Decisions in Economics and Finance
https://doi.org/10.1007/s10203-025-00549-2

Effects of financial intermediation on real variables: a discrete-time dynamical framework

Serena Brianzoni[1] · **Giovanni Campisi**[1] · **Antonio Palestrini**[2]

Received: 31 March 2025 / Accepted: 7 November 2025

Abstract

This paper develops a discrete-time model to investigate the impact of financial intermediation on economic growth in an economy composed of households, firms, and banks. The model is initially formulated in a general framework, allowing for flexibility in both the utility function and the financial intermediation technology. In a subsequent specification, we adopt a modified Constant Elasticity of Substitution (CES) utility function and assume a constant share of labor employed by each bank, leading to a two-dimensional dynamical system. Our analysis shows that the model's behavior is sensitive to the choice of utility function: multiple dynamical scenarios emerge as the savings rate changes. We identify key economic parameters – specifically, the number of banks, the rate of change in the share of employment per bank, and the rate of change in savings – and assess their influence through sensitivity analysis. Additionally, we explore the joint effects of these parameters using Monte Carlo simulations. The results highlight the complex interplay between financial intermediation and macroeconomic dynamics, offering new insights into the structural determinants of economic growth.

Keywords Discrete-time dynamical systems · Local dynamics · Endogenous growth · Financial intermediation

JEL Classification C61 · C62 · C63 · G21 · O43

✉ Giovanni Campisi
g.campisi@univpm.it

Serena Brianzoni
s.brianzoni@univpm.it

Antonio Palestrini
a.palestrini@univpm.it

[1] Department of Management, Polytechnic University of Marche, Piazzale R. Martelli 8, Ancona 60121, Italy

[2] Department of Economics and Social Sciences, Polytechnic University of Marche, Piazzale R. Martelli 8, Ancona 60121, Italy

Published online: 20 November 2025

1 Introduction

Studies investigating the key role of financial factors in economic growth have a long-standing tradition. In this regard, we refer to seminal contributions such as Bencivenga and Smith (1991); Benhabib and Spiegel (2000); Demetriades and Hussein (2000); Greenwood and Jovanovic (1990); Levine (2005); Levine et al. (2000); McKinnon (1973); Pagano (1993), among others. More recent literature has deepened this analysis by incorporating a wider array of methodologies and highlighting complex interactions between financial development, regulation, and macroeconomic stability. For example, King and Levine (1993) and Rajan and Zingales (1998) reinforced the empirical basis of the relationship, while (Levine 2005) offered a comprehensive review of mechanisms. Other recent studies, such as Aghion et al. (2019) and Beck et al. (2021), highlight nonlinearities and threshold effects in the finance-growth nexus, showing how institutional quality and inequality may alter the effectiveness of financial intermediation in promoting growth. From a macro-financial perspective, models that incorporate financial frictions and market imperfections (e.g., Gertler and Kiyotaki (2010); Brunnermeier and Sannikov (2016)) have provided new insights into the mechanisms through which financial development influences economic stability and long-run performance. Notably, Gennaioli et al. (2012) examine behavioral finance aspects and their macroeconomic consequences, Adrian and Shin (2010) analyze the interplay of liquidity and leverage in financial stability, and Dell'Ariccia et al. (2017) explore the impact of bank leverage regulation on economic performance. These contributions underscore that the relationship between financial development and growth is multifaceted, influenced by regulatory frameworks, institutional quality, and the evolving structure of the financial system, thus motivating further theoretical exploration as in our work. The central idea of this line of research is that financial intermediation acts as a key driver of economic growth. More specifically, the financial sector influences both savings and capital accumulation, and increases in these variables are associated with positive effects on economic growth (see Levine (2005) and Pagano (1993) for discussions on the role of the financial sector in economic growth). In this regard, financial development plays an important role in long-term economic growth.

Although these studies generally confirm the existence of a link between financial development and economic growth, their approach is predominantly empirical. In addition, their findings are not always consistent. For example, Levine et al. (2000) find that exogenous components of financial intermediary development are positively associated with economic growth, while (Benhabib and Spiegel 2000) argue that indicators correlated with the growth of the productivity of total factors differ from those that encourage investment, which implies a weaker relationship between financial development and growth.

However, it remains reasonable to assume that banks play a fundamental role in the growth process through their intermediation between households and firms.

In recent years, banking systems have been analyzed using duopoly frameworks, primarily to explore issues related to stability in the banking sector, although with a focus on different aspects. For example, Fanti (2014) studies the impact of exogenous capital regulation on loan demand, Brianzoni and Campisi (2021) and Brianzoni et al.

(2022) apply a similar framework to Italian banking systems, Ansori et al. (2024) examine macroprudential policy in Indonesia.

Given that most contributions studying the role of banks as intermediaries between firms and households remain empirical rather than theoretical, Eggoh and Villieu (2014) propose an endogenous growth model in continuous time in which banks serve as the sole intermediaries between firms (the productive sector) and households. In their framework, household savings are channeled through banks and transformed into investment by firms. Financial intermediation is thus modeled as a process that converts savings into loans for productive investment. Banks operate under monopolistic competition, while the final goods sector is perfectly competitive and features a constant returns-to-scale technology. The authors identify two distinct balanced growth paths in the long run. Extending this approach, Byrska et al. (2019) assume that financial intermediation technology depends on the share of labor employed by banks, which is endogenous. From a mathematical perspective, the model in Eggoh and Villieu (2014) is two-dimensional, whereas Byrska et al. (2019) extend the analysis to a three-dimensional system. Both frameworks are developed in continuous time.

Our model contributes to this growing literature by providing a discrete-time theoretical framework that explicitly incorporates financial intermediation as a key mechanism linking household savings to productive investment, extending and complementing previous continuous-time models such as Eggoh and Villieu (2014) and Byrska et al. (2019). Our results resonate with recent empirical findings regarding the heterogeneous effects of financial sector expansion. For instance, the finding that an increase in the number of banks can have opposing effects on consumption depending on the equilibrium level echoes the dual role of financial development highlighted in Dell'Ariccia et al. (2017) and Adrian and Shin (2010), where financial deepening can both foster growth and introduce vulnerabilities. From a practical standpoint, our framework could help interpret economic scenarios observed in banking systems undergoing structural changes, such as those experienced in emerging markets or in economies subject to macroprudential regulation adjustments (e.g., Fanti (2014), Brianzoni and Campisi (2021)). By bridging theoretical modeling with empirical observations and policy-relevant contexts, our work aims to enrich the understanding of the financial-growth nexus and support the design of effective macro-financial policies.

Our paper falls within this research area and is primarily motivated by two considerations: i) effects of financial intermediation are still not fully understood, ii) the contrasting results in the literature often stem from the different methodological approaches employed, which may yield progress along specific dimensions while leaving others unexplored.

Motivated by these insights and by the broader economic relevance of the topic, we develop a discrete-time growth model with financial intermediation. As in Byrska et al. (2019) and Eggoh and Villieu (2014), the economy is composed of households, firms (the productive sector) and banks (the financial sector). Households and companies cannot interact directly; instead, banks intermediate and transform savings into investment in the productive sector.

More precisely, in Section 2, we introduce a discrete-time economic framework that generalizes previous continuous-time models by incorporating a flexible intermediation technology and a broad class of utility functions. This generality allows us to capture a richer set of dynamics in the financial intermediation-growth nexus. Later, in Section 3, we specialize the model using a modified Constant Elasticity of Substitution (CES) utility function and a constant rate of change in the share of employment per bank to facilitate analytical and numerical tractability. Our analysis reveals novel qualitative dynamics–including the existence of multiple equilibria and bifurcation phenomena–that arise naturally in discrete time and extend the insights provided by earlier frameworks. This approach offers a new theoretical perspective on how financial intermediation shapes economic growth patterns and complements existing empirical and theoretical contributions in the literature.

Our contribution to the literature is twofold. First, by developing a discrete-time framework with endogenous financial intermediation technology, we offer a novel perspective on dynamic phenomena–such as fold bifurcation and multiple steady states–that complement and enrich the insights from existing continuous-time models. This allows for a richer characterization of the finance-growth nexus and its inherent nonlinearities. Second, our flexible modeling approach accommodates a broad class of utility functions and parameter variations, enabling a more nuanced analysis of how changes in banking structure and savings behavior affect long-term growth and stability. These features make our framework particularly relevant for understanding real-world financial systems and their complex dynamics, thus offering new theoretical insights and potential policy implications.

Given that the findings in the existing literature depend heavily on the financial indicators selected for analysis, in Section 4, we examine the role of the most relevant economic parameters from two complementary perspectives. First, we perform a sensitivity analysis to evaluate the impact of individual parameters of interest, focusing on three main factors: the number of banks, the rate of change in the share of employment per bank, and the rate of change in savings. Second, we explore the joint effects of these parameters through Monte Carlo simulations.

Our dynamical analysis primarily focuses on the existence of acceptable equilibria depending on parameter values. It reveals that the system can exhibit a continuum of steady states, no equilibrium at all, or up to two fixed points generated through a fold bifurcation. Local stability analysis confirms the existence of a saddle point under suitable parameter values. Furthermore, the numerical analysis shows that an increase in financial intermediation activity can boost overall consumption without affecting deposits – though this occurs only at a low-consumption equilibrium level. Monte Carlo simulations suggest that an increase in the number of banks can have a positive impact on the economy at high-consumption equilibrium levels, while having a negative effect at low-consumption levels.

2 The general setting

The economy consists of households, firms, and banks. Banks perform financial intermediation by collecting deposits from households and lending these funds to firms. In other words, banks transform household savings into investment.

Firms. The output produced at time t by firm i is described by the Cobb-Douglas production function: $Y_{it} = K_{it}^{\alpha}(L_{it}K_t)^{1-\alpha}$ $(0 < \alpha < 1)$,[1] where K_t denotes the aggregate capital stock of the economy at time t, while L_{it}, K_{it}, and Y_{it} represent, respectively, the labor, capital, and output of firm i at time t. Capital is obtained from bank j at the interest rate r_{jt}.

As in Eggoh and Villieu (2014) and in line with Romer (1986), technological progress is embedded in the aggregate capital stock K_t, which is treated as external to the production function of firm i.

The production function, consistent with the relevant literature (Eggoh and Villieu 2014; Byrska et al. 2019), exhibits increasing returns to scale, driven by positive externalities arising from the expanding scale of financial intermediation. This key feature is interpreted as reflecting organizational synergies (Farmer 1999).

Firm i maximizes its profits at time t according to the following objective function: $\Pi_{it}^F = K_{it}^{\alpha}(L_{it}K_t)^{1-\alpha} - r_{jt}K_{it} - w_tL_{it}$, where w_t is the wage level at time t.

The first-order conditions (FOC) for the maximization problem of firm i are:

$$\frac{\partial \Pi_{it}^F}{\partial K_{it}} = \alpha K_{it}^{\alpha-1}(K_tL_{it})^{1-\alpha} - r_{jt} = 0, \tag{1}$$

and

$$\frac{\partial \Pi_{it}^F}{\partial L_{it}} = K_{it}^{\alpha}(1-\alpha)(K_tL_{it})^{-\alpha}K_t - w_t = 0. \tag{2}$$

Banks. Consider an economy with n banks. As previously noted, banks collect deposits from households and lend them to firms. Each bank j maximizes its profits according to the following objective function: $\Pi_{jt}^B = r_{jt}K_{jt} - r^BB_{jt} - w_t\theta_{jt}$, where r^BB_{jt} denotes the interest paid on deposits, and θ_{jt} represents the share of labor employed by bank j to transform household savings into loans. Let $\varphi(\theta_{jt})$ be the financial intermediation technology, assumed to satisfy the following properties: $\varphi(\theta_{jt}) \leq 1$, $\varphi'(\theta_{jt}) > 0$ and $\varphi''(\theta_{jt}) < 0$). Each bank maximizes profits subject to the following equation:

$$K_{jt+1} - K_{jt} = \varphi(\theta_{jt})(B_{jt+1} - B_{jt}). \tag{3}$$

As in Eggoh and Villieu (2014), we assume that the total labor force is allocated between the real and financial sectors. For the sake of tractability, it is therefore reasonable to assume a common wage across both sectors, leaving a more detailed treatment of sectoral wage differentiation for future research.

In this work **banks are assumed to be homogeneous**, meaning they adopt the same strategy and behave in the same way. As a result, the total labor employed in the financial sector is given by $\sum_{j=1}^{n}\theta_{jt} = n\theta_t$ and the interest rate is the same across all banks, i.e., $r_{jt} = r_t$ $(\forall j = 1, ..., n)$. This assumption is justified by the fact that, in a symmetric equilibrium, credit interest rates are identical across banks, leading to similar employment patterns across them. Then, from the the first-order condition (1):

[1] See Layson (2015) and Shanks (2024) for an in-depth analysis of increasing returns to scale.

$$r_t = \alpha K_{it}^{\alpha-1}(K_t L_{it})^{1-\alpha}.$$

Hence, in a symmetric equilibrium – where all banks behave identically and charge the same interest rate on credit – it follows that $K_{it} = K_t$ and we obtain:

$$r_t = \alpha L_{it}^{1-\alpha}.$$

Since L_{it} and $\theta_{jt} = n\theta_t$ are the proportions of the workforce in the productive and financial sectors, respectively, we have the following:

$$r_t = \alpha L_{it}^{1-\alpha} = \alpha(1 - n\theta_t)^{1-\alpha}.$$

Moreover, under symmetric equilibrium (such that $K_{it} = K_t$), the equality between investment and savings in the goods market becomes: $K_{t+1} - K_t = Y_{it} - C_t = K_t^{\alpha}(L_{it}K_t)^{1-\alpha} - C_t = K_t(1 - n\theta_t)^{1-\alpha} - C_t$, where C_t represents the consumption of a representative household.

The last equation yields the following:

$$\frac{K_{t+1} - K_t}{K_t} = (1 - n\theta_t)^{1-\alpha} - c_t \tag{4}$$

where $c_t = \frac{C_t}{K_t}$ is the consumption-to-capital ratio.

Now, consider the bond-to-capital ratio $b_t = \frac{B_t}{K_t}$. Then:

$$\begin{aligned}
\frac{b_{t+1} - b_t}{b_t} &= \frac{b_{t+1}}{b_t} - 1 \\
&= \frac{B_{t+1}}{K_{t+1}} \frac{K_t}{B_t} - 1 \\
&= \frac{K_t}{K_{t+1}} \left(\frac{B_{t+1}}{B_t} - \frac{K_{t+1}}{K_t} \right) \\
&= \frac{K_t}{K_{t+1}} \left(\frac{B_{t+1} - B_t}{B_t} - \frac{K_{t+1} - K_t}{K_t} \right).
\end{aligned}$$

Taking into account (3), the previous equation becomes:

$$\frac{b_{t+1} - b_t}{b_t} = \frac{K_t}{K_{t+1}} \cdot \left(\frac{1}{\varphi(\theta_t) b_t} - 1 \right) \cdot \frac{K_{t+1} - K_t}{K_t}. \tag{5}$$

Finally, we use Equation (4):

$$\frac{b_{t+1} - b_t}{b_t} = \frac{1}{(1 - n\theta_t)^{1-\alpha} - c_t + 1} \left(\frac{1}{\varphi(\theta_t) b_t} - 1 \right) \left[(1 - n\theta_t)^{1-\alpha} - c_t \right]. \tag{6}$$

Households. Assuming a discount rate $\rho > 0$ (the discount factor is $(1 + \rho)^{-t}$), the representative household has preferences over consumption represented by the

following intertemporal utility function: U$=\sum_{t=0}^{\infty}(1+\rho)^{-t}u(c_t)$, where we assume that the total supply of labor is constant and normalized to unity.
In each period, households earn labor income w_t, receive interest payments $r^B B_t$ and dividends Δ_t from banks. Consequently, the budget constraint is as follows:

$$B_{t+1} = (1+r^B)B_t + w_t - c_t + \Delta_t. \tag{7}$$

Solving the optimization problem via the substitution method - specifically, by expressing c_t and c_{t+1} from the constraint and substituting into the objective function - the first-order condition ($\frac{\partial U}{\partial B_{t+1}}=0$) is:

$$u'(c_t) = (1+\rho)^{-1}u'(c_{t+1})(1+r^B). \tag{8}$$

Note that $r^B-\rho>0$ represents the net interest rate spread.

At this stage, two of the equations forming the final discrete-time dynamical system have been derived; these are given by (6) and (8).

Regarding the rate of change in the employment share at each bank, θ_t, Byrska et al. (2019) propose that it depends on the rate of change in savings: $\frac{\theta_{t+1}-\theta_t}{\theta_t} = (a-\theta_t)\cdot\frac{B_{t+1}-B_t}{B_t}$, where $a\in(0,1)$). By contrast, Eggoh and Villieu (2014) assume that θ_t remains constant over time.

From a mathematical perspective, the former assumption leads to a three-dimensional system of difference equations, whereas the latter yields a two-dimensional system. In this work, we develop the model following the approach of Eggoh and Villieu (2014), postponing the analysis of an endogenous rate of change in the employment share to future research.[2]

Furthermore, in the next section, we introduce the Constant Elasticity of Substitution (CES) utility function.

3 A bidimensional system's analysis

The utility function considered here is a modified CES (Constant Elasticity of Substitution) form: $u(c)=(1+c^s)^{\frac{1}{s}}$, $s\in(-\infty,1)-\{0\}$. **Parameter s is related to the elasticity of substitution, given by $\sigma=\frac{1}{1-s}$. For $s>0\,(s<0)$, there is low (high) substitutability between factors.** This specification reflects the assumption that utility depends solely on consumption, c, and remains strictly positive even when consumption is zero.[3].

It is important to note that the CES function does not satisfy the weak Inada conditions (see Brianzoni et al. (2007) and Brianzoni et al. (2009)).

Substituting this utility function into $u'(c_t)=(1+\rho)^{-1}u'(c_{t+1})(1+r^B)$, one can obtain:

[2] Observe that the expression $\frac{\theta_{t+1}-\theta_t}{\theta_t}=(a-\theta_t)\cdot\frac{B_{t+1}-B_t}{B_t}$ can be rewritten as $\frac{\theta_{t+1}-\theta_t}{\theta_t}=(a-\theta_t)\cdot\frac{1}{\varphi(\theta_t)b_t}\cdot\left[(1-n\theta_t)^{1-\alpha}-c_t\right]$, implying that the plane $\{(c_t,b_t,\theta_t):\theta_t=a\}$ is positively invariant.

[3] The utility function incorporates external factors (normalized to 1) that act as consumption reference points.

$$c_{t+1} = \left[\left(\frac{1+\rho}{1+r^B} \right)^{\frac{s}{1-s}} (c_t^{-s} + 1) - 1 \right]^{-\frac{1}{s}}.$$

Assuming $\theta_t = \theta \ \forall t$, the model reduces to the following two-dimensional system:

$$T_2 = \begin{cases} c_{t+1} = \left[\left(\frac{1+\rho}{1+r^B} \right)^{\frac{s}{1-s}} (c_t^{-s} + 1) - 1 \right]^{-\frac{1}{s}} - (1-n\theta)^{1-\alpha} c_t + c_t^2 + c_t = f(c_t) \\ b_{t+1} = \frac{1}{(1-n\theta)^{1-\alpha} - c_t + 1} \cdot \left(\frac{1}{\varphi(\theta)} - b_t \right) \cdot \left[(1-n\theta)^{1-\alpha} - c_t \right] + b_t = g(c_t, b_t) \end{cases}. \quad (9)$$

In what follows, we derive conditions on the parameter values under which the system admits feasible steady states. To this end, the following preliminary result is established.

Proposition 1 *Let $s > 0$. For parameter values such that $(1-n\theta)^{s(\alpha-1)} + 1 = \left(\frac{1+\rho}{1+r^B} \right)^{\frac{-s}{1-s}}$, System (9) admits a continuum of fixed points defined by $c = (1-n\theta)^{1-\alpha}$, for any b.*

Proof After imposing equilibrium conditions $c_t = c$ and $b_t = b$ $(\forall t)$ in the System (9), the second equation says that $c = (1-n\theta)^{1-\alpha}$ or $b = \frac{1}{\varphi(\theta)}$. Considering the former solution and substituting it into the first equation of System (9), we find that it is verified iff $(1-n\theta)^{s(\alpha-1)} + 1 = \left(\frac{1+\rho}{1+r^B} \right)^{\frac{-s}{1-s}}$. For $s < 0$ the continuum of steady states does not exist, because the condition on parameter values defined in Proposition 1 cannot be verified (being $r^B > \rho$). □

Thanks to the triangular structure of System (9), the eigenvalues of the Jacobian matrix at a steady state are given by: $\lambda_1 = \frac{\partial f}{\partial c}(c^\star, b^\star)$ and $\lambda_2 = \frac{\partial g}{\partial b}(c^\star, b^\star)$. Since $\frac{\partial g}{\partial b}(c^\star, b^\star) = 1$ for $c^\star = (1-n\theta)^{1-\alpha}$, all existing fixed points are therefore non-hyperbolic. Figure 1 provides clear empirical support for Proposition (1), demonstrating that the economy admits a continuum of steady states. In particular, the deposit variable b_t varies according to initial conditions, reflecting path dependence and the multiplicity of equilibria in the model. Meanwhile, the consumption variable c_t spans continuously all values along the horizontal line (shown in black), indicating that a wide range of consumption levels can persist in the long run. This continuum of fixed points highlights the model's rich dynamic structure and suggests that long-term economic outcomes are highly sensitive to initial endowments and shocks, with important implications for policy design and economic forecasting.

Now, consider the case where $(1-n\theta)^{s(\alpha-1)} + 1 \neq \left(\frac{1+\rho}{1+r^B} \right)^{\frac{-s}{1-s}}$, so that $c \neq (1-n\theta)^{1-\alpha}$. From the second equation of System (9) it immediately follows that the equilibrium condition is $b = \frac{1}{\varphi(\theta)}$. To analyze the first equation, we distinguish between two cases: $i)$ $s \in (0, 1)$, $ii)$ $s < 0$.

We begin with case $i)$ $s \in (0, 1)$. The following proposition addresses the existence of fixed points for positive values of s.

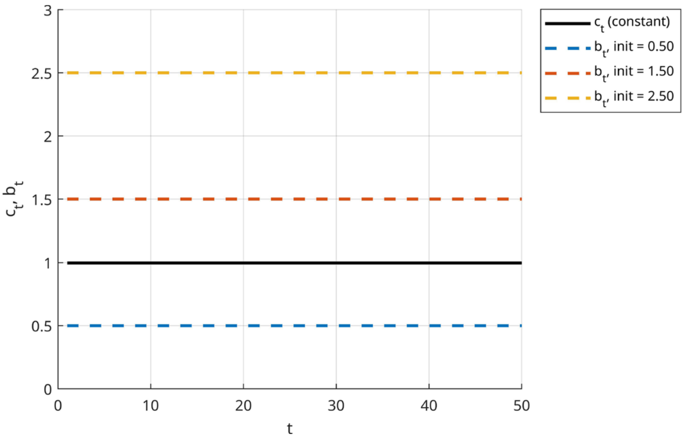

Fig. 1 Time series of c_t and b_t showing the existence of a continuum of fixed points

Proposition 2 *For* $s \in (0,1)$ *and* $(1-n\theta)^{s(\alpha-1)}+1 \neq \left(\frac{1+\rho}{1+r^B}\right)^{\frac{-s}{1-s}}$*, the system cannot accept economically meaningful steady-states.*

Proof Consider $c_t = c\ \forall t$, hence the first equation of System (9) can be written as:

$$\left\{\left[\left(\frac{1+\rho}{1+r^B}\right)^{\frac{s}{1-s}}-1\right]c^s+\left(\frac{1+\rho}{1+r^B}\right)^{\frac{s}{1-s}}\right\}^{-\frac{1}{s}}=(1-n\theta)^{1-\alpha}-c.$$

Let $H_1(c)=\left\{\left[\left(\frac{1+\rho}{1+r^B}\right)^{\frac{s}{1-s}}-1\right]c^s+\left(\frac{1+\rho}{1+r^B}\right)^{\frac{s}{1-s}}\right\}^{-\frac{1}{s}}$ and $G_1(c)=(1-n\theta)^{1-\alpha}-c$. Then, G_1 is a decreasing linear function that intersects both axes at the point $(1-n\theta)^{1-\alpha}<1$. Function H_1 is such that $H_1(0)=\left(\frac{1+r^B}{1+\rho}\right)^{\frac{1}{1-s}}>1$. In order to guarantee the existence of function H_1 for all admissible parameter values, we consider $c<c^\star$ with $c^\star=\left[\frac{(1+\rho)^{\frac{s}{1-s}}}{(1+r^B)^{\frac{s}{1-s}}-(1+\rho)^{\frac{s}{1-s}}}\right]^{\frac{1}{s}}>1$. Observing that $H_1'(c)>0, \forall c<c^\star$, we can conclude that there are no fixed points.

□

Proposition 2 establishes that for $s \in (0,1)$, the model fails to generate economically meaningful steady states unless a specific functional condition is met. Economically, the restriction $s \in (0,1)$ implies a very low intertemporal elasticity of substitution (i.e., households strongly prefer smoothing consumption over time). However, under such preferences, the dynamic interaction between consumption and deposits cannot reconcile the agents' optimal saving behavior with the constraints imposed by the production technology and financial frictions. The failure of the condition $(1-n\theta)^{s(\alpha-1)}+1 \neq \left(\frac{1+\rho}{1+r^B}\right)^{\frac{-s}{1-s}}$ leads to inconsistencies in the steady-state equations, making it impossible for the system to settle at a fixed point where both

consumption and deposits are positive and finite. This result complements the findings of Eggoh and Villieu (2014), who also highlight the sensitivity of steady-state existence to preference parameters in models with financial frictions. However, while their framework emphasizes determinacy and indeterminacy through dynamic inefficiencies, Proposition 2 shows that even before addressing equilibrium uniqueness, the model may simply fail to generate any economically feasible long-run outcome if preferences are not well aligned with structural fundamentals.
Let us now turn to case $ii)$ $s < 0$. The following proposition characterizes the number of steady states under this condition.

Proposition 3 *For $s < 0$ and $(1-n\theta)^{s(\alpha-1)}+1 \neq \left(\frac{1+\rho}{1+r^B}\right)^{\frac{-s}{1-s}}$ there are two scenarios:*

1. *If $s \in (-1, 0)$ then there are no equilibria*
2. *If $s \leq -1$ then the system can admit up to 2 fixed points.*

Proof We rewrite the first equation of System (9) for $c_t = c \; \forall t$ as:

$$\left\{\left(\frac{1+\rho}{1+r^B}\right)^{\frac{s}{1-s}} \cdot (c^{-s}+1) - 1\right\}^{-\frac{1}{s}} = (1-n\theta)^{1-\alpha}c - c^2.$$

Define $H_2(c) = \left\{\left(\frac{1+\rho}{1+r^B}\right)^{\frac{s}{1-s}} \cdot (c^{-s}+1) - 1\right\}^{-\frac{1}{s}}$ and $G_2(c) = (1-n\theta)^{1-\alpha}c - c^2$. G_2 is a concave function intersecting the horizontal axes in $c = 0$ and $c = (1-n\theta)^{1-\alpha}$, with maximum point for $c_M = \frac{1}{2}(1-n\theta)^{1-\alpha}$. Moreover, $H_2(0) = \left\{\left(\frac{1+r^B}{1+\rho}\right)^{\frac{-s}{1-s}} - 1\right\}^{-\frac{1}{s}} > 0$ and $H_2'(c) > 0 \; \forall c > 0$.

To complete the proof, we observe that $\lim_{c\to 0} H_2'(c) = +\infty$ for $s \in (-1, 0)$, while $\lim_{c\to 0} H_2'(c) = 0$ $s \leq -1$. This means that for $s < -1$ functions H_2 and G_2 cannot intersect for $c \geq 0$; while, for $s \leq -1$ there can be no or one or two equilibria such that $c^\star < (1-n\theta)^{1-\alpha}$. ◻

Proposition 3 establishes that the existence and multiplicity of fixed points hinge on the validity of the algebraic condition $(1-n\theta)^{s(\alpha-1)}+1 \neq \left(\frac{1+\rho}{1+r_b}\right)^{\frac{-s}{1-s}}$. The left-hand side of this expression captures the combined effect of production efficiency, labor market frictions, and preference parameters on the economy's long-run capacity to sustain consumption and deposits. Specifically, the term $(1-n\theta)^{s(\alpha-1)}$ reflects how labor adjustment costs and the elasticity of substitution influence steady-state outcomes. The right-hand side, instead, summarizes the intertemporal trade-offs between impatience and financial returns, governing agents' saving and consumption decisions over time. When this equality fails to hold and the parameter s, representing intertemporal substitution, lies in the interval $(-1, 0)$, the model admits no fixed points (Case 1). This suggests that under moderate substitutability preferences, the dynamic forces in the economy prevent the system from settling into a steady state. Conversely, when $s \leq -1$, the equality may hold, and the system can exhibit up to two distinct fixed

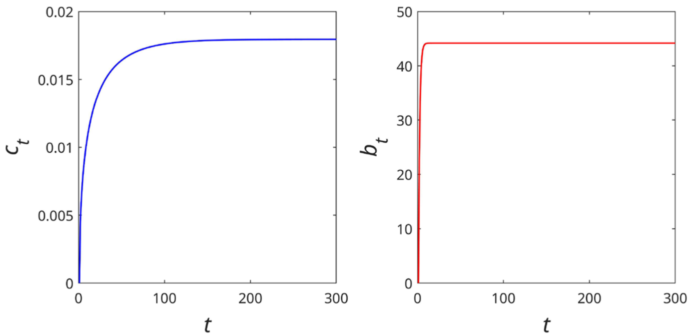

Fig. 2 Time series of c_t (left panel) and b_t (right panel) showing the existence of a unique equilibrium when $r_b = 0.02$, $\rho = 0.01999$, $\alpha = 1.21903796$, $n = 2$, $s = -2.2$, $\theta = 0.08$, $\beta = 1.5$. Initial conditions $c_0 = 0.00003$, $b_0 = 0.00002$

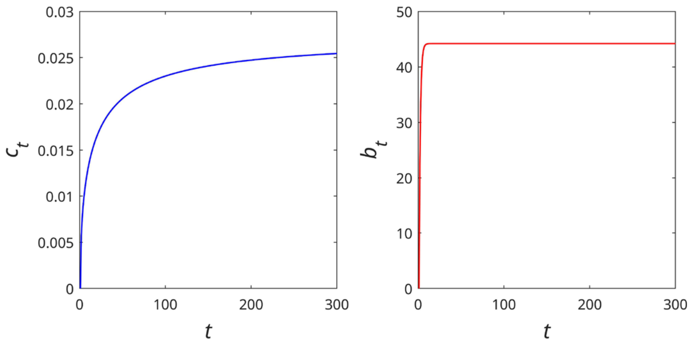

Fig. 3 Time series of c_t (left panel) and b_t (rigth panel) showing the existence of the first equilibrium when $r_b = 0.02$, $\rho = 0.01999$, $\alpha = 1.21903796$, $n = 2$, $s = -2.299999996$, $\theta = 0.08$, $\beta = 1.5$. Initial conditions $c_0 = 0.0004$, $b_0 = 0.0008$

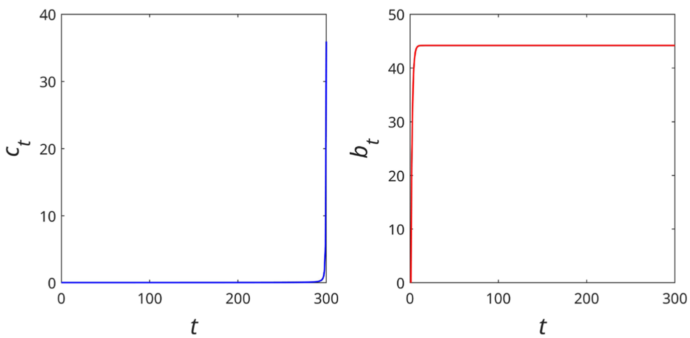

Fig. 4 Time series of c_t (left panel) and b_t (right panel) showing the existence of the second equilibrium when $r_b = 0.02$, $\rho = 0.01999$, $\alpha = 1.21903796$, $n = 2$, $s = -2.299999996$, $\theta = 0.08$, $\beta = 1.5$. Initial conditions $c_0 = 0.00008$, $b_0 = 0.1$

points (Case 2), indicating the possibility of multiple long-run equilibria driven by stronger substitution effects. These findings align with the insights of Eggoh and Villieu (2014), who also emphasize how preference parameters can generate multiplicity through bifurcation mechanisms. However, our framework differs by embedding this mechanism into a setting where labor market frictions and financial parameters jointly determine the dynamics. This highlights how both institutional and behavioral features interact to shape the existence and multiplicity of steady states in the economy. In particular, Proposition 3, Case 2, highlights the emergence of multiple steady states as a result of a fold bifurcation driven by the preference parameters s, which captures the intertemporal elasticity of substitution. When s crosses a critical threshold (i.e., becomes more negative), the system admits up to two distinct fixed points for consumption, while the level of deposits b_t remains constant across equilibria. Figure 2 illustrates the case with a single equilibrium, whereas Figures 3 and 4 depict the scenario with two equilibria, where the long-run outcome depends on the initial conditions. Economically, this implies that long-run consumption can settle at either a low or high level depending on initial conditions, despite the financial side of the economy (i.e., deposits) remaining unchanged. This reflects a form of non-linear adjustment where preferences alone can generate regime-switching behavior in real economic activity without any corresponding change in financial accumulation. Such dynamics emphasize the importance of expectation-driven outcomes and suggest that even in the absence of financial policy shifts, multiple long-run consumption paths may exist. Policy interventions that affect expectations or directly shift initial conditions could be key to steering the economy toward the more desirable high-consumption equilibrium.

From the geometric analysis presented in the previous proof, we emphasize that the term $\left\{\left(\frac{1+r^B}{1+\rho}\right)^{\frac{-s}{1-s}} - 1\right\}^{-\frac{1}{s}}$ must be sufficiently small to guarantee the existence of two distinct fixed points, which emerge through a fold bifurcation. Notably, these equilibria satisfy $c^\star < (1-n\theta)^{1-\alpha}$.

Given $c^\star < (1-n\theta)^{1-\alpha}$, the second equation of System (9) implies that, at steady state, $b^\star = \frac{1}{\varphi(\theta)}$.

The eigenvalues of the Jacobian evaluated at $(c^\star, b^\star)$ are:

- $\lambda_1 = \frac{\partial f}{\partial c}(c^\star, b^\star)$
- $\lambda_2 = \frac{\partial g}{\partial b}(c^\star, b^\star) = -\frac{(1-n\theta)^{1-\alpha} - c^\star}{(1-n\theta)^{1-\alpha} - c^\star + 1} + 1 \in (0, 1)$

When $|\lambda_1| > 1$, i.e., the fixed point is a saddle, the eigenvalues split into two sets based on their moduli – one greater than one and the other less than one. Consequently, the state space can be decomposed locally into two invariant subspaces: a stable invariant manifold and an unstable invariant manifold.[4] In Figure 5, we prove that the fixed point is a saddle. To this purpose, the diagram shows the stable (the solid curve in blue) and the unstable (the dashed line in red) manifolds. We can see, with the help of arrows, that the consumption variable is converging while deposits are diverging.

[4] See Medio and Lines (2001) for details.

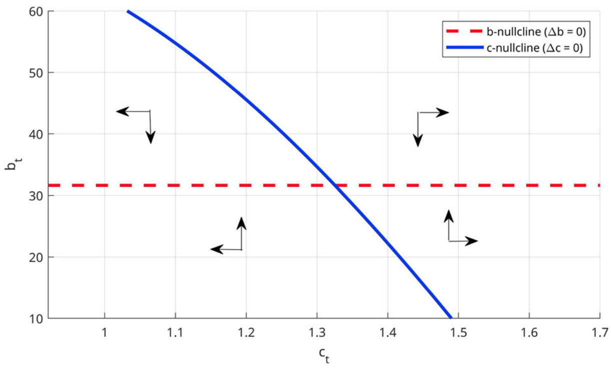

Fig. 5 Transitional dynamics of System (9) showing that the fixed point is a saddle point

Table 1 Parameter setting

r_b	ρ	α	s	θ	β
0.02	0.01999	0.9	–2.47	0.08	1.5

4 Numerical analysis

In this section, we present several numerical simulations, including sensitivity analyses on individual parameters and Monte Carlo simulations that allow parameters to vary across their entire plausible ranges. Our primary focus is on the impact of three key parameters - the number of banks n, the rate of change in the share of employment in each bank θ, and the rate of change in savings s - on the dynamics of consumption and deposits. The selection of these parameters aligns with the core objective of our analysis: to understand how financial intermediation influences economic growth. Specifically, n and θ relate directly to the financial sector, while s captures features of the economic sector. Following the approach of Brianzoni et al. (2018), we conduct parameter variations by increasing and decreasing each parameter by 50% relative to the benchmark values to facilitate meaningful comparisons. The set of parameters used in the simulations are listed in Table 1. In detail, r_b represents the cost of banks to collect resources from households, ρ is the discount rate and measures the degree of impatience. High discount rate implies a strong preference for current consumption, while low discount rate indicates that households are willing to postpone consumption. α indicates how important one input (i.e., capital) is relative to another (i.e., labor) in the production process, s is the parameter related to the elasticity of substitution (as outlined in Section 3), θ is the rate of change in the share of employment in each bank, β is a positive parameter capturing external shock on the technology of intermediation.

Consistent with Proposition 3, two values of s are considered depending on the number of equilibria: for $s = -2.3$ the model admits two equilibria, whereas for $s = -2.47$ there is a unique equilibrium. With sensitivity analysis each parameter is perturbed within a symmetric interval (plus and minus fifty percent from the central value reported in Table 1, and for each new value, we numerically compute the fixed points of the dynamical system. Among these, we retain only the economically

Table 2 Sensitivity analysis on parameter n, when $s = -2.47$

n	c_1^*	c_2^*	b_1^*	b_2^*
1	0.0333	–	41.9744	–
2	0.0388	–	41.9586	–
3	0.0471	–	41.2950	–
4	0.0612	–	41.2783	–
5	0.0905	–	41.2592	–
6	0.1917	–	41.2371	–

Table 3 Sensitivity analysis on parameter n, when $s = -2.3$

n	c_1^*	c_2^*	b_1^*	b_2^*
1	0.0333	38.5822	41.9744	41.9744
2	0.0388	38.5901	41.9586	41.9586
3	0.0471	38.6144	41.2950	41.2950
4	0.0612	38.6189	41.2783	41.2783
5	0.0905	38.6273	41.2592	41.2592
6	0.1917	38.6290	41.2371	41.2371

meaningful (i.e., non-negative and stable) steady states, and we track the resulting levels of consumption and deposit at equilibrium as the final output of the sensitivity analysis. In the Monte Carlo simulation, we jointly vary the same three parameters by drawing 10,000 random combinations from independent uniform distributions over pre-specified intervals. For each draw, we solve the system numerically to identify possible steady states. When multiple equilibria exist, we retain all admissible steady states and record the associated consumption and deposit levels.

Tables 2,3 report the results of the sensitivity analysis as the number of banks operating in the system varies between 1 and 6. **Table 2 corresponds to the single-equilibrium case, with consumption increasing and deposits remaining essentially constant over the range of** n.

In contrast, Table 3, which refers to the two-equilibria scenario, reveals the emergence of a new steady state characterized by a higher and constant consumption level, while deposits remain unchanged.

This finding is in line with the results of Huang and Lin (2009), who emphasize that the positive effects of financial development on growth are generally more pronounced and significant in low-income countries (represented in our model by lower consumption levels) than in high-income economies.

Tables 4 and 5 illustrate the effects of the share of employment in each bank, represented by the parameter θ, on the levels of consumption and deposits.

In Table 4, a clear divergence in the behavior of consumption and deposits emerges. Specifically, as θ increases, the level of deposits declines sharply. In contrast, consumption experiences a period of strong growth as θ increases. Unlike the scenario with one equilibrium, the emergence of a second equilibrium leads to a notable reduction in both consumption and deposits.

Table 4 Sensitivity analysis on parameter θ when $s = -2.47$

θ	c_1^*	c_2^*	b_1^*	b_2^*
0.04	0.0589	–	118.7062	–
0.05	0.0653	–	84.9342	–
0.06	0.0736	–	64.6077	–
0.08	**0.1027**	**–**	**41.9586**	**–**
0.09	0.1321	–	35.1612	–
0.1	0.1925	–	30.0191	–
0.12	1.2540	–	22.8331	–

Table 5 Sensitivity analysis on parameter θ when $s = -2.3$

θ	c_1^*	c_2^*	b_1^*	b_2^*
0.04	0.0589	50.0327	118.7062	118.7062
0.05	0.0653	42.0347	84.9342	84.9342
0.06	0.0736	39.0369	64.6077	64.6077
0.08	**0.1027**	**38.5901**	**41.9586**	**41.9586**
0.09	0.1321	21.0464	35.1612	35.1612
0.1	0.1925	17.0510	30.0191	30.0191
0.12	1.2540	10.0644	22.8331	22.8331

Table 6 Sensitivity analysis on parameter s

s	c_1^*	c_2^*	b_1^*	b_2^*
–3.7	0.5396	–	50.1215	–
–3.2	0.0919	–	46.3355	–
–2.7	0.0375	–	43.7895	–
–2.47	**0.1027**	–	**41.2783**	–
–2.3	**0.1027**	**38.5901**	**41.2783**	**41.2783**
–1.6	0.006	31.1326	39.7257	39.7257
–1.24	0.0051	30.1489	39.7191	39.7191

Simulation results concerning the elasticity parameter s reveal more ambiguous patterns. When the model admits a unique equilibrium, consumption exhibits a fluctuating pattern, while deposit levels tend to decline. However, once s falls below the threshold value of -2.3, a second equilibrium arises, characterized by falling consumption and deposit levels.

We now turn to the numerical simulations that examine the joint effects of the key parameters under consideration.Table6 To this end, Tables 7 and 8 present results from a Monte Carlo analysis based on various parameter combinations. Specifically, we compute the average levels of consumption and deposits by varying two parameters across their respective ranges, while holding the third parameter either above or below its central (benchmark) value.

As in the sensitivity analysis, we distinguish between the case with a unique equilibrium (Table 7) and the case with two equilibria (Table 8).

Table 7 Monte Carlo simulation results with one equilibrium

	c^*	b^*
$n \geq 4$	0,0015	41,2076
$\theta \geq 0, 08$	0,0202	41,1197
$s \geq -2.47$	0,0316	41,1065
$n \leq 4$	4,3238	41,2015
$\theta \leq 0, 08$	0,0039	41,2385
$s \leq -2, 47$	0,0836	41,1757

Table 8 Monte Carlo simulation results with two equilibria

	c_1^*	c_2^*	b_1^*	b_2^*
$n \geq 4$	0,0937	51,9407	41,2076	41,2076
$\theta \geq 0, 08$	0,0251	39,2013	41,1703	41,1703
$n \leq 4$	0,3925	47,6790	41,0460	41,0460
$\theta \leq 0, 08$	0,0068	46,7961	41,2317	41,2317
$s \leq -2, 3$	0,0168	28,8174	41,1986	41,1986

As shown in Table 7, when the model is characterized by a unique equilibrium, the behavior of average consumption and deposits varies notably across parameter combinations. Specifically, for values of $n \geq 4$ and with θ and s varying within their respective ranges, the average consumption is very low. Conversely, when $n \leq 4$ the average consumption increases, while deposits remain at the same level.

Focusing on θ, deposits remain constant for all combinations, whereas consumption is higher when $\theta \geq 0.08$. As for the savings elasticity parameter s, it behaves similarly to n: higher average values are observed when it is less than -2.47.

In Table 8, which corresponds to the case with two equilibria, the level of deposits is again constant across all parameter combinations. Relative to the unique equilibrium case, when $n \geq 4$, the low-consumption equilibrium value (c_1^*) is lower than for $n \leq 4$, while the opposite occurs for the high-consumption equilibrium value (c_2^*).

When $\theta \geq 0.08$, the low-consumption equilibrium c_1^* tends to be higher than when $\theta \leq 0.08$, while the high-consumption equilibrium c_2^* is lower in comparison.

Finally, for values of $s \leq -2.3$, we observe that the low-consumption equilibrium value is lower than its counterparts in both cases with a single equilibrium. Moreover, the high-consumption equilibrium value reaches its minimum compared to the other cases.

We can summarize our results from two perspectives. In the first case, we separately consider the roles of financial intermediation (captured by the parameters n and θ) and of the economic sector (with the parameter s). In particular, in economies characterized by a low-consumption equilibrium level, an increase in financial intermediation activity can lead to a significant improvement in consumption, with no substantial impact on deposit levels. However, with a high-consumption equilibrium level, although an increase in the number of banks seems to have no effect on either consumption or deposits, raising the level of employment in each bank tends to reduce consumption. Second, when considering the joint effect of the parameters, different results emerge. Specifically, an increase in the number of banks may be beneficial in a high-consumption equilibrium but may have adverse effects in a low-consumption regime. Conversely, a greater level of employment in each bank seems to expand (reduce) the low (high)-consumption equilibrium level. Our theoretical findings suggest several policy-relevant insights. First, we show that the macroeconomic impact of financial intermediation is highly regime-dependent. In economies operating near a low-consumption equilibrium, increasing the number of banks or enhancing labor intensity within banks can improve aggregate consumption without reducing deposits. In contrast, in high-consumption regimes, further increases in financial sector employment may have a negative effect on consumption. These results imply that one-size-fits-all financial sector policies may be ineffective or even counterproductive, depending on the economy's underlying equilibrium. Second, the joint variation of financial parameters reveals non-linear and asymmetric effects, pointing to the existence of threshold dynamics in the finance-growth relationship. For policymakers, this underscores the importance of identifying an economy's position relative to possible multiple equilibria before designing financial reforms. Improving the quality and efficiency of financial intermediation–rather than simply expanding the number of banks–may be a more effective lever for long-run growth. Although our model is theoretical, it is consistent with several real-world contexts. For instance, Italy's banking sector is characterized by high fragmentation and uneven efficiency across regions, with implications for consumption and credit availability (Guiso et al. 2004). Similarly, recent cross-country evidence suggests that the benefits of financial development diminish–and may turn negative–once a certain threshold is exceeded (Arcand et al. 2015). These empirical cases suggest that our framework can help interpret the non-linear relationship between financial structure and economic performance observed in practice.

In conclusion, our results underscore the critical role of financial intermediation in supporting economic growth, particularly in less developed scenarios. These findings are consistent with the existing literature, which emphasizes the positive but complex interplay between financial development and macroeconomic performance.

5 Conclusions

This study investigates the impact of financial intermediation on the production sector by modeling the real economy—comprising firms and households—alongside a financial sector represented by banks. The primary objective is to explore how financial intermediation influences economic growth. We contribute to the existing literature by formulating the model in discrete time, which allows for the application of advanced analytical tools and facilitates a richer dynamical analysis. Furthermore, the model is developed in a general framework that captures the role of both the utility function and the financial intermediation technology in shaping long-term outcomes. While we eventually adopt a Constant Elasticity of Substitution (CES) utility function and assume a constant rate of change in the share of employment in each bank, these assumptions are introduced progressively, allowing the model to remain flexible and broadly applicable.

Our dynamical analysis uncovers a variety of equilibrium scenarios, depending on the parameters governing preferences. The system may exhibit no equilibria, a continuum of steady states, or up to two fixed points arising through fold bifurcation.

We also examine the most relevant economic parameters through a sensitivity analysis, focusing on three key factors: the number of banks, the rate of change in the share of employment per bank, and the rate of change in savings. Numerical simulations reveal that increased financial intermediation can enhance consumption without affecting deposit levels, but only in the presence of a low-consumption equilibrium.

Additionally, Monte Carlo simulations highlight the complex joint effects of the parameters. In particular, increasing the number of banks may positively affect the economy in high-consumption equilibria while negatively affecting it in low-consumption regimes.

Finally, the generality of our framework makes it well-suited for future extensions. In particular, the model can accommodate alternative utility specifications or endogenize the evolution of the employment share in the financial sector, paving the way for deeper insights into the finance–growth nexus.

Acknowledgements The authors thank the Editor and the anonymous reviewers for their helpful and valuable comments. Open access funding provided by Università Politecnica delle Marche within the CRUI-CARE Agreement.

Funding Open access funding provided by Università Politecnica delle Marche within the CRUI-CARE Agreement.

Data Availability Not applicable.

Declarations

Conflicts of interest The authors have no relevant financial or non-financial interests to disclose.

References

Adrian, T., Shin, H.S.: Liquidity and leverage. J. Financ. Intermed. **19**(3), 418–437 (2010)

Aghion, P., Bachetta, P., Ranciere, R., Rogoff, K.: A theory of financial development and social inequality. Econometrica **87**(5), 1621–1663 (2019)

Ansori, M.F., Brianzoni, S., Campisi, G.: Bifurcations and complex dynamics in a banking duopoly model with macroprudential policy. Physica A **641**, 129730 (2024)

Arcand, J.-L., Berkes, E., Panizza, U.: Too much finance? J. Econ. Growth **20**(2), 105–148 (2015)

Beck, T., Levine, R., Levkov, A.: Financial development and economic growth: The role of policy and institutions. J. Financ. Econ. **141**(3), 1255–1279 (2021)

Bencivenga, V., Smith, B.: Financial intermediation and endogenous growth. Rev. Econ. Stud. **58**, 195–209 (1991)

Benhabib, J., Spiegel, M.: The role of financial development in growth and investment. J. Econ. Growth **5**(4), 341–360 (2000)
Brianzoni, S., Campisi, G.: Dynamical analysis of a banking duopoly model with capital regulation and asymmetric costs. Discrete Continuous Dyn. Syst. Ser. B. **26**, 1288–1302 (2021)
Brianzoni, S., Mammana, C., Michetti, E.: Complex dynamics in the neoclassical growth model with differential savings and non-constant labor force growth. Stud. Nonlinear Dyn. Econom. **11**(3), 1–17 (2007)
Brianzoni, S., Mammana, C., Michetti, E.: Nonlinear dynamics in a business-cycle model with logistic population growth. Chaos, Solitons & Fractals **40**(2), 717–730 (2009)
Brianzoni, S., Campisi, G., Russo, A.: Corruption and economic growth with non constant labor force growth. Commun. Nonlinear Sci. Numer. Simul. **58**, 202–219 (2018)
Brianzoni, S., Campisi, G., Colasante, A.: Nonlinear banking duopoly model with capital regulation: The case of italy. Chaos, Solitons & Fractals **160**, 112209 (2022)
Brunnermeier, M.K., Sannikov, Y.: The financial crisis and the policy responses: an empirical analysis of what went wrong. Am. Econ. Rev. **106**(5), 126–130 (2016)
Byrska, D., Krawiec, A., Szydlowski, M.: Dynamics os a simple endogenous growth model with financial intermediation. Macroecon. Dyn. **25**(5), 1288–1302 (2019)
Dell'Ariccia, G., Laeven, L., Suarez, G.A.: Bank leverage and monetary policy's risk-taking channel: Evidence from the united states. J. Financ. **72**(2), 613–654 (2017)
Demetriades, P., Hussein, K.: Does financial development cause economic growth? time series evidence from 16 countries. Journal of Developments Economics **51**, 387–411 (2000)
Eggoh, J.C., Villieu, P.: A simple endogenous growth model of financial intermediation with multiplicity and indeterminacy. Econ. Model. **38**, 357–366 (2014)
Fanti, L.: The dynamics of a banking duopoly with capital regulations. Econ. Model. **37**, 340–349 (2014)
Farmer, R.E.: The Macroeconomics of Self-fulfilling Prophecies. MIT Press, Cambridge (1999)
Gennaioli, N., Shleifer, A., Vishny, R.: A modern approach to behavioral finance. J. Financ. Econ. **104**(2), 452–472 (2012)
Gertler, M., Kiyotaki, N.: Financial intermediation and macroeconomic fluctuations. Handb. Mon. Econ. **3**, 547–599 (2010)
Greenwood, J., Jovanovic, B.: Financial development, growth, and the distribution of income. J. Pol. Econ. **98**(5, Part 1), 1076–1107 (1990)
Guiso, L., Sapienza, P., Zingales, L.: Does local financial development matter? Quart. J. Econ. **119**(3), 929–969 (2004)
Huang, H.-C., Lin, S.-C.: Non-linear finance-growth nexus: a threshold with instrumental variable approach 1. Econ. Transit. **17**(3), 439–466 (2009)
King, R.G., Levine, R.: Finance and growth: schumpeter might be right. Quart. J. Econ. **108**(3), 717–737 (1993)
Layson, S.K.: The increasing returns to scale ces production function and the law of diminishing marginal returns. South. Econ. J. **82**(2), 408–415 (2015)
Levine, R.: Finance and growth: Theory and evidence. Handb. Econ. Growth. **1**, 865–934 (2005)
Levine, R.: Chapter 12 finance and growth: theory and evidence. Handbook of Economic Growth **1**(A), 865–934 (2005)
Levine, R., Loayza, N., Beck, T.: Financial intermediation and growth: causality and causes. J. Monet. Econ. **46**(1), 31–77 (2000)
McKinnon, R.: Money and capital in economic development. The Brookings Institution, Washington, D.C. (1973)
Medio, A., Lines, M.: Nonlinear dynamics, a primer. Cambridge University Press, Cambridge (2001)
Pagano, M.: Financial market and growth: an overview. Eur. Econ. Rev. **37**(2–3), 613–622 (1993)
Rajan, R.G., Zingales, L.: Financial dependence and growth. Am. Econ. Rev. **88**(3), 559–586 (1998)
Romer, P.M.: Increasing returns and long-run growth. J. Political Econ. **94**(5), 1002–1037 (1986)
Shanks, O.: Increasing returns to scale and markups. Struct. Chang. Econ. Dyn. **70**, 257–267 (2024)