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RESEARCH

# Heterogeneous traders: endogenous uncertainty in financial markets

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**Abstract** This paper presents a model in which fundamentalists, absolute momentum traders, and cross-sectional momentum traders interact in a financial market with two risky assets. We assume that the excess demand of fundamentalists is described by a general polynomial function, with terms up to cubic order. We demonstrate that the combined effect of nonlinearity, fundamentalists, and momentum traders is effective in generating both bull and bear market dynamics. Additionally, we employ a stochastic version of our deterministic model to generate simulated time series that replicate the stylized facts—such as asymmetry, excess kurtosis, and volatility clustering—of two energy markets: the Pennsylvania–New Jersey–Maryland (PJM) electricity market and the Queensland electricity market (QLD).

**Keywords** Heterogeneous agents · Multi-asset · Bifurcations · Energy markets · Stylized facts · Time series

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## 1 Introduction

The role of heterogenous agents models (HAM) in explaining many stylized facts and market anomalies has demonstrated its effectiveness in detecting the impact of behavioral forces on market stability (see, e.g., [1] and [2] for a survey). [3] and [4], among others, have documented the key stylized facts observed in financial markets, which are the main focus of HAM (see [5–7], for example). The underlying mechanism that explains these empirical findings is the nonlinear building-block introduced in the models, which serves as the basis for understanding the dynamics of the endogenous variables. In this context, this paper analyses a nonlinear HAM with fundamentalists, momentum and cross-sectional traders to account for empirical evidence in the energy market sector, specifically the electricity market. Since 1990, following a period of liberalization in the electricity sector, the electricity has become a traded commodity ([8,9]), making it important to study its statistical properties using a behavioral model. We follow the literature on HAM to develop a nonlinear dynamical system with three types of agents trading in a financial market with two risky assets: fundamentalists, absolute momentum and cross-sectional momentum traders. The link between these two assets is provided by the cross-sectional momentum traders. The financial literature has established the destabilizing role of momentum traders in markets (see [10,11] and [12], for example). Moreover, given the nature of the trading strategy of the cross-sectional momentum agents, who act simultaneously in both assets, the instability of the system could be amplified. An additional feature of our model is the functional form of the excess of demand of fundamentalists. This approach is inspired by [13], who considers a general polynomial functional form with entries up to cubic order in the excess of demand of fundamentalists and chartists when estimating the parameters of a collection of behavioral models. To preserve the analytical tractability of our model, we apply this functional specification only to fundamentalists. It is important to note that a nonlinear trading rule for fundamentalists has also been considered in [14]. A higher-order nonlinear trading rule for fundamentalists leads to a more pronounced reaction to large mispricings of the price from its fundamental value. In this regard, the greater the deviation of the price from its fundamental value, the more aggressive the fundamentalists become. The final model results in a system of second-order difference equations which, when necessary, can be reduced to a system of first-order difference equations. We demonstrate several interesting results. First, our final system can admit between 1 and 9 fixed points, depending on parameter values. This results in the coexistence of the fundamental equilibrium with other types of fundamental equilibria, introducing uncertainty about the fundamental value (see [15,16], for example). Second, through our bifurcation analysis, we confirm the occurrence of the Neimark–Sacker bifurcation, which is responsible for the market price fluctuations around the fundamental equilibrium and justifies the time-series momentum in the short-run and mean reversion in long-run, in line with [11] and [10]. Third, we find results consistent with the analysis of [13] regarding the impact of the different polynomial terms. Specifically, we do not exclude the relevance of the linear component in explaining key dynamics, and by using different model specifications that incorporate a stochastic component, we confirm the joint impact of the quadratic and cubic terms in facilitating the matching of stylized facts.

Our contributions to the existing literature are as follows. We build a dynamic model with fundamentalists, as well as both absolute and cross-sectional momentum traders, specifying a general functional form for the excess demand of fundamentalists, including nonlinear terms up to cubic order, following [13]. This component, which introduces significant nonlinearity into the model, combined with the destabilizing role of cross-sectional momentum traders, generates interesting and complex dynamics in electricity market returns. Second, based on the results of our stability analysis, we find that the model can admit multiple fundamental equilibria. This is an important contribution to the literature examining the role of sentiment indices in the presence of uncertainty about the fundamental value ([15,16]). However, while in these models the uncertainty in the fundamental value is assumed by the authors, in our model this uncertainty is intrinsic and arises directly from the tuning of the parameters. Third, we complement the literature on HAM by analyzing the empirical distribution of log-returns from two international energy markets: the Pennsylvania–New Jersey–Maryland (PJM) power market, and the Queensland electricity market (QLD). While most of the HAM literature focuses on financial markets, we validate our results by considering the stylized facts of energy markets. We account for a stochastic version of our model in order to match their empirical evidence.

The paper is organized as follows. In Sect. 2 we introduce the model. In Sect. 3 we analytically study the model's properties and verify them through several numerical simulations. Section 4 presents a stochastic version of the model to match the stylized facts of the PJM and QLD markets, particularly the asymmetry, excess kurtosis, and volatility clustering. Section 5 concludes.

## 2 The model

We work in a financial market with two risky assets, denoted as $A$ and $B$.[1] Both fundamental values are assumed to be constant: $F_t^A = F^A$ and $F_t^B = F^B$, $\forall t$.

The market is populated by three types of inverstors: fundamentalists, absolute momentum (chartists) and cross-sectional momentum.

Following [13], we introduce the following excess demand functions for each type of traders:

1. Fundamentalists react on deviations between the fundamental value and the market price:

$$p_{t+1}^i - p_t^i = \beta_{f1}(F^i - p_t^i) + \beta_{f2}(F^i - p_t^i)^2 + \beta_{f3}(F^i - p_t^i)^3$$

[1] $A$ and $B$ could represent two markets for the same asset.

2. Absolute momentum agents act according to price changes in the short run:

$$p_{t+1}^i - p_t^i = \beta_a(p_t^i - p_{t-1}^i)$$

3. Cross-sectional momentum traders take a long position in one asset and a short position in the other[2]:

$$p_{t+1}^i - p_t^i = \beta_c[(p_t^i - p_{t-1}^i) - (p_t^j - p_{t-1}^j)]$$

with $i, j = A, B$ and $i \neq j$, $\beta_{f1}, \beta_{f2}, \beta_{f3}, \beta_a, \beta_c > 0$.

Assuming fixed proportions of agents ($\alpha_f^i, \alpha_a^i, \alpha_c^i \in (0, 1)$ such that $\alpha_f^i + \alpha_a^i + \alpha_c^i = 1$) and working in terms of deviations of fundamentals ($x_t^i = p_t^i - F^i$), the overall excess demand is given by:

$$\begin{aligned} x_{t+1}^i - x_t^i = &- \gamma_{f1}^i x_t^i + \gamma_{f2}^i (x_t^i)^2 - \gamma_{f3}^i (x_t^i)^3 + \\ &+ \gamma_c^i (x_t^i - x_{t-1}^i) - \gamma_c^i (x_t^j - x_{t-1}^j) + \\ &+ \gamma_a^i (x_t^i - x_{t-1}^i) \end{aligned}$$

where $\mu^i > 0$ is the speed of the price adjustment made by the market maker and $\gamma_{f1}^i = \mu^i \alpha_f^i \beta_{f1}$, $\gamma_{f2}^i = \mu^i \alpha_f^i \beta_{f2}$, $\gamma_{f3}^i = \mu^i \alpha_f^i \beta_{f3}$, $\gamma_a^i = \mu^i \alpha_a^i \beta_a$, $\gamma_c^i = \mu^i \alpha_c^i \beta_c$.

Thus, the final system $S$ is given by:

$$\begin{cases} x_{t+1}^A &= H_A(x_t^A, x_t^B, x_{t-1}^A, x_{t-1}^B) = -\gamma_{f3}^A (x_t^A)^3 + \\ &\quad + \gamma_{f2}^A (x_t^A)^2 + (1 - \gamma_{f1}^A) x_t^A \\ &\quad + (\gamma_c^A + \gamma_a^A)(x_t^A - x_{t-1}^A) - \gamma_c^A (x_t^B - x_{t-1}^B) \\ x_{t+1}^B &= H_B(x_t^A, x_t^B, x_{t-1}^A, x_{t-1}^B) = -\gamma_{f3}^B (x_t^B)^3 \\ &\quad + \gamma_{f2}^B (x_t^B)^2 + (1 - \gamma_{f1}^B) x_t^B \\ &\quad + (\gamma_c^B + \gamma_a^B)(x_t^B - x_{t-1}^B) - \gamma_c^B (x_t^A - x_{t-1}^A) \end{cases} \tag{1}$$

When necessary, we introduce the auxiliary variables $y_{t+1}^A = x_t^A$ and $y_{t+1}^B = x_t^B$ to transform it into a first-order difference equation system:

$$\begin{cases} x_{t+1}^A = H_A(x_t^A, x_t^B, y_t^A, y_t^B) \\ x_{t+1}^B = H_B(x_t^A, x_t^B, y_t^A, y_t^B) \\ y_{t+1}^A = x_t^A \\ y_{t+1}^B = x_t^B \end{cases} \tag{2}$$

In the following section, we perform a dynamical analysis using both analytical and numerical tools.

## 3 Dynamical analysis

Before investigating the existence of steady states owned by the system, we provide the following definition to classify the different types of equilibrium.

**Definition 1** The fixed points $(x^A, x^B)$ of System (1) are classified as:

- Fundamental steady state iff $x_A = 0$ and $x_B = 0$
- $A$-fundamental steady state iff $x_A = 0$ and $x_B \neq 0$
- $B$-fundamental steady state iff $x_A \neq 0$ and $x_B = 0$
- Non-fundamental steady state iff $x_A \neq 0$ and $x_B \neq 0$

As a preliminary result, we observe that the fundamental steady state $(0, 0)$ exists for any range of parameter values. In contrast, for the other types of equilibria, the following proposition holds.

**Proposition 1** *Let* $\Delta_A = (\gamma_{f2}^A)^2 - 4\gamma_{f3}^A \gamma_{f1}^A$ *and* $\Delta_B = (\gamma_{f2}^B)^2 - 4\gamma_{f3}^B \gamma_{f1}^B$. *The system defined by (1) can admit up to nine fixed points:*

- *The* ***fundamental*** *equilibrium:* $(x_F^A, x_F^B) = (0, 0)$, *for any range of the parameter values*
- *Two A*−***fundamental*** *equilibria:* $(0, x_\star^B)$ *with* $x_\star^B = \frac{\gamma_{f2}^B \pm \sqrt{\Delta_B}}{2\gamma_{f3}^B}$, *iff* $\Delta_B \geq 0$
- *Two B*−***fundamental*** *equilibria:* $(x_\star^A, 0)$ *with* $x_\star^A = \frac{\gamma_{f2}^A \pm \sqrt{\Delta_A}}{2\gamma_{f3}^A}$, *iff* $\Delta_A \geq 0$
- *Four* ***non-fundamental*** *equilibria:* $(x_\star^A, x_\star^B)$ *with* $x_\star^A = \frac{\gamma_{f2}^A \pm \sqrt{\Delta_A}}{2\gamma_{f3}^A}$ *and* $x_\star^B = \frac{\gamma_{f2}^B \pm \sqrt{\Delta_B}}{2\gamma_{f3}^B}$ *iff* $\Delta_A, \Delta_B \geq 0$

*Proof* After imposing equilibrium conditions $x_t^A = x^A$ and $x_t^B = x^B$ $\forall t$, System (1) becomes:

$$\begin{cases} x^A = -\gamma_{f3}^A (x^A)^3 + \gamma_{f2}^A (x^A)^2 + (1 - \gamma_{f1}^A) x^A \\ x^B = -\gamma_{f3}^B (x^B)^3 + \gamma_{f2}^B (x^B)^2 + (1 - \gamma_{f1}^B) x^B \end{cases}$$

[2] The cross-sectional momentum strategy is well documented in [10].

**Table 1** Number and type of fixed points in different parameter regions, where $\Delta_A = (\gamma_{f2}^A)^2 - 4\gamma_{f3}^A\gamma_{f1}^A$ and $\Delta_B = (\gamma_{f2}^B)^2 - 4\gamma_{f3}^B\gamma_{f1}^B$

| Parameter regions | $\Delta_A > 0$ | $\Delta_A = 0$ | $\Delta_A < 0$ |
|---|---|---|---|
| $\Delta_B > 0$ | 1 fundamental | 1 fundamental | 1 fundamental |
| | 2 $A$-fundamental | 2 $A$-fundamental | 2 $A$-fundamental |
| | 2 $B$-fundamental | 1 $B$-fundamental | |
| | 4 non-fundamental | 2 non-fundamental | |
| $\Delta_B = 0$ | 1 fundamental | 1 fundamental | 1 fundamental |
| | 1 $A$-fundamental | 1 $A$-fundamental | 1 $A$-fundamental |
| | 2 $B$-fundamental | 1 $B$-fundamental | |
| | 2 non-fundamental | 1 non-fundamental | |
| $\Delta_B < 0$ | 1 fundamental | 1 fundamental | 1 fundamental |
| | 2 $B$-fundamental | 1 $B$-fundamental | |

whose solutions get to the vectors defined in the proposition. □

According to Proposition 1, our model can admit between 1 and 9 fixed points, depending on the parameter values. Table 1 summarizes the existence of steady states in the corresponding parameter regions, according to the signs of $\Delta_{A,B}$.

Note that once $\Delta_B$ is fixed, the number of $A$-fundamental steady state remains constant. In contrast, the number of B-fundamental fixed points varies. Specifically, for any $\Delta_B$, there are two $B$-fundamental steady states if $\Delta_A > 0$. As $\Delta_A$ decreases through zero and becomes negative, the two $B$-fundamental fixed points coalesce and disappear. The same story holds if $A$ and $B$ interchange. Therefore, the qualitative change in the number of $A-$ (or $B-$) fundamental equilibria is associated with fold bifurcations, that occur at $\Delta_B = 0$ (or $\Delta_A = 0$).

Moreover, from Proposition 1, we can immediately derive conditions on the parameter values that guarantee the uniqueness of the fundamental steady state, as stated in the following corollary.

**Corollary 1** *Let* $\Delta_A = (\gamma_{f2}^A)^2 - 4\gamma_{f3}^A\gamma_{f1}^A$ *and* $\Delta_B = (\gamma_{f2}^B)^2 - 4\gamma_{f3}^B\gamma_{f1}^B$. *The fundamental equilibrium* $(0, 0)$ *is the unique steady state of the system iff* $\Delta_A < 0$ *and* $\Delta_B < 0$.

Now, we turn our attention to the local stability analysis of the fundamental equilibrium. To this end, we need to consider the system as defined in (2). The trace and determinant of the Jacobian matrix are given by: $Tr(x_A, x_B) = \frac{\partial H_A}{\partial x_t^A}(x_A, x_B) + \frac{\partial H_B}{\partial x_t^B}(x_A, x_B)$, $Det(x_A, x_B) = \frac{\partial H_A}{\partial y_t^A}(x_A, x_B) \cdot \frac{\partial H_B}{\partial y_t^B}(x_A, x_B) - \frac{\partial H_A}{\partial y_t^B}(x_A, x_B) \cdot \frac{\partial H_B}{\partial y_t^A}(x_A, x_B)$. Hence:

$$Tr(0, 0) = 2 + \gamma_c^A + \gamma_a^A + \gamma_c^B + \gamma_a^B - \gamma_{f1}^A - \gamma_{f1}^B$$

$$Det(0, 0) = (\gamma_c^A + \gamma_a^A) \cdot (\gamma_c^B + \gamma_a^B) - \gamma_c^A\gamma_c^B$$

The following proposition proves that the fundamental equilibrium undergoes a Neimark–Sacker bifurcation.

**Proposition 2** *The Neimark–Sacker bifurcation of the fundamental steady state occurs at* $\gamma_c^A\gamma_a^B + \gamma_a^A\gamma_c^B + \gamma_a^A\gamma_a^B = 1$, *if* $\gamma_{f1}^A + \gamma_{f1}^B < 4 + \gamma_c^A + \gamma_a^A + \gamma_c^B + \gamma_a^B$ *and* $\gamma_{f1}^A + \gamma_{f1}^B > \gamma_c^A + \gamma_a^A + \gamma_c^B + \gamma_a^B$.

*Proof* The fixed point $(0, 0)$ is locally asymptotically stable iff:

$$\begin{cases} 1 + Tr(0,0) + Det(0,0) > 0 \\ 1 - Tr(0,0) + Det(0,0) > 0 \\ 1 - Det(0,0) > 0 \end{cases}$$

and a bifurcation results from the violation of one of such conditions. In particular, the Neimark-Sacker occurs when $Det(0, 0) = 1$ while $1 + Tr(0, 0) + Det(0, 0) > 0$ and $1 - Tr(0, 0) + Det(0, 0) > 0$. The last two conditions are guaranteed if $Tr(0, 0) \in (-2, 2)$, that in our case are $\gamma_c^A + \gamma_a^A + \gamma_c^B + \gamma_a^B > \gamma_{f1}^A + \gamma_{f1}^B - 4$ and $\gamma_c^A + \gamma_a^A + \gamma_c^B + \gamma_a^B < \gamma_{f1}^A + \gamma_{f1}^B$, while $\gamma_c^A\gamma_a^B + \gamma_a^A\gamma_c^B + \gamma_a^A\gamma_a^B = 1$ means $Det(0, 0) = 1$. □

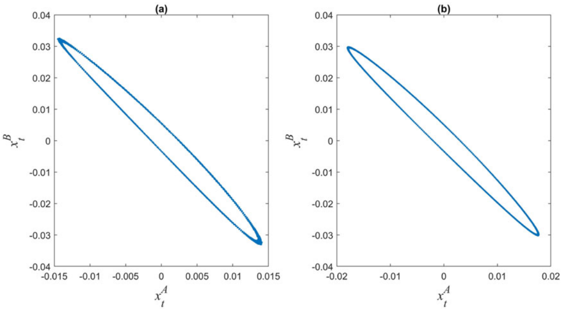


**Fig. 1** Attractor in the plane $(x_t^A, x_t^B)$ when conditions $\gamma_c^A\gamma_a^B + \gamma_a^A\gamma_c^B + \gamma_a^A\gamma_a^B < 1$, $\gamma_{f1}^A + \gamma_{f1}^B < 4 + \gamma_c^A + \gamma_a^A + \gamma_c^B + \gamma_a^B$ and $\gamma_{f1}^A + \gamma_{f1}^B > \gamma_c^A + \gamma_a^A + \gamma_c^B + \gamma_a^B$ hold. Parameter values: $\gamma_{f3}^A = 0.45$, $\gamma_{f3}^B = 0.59$, $\gamma_{f2}^A = 0.7$, $\gamma_{f2}^B = 0.6$, $\gamma_{f1}^B = 2.2$, $\gamma_a^A = 0.4142$, $\gamma_a^B = 0.4$, $\gamma_c^A = 0.201$, $\gamma_c^B = 0.4$, and $\gamma_{f1}^A = 2.4$ in (**a**), while $\gamma_{f1}^A = 2.026$ in (**b**). Initial conditions $x_0^A = 0.2$, $x_0^B = 0.2$

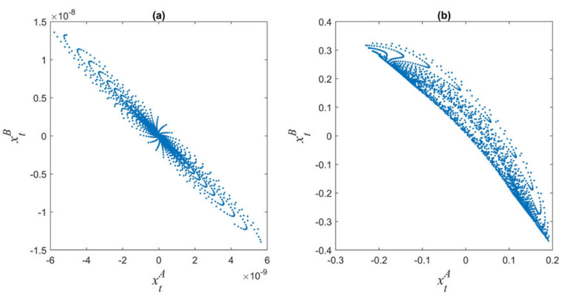


**Fig. 2** Attractor in the plane $(x_t^A, x_t^B)$ when conditions $\gamma_c^A\gamma_a^B + \gamma_a^A\gamma_c^B + \gamma_a^A\gamma_a^B < 1$, $\gamma_{f1}^A + \gamma_{f1}^B < 4 + \gamma_c^A + \gamma_a^A + \gamma_c^B + \gamma_a^B$ and $\gamma_{f1}^A + \gamma_{f1}^B > \gamma_c^A + \gamma_a^A + \gamma_c^B + \gamma_a^B$ hold. Parameter values: $\gamma_{f3}^A = 0.45$, $\gamma_{f3}^B = 0.59$, $\gamma_{f2}^A = 0.7$, $\gamma_{f2}^B = 0.6$, $\gamma_{f1}^B = 2.2$, $\gamma_a^A = 0.4142$, $\gamma_a^B = 0.4$, $\gamma_c^A = 0.201$, $\gamma_c^B = 0.4$, and $\gamma_{f1}^A = 2.46$ in (**a**), while $\gamma_{f1}^A = 2.033$ in (**b**). Initial conditions $x_0^A = 0.2$, $x_0^B = 0.2$

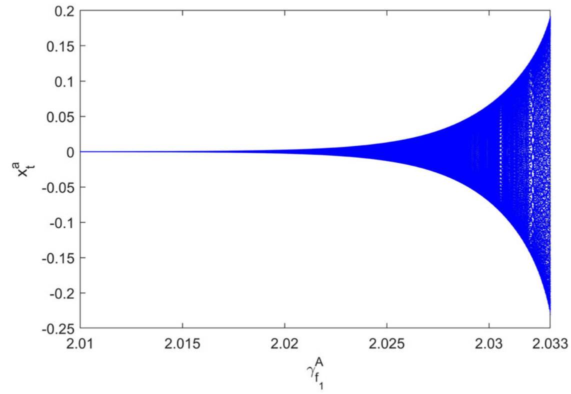

**Fig. 3** Bifurcation diagram on varying $\gamma_{f1}^A \in [2.01, 2.033]$ for parameter values: $\gamma_{f3}^A = 0.45$, $\gamma_{f3}^B = 0.59$, $\gamma_{f2}^A = 0.7$, $\gamma_{f2}^B = 0.6$, $\gamma_{f1}^B = 2.2$, $\gamma_a^A = 0.4142$, $\gamma_a^B = 0.4$, $\gamma_c^A = 0.201$, $\gamma_c^B = 0.4$. Initial conditions $x_0^A = 0.2$, $x_0^B = 0.2$

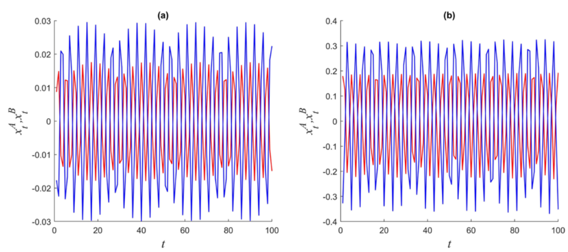


**Fig. 4** Trajectories of asset A (in red) and B (in blue) in the plane $(t, x_t^i)$, with $i = A, B$. Parameter values: $\gamma_{f3}^A = 0.45$, $\gamma_{f3}^B = 0.59$, $\gamma_{f2}^A = 0.7$, $\gamma_{f2}^B = 0.6$, $\gamma_{f1}^B = 2.2$, $\gamma_a^A = 0.4142$, $\gamma_a^B = 0.4$, $\gamma_c^A = 0.201$, $\gamma_c^B = 0.4$, and $\gamma_{f1}^A = 2.026$ in (**a**), while $\gamma_{f1}^A = 2.033$ in (**b**). Initial conditions $x_0^A = 0.2$, $x_0^B = 0.2$

Given the importance of this kind of bifurcation in the literature, we highlight the dynamics of the price in Figs. 1, 2 and 3. Figure 1 illustrates the loss of stability of the fundamental fixed point and the appearance of a closed invariant curve around it, a typical pattern of the Neimark–Sacker bifurcation. Figure 2 shows the complex attractors that arise when the level of activity of fundamentalists it too high. The same result is confirmed in Fig. 3 via a bifurcation diagram when $\gamma_{f1}^A$, i.e. the level of activity of fundamentalists in market A, is varied. Finally, Fig. 4, illustrates the price dynamics of both markets as $\gamma_{f1}^A$ increases. One can observe that the trajectories change from periodic (Fig. 4a) to quasi-periodic (Fig. 4b).

We would like to highlight that the fixed-point analysis conducted thus far demonstrates that our model can exhibit the coexistence of multiple locally stable equilibria. Specifically, following a saddle-node (or fold) bifurcation, at least one stable equilibrium emerges, either an A- or B- fundamental fixed point, depending on the parameters $\Delta_A$ and $\Delta_B$. In this context, the fundamental steady state can be locally stable, in accordance with the conditions established in the proof of Proposition 2. In other words, there may exist a range of control parameter values for which the model admits multiple stable equilibria.

Interestingly, this result still holds even in the absence of cross-sectional momentum trading ($\gamma_c^{A,B} =$

0), when the two assets are independent of each other. In this case, our nonlinear demand function can generate up to three equilibria for each asset. Similarly to the previous case, for $i = A, B$, if $\Delta_i = (\gamma^i_{f_2})^2 - 4\gamma^i_{f_3}\gamma^i_{f_1} \geq 0$ then asset $i$ admits two non-fundamental steady state (possibly coinciding) in addition to the fundamental one; otherwise, the latter is unique. A saddle-node bifurcation occurs when $\Delta_i = 0$. Also in this case, the parameter values that ensure the local stability of the fundamental equilibrium are consistent with those of the saddle-node bifurcation, which leads to the emergence of an additional locally stable steady state.

However, Proposition 2 highlights the destabilizing effect of cross-sectional momentum trading on the fundamental equilibrium, as indicated by the fact that the determinant $Det(0, 0)$ increases as the parameters $\gamma_c^{A,B}$ increase, once the other parameters are fixed. This implies that the presence of such agents pushes the fixed point toward the Neimark–Sacker bifurcation.

From Proposition 2, one can directly obtain the following bifurcation curve in the parameter plane $(\gamma_c^A, \gamma_c^B)$, once the other parameters are fixed:

**Corollary 2** *The Neimark–Sacker bifurcation curve in the parameter plane $(\gamma_c^A, \gamma_c^B)$ is defined by: $\gamma_c^A\gamma_a^B + \gamma_a^A\gamma_c^B + \gamma_a^A\gamma_a^B = 1$ such that $\gamma^A_{f1} + \gamma^B_{f1} - \gamma_a^A - \gamma_a^B - 4 < \gamma_c^A + \gamma_c^B < \gamma^A_{f1} + \gamma^B_{f1} - \gamma_a^A - \gamma_a^B$.*

For what concerns the Neimark–Sacker bifurcation, the parameters $\gamma^A_{f1}$ and $\gamma^B_{f1}$ do not directly influence the Neimark–Sacker bifurcation curve $1 - Det(0, 0) = 0$, but they they contribute to the emergence of more complex dynamics immediately after the bifurcation. Moreover, they are the leading actors in the period-doubling bifurcation of the fundamental equilibrium, as proved in the following proposition.

**Proposition 3** *Let $(\gamma_c^A + \gamma_a^A)\cdot(\gamma_c^B + \gamma_a^B) - \gamma_c^A\gamma_c^B < 1$, then the fundamental steady state undergoes a period doubling bifurcation if $\gamma^A_{f1} + \gamma^B_{f1}$ is big enough.*

*Proof* The hypotheses $(\gamma_c^A + \gamma_a^A) \cdot (\gamma_c^B + \gamma_a^B) - \gamma_c^A\gamma_c^B < 1$ means $1 - Det(0, 0) > 0$. Moreover, observe that $Tr(0, 0)$ goes to $-\infty$ when $\gamma^A_{f1} + \gamma^B_{f1}$ tends to $+\infty$, in other terms $1 - Tr(0, 0) + Det(0, 0) > 0$ while $1 + Tr(0, 0) + Det(0, 0)$ becomes negative if $\gamma^A_{f1} + \gamma^B_{f1}$ is big enough. □

In the next subsection we investigate the role of different market agents in the final dynamics, applying our theoretical results, and performing numerical analysis, as well.

### 3.1 The role of heterogeneous traders

We would like to highlight the role of cross-sectional momentum traders in our model, given their significance in the literature ([10,11]). In fact, as shown in Corollary 2, the destabilizing effect of cross-sectional momentum traders is mitigated by the action of fundamentalists. Two key points should be emphasized. First, the occurrence of the Neimark–Sacker bifurcation requires the coordination of the cross-sectional momentum traders across both markets. Second, the bifurcation can also occur if the overall activity of the absolute momentum traders is lower than that of fundamentalists in both markets. If we read Proposition 3 in conjunction with the result of Corollary 2, we find two routes to complicated dynamics generated by fundamentalists. In the first case, when fundamentalists are excessively aggressive in their trading strategies, they may destabilize the market, leading to the occurrence of a period-doubling bifurcation. This corresponds to the case in which $\gamma^A_{f_1} + \gamma^B_{f_1}$ is sufficiently large. However, another route emerges when the overall activity of absolute momentum traders is lower than that of fundamentalists in both markets. In this scenario, the fundamentalists' relative aggressiveness is further amplified by the influence of cross-sectional momentum traders, resulting in complex market dynamics.

To better understand the role of different market agents in ensuring stability, Figs. 5, 6 and 7 illustrate the joint effects of their trading activity through a two-dimensional bifurcation diagram. Our focus here is on the role of fundamentalists for two main reasons. First, according to the literature on HAM, fundamentalists are viewed as stabilizing agents in the market ([17,18]). Second, we have introduced a specific polynomial excess demand function for these traders, which includes terms up to cubic order (following [13]). The nonlinear terms can be interpreted as reflecting the level of aggressiveness exhibited by fundamentalists in the market. Additionally, we note that our model does not incorporate the switching mechanism of traders (see, for example, [19] and [20]) but instead focuses on their trading strategies as outlined in the seminal work of [17]. We leave the exploration of this aspect for future developments in our work.

Figures 5 and 6 represent the stability region (in blue) when different combinations of the parameters for fundamentalists operating within the same market are considered. Specifically, we examine the stability

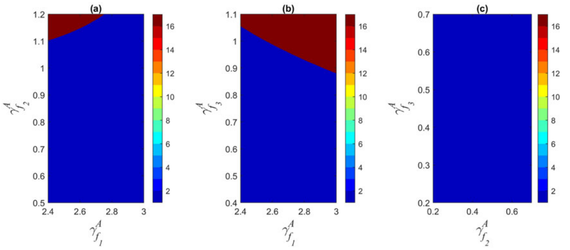

**Fig. 5** Two-dimensional bifurcation diagram in the plane $(\gamma_{f1}^A, \gamma_{f2}^A)$ in panel (**a**), $(\gamma_{f1}^A, \gamma_{f3}^A)$ in panel (**b**), $(\gamma_{f2}^A, \gamma_{f3}^A)$ in panel (**c**). Parameter values: $\gamma_{f3}^A = 0.45, \gamma_{f3}^B = 0.59, \gamma_{f2}^A = 0.7, \gamma_{f2}^B = 0.6, \gamma_{f1}^B = 2.2, \gamma_a^A = 0.4142, \gamma_a^B = 0.4, \gamma_c^A = 0.201, \gamma_c^B = 0.4$, and $\gamma_{f1}^A = 2.026$. Initial conditions $x_0^A = 0.2, x_0^B = 0.2$

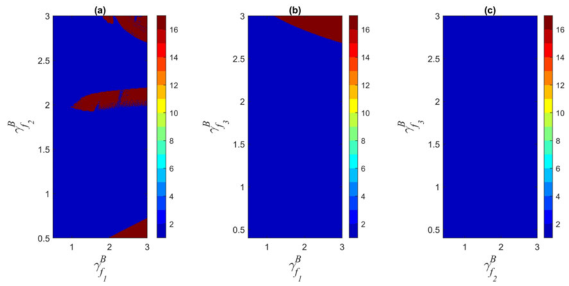

**Fig. 6** Two-dimensional bifurcation diagram in the plane $(\gamma_{f1}^B, \gamma_{f2}^B)$ in panel (**a**), $(\gamma_{f1}^B, \gamma_{f3}^B)$ in panel (**b**), $(\gamma_{f2}^B, \gamma_{f3}^B)$ in panel (**c**). Parameter values: $\gamma_{f3}^A = 0.45, \gamma_{f3}^B = 0.59, \gamma_{f2}^A = 0.7, \gamma_{f2}^B = 0.6, \gamma_{f1}^B = 2.2, \gamma_a^A = 0.4142, \gamma_a^B = 0.4, \gamma_c^A = 0.201, \gamma_c^B = 0.4$, and $\gamma_{f1}^A = 2.026$. Initial conditions $x_0^A = 0.2, x_0^B = 0.2$

effects in market A of a simultaneous increase in the linear and quadratic terms (Fig. 5a), the linear and cubic terms (Fig. 5b), and the quadratic and cubic terms (Fig. 5c). The same sequence of combinations is presented in Fig. 6 for market B. It is evident that the effects on stability are not symmetric across both markets, especially when the linear term interacts with the quadratic term. In this case, market A exhibits instability for large values of the quadratic term, while market B faces periods of instability even for low values of the quadratic term, provided the linear term is sufficiently high. A clear pattern emerges when the linear term interacts with the cubic term in both markets and when the quadratic and cubic terms are considered together. In the first scenario, both markets A and B undergo periods of instability if the linear and cubic terms are large enough. Notably, the adjustment of the quadratic and cubic terms ensures stability in the market.

Finally, in Fig. 7, we analyze the simultaneous interaction of the parameters for fundamentalists trading across different markets. A closer inspection of panels (a) and (b) of Fig. 7 reveals that the loss of stability may be attributed to higher values of the higher-order components (quadratic and cubic terms) in the model. This suggests an overreaction by fundamentalists to mispricing relative to the fundamental value. In contrast, Fig. 7(c) confirms the continued stability when the quadratic and cubic components are adjusted together.

Overall, our analysis provides evidence of the destabilizing role of cross-sectional momentum traders, supporting the findings in the existing literature on this topic. Furthermore, fundamentalists may contribute to instability if their activity level becomes sufficiently high. To maintain the proper equilibrium in the system, fundamentalists must avoid overreacting to mispricing relative to the fundamental value. Lastly, our results are consistent with the evidence presented by [13] regarding the impact of higher-order components in the excess demand of traders. In the next section, we will show that it is the combined effect of all polynomial terms that ensures the model works correctly, capturing the stylized facts.

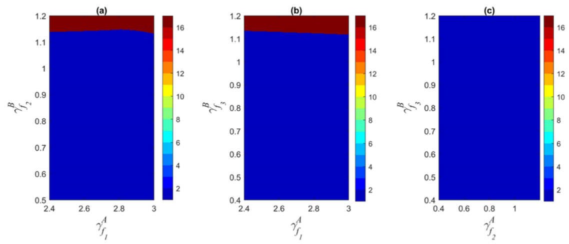

**Fig. 7** Two-dimensional bifurcation diagram in the plane $(\gamma_{f1}^A, \gamma_{f2}^B)$ in panel (**a**), $(\gamma_{f1}^A, \gamma_{f3}^B)$ in panel (**b**), $(\gamma_{f2}^A, \gamma_{f3}^B)$ in panel (**c**). Parameter values: $\gamma_{f3}^A = 0.45, \gamma_{f3}^B = 0.59, \gamma_{f2}^A = 0.7, \gamma_{f2}^B = 0.6, \gamma_{f1}^B = 2.2, \gamma_a^A = 0.4142, \gamma_a^B = 0.4, \gamma_c^A = 0.201, \gamma_c^B = 0.4$, and $\gamma_{f1}^A = 2.026$. Initial conditions $x_0^A = 0.2, x_0^B = 0.2$

## 4 Empirical model

To gain a better understanding of the properties of our model, we conduct a more comprehensive analysis to evaluate its ability to match the statistical properties of two key energy markets: the Pennsylvania–New Jersey–Maryland (PJM) and the Queensland (QLD)

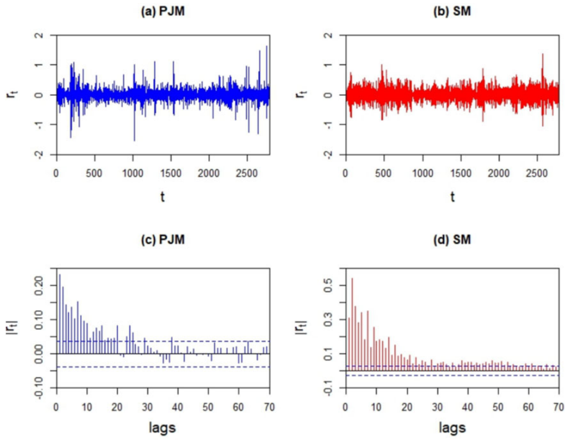


**Fig. 8** Historical behavior of log-returns of PJM (panel (a) and our stochastic model (panel (b)), while in panels (c) and (d) the corresponding autocorrelations of absolute returns (ACF)

electricity markets. We begin by analyzing the PJM market. Specifically, we use the time series of daily electricity prices, which are weighted averages of the 24 hly market prices expressed in nominal dollars per megawatt-hour ($/MWh), from April 2014 to March 2024 in the PJM power market (US Northeast region). The data are freely available for download at www.eia.gov/electricity/wholesale. In our analysis, we focus on daily log-returns. We added a noise component to each equation of System 1, extracted from a normal distribution with a mean of zero and a standard deviation of $\sigma = 5$. The set of parameters used to simulate the stochastic returns comes from the analysis performed in Section 3, where it is shown that these parameters were effective in generating interesting dynamics in financial markets. For this purpose, we use the following parameter set: $\gamma_{f3}^A = 0.45$, $\gamma_{f3}^B = 0.59$, $\gamma_{f2}^A = 0.7$, $\gamma_{f2}^B = 0.6$, $\gamma_{f1}^B = 2.2$, $\gamma_a^A = 0.4142$, $\gamma_a^B = 0.4$, $\gamma_c^A = 0.1$, $\gamma_c^B = 0.4$, $\gamma_{f1}^A = 2.033$, $\sigma = 5$, and initial conditions $x_0^A = 0.2$, $x_0^B = 0.2$. The value of $\sigma$ aligns with the results of [21], representing the mean of low-frequency volatility (i.e. the standard deviation of logarithmic daily price changes). Panels (a) and (b) of Fig. 8 display the returns of PJM market and those from our stochastic model (SM), respectively. It is evident that temporary deviations of market returns from their mean can be significant at times. Indeed, the time series exhibit intermittent and large fluctuations corresponding to periods of high and low volatility. This characteristic is known as volatility clustering. This property

**Table 2** Several specification of the SM model accounting for different combinations of quadratic and cubic terms of fundamentalists

| | Model specification |
|---|---|
| Model 1 | $x_{t+1} = \gamma_{f2}^i (x_t^i)^2 + \varepsilon_t^i$ |
| Model 2 | $x_{t+1} = -\gamma_{f3}^i (x_t^i)^3 + \varepsilon_t^i$ |
| Model 3 | $x_{t+1} = -\gamma_{f3}^i (x_t^i)^3 + \gamma_{f2}^i (x_t^i)^2 + \varepsilon_t^i$ |

is confirmed by the sample autocorrelation functions (ACF), which show statistically significant long-term correlations in the absolute returns, as seen in Fig. 8(c–d). Based on the analysis of Fig. 8, we conclude that our model replicates the volatility clustering observed in the PJM market quite well.

In Table 2, we present three models that include only fundamentalists. In the literature, models with only fundamentalists assume that traders have different beliefs about the fundamental value, which introduces a degree of uncertainty into the dynamics of price over time (see, for example, [15]). Regarding our models 1–3, the inclusion of the cubic, quadratic, or both components of $\gamma$ account for the overall activity of fundamentalists in the market: when we consider both the cubic and the quadratic components of $\gamma$ the influence of fundamentalists in driving the price toward equilibrium is amplified. The role of fundamentalists in our model is significant, as highlighted in the analysis performed in Section 3, where different fundamental equilibria emerged. To this end, in the empirical analysis, we include models 1–3 to assess whether fundamentalists alone can explain the stylized facts of electricity markets.

The results for Models 1–3 are reported in Table 3, where they are compared with the real data from the PJM market. As shown, considering the cubic, quadratic, or both components of $\gamma$ drives the price towards the equilibrium but worsens the fit with the real data. This finding is consistent with the results of [13], who observed relatively small improvements in the goodness-of-fit when considering only a quadratic or cubic term.

Based on the results obtained in Table 3, we expand our analysis with additional specifications of the SM model, as reported in Table 4. In this context, we aim to highlight two key points. First, our model is capable of replicating other stylized facts of the PJM market,

**Table 3** Summary statistics for PJM and Models 1–3

| | PJM | Model 1 | Model 2 | Model 3 |
|---|---|---|---|---|
| $r_{min}$ | – 1.5302 | – 0.4292 | – 0.3026 | – 0.3027 |
| $r_{max}$ | 1.6373 | 0.3967 | 0.3095 | 0.3101 |
| sd | 0.2075 | 0.1102 | 0.0852 | 0.0852 |
| Skewness | 0.1317 | 0.0203 | 0.0072 | 0.0078 |
| Kurtosis | 8.8662 | 3.0554 | 3.0135 | 3.0170 |
| J–B statistic | 9157.52 | 0.9358 | 0.0727 | 0.0999 |
| Tsay test | 2.3724 | 1.5371 | 42.213 | 41.293 |
| $H_r$ | 0.2526 | 0.3947 | 0.5058 | 0.5055 |
| $H_{\|r\|}$ | 0.6887 | 0.5749 | 0.4661 | 0.4666 |

The table reports the summary statistics including minimum ($r_{min}$) and maximum ($r_{max}$) returns, standard deviation (sd), skewness, kurtosis, Jarque–Bera statistic (J–B statistic), the Tsay nonlinearity test (Tsay test), the Hurst exponent for returns ($H_r$) and absolute returns ($H_{|r|}$), respectively

**Table 4** Several specification of the SM model accounting for the quadratic and cubic terms of fundamentalists

| | Model specification |
|---|---|
| Model 4 | $x^i_{t+1} = x^i_t + \varepsilon^i_t$ |
| Model 5 | $x^i_{t+1} = (1 - \gamma^i_{f1})x^i_t + (\gamma^i_c + \gamma^i_a)(x^i_t - x^i_{t-1}) - \gamma^i_c(x^i_t - x^i_{t-1}) + \varepsilon^i_t$ |
| Model 6 | $x^i_{t+1} = \gamma^i_{f2}(x^i_t)^2 + (1 - \gamma^i_{f1})x^i_t + (\gamma^i_c + \gamma^i_a)(x^i_t - x^i_{t-1}) - \gamma^i_c(x^i_t - x^i_{t-1}) + \varepsilon^i_t$ |
| Model 7 | $x^i_{t+1} = -\gamma^i_{f3}(x^i_t)^3 + (1 - \gamma^i_{f1})x^i_t + (\gamma^i_c + \gamma^i_a)(x^i_t - x^i_{t-1}) - \gamma^i_c(x^i_t - x^i_{t-1}) + \varepsilon^i_t$ |
| Model 8 | $x^i_{t+1} = -\gamma^i_{f3}(x^i_t)^3 + \gamma^i_{f2}(x^i_t)^2 + (1 - \gamma^i_{f1})x^i_t + (\gamma^i_c + \gamma^i_a)(x^i_t - x^i_{t-1}) - \gamma^i_c(x^i_t - x^i_{t-1}) + \varepsilon^i_t$ |

in addition to volatility clustering. Second, we seek to demonstrate the explanatory power of our complete nonlinear agent-based model in comparison to its linear or simplified counterpart. Our simulations consider several specifications of the model (Model 8). As a robustness check, we also include a simple random walk (Model 4) to assess whether the replication of empirical stylized facts can be attributed to both the nonlinear specification and the behavioral component. Regarding the other models, we proceed as follows. Model 5 incorporates only one "behavioral" component, namely the linear one. Models 6 and 7 consider combinations of two components at once: quadratic and linear (Model 6), cubic and linear (Model 7).

A closer inspection of Table 5 reveals that only our model is able to match all the empirical evidence from the PJM market. When considering the minimum ($r_{min}$) and maximum ($r_{max}$) values of returns, we note that only Model 8 achieves values that are closest to those of the PJM market in both cases simultaneously. Indeed, Model 4 exhibits a larger negative value for $r_{min}$, while Models 5–7 underestimate both $r_{min}$ and $r_{max}$. The same pattern holds for the standard deviation ($sd$).

Another key characteristic of the PJM time series is the distribution of returns, which differs from that of a normal distribution. In fact, the kurtosis and skewness indices show that it is characterized by fat tails and a right-skewed distribution. Moreover, the Jarque–Bera test (J–B statistic) confirms the non-normality of PJM returns at the 1% significance level. Beyond Model 4, which exhibits negative skewness, Models 5–7 present skewness indices close to zero, while the skewness of Model 8 closely matches the empirical distribution. Model 8 has a kurtosis index greater than 3 and, as a result, has heavier tails compared to the Normal distribution when compared to the other models, and therefore, better captures real data. Models 4 and 8 reject the null hypothesis that the simulated distribution of returns is normal. It is worth noting that a simple random walk outperforms Models 5–7 in the J–B test. We have conducted the Tsay nonlinearity test to verify that the nonlinear specification of our models is appropriate

**Table 5** Summary statistics for PJM and Models 4–8

| | *PJM* | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 |
|---|---|---|---|---|---|---|
| $r_{min}$ | − 1.5302 | − 5.39 | − 0.4292 | − 0.4516 | − 0.4016 | − 1.1860 |
| $r_{max}$ | 1.6373 | 1.5778 | 0.3967 | 0.3933 | 0.3784 | 1.7798 |
| sd | 0.2075 | 1.7284 | 0.1102 | 0.1119 | 0.1111 | 0.2668 |
| Skewness | 0.1317 | − 0.4157 | 0.0203 | 0.0043 | 0.0079 | 0.1178 |
| Kurtosis | 8.8662 | 1.8253 | 3.0554 | 3.0915 | 3.0650 | 4.7559 |
| J–B statistic | 9157.52 | 422.85 | 0.9358 | 1.6753 | 0.8791 | 638.77 |
| Tsay test | 2.3724 | 0.0049 | 0.9657 | 1.6038 | 1.6524 | 3.2135 |
| $H_r$ | 0.2526 | 0.9895 | 0.3947 | 0.3889 | 0.3928 | 0.4198 |
| $H_{\|r\|}$ | 0.6887 | 0.9822 | 0.5749 | 0.5918 | 0.5670 | 0.7289 |

The table reports the summary statistics including minimum ($r_{min}$) and maximum ($r_{max}$) returns, standard deviation (sd), skewness, kurtosis, Jarque–Bera statistic (J–B statistic), the Tsay nonlinearity test (Tsay test), the Hurst exponent for returns ($H_r$) and absolute returns ($H_{|r|}$), respectively

for this analysis. Specifically, the Tsay test is based on the F-statistic: under the null hypothesis, the process is linear, whereas a significant F-statistic provides evidence of nonlinearity in the series. The results indicate that the test detects significant nonlinearity in Models 7 and 8. Finally, the last two rows of Table 5 confirm the absence of autocorrelation in the return series and the persistence of correlations in absolute returns for PJM, employing the Hurst exponent of returns ($H_r$) and absolute returns ($H_{|r|}$). According to our simulations, Models 5, 6, 7 and 8 are able to fit the Hurst exponents of PJM. Overall, our results demonstrate that the proposed model is capable of replicating the stylized facts of electricity market returns using the PJM index data. When compared with other nested models, the only one that can simultaneously replicate all the empirical evidence from the PJM index is the nonlinear model that incorporates both cubic and quadratic behavioral components, consistent with the findings of [13].

We have conducted a similar analysis for the Australian electricity market, in particular the Queensland (QLD) electricity market. The inclusion of another market allows for an international comparison between electricity markets. The data for the Australian market were obtained from the Australian Energy Market Operator (AEMO) and are freely available at https://www.aemo.com.au. As with the PJM market, we use the daily average spot prices, which are calculated as weighted averages of the 24 hly market prices, expressed in nominal dollars per megawatt-hour ($/MWh). The data cover the period from April 2014 to March 2024. We use the same parameter values as in

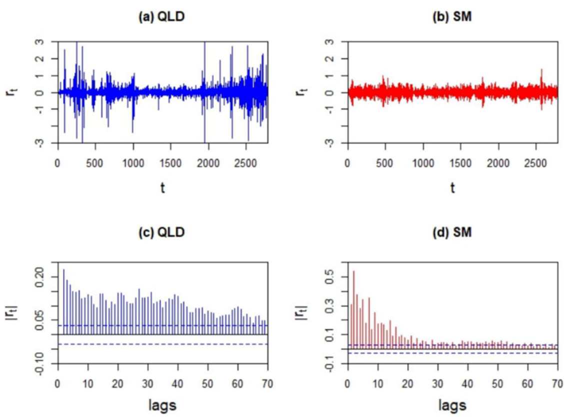


**Fig. 9** Historical behavior of log-returns of QLD (panel (a) and our stochastic model (panel (b)), while in panels (c) and (d) the corresponding autocorrelations of absolute returns (ACF)

the PJM case with $\sigma = 7.79$ for QLD (in line with the analysis of [21]). In Fig. 9, panels (a) and (b), we note that the volatility of QLD market is larger than those of PJM market and our model. Moreover, panels (c) and (d) of Fig. 9 highlight that the SM model is able to capture the correlations in the absolute returns of QLD but only for the first 25 lags. In Tables 6 and 7, we report the summary statistics for the Queensland (QLD) electricity market. Table 6 presents the results for a model that includes only fundamentalists in the market. Once again, we find evidence that the magnitude of the exponent $\gamma$ influences the dynamics towards equilibrium, with fundamentalists driving the price toward equilibrium when they trade more aggressively. This also sug-

**Table 6** Summary statistics for Queensland and Models 1–3

| | QLD | Model 1 | Model 2 | Model 3 |
|---|---|---|---|---|
| $r_{min}$ | − 3.7144 | − 0.5689 | − 0.3919 | − 0.3921 |
| $r_{max}$ | 4.0443 | 0.5309 | 0.4046 | 0.4057 |
| sd | 0.4070 | 0.1460 | 0.1108 | 0.1109 |
| Skewness | 0.1733 | 0.0165 | 0.0088 | 0.0097 |
| Kurtosis | 15.97 | 0.0411 | 0.0139 | 0.0200 |
| J–B statistic | 37551.96 | 0.5541 | 0.0854 | 0.1333 |
| Tsay test | 2.7321 | 0.9083 | 9.4982 | 66.223 |
| $H_r$ | 0.3459 | 0.4391 | 0.5359 | 0.5357 |
| $H_{\|r\|}$ | 0.8422 | 0.6157 | 0.4967 | 0.4963 |

The table reports the summary statistics including minimum ($r_{min}$) and maximum ($r_{max}$) returns, standard deviation (sd), skewness, kurtosis, Jarque–Bera statistic (J–B statistic), the Tsay nonlinearity test (Tsay test), the Hurst exponent for returns ($H_r$) and absolute returns ($H_{|r|}$), respectively

**Table 7** Summary statistics for Queensland and Models 4–8

| | QLD | Model 4 | Model 5 | Model 6 | Model 7 | Model 8 |
|---|---|---|---|---|---|---|
| $r_{min}$ | − 3.7144 | − 10.5921 | − 0.5689 | − 0.6314 | − 0.6616 | − 1.0818 |
| $r_{max}$ | 4.0443 | 2.9685 | 0.5309 | 0.5567 | 0.4855 | 1.6137 |
| sd | 0.4070 | 3.3661 | 0.1460 | 0.1506 | 0.1481 | 0.2636 |
| Skewness | 0.1733 | − 0.4154 | 0.0165 | 0.004 | 0.0001 | 0.0772 |
| Kurtosis | 15.97 | − 0.4646 | 0.0411 | 0.0830 | 0.0914 | 1.2538 |
| J–B statistic | 37551.96 | 422.85 | 0.5541 | 1.4145 | 1.6559 | 319.80 |
| Tsay test | 2.7321 | 0.0049 | 0.8214 | 2.1893 | 0.9870 | 3.6407 |
| $H_r$ | 0.3459 | 1.02 | 0.4391 | 0.4323 | 0.4368 | 0.4894 |
| $H_{\|r\|}$ | 0.8422 | 0.9961 | 0.6157 | 0.6402 | 0.6168 | 0.7680 |

The table reports the summary statistics including minimum ($r_{min}$) and maximum ($r_{max}$) returns, standard deviation (sd), skewness, kurtosis, Jarque–Bera statistic (J–B statistic), the Tsay nonlinearity test (Tsay test), the Hurst exponent for returns ($H_r$) and absolute returns ($H_{|r|}$), respectively

gests that an appropriate model should account for the effects of all market participants.

In this respect, Table 7 presents additional specifications of the SM model, allowing us to compare our results with the real data of QLD electricity market. The results show that Model 8 remains the best model among those considered in replicating the stylized facts of Queensland electricity market. When comparing these results to those of the PJM market, we find that our model aligns more closely with the stylized facts of the PJM market than with those of the Queensland market probably due to the different volatiilty of the two markets.

From our analysis, we can draw some important conclusions. First, the choice of the electricity market serves to test the ability of our model to replicate empirical facts. The rationale behind this choice is the structure of the electricity market, which resembles a fundamentalists vs. chartists configuration. In this context, regulated electricity companies behave similarly to fundamentalists, while private companies are more akin to chartists. Second, as shown in the results of Tables 5 and 7, the model successfully describes market dynamics characterized by a moderate level of volatility. This explains the superior fit of our model with the PJM market, as compared to the Queensland market.

## 5 Conclusions

In this paper, we investigated the price dynamics of a multi-asset model based on the literature of heterogeneous agents. A key feature of our heterogeneous agent model (HAM) is that it incorporates a general polynomial excess demand function for fundamentalists, specifically including polynomial terms up to the third order. This results in a high level of aggressiveness by fundamentalists in relation to the mispricing with respect to the fundamental value. Additionally, we account for the presence of two types of momentum traders: absolute momentum and cross-sectional momentum traders. There is a large consensus in the literature on the destabilizing role of cross-sectional momentum traders. This characteristic, combined with the nonlinear nature of the excess of demand function of fundamentalists, gives rise to interesting dynamics in our model. In particular, our stability analysis reveals that a Neimark–Sacker bifurcation can occur if there is coordination among cross-sectional momentum traders across both assets and if the overall activity of absolute momentum traders is lower than that of fundamentalists. Moreover, the market can be destabilized via period-doubling bifurcation if fundamentalists overreact to mispricing with respect to the fundamental value. Our findings align with the results of [13] on the impact of linear and nonlinear terms in the model. Finally, we focus specifically on the stylized facts of electricity markets, comparing them with the time series of returns generated by our model. We demonstrated that, among the two electricity markets considered, our model was able to replicate key empirical evidences of PJM market, including asymmetry, excess of kurtosis and volatility clustering. This is due to the model's moderate volatility and its structure, which closely resemble markets where traders act as both fundamentalists and speculators.

Given the interesting results of our model, we think that it can offer numerous opportunities for further development, for example the investigation of the local stability of the multiple equilibria. Since the analysis is mathematically rather complex, it may be carried out with the support of numerical methods in future research.

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**Author contributions** Formal analysis, SB; investigation, SB and GC; software, GC; supervision, GP, SB; conceptualization, GP, SB, GC; visualization, GP; writing—original draft, SB and GC; writing—review and editing, GP, SB, GC. All authors reviewed the manuscript.

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**Data Availability Statement** No datasets were generated or analysed during the current study.

### Declarations

**Conflict of interest** The authors have no relevant financial or non-financial interests to disclose.

**Code availability** Not applicable.



## References

1. Hommes, C.: Behavioral Rationality and Heterogeneous Expectations in Complex Economic Systems. Cambridge University Press, Cambridge (2013)
2. Dieci, R., He, X.-Z.: Heterogeneous agent models in finance. Handb. Comput. Econ. **4**, 257–328 (2018)
3. Lux, T., Marchesi, M.: Volatility clustering in financial markets: a microsimulation of interacting agents. Int. J. Theor. Appl. Finance **3**(04), 675–702 (2000)
4. Cont, R.: Empirical properties of asset returns: stylized facts and statistical issues. Quant. Finance **1**, 223–236 (2001)

5. Chiarella, C., He, X.-Z., Wang, D.: Statistical Properties of a Heterogeneous Asset Pricing Model with Time-varying Second Moment, vol. 567, pp. 109–123. Springer, Berlin (2006)
6. He, X.-Z., Li, Y.: Power-law behaviour, heterogeneity, and trend chasing. J. Econ. Dyn. Control **31**(10), 3396–3426 (2007)
7. Alfarano, S., Lux, T., Wagner, F.: Time variation of higher moments in a financial market with heterogeneous agents: an analytical approach. J. Econ. Dyn. Control **32**(1), 101–136 (2008)
8. Mari, C.: Random movements of power prices in competitive markets: a hybrid model approach. J. Energy Mark. **1**(2), 87–103 (2008)
9. Mari, C.: Short-term stochastic movements of electricity prices and long-term investments in power generating technologies. Energy Syst. **12**(3), 737–772 (2021)
10. He, X.-Z., Li, K., Wang, C.: Time-varying economic dominance in financial markets: a bistable dynamics approach. Chaos Interdiscip. J. Nonlinear Sci. **28**(5), 055903 (2018)
11. Dieci, R., He, X.-Z.: Cross-section instability in financial markets: impatience, extrapolation, and switching. Decis. Econ. Finance **44**(2), 727–754 (2021)
12. Brianzoni, S., Campisi, G., Pacelli, G.: Coexisting attractors in a heterogeneous agent model in discrete time. Mathematics **11**(10), 2348 (2023)
13. Lux, T.: Can heterogeneous agent models explain the alleged mispricing of the S&P 500? Quant. Finance **21**(9), 1413–1433 (2021)
14. Tramontana, F., Gardini, L., Dieci, R., Westerhoff, F.: The emergence of bull and bear dynamics in a nonlinear model of interacting markets. Discrete Dyn. Nat. Soc. **2009**(1), 310471 (2009)
15. Kaltwasser, P.R.: Uncertainty about fundamentals and herding behavior in the forex market. XXPhys. A **389**(6), 1215–1222 (2010)
16. He, X.-Z., Zheng, H.: Trading heterogeneity under information uncertainty. J. Econ. Behav. Organ. **130**, 64–80 (2016)
17. Day, R.H., Huang, W.: Bulls, bears and market sheep. J. Econ. Behav. Organ. **14**(3), 299–329 (1990)
18. Brock, W.A., Hommes, C.H.: Heterogeneous beliefs and routes to chaos in a simple asset pricing model. J. Econ. Dyn. Control **22**(8), 1235–1274 (1998)
19. Huang, W., Chen, Z.: Modeling regional linkage of financial markets. J. Econ. Behav. Organ. **99**, 18–31 (2014)
20. Westerhoff, F.H., Dieci, R.: The effectiveness of Keynes–Tobin transaction taxes when heterogeneous agents can trade in different markets: a behavioral finance approach. J. Econ. Dyn. Control **30**(2), 293–322 (2006)
21. Ullrich, C.J.: Realized volatility and price spikes in electricity markets: the importance of observation frequency. Energy Econ. **34**(6), 1809–1818 (2012)