 3\}$, while Greek indices take values in $\{1, 2\}$. Moreover, we introduce the following notations for the scalar and dyadic products: $\mathbf{a} \cdot \mathbf{b} := a_i b_i$ and $\mathbf{a} \otimes \mathbf{b} := (a_i b_j)$, for all vectors $\mathbf{a} = (a_i)$ and $\mathbf{b} = (b_i)$ in $\mathbb{R}^3$.

The composite structure consists of three solids: a thin layer (adhesive interphase) of constant thickness ε is perfectly embedded between two adherents (see Fig. 1-left). This interphase occupies a domain $B^\varepsilon = S \times (-\varepsilon/2, \varepsilon/2) \subset \mathbb{R}^3$ with cross-section $S \subset \mathbb{R}^2$, a bounded open set with smooth boundary. We introduce an orthonormal Cartesian basis $(O; \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ and we take (x_1, x_2, x_3) to designate the three coordinates of a particle. The origin of the basis belongs to S . To simplify notation, the surface S is chosen in the plane $(O; \mathbf{e}_1, \mathbf{e}_2)$. The characteristic sizes of the adherents are assumed to be much larger than those of the adhesive. The interphase thickness ε is taken in the following as a small parameter for the asymptotic expansions of the problem. The two (adherents) solids occupy the reference configurations $\Omega_\pm^\varepsilon \subset \mathbb{R}^3$. Let $S_\pm^\varepsilon \subset \mathbb{R}^2$ denote the planar interfaces between the interphase and the adherents and let $\Omega^\varepsilon = \Omega_\pm^\varepsilon \cup S_\pm^\varepsilon \cup B^\varepsilon$ denote the

whole composite system. The displacement and stress vector fields are assumed to be continuous through $S_{\pm}^{\varepsilon}$. Classically, for vanishing ε , the thin layer can be replaced by a surface separating the adherents called *interface* through which certain conditions on displacements and tractions prevail [16]. The herein adopted

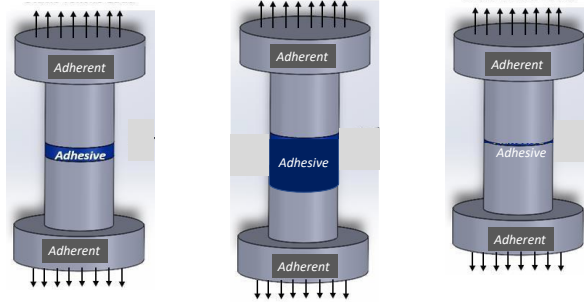


Fig. 1: Left: Initial problem of bonding; Center: Rescaled configuration; Right: Final configuration.

matched asymptotic expansion theory is founded on the use of the asymptotic analysis to derive mechanical laws governing imperfect interface conditions [7]. The five-step methodology can be summarized as follows:

- Expansion: primal and dual variable fields (displacement, stress, heat flux, electric displacement, etc.) are expanded as series of powers with respect to the thickness variable ε .
- Rescaling: a rescaling in the x_3 -direction is performed; the adhesive thickness ε becomes of length 1 (Fig. 1-center); the adherents are translated along x_3 . The domains no longer depend on the thickness.
- Analysis: these fields are transferred to the equilibrium and constitutive equations. The solutions at various orders of ε are identified and relations between primal and dual variables are finally obtained.
- Matching: matching conditions, in various configurations, are then applied to the fields.
- Model: the solution is given in the final configuration (see Fig. 1-right).

The following notation is adopted:

- $[[f]] := f(x_\alpha, 0^+) - f(x_\alpha, 0^-)$: jump in the limit configuration (Fig. 1-right);

– $\langle\langle f \rangle\rangle := \frac{1}{2}(f(x_\alpha, 0^+) + f(x_\alpha, 0^-))$: average in the limit configuration (Fig. 1-right);

where f is a generic function and $x_\alpha = (x_1, x_2)$ are the in-plane coordinates.

2.2 Soft imperfect interface law for multiphysic composites

In this section, we recall the soft imperfect interface laws obtained through an asymptotic analysis in a general multiphysic framework [23]. These conditions can be then adapted to specific cases, such as elasticity, piezoelectricity, etc. Let us consider a composite material presenting a linear coupled multiphysic behaviour. The state at the equilibrium of this multiphysic material is characterized by a collection of N vector parameters, called the primal state variables, $\mathbf{u}_1^\varepsilon, \dots, \mathbf{u}_N^\varepsilon$. We denote by $\mathbf{s}^\varepsilon := (\mathbf{u}_1^\varepsilon, \dots, \mathbf{u}_N^\varepsilon)$, a generalized vector field. With the multiphysic state $\mathbf{s}^\varepsilon$, we associate its dual physical quantity $\mathbf{t}^\varepsilon = \mathbf{t}^\varepsilon(\nabla^\varepsilon \mathbf{s}^\varepsilon)$, where $(\nabla^\varepsilon \mathbf{s}^\varepsilon)_i := \mathbf{s}_{,i}^\varepsilon = (\mathbf{u}_{1,i}^\varepsilon, \dots, \mathbf{u}_{N,i}^\varepsilon)$ denotes the gradient of $\mathbf{s}^\varepsilon$. The vector field $\mathbf{t}^\varepsilon := (\boldsymbol{\sigma}_1^\varepsilon, \dots, \boldsymbol{\sigma}_N^\varepsilon)$ represents a generalized stress field. We consider the following homogeneous and linear constitutive law, $\mathbf{t}^\varepsilon = \mathbb{K}^\varepsilon \nabla^\varepsilon \mathbf{s}^\varepsilon$, where $\mathbb{K}^\varepsilon$ is a generalized constitutive matrix, satisfying suitable symmetry and positivity properties. We assume that the adherents are subjected to a generalized system of body forces $\mathbf{f}^\varepsilon$ and surface forces $\mathbf{g}^\varepsilon$. Body and surface forces are neglected in adhesive layer. Homogeneous boundary conditions are prescribed on a partition of the composite external boundary Γ_u^ε , far from the thin layer. It is assumed that the adhesive and the adherents are perfectly bonded on $S_\pm^\varepsilon$. The differential formulation of the governing equations has the following structure:

$$\begin{cases} -\operatorname{div} \mathbf{t}^\varepsilon = \mathbf{f}^\varepsilon & \text{in } \Omega^\varepsilon, \\ \mathbf{t}^\varepsilon \mathbf{n}^\varepsilon = \mathbf{g}^\varepsilon & \text{on } \Gamma_g^\varepsilon, \\ \mathbf{s}^\varepsilon = \mathbf{0} & \text{on } \Gamma_u^\varepsilon, \end{cases}$$

where $\mathbf{t}^\varepsilon \mathbf{n}^\varepsilon := (\boldsymbol{\sigma}_1^\varepsilon \mathbf{n}^\varepsilon, \dots, \boldsymbol{\sigma}_N^\varepsilon \mathbf{n}^\varepsilon)$ represents the generalized stress vector on the boundary Γ_g^ε and $\mathbf{n}^\varepsilon$ the outer normal unit vector to Γ_g^ε .

By applying the asymptotic expansions method and developing in series of powers of ε the fields $\mathbf{t}^\varepsilon = \mathbf{t}^0 + \varepsilon \mathbf{t}^1 + o(\varepsilon)$ and $\mathbf{s}^\varepsilon = \mathbf{s}^0 + \varepsilon \mathbf{s}^1 + o(\varepsilon)$, as described in Section 2.1, the imperfect interface laws can be characterized. Here, we recall the case in which the adhesive has mechanical characteristics weaker than those of the adherents (soft interface model). The constitutive coefficients of the adherents are assumed independent of ε , i.e., $\mathbb{K}^\varepsilon = \mathbb{K}$, while the material parameters of the adhesive depend linearly on ε , so that $\hat{\mathbb{K}}^\varepsilon = \varepsilon \hat{\mathbb{K}}$. In the following, $\hat{\mathbf{K}}_{ij}$ denotes the submatrices of tensor $\hat{\mathbb{K}}$, as defined in [23], whose components are $\hat{K}_{ij}^{kl} := \hat{\mathbb{K}}_{likj}$. The transmission problems at order 0 and order 1 can be summarized as follows:

– Order 0

Governing equations

$$\begin{cases} -\operatorname{div} \mathbf{t}^0 = \mathbf{f} & \text{in } \Omega_\pm, \\ \mathbf{t}^0 \mathbf{n} = \mathbf{g} & \text{on } \Gamma_g, \\ \mathbf{s}^0 = \mathbf{0} & \text{on } \Gamma_u, \end{cases}$$

Transmission conditions on $S_\pm$

$$\begin{cases} [[\mathbf{s}^0]] = (\hat{\mathbf{K}}_{33})^{-1} \langle\langle \mathbf{t}^0 \mathbf{e}_3 \rangle\rangle, \\ [[\mathbf{t}^0 \mathbf{e}_3]] = \mathbf{0}. \end{cases} \quad (1)$$

– Order 1

Governing equations

$$\begin{cases} -\operatorname{div} \mathbf{t}^1 = \mathbf{0} & \text{in } \Omega_{\pm}, \\ \mathbf{t}^1 \mathbf{n} = \mathbf{0} & \text{on } \Gamma_g, \\ \mathbf{s}^1 = \mathbf{0} & \text{on } \Gamma_u, \end{cases}$$

Transmission conditions on $S_{\pm}$

$$\begin{cases} [[\mathbf{s}^1]] = (\hat{\mathbf{K}}_{33})^{-1} \left(\langle \langle \mathbf{t}^1 \mathbf{e}_3 \rangle \rangle - \hat{\mathbf{K}}_{\alpha 3} \langle \langle \mathbf{s}^0 \rangle \rangle_{,\alpha} \right), \\ [[\mathbf{t}^1 \mathbf{e}_3]] = -\hat{\mathbf{K}}_{3\alpha} [[\mathbf{s}^0]]_{,\alpha}, \end{cases}$$

where $\mp \mathbf{e}_3$ denote the unit normal vectors of the interfaces $S_{\pm}$. The reader can refer to [23] for the characterization of other contact laws, namely the hard and rigid interface models. At order 0, the interface behaves from a mechanical point of view as a series of linear springs, reacting to the jump between the top and bottom multiphysics states, while, at order 1, a mixed interface model is obtained with the appearance of nonlocal terms, depending on the in-plane derivatives of the jump and mean values of $\mathbf{s}^0$.

2.3 A model of soft damaging imperfect interface

In this section, we review a model of soft damaging elastic imperfect interface proposed in [4]. The model has been also generalized to the case of hard interface in [20]. This linear elastic model can be considered an adaptation of the multiphysics one, introduced in Section 2.2, by considering one state variable, namely the displacement field $\mathbf{s}^{\varepsilon} \equiv \mathbf{u}^{\varepsilon}$ and its associated Cauchy's stress field $\mathbf{t}^{\varepsilon} \equiv \boldsymbol{\sigma}^{\varepsilon}$, and introducing the additional damage evolution problem within the adhesive. The adherents of the composite system are now assumed to be homogeneous and linearly elastic, with $\mathbb{K}$ being the fourth-rank elasticity tensor. Tensor $\mathbb{K}$ has the usual symmetry properties, with the minor and major symmetries, and are positive definite. The material of the adhesive is supposed to be damaging and obey the homogeneous and linear constitutive law $\boldsymbol{\sigma}^{\varepsilon} = \mathbb{K}^{\varepsilon}(R^{\varepsilon}) \mathbf{e}^{\varepsilon}(\mathbf{u}^{\varepsilon})$, where $\mathbb{K}^{\varepsilon}(R^{\varepsilon})$ is the elasticity tensor depending on a damage parameter $R^{\varepsilon} \in [0, 1]$, and $\mathbf{e}^{\varepsilon}(\mathbf{u}^{\varepsilon}) := \frac{1}{2} (\nabla^{\varepsilon} \mathbf{u}^{\varepsilon} + (\nabla^{\varepsilon} \mathbf{u}^{\varepsilon})^T)$ is the linearized strain tensor. The damage parameter evolves according to the equation:

$$\eta^{\varepsilon} (\dot{R}^{\varepsilon})^n = - \left\{ \omega^{\varepsilon} + \frac{1}{2} \hat{\mathbb{K}}_{,R}^{\varepsilon}(R^{\varepsilon}) \mathbf{e}^{\varepsilon}(\mathbf{u}^{\varepsilon}) \cdot \mathbf{e}^{\varepsilon}(\mathbf{u}^{\varepsilon}) + \alpha^{\varepsilon} \Delta_p^{\varepsilon} R^{\varepsilon} \right\}_-, \quad R^{\varepsilon}(0) = R_0^{\varepsilon} \quad (2)$$

where R_0^{ε} is the initial damage value, $\eta^{\varepsilon} = \tilde{\eta} \varepsilon^{-1}$ is a damage viscosity, $\omega^{\varepsilon} = \tilde{\omega} \varepsilon^{-1}$ represents an energy threshold for the damage initiation, and $\alpha^{\varepsilon} = \tilde{\alpha} \varepsilon^{-1}$, n and p are positive material parameters, depending eventually on R^{ε} . In (2), Δ_p denotes the p -Laplacian operator

$$\Delta_p^{\varepsilon} R^{\varepsilon} = \nabla^{\varepsilon} \cdot (|\nabla^{\varepsilon} R^{\varepsilon}|^{p-2} \nabla^{\varepsilon} R^{\varepsilon}), \quad (3)$$

and $\{\cdot\}_-$ the negative part of a function. As before, adherents are subjected to a body force density $\mathbf{f}$ in $\Omega_{\pm}$ and to a surface force density $\mathbf{g}$ on Γ_g . Body and surface forces in the glue are neglected. On a part Γ_u of the boundary, homogeneous boundary conditions concerning the displacements are prescribed.

The case reviewed here is that of a soft adhesive: $\mathbb{K}^\varepsilon = \varepsilon \mathbb{K}$. For simplicity, only the results at the lowest order are presented. The soft contact conditions (1) can be equivalently recovered from the asymptotic analysis relative to a damaging adhesive. By taking an approximation of the initial displacement field such that $\mathbf{u}^\varepsilon \approx \mathbf{u}^0$, the equation for damage evolution in the glue takes the following form:

$$\tilde{\eta}(R)\dot{R}^n = - \left\{ \tilde{\omega}(R) + \frac{1}{2} \hat{\mathbf{K}}_{33,R}(R)[[\mathbf{u}^\varepsilon]] \cdot [[\mathbf{u}^\varepsilon]] + \tilde{\alpha} \Delta_p^2 R \right\}_- . \quad (4)$$

3 Case of study: a composite block under a combined tensile-shear load

In this section, we study the equilibrium problem of a composite made of two adherents joined by a damaging adhesive under the combined loads of traction and shear. The adherents are two identical elastic isotropic blocks, Ω_- and Ω_+ , with the same length and height, l and h respectively, subjected to simple shear and traction loads on the boundary in the plane $(O; x_2, x_3)$, as indicated in Figure 2. The two adherent blocks are joined by a thin elastic isotropic damaging glue modeled as an imperfect interface obeying the transmission conditions in (1) and the damage evolution equation (4), with

$$\hat{\mathbf{K}}_{33} = \varepsilon^{-1} \mu^\varepsilon(R)(\mathbf{e}_1 \otimes \mathbf{e}_1 + \mathbf{e}_2 \otimes \mathbf{e}_2) + \varepsilon^{-1} (2\mu^\varepsilon(R) + \lambda^\varepsilon(R))(\mathbf{e}_3 \otimes \mathbf{e}_3) \quad (5)$$

arising from an isotropic adhesive material behavior, cf. [24]. In (5), $\lambda^\varepsilon(R)$ and $\mu^\varepsilon(R)$ are the Lamé's parameters of the adhesive, measured in the reference configuration, i.e. before rescaling. The micromechanical homogenization approach proposed by Kachanov and Sevostianov [12, 13], based on the approximation of non-interacting microcracks, is here applied to model the micro-cracking damaging behavior of the adhesive. In the 2-D case, within the hypotheses of linear elastic isotropic matrix and of randomly-distributed microcracks, the complementary energy density of the effective medium gives for the effective Young's modulus $E(R)$ and Poisson's ratio $\nu(R)$:

$$E^\varepsilon(R) = E_0^\varepsilon(1 + C_E R)^{-1}, \quad \nu^\varepsilon(R) = \nu_0^\varepsilon(1 + C_\nu R)^{-1}, \quad (6)$$

where E_0^ε and ν_0^ε are the elasticity parameters of the adhesive before damage, and the constants C_E and C_ν depend on the orientation distribution of defects. Here, $C_E = \pi = C_\nu$. The elastic parameters (6) can also be expressed in terms of Lamé's parameters, as follows:

$$\lambda^\varepsilon(R) = \frac{E_0^\varepsilon \nu_0^\varepsilon}{(1 - 2\nu_0^\varepsilon + \pi R)(1 + \nu_0^\varepsilon + \pi R)}, \quad (7)$$

$$\mu^\varepsilon(R) = \frac{E_0^\varepsilon}{2(1 + \nu_0^\varepsilon + \pi R)}. \quad (8)$$

The stress tensor in the two adherents is assumed to take the simple form

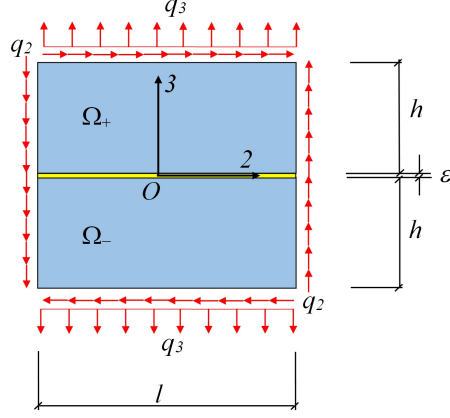


Fig. 2: Geometric and loading configuration of a composite made of two adherent blocks joined by a damaging adhesive under a combined tensile-shear load.

$$\boldsymbol{\sigma}^\varepsilon = q_2(\mathbf{e}_2 \otimes \mathbf{e}_3 + \mathbf{e}_3 \otimes \mathbf{e}_2) + q_3 \mathbf{e}_3 \otimes \mathbf{e}_3, \quad (9)$$

with q_2 and q_3 the shear and tensile loads, respectively. Note that with this choice, the second equation of the transmission conditions in (1) is identically satisfied. Equilibrium, compatibility, constitutive equations in the adherents and the first equation of the transmission conditions in (1) require the displacement fields in Ω_+ and Ω_- , denoted by $\mathbf{u}_+^\varepsilon$ and $\mathbf{u}_-^\varepsilon$, respectively, to be of the form

$$\mathbf{u}_+^\varepsilon(x_1, x_2, x_3) = c_1 x_1 \mathbf{e}_1 + (c_1 x_2 + c_3 x_3 + c_4) \mathbf{e}_2 + (c_5 + c_2 x_3) \mathbf{e}_3, \quad (10)$$

$$\mathbf{u}_-^\varepsilon(x_1, x_2, x_3) = c_1 x_1 \mathbf{e}_1 + (c_1 x_2 + c_3 x_3 + c_3 h) \mathbf{e}_2 + (c_2 h + c_2 x_3) \mathbf{e}_3, \quad (11)$$

where $c_i, i = 1, 2, 3, 4, 5$ are the following combinations of material and loading parameters

$$\begin{aligned} c_1 &= -\frac{\nu q_3}{E}, \quad c_2 = \frac{q_3}{E}, \quad c_3 = \frac{q_2}{\mu}, \\ c_4 &= \frac{q_2}{\mu} h + \frac{q_2}{\mu^\varepsilon(R)} \varepsilon, \quad c_5 = \frac{q_3}{E} h + \frac{q_3}{2\mu^\varepsilon(R) + \lambda^\varepsilon(R)} \varepsilon, \end{aligned}$$

being E , ν and μ the Young's modulus, the Poisson's ratio, and the shear modulus of the isotropic material composing the adherents, respectively. Note that the displacement fields $\mathbf{u}_\pm^\varepsilon$ have been calculated so as to prevent the lower basis of the composite block from translating along the vertical direction.

In view of (10) and (11), the equation (4) for damage evolution takes the form

$$\tilde{\eta} \dot{R} = - \left\{ \tilde{\omega} + \frac{\varepsilon}{2} \left(\frac{\mu_{,R}^\varepsilon(R)}{(\mu^\varepsilon(R))^2} q_2^2 + \frac{2\mu_{,R}^\varepsilon(R) + \lambda_{,R}^\varepsilon(R)}{(2\mu^\varepsilon(R) + \lambda^\varepsilon(R))^2} q_3^2 \right) \right\}_-, \quad (12)$$

where, for simplicity, we have chosen $n = 1$ and $\alpha = 0$, $\tilde{\eta}$ and $\tilde{\omega}$ have been assumed to be independent of R , and a comma denotes the first derivative with respect to the damage variable R .

Substituting (7) and (8) into (12) gives

$$\tilde{\eta}\dot{R} = - \left\{ \tilde{\omega} - \frac{\varepsilon \pi}{E_0^\varepsilon} \left(q_2^2 + \left(\frac{1}{2} + \frac{(\nu_0^\varepsilon)^2}{(1 - \nu_0^\varepsilon + \pi R)^2} \right) q_3^2 \right) \right\}_-, \quad (13)$$

As a measure of the macroscopic deformation of the composite block, we introduce the macroscopic shear strain defined as

$$\gamma := \frac{1}{2h} (\mathbf{u}_+^\varepsilon(x_1, x_2, h) - \mathbf{u}_-^\varepsilon(x_1, x_2, -h)) \cdot \mathbf{e}_2. \quad (14)$$

Substituting Equations (10) and (11) into (14), we obtain that

$$\gamma = \frac{q_2}{\mu} + \frac{\varepsilon q_2}{2\mu^\varepsilon(R)h}. \quad (15)$$

Solving the latter Equation with respect to q_2 gives the following macroscopic shear load-shear strain response

$$q_2 = \frac{1}{\left(\frac{1}{\mu} + \frac{\varepsilon}{2\mu^\varepsilon(R)h} \right)} \gamma. \quad (16)$$

Finally, the axial strain along the 3-axis defined as

$$\epsilon := \frac{1}{2h} (\mathbf{u}_+^\varepsilon(x_1, x_2, h) - \mathbf{u}_-^\varepsilon(x_1, x_2, -h)) \cdot \mathbf{e}_3, \quad (17)$$

can be simply calculated as

$$\epsilon = \frac{q_3}{E} + \frac{q_3}{2\mu^\varepsilon(R) + \lambda^\varepsilon(R)} \frac{\varepsilon}{2h}. \quad (18)$$

In the following section, we will consider the two different cases of soft and hard devices, meaning that in the first instance a history for the shear load q_2 will be prescribed, while in the second one a history for the shear strain γ will be prescribed. In both cases, the traction load q_3 will be kept fixed.

3.1 Numerical results

In this section, the differential problem given by (13) together with the initial condition $R(0) = R_0$, which specifies the initial value of the damage parameter, R_0 , is numerically solved by using the built-in function *ode45* of the commercial software MATLAB [18]. For simplicity, in all simulations we have assumed $R_0 = 0$.

Table 1 shows the geometrical, material and loading parameters chosen for the numerical simulations. In particular, the adherents are assumed to be two

Table 1: Geometrical, material and loading parameters

	Quantity	Symbol	Value	Unit
Adherents	Height	h	30.0	mm
	Young's modulus	E	200.00	GPa
	Poisson's ratio	ν	0.30	–
Adhesive	Thickness	ε	0.3	mm
	Young's modulus	E_0	4.53	GPa
	Poisson's ratio	ν_0	0.36	–
	Viscosity	$\tilde{\eta}$	$4.0 \cdot 10^{-4}$	MPa s m
	Activation energy	$\tilde{\omega}$	$2.0 \cdot 10^{-5}$	MPa m
	Initial damage	R_0	0	–
Soft device	Shear stress rate	$\dot{q}_2$	6.25	MPa s ⁻¹
	Traction load	q_3	0, 20, 50	MPa
Hard device	Shear strain rate	$\dot{\gamma}$	10^{-4}	s ⁻¹
	Traction load	q_3	0, 20, 50	MPa

S45C carbon steel blocks joined by an adhesive-one-component epoxy adhesive [21].

In the soft device case, a shear stress linear ramp is prescribed

$$q_2(t) = \dot{q}_2 t, \quad (19)$$

while in the hard device case a shear strain linear ramp is prescribed

$$\gamma(t) = \dot{\gamma} t. \quad (20)$$

Figure 3 shows the evolution of the damage parameter in the two cases of soft and hard device. In the case of vanishing q_3 , damage evolution starts ($R = 0.001$) at $t = 1.78$ s for both the soft device case and the hard device case. This occurs for because the same time instant makes to vanish the left-hand side of the evolution equation (13), due the choice of the activation energy density ($\tilde{\omega}$) is the same in both cases, and the choice of $\dot{q}_2$ and $\dot{\gamma}$ so as to satisfy the relation

$$\dot{q}_2 = \frac{1}{\left(\frac{1}{\mu} + \frac{\varepsilon}{2\mu^\varepsilon(R_0)h}\right)} \dot{\gamma}. \quad (21)$$

Figure 3 also shows that increasing values of q_3 increase the damage evolution inside the adhesive, coherently with the left-hand side of the evolution equation (13). Damage evolution is always more rapid in the soft device case: in the soft device case, full damage ($R = 1$) is reached at $t = 5.67$ s for $q_3 = 0$ MPa, at $t = 4.5$ s for $q_3 = 20$ MPa, and at $t = 1.37$ s for $q_3 = 50$ MPa, while in the hard

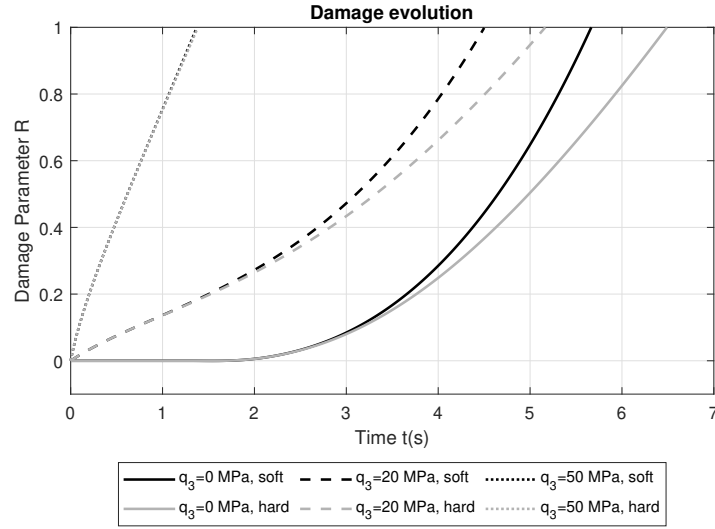


Fig. 3: Damage evolution in the adhesive of the composite block subjected to shear in a soft device (black lines) and in a hard device (gray lines). For both cases, three different levels of the traction load are considered: $q_3 = 0, 20, 50$ MPa.

device case, it is reached at $t = 6.49$ s for $q_3 = 0$ MPa, at $t = 5.17$ s for $q_3 = 20$ MPa, and at $t = 1.38$ s for $q_3 = 50$ MPa.

Figure 4 shows the shear stress-shear strain responses for increasing traction loads in both cases. The curves have been stopped at $R = 1$, corresponding to full damage of the adhesive. Note that the response is always more rigid in the hard device case, as expected.

Figure 5 illustrates the evolution of the macroscopic axial strain ϵ along the 3-axis, cf. (18), with respect to the shear strain. For $q_3 = 0$ MPa, ϵ vanishes, being the block subjected to pure shear. For $q_3 = 20$ MPa and $q_3 = 50$ MPa, ϵ is initially non-zero (cf. the horizontal lines in Fig.5), due to the elasticity of the adherents' material and the initial elastic response of the adhesive. As time increases, the adhesive material damages and softens, causing a rise of ϵ with increasing values of the shear strain. This Poynting-type effect is clearly due the damaging non-linear behavior of the adhesive and is larger for higher values of q_3 , that cause more damage.

4 Conclusion and perspectives

In this paper, a review of some recent models for soft imperfect interfaces in a multiphysics framework is presented. Two main cases are considered: a linear

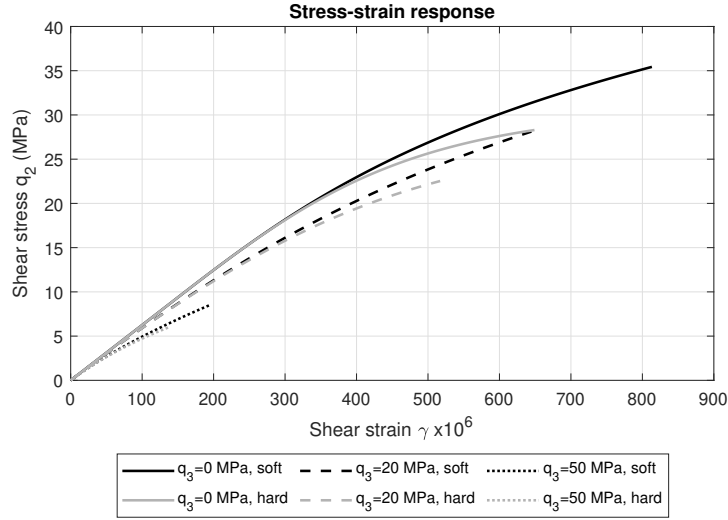


Fig. 4: Shear stress-shear strain macroscopic response of the composite block subjected to shear in a soft device (black lines) and in a hard device (gray lines). For both cases, three different levels of the traction load are considered: $q_3 = 0, 20, 50$ MPa.

multiphysics framework with three limit models (soft, hard, and rigid interfaces) and the behavior of damaged structural adhesives. The latter combines asymptotic theory and micromechanical homogenization to describe ductile and brittle damage in adhesives. A novel example of a composite block with an adhesive characterized by elastic soft damaging behavior under combined shear and traction loads is provided. Two different cases of soft and hard devices are considered: for the first one, a history of the shear load is prescribed, while for the second one a history for the shear strain is prescribed. In both cases, the traction load is kept fixed. For the two cases, the evolution of the damage parameter and the macroscopic shear stress-shear strain response are calculated and compared.

Key points of this work can be summarised as follows

- The proposed asymptotic approach in the case of multiphysics and damageable thin layer, represents a suitable tool to derive complex interface laws in a unified way.
- The interface laws that have been derived are reduced models, which can be implemented in finite element models of complex structures to avoid a volumic description of thin layers.
- In the proposed numerical example, the evolution of the damage parameter and the macroscopic shear stress-shear strain response are calculated and

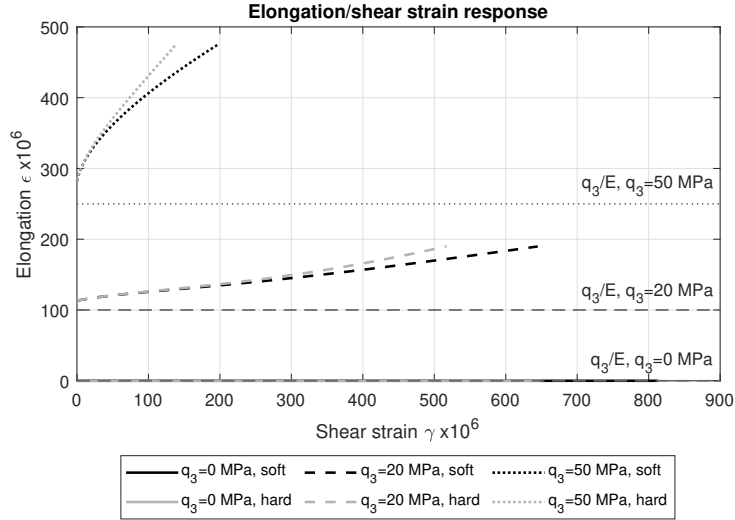


Fig. 5: Axial strain versus shear strain macroscopic response of the composite block subjected to shear in a soft device (black lines) and in a hard device (gray lines). For both cases, three different levels of the traction load are considered: $q_3 = 0, 20, 50$ MPa.

compared. Interestingly, a shear-axial coupling of the Poynting-type emerges, due to the softening behavior of the adhesive.

This study presents some limitations. First, a main limitation of the proposed interface models is the assumption of uniform thickness for the adhesive. More realistic geometries can be taken into account by describing the adhesive as a smooth rough interphase, see for example [8]. Second, a limitation of the provided example is the basic assumption of uniform stress (see Eq. (9)), that on one side allows to calculate a simple closed-form solution, on the other neglects stresses concentrations near the boundary. These aspects, together with more general material behaviors for the adhesive, like viscoplasticity and viscoelasticity, could be the object of further works.

References

1. Alfano, G., Crisfield, M. Finite element interface models for the delamination analysis of laminated composites: mechanical and computational issues. *International Journal for Numerical Methods in Engineering*, 50(7), 1701–1736 (2001). doi:10.1002/nme.93
2. Alfano, G., Sacco, E. Combining interface damage and friction in a cohesive-zone model. *International Journal for Numerical Methods in Engineering*, 68(5), 542–582 (2006). doi:10.1002/nme.1728

3. Bambole, A. N., Desai, Y. M. Hybrid-interface finite element for laminated composite and sandwich beams. *Finite Elements in Analysis and Design*, 43(13), 1023–1036 (2007). doi:10.1016/j.finel.2007.06.013
4. Bonetti, E., Bonfanti, G., Lebon F., Rizzoni, R. A model of imperfect interface with damage, *Meccanica* 52, 1911–1922 (2017). doi:10.1007/s11012-016-0520-1
5. Cerfontaine, B., Dieudonné, A. C., Radu, J. P., Collin, F., Charlier, R. 3D zero-thickness coupled interface finite element: Formulation and application. *Computers and Geotechnics*, 69, 124–140 (2015). doi:10.1016/j.compgeo.2015.04.016
6. Chen, Z., Xiao, Y., Zhang, L. The adaptive immersed interface finite element method for elliptic and Maxwell interface problems. *Journal of Computational Physics*, 228(14), 5000–5019 (2009). doi:10.1016/j.jcp.2009.03.044
7. Dumont, S., Lebon, F., Rizzoni, R. An asymptotic approach to the adhesion of thin stiff films, *Mech. Res. Commun.* 58, 24–35 (2014).doi:10.1016/j.mechrescom.2014.01.007
8. Dumont, S., Lebon, F., Raffa, M. L., Rizzoni, R. Towards nonlinear imperfect interface models including microcracks and smooth roughness. *Ann. Solid Struct. Mech.* 9 (1-2), 13–27, (2017).
9. Felippa, C. A. Interfacing finite element and boundary element discretizations. In *Boundary Element Methods: Proceedings of the Third International Seminar, Irvine, California, July 1981* (pp. 541-551) Springer Berlin Heidelberg (1981) doi: 10.1016/S0307-904X(81)80062-5
10. Funari, M. F., Greco, F., Lonetti, P. A moving interface finite element formulation for layered structures. *Composites Part B: Engineering*, 96, 325–337 (2016). doi: 10.1016/j.compositesb.2016.04.047
11. Giampieri, A., Perego, U. An interface finite element for the simulation of localized membrane-bending deformation in shells. *Computer Methods in Applied Mechanics and Engineering*, 200(29-32), 2378–2396 (2011). doi:10.1016/j.cma.2011.04.009
12. Kachanov, M. Elastic solids with many cracks and related problems, *Adv. Appl. Mech.* 30, 259–445 (1994). doi:10.1016/S0065-2156(08)70176-5
13. Kachanov, M., Sevostianov, I. *Micromechanics of materials, with applications*. Cham: Springer (2018). doi:10.1007/978-3-319-76204-3
14. Khaledi, K., Rezaei, S., Wulfinghoff, S., Reese, S. Modeling of joining by plastic deformation using a bonding interface finite element. *International Journal of Solids and Structures*, 160, 68–79 (2019). doi:10.1016/j.ijsolstr.2018.10.014
15. Lebon, F. and Rizzoni R.. Asymptotic behavior of a hard thin linear elastic interphase: An energy approach, *Int. J. Sol. Struct.*, 48, (2011). doi.org/10.1016/j.ijsolstr.2010.10.006
16. Licht, C. Asymptotic modeling of assemblies of thin linearly elastic plates, *C. R. - Mec.* 335, 775–780 (2007).doi:10.1016/j.crme.2007.10.008
17. Lorentz, E. A mixed interface finite element for cohesive zone models. *Computer methods in applied mechanics and engineering*, 198(2), 302–317 (2008). doi:10.1016/j.cma.2008.08.006
18. MATLAB version: 24.1.0.2578822 (R2024a), Natick, Massachusetts: The MathWorks Inc.; 2024.
19. Poynting J. H. On changes in the dimensions of a steel wire when twisted, and on the pressure of distortional waves in steel. *Proc. R. Soc. London, Ser A* 86: 534–561 (1912).
20. Raffa, M. L., Lebon, F., Rizzoni, R. A micromechanical model of a hard interface with micro-cracking damage, *Int. J. Mech. Sci.* 106974 (2022). doi:10.1016/j.ijmecsci.2021.106974

21. Raffa, M.L., Rizzoni, R., Lebon, F. A Model of Damage for Brittle and Ductile Adhesives in Glued Butt Joints, *Technologies* 9, 19 (2021). doi:10.3390/technologies9010019
22. Segurado, J., LLorca, J. A new three-dimensional interface finite element to simulate fracture in composites. *International journal of solids and structures*, 41(11-12), 2977–2993 (2004). doi:10.1016/j.ijsolstr.2004.01.007
23. Serpilli, M., Rizzoni, R., Lebon, F., Dumont, S. An asymptotic derivation of a general imperfect interface law for linear multiphysics composites, *Int. J. Solids Struct.* 180–181, 97–107 (2019). doi:10.1016/j.ijsolstr.2019.07.014
24. Rizzoni, R., Dumont, S., Lebon, F., Sacco, E. Higher order model for soft and hard elastic interfaces. *Int. J. Solid Struct.* 51 (23 – 24), 4137 – 4148 (2014). doi.org/10.1016/j.ijsolstr.2014.08.005
25. Truesdell C., Noll W. Antman S.S. (Ed.), *The Non-Linear Field Theories of Mechanics*, Springer-Verlag, Berlin (2004) 3rd edition.