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Stability in parametric resonance of a controlled stay cable with time delay

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14 ABSTRACT

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The stability of the parametric resonance of the semiactive control of a stay cable with time 15 delay is investigated. The in-plane nonlinear equations of motion are initially obtained via the 16 Hamilton principle. Then, utilizing the method of multiple scales, the modulation equations that 17 govern the nonlinear dynamics are obtained. These equations are then utilized to investigate the 18 effect of time delays on the amplitude and frequency-response behavior and, subsequently, on the 19 stability of the parametric resonance of the controlled cable, that it is shown to depend on the 20 excitation amplitude and the commensurability of the delayed-response frequency to the excitation 21 frequency. The stability region of the parameteric resonance is shifted, and the effects of control on 22 the cable become worse by increasing time delay. The work plays a guiding role in the parametric 23 design of the control system for stay cables. 24

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25 INTRODUCTION

As the main bearing member of long-span structure, the stayed cable is characterized by light weight and small damping, hence it vibrates easily due to external excitation, such as wind, rain, traffic or earthquake (Irvine, 1981; Warminski et al., 2016; Ni et al., 2007). In recent years, largescale vibrations of the stayed cables of bridges have been observed at low wind speeds, which is generally considered to be the result of a parametric resonance phenomenon (Ni et al., 2007; da Costa et al., 1996). Therefore, it is important to investigate the vibration mechanism of cables.

Hikami and Shiraishi (1988) and Matsumoto et al. (1992) investigated the mechanism of rain-32 wind induced vibration of cable of cable-stayed bridges and proposed aerodynamic countermeasures 33 to suppress the vibration. Jafari et al. (2020) reviewed the past studies about different types of wind-34 induced cable vibration. Zhao et al. (2014) discussed the analytical solutions for resonant response 35 of suspended cables subjected to external excitation. Lenci and Ruzziconi (2009) studied nonlinear 36 phenomena in the single-mode dynamics of a cable-supported beam. Gattulli et al. (2019) analyzed 37 the modal interactions in the nonlinear dynamics of a beam-cable-beam. It is worth pointing out 38 that the parametric vibration of the stay cable is one of the main aspects. Wang and Zhao (2009) 39 addressed the large amplitude motion mechanism and the non-planar vibration character of stay 40 cables subject to the support motions. Ying et al. (2006) investigated the parametrically excited 41 instability of a cable under two support motions. Guo and Rega (2021a,b) studied the modal 42 dynamics of boundary-interior coupled structures. Cong and Kang (2019) considered the planar 43 nonlinear dynamic behavior of a cable-stayed bridge under excitation of tower motion. Lu et al. 44 (2020) studied nonlinear parametric vibration with different orders of small parameters for stayed 45 cables. 46

In parallel the the previous works, that are focused on understanding complex nonlinear phenomena, other studies focused on the vibration control of cables. Fujino and Susumpow (1994) studied active control of in-plane cable vibration by axial support motion via experiments. Wang et al. (2005) investigated optimal design of viscous dampers for multimode vibration control of bridge cables. Ying et al. (2007) studied parametrically excited instability analysis of a semi-

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actively controlled cable. Dai et al. (2014) addressed an extended nonlinear elastic cable with
 an active vibration control strategy. Tehrani and Kalkowski (2016) investigated active control of
 parametrically excited systems. Raftoyiannis and Michaltsos (2016) studied movable anchorage
 system for vibration control of stay-cables in bridges. Huang et al. (2019) evaluated the perfor mances of inerter-based damping devices for structural vibration control of stay cables. Peng et al.
 (2020) investigated nonlinear primary resonance in vibration control of cable-stayed beam via time
 delayed feedback control.

It has been shown (Hu and Wang, 2002; Sipahi et al., 2011) that in the vibration control sys-59 tem the time delay is *not* negligible. Cha et al. (2012) studied time delay effects on large-scale 60 MR damper based semi-active control strategies. Yan et al. (2020) considered energy determin-61 ing multiple stability in time-delayed systems. Udwadia et al. (2007) presented principles and 62 applications of time-delayed control design for active control of structures. Ji and Zhou (2017) 63 investigated coexistence of two families of sub-harmonic resonances in a time-delayed nonlinear 64 system at different forcing frequencies. Wang et al. (2017) and Wang and Xu (2017) studied effect 65 of delay combinations on stability and Hopf bifurcation of an oscillator with acceleration-derivative 66 feedback and sway reduction of a pendulum on a movable support using a delayed proportional-67 derivative or derivative-acceleration feedback. Sun et al. (2018) studied parameter design of a 68 multi-delayed isolator with asymmetrical nonlinearity. Their results showed that time delay can 69 affect the damping performance of the control system, and, on the other hand, making good use of 70 it can provide another control idea and improve control performance. 71

As a matter of fact, very few studies concerned with the time delay effects in nonlinear parametric
 resonance of controlled cables, and filling this gap is the main goal of this work. It leads to interesting
 and partially unexpected results in terms of performance (or better, loss of performance) of the
 considered control.

(They should be better underlined the differences with respect to our previous paper (Peng et al.,
2020))

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The mechanical model of controlled cable under axial excitation is considered. The method of

multiple scales is used to analyze the parametric vibration under the influence of time delay. The
 stability of parametric resonance of the controlled stay cable is discussed, and the time delay effect
 of the parametric vibration system is discussed by numerical examples.

82 CONTROLLED CABLE MODEL AND EQUATIONS OF MOTION

As shown in Fig. 1, a stayed cable subject to a vertical sinusoidal support motion $Z \sin \omega t$ (where Z and ω denote the amplitude and frequency, respectively), is considered. A Cartesian coordinate system O - xy is chosen, with the origin O placed at the left fixed support A of the cable. The displacements of the points are denoted by u(x, t) and v(x, t) along the x and y directions, respectively. a is the distance between the right oscillating boundary B and the MR damper.

⁸⁸ The axial Lagrangian strain of the inclined cable can be written as

$$\varepsilon(x,t) = u' + y'v' + \frac{{v'}^2}{2},$$
 (1)

where prime indicates differentiation with respect to the spatial coordinate x and y(x) is the static configuration of the cable, that can be approximately written as $y(x) = \frac{mgl\cos\theta}{2H}x(1-x)$. The equations of motions can be obtained by means of the Hamilton principle (Wang and Zhao, 2009)

$$m\ddot{u} + c_u\dot{u} - \left\{ EA\left[u' + y'v' + \frac{{v'}^2}{2}\right] \right\}' = 0,$$
(2)

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$$m\ddot{v} + c_v\dot{v} - \left\{Hv' + EA(y'+v')\left[u'+y'v'+\frac{{v'}^2}{2}\right]\right\}' = 0,$$
(3)

where dot indicates differentiation with respect to time *t*, *m* is the mass per unit length; *E* is the Young modulus, *A* is the area of the cross-section, c_u and c_v are the viscous damping coefficients per unit length, *H* is the axial component of the initial tension ($H \ll EA$) and *g* is the gravity acceleration. The boundary conditions can be written as

$$u(0,t) = v(0,t) = 0, \quad u(l,t) = Z\sin\theta\sin(\omega t), \quad v(l,t) = Z\cos\theta\sin(\omega t), \quad (4)$$

where *l* is the cable span and θ is the angle of inclination of the cable (see Fig. 1). It is worth to remark that the boundary conditions are nonhomogeneous both in the axial displacement component u(x, t) and in-plane transverse displacement component v(x, t).

¹⁰⁴ Under the quasi-static assumption in the axial direction, i.e, neglecting the acceleration and ¹⁰⁵ velocity term in Eq. (2), and taking into account the boundary conditions, the displacement u(x, t)¹⁰⁶ can be expressed by

$$u(x,t) = Z\sin\theta\sin(\omega t)\frac{x}{l} + \frac{x}{l}\int_0^l \left(y'v' + \frac{v'^2}{2}\right)dx - \int_0^x \left(y'v' + \frac{v'^2}{2}\right)dx.$$
 (5)

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Inserting Eq. (5) in Eq. (3) it is possible to obtain an equation in the primary unknown v(x, t). Then, considering the concentrated force at x = l - a due to the damper (introduced to reduce the cable oscillations) and the distributed external load, and proceeding in a manner similar to (Peng et al., 2020), the non-dimensional equations of motion can be written as

¹¹²
$$\ddot{v} + c_v \dot{v} - v'' - \alpha (y'' + v'') \left\{ z_0 \sin \theta \sin(\Omega t) + \int_0^l \left(y' v' + \frac{v'^2}{2} \right) dx \right\} = F_d \delta(x - (l - a)) + F \cos(\Omega t),$$
(6)

where $F_d = -C_{eq}\dot{v}(t-\tau)$ is the control force of the damper, τ the time delay of the control system, F(x) the spatial distribution of the distributed force and δ is the Dirac delta function. The non-dimensional variables are $x^* = x/l$, $a^* = a/l$, $y^* = y/l$, $z_0 = Z/l$, $v^* = v/l$, $\alpha = EA/H$, $t^* = t/l\sqrt{H/m}$, $\Omega = \omega l/\sqrt{m/H}$, $c_v^* = c_v l/(m)\sqrt{m/H}$. Asterisks in Eq. (6) are dropped for simplicity.

For the nonhomogeneous boundary value problem, it is convenient to introduce a suitable chosen particular solution, which satisfies the nonhomogeneous boundary conditions, to transform the nonhomogeneous problem to a homogeneous one. Then, the solution of the homogeneous problem can be approximated by a time-varying linear combination of known (and fixed) spatial functions, which are assumed to be the eigenfunctions of the homogeneous problem. In this study, according to the boundary condition of the inclined cable, the non-dimensional displacements v(x, t) is sough after in the form

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$$v(x,t) = \sum_{i=1}^{N} \phi_i(x) q_i(t) + x z_0 \cos \theta \sin(\Omega t), \tag{7}$$

where $q_i(t)$ are the generalized displacements, and $\phi_i(x) = \sqrt{2} \sin(i\pi x)$ the *i*th in-plane mode shapes. Substitution of Eq. (7) into Eq. (6) and application of the Galerkin method yield a set of nonlinear ordinary differential equations

(in the following equation:

• the term due to the damper (proportional to F_d) is missing;

• F is missing;

• if the $\phi_i(x)$ are the linear normal modes, the linear part (without excitation) should be decoupled, i.e. the Γ_{2ij} should be 0 for $i \neq j$;

• the definition of the coefficients Γ is strange/incosistent: the Γ_1 are the time dependent, while all other not. I suggest to rewrite in such a way that all Γ s are time independent, and the harmonic terms appear explicitly in the equation.

¹³⁷ Please check carefully the previous points)

$$\ddot{q}_{i} + 2\omega_{i}\xi_{i}\dot{q}_{i} + \Gamma_{1i}q_{i} + \sum_{j=1}^{N} (\Gamma_{2ij}q_{j} + \Gamma_{3ij}q_{j}^{2} + \Gamma_{4ij}q_{j}q_{i} + \Gamma_{5ij}q_{j}^{2}q_{i})$$

$$= \Gamma_{6i}\sin(\Omega t) + \Gamma_{7i}\cos(\Omega t) + \Gamma_{8i}\sin^{2}(\Omega t), \quad i = 1, 2, ..., N,$$
(8)

where ξ_i are the viscous damping ratios, $\omega_i = \sqrt{\Gamma_{1i} + \Gamma_{2ii}}$ (please check this) the *i*th in-plane natural

frequencies, and the other coefficients are given by

$$\Gamma_{1i} = i^{2} (1 + \alpha z_{0} \sin \theta \sin(\Omega t) + \frac{1}{2} \alpha z_{0}^{2} \cos^{2} \theta \sin^{2}(\Omega t)),$$

$$\Gamma_{2ij} = \alpha \int_{0}^{1} y' \phi_{i}'(x) dx \int_{0}^{1} y' \phi_{j}'(x) dx,$$

$$\Gamma_{3ij} = \frac{\alpha}{2} j^{2} \int_{0}^{1} y' \phi_{i}'(x) dx,$$

$$\Gamma_{4ij} = \alpha i^{2} \int_{0}^{1} y' \phi_{j}'(x) dx,$$

$$\Gamma_{5ij} = \frac{\alpha}{2} i^{2} j^{2},$$

$$\Gamma_{6i} = z_{0} \Omega^{2} \cos \theta \int_{0}^{1} x \phi_{i}(x) dx - \alpha z_{0} \sin \theta \int_{0}^{1} y' \phi_{i}'(x) dx,$$

$$\Gamma_{7i} = -2\xi_{i} i z_{0} \Omega \cos \theta \int_{0}^{1} x \phi_{i}(x) dx.$$
(9)

139 LINEAR STABILITY ANALYSIS

In this section, the linear stability analysis of the single degree of freedom vibration mode is investigated. Considering only one equation i = n in Eq. (8), neglecting the nonlinear terms and the external excitation ($z_0 = 0$) the following equation is obtained ($\mu_n = 2\omega_n\xi_n$) (in the following equation k_n is not defined)

$$\ddot{q}_n(t) + \mu_n \dot{q}_n(t) + \omega_n^2 q_n(t) = -k_n \dot{q}_n(t - \tau).$$
(10)

The solution of Eq. (10) is given by

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$$q_n = A_n e^{(\xi_n + i\lambda_n)t} \tag{11}$$

where A_n , ξ_n and λ_n are amplitude, damping coefficient and response frequency, respectively. All

are real numbers. Substituting Eq. (11) in Eq. (10), and separating real and imaginary parts, gives

$$\lambda_n \left(2\xi_n + \mu_n\right) e^{\xi_n \tau} + k_n \left[\lambda_n \cos\left(\lambda_n \tau\right) - \xi_n \sin\left(\lambda_n \tau\right)\right] = 0 \tag{12}$$

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$$\left(\lambda_n^2 - \xi_n^2 - \mu_n \xi_n - \omega_n^2\right) e^{\xi_n \tau} - k_n \left[\xi_n \cos\left(\lambda_n \tau\right) + \lambda_n \sin\left(\lambda_n \tau\right)\right] = 0.$$
(13)

When $\xi_n < 0$ the solution (11) converges to 0 for $t \to \infty$ and thus is stable, while for $\xi_n > 0$ the solution diverges to infinity and thus is unstable. The stability limit is then given by $\xi_n = 0$. Substituting this value in Eq. (12) and Eq. (13) we obtain

$$\cos(\lambda_n \tau) = -\frac{\mu_n}{k_n}, \quad \sin(\lambda_n \tau) = \frac{\lambda_n^2 - \omega_n^2}{k_n \lambda_n}, \tag{14}$$

and thus the boundary of linear stability are

$$\tau = \frac{1}{\lambda_n} \left[\tan^{-1} \left(-\frac{\lambda_n^2 - \omega_n^2}{\lambda_n \mu_n} \right) + j\pi \right], \quad j = 0, 1, \cdots, \quad k_n = \pm \frac{\sqrt{\lambda_n^2 \mu_n^2 + (\lambda_n^2 - \omega_n^2)^2}}{\lambda_n}.$$
(15)

The stability regions described by Eq. (15) are shown in Fig. 2, where regions i, ii and iii 158 corresponds to a small, medium and large values of time delay, respectively. The figure clearly 159 shows that for small values of the delay the system is stable, and thus the control effective, even for 160 very large values of the gain k_n . For medium and large values of τ , on the other hand, the stability 161 region is a narrow strip around $k_n = 0$, namely the system is stable only for very low values of k_n , 162 giving not good performance because with small values of the gain the damping is low and the 163 vibration reduction is ineffective. For quite large values of k_n , the system is stable for low values of 164 the delay, and loses stability for increasing τ . This could be very dangerous from a practical point 165 of view, because unplanned increasing delay of the control, due for example to the ageing of the 166 structure, can destabilize the system, with unwanted phenomena up to collapse. 167

168 STABILITY OF THE PARAMETRICALLY RESONANCE RESPONSE



In this section, we continue to consider the single degree of freedom vibration mode, but extend

the analysis to the nonlinear regime, utilizing the method of multiple scales (Nayfeh and Mook,
1979).

It is convenient to introduce a small bookkeeping parameter ε to obtain the solution. The equation of the motion can be written as ((16) is not consistent with (8): here - correctly from my point of view - the Γ are not time dependent, see my previous comments just before Eq. (8). Please check and modify)

$$\ddot{q}_{n} + \omega_{n}^{2} q_{n} + \varepsilon \Gamma_{1nn} q_{n} \cos(\Omega t) + \varepsilon \mu \dot{q}_{n} + \varepsilon (\Gamma_{3nn} + \Gamma_{4nn}) q_{n}^{2} + \varepsilon \Gamma_{5nn} q_{n}^{3} = -\varepsilon k_{n} \dot{q}_{n} (t - \tau) + \varepsilon \Gamma_{6nn} \sin(\Omega t) + \varepsilon \Gamma_{7nn} \cos(\Omega t) + \varepsilon \Gamma_{8nn} \sin^{2}(\Omega t),$$
(16)

177 The solution of Eq. (16) is sought after in the form

$$q_n(t;\varepsilon) = q_{n0}(T_0, T_1, ...) + \varepsilon q_{n1}(T_0, T_1, ...) + \cdots$$
(17)

where $T_n = \varepsilon^n t$, n = 0, 1, 2. It is further assumed that

$$\omega_n = \frac{\Omega}{2} + \varepsilon \omega_1 + \varepsilon^2 \omega_2 + \cdots$$
 (18)

¹⁸¹ Substituting Eq. (17) and Eq. (18) in Eq. (16), and equating the coefficients of ε^0 and ε^1 on both ¹⁸² sides, we obtain

$$D_0^2 q_{n0} + \frac{\Omega^2}{4} q_{n0} = 0, (19)$$

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$$D_{0}^{2}q_{n1} + \frac{\Omega^{2}}{4}q_{n1} = -2D_{0}D_{1}q_{n0} - \Omega\omega_{1}q_{n0} - \Gamma_{1nn}q_{n0}\cos(\Omega t) - \mu D_{0}q_{n0} - (\Gamma_{3nn} + \Gamma_{4nn})q_{n0}^{2}$$

$$-\Gamma_{5nn}q_{n0}^{3} - k_{n}\dot{q}_{n0}(t-\tau) + \Gamma_{6nn}\sin(\Omega t) + \Gamma_{7nn}\cos(\Omega t) + \Gamma_{8nn}\sin^{2}(\Omega t),$$
(20)

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where D_n denotes the derivatives with respect to T_n .

¹⁸⁷ The general solution of Eq. (19) can be written as

$$q_{n0} = A_n(T_1) \exp\left(\frac{i\Omega T_0}{2}\right) + \bar{A}_n(T_1) \exp\left(\frac{-i\Omega T_0}{2}\right).$$
(21)

¹⁸⁹ Substituting Eq. (21) in Eq. (20) we obtain (in the following equation $sin(\Omega t)$ must be transformed ¹⁹⁰ in the exponential form. Furthermore, it is convenient to collect terms multiplying the same ¹⁹¹ exponential terms (as is has been done for $exp(\frac{i\Omega T_0}{2})$). Please check.)

$$D_{0}^{2}q_{n1} + \frac{\Omega^{2}}{4}q_{n1} = -\left[i\Omega A_{n}' + \Omega\omega_{1}A_{n} + \frac{\Gamma_{1nn}}{2}\bar{A}_{n} + \frac{1}{2}i\mu\Omega A_{n} + 3\Gamma_{5nn}A_{n}^{2}\bar{A}_{n} + \frac{1}{2}k_{n}i\Omega A_{n}\exp\left(-\frac{i\Omega\tau}{2}\right)\right]$$

$$\exp\left(\frac{i\Omega T_{0}}{2}\right) - \frac{\Gamma_{1nn}}{2}A_{n}\exp\left(\frac{3i\Omega T_{0}}{2}\right) - (\Gamma_{3nn} + \Gamma_{4nn})A_{n}^{2}\exp(i\Omega T_{0}) - (\Gamma_{3nn} + \Gamma_{4nn})$$

$$A_{n}\bar{A}_{n} - \Gamma_{5nn}A_{n}^{3}\exp\left(\frac{3i\Omega T_{0}}{2}\right) + \frac{\Gamma_{7nn}}{2}\exp(i\Omega T_{0}) + \Gamma_{6nn}\sin(\Omega t) + \Gamma_{8nn}\sin^{2}(\Omega t) + cc,$$
(22)

where *cc* denotes the complex conjugate of the preceding terms. To eliminate secular terms from q_{n1} we must put

$$i\Omega A_{n}' + \Omega \omega_{1} A_{n} + \frac{\Gamma_{1nn}}{2} \bar{A}_{n} + \frac{1}{2} i\mu \Omega A_{n} + 3\Gamma_{5nn} A_{n}^{2} \bar{A}_{n} + \frac{1}{2} k_{n} i\Omega A_{n} \exp\left(-\frac{i\Omega\tau}{2}\right) = 0.$$
(23)

¹⁹⁶ To solve Eq. (23), we write A_n in the polar form:

$$A_n = \frac{1}{2}a_n \exp(i\beta_n),\tag{24}$$

where a_n and β_n are real functions of T_1 . Substituting Eq. (24) in Eq. (23) and separating real and imaginary parts, we have

$$a'_n = -\frac{1}{2}\mu_e a_n + \frac{\Gamma_{1nn}a_n}{2\Omega}\sin 2\beta_n,$$
(25)

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$$\beta_n' = \omega_1 + \frac{k_n}{2} \sin\left(\frac{\Omega\tau}{2}\right) + \frac{3\Gamma_{5nn}}{4\Omega}a_n^2 + \frac{2\Gamma_{1nn}}{\Omega}\cos 2\beta_n,\tag{26}$$

where $\mu_e = \mu + k_n \cos\left(\frac{\Omega \tau}{2}\right)$.

When $a'_n = \gamma'_n = 0$, the sought periodic solution is obtained. Considering the nontrivial

solutions $(a_n \neq 0)$, from Eq. (25) and Eq. (26) we can then obtain

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$$\sin 2\beta_n = \frac{\Omega\mu_e}{\Gamma_{1nn}} \tag{27}$$

Remembering that $\cos(2\beta_n) = \pm \sqrt{1 - \sin^2(2\beta_n)}$ and substituting Eq. (27) in Eq. (26), we obtain the amplitude of the steady solution

$$a_n^2 = -\frac{4\Omega}{3\Gamma_{5nn}} \left[\omega_1 + \frac{k_n}{2} \sin\left(\frac{\Omega\tau}{2}\right) \right] \pm \frac{8\Gamma_{1nn}}{3\Gamma_{5nn}} \sqrt{1 - \frac{\Omega^2 \mu_e^2}{\Gamma_{1nn}^2}},\tag{28}$$

which is the frequency-response equation, since the excitation amplitude z_0 in within Γ_{1nn} .

It is worth to underline that Γ_{6nn} , Γ_{7nn} and Γ_{8nn} do not appear in Eq. (28) because we are focusing on the parametric excitation (this is reflected in the choice (18)). They would appear if one consider the external resonance, i.e. $\omega_n \approx \Omega$. This is left for future work.

Since a_n is a real function, from $a_n^2 > 0$ we obtain first order approximate region of existence of the periodic solution

 $\omega_1 < -\frac{k_n}{2} \sin\left(\frac{\Omega\tau}{2}\right) \pm \frac{2\Gamma_{1nn}}{\Omega} \sqrt{1 - \frac{\Omega^2 \mu_e^2}{\Gamma_{1nn}^2}}.$ (29)

Inserting this expression in Eq. (18), and remembering that $\varepsilon k_n = \hat{k}_n$, $\varepsilon \mu_e = \hat{\mu}_e$ and $\varepsilon \Gamma_{1nn} = \hat{\Gamma}_{1nn}$, yields

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$$\frac{2\omega}{\Omega} < 1 - \frac{\hat{k}_n}{\Omega} \sin\left(\frac{\Omega\tau}{2}\right) \pm \frac{4\hat{\Gamma}_{1nn}}{\Omega^2} \sqrt{1 - \frac{\Omega^2\hat{\mu}_e^2}{\hat{\Gamma}_{1nn}^2}}.$$
(30)

In the frequency/amplitude parameter space (Ω, z_0) the boundary of the existence region, which actually coincides with the stability region, is obtained by considering the equality instead of the inequality in Eq. (30). It has the classical V-shape with vertex in $\omega = \Omega/2$ (see for example forthcoming Fig. 3).

224 NUMERICAL RESULTS AND DISCUSSIONS

A stay cable of the Dongting Lake Bridge, in China, was chosen as an example to verify the

spatial motions of the cable. The dimensional parameters and material properties of the sample stay cable are (Wang and Zhao, 2009): span l = 121.9m; inclination angle $\theta = 35.2^{\circ}$; cross-sectional area $A = 6\ 237 \times 10^{-6}$ m²; initial tension $H = 3\ 150$ kN; elastic modulus $E = 2.0 \times 10^{5}$ MPa; mass per unit length m = 51.8kg/m. (It could be helpful for the reader to report the numerical values of the coefficients appearing in (16))

Figures 3-5 show the stability regions Eq. (30) of the controlled cable for different values of the parameters.

The effect of the time delay on the stability of the parametric resonance of the controlled cable is shown in Fig. 3. It is clear that increasing the delay τ the unstable region (that above the stability boundary) increases is magnitude, confirming the findings of Sect. 3 that the delay has a destabilizing effect. Actually, τ has a strong effect on the minimum values of the curve, while mildly affects the frequency where this minimum occurs (always in the neighborhood of the perfect parametric resonance $\omega = \Omega/2$).

Figure 4 analyzes the effect of the control gain on the stability of the controlled cable. According to the common sense, by increasing the absolute value of the feedback control gain k_n , the unstable region moves up. The minimum value of the limit curve is almost proportional to k_n , showing the effectiveness of control in reducing the parametric resonance instability. The frequency where the minimum occurs slightly increases, even if this is not expected to be relevant in practical applications.

In the frequency/damping parameter space (Ω, μ) the effect of the amplitude z_0 of the excitation on the stability of the controlled cable is shown in Fig. 5. As expected, the larger z_0 the larger is the instability region (now below the reported curves), meaning that a large gain is needed to control large excitation amplitudes. The case with no control ($k_1 = 0$) is also reported to appreciate the beneficial effect of control.

We now illustrated the effect of control gain (Fig. 6) and control delay (Fig. 7) on the frequency response curves, which are very important for practical applications and for design. Figure 6 shows that the frequency response curve shifts to the right when the control gain increases (in absolute

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value), confirming the beneficial effect of control on increasing the instability threshold of the rest position. The stable curve occurring for "large" displacements (that experienced by the system after the loss of the stability of the rest position), on the other hand, is not affected that much by k_n , apart from the left Saddle-Node bifurcation where it is born. This curve is instead much more influenced by the delay τ , as shown in Fig. 7, that also confirms that increasing the delay destabilizes the rest position.

Finally, Fig. 8 shows the comparison of the time history of the uncontrolled and controlled cable with different time delays. Comparing Fig. 8(a) with Fig. 8(b) it can be seen that with a small delay the control is very effective in reducing the vibration amplitudes of the cable. Increasing the delay, the destabilizing effect, already illustrated, can be seen also in the time history of Fig. 8(c). Note that the maximum displacement of Fig. 8(c) is quite similar to that of Fig. 8(a), showing how the large delay nullifies the effect of control.

(I noted that in all simulations the gain k_n is assumed to be negative. What happens for positive values?)

267 CONCLUSIONS

The stability of the parametric resonance of the controlled cable under the influence of time delay has been investigated both in the linear (with the exact solution) and in the nonlinear (by using the multi-scale method) regimes.

The influence of the control gain, the time delay and the amplitude of the external excitation on the stability of the controlled region is analyzed. The results show that the unstable region increases with the time delay and decreases with the increase of the absolute value of the control gain. These findings have been obtained theoretically analyzing the closed form solutions, and have been confirmed by numerical simulations.

The general conclusion of this paper is that when carrying out control design, especially when considering active and semi-active control, it is very important to properly take into account the influence of the time delay. 279

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284 **REFERENCES**

- Cha, Y.-J., Agrawal, A. K., and Dyke, S. J. (2012). "Time delay effects on large-scale MR damper
 based semi-active control strategies." *Smart Materials and Structures*, 22(1), 015011.
- ²⁸⁷ Cong, Y. and Kang, H. (2019). "Planar nonlinear dynamic behavior of a cable-stayed bridge under
 ²⁸⁸ excitation of tower motion." *European Journal of Mechanics A/Solids*, 76, 91–107.
- da Costa, A. P., Martins, J. A. C., Branco, F., and Lilien, J. L. (1996). "Oscillations of bridge stay
 cables induced by periodic motions of deck and/or towers." *Journal of Engineering Mechanics*, 122(7), 613–622.
- Dai, L., Sun, L., and Chen, C. (2014). "Control of an extending nonlinear elastic cable with an active
 vibration control strategy." *Communications in Nonlinear Science and Numerical Simulation*,
 19(10), 3901–3912.
- Fujino, Y. and Susumpow, T. (1994). "An experimental study on active control of in-plane cable
 vibration by axial support motion." *Earthquake Engineering & Structural Dynamics*, 23(12),
 1283–1297.
- Gattulli, V., Lepidi, M., Potenza, F., and Sabatino, U. (2019). "Modal interactions in the nonlinear
 dynamics of a beam-cable-beam." *Nonlinear Dynamics*, 96(4), 2547–2566.
- Guo, T. and Rega, G. (2021a). "Modal dynamics of boundary-interior coupled structures. part
 1: A general approach using components green's function." *Mechanical Systems and Signal Processing*, 149, 107230.

303	Guo, T. and Rega, G. (2021b). "Modal dynamics of boundary-interior coupled structures. part 2:
304	An asymptotic interpretation of mode localization." Mechanical Systems and Signal Processing,
305	149, 107248.

- Hikami, Y. and Shiraishi, N. (1988). "Rain-wind induced vibrations of cables stayed bridges."
 Journal of Wind Engineering and Industrial Aerodynamics, 29(1), 409–418.
- Hu, H. and Wang, Z. (2002). *Dynamics of controlled mechanical systems with delayed feedback*.
 Springer-Verlag, Berlin.
- Huang, Z. W., Hua, X. G., Chen, Z. Q., and Niu, H. W. (2019). "Performance evaluation of
 inerter-based damping devices for structural vibration control of stay cables." *Smart Structures and Systems*, 23(6), 615–626.
- ³¹³ Irvine, H. (1981). *Cable Structures*. MIT Press Series in Structural Mechanics, Cambridge.
- Jafari, M., Hou, F., and Abdelkefi, A. (2020). "Wind-induced vibration of structural cables." *Nonlinear Dynamics*, 100(5), 351–421.
- Ji, J. C. and Zhou, J. (2017). "Coexistence of two families of sub-harmonic resonances in a time-delayed nonlinear system at different forcing frequencies." *Mechanical Systems and Signal Processing*, 93, 151–163.
- Lenci, S. and Ruzziconi, L. (2009). "Nonlinear phenomena in the single-mode dynamics of a cable-supported beam." *International Journal of Bifurcation and Chaos*, 19(3), 923–945.
- Lu, Q., Sun, Z., and Zhang, W. (2020). "Nonlinear parametric vibration with different orders of small parameters for stayed cables." *Engineering Structures*, 224, 111198.
- Matsumoto, M., Shiraishi, N., and Shirato, H. (1992). "Rain-wind induced vibration of cables of cable-stayed bridges." *Journal of Wind Engineering and Industrial Aerodynamics*, 43(1), 2011–2022 International Conference on Wind Engineering.

326	Nayfeh, A. H. and Mook, D. T. (1979). Nonlinear Oscillations. Wiley, New York.
327 328	Ni, Y., Wang, X., Chen, Z., and Ko, J. (2007). "Field observations of rain-wind-induced cable vibration in cable-stayed dongting lake bridge." <i>Journal of Wind Engineering and Industrial</i>
329	Aerodynamics, 95(5), 303–328.
330	Peng, J., Xiang, M., Wang, L., Xie, X., Sun, H., and Yu, J. (2020). "Nonlinear primary resonance
331	in vibration control of cable-stayed beam with time delay feedback." Mechanical Systems and
332	Signal Processing, 137, 106488.
333	Raftoyiannis, I. G. and Michaltsos, G. T. (2016). "Movable anchorage systems for vibration control
334	of stay-cables in bridges." Engineering Structures, 112, 162–171.
335	Sipahi, R., Niculescu, S., Abdallah, C. T., Michiels, W., and Gu, K. (2011). "Stability and
336	stabilization of systems with time delay." IEEE Control Systems Magazine, 31(1), 38-65.
337	Sun, X., Zhang, S., and Xu, J. (2018). "Parameter design of a multi-delayed isolator with asym-
338	metrical nonlinearity." International Journal of Mechanical Sciences, 138-139, 398-408.
339	Tehrani, M. G. and Kalkowski, M. K. (2016). "Active control of parametrically excited systems."
340	Journal of Intelligent Material Systems and Structures, 27(9), 1218–1230.
341	Udwadia, F. E., von Bremen, H., and Phohomsiri, P. (2007). "Time-delayed control design for active
342	control of structures: principles and applications." Structural Control and Health Monitoring,
343	14(1), 27–61.
344	Wang, L. H. and Zhao, Y. Y. (2009). "Large amplitude motion mechanism and non-planar vibration
345	character of stay cables subject to the support motions." Journal of Sound and Vibration, 327,
346	121–133.
347	Wang, X., Ni, Y., Ko, J., and Chen, Z. (2005). "Optimal design of viscous dampers for multi-mode

vibration control of bridge cables." *Engineering Structures*, 27(5), 792–800.

349	Wang, Z., Hu, H., Xu, Q., and Stepan, G. (2017). "Effect of delay combinations on stability and
350	hopf bifurcation of an oscillator with acceleration-derivative feedback." International Journal
351	of Non-Linear Mechanics, 94, 392–399 A Conspectus of Nonlinear Mechanics: A Tribute to the
352	Oeuvres of Professors G. Rega and F. Vestroni.
353	Wang, Z. and Xu, Q. (2017). "Sway reduction of a pendulum on a movable support using a delayed
354	proportional-derivative or derivative-acceleration feedback." Procedia IUTAM, 22, 176-183
355	IUTAM Symposium on Nonlinear and Delayed Dynamics of Mechatronic Systems.
356	Warminski, J., Zulli, D., Rega, G., and Latalski, J. (2016). "Revisited modelling and multimodal
357	nonlinear oscillations of a sagged cable under support motion." Meccanica, 51, 2541–2575.
358	Yan, Y., Zhang, S., Guo, Q., Xu, J., and Kim, K. C. (2020). "Energy determines multiple stability
359	in time-delayed systems." Nonlinear Dynamics, 102, 2399–2416.
360	Ying, Z. G., Ni, Y. Q., and Ko, J. M. (2006). "Parametrically excited instability of a cable under two
361	support motions." International Journal of Structural Stability and Dynamics, 06(01), 43-58.
362	Ying, Z. G., Ni, Y. Q., and Ko, J. M. (2007). "Parametrically excited instability analysis of a
363	semi-actively controlled cable." Engineering Structures, 29(4), 567-575.
364	Zhao, Y., Sun, C., Wang, Z., and Wang, L. (2014). "Analytical solutions for resonant response of
365	suspended cables subjected to external excitation." Nonlinear Dynamics, 78(2), 1017-1032.

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Fig. 1. The configuration of the controlled cable model.



Fig. 2. Stability region (in white) of the single mode response of the controlled cable system.



Fig. 3. The effect of the time delay on the stability of the controlled cable.



Fig. 4. The effect of the control gain on the stability of the controlled cable.



Fig. 5. The effect of the amplitude of the excitation on the stability of the controlled cable.



Fig. 6. The frequency response curve of the controlled cable with time delay $\tau = \pi/16$. (write k_1 instead of k (as in the previous figures). Write a_n instead of a (as reported in the text of the paper). Who is f_1 ? It is equal to $\Omega/2\pi$? This should be said. Report the value of z_0 used in this curves)



Fig. 7. The frequency response curve of the controlled cable with control gain $k_1 = -0.15$. (The same comments on the previous figure apply. Furthermore, I believe that $\tau = \pi/32$ refers to the green curve, not to the red one)



Fig. 8. Comparison of the time history of the controlled cable. (a) no control; (b) $k_1 = -1$, $\tau = \pi/2$; (c) $k_1 = -1$, $\tau = \pi$. (report the values of Ω and z_0 used for these figures)