Stability in Parametric Resonance of a Controlled Stay Cable with Time Delay

Stability in parametric resonance of a controlled stay cable with time delay

Jian Peng[1], Yongyin Zhang[2], Luxin Li[3], Hongxin Sun[4], and Stefano Lenci[5]

[1]School of Civil Engineering, Hunan University of Science and Technology, Xiangtan, Hunan 411201, PR China . (Corresponding author, E-mail:pengjian@hnu.edu.cn).

[2]M.S.Student, School of Civil Engineering, Hunan University of Science and Technology, Xiangtan, Hunan 411201, PR China. E-mail:zhangyongyin@mail.hnust.edu.cn.

[3]PhD.Student, State Key Laboratory of Structural Analysis for Industrial Equipment, Department of Engineering Mechanics, Dalian University of Technology, Dalian 116024, China. E-mail:liluxin93@163.com.

[4]School of Civil Engineering, Hunan University of Science and Technology, Xiangtan, Hunan 411201, PR China.E-mail: cehxsun@hnu.edu.cn.

[5]Department of Civil and Building Engineering and Architecture, Polytechnic University of Marche, Ancona 60131, Italy. E-mail:lenci@univpm.it

ABSTRACT

The stability of the parametric resonance of the semiactive control of a stay cable with time delay is investigated. The in-plane nonlinear equations of motion are initially obtained via the Hamilton principle. Then, utilizing the method of multiple scales, the modulation equations that govern the nonlinear dynamics are obtained. These equations are then utilized to investigate the effect of time delays on the amplitude and frequency-response behavior and, subsequently, on the stability of the parametric resonance of the controlled cable, that it is shown to depend on the excitation amplitude and the commensurability of the delayed-response frequency to the excitation frequency. The stability region of the parameteric resonance is shifted, and the effects of control on the cable become worse by increasing time delay. The work plays a guiding role in the parametric design of the control system for stay cables.

INTRODUCTION

As the main bearing member of long-span structure, the stayed cable is characterized by light weight and small damping, hence it vibrates easily due to external excitation, such as wind, rain, traffic or earthquake (Irvine, 1981; Warminski et al., 2016; Ni et al., 2007). In recent years, large-scale vibrations of the stayed cables of bridges have been observed at low wind speeds, which is generally considered to be the result of a parametric resonance phenomenon (Ni et al., 2007; da Costa et al., 1996). Therefore, it is important to investigate the vibration mechanism of cables.

Hikami and Shiraishi (1988) and Matsumoto et al. (1992) investigated the mechanism of rain-wind induced vibration of cable of cable-stayed bridges and proposed aerodynamic countermeasures to suppress the vibration. Jafari et al. (2020) reviewed the past studies about different types of wind-induced cable vibration. Zhao et al. (2014) discussed the analytical solutions for resonant response of suspended cables subjected to external excitation. Lenci and Ruzziconi (2009) studied nonlinear phenomena in the single-mode dynamics of a cable-supported beam. Gattulli et al. (2019) analyzed the modal interactions in the nonlinear dynamics of a beam-cable-beam. It is worth pointing out that the parametric vibration of the stay cable is one of the main aspects. Wang and Zhao (2009) addressed the large amplitude motion mechanism and the non-planar vibration character of stay cables subject to the support motions. Ying et al. (2006) investigated the parametrically excited instability of a cable under two support motions. Guo and Rega (2021a,b) studied the modal dynamics of boundary-interior coupled structures. Cong and Kang (2019) considered the planar nonlinear dynamic behavior of a cable-stayed bridge under excitation of tower motion. Lu et al. (2020) studied nonlinear parametric vibration with different orders of small parameters for stayed cables.

In parallel the the previous works, that are focused on understanding complex nonlinear phenomena, other studies focused on the vibration control of cables. Fujino and Susumpow (1994) studied active control of in-plane cable vibration by axial support motion via experiments. Wang et al. (2005) investigated optimal design of viscous dampers for multimode vibration control of bridge cables. Ying et al. (2007) studied parametrically excited instability analysis of a semi-actively controlled cable. Dai et al. (2014) addressed an extended nonlinear elastic cable with an active vibration control strategy. Tehrani and Kalkowski (2016) investigated active control of parametrically excited systems. Raftoyiannis and Michaltsos (2016) studied movable anchorage system for vibration control of stay-cables in bridges. Huang et al. (2019) evaluated the performances of inerter-based damping devices for structural vibration control of stay cables. Peng et al. (2020) investigated nonlinear primary resonance in vibration control of cable-stayed beam via time delayed feedback control.

It has been shown (Hu and Wang, 2002; Sipahi et al., 2011) that in the vibration control system the time delay is *not* negligible. Cha et al. (2012) studied time delay effects on large-scale MR damper based semi-active control strategies. Yan et al. (2020) considered energy determining multiple stability in time-delayed systems. Udwadia et al. (2007) presented principles and applications of time-delayed control design for active control of structures. Ji and Zhou (2017) investigated coexistence of two families of sub-harmonic resonances in a time-delayed nonlinear system at different forcing frequencies. Wang et al. (2017) and Wang and Xu (2017) studied effect of delay combinations on stability and Hopf bifurcation of an oscillator with acceleration-derivative feedback and sway reduction of a pendulum on a movable support using a delayed proportional-derivative or derivative-acceleration feedback. Sun et al. (2018) studied parameter design of a multi-delayed isolator with asymmetrical nonlinearity. Their results showed that time delay can affect the damping performance of the control system, and, on the other hand, making good use of it can provide another control idea and improve control performance.

As a matter of fact, very few studies concerned with the time delay effects in nonlinear parametric resonance of controlled cables, and filling this gap is the main goal of this work. It leads to interesting and partially unexpected results in terms of performance (or better, loss of performance) of the considered control.

(They should be better underlined the differences with respect to our previous paper (Peng et al., 2020))

The mechanical model of controlled cable under axial excitation is considered. The method of multiple scales is used to analyze the parametric vibration under the influence of time delay. The stability of parametric resonance of the controlled stay cable is discussed, and the time delay effect of the parametric vibration system is discussed by numerical examples.

CONTROLLED CABLE MODEL AND EQUATIONS OF MOTION

As shown in Fig. 1, a stayed cable subject to a vertical sinusoidal support motion $Z \sin \omega t$ (where Z and ω denote the amplitude and frequency, respectively), is considered. A Cartesian coordinate system $O - xy$ is chosen, with the origin O placed at the left fixed support A of the cable. The displacements of the points are denoted by $u(x,t)$ and $v(x,t)$ along the x and y directions, respectively. a is the distance between the right oscillating boundary B and the MR damper.

The axial Lagrangian strain of the inclined cable can be written as

$$\varepsilon(x,t) = u' + y'v' + \frac{v'^2}{2}, \tag{1}$$

where prime indicates differentiation with respect to the spatial coordinate x and $y(x)$ is the static configuration of the cable, that can be approximately written as $y(x) = \frac{mgl\cos\theta}{2H} x(1-x)$. The equations of motions can be obtained by means of the Hamilton principle (Wang and Zhao, 2009)

$$m\ddot{u} + c_u\dot{u} - \left\{ EA\left[u' + y'v' + \frac{v'^2}{2} \right] \right\}' = 0, \tag{2}$$

$$m\ddot{v} + c_v\dot{v} - \left\{ Hv' + EA(y'+v')\left[u' + y'v' + \frac{v'^2}{2} \right] \right\}' = 0, \tag{3}$$

where dot indicates differentiation with respect to time t, m is the mass per unit length; E is the Young modulus, A is the area of the cross-section, c_u and c_v are the viscous damping coefficients per unit length, H is the axial component of the initial tension ($H \ll EA$) and g is the gravity acceleration. The boundary conditions can be written as

$$u(0,t) = v(0,t) = 0, \quad u(l,t) = Z\sin\theta\sin(\omega t), \quad v(l,t) = Z\cos\theta\sin(\omega t), \tag{4}$$

where l is the cable span and θ is the angle of inclination of the cable (see Fig. 1). It is worth to remark that the boundary conditions are nonhomogeneous both in the axial displacement component $u(x,t)$ and in-plane transverse displacement component $v(x,t)$.

Under the quasi-static assumption in the axial direction, i.e, neglecting the acceleration and velocity term in Eq. (2), and taking into account the boundary conditions, the displacement $u(x,t)$ can be expressed by

$$u(x,t) = Z\sin\theta\sin(\omega t)\frac{x}{l} + \frac{x}{l}\int_0^l\left(y'v' + \frac{v'^2}{2}\right)dx - \int_0^x\left(y'v' + \frac{v'^2}{2}\right)dx. \quad (5)$$

Inserting Eq. (5) in Eq. (3) it is possible to obtain an equation in the primary unknown $v(x,t)$. Then, considering the concentrated force at $x = l - a$ due to the damper (introduced to reduce the cable oscillations) and the distributed external load, and proceeding in a manner similar to (Peng et al., 2020), the non-dimensional equations of motion can be written as

$$\ddot{v}+c_v\dot{v}-v''-\alpha(y''+v'')\left\{z_0\sin\theta\sin(\Omega t) + \int_0^l\left(y'v' + \frac{v'^2}{2}\right)dx\right\} = F_d\delta(x-(l-a))+F\cos(\Omega t), \quad (6)$$

where $F_d = -C_{eq}\dot{v}(t-\tau)$ is the control force of the damper, τ the time delay of the control system, $F(x)$ the spatial distribution of the distributed force and δ is the Dirac delta function. The non-dimensional variables are $x^* = x/l$, $a^* = a/l$, $y^* = y/l$, $z_0 = Z/l$, $v^* = v/l$, $\alpha = EA/H$, $t^* = t/l\sqrt{H/m}$, $\Omega = \omega l/\sqrt{m/H}$, $c_v^* = c_v l/(m)\sqrt{m/H}$. Asterisks in Eq. (6) are dropped for simplicity.

For the nonhomogeneous boundary value problem, it is convenient to introduce a suitable chosen particular solution, which satisfies the nonhomogeneous boundary conditions, to transform the nonhomogeneous problem to a homogeneous one. Then, the solution of the homogeneous problem can be approximated by a time-varying linear combination of known (and fixed) spatial functions, which are assumed to be the eigenfunctions of the homogeneous problem. In this study, according to the boundary condition of the inclined cable, the non-dimensional displacements

$v(x,t)$ is sougth after in the form

$$v(x,t) = \sum_{i=1}^{N} \phi_i(x) q_i(t) + x z_0 \cos\theta \sin(\Omega t), \tag{7}$$

where $q_i(t)$ are the generalized displacements, and $\phi_i(x) = \sqrt{2}\sin(i\pi x)$ the ith in-plane mode shapes. Substitution of Eq. (7) into Eq. (6) and application of the Galerkin method yield a set of nonlinear ordinary differential equations

(in the following equation:

- the term due to the damper (proportional to F_d) is missing;
- F is missing;
- if the $\phi_i(x)$ are the linear normal modes, the linear part (without excitation) should be decoupled, i.e. the Γ_{2ij} should be 0 for $i \neq j$;
- the definition of the coefficients Γ is strange/incosistent: the Γ_1 are the time dependent, while all other not. I suggest to rewrite in such a way that all Γs are time independent, and the harmonic terms appear explicitly in the equation.

Please check carefully the previous points)

$$\begin{aligned} &\ddot{q}_i + 2\omega_i \xi_i \dot{q}_i + \Gamma_{1i} q_i + \sum_{j=1}^{N} (\Gamma_{2ij} q_j + \Gamma_{3ij} q_j^2 + \Gamma_{4ij} q_j q_i + \Gamma_{5ij} q_j^2 q_i) \\ &= \Gamma_{6i} \sin(\Omega t) + \Gamma_{7i} \cos(\Omega t) + \Gamma_{8i} \sin^2(\Omega t), \quad i = 1, 2, ..., N, \end{aligned} \tag{8}$$

where ξ_i are the viscous damping ratios, $\omega_i = \sqrt{\Gamma_{1i} + \Gamma_{2ii}}$ (please check this) the ith in-plane natural frequencies, and the other coefficients are given by

$$
\begin{aligned}
\Gamma_{1i} &= i^2(1+\alpha z_0 \sin\theta \sin(\Omega t) + \frac{1}{2}\alpha z_0^2 \cos^2\theta \sin^2(\Omega t)),\\
\Gamma_{2ij} &= \alpha \int_0^1 y'\phi_i'(x)dx \int_0^1 y'\phi_j'(x)dx,\\
\Gamma_{3ij} &= \frac{\alpha}{2} j^2 \int_0^1 y'\phi_i'(x)dx,\\
\Gamma_{4ij} &= \alpha i^2 \int_0^1 y'\phi_j'(x)dx,\\
\Gamma_{5ij} &= \frac{\alpha}{2} i^2 j^2,\\
\Gamma_{6i} &= z_0\Omega^2 \cos\theta \int_0^1 x\phi_i(x)dx - \alpha z_0 \sin\theta \int_0^1 y'\phi_i'(x)dx,\\
\Gamma_{7i} &= -2\xi_i i z_0 \Omega \cos\theta \int_0^1 x\phi_i(x)dx,\\
\Gamma_{8i} &= \frac{\alpha}{2} z_0^2 \cos^2\theta \int_0^1 y''\phi_i(x)dx.
\end{aligned}
\tag{9}
$$

LINEAR STABILITY ANALYSIS

In this section, the linear stability analysis of the single degree of freedom vibration mode is investigated. Considering only one equation $i = n$ in Eq. (8), neglecting the nonlinear terms and the external excitation ($z_0 = 0$) the following equation is obtained ($\mu_n = 2\omega_n\xi_n$) (in the following equation k_n is not defined)

$$\ddot{q}_n(t) + \mu_n\dot{q}_n(t) + \omega_n^2 q_n(t) = -k_n\dot{q}_n(t-\tau). \tag{10}$$

The solution of Eq. (10) is given by

$$q_n = A_n e^{(\xi_n + i\lambda_n)t} \tag{11}$$

where A_n, ξ_n and λ_n are amplitude, damping coefficient and response frequency, respectively. All are real numbers. Substituting Eq. (11) in Eq. (10), and separating real and imaginary parts, gives

$$\lambda_n \left(2\xi_n + \mu_n\right) e^{\xi_n \tau} + k_n \left[\lambda_n \cos\left(\lambda_n \tau\right) - \xi_n \sin\left(\lambda_n \tau\right)\right] = 0 \tag{12}$$

$$\left(\lambda_n^2 - \xi_n^2 - \mu_n \xi_n - \omega_n^2\right) e^{\xi_n \tau} - k_n \left[\xi_n \cos\left(\lambda_n \tau\right) + \lambda_n \sin\left(\lambda_n \tau\right)\right] = 0. \tag{13}$$

When $\xi_n < 0$ the solution (11) converges to 0 for $t \to \infty$ and thus is stable, while for $\xi_n > 0$ the solution diverges to infinity and thus is unstable. The stability limit is then given by $\xi_n = 0$. Substituting this value in Eq. (12) and Eq. (13) we obtain

$$\cos\left(\lambda_n \tau\right) = -\frac{\mu_n}{k_n}, \quad \sin\left(\lambda_n \tau\right) = \frac{\lambda_n^2 - \omega_n^2}{k_n \lambda_n}, \tag{14}$$

and thus the boundary of linear stability are

$$\tau = \frac{1}{\lambda_n}\left[\tan^{-1}\left(-\frac{\lambda_n^2 - \omega_n^2}{\lambda_n \mu_n}\right) + j\pi\right], j = 0, 1, \cdots, \quad k_n = \pm\frac{\sqrt{\lambda_n^2 \mu_n^2 + (\lambda_n^2 - \omega_n^2)^2}}{\lambda_n}. \tag{15}$$

The stability regions described by Eq. (15) are shown in Fig. 2, where regions i, ii and iii corresponds to a small, medium and large values of time delay, respectively. The figure clearly shows that for small values of the delay the system is stable, and thus the control effective, even for very large values of the gain k_n. For medium and large values of τ, on the other hand, the stability region is a narrow strip around $k_n = 0$, namely the system is stable only for very low values of k_n, giving not good performance because with small values of the gain the damping is low and the vibration reduction is ineffective. For quite large values of k_n, the system is stable for low values of the delay, and loses stability for increasing τ. This could be very dangerous from a practical point of view, because unplanned increasing delay of the control, due for example to the ageing of the structure, can destabilize the system, with unwanted phenomena up to collapse.

STABILITY OF THE PARAMETRICALLY RESONANCE RESPONSE

In this section, we continue to consider the single degree of freedom vibration mode, but extend the analysis to the nonlinear regime, utilizing the method of multiple scales (Nayfeh and Mook, 1979).

It is convenient to introduce a small bookkeeping parameter ε to obtain the solution. The equation of the motion can be written as ((16) is not consistent with (8): here - correctly from my point of view - the Γ are not time dependent, see my previous comments just before Eq. (8). Please check and modify)

$$\begin{aligned}\ddot{q}_n + {\omega_n}^2 q_n + \varepsilon\Gamma_{1nn} q_n \cos(\Omega t) + \varepsilon\mu\dot{q}_n + \varepsilon(\Gamma_{3nn} + \Gamma_{4nn}) q_n^2 + \varepsilon\Gamma_{5nn} q_n^3 = \\ -\varepsilon k_n \dot{q}_n(t-\tau) + \varepsilon\Gamma_{6nn}\sin(\Omega t) + \varepsilon\Gamma_{7nn}\cos(\Omega t) + \varepsilon\Gamma_{8nn}\sin^2(\Omega t),\end{aligned} \tag{16}$$

The solution of Eq. (16) is sought after in the form

$$q_n(t;\varepsilon) = q_{n0}(T_0, T_1, \ldots) + \varepsilon q_{n1}(T_0, T_1, \ldots) + \cdots \tag{17}$$

where $T_n = \varepsilon^n t, n = 0, 1, 2,$. It is further assumed that

$$\omega_n = \frac{\Omega}{2} + \varepsilon\omega_1 + \varepsilon^2\omega_2 + \cdots \tag{18}$$

Substituting Eq. (17) and Eq. (18) in Eq. (16), and equating the coefficients of ε^0 and ε^1 on both sides, we obtain

$$D_0^2 q_{n0} + \frac{\Omega^2}{4} q_{n0} = 0, \tag{19}$$

$$\begin{aligned}D_0^2 q_{n1} + \frac{\Omega^2}{4} q_{n1} = -2D_0 D_1 q_{n0} - \Omega\omega_1 q_{n0} - \Gamma_{1nn} q_{n0}\cos(\Omega t) - \mu D_0 q_{n0} - (\Gamma_{3nn} + \Gamma_{4nn}) q_{n0}^2 \\ -\Gamma_{5nn} q_{n0}^3 - k_n \dot{q}_{n0}(t-\tau) + \Gamma_{6nn}\sin(\Omega t) + \Gamma_{7nn}\cos(\Omega t) + \Gamma_{8nn}\sin^2(\Omega t),\end{aligned} \tag{20}$$

where D_n denotes the derivatives with respect to T_n.

The general solution of Eq. (19) can be written as

$$q_{n0} = A_n(T_1)\exp\left(\frac{i\Omega T_0}{2}\right) + \bar{A}_n(T_1)\exp\left(\frac{-i\Omega T_0}{2}\right). \tag{21}$$

Substituting Eq. (21) in Eq. (20) we obtain (in the following equation $\sin(\Omega t)$ must be transformed in the exponential form. Furthermore, it is convenient to collect terms multiplying the same exponential terms (as is has been done for $\exp\left(\frac{i\Omega T_0}{2}\right)$). Please check.)

$$D_0^2 q_{n1} + \frac{\Omega^2}{4} q_{n1} = -\left[i\Omega A_n' + \Omega\omega_1 A_n + \frac{\Gamma_{1nn}}{2}\bar{A}_n + \frac{1}{2}i\mu\Omega A_n + 3\Gamma_{5nn}A_n^2\bar{A}_n + \frac{1}{2}k_n i\Omega A_n \exp\left(-\frac{i\Omega\tau}{2}\right)\right] \exp\left(\frac{i\Omega T_0}{2}\right) - \frac{\Gamma_{1nn}}{2}A_n \exp\left(\frac{3i\Omega T_0}{2}\right) - (\Gamma_{3nn}+\Gamma_{4nn})A_n^2 \exp(i\Omega T_0) - (\Gamma_{3nn}+\Gamma_{4nn}) A_n\bar{A}_n - \Gamma_{5nn}A_n^3 \exp\left(\frac{3i\Omega T_0}{2}\right) + \frac{\Gamma_{7nn}}{2}\exp(i\Omega T_0) + \Gamma_{6nn}\sin(\Omega t) + \Gamma_{8nn}\sin^2(\Omega t) + cc, \tag{22}$$

where cc denotes the complex conjugate of the preceding terms. To eliminate secular terms from q_{n1} we must put

$$i\Omega A_n' + \Omega\omega_1 A_n + \frac{\Gamma_{1nn}}{2}\bar{A}_n + \frac{1}{2}i\mu\Omega A_n + 3\Gamma_{5nn}A_n^2\bar{A}_n + \frac{1}{2}k_n i\Omega A_n \exp\left(-\frac{i\Omega\tau}{2}\right) = 0. \tag{23}$$

To solve Eq. (23), we write A_n in the polar form:

$$A_n = \frac{1}{2}a_n \exp(i\beta_n), \tag{24}$$

where a_n and β_n are real functions of T_1. Substituting Eq. (24) in Eq. (23) and separating real and imaginary parts, we have

$$a_n' = -\frac{1}{2}\mu_e a_n + \frac{\Gamma_{1nn}a_n}{2\Omega}\sin 2\beta_n, \tag{25}$$

$$\beta_n' = \omega_1 + \frac{k_n}{2}\sin\left(\frac{\Omega\tau}{2}\right) + \frac{3\Gamma_{5nn}}{4\Omega}a_n^2 + \frac{2\Gamma_{1nn}}{\Omega}\cos 2\beta_n, \tag{26}$$

where $\mu_e = \mu + k_n\cos\left(\frac{\Omega\tau}{2}\right)$.

When $a_n' = \gamma_n' = 0$, the sought periodic solution is obtained. Considering the nontrivial solutions ($a_n \neq 0$), from Eq. (25) and Eq. (26) we can then obtain

$$\sin 2\beta_n = \frac{\Omega\mu_e}{\Gamma_{1nn}} \tag{27}$$

Remembering that $\cos(2\beta_n) = \pm\sqrt{1-\sin^2(2\beta_n)}$ and substituting Eq. (27) in Eq. (26), we obtain the amplitude of the steady solution

$$a_n^2 = -\frac{4\Omega}{3\Gamma_{5nn}}\left[\omega_1 + \frac{k_n}{2}\sin\left(\frac{\Omega\tau}{2}\right)\right] \pm \frac{8\Gamma_{1nn}}{3\Gamma_{5nn}}\sqrt{1-\frac{\Omega^2\mu_e^2}{\Gamma_{1nn}^2}}, \tag{28}$$

which is the frequency-response equation, since the excitation amplitude z_0 in within Γ_{1nn}.

It is worth to underline that Γ_{6nn}, Γ_{7nn} and Γ_{8nn} do not appear in Eq. (28) because we are focusing on the parametric excitation (this is reflected in the choice (18)). They would appear if one consider the external resonance, i.e. $\omega_n \approx \Omega$. This is left for future work.

Since a_n is a real function, from $a_n^2 > 0$ we obtain first order approximate region of existence of the periodic solution

$$\omega_1 < -\frac{k_n}{2}\sin\left(\frac{\Omega\tau}{2}\right) \pm \frac{2\Gamma_{1nn}}{\Omega}\sqrt{1-\frac{\Omega^2\mu_e^2}{\Gamma_{1nn}^2}}. \tag{29}$$

Inserting this expression in Eq. (18), and remembering that $\varepsilon k_n = \hat{k}_n$, $\varepsilon\mu_e = \hat{\mu}_e$ and $\varepsilon\Gamma_{1nn} = \hat{\Gamma}_{1nn}$, yields

$$\frac{2\omega}{\Omega} < 1 - \frac{\hat{k}_n}{\Omega}\sin\left(\frac{\Omega\tau}{2}\right) \pm \frac{4\hat{\Gamma}_{1nn}}{\Omega^2}\sqrt{1-\frac{\Omega^2\hat{\mu}_e{}^2}{\hat{\Gamma}_{1nn}{}^2}}. \tag{30}$$

In the frequency/amplitude parameter space (Ω, z_0) the boundary of the existence region, which actually coincides with the stability region, is obtained by considering the equality instead of the inequality in Eq. (30). It has the classical V-shape with vertex in $\omega = \Omega/2$ (see for example forthcoming Fig. 3).

NUMERICAL RESULTS AND DISCUSSIONS

A stay cable of the Dongting Lake Bridge, in China, was chosen as an example to verify the spatial motions of the cable. The dimensional parameters and material properties of the sample stay cable are (Wang and Zhao, 2009): span $l = 121.9$m; inclination angle $\theta = 35.2^o$; cross-sectional area $A = 6\,237 \times 10^{-6}$m^2; initial tension $H = 3\,150$kN; elastic modulus $E = 2.0 \times 10^5$MPa; mass per unit length $m = 51.8$kg/m. (It could be helpful for the reader to report the numerical values of the coefficients appearing in (16))

Figures 3-5 show the stability regions Eq. (30) of the controlled cable for different values of the parameters.

The effect of the time delay on the stability of the parametric resonance of the controlled cable is shown in Fig. 3. It is clear that increasing the delay τ the unstable region (that above the stability boundary) increases is magnitude, confirming the findings of Sect. 3 that the delay has a destabilizing effect. Actually, τ has a strong effect on the minimum values of the curve, while mildly affects the frequency where this minimum occurs (always in the neighborhood of the perfect parametric resonance $\omega = \Omega/2$).

Figure 4 analyzes the effect of the control gain on the stability of the controlled cable. According to the common sense, by increasing the absolute value of the feedback control gain k_n, the unstable region moves up. The minimum value of the limit curve is almost proportional to k_n, showing the effectiveness of control in reducing the parametric resonance instability. The frequency where the minimium occurs slightly increases, even if this is not expected to be relevant in practical applications.

In the frequency/damping parameter space (Ω, μ) the effect of the amplitude z_0 of the excitation on the stability of the controlled cable is shown in Fig. 5. As expected, the larger z_0 the larger is the instability region (now below the reported curves), meaning that a large gain is needed to control large excitation amplitudes. The case with no control ($k_1 = 0$) is also reported to appreciate the beneficial effect of control.

We now illustrated the effect of control gain (Fig. 6) and control delay (Fig. 7) on the frequency response curves, which are very important for practical applications and for design. Figure 6 shows that the frequency response curve shifts to the right when the control gain increases (in absolute value), confirming the beneficial effect of control on increasing the instability threshold of the rest position. The stable curve occurring for “large" displacements (that experienced by the system after the loss of the stability of the rest position), on the other hand, is not affected that much by k_n, apart from the left Saddle-Node bifurcation where it is born. This curve is instead much more influenced by the delay τ, as shown in Fig. 7, that also confirms that increasing the delay destabilizes the rest position.

Finally, Fig. 8 shows the comparison of the time history of the uncontrolled and controlled cable with different time delays. Comparing Fig. 8(a) with Fig. 8(b) it can be seen that with a small delay the control is very effective in reducing the vibration amplitudes of the cable. Increasing the delay, the destabilizing effect, already illustrated, can be seen also in the time history of Fig. 8(c). Note that the maximum displacement of Fig. 8(c) is quite similar to that of Fig. 8(a), showing how the large delay nullifies the effect of control.

(I noted that in all simulations the gain k_n is assumed to be negative. What happens for positive values?)

CONCLUSIONS

The stability of the parametric resonance of the controlled cable under the influence of time delay has been investigated both in the linear (with the exact solution) and in the nonlinear (by using the multi-scale method) regimes.

The influence of the control gain, the time delay and the amplitude of the external excitation on the stability of the controlled region is analyzed. The results show that the unstable region increases with the time delay and decreases with the increase of the absolute value of the control gain. These findings have been obtained theoretically analyzing the closed form solutions, and have been confirmed by numerical simulations.

The general conclusion of this paper is that when carrying out control design, especially when considering active and semi-active control, it is very important to properly take into account the influence of the time delay.

ACKNOWLEDGEMENTS

The study was supported by the National Natural Science Foundation of China (Grant No.52078210) and Provincial Natural Science Foundation of Hunan (Grant No.2021JJ10003). The work of Prof. Lenci has been partially done within his belonging to the “Gruppo Nazionale per la Fisica Matematica".

REFERENCES

Cha, Y.-J., Agrawal, A. K., and Dyke, S. J. (2012). “Time delay effects on large-scale MR damper based semi-active control strategies.” *Smart Materials and Structures*, 22(1), 015011.

Cong, Y. and Kang, H. (2019). “Planar nonlinear dynamic behavior of a cable-stayed bridge under excitation of tower motion.” *European Journal of Mechanics - A/Solids*, 76, 91–107.

da Costa, A. P., Martins, J. A. C., Branco, F., and Lilien, J. L. (1996). “Oscillations of bridge stay cables induced by periodic motions of deck and/or towers.” *Journal of Engineering Mechanics*, 122(7), 613–622.

Dai, L., Sun, L., and Chen, C. (2014). “Control of an extending nonlinear elastic cable with an active vibration control strategy.” *Communications in Nonlinear Science and Numerical Simulation*, 19(10), 3901–3912.

Fujino, Y. and Susumpow, T. (1994). “An experimental study on active control of in-plane cable vibration by axial support motion.” *Earthquake Engineering & Structural Dynamics*, 23(12), 1283–1297.

Gattulli, V., Lepidi, M., Potenza, F., and Sabatino, U. (2019). “Modal interactions in the nonlinear dynamics of a beam-cable-beam.” *Nonlinear Dynamics*, 96(4), 2547–2566.

Guo, T. and Rega, G. (2021a). “Modal dynamics of boundary-interior coupled structures. part 1: A general approach using components green’s function.” *Mechanical Systems and Signal Processing*, 149, 107230.

Guo, T. and Rega, G. (2021b). "Modal dynamics of boundary-interior coupled structures. part 2: An asymptotic interpretation of mode localization." *Mechanical Systems and Signal Processing*, 149, 107248.

Hikami, Y. and Shiraishi, N. (1988). "Rain-wind induced vibrations of cables stayed bridges." *Journal of Wind Engineering and Industrial Aerodynamics*, 29(1), 409–418.

Hu, H. and Wang, Z. (2002). *Dynamics of controlled mechanical systems with delayed feedback*. Springer-Verlag, Berlin.

Huang, Z. W., Hua, X. G., Chen, Z. Q., and Niu, H. W. (2019). "Performance evaluation of inerter-based damping devices for structural vibration control of stay cables." *Smart Structures and Systems*, 23(6), 615–626.

Irvine, H. (1981). *Cable Structures*. MIT Press Series in Structural Mechanics, Cambridge.

Jafari, M., Hou, F., and Abdelkefi, A. (2020). "Wind-induced vibration of structural cables." *Nonlinear Dynamics*, 100(5), 351–421.

Ji, J. C. and Zhou, J. (2017). "Coexistence of two families of sub-harmonic resonances in a time-delayed nonlinear system at different forcing frequencies." *Mechanical Systems and Signal Processing*, 93, 151–163.

Lenci, S. and Ruzziconi, L. (2009). "Nonlinear phenomena in the single-mode dynamics of a cable-supported beam." *International Journal of Bifurcation and Chaos*, 19(3), 923–945.

Lu, Q., Sun, Z., and Zhang, W. (2020). "Nonlinear parametric vibration with different orders of small parameters for stayed cables." *Engineering Structures*, 224, 111198.

Matsumoto, M., Shiraishi, N., and Shirato, H. (1992). "Rain-wind induced vibration of cables of cable-stayed bridges." *Journal of Wind Engineering and Industrial Aerodynamics*, 43(1), 2011–2022 International Conference on Wind Engineering.

Nayfeh, A. H. and Mook, D. T. (1979). *Nonlinear Oscillations*. Wiley, New York.

Ni, Y., Wang, X., Chen, Z., and Ko, J. (2007). "Field observations of rain-wind-induced cable vibration in cable-stayed dongting lake bridge." *Journal of Wind Engineering and Industrial Aerodynamics*, 95(5), 303–328.

Peng, J., Xiang, M., Wang, L., Xie, X., Sun, H., and Yu, J. (2020). "Nonlinear primary resonance in vibration control of cable-stayed beam with time delay feedback." *Mechanical Systems and Signal Processing*, 137, 106488.

Raftoyiannis, I. G. and Michaltsos, G. T. (2016). "Movable anchorage systems for vibration control of stay-cables in bridges." *Engineering Structures*, 112, 162–171.

Sipahi, R., Niculescu, S., Abdallah, C. T., Michiels, W., and Gu, K. (2011). "Stability and stabilization of systems with time delay." *IEEE Control Systems Magazine*, 31(1), 38–65.

Sun, X., Zhang, S., and Xu, J. (2018). "Parameter design of a multi-delayed isolator with asymmetrical nonlinearity." *International Journal of Mechanical Sciences*, 138-139, 398–408.

Tehrani, M. G. and Kalkowski, M. K. (2016). "Active control of parametrically excited systems." *Journal of Intelligent Material Systems and Structures*, 27(9), 1218–1230.

Udwadia, F. E., von Bremen, H., and Phohomsiri, P. (2007). "Time-delayed control design for active control of structures: principles and applications." *Structural Control and Health Monitoring*, 14(1), 27–61.

Wang, L. H. and Zhao, Y. Y. (2009). "Large amplitude motion mechanism and non-planar vibration character of stay cables subject to the support motions." *Journal of Sound and Vibration*, 327, 121–133.

Wang, X., Ni, Y., Ko, J., and Chen, Z. (2005). "Optimal design of viscous dampers for multi-mode vibration control of bridge cables." *Engineering Structures*, 27(5), 792–800.

Wang, Z., Hu, H., Xu, Q., and Stepan, G. (2017). "Effect of delay combinations on stability and hopf bifurcation of an oscillator with acceleration-derivative feedback." *International Journal of Non-Linear Mechanics*, 94, 392–399 A Conspectus of Nonlinear Mechanics: A Tribute to the Oeuvres of Professors G. Rega and F. Vestroni.

Wang, Z. and Xu, Q. (2017). "Sway reduction of a pendulum on a movable support using a delayed proportional-derivative or derivative-acceleration feedback." *Procedia IUTAM*, 22, 176–183 IUTAM Symposium on Nonlinear and Delayed Dynamics of Mechatronic Systems.

Warminski, J., Zulli, D., Rega, G., and Latalski, J. (2016). "Revisited modelling and multimodal nonlinear oscillations of a sagged cable under support motion." *Meccanica*, 51, 2541–2575.

Yan, Y., Zhang, S., Guo, Q., Xu, J., and Kim, K. C. (2020). "Energy determines multiple stability in time-delayed systems." *Nonlinear Dynamics*, 102, 2399–2416.

Ying, Z. G., Ni, Y. Q., and Ko, J. M. (2006). "Parametrically excited instability of a cable under two support motions." *International Journal of Structural Stability and Dynamics*, 06(01), 43–58.

Ying, Z. G., Ni, Y. Q., and Ko, J. M. (2007). "Parametrically excited instability analysis of a semi-actively controlled cable." *Engineering Structures*, 29(4), 567–575.

Zhao, Y., Sun, C., Wang, Z., and Wang, L. (2014). "Analytical solutions for resonant response of suspended cables subjected to external excitation." *Nonlinear Dynamics*, 78(2), 1017–1032.

List of Figures

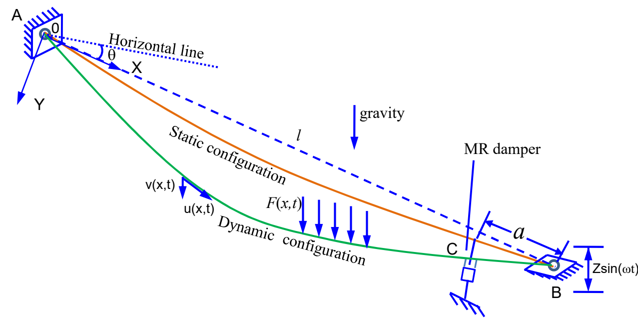

Fig. 1. The configuration of the controlled cable model.

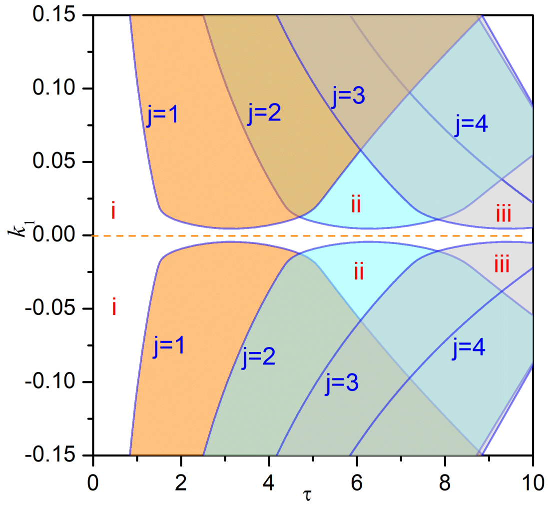

Fig. 2. Stability region (in white) of the single mode response of the controlled cable system.

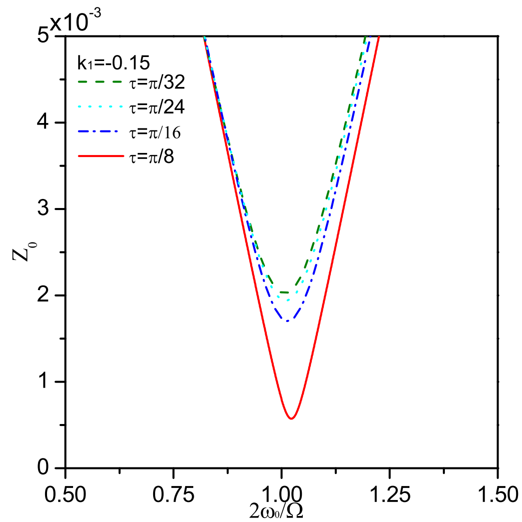

Fig. 3. The effect of the time delay on the stability of the controlled cable.

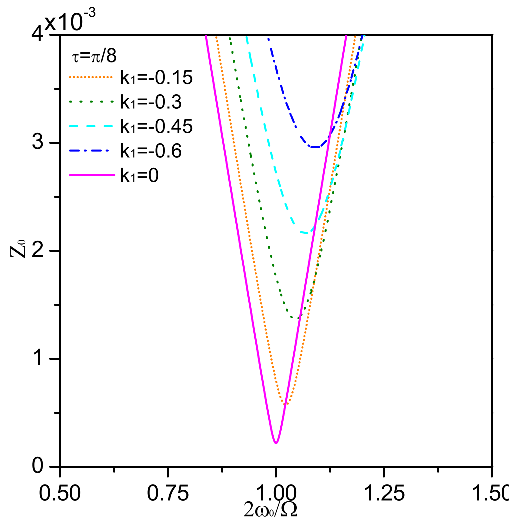

Fig. 4. The effect of the control gain on the stability of the controlled cable.

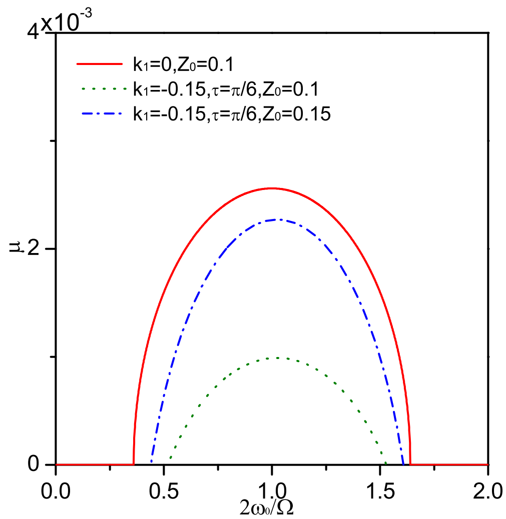

Fig. 5. The effect of the amplitude of the excitation on the stability of the controlled cable.

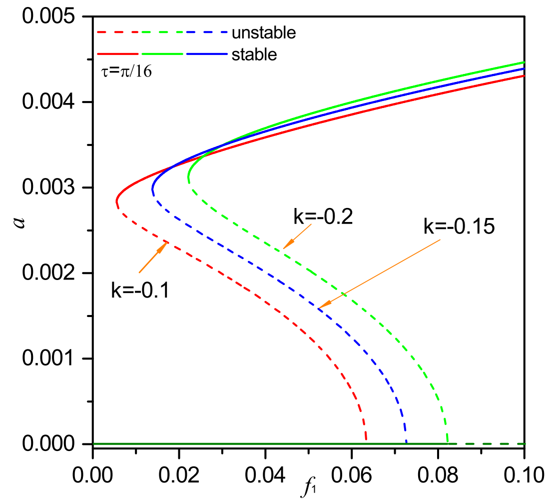

Fig. 6. The frequency response curve of the controlled cable with time delay $\tau = \pi/16$. (write k_1 instead of k (as in the previous figures). Write a_n instead of a (as reported in the text of the paper). Who is f_1? It is equal to $\Omega/2\pi$? This should be said. Report the value of z_0 used in this curves)

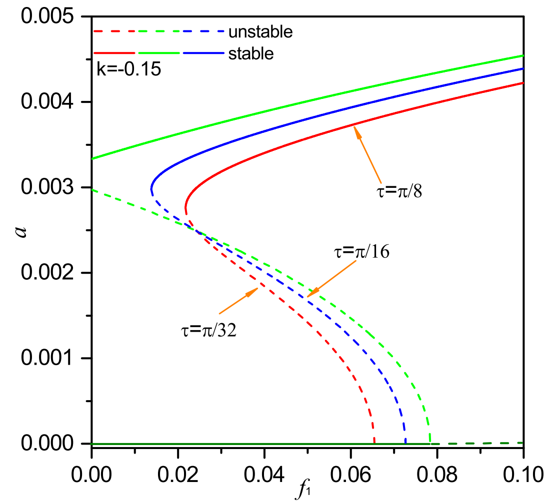

Fig. 7. The frequency response curve of the controlled cable with control gain $k_1 = -0.15$. (The same comments on the previous figure apply. Furthermore, I believe that $\tau = \pi/32$ refers to the green curve, not to the red one)

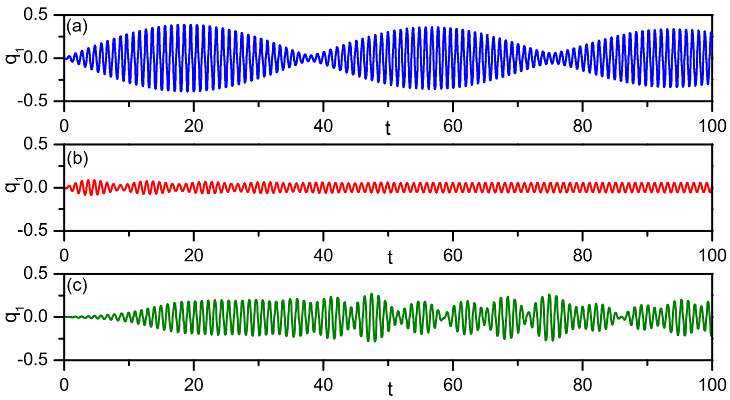

Fig. 8. Comparison of the time history of the controlled cable. (a) no control; (b) $k_1 = -1, \tau = \pi/2$; (c) $k_1 = -1, \tau = \pi$. (report the values of Ω and z_0 used for these figures)