UNIVERSITÀ POLITECNICA DELLE MARCHE
Repository ISTITUZIONALE

Quantum Computational Methods for Higher Order Modes Detection in Transmission Lines

This is the peer reviewd version of the followng article:

Original
Quantum Computational Methods for Higher Order Modes Detection in Transmission Lines / Colella, E.; Bastianelli, L.; Primiani, V. Mariani; Moglie, F.; Gradoni, G.. - ELETTRONICO. - (2025), pp. 1288-1293. (2025 International Symposium on Electromagnetic Compatibility, EMC Europe 2025 Paris 1 - 5 September 2025) [10.1109/emceurope61644.2025.11176301].

Availability:
This version is available at: 11566/350154 since: 2025-11-18T12:31:53Z

Publisher:
Institute of Electrical and Electronics Engineers Inc.

Published
DOI:10.1109/emceurope61644.2025.11176301

Terms of use:

The terms and conditions for the reuse of this version of the manuscript are specified in the publishing policy. The use of copyrighted works requires the consent of the rights’ holder (author or publisher). Works made available under a Creative Commons license or a Publisher's custom-made license can be used according to the terms and conditions contained therein. See editor’s website for further information and terms and conditions.
This item was downloaded from IRIS Università Politecnica delle Marche (https://iris.univpm.it). When citing, please refer to the published version.

Publisher copyright:

IEEE - Postprint/Author's Accepted Manuscript

©2025 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works. To access the final edited and published work see 10.1109/emceurope61644.2025.11176301

(Article begins on next page)

25 April 2026

Quantum Computational Methods for Higher Order Modes Detection in Transmission Lines

E. Colella [1, 2], L. Bastianelli [1], V. Mariani Primiani [1], F. Moglie [1], G. Gradoni [2],
[1] Università Politecnica delle Marche, Marche, Italy
[2] University of Surrey, Guildford, UK

Abstract— **The efficient computation of higher-order modes in multiconductor transmission lines is crucial, as these modes alter the distribution of TEM modes and increase cross-talk, affecting electromagnetic compatibility and signal integrity in high-frequency circuits. Traditional numerical methods face challenges in handling large-scale eigenvalue problems due to increasing computational complexity. Quantum computing offers a promising alternative by leveraging quantum principles such as superposition and entanglement to solve large eigenvalue problems more efficiently than classical solvers. In this work, we explore the variational quantum eigensolver as a quantum-assisted method for waveguide modal analysis. Starting from the Helmholtz equation for TM modes, we discretize the system using the finite difference method, map the Hamiltonian onto the Pauli basis, and implement the VQE with a hardware-efficient ansatz optimized via BFGS on the Qiskit statevector simulator of IBM [1]. As a test case, we analyze a shielded stripline. The quantum eigensolver successfully computes the first two TM modes and their cutoff frequencies while reconstructing the E_z and E_x field distributions at 1 GHz. This preliminary study shows the feasibility of quantum algorithms for solving large eigenvalue problems in computational electromagnetics where classical computing can fail, opening new possibilities for the efficient analysis of shielded multiconductor transmission lines, where higher-order modes significantly impact cross-talk and signal integrity. Future work will focus on scaling this approach to analyze multiconductor propagation in complex transmission-line structures.**

Keywords— **EMC, Helmholtz equation, shielded stripline, variational quantum eigensolver, waveguide modes.**

I. INTRODUCTION

The advent of quantum computing has ushered in a new era of computational possibilities, with the potential to have brought important contributions in numerous scientific and engineering disciplines [2]. While fault-tolerant quantum computers remain a long-term goal, the current generation of quantum devices, known as noisy intermediate-scale quantum (NISQ) devices, has already demonstrated promise in tackling complex problems that are intractable for classical computers [3]. Among the various quantum algorithms developed for NISQ-era devices, variational quantum algorithms (VQAs) have emerged as a powerful approach to solving optimization and eigenvalue problems [4], [5]. These hybrid quantum-classical algorithms leverage the strengths of quantum computing while mitigating the impact of hardware limitations by using classical optimization techniques to refine quantum circuit parameters.

One of the most prominent VQAs is the variational quantum eigensolver (VQE), originally developed to determine the ground-state energy of molecular systems [6]. The core idea behind VQE is to approximate the lowest eigenvalues of an operator by encoding trial wavefunctions into parameterized quantum circuits and iteratively optimizing these parameters using classical feedback loops [7]. Beyond its initial applications in quantum chemistry, VQE has been successfully extended to other eigenvalue problems, including those arising in physics and engineering [8]. In this work, we apply VQE to solve an important problem in electromagnetics, determining the transverse magnetic (TM) modes of a shielded stripline , as an extension of the work proposed in [9].

Waveguides, and more in general shielded transmission lines, are fundamental components in modern electromagnetic and microwave engineering, serving as transmission structures that confine and guide electromagnetic waves [10]. The analysis of their modal properties is essential for ensuring efficient signal propagation and minimizing unwanted interference, a crucial aspect of electromagnetic compatibility (EMC) [11]. Higher order modes are crucial in signal integrity and cross talk analysis[12]–[15]. The behavior of electromagnetic waves within a waveguide is governed by the Helmholtz equation, which describes how fields propagate under given boundary conditions [16]. For a shielded stripline ([17], Figure 5.7), which consists of a rectangular waveguide with a central conductor embedded within a dielectric medium, the TM modes must satisfy Dirichlet boundary conditions along the conducting surfaces [18]. These boundary conditions ensure that the tangential electric field vanishes at the metallic interfaces, leading to a discrete set of eigenmodes characterized by specific cutoff frequencies [10].

To solve this problem numerically, we discretize the Helmholtz equation using the central finite difference method, which approximates the second-order spatial derivatives over a discretized grid [19]. This discretization transforms the continuous wave equation into a matrix eigenvalue problem, where the eigenvalues correspond to the squared wave numbers of the permissible modes [20]. The cutoff frequencies, which define the lowest frequency at which each mode can propagate, are directly obtained from these eigenvalues [7]. Instead of relying on traditional numerical solvers, we employ the VQE framework to compute these eigenvalues, showcasing how quantum algorithms can be leveraged to solve fundamental problems in electromagnetics.

Beyond its theoretical significance, this approach aligns with ongoing efforts to explore quantum computing applications in engineering and physics. The integration of quantum algorithms into electromagnetic analysis offers a novel perspective on classical computational electromagnetics, potentially leading to more efficient and scalable solutions for complex wave propagation in high dimensional problems [21]. Moreover, by leveraging quantum computing in EMC-related studies, we can develop new methodologies to assess signal integrity, interference, and multiconductor line design in high-frequency applications [22].

In this work, we show how the VQE algorithm can be utilized to determine the TM modes and cutoff frequencies of a closed stripline . We begin by formulating the problem from the Helmholtz equation, incorporating Dirichlet boundary conditions to model the conducting surfaces [23] and internal metallic conductor. The finite difference method is then employed to transform the wave equation into a discretized eigenvalue problem. Finally, we solve this problem using the VQE framework, illustrating the potential of quantum variational algorithms in electromagnetics and computational wave physics.

II. HAMILTONIAN FORMULATION

Figure 1 shows the considered geometry for the present test case, a shielded striplines. The propagation of electromagnetic waves in a waveguide is governed by Maxwell's equations. By considering time-harmonic fields of the form $\mathbf{E}(\mathbf{r},t) = \mathbf{E}(\mathbf{r})e^{j\omega t}$ and assuming a lossless, homogeneous medium, the wave equation for the electric field can be derived. In particular, for the transverse magnetic (TM) modes, where the longitudinal electric field component E_z exists while the transverse components E_x and E_y vanish, the governing equation reduces to the scalar Helmholtz equation:

$$\nabla^2 E_z + k^2 E_z = 0 \tag{1}$$

where $k = \frac{\omega}{c}$ is the wavenumber in the medium. For a shielded stripline with a conducting structure embedded within a dielectric medium, the wave equation must satisfy the appropriate boundary conditions. The Dirichlet boundary condition is imposed at the metallic surfaces of the stripline, ensuring that the tangential component of the electric field vanishes:

$$E_z = 0 \quad \text{on the conducting surfaces.} \tag{2}$$

To solve the Helmholtz equation numerically, the domain is discretized into a uniform grid, and the central finite difference method is applied to approximate the second-order spatial derivatives. In a two-dimensional rectangular grid with spatial steps Δx and Δy, the second-order derivatives are approximated as:

$$\frac{\partial^2 E_z}{\partial x^2} \approx \frac{E_{i+1,j} - 2E_{i,j} + E_{i-1,j}}{\Delta x^2} \tag{3}$$

$$\frac{\partial^2 E_z}{\partial y^2} \approx \frac{E_{i,j+1} - 2E_{i,j} + E_{i,j-1}}{\Delta y^2}. \tag{4}$$

Substituting these into the Helmholtz equation results in the discretized form:

$$\frac{E_{i+1,j} - 2E_{i,j} + E_{i-1,j}}{\Delta x^2} + \frac{E_{i,j+1} - 2E_{i,j} + E_{i,j-1}}{\Delta y^2} + k^2 E_{i,j} = 0. \tag{5}$$

Rearranging, this equation can be rewritten as a matrix eigenvalue problem:

$$\mathbf{A}\mathbf{E} = k^2 \mathbf{E}, \tag{6}$$

where $\mathbf{A}$ is a sparse matrix representing the finite difference discretization of the Laplacian operator, and $\mathbf{E}$ is the vectorized form of E_z over the grid points. The eigenvalues k^2 determine the propagation constants, from which the cutoff frequencies of the TM modes can be extracted. The matrix $\mathbf{A}$ takes the form of a discrete Laplacian operator in two dimensions. Assuming a uniform grid of size $N_x \times N_y$, the matrix $\mathbf{A}$ can be written as a block tridiagonal matrix:

$$\mathbf{A} = \frac{1}{\Delta x^2}\mathbf{I}_{N_y} \otimes \mathbf{T}_{N_x} + \frac{1}{\Delta y^2}\mathbf{T}_{N_y} \otimes \mathbf{I}_{N_x}, \tag{7}$$

where $\otimes$ denotes the Kronecker product, and the tridiagonal matrix $\mathbf{T}_N$ is defined as:

$$\mathbf{T}_N = \begin{bmatrix} -2 & 1 & 0 & \dots & 0 \\ 1 & -2 & 1 & \dots & 0 \\ 0 & 1 & -2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & -2 \end{bmatrix} \in R^{N \times N}. \tag{8}$$

This formulation captures the discrete approximation of the Laplacian operator with Dirichlet boundary conditions, ensuring that the field values at the conducting surfaces remain zero. The eigenvalues of $\mathbf{A}$ correspond to the squared wavenumbers k^2, and solving for these eigenvalues provides the modal distribution and cutoff frequencies of the shielded stripline.

The formulation of this problem in a Hamiltonian framework is particularly useful for quantum computing applications. The matrix $\mathbf{A}$ can be mapped onto a quantum Hamiltonian, allowing the use of variational quantum algorithms such as the VQE to compute its eigenvalues efficiently. In this representation, the discretized wave equation takes the form of the time-indipendent Schrodinger Equation:

$$\hat{H}\left|\psi\right\rangle = \lambda\left|\psi\right\rangle, \tag{9}$$

where $\hat{H}$ corresponds to the Hamiltonian encoding the Laplacian operator, and $|\psi\rangle$ represents the quantum state encoding the wavefunction E_z. By solving this eigenvalue problem using quantum optimization techniques, the mode structures and their corresponding cutoff frequencies can be obtained in a computationally efficient manner.

III. VQE IMPLEMENTATION

The VQE is a hybrid quantum-classical algorithm designed to approximate the lowest eigenvalues of a given Hamiltonian. Originally developed for quantum chemistry applications [6], VQE has since been adapted to solve general eigenvalue problems, making it a powerful tool for computational physics and engineering. Unlike traditional quantum phase estimation algorithms, which require deep quantum circuits and long coherence times, VQE is well-suited for NISQ devices [3]. It leverages parameterized quantum circuits (also known as ansatz) and classical optimization techniques to iteratively minimize a cost function that represents the expectation value of the Hamiltonian.

In the context of our problem, the goal of VQE is to find the lowest eigenvalues of the discretized Helmholtz equation in the analysed structure, formulated as the matrix eigenvalue equation. Since quantum computers naturally operate on qubits using Pauli matrices, we first map the matrix $\mathbf{A}$ onto a quantum Hamiltonian expressed as a sum of tensor products of Pauli operators $\{X, Y, Z, I\}$. Quantum computers work with Hamiltonians represented in terms of Pauli matrices, which form a complete basis for the space of Hermitian operators. Given the sparse nature of the Laplacian matrix $\mathbf{A}$, we decompose it into a linear combination of Pauli strings using the Jordan-Wigner transformation [8]. This results in a Hamiltonian of the form:

$$\hat{H} = \sum_i c_i P_i, \tag{10}$$

where P_i are tensor products of Pauli matrices I, X, Y, Z, and c_i are real coefficients. By expressing the eigenvalue problem in this way, the expectation value of $\hat{H}$ can be computed on a quantum processor by measuring the individual Pauli terms and summing their weighted contributions. A crucial step in the VQE algorithm is selecting a parameterized quantum circuit (ansatz) that approximates the target eigenstates of the Hamiltonian. In this work, we adopt the hardware-efficient ansatz, which consists of a sequence of parameterized single-qubit rotations and entangling gates suited to the connectivity of the quantum hardware (see Fig. 1), [9]. This ansatz takes the form:

$$U(\boldsymbol{\theta}) = U_{\text{ent}} \prod_i R_y(\theta_i), \tag{11}$$

where $R_y(\theta_i) = e^{-i\theta_i Y/2}$ are parameterized rotations, and U_{ent} is a set of entangling gates that introduce quantum correlations among the qubits. The depth of the ansatz controls the expressiveness of the variational wavefunction; deeper circuits provide better approximations but increase the risk of noise and hardware limitations. The VQE algorithm minimizes the expectation value of the Hamiltonian with respect to the trial wavefunction $|\psi(\boldsymbol{\theta})\rangle$, given by:

$$E(\boldsymbol{\theta}) = \langle\psi(\boldsymbol{\theta})|\hat{H}|\psi(\boldsymbol{\theta})\rangle + \sum_{i=0}^{k-1} \langle\psi(\boldsymbol{\theta})|\,\psi(\boldsymbol{\theta}^{(i)})\rangle. \tag{12}$$

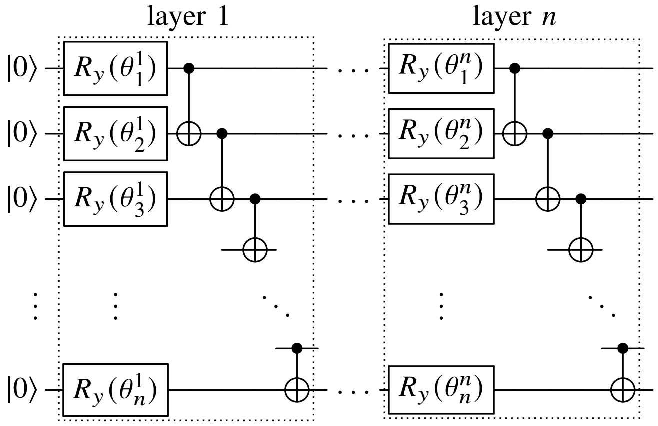

Fig. 1. Hardware-efficient ansatz circuit used in the VQE implementation. The ansatz consists of layers of parameterized single-qubit rotations and entangling gates (CNOTs), designed to efficiently explore the solution space while maintaining compatibility with quantum hardware constraints.

where for each $i \in \{0, \dots, k-1\}, \beta_i$ is chosen to be any large constant satisfying $\beta_i > E_k - E_i$, and $\theta^{(i)}$ is defined as the (previously-found) angle that minimizes (or approximately minimizes) the cost function. This function serves as the cost function for the classical optimizer. By iteratively updating the variational parameters $\boldsymbol{\theta}$, the optimizer seeks to find the minimum eigenvalue of $\hat{H}$, which corresponds to the lowest propagating higher order mode in the waveguide.

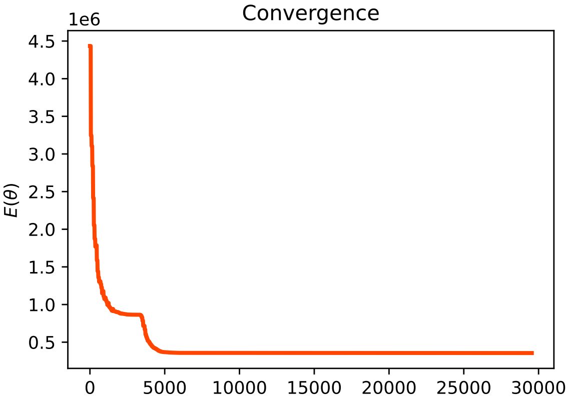

Fig. 2. Convergence of the VQE algorithm to the minimum of the cost function.

To minimize the cost function, we employ a classical optimization algorithm that updates the parameters $\boldsymbol{\theta}$ based on quantum circuit measurements. Common optimization methods include gradient-free approaches such as the Cobyla and Nelder-Mead algorithms, as well as gradient-based methods like BFGS or Adam. Each iteration of the VQE loop consists of the following steps:

1) Prepare the quantum state $|\psi(\boldsymbol{\theta})\rangle = U(\boldsymbol{\theta})\,|0\rangle^{\otimes n}$ using the ansatz $U(\boldsymbol{\theta})$ on the initilized quibits $|0\rangle^{\otimes n}$, with n total number of qubits.

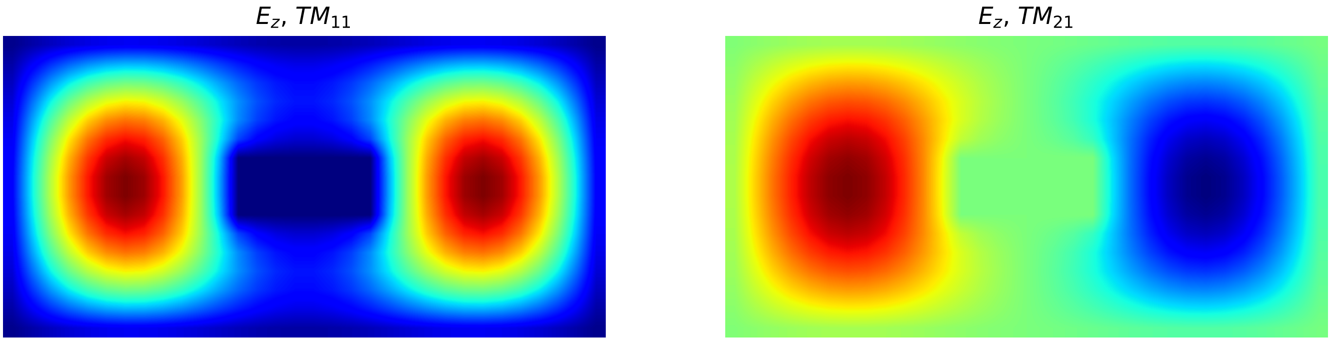

Fig. 3. Distribution of the longitudinal electric field component E_z for the first two transverse magnetic (TM) modes in the shielded striplines. On the left TM_{11} mode. On the right TM_{21} mode. The computed E_z is null at the box and center conductor boundaries.

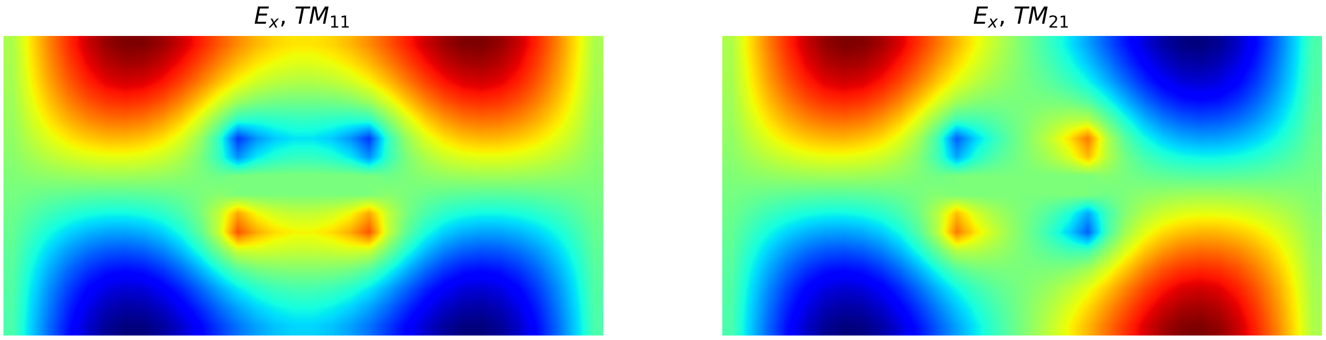

Fig. 4. Distribution of the transverse electric field component E_x for the first two transverse magnetic (TM) modes in the shielded stripline . On the left TM_{11} mode. On the right TM_{21} mode.

2) Compute the cost function $E(\boldsymbol{\theta})$ based on quantum measurements.
3) Use the classical optimizer to update the parameters $\boldsymbol{\theta}$ to minimize $E(\boldsymbol{\theta})$.
4) Repeat until convergence to the lowest eigenvalue.

After running the VQE optimization, we obtain an approximation of the lowest eigenvalue of the Hamiltonian, which corresponds to the smallest eigenvalue k_{min}^2 of the matrix $\mathbf{A}$. From this, we compute the lowest cutoff frequency of the stripline shielded stripline:

$$f_c = \frac{k_{\text{min}} c}{2\pi\sqrt{\varepsilon_r}}, \tag{13}$$

where c is the speed of light, and ε_r is the relative permittivity of the dielectric medium.

In the next section we show the application of the VQE for solving the waveguide modes in a shielded stripline, considering a single conductor transmission line, using the discretization reported in the previous section.

IV. RESULTS

To validate the proposed VQE approach, we applied it to analyze a shielded stripline of dimensions 16×8 mm, filled with air and bounded by metallic surfaces, figure 1. The stripline structure included a central metallic conductor occupying 20% of the waveguide width, ensuring a realistic configuration for guided-wave propagation. The system was discretized using a uniform grid with spatial steps $\Delta x = \Delta y = 1$ mm, leading to a computational domain of $16 \times 8 = 128$ grid points.

After defining the grid parameters, we formulated the Hamiltonian corresponding to the finite-difference discretization of the Helmholtz equation for TM modes. The resulting sparse matrix $\mathbf{A}$ was mapped onto a quantum Hamiltonian by decomposing it into a linear combination of Pauli operators, using a procedure such as the Jordan-Wigner transformation.

Given the dimensionality of the problem, the Hamiltonian was encoded into a 7-qubit register, ensuring sufficient resolution while maintaining computational feasibility on current quantum hardware. The VQE routine was then executed to minimize the cost function, corresponding to the expectation value of the Hamiltonian over the parameterized ansatz state. We employed the BFGS (Broyden-Fletcher-Goldfarb-Shanno) algorithm as the classical optimizer, due to its efficiency in handling smooth and differentiable cost landscapes. The BFGS algorithm updates the variational parameters $\boldsymbol{\theta}$ iteratively by computing an approximation of the inverse Hessian matrix, following the update rule:

$$\boldsymbol{\theta}_{n+1} = \boldsymbol{\theta}_n - \alpha B_n^{-1} \nabla E(\boldsymbol{\theta}_n), \tag{14}$$

where α is the step size, $\nabla E(\boldsymbol{\theta}_n)$ is the gradient of the cost function at step n, B_n^{-1} is an approximation of the inverse Hessian matrix. The optimization process iteratively updated the variational parameters $\boldsymbol{\theta}$ of the hardware-efficient ansatz, systematically reducing the cost function until convergence was achieved.

The Qiskit Statevector simulator of IBM was used to run the VQE routine, allowing for an exact simulation of quantum state evolution without noise effects. At the end of the minimization, the optimal parameters were assigned to the ansatz circuit, allowing us to extract the physical quantities of interest.

The cutoff frequencies of the shielded stripline were determined from the expectation value of the optimized ansatz state according to (11) where k^2_{min} is the lowest eigenvalue obtained from the VQE routine, c is the speed of light, and $\varepsilon_r = 1$ for air-filled waveguides. After around 30000 running steps, the VQE algorithm converged to the minimum of the cost function (Fig. 2). As a preliminary example, we computed the first two TM modes for E_z and E_x , and their corresponding cutoff frequencies, as shown in Fig. 3 and Fig. 4. Differencies on the transverse component Ex are well evident respect to the TEM mode only. The presence of the longitudinal component Ez can be responsible for crosstalk enhancement, respect to that due only to TEM mode, in case of presence of other conductors.

The computed cutoff frequencies were $f_{c_1} = 29.37$ GHz and $f_{c_2} = 29.51$ GHz, where the adopted discretization fulfilled the $\lambda/10$ condition. Furthermore, the eigenmodes of the system were reconstructed by evaluating the probability distribution of the optimal ansatz state over the grid points. This allowed us to map the distribution of the longitudinal electric field component E_z across the cross-section of the stripline, providing insight into the spatial characteristics of the computed TM modes.

V. CONCLUSION

In this work, we have demonstrated the application of the Variational Quantum Eigensolver (VQE) to determine the higher order modes and cutoff frequencies of a shielded stripline. Starting from the Helmholtz equation for TM modes, we discretized the system using the finite difference method, formulated the problem as a Hamiltonian eigenvalue problem, and mapped it onto a quantum Hamiltonian expressed in the Pauli basis.

The resulting eigenvalue problem was then solved using the VQE algorithm implemented on the Qiskit Statevector simulator (IBM), employing a hardware-efficient ansatz optimized via the BFGS classical minimization algorithm. By running the VQE optimization, we successfully extracted the first two TM eigenmodes of the considered stripline, together with their cutoff frequencies.

These preliminary results validate the potential of quantum computing for solving eigenvalue problems in electromagnetics, particularly in waveguide modal analysis.

The broader goal of this research is to efficiently compute higher-order modes in shielded multiconductor transmission lines, where the presence of higher order modes modifies the distribution of Transverse Electromagnetic (TEM) filed, so increasing cross-talk between transmission lines. Accurately resolving these higher-order modes is crucial for electromagnetic compatibility (EMC) studies, as they play a key role in signal integrity, interference mitigation, and high-frequency circuit design.

While this work focused on a single-conductor stripline, it serves as a foundation for extending the method to more complex multiconductor configurations, where coupling effects and mode interactions become significant. Our next step is to scale this approach to analyze multimodal propagation in multiconductor shielded striplins, which will be presented in an upcoming conference contribution. Subsequently, the metodology migth be extended to open multiconductor transmission lines such as complex PCB traces.

This research paves the way for leveraging quantum computing in computational electromagnetics, with the long-term goal of improving transmission line analysis and design, signal integrity, and interference reduction in high-speed electronic circuits.

The proposed quantum-classical workflow pioneers the use of VQE for waveguide modal analysis, representing the first demonstration of quantum algorithms to extract higher-order transverse magnetic modes in a shielded stripline. By discretizing the Helmholtz equation with a finite-difference scheme and mapping the resulting eigenvalue problem onto a sum of Pauli operators, the method leverages quantum superposition to encode and solve high-dimensional, complex electromagnetic-compatibility problems using only a few number of qubits. This end-to-end formulation not only opens the door to tackling scenarios that can overwhelm classical solvers but also establishes a clear pathway for scaling toward multiconductor and intricate PCB interconnect geometries.

However, all validations to date rely on noise-free, statevector simulations, leaving open key questions about real-world performance. To show genuine quantum advantage, the approach must be ported to hardware platforms—complete with error mitigation strategies—and extended beyond the second mode to include higher-order interactions and multiconductor coupling effects. Such on-device experiments will be crucial for assessing resource overheads (Pauli-term counts, measurement shots, ansatz depth) and for validating the method's applicability to realistic crosstalk and EMC challenges.

REFERENCES

[1] A. Cross, "The ibm q experience and qiskit open-source quantum computing software," in *APS March meeting abstracts*, vol. 2018, 2018, pp. L58–003.

[2] J. Preskill, "Quantum computing in the nisq era and beyond," *Quantum*, vol. 2, p. 79, 2018. DOI: 10.22331/q-2018-08-06-79.

[3] K. Bharti, A. Cervera-Lierta, T. T. Kyaw, *et al.*, "Noisy intermediate-scale quantum algorithms," *Reviews of Modern Physics*, vol. 94, p. 015 004, 2022. DOI: 10.1103/RevModPhys.94.015004.

[4] M Cerezo, A. Arrasmith, R. Babbush, *et al.*, "Variational quantum algorithms," *Nature Reviews Physics*, vol. 3, no. 9, pp. 625–644, 2021. DOI: 10.1038/s42254-021-00348-9.

[5] E. Colella, L. Bastianelli, V. M. Primiani, Z. Peng, F. Moglie, and G. Gradoni, "Quantum optimization of reconfigurable intelligent surfaces for mitigating multipath fading in wireless networks," *IEEE Journal on Multiscale and Multiphysics Computational Techniques*, 2024.

[6] A. Peruzzo, J. McClean, P. Shadbolt, *et al.*, "A variational eigenvalue solver on a photonic quantum processor," *Nature Communications*, vol. 5, p. 4213, 2014. DOI: 10.1038/ncomms5213.

[7] R. C. Rumpf, *Electromagnetic and Photonic Simulation for the Beginner: Finite-Difference Frequency-Domain in MATLAB*. Artech House, 2022.

[8] J. R. McClean, J. Romero, R. Babbush, and A. Aspuru-Guzik, "The theory of variational hybrid quantum-classical algorithms," *New Journal of Physics*, vol. 18, no. 2, p. 023 023, 2016. DOI: 10.1088/1367-2630/18/2/023023.

[9] E. Colella, S. Beloin, L. Bastianelli, V. M. Primiani, F. Moglie, and G. Gradoni, "Variational quantum shot-based simulations for waveguide modes," *IEEE Transactions on Microwave Theory and Techniques*, vol. 72, no. 4, pp. 2084–2094, 2024. DOI: 10.1109/TMTT.2023.3339243.

[10] D. M. Pozar, *Microwave Engineering*. John Wiley & Sons, 2011.

[11] C. R. Paul, *Introduction to Electromagnetic Compatibility*. John Wiley & Sons, 2006.

[12] C. Li, K. Cai, M. Ouyang, *et al.*, "Mode-decomposition-based equivalent via (mev) model and mev model application range analysis," in *2023 Joint Asia-Pacific International Symposium on Electromagnetic Compatibility and International Conference on ElectroMagnetic Interference & Compatibility (APEMC/INCEMIC)*, 2023, pp. 1–4. DOI: 10.1109/APEMC57782.2023.10217672.

[13] M. Alam, M. Koshiba, K. Hirayama, and Y. Hayashi, "Hybrid-mode analysis of multilayered and multiconductor transmission lines," *IEEE Transactions on Microwave Theory and Techniques*, vol. 45, no. 2, pp. 205–211, 1997. DOI: 10.1109/22.557601.

[14] Olyslager, Femke and Franchois, Ann, "Transmission line representations for higher order modes in lossy waveguides," und, in *2004 URSI EMTS - International Symposium on Electromagnetic Theory*, 2004, 263–265, ISBN: 88-8492-252-6.

[15] D. Williams and F. Olyslager, "Modal cross power in quasi-tem transmission lines," *IEEE Microwave and Guided Wave Letters*, vol. 6, no. 11, pp. 413–415, 1996. DOI: 10.1109/75.541457.

[16] R. F. Harrington, *Time-harmonic Electromagnetic Fields*. McGraw-Hill, 2001.

[17] R. Chatterjee, *Elements of Microwave Engineering* (Ellis Horwood series in electrical and electronic engineering). Ellis Horwood, 1986, ISBN: 9780745800608. [Online]. Available: https://books.google.it/books?id=rmGSQgAACAAJ.

[18] R. E. Collin, *Field Theory of Guided Waves*. McGraw-Hill, 1991.

[19] K. Wang, Y. Zhang, and R. Guo, "Finite difference methods for the helmholtz equation: A brief review," *Mathematica Numerica Sinica*, vol. 40, no. 2, pp. 171–190, 2018.

[20] A. Taflove and S. C. Hagness, *Computational Electrodynamics: The Finite-Difference Time-Domain Method*. Artech House, 2005.

[21] A. Montanaro, "Quantum algorithms: An overview," *npj Quantum Information*, vol. 2, no. 1, pp. 1–8, 2016.

[22] L. F. Knockaert and D. De Zutter, "On the complex symmetry of the poincaré-steklov operator," *Progress in Electromagnetics Research B*, vol. 6, pp. 41–53, 2008.

[23] L. Zhang, "The finite difference method for the helmholtz equation with applications to cloaking," 2009.