Nonlinear banking duopoly model with capital regulation: The case of Italy

(Article begins on next page)

Nonlinear banking duopoly model with capital regulation: The case of Italy

Serena Brianzoni[a], Giovanni Campisi[a,*], Annarita Colasante[b]

[a] *Department of Management, Polytechnic University of Marche, Piazzale Martelli 8, 60121, Ancona (AN), Italy*
[b] *Università degli Studi di Roma "Unitelma Sapienza", Roma (RM), Italy*

Abstract

We analyse a nonlinear banking duopoly model with capital regulation and asymmetric costs. We follow the literature on banking and capital regulation focusing on Italian banks. We extend the banking duopoly model with nonlinear costs of Brianzoni and Campisi (2021), by introducing the hyperbolic inverse demand function, following Puu (1991). In this way, we include a further nonlinear component in the model consisting of nonlinear demand of loans. We proceed in two parts. First, we concentrate on the analysis of the local stability of the model. Given the high number of parameters, we support the analytical study with several numerical simulations. In the second part, we focus on the conditions under which small banks are more efficient than large banks. For this purpose, we study the dynamics of loans when different parameters vary simultaneously. Our results confirm the empirical evidence that small banks play a central role in supporting local firms and

*Corresponding author
Email addresses: s.brianzoni@univpm.it (Serena Brianzoni), g.campisi@univpm.it (Giovanni Campisi), annarita.colasante@unitelmasapienza.it (Annarita Colasante)

families more than large banks.
Keywords: Duopoly, Bifurcations, Capital Regulation, Nonlinear Dynamics

1. Introduction

The financial crises of recent years have threatened the stability of all banks. Certainly, the role of banks to foster economy and economic growth has been of primary importance. The present work aims at analyzing the efficiency and regulation of Italian banks in the light of recent financial crisis taking into account the heterogeneity characterizing them. According to Banca d'Italia, Italian banks are classified in five categories (major, large, medium, small and minor) depending on their size related to the amount of deposits, capital and managed external funds. We start from this classification and we take into account two categories of banks, large and small. To this purpose, relying on the work of Alessandrini and Papi (2018a), we want to stress the role of small banks in supporting local firms and families thanks to their long and stable relationship. This important function of small banks is also remarked empirically by Stefani et al. (2016), indeed the authors find that in the period $2007-2014$ this kind of banks increased their loans supply in favor of families and small firms. An interesting work supporting the role of local banks is that of Barboni and Rossi (2019), where the authors analyse the role of local banks versus non-local banks in supporting local economies by considering a data-set of loans granted by 348 Italian banks in the period before and after the 2007-2008 Financial Crisis. In particular, they underline the importance of soft information collected by local banks thanks to their personal relationship established with funded entrepreneurs. According to the authors, soft information owned by local banks mitigates information asymmetries in credit markets, allowing them a lower credit rationing and a greater customer support. Moreover, the interdependency between efficiency and size is analysed by Aiello and Bonanno (2013) showing that efficiency decreases with size. Mesa et al. (2014) remark that local banking systems with a lower number of entities show better efficiency ratios, too. However, efficiency and size have to be considered in the light of the growing pressure of regulation due to the occurrence of the financial crisis. As highlighted by **?**, regulation plays a predominant role in the period of financial distress with the main function to stabilize the economic and financial system. In particular, while in the period of stability of the financial system, banking system aims at pursuit efficiency, periods of crises imply a prevalent impact of regulation.

For what it concerns efficiency, it is worth to stress that in the banking literature there is a debate about the implementation of a cost efficiency or a profit efficiency without consensus on them. For example, Aiello and Bonanno (2013) evaluate both cost and profit efficiency analysing the Italian banking industry in the period between $2006 - 2011$. Maudos et al. (2002) find evidence of a higher level of profit inefficiency than of cost inefficiency focusing on a sample of ten countries of the European Union. Assaf et al. (2019) find that improving bank cost efficiency during normal times may promote better financial crisis performance, while profit efficiency has limited benefits. Rossi et al. (2009), analysing the link between diversification and bank efficiency, argument that on the one hand diversification decreases cost inefficiency and, on the other hand, increases profit efficiency. In this paper we consider cost efficiency, that is we consider that banks offer the maximum level of loans at the minimum cost. However, we stress that in the paper we take into consideration also a broader definition of efficiency. In particular, we consider *efficient* a bank that is able to guarantee a constant level of loans over time. On the contrary, we consider *inefficient* a bank providing volatile level of loans over time.

Our analysis evaluates the cost efficiency of banks of different size, starting from the work of Brianzoni and Campisi (2021). Most fully, the authors investigate the role of efficiency in the Italian banking sector considering a simplified version of the model of Monti (1972) and Klein (1971). In particular, they analyse a banking duopoly model with linear demand of loans and asymmetric costs, assuming costs of large banks higher than those of small banks. Further, banks are assumed to be heterogeneous in their beliefs, large bank's approach is based on a rule-of-thumb (the gradient mechanism), while the small bank adopts a best response mechanism with boundedly rational expectations. In other words, both banks are endowed with bounded rationality but they use different boundedly rational approaches.

The choice of different beliefs is justified by the empirical evidence that small banks have maintained their level of loans almost constant over time, increasing its volume especially in the period of crisis, while large banks have rationed the credit supply. Brianzoni and Campisi (2021), relying on the empirical facts resumed above, confirm the higher efficiency of small banks to manage loans demand in local economies, moreover, a particular case is studied. They consider a scenario where only small banks operate in the market and in this case they show that the stability of the economic system increases. The key parameters of the model are the costs of the two banks and the impact of regulation. They show by means of local and global properties of their map that higher level of costs of large banks ensures higher stability, while it is confirmed the stabilizing role of regulation as in Fanti (2014), too.

This work is dedicated to Tönu Puu for his huge contributions in the field of economic dynamics (Puu (1991, 1995, 1998, 2013), Puu and Sushko (2002)). In particular we consider the work of Puu (1991) where a Cournot duopoly model based on an isoelastic demand function and constant marginal costs for the competitors is studied. In his analysis, Puu finds a period-doubling cascade towards chaos.

In the present work, following Puu (1991), we include a further nonlinear component in the model consisting of nonlinear demand of loans, in order to extend the framework of Brianzoni and Campisi (2021) which considers a banking duopoly with capital regulation and asymmetric costs but adopting linear inverse demand function. Given the high nonlinearity of our model, the analytical part is deeply extended via numerical simulations. In details, we analyse stability of fixed points via local properties of our map.

The present work aims at analysing the cost efficiency of Italian banks in the presence of capital regulation. To achieve our goal, we consider a banking duopoly model with nonlinear demand and asymmetric cost functions. Moreover, in order to take into account the empirical evidence of the greater efficiency of small banks with respect to the large banks (see Alessandrini and Papi (2018a,b), Aiello and Bonanno (2013), Giordano and Lopes (2006)), we endow large banks with quadratic cost function implying a decreasing return to scale, while we assume that small banks face constant marginal costs. This set-up allows us to consider the role played by small banks in supporting local communities increasing the volume of loans especially in the period of crisis. Conversely, large banks have rationed the credit supply (see Stefani et al. (2016) for more details).

Additionally, the present paper stresses similarities and differences with respect to the model with linear demand of Brianzoni and Campisi (2021). The comparison between this framework in honor of Tönu Puu who inspired this Special Issue and the linear demand's case has a twofold interest.
On the one hand, some results confirm and, hence, reinforce the link between asymmetric costs and the level of the total demand of loans. In fact, as we will see, large banks should increase their costs more than small banks, in order to support the total demand of loans.
On the other hand, we contribute to the field with different results and new evidences, which can be summarized as follows.

In our simulations, unlike Brianzoni and Campisi (2021), we focus on the effects of regulation considering the case of homogeneous and heterogeneous regulation. In both cases, it emerges that small banks are more efficient than large banks. Moreover, we consider a regulation parameter coherent with the total capital ratio established by the Basel Accords which must be no lower than 8%. In addition, in this model all the banks are always active in the market unlike Brianzoni and Campisi (2021), where a scenario with only small banks operating in the market was possible.

The paper is organized as follows. In Section 2 we describe the model highlighting the new ingredients introduced. In Section 3 we prove the existence of the fixed point, afterward we perform the dynamical analysis of the most interesting economic scenarios. In Section 4 we conduct a numerical analysis which aims at stressing the joint effect of parameters under study. Section 5 concludes our work.

2. The model

Banking models have become of greater importance in the oligopoly literature. In particular, Fanti (2014) and Brianzoni and Campisi (2021) make use of banking duopoly models in order to focus on two main issues, i.e. efficiency and regulation. In this way, the study of nonlinear dynamics in oligopolies (see, among others, Puu (1991) and Puu and Sushko (2002)) meets the banking sector.

In this paper, following the above mentioned works on the field, we consider a banking duopoly model, with absence of open positions between banks in the interbank market, as in Fanti (2014) and Brianzoni and Campisi (2021).

Consequently, the balance sheet of bank i is composed by loans (L_i) on the asset side, capital (K_i) and deposits (D_i) on the liability side, (with $i = 1, 2$).

We extend the work of Brianzoni and Campisi (2021) by assuming that the total demand function for loans is nonlinear, leading to the inverse demand function f:

$$f(L) = \frac{1}{L}, \qquad L = L_1 + L_2$$

The functional form of $f(L)$ consists of a decreasing, concave and hyperbolic inverse demand. This map has been analysed by Puu (1991) and Tramontana (2010). The authors show the emergence of different routes to complicated dynamics. Moreover, Fanti et al. (2015) use a more general functional form for the demand function. In particular, the authors, in their nonlinear Cournot duopoly model, assume that the price elasticity of demand is different from one, causing interesting local and global dynamic events that cannot be observed in the case of unit-elastic demand and homogeneous players.

As in Brianzoni and Campisi (2021), in this paper we consider heterogeneity in the bank size. More precisely, we recall that quadratic costs are assumed for large banks ($i = 1$), while linear costs for small banks ($i = 2$). This last assumption is due to empirical results coming from the recent bank literature (e.g. Alessandrini and Papi (2018a,b), Aiello and Bonanno (2013), Giordano and Lopes (2006)) which find evidence of the greater efficiency of small banks with respect to the larger ones. These studies show that the efficiency reflects the role played by small banks to economically sustain local firms and families.
As a consequence, the cost functions for loans are respectively given by $c_1L_1^2$ and c_2L_2, so that marginal costs for loans are increasing (decreasing returns to scale) for large banks ($i = 1$) and constant for small banks ($i = 2$).

With regard to the costs for deposits, they are assumed to be linear for both banks. This choice follows Fanti (2014), which assumes that capital regulation is based on the supply of loans. In this case, the deposit remuneration is not relevant. However, the optimal volume of loans does not depend on the properties of the deposit market in the case of separable cost functions. Small banks have the same constant marginal cost for deposits and loans, while large banks face greater marginal costs for loans than deposits only for $L_1 > 0.5$, indeed marginal costs of large banks are equal to $2c_1L_1$. To make our analysis clearer, in the rest of the paper, we refer to c_1 and c_2 as parameter costs keeping in mind that the marginal cost for large banks is $2c_1L_1$ while for small banks is c_2.

In this way, we arrive to the following profit functions:

$$\pi_1 = \left(\frac{1}{L_1 + L_2}\right) L_1 - rK_1 - c_1D_1 - c_1L_1^2$$

$$\pi_2 = \left(\frac{1}{L_1 + L_2}\right) L_2 - rK_2 - c_2D_2 - c_2L_2$$

for large and small banks, respectively. The parameter $r > 0$ is the exogenous capital remuneration which has to be high enough for having capital remuneration higher than marginal costs,[1] i.e. $r > \max[c_1, c_2]$.

The capital requirement is binding, i.e. $K_i = \gamma L_i$ (γ is the percentage determined by the regulator) and $D_i = L_i - K_i = (1 - \gamma)L_i$, $(i = 1, 2)$. Unlike Fanti (2014) and Brianzoni and Campisi (2021), we consider a regulation parameter coherent with the total capital ratio established by the Basel Accords which must be no lower than 8%.

As a consequence, profit functions become:

$$\begin{aligned} \pi_1(L_1, L_2) &= L_1 \cdot \left\{\tfrac{1}{L_1+L_2} - [c_1(1 + L_1) + \gamma(r - c_1)]\right\} \\ \pi_2(L_1, L_2) &= L_2 \cdot \left\{\tfrac{1}{L_1+L_2} - [2c_2 + \gamma(r - c_2)]\right\} \end{aligned} \tag{1}$$

[1]Considering an endogenous capital remuneration such that it is higher than marginal costs for any loan level can be an interesting development.

and marginal profits are given by:

$$\begin{array}{l} \frac{\partial \pi_1}{\partial L_1}(L_1, L_2) = \frac{L_2}{(L_1+L_2)^2} - \gamma r - (1-\gamma)c_1 - 2c_1 L_1 \\ \frac{\partial \pi_2}{\partial L_2}(L_1, L_2) = \frac{L_1}{(L_1+L_2)^2} - \gamma r - (2-\gamma)c_2 \end{array} \tag{2}$$

Dynamical setup. Let us go to introduce a dynamical setting where time is indexed by $t \in \mathbb{Z}_+$. Following the related literature (see Bischi et al. (1998), Fanti et al. (2012, 2013), Tramontana (2010), Brianzoni et al. (2015)) we assume that bank 1 has limited information, in other words large banks set the level of loans between two periods according to the following adjustment process:

$$L_{1,t+1} = L_{1,t} + \alpha L_{1,t} \frac{\partial \pi_1}{\partial L_{1,t}}(L_{1,t}, L_{2,t}), \quad t \in \mathbb{Z}_+$$

where $\alpha > 0$ is the speed of adjustment. As in Brianzoni and Campisi (2021), large banks increase or decrease their loans according to the marginal profit of the last period. In fact, the empirical literature (see Stefani et al. (2016)) finds that large banks have cut supply of loans following the market trend in financial crisis and looking at the profitability of their investments. Moreover, it points out that small banks have maintained their level of loans almost constant over time, increasing its volume especially in the period of crisis. Coherently, small banks expect that the level of loans of large banks will be equal to the last period's one and, then they maximize their expected profit, i.e.:

$$\frac{L_{1,t}}{(L_{1,t} + L_{2,t+1})^2} - \gamma r - (2-\gamma)c_2 = 0.$$

The final discrete time dynamical system $(T, \mathbb{R}_+^2)$ describing the dynamics of loans, is given by :

$$\begin{cases} L_{1,t+1} = f(L_{1,t}, L_{2,t}) = L_{1,t} + \alpha L_{1,t} \left[\frac{L_{2,t}}{(L_{1,t}+L_{2,t})^2} - \gamma r - (1-\gamma)c_1 - 2c_1 L_{1,t} \right] \\ \\ L_{2,t+1} = g(L_{1,t}) = \sqrt{\frac{L_{1,t}}{\gamma r + (2-\gamma)c_2}} - L_{1,t} \end{cases} \tag{3}$$

where: $\alpha > 0$, $r > 0$, $\gamma \in [0,1]$, $c_1, c_2 \geq 0$, so that $\gamma r + (2-\gamma)c_2 > 0$. Note that the condition $\gamma r + (2-\gamma)c_2 > 0$ ensures the survival of small banks in the market.

3. Analysis of equilibrium points

Differently from Brianzoni and Campisi (2021), in this model it is not possible to derive the solutions of the algebraic system $T(L_1, L_2) = (L_1, L_2)$. Hence, in this section we analyse the existence and the number of fixed points owned by the system, taking into account the positivity of the equilibrium level of loan.

Please notice that, even though steady states are not explicitly defined, we can obtain very interesting results about their existence and localization, as described by the following proposition.

Proposition 1. *System (3) always admits a unique interior steady state* $L^\star = (L_1^\star, L_2^\star) > (0,0)$, *which is located on the following curve of the* $(L_1, L_2)-$*plane:*

$$L_2 = \phi(L_1) = \frac{\gamma r + (1-\gamma)c_1 + 2c_1 L_1}{\gamma r + (2-\gamma)c_2} \cdot L_1 \tag{4}$$

Proof. Once the equilibrium conditions $L_{i,t} = L_i \ \forall i = 1, 2$ are set, from the second equation of System (3), we get $(L_1 + L_2)^2 = \frac{L_1}{\gamma r + (2-\gamma)c_2}$. By substituting it into the first equation, it is possible to arrive to the curve ϕ of Formula (4) by making use of some algebra.

After that, we substitute (4) into the second equation of the System (3) in equilibrium:
$\left(\frac{2\gamma r L_1 + (2-\gamma)c_2 L_1 + (1-\gamma)c_1 L_1 + 2c_1 L_1^2}{\gamma r + (2-\gamma)c_2}\right)^2 = \frac{L_1}{\gamma r + (2-\gamma)c_2}$. From the last equation, we arrive to the following third degree equation: $AL_1^3 + BL_1^2 + CL_1 + D = 0$, with $A = 4c_1^2 > 0$, $B = 4c_1[2\gamma r + (2-\gamma)c_2 + (1-\gamma)c_1] > 0$, $C = [2\gamma r + (2-\gamma)c_2 + (1-\gamma)c_1]^2 > 0$, $D = -[\gamma r + (2-\gamma)c_2] < 0$, which corresponds to one positive solution (for the Descartes' rule). □

The curve ϕ defined by Proposition 1 is very helpful in order to compare the equilibrium levels for loans of small and large banks. In fact, we find a condition on the parameter values under which small banks are characterized by a greater equilibrium value of loans with respect to large banks, as proved by the following result.

Proposition 2. *If* $(1-\gamma)c_1 > (2-\gamma)c_2$ *then* $L_2^\star > L_1^\star$.

Proof. The curve ϕ defined by (4), along which the unique fixed point lies, has got the following geometrical properties: (i) it always intersects the axis at the origin, i.e. $\phi(0) = 0$ and (ii) it is strictly increasing for any admissible set of the parameter values. As a consequence, if $\phi'(0) > 1$ then the result holds. From this, we obtain the sufficient condition $(1-\gamma)c_1 > (2-\gamma)c_2$ of the proposition. □

This proposition shows that the small bank performs better than the large one at the equilibrium, just when c_2 is not excessive with respect to c_1, taking into account also the regulation parameter γ. Moreover, the mathematical result is important because it reflects the main empirical assumptions of the model (in line with Barboni and Rossi (2019), Alessandrini and Papi (2018a), Stefani et al. (2016)) . In particular, small banks support their customers (firms and families) increasing their loans supply during financial crisis. The key of their successful performance with respect to that of the large banks, relies on the knowledge of the relevant information that facilitates lending decisions (soft information).

It is interesting to consider a parameter configuration for which the condition in Proposition 2 is violated, but the level of L_2 is higher than the level of L_1 also outside the equilibrium. To this end, we present Figure 1, where a trajectory of the system shows such a result for the appropriate parameter values. In the case considered, it is relevant the fact that small banks perform better than large banks but numerical simulations show that c_2 has to be consistently smaller than c_1.

Let us go to move to the local stability analysis of the steady state. According to the well known stability conditions (see e.g. Medio and Lines (2001)):

1. $1 + \text{tr}(J) + \det(J) > 0$
2. $1 - \text{tr}(J) + \det(J) > 0$
3. $1 - \det(J) > 0$

we have to compute the Jacobian matrix J evaluated at the equilibrium point. After some algebra, the partial derivatives of System (3) are given by:

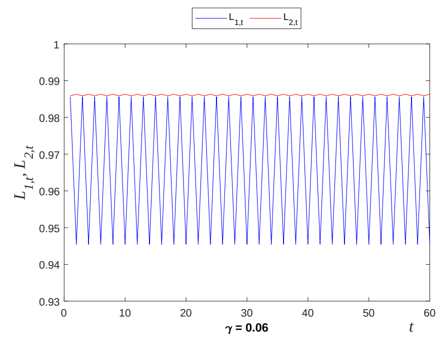

Figure 1: Trajectories of the system when Proposition 2 is violated. Parameters values: $\gamma = 0.06$, $\alpha = 3.8$, $r = 2$, $c_1 = 0.14$, $c_2 = 0.0688$ and i.c. $L_{1,0} = 0.6$, $L_{2,0} = 0.55$.

$$\frac{\partial f}{\partial L_1}(L_1, L_2) = 1 - \alpha\left\{\gamma r + (1-\gamma)c_1 + 4c_1 L_1 + \frac{L_1 - L_2}{(L_1+L_2)^3} L_2\right\}$$
$$\frac{\partial f}{\partial L_2}(L_1, L_2) = \alpha L_1 \frac{L_1 - L_2}{(L_1+L_2)^3}$$
$$\frac{\partial g}{\partial L_1}(L_1, L_2) = -1 + \frac{1}{2}\frac{1}{\gamma r + (2-\gamma)c_2}\sqrt{\frac{\gamma r + (2-\gamma)c_2}{L_1}}$$
$$\frac{\partial g}{\partial L_2}(L_1, L_2) = 0.$$

As a consequence, the determinant of the Jacobian matrix is given by $\det(J) = -\alpha\frac{L_1 - L_2}{(L_1+L_2)^3} \cdot \left[-L_1 + \frac{1}{2}\sqrt{\frac{L_1}{\gamma r + (2-\gamma)c_2}}\right]$. Taking into account the second equation of the system in equilibrium, it can be rewritten as $\det(J) = \frac{\alpha}{2}\frac{(L_1-L_2)^2}{(L_1+L_2)^3} > 0$. About the trace, imposing the first equation of the system in equilibrium into $\mathrm{tr}(J) = \frac{\partial f}{\partial L_1}(L_1, L_2)$, we find that $\mathrm{tr}(J) = 1 - \alpha \cdot \left[\frac{L_2}{(L_1+L_2)^2} + 2c_1 L_1 + \frac{(L_1-L_2)\cdot L_2}{(L_1+L_2)^3}\right] = 1 - 2\alpha L_1\left[c_1 + \frac{L_2}{(L_1+L_2)^3}\right]$.

As a first result we observe that the second stability condition $1 - \mathrm{tr}(J) + \det(J) > 0$ is always fulfilled. Mathematically speaking, this means that the fold bifurcation is ruled out, since the necessary condition for its existence cannot be verified.

In order to study the first and the third stability conditions, we are faced with the complicated structure of the system, characterized by a high number

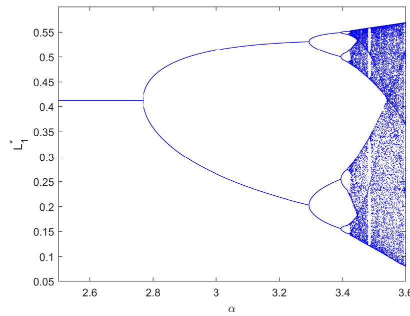

Figure 2: Bifurcation diagram on varying $\alpha \in (2.5, 3.6)$. Parameters values: $\gamma = 0.08$, $r = 2$, $c_1 = 0.25$, $c_2 = 0.16$, and i.c. $L_{1,0} = 0.36$, $L_{2,0} = 4$.

of parameters and a not explicitly defined fixed point. Nevertheless, we can observe that the equilibrium point $L^\star$ does not depend on the parameter α. Hence, once the other parameter values are fixed, we can focus on α's values such that the stability conditions are violated. This, together with the conditions on the other parameters coming from our analysis, allows us to continue the study by performing simulations.

Numerical evidence shows that for high values of the speed of adjustment α, the primary bifurcation through which the fixed point loses stability is the period doubling (see Figure 2). Note that in Figure 2 the value of α causing the emergence of complex dynamics is large (i.e. $\alpha \simeq 3.4$). However, in Figures 3 and 4 we can understand two facts. From a mathematical point of view, other parameters make the fixed point to lose its stability, that is, the parameter cost of small banks (Figure 3) and the parameter cost of large banks (Figure 4). In addition, in both cases, the value of α is smaller than the bifurcation value observed in Figure 2, being $\alpha = 2.35$ in Figure 3 and $\alpha = 2.02$ in Figure 4. On the other hand, the configuration of parameters

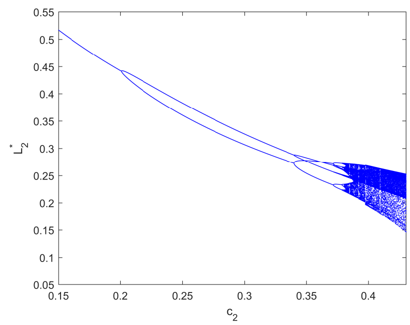

Figure 3: Bifurcation diagram on varying $c_2 \in (0.15, 0.43)$. Parameters values: $\gamma = 0.12$, $\alpha = 2.35$, $r = 1.5$, $c_1 = 0.34$, and i.c. $L_{1,0} = 0.3$, $L_{2,0} = 0.5$.

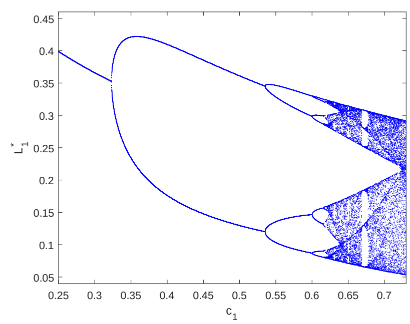

Figure 4: Bifurcation diagram on varying $c_1 \in (0.25, 0.73)$. Parameters values: $\gamma = 0.12$, $\alpha = 2.02$, $r = 1.59$, $c_2 = 0.35$, and i.c. $L_{1,0} = 0.15$, $L_{2,0} = 0.63$.

used reveals that large banks are able to bear a growth of costs better than small banks as in consequence of economy of scale. In this respect, there is the need of some policy intervention in order to equally distribute the costs between all the banks, and for this purpose the regulation plays a key role.

In order to highlight the role of regulation in our model, in Figure 5 we analyse the role of regulation parameter when we have a stable two cycle. From an economic point of view, we are interested in understanding what

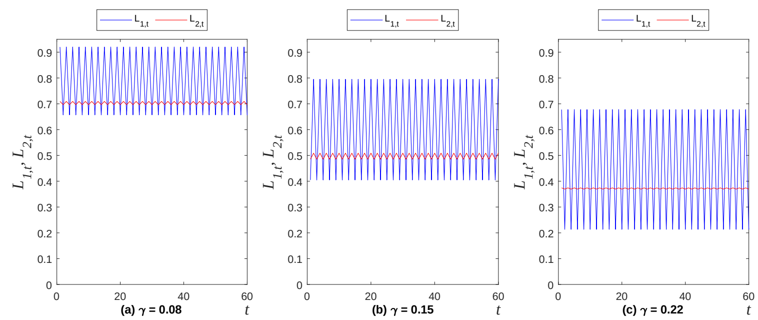

Figure 5: The volatility of the demand of loans. Trajectories of the system for different values of the cost of regulation $\gamma = 0.08$ (Panel (a)), $\gamma = 0.15$ (Panel (b)), $\gamma = 0.22$ (Panel (c)) when $\alpha = 3$, $r = 2$, $c_1 = 0.22$, $c_2 = 0.1$ and i.c. $L_{1,0} = 0.8$, $L_{2,0} = 0.7$.

kind of dynamics appears when the system is stable tuning some key parameter. In detail, in Figure 5 we plot the trajectories of the loans of the two types of banks for different values of γ (representing the exogenous banking regulation parameter). Note that all the trajectories reported in Figure 5 show lower volatility in the demand of loans for small banks than large banks. In addition, increasing values of γ lead to an average decrease in the demand of loans for all banks.

To sum up, when the market is composed by two types of banks, large and small, a stability period is always admissible. In our analysis we have concentrated the attention on three parameters: the intensity of bank regulation γ, and the costs of the two banks c_1 (for large banks) and c_2 (for small banks). From numerical simulations it has emerged that for moderate growth of the parameter costs, all the banks are able to provide the demand of loans at the equilibrium. Otherwise, when the costs grow consistently, large banks are able to better manage them than small banks, due to the favorable economy of scale. On the other hand, when we consider the effects of regulation on the demand of loans of all banks we find that, generally speaking, an increase of the regulation parameter causes a decrease in the demand of loans. In addition, the demand of loans of small banks exhibits lower volatility than the demand of loans of large banks, for all the values of γ which we considered. Given the importance of regulation and costs in the model, in the next section we devote our attention to the study of the combined effect of these parameters.

4. Numerical Simulations

This section collects several numerical simulations to help in the understanding the behavior of the main variables of interest, under different parameter settings. As we already explained, this model has no algebraic solution so that we analyse the main results by implementing simulations. Unlike Brianzoni and Campisi (2021), beyond the effects of asymmetric costs on the equilibria of the model we also focus on the effects of regulation. We proceed in two parts. First, we run simulations in a two dimensional parameter space, in other words we compute values of loans for simultaneous variations of both c_1 and c_2 (Figure 6 (a)), γ and c_1 (Figure 6 (b)), γ and c_2 (Figure 6 (c)). Second, we analyse the effects of regulation on the demand of loans in two scenarios: homogeneous regulation (Figure 7), and heterogeneous regulation (Figure 8).

To get a general view of our model, we present a 2D-bifurcation diagram for different parameter configurations in the plane (Figure 6). Here, different colors correspond to attracting cycles of different periods k, with $k < 16$, and red is related either to chaotic attractors, or to cycles of higher preiodicity. In particular, in Figure 6 (a), we present a 2D-bifurcation diagram in the (c_1, c_2)-parameter plane. It results that a stability region does exist until the cost of small banks belongs to the range $(0.1, 0.23)$. After that, only an increase of the costs of large banks may ensure the existence of a stability region. Similarly, this happens in Brianzoni and Campisi (2021), and in this sense our work confirms and reinforces the link between asymmetric costs and the level of the total demand of loans. Furthermore, when we consider the effects of regulation on both the costs of large and small banks, we find that large banks are better able to handle increased regulation. Indeed, in Figure 6 (b) we present a 2D-bifurcation diagram in the (γ, c_1)-plane, and we can see that for $\gamma \in (0.05, 0.15)$ only a fixed point exists. Differently, when we consider the joint effect of regulation and the cost of small banks in the 2D-bifurcation diagram of Figure 6 (c), for $\gamma \in (0.05, 0.15)$ we see a transition from a fixed point to a $2-$cycle.

The last aspect we are interested in is the effect of different levels of regulation. To this purpose, we consider two scenarios. First we analyse the case of homogeneous regulation and, then, we consider the case in which the two banks face heterogeneous regulation levels. In order to be coherent with the Basel Accords, which establish a total capital ratio no lower than 8%, we check the effects due to different levels of regulation on the demand of loans that lie close to the initial one. Accordingly, we consider γ equal to: 0.05, 0.08, 0.1. Figure 7 shows the time evolution of L_1 (panel (a)) and L_2 (panel (b)), respectively, considered for $c_1 = 0.2$ and $c_2 = 0.01$: each line represents the time series corresponding to different values of γ. It emerges that, for

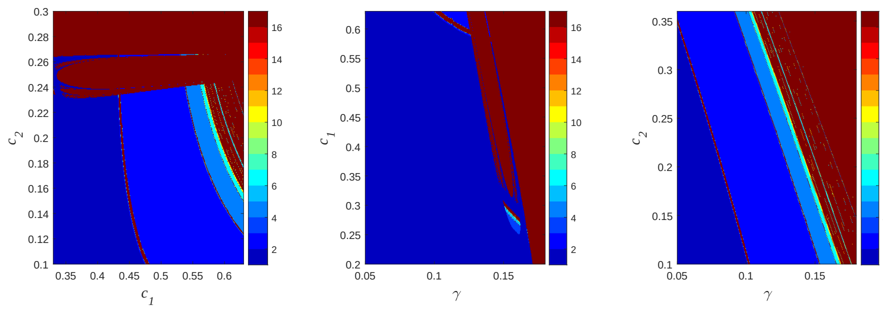

Figure 6: In (a) 2D bifurcation diagram in the (c_1, c_2)-parameter plane for $\gamma = 0.08$, $\alpha = 2.4$, $r = 2$, $c_1 \in (0.33, 0.63)$ and $c_2 \in (0.1, 0.3)$, and initial condition $L_{1,0} = 0.36$ $L_{2,0} = 0.4$. In (b) 2D bifurcation diagram in the (γ, c_1)-parameter plane for $c_2 = 0.1$, $\alpha = 2.4$, $r = 2$, $\gamma \in (0.05, 0.18)$ and $c_1 \in (0.2, 0.63)$, and initial condition $L_{1,0} = 0.38$ $L_{2,0} = 0.42$. In (c) 2D bifurcation diagram in the (γ, c_2)-parameter plane for $c_1 = 0.4$, $\alpha = 2.4$, $r = 2$, $\gamma \in (0.05, 0.18)$ and $c_2 \in (0.1, 0.36)$, and initial condition $L_{1,0} = 0.37$ $L_{2,0} = 0.41$.

all the values of γ, the level of the demand of loans of small banks is higher than the level of the loans of large banks.

When the scenario of heterogeneous regulation is considered, we have to take into account the following map:

$$\begin{cases} L_{1,t+1} = f(L_{1,t}, L_{2,t}) = L_{1,t} + \alpha L_{1,t}\left[\frac{L_{2,t}}{(L_{1,t}+L_{2,t})^2} - \gamma_1 r - (1-\gamma_1)c_1 - 2c_1 L_{1,t}\right] \\ \\ L_{2,t+1} = g(L_{1,t}) = \sqrt{\frac{L_{1,t}}{\gamma_2 r + (2-\gamma_2)c_2}} - L_{1,t} \end{cases} \tag{5}$$

where: $\alpha > 0$, $r > 0$, $\gamma_1, \gamma_2 \in [0, 1]$, $c_1, c_2 \geq 0$, so that $\gamma_2 r + (2-\gamma_2)c_2 > 0$ which represents the survival condition of small banks.

In Figure 8 we plot the time series of L_1 and L_2 when the two banks

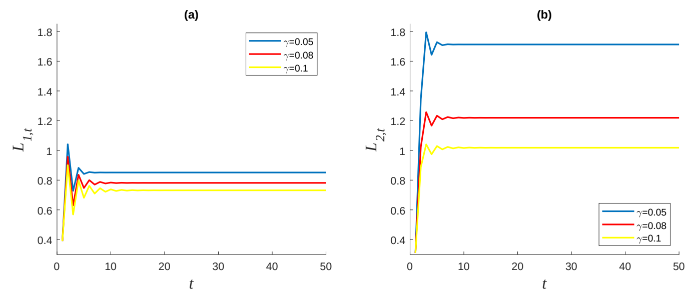

Figure 7: Time series of $L_{1,t}$ (in (a)) and $L_{2,t}$ (in (b)) for three values of γ. In both panels, we use the following parameter values: $\alpha = 3$, $c_1 = 0.2$, $c_2 = 0.01$, $r = 2.2$. Initial conditions $L_{1,0} = 0.39$, $L_{2,0} = 0.31$.

face different costs of regulation. In particular, in Figure 8 (a), we assume that the parameter cost of regulation for large banks (γ_1) is smaller than the parameter cost of regulation of small banks (γ_2), while in Figure 8 (b), we consider a larger parameter cost of regulation for large banks than for small banks. When $\gamma_1 < \gamma_2$, we see a larger volatility of the demand of loans for large banks with respect to small banks. Conversely, when $\gamma_1 > \gamma_2$, both banks offer a constant level of loans in the economy and, additionally, small banks are able to lend more than large banks. From the results of Figures 7-8 it emerges that, if the configuration of the costs of both banks is different enough (as in the case of Figure 7), then in the case of homogeneous regulation between banks, the demand of loans of small banks is higher with respect to large banks, independently of the considered value of γ. Otherwise, if regulation is heterogeneous between banks, it results that small banks are more efficient than large banks, in contrast with the evidence shown in Figure 6. Indeed, in one case ($\gamma_1 < \gamma_2$) small banks are able to maintain a constant level of loans, unlike large banks whose demand of loans is characterized by

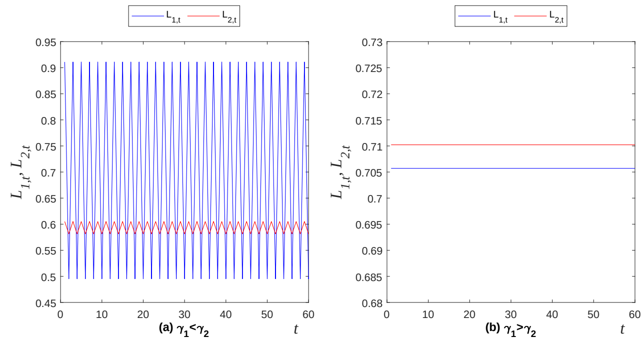

Figure 8: Trajectories of the system for heterogeneous cost of regulation. In (a), $\gamma_1 = 0.09$, $\gamma_2 = 0.11$, $\alpha = 2.8$, $r = 2$, $c_1 = 0.27$, $c_2 = 0.1$ and i.c. $L_{1,0} = 0.6$, $L_{2,0} = 0.55$. In (b) $\gamma_1 = 0.12$, $\gamma_2 = 0.08$, $\alpha = 2.8$, $r = 2$, $c_1 = 0.215$, $c_2 = 0.1$ and i.c. $L_{1,0} = 0.6$, $L_{2,0} = 0.55$.

high volatility (in line with empirical evidence of Stefani et al. (2016)). In the other scenario ($\gamma_1 > \gamma_2$), small banks are able to offer more than large banks. This case shows that it would be desirable to divide the costs of regulation between banks proportionally, according to their size and typology (in line with the proportionality principle) in order to guarantee a stable demand of loans of the entire banking system.

5. Conclusions

This paper has investigated the effects that bank diversification across size and costs can have on efficiency. Our analysis makes use of the theory of discrete dynamical systems, in particular, local stability analysis of equilibria and numerical simulations have been the essential instruments which helped us to deeper analyse the model. Although the high nonlinearity of the system, the conditions found on the parameter values by the analytical part have been crucial to address both the bifurcation analysis and the numerical simulations. Moreover, the latter has allowed us to study the joint effect of different parameters of the model, in particular the costs of both banks and the regulation parameter.

In our theoretical investigation we found that, under suitable conditions, the role of small banks in supporting local communities is a key factor for the growth and the sustainability of the local territories. This emerged in both the analytical and numerical analysis. Indeed, comparing the effects of regulation on the costs of both banks, large banks are better able to handle increased regulation. In contrast, when we focus on the demand of loans of large and small banks, we have seen that in the scenario of homogeneous regulation, the level of the demand of loans of small banks is higher than the level of the loans of large banks. The higher efficiency of small banks with respect to large banks holds also in the scenario of heterogeneous regulation. Indeed, when $\gamma_1 < \gamma_2$ the demand of loans of small banks exhibits lower volatility than the demand of loans of large banks. When $\gamma_1 > \gamma_2$, small banks are able to offer more than large banks. Finally, we have observed that if the costs of regulation are proportionally shared among banks with respect to their size and typology (in line with the proportionality principle), then it is possible to tune the level of regulation allowing an increase of banks' loans (especially those of small banks).

Our work highlights the importance of relationship lending, particularly in a period of financial distress focusing on the role of local banks, which rely more on soft information than not local banks.

Conflict of interest

The authors declare that they have no conflict of interest.

Acknowledgements

We thank all the participants to the 12th Nonlinear Economic Dynamics conference (NED-2021) in Milan, for their useful suggestions. We thank also the participants to the 11th Nonlinear Economic Dynamics conference (NED-2019) in Ukraine where an earlier version of the model was presented.

References

Aiello, F., Bonanno, G., 2013. Profit and cost efficiency in the Italian banking industry (2006-2011). Economics and Business Letters 2, 190–205.

Alessandrini, P., Papi, L., 2018a. Banche locali e piccole imprese dopo la crisi tra nuove regole e innovazioni digitali. Money and Finance Research group (Mo. Fi. R.) Working Papers No 148.

Alessandrini, P., Papi, L., 2018b. L'impatto della" bolla regolamentare" sulle banche: alcune valutazioni. Money and Finance Research group (Mo. Fi. R.) Working Papers No 150.

Assaf, A.G., Berger, A.N., Roman, R.A., Tsionas, M.G., 2019. Does efficiency help banks survive and thrive during financial crises? Journal of Banking & Finance 106, 445–470.

Barboni, G., Rossi, C., 2019. Does your neighbour know you better? The supportive role of local banks in the financial crisis. Journal of Banking & Finance 106, 514–526.

Bischi, G.I., Stefanini, L., Gardini, L., 1998. Synchronization, intermittency and critical curves in a duopoly game. Mathematics and Computers in Simulation 44, 559–585.

Brianzoni, S., Campisi, G., 2021. Dynamical analysis of a banking duopoly model with capital regulation and asymmetric costs. Discrete & Continuous Dynamical Systems - B 26, 5807–5825.

Brianzoni, S., Gori, L., Michetti, E., 2015. Dynamics of a Bertrand duopoly with differentiated products and nonlinear costs: Analysis, comparisons and new evidences. Chaos, Solitons & Fractals 79, 191–203.

Fanti, L., 2014. The dynamics of a banking duopoly with capital regulations. Economic Modelling 37, 340–349.

Fanti, L., Gori, L., Mammana, C., Michetti, E., 2013. The dynamics of a Bertrand duopoly with differentiated products: synchronization, intermittency and global dynamics. Chaos, Solitons & Fractals 52, 73–86.

Fanti, L., Gori, L., Sodini, M., 2012. Nonlinear dynamics in a Cournot duopoly with relative profit delegation. Chaos, Solitons & Fractals 45, 1469–1478.

Fanti, L., Gori, L., Sodini, M., 2015. Nonlinear dynamics in a Cournot duopoly with isoelastic demand. Mathematics and Computers in Simulation 108, 129–143.

Giordano, L., Lopes, A., 2006. Preferenza al rischio e qualità degli impieghi come determinanti dell'efficienza del sistema bancario Italiano. In: A. Giannola (a cura di), Riforme e mutamento strutturale in Italia. Mercato, imprese ed istituzioni in un sistema dualistico, Carocci Editore, Roma .

Klein, M.A., 1971. A theory of the banking firm. Journal of Money, Credit and Banking 3, 205–218.

Maudos, J., Pastor, J.M., Perez, F., Quesada, J., 2002. Cost and profit efficiency in European banks. Journal of International Financial Markets, Institutions and Money 12, 33–58.

Medio, A., Lines, M., 2001. Nonlinear Dynamics: A Primer. Cambridge University Press.

Mesa, R.B., Sánchez, H.M., Sobrino, J.N.R., 2014. Main determinants of efficiency and implications on banking concentration in the European Union. Revista de Contabilidad 17, 78–87.

Monti, M., 1972. Deposit, credit and interest rate determination under alternative bank objective function. North-Holland/American Elsevier.

Puu, T., 1991. Chaos in duopoly pricing. Chaos, Solitons & Fractals 1, 573–581.

Puu, T., 1995. The chaotic monopolist. Chaos, Solitons & Fractals 5, 35–44.

Puu, T., 1998. The chaotic duopolists revisited. Journal of Economic Behavior & Organization 33, 385–394.

Puu, T., 2013. Attractors, bifurcations, & chaos: Nonlinear phenomena in economics. Springer Science & Business Media.

Puu, T., Sushko, I., 2002. Oligopoly dynamics: Models and tools. Springer-Verlag.

Rossi, S.P., Schwaiger, M.S., Winkler, G., 2009. How loan portfolio diversification affects risk, efficiency and capitalization: A managerial behavior model for austrian banks. Journal of Banking & Finance 33, 2218–2226.

Stefani, M.L., Vacca, V., Coin, D., Del Prete, S., Demma, C., Galardo, M., Garri, I., Mocetti, S., Pellegrino, D., 2016. Le banche locali e il finanziamento dei territori: evidenze per l'Italia (2007-2014). 324, Banca d'Italia.

Tramontana, F., 2010. Heterogeneous duopoly with isoelastic demand function. Economic Modelling 27, 350 – 357. doi:https://doi.org/10.1016/j.econmod.2009.09.014.