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# Wall heating effects on aeroacoustic fields radiated by rigid bodies at different flow regimes

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## 1 Abstract

2 This study presents several direct numerical computations concerning wall heating  
3 effects on aeroacoustic fields derived from flow-rigid body interaction. Both laminar  
4 and turbulent flow configurations involving isolated and multibody arrangements  
5 have been addressed. Some insights into the physical mechanisms concerning these  
6 problems are addressed and discussed. In particular, we observed that the aeroacous-  
7 tic fields produced by laminar flows can be more easily controlled and practically  
8 suppressed in terms of acoustic emission by wall heating. By contrast, in turbu-  
9 lent flows, the effectiveness of the analysed technique is more limited. Indeed, wall  
10 heating produces a slight increase in overall emissions. However, at the tonal peak  
11 frequency derived from adiabatic configuration, the acoustic contribution is consid-  
12 erably reduced.

13 *Key words:*

14 Wall heating, OpenFOAM, Aeroacoustic fields, Direct Numerical Simulation

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## 15 **1 Introduction**

16 Self-noise produced by aerodynamic bodies is an important issue in many ap-  
17 plications ranging from wind turbines and helicopter rotors to fan blades and  
18 airframes, [1]. Similar problems can also be found in broad sense for bluff bod-  
19 ies with reference to built environment noise pollution, [2].

20 The flow fields derived from aforementioned application areas share unstead-  
21 iness in the boundary layer and/or in the wake region which are responsible  
22 for the creation of acoustic perturbations, [1, 3]. In this context, it is easy to  
23 understand that an aeroacoustic field can be controlled by altering the wake  
24 geometry as well as boundary layer (if present) detachment features.

25 Passive control techniques have been widely studied and reported. These ap-  
26 proaches use additional objects near the reference body to mitigate acoustic  
27 disturbance, [4, 5]. Active control techniques have also received significant at-  
28 tention by several authors, mainly in relation to flow-field modification. Specif-  
29 ically, we can find some studies treating flow-field control through the impo-  
30 sition of line, transverse, or rotary oscillations, [6, 7]. Other techniques have  
31 even considered synthetic jets, [8]. Nevertheless, the effect of forced transverse  
32 oscillations on the sound generation and propagation from a circular cylinder  
33 immersed in uniform flow was only addressed very recently, [9, 10].

34 It is worth emphasising that wake geometry can also be controlled by heating  
35 the body's surface. This phenomenon was experimentally observed in a circu-  
36 lar cylinder by Lecordier et al. [11, 12]. However, the acoustic effects related to  
37 wall heating were investigated preliminarily by D'Alessandro et al. [13], who  
38 extended their analyses to non-circular geometry sections as well. It was re-  
39 vealed that the wall temperature increment can reduce the aerodynamic force  
40 pulsations generated by the Karman vortex street, which is shed over bluff  
41 bodies in laminar flows. In this context, any reduction in lift pulsations is  
42 crucial because it leads to aeroacoustic perturbations decay. With the present  
43 study, we want to advance the state-of-art concerning the wall heating conse-

44 quences on the aeroacoustic sound generation and propagation. In particular,  
 45 the role of dynamic viscosity and flow/sound problems involving more than  
 46 one body are investigated. In addition, we investigated the thermal effect on  
 47 the aeroacoustic sound emitted using an NACA 0012 airfoil under turbulent  
 48 flow. This is a particular aspect of the novelty of this study. In fact, at the  
 49 time of writing, the impact of wall heating on aeroacoustic fields had not yet  
 50 been studied in a similar flow regime.

51 Direct computation of the flow and acoustic fields was adopted as an analysis  
 52 tool. In particular, an open-source solver called `caafoam`, developed by the  
 53 authors themselves, [13], was adopted.

54 The remainder of this paper is organised as follows, the governing equations  
 55 are presented in Section 2, while the adopted numerical discretisation tech-  
 56 niques are briefly discussed in Section 3. Section 4 is devoted to numerical  
 57 results. The conclusions are presented in Section 5 .

## 58 **2 Flow model**

59 The flow model equations, based on the conservative variables  $\mathbf{u} = (\rho, \rho u_i, \rho E)^T$ ,  
 60 are as follows:

$$61 \quad \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot \mathbf{F}_c(\mathbf{u}) - \nabla \cdot \mathbf{F}_v(\mathbf{u}, \nabla \mathbf{u}) = \sigma(\mathbf{u}_{ref} - \mathbf{u}), \quad (1)$$

62 where  $\mathbf{F}_c(\mathbf{u})$  and  $\mathbf{F}_v(\mathbf{u}, \nabla \mathbf{u})$  are convective and viscous fluxes, respectively,  
 63 given by:

$$64 \quad \mathbf{F}_{c,j} = \begin{pmatrix} \rho u_j \\ \rho u_1 u_j + p \delta_{1j} \\ \rho u_2 u_j + p \delta_{2j} \\ \rho u_3 u_j + p \delta_{3j} \\ \rho u_j H \end{pmatrix}, \quad \mathbf{F}_{v,j} = \begin{pmatrix} 0 \\ \tau_{1j} \\ \tau_{2j} \\ \tau_{3j} \\ \tau_{ji} u_i - q_j \end{pmatrix}. \quad (2)$$

65 The viscous stress tensor is computed using the constitutive relation for New-  
 66 tonian fluids, whereas for heat flux vector components, the Fourier postulate  
 67 is used

$$68 \quad \tau_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \right), \quad q_j = -\lambda \frac{\partial T}{\partial x_j}. \quad (3)$$

69 In this study, the dynamic viscosity  $\mu$  was handled in two different ways. In  
 70 the first case, a constant value was imposed, whereas the Sutherland law [14]  
 71 was adopted in the second approach. However, the thermal conductivity was  
 72 obtained using the Prandtl number.

73 The pressure was computed by assuming the ideal gas equation of state as  
 74 a thermodynamic model. By contrast, the fluid temperature was evaluated  
 75 employing the internal energy equation according to:

$$76 \quad p = \rho(\gamma - 1) \left( E - \frac{1}{2} u_k u_k \right). \quad c_v T = E - \frac{1}{2} u_k u_k. \quad (4)$$

77 The last term, in the right-hand side of eq. 1 is not physical and represents  
 78 the expression of the artificial sponge layer [15, 16]; this term avoids spurious  
 79 wave reflections at the external boundaries, producing damping of the flow  
 80 variables to a fixed reference solution,  $\mathbf{u}_{ref}$ .

81 In our approach the spatially varying coefficient  $\sigma$  was defined as follows:

$$82 \quad \sigma = \sigma_0 \left( \frac{L_{sp} - d}{L_{sp}} \right)^n \quad (5)$$

83 where  $L_{sp}$  denotes the layer width,  $d$  denotes the minimum distance from  
 84 the inflow/outflow boundaries,  $\sigma_0$  is a constant value, and  $n$  is an integer  
 85 parameter controlling the shape of the sponge profile.

86 The optimal sponge layer design is a complex problem, [17]. The two main  
 87 parameters to consider in order to avoid sponge failure are  $-\eta_{target}$ , which  
 88 defines the sponge strength, and the sponge width. The first one is defined as  
 89 follows:

$$90 \quad -\eta_{target} = 20 \frac{2 \log_{10} e}{1 - M_\infty^2} \int_{L_{sp}} \sigma d\mathbf{x}. \quad (6)$$

91 and it is expressed in dB. By contrast, the sponge width must be set with the  
 92 following constraint for its dimensionless expression:

$$93 \quad 0.5 \leq \frac{L_{sp} \cdot f}{a_\infty} \leq 2 \quad (7)$$

94 where  $f$  is the sound disturbance frequency and  $a_\infty$  is the speed of sound, [17].  
 95 For all the laminar flow computations presented in this study, we fixed  $n = 2$   
 96 in eq. 5, while the dimensionless parameter  $(L_{sp} \cdot f) / a_\infty$  was 0.5 in order to  
 97 limit the computational load. It is important to point out that the sponge layer  
 98 dimensionless width is based on the characteristic frequency derived from the  
 99 spectral analysis of the lift coefficient time-history obtained using the adiabatic  
 100 wall condition. The sponge strength is provided for every configuration in their  
 101 respective sections. Finally, the sponge layer size for the turbulent flow case is  
 102 described in detail in Sec. 4.4.

### 103 3 Numerical approximation

104 The flow governing equations were solved using a collocated cell-centred fi-  
105 nite volume method, available within the OpenFOAM v2.3.x library [18]. The  
106 code’s object-oriented structure enables users to implement their own mod-  
107 els and solvers in the baseline codes with relatively little effort (for exam-  
108 ple [19, 20, 21]), and that is why it has been receiving significant interest from  
109 the CFD community in recent years.

110 In particular, our solution strategy relies on a solver, called `caafoam`, devel-  
111 oped by our research group [13], and has already proven to be reliable for  
112 direct computations of the aeroacoustic sound.

113 The convective part of the Eulerian flux was computed by following Piroz-  
114 zoli’s energy-conserving scheme [22], whereas standard central schemes for  
115 diffusive contributions were adopted. For time integration, 2N storage explicit  
116 Runge–Kutta (ERK) was employed. In particular, we considered a five-stage,  
117 fourth-order accurate ERK scheme available in the open literature [23]. In our  
118 computational experience, this scheme was appealing showed very good par-  
119 allel performance and it allowed for the use of a maximum Courant number  
120 equal to 1.

### 121 4 Results

122 In this section we present several low Mach number cases involving uniform  
123 velocity inlet boundary condition. The cases of a single circular and two square  
124 cylinders placed side by side, as well as in a tandem configuration, were anal-  
125 ysed. In all the aforementioned cases, the Mach number of the undisturbed  
126 flow was  $M_\infty = 0.2$ ,  $\gamma = 1.4$ , and the Prandtl number  $Pr$  was 0.75. The  
127 Reynolds number, based on a reference length  $L_{ref}$ , is defined in its standard  
128 form  $Re = \rho_\infty u_\infty L_{ref} / \mu_\infty$ .

129 In addition, the flow/noise problem correlated with the NACA 0012 airfoil  
 130 at a chord-based Reynolds number equal to  $5 \cdot 10^4$ ,  $M_\infty = 0.4$  and  $\alpha = 5^\circ$   
 131 was investigated to assess the impact of wall heating in turbulent flow regime.  
 132 Undisturbed flow parameters for NACA problem are discussed in Sec. 4.4.  
 133 The acoustic field was analysed in terms of the dimensionless fluctuating pres-  
 134 sure, defined as

$$135 \quad p' = \frac{p - \bar{p}}{\rho_\infty u_{ref}^2} \quad (8)$$

136 where  $\bar{p}$  denotes average pressure field. For comparison with literature data,  
 137  $u_{ref} = a_\infty$ , for laminar flow cases. By contrast, for NACA airfoil we set  $u_{ref} =$   
 138  $u_\infty$ . Polar plots containing the root mean square of  $p'$  are shown to provide  
 139 evidence of sound features in the far field. The dilatation rate field  $\partial u_j / \partial x_j$   
 140 was also used to visualise the acoustic waves.

141 Aerodynamic performance were evaluated using dimensionless drag and lift  
 142 coefficients given by eq. 9:

$$143 \quad C_D = \frac{2D'}{\rho u_\infty^2 A_{ref}}, \quad C_L = \frac{2L'}{\rho u_\infty^2 A_{ref}}. \quad (9)$$

144 Standard statistics were used to analyse  $C_D$  and  $C_L$  behaviours, such as the  
 145 mean drag coefficient  $\overline{C_D}$ , mean lift coefficient  $\overline{C_L}$ , root mean square of the  
 146 lift coefficient  $C_{L,rms}$ , root mean square of the drag coefficient  $C_{D,rms}$ , and the  
 147 oscillation amplitudes of the force coefficients ( $\Delta C_D = (C_{D,max} - C_{D,min})/2$   
 148 and  $\Delta C_L = (C_{L,max} - C_{L,min})/2$ ). The Strouhal number is defined as:

$$149 \quad St = \frac{f L_{ref}}{u_\infty}. \quad (10)$$

150 All the solutions were obtained on distributed-memory parallel machines and  
 151 the computations requiring a lower load were run on a Linux cluster with  
 152 16 Intel Xeon E5-2603v3-based nodes for 192 CPU cores operating at 1.6  
 153 GHz. The code was built using Intel compilers and an MPI library version  
 154 developed by Intel. Larger cases were run on the MARCONI-100 system hosted



155 by CINECA. In the PowerPC-based system, OpenFOAM was built using GNU  
156 compilers and IBM Spectrum MPI.

#### 157 4.1 Circular cylinder

158 The first test case considered in this study was the sound generation by the  
159 flow past a circular cylinder at  $Re = 150$  where the cylinder diameter  $D$  is the  
160 reference length. We used a fully-structured O-type grid with far-field bound-  
161 aries placed at 150 times  $D$ , and the height of the first cell next to the wall  
162  $y_c$  was set to  $y_c/D = 5 \cdot 10^{-3}$ . To discretise the radial direction, we used 750  
163 cells, whereas 700 were used in the azimuthal direction. Thus, the total num-  
164 ber of cells  $N_c$  was  $5.25 \cdot 10^5$ . It is also important to note that this grid was  
165 already benchmarked by the authors for this kind of flow/sound field, [13].  
166 The sponge's strength was set at 40 dB.

167 Two different settings were considered to investigate the effects of the thermal  
168 boundary conditions at the wall on the acoustic field, fixed wall tempera-  
169 tures ( $T_w = 2T_\infty$  and  $T_w = 3T_\infty$ ) and a fixed wall temperature gradient  
170 ( $\partial T/\partial n|_w = 0.5$ ,  $\partial T/\partial n|_w = 1$  and  $\partial T/\partial n|_w = 2$ ). A baseline configuration  
171 involving an adiabatic wall was also computed.

172 Fig. 1 shows the directivity plot based on  $p'_{rms}$  evaluated on a circle having  
173 a radius  $r/D = 75$ . The effectiveness of the proposed grid and solver was  
174 demonstrated by comparing our predictions with the DNS data published by  
175 Inoue and Hakateyama, [3]. A key element is the clear dominance of lift fluc-  
176 tuations which yield a typical dipolar acoustic field. Fig. 2 shows  $p'_{rms}$  polar  
177 plot on a circle having a radius  $r = 40 D$  for different fixed wall temperatures.  
178 Note that we selected  $r = 40 D$  because we wanted to prevent excessive sound  
179 wave decay in the far field in order to estimate wall thermal effects on the  
180 acoustic field. The impact of the thermophysical model on dynamic viscosity  
181 was also assessed; the data reported in Fig. 2, labelled with  $\mu(T)$  are related

182 to the Sutherland viscosity model results, whereas the other data refer to the  
 183 temperature-independent viscosity. Note that the dimensionless Sutherland  
 184 constant is  $S/T_\infty = 0.3855$ . **It is worth noting that the considered flow/sound**  
 185 **problems, which relied on adiabatic wall conditions, experienced very limited**  
 186 **magnitude of the temperature field gradient. This effect was the result of the**  
 187 **wall pressure gradient distribution. Therefore,** it is very clear that thermal  
 188 dependent viscosity produces negligible effects when adiabatic wall boundary  
 189 conditions are adopted. For non-adiabatic cases, the effect is the opposite.  
 190 Tab. 1 and Tab. 2 show that non-adiabatic wall conditions produce contrast-  
 191 ing effects on the  $\overline{C_D}$ . Conversely, the force coefficient pulsations and St are  
 192 reduced as a result of the wall heating. It should be noted that similar results  
 193 agree with those of Lecordier et al. [11, 12], who experimentally observed  
 194 vortex shedding damping behind a heated circular cylinder. Moreover, vortex  
 195 shedding damping, related to wall heating was confirmed for both thermo-  
 196 physical models used in this study. This is extremely important to explain  
 197 why far-field sound abatement occurs at higher wall temperatures. Indeed,  
 198 the acoustic field derived from the fluid-body interaction is related to the  
 199 sound sources damping, *i.e.*  $\Delta C_D$  and  $\Delta C_L$ . Furthermore, it is evident that  
 200 the adoption of the Sutherland model amplified sound abatement. In fact,  
 201 with the temperature-independent viscosity model, we can only account for  
 202 the modification of the pressure owing to the thermal effect, that is,  $p \propto T$ .  
 203 By contrast, when the Sutherland model was enabled, the viscosity increased  
 204 with temperature; thus, we obtained  $\tau_w \propto T$  (where  $\tau_w$  denotes wall shear  
 205 stress). Consequently, in the latter case, we increased both the wall shear  
 206 stresses and the pressure gradient magnitude acting on the wall temperature.  
 207 When a fixed viscosity was used, only the pressure gradient magnitude was af-  
 208 fected by the temperature field. Owing to the previous reasons we believe that  
 209 the Sutherland model-based predictions produced more significant dissipation  
 210 on the flow properties in the wall region, as well as in the near-wall region.  
 211 Hence, the final effect was the diminishing force oscillations with a consequent

212 stronger  $p'_{rms}$  average value abatement.

213 Fig. 3 shows acoustic pressure polar plots obtained using fixed wall temper-  
214 ature gradient on the cylinder surface. Clearly, it can be observed that  $p'_{rms}$   
215 exhibits a behaviour similar to that of the fixed temperature case. Neverthe-  
216 less, the impact of the thermophysical model assumes even greater importance  
217 given by the magnitude of the wall temperature gradient.

218 Therefore, in the following, because of the augmentations reported above, we  
219 used the Sutherland model which is more physical than other models. Simul-  
220 taneously, fixed-gradient boundary conditions for the temperature field were  
221 employed because they were simpler to reproduce experimentally (through  
222 Joule heating) for the sake of benchmarking.

#### 223 4.1.1 *Effect of local wall heating*

224 In order to improve the understanding of the aerodynamic sound active ther-  
225 mal control we analysed the flow/sound fields performing localised wall heat-  
226 ing. In other words, we generated three different configurations, namely C1,  
227 C2, and C3, and thereafter applied the wall temperature gradient to the up-  
228 per and lower sides of the cylinder. The aforementioned cases differed from  
229 the angular sector extension where the conditions were imposed. In the C1  
230 configuration, we assessed two different cases. First, we applied  $\partial T/\partial n|_w > 0$   
231 for  $0^\circ \leq \theta \leq 90^\circ$ , whereas  $\partial T/\partial n|_w = 0$  for  $90^\circ < \theta \leq 180^\circ$ . In the latter, we  
232 used  $\partial T/\partial n|_w = 0$  for  $0^\circ \leq \theta \leq 90^\circ$ , whereas  $\partial T/\partial n|_w > 0$  for  $90^\circ < \theta \leq 180^\circ$ .  
233 Note that  $\theta$  is clockwise positive and equal to  $0^\circ$  at the cylinder's leading  
234 edge (LE), whereas is  $180^\circ$  at the trailing edge (TE). On the bottom side,  
235 the boundary conditions for the temperature were symmetric with respect to  
236 the upper one. For case C2, we divided both cylinder sides into three equal  
237 parts with an angular extension  $\Delta\theta = 60^\circ$ , to obtain three angular sector  
238 couples. Successively, we did not apply a null temperature gradient to each of  
239 the three couples, and the remaining part of the surface of the cylinder was

240 adiabatic. Finally, for C3, we defined the wall boundary conditions similar to  
 241 C2 with  $\Delta\theta = 30^\circ$ ; hence, we obtained six different cases to study. The results  
 242 related to the aforementioned configurations, are showed in Fig. 4 – 6. To  
 243 better estimate the acoustic damping provided by local wall heating, all polar  
 244 plots also contained  $p'_{rms}$  for the fully adiabatic and heated cases, respectively,  
 245  $\partial T/\partial n|_w = 0$  and  $\partial T/\partial n|_w = 1$ .

246 Looking at Fig. 4, it is interesting to note that the application of the heat flux  
 247 corresponding to  $\partial T/\partial n|_w = 1$  only for the cylinder forepart,  $0^\circ \leq \theta \leq 90^\circ$ ,  
 248 had negligible effects on the sound field. In fact, this configuration produced  
 249 very similar results to a fully adiabatic condition. By contrast,  $\partial T/\partial n|_w = 1$   
 250 for  $90^\circ < \theta \leq 180^\circ$  was equivalent to the temperature gradient application on  
 251 the full surface. Further shrinkage in the thermal flux application produced  
 252 in the C2 case, provided evidence that almost the entire sound reduction ef-  
 253 fect can be assigned to the  $120^\circ < \theta \leq 180^\circ$  zone, as depicted in Fig. 5.  
 254 The adoption of an angular step equal to  $30^\circ$  improved our understanding of  
 255 the discussed phenomenon. Indeed, from Fig. 6, it is clear in the two zones:  
 256  $120^\circ < \theta \leq 150^\circ$  and  $150^\circ < \theta \leq 180^\circ$  both contribute to sound emission  
 257 reduction. However, the latter one has grater weight than the former in the  
 258 overall context.

259 Therefore, it can be stated that wall heating focused on separated flow region  
 260 is the key aspect for adequately controlling force oscillations and related sound  
 261 emission.

## 262 4.2 *Square cylinders arranged side by side*

263 The subject of this subsection is sound generation around two square cylinders  
 264 placed side by side, as shown in Fig. 7.  $L$  is the spacing between the centres  
 265 of the two cylinders and  $D$  is their diameter. In this study, the ratio  $L/D$  was  
 266 set to 3, while the Reynolds number, based on the cylinders' diameter, is 150.

267 This flow configuration generates a bifurcation of the wake patterns with dif-  
268 ferent acoustic responses, [24]. In the following, we refer to a flow field called  
269 in literature as in-phase because it exhibits synchronised lift coefficients. It is  
270 obtained using a initial field consisting of a clockwise vortex placed behind  
271 both the upper and the lower cylinder, [24]. A fully-structured orthogonal  
272 computational grid was used, which adopted a sponge layer with a strength  
273 of 45 dB. This grid was already successfully tested for similar problems, [13].  
274 The far field was placed 200  $D$  from the midpoint of the two cylinders (see  
275 Fig. 7), grid cells were clustered near the cylinder wall with a dimensionless  
276 first-cell height of  $10^{-2}$ . The number of cells had a total number of  $4.44 \cdot 10^6$ .  
277 The typical run was performed on MARCONI-100 HPC system using 192  
278 CPU cores.

279 Fig. 8(a) shows a directivity plot evaluated on a circle having dimensionless  
280 radius  $r/D = 75$ . Overall, the results showed good agreement with the find-  
281 ings of Inoue et al. [24]. Thus, the usefulness of both the grid and solver was  
282 proven.

283 Fig. 8(b) shows acoustic pressure polar plots obtained with varying wall tem-  
284 perature gradient on the cylinders' surface. It can be clearly observed that in  
285 this case, the sound emission was significantly reduced owing to thermal wall  
286 flux application. Furthermore, this evidence was in good agreement with force  
287 coefficient modifications which are highlighted in Tab. 3.

### 288 4.3 *Square cylinders in tandem*

289 With the same aim as for the side-by-side arrangement, we also considered the  
290 flow and sound generation around two square cylinders in a tandem configura-  
291 tion at  $Re = 150$  with  $L/D = 2$ , where  $L$  and  $D$  have the meaning expressed  
292 in Fig. 9.

293 Also in this case we have used a square computational domain with a far field

294 placed at  $200 D$  from the origin. Quadrilateral orthogonal cells were used to  
295 discretize the flow domain. The total number of cells,  $N_c$ , was about  $4.2 \cdot 10^6$ ,  
296 and a grid refinement was performed near the walls of the cylinders adopting  
297  $y_c/D = 10^{-2}$  as in our previous work [13]. Acoustic wave reflections at the far  
298 boundaries were removed using a configuration derived from the previous test  
299 cases; the sponge layer's strength was 45 dB. The typical run was performed  
300 on MARCONI-100 HPC system using 192 CPU-core.

301 Once again, `caafoam` solver, as well as the selected grid, are in very good  
302 agreement with DNS data of Inoue [25], as showed in Fig. 10(a). In Fig. 10(b)  
303 can be noted that acoustic pressure is significantly reduced by wall heating  
304 also in this configuration. This element is corroborated by force coefficients  
305 modifications reported in Tab. 4.

306 It is very clear that this specific configuration put in evidence a noticeable  
307 abatement of the emitted sound if compared with the other ones previously  
308 described. In the authors' opinion this result is connected to the aerodynamic  
309 coefficients fluctuations magnitude which is the lowest for the present arrange-  
310 ment. Thus, the effect of the wall thermal flux is more marked than in the  
311 other flow/sound problems addressed in this work.

#### 312 4.4 Turbulent flow past NACA 0012 at $Re = 5 \cdot 10^4$ , $M_\infty = 0.4$ and $\alpha = 5^\circ$

313 In this subsection, the numerical results of turbulent flow past NACA 0012  
314 are presented and discussed. Owing to the strong spanwise coherence of the  
315 pressure fluctuation for turbulent flows over airfoils operating at moderate  
316 Reynolds numbers [26], the aeroacoustic field was examined through 2-D  
317 DNS, [26, 27]. In the following, the undisturbed flow parameters were set for  
318 validation, with reference to Sandberg et al. [26] and Jones et al. [28]. In par-  
319 ticular, we adopted the specific heat ratio, Prandtl number, and dimensionless  
320 Sutherland's constant as  $\gamma = 1.4$ ,  $Pr = 0.72$ , and  $S/T_\infty = 0.3686$ , respectively.

321 The adiabatic wall condition was fixed on the airfoil surface.  
 322 The computational domain is a C-type region that extended 7.3 times the  
 323 airfoil chord length  $c$  in the radial direction and  $5 c$  in the wake region. Four  
 324 fully-structured computational grids were used to achieve grid independence  
 325 in terms of both aerodynamic coefficients and acoustic pressure. All the afore-  
 326 mentioned grids had a dimensionless height for the first cell next to the wall,  
 327  $y_p/c$ , equal to  $10^{-4}$ . Considering the complexity of the involved phenomena, it  
 328 was evident that the correct definition of the grid resolution requirements was  
 329 mandatory. In this study, an iterative grid generation method was adopted  
 330 to achieve a suitable mesh configuration. Cell distribution details are listed  
 331 in Tab. 5. Moreover, the following numerical computations were run on the  
 332 MARCONI-100 system using 192 CPU cores for the coarsest grid, whereas  
 333 the finest grid typical run employed 320 CPU cores.  
 334 As briefly introduced above, the configuration considered in this study pro-  
 335 duced a complex variety of fluid phenomena. In particular, a long laminar  
 336 separation bubble (LSB) with an approximate extension of  $0.5 c$  was present  
 337 on the suction side. Moreover, LSB reattachment led to vortex shedding that  
 338 evolved until the TE, where the airfoil-vortices interaction generated a dipole-  
 339 lar tonal noise. The vortical structures that convected on the suction side are  
 340 shown in Fig. 11(a), whereas Fig. 11(b) provides a view of the acoustic waves  
 341 through the dilatation rate field  $\partial u_j / \partial x_j$ . In addition, it can be clearly ob-  
 342 served that the main acoustic source was located immediately after the airfoil  
 343 TE. Table 6 provides evidence that for the coarsest grid, G1, the time-averaged  
 344 lift coefficient,  $\overline{C_L}$ , is underestimated; however, the time-averaged drag coef-  
 345 ficient,  $\overline{C_D}$ , is slightly overestimated compared with the literature data. The  
 346 aerodynamic coefficients converged to those in the literature when finer grids  
 347 were adopted, particularly in terms of  $\overline{C_L}$ . Globally, a good agreement with  
 348 Jones [28] was reached for G3 and G4.  
 349 The local aerodynamic performance was estimated using the pressure coeffi-  
 350 cient  $c_p = 2(p - p_\infty) / \rho_\infty u_\infty^2$ , and skin friction coefficient,  $c_f = 2\tau_w / \rho_\infty u_\infty^2$ . In

351 Fig. 12(a) the mean pressure coefficient,  $\overline{c_p}$ , distribution is represented. It is  
 352 confirmed that the G3 grid results provide an appropriate description of the  
 353 flow around the NACA 0012 airfoil. In particular, the wide extension of the  
 354 LSB was noticeable when looking at the plateau on the suction side, whereas  
 355  $\overline{c_p}$  on the pressure side suggested that for the considered angle of attack, the  
 356 airfoil lower surface had a flat plate-like behaviour.

357 On the contrary, Fig. 12(b) shows the mean skin friction coefficient,  $\overline{c_f}$  distri-  
 358 bution, along the airfoil chord. In this framework, it is interesting to note that  
 359 as the grid resolution increased, the reattachment point moved toward the LE  
 360 and the secondary separation region was better described.

361 In order to isolate the vortex shedding characteristic frequency and its related  
 362 sound emission, a sampling probe was used to collect the pressure data. It was  
 363 placed at  $x/c = 0.5$  and  $y/c = 0.6$ . Note that the origin of the reference frame  
 364 was placed on the airfoil LE, and the  $x$ -axis was aligned with the airfoil chord,  
 365 as in Sandberg et al. [26]. Pressure data were sampled over a dimensionless  
 366 time fixed at 120 and Fourier transformed. As shown in Fig. 13(a), a tonal peak  
 367 was observed corresponding to a reduced frequency  $\mu_0 = \pi f c / a_\infty (1 - M_\infty^2)$ ,  
 368 equal to 5.036. This holds true for all the considered grids.

369 It is worth emphasising that a critical point of this numerical computation is  
 370 correlated with proper sponge layer calibration. This flow problem was per-  
 371 formed based on the lowest significant tonal peak exhibited by the spectrum  
 372 reported in Fig. 13(a). This frequency was selected to obtain  $L_{sp} \cdot f / a_\infty = 1$ ,  
 373 which implies a thicker sponge layer than in laminar configuration. A similar  
 374 condition is strictly required to dissipate the vortical flow structures shed by  
 375 airfoil [17]. As  $L_{sp} \cdot f / a_\infty \geq 2$  can produce sponge-layer auto-reflection issues,  
 376 we decided to use a compromise value to avoid an excessive overcoming of  
 377 the sponge thickness upper bound. The efficacy of the previous choice coupled  
 378 with  $-\eta_{target} = 40$  dB is evident from Fig. 11(b), where unphysical pressure  
 379 waves are not present.

380 The aeroacoustic noise produced in this configuration was also investigated



381 using the polar distribution of acoustic pressure in the mean field. In this spe-  
 382 cific case, the pressure-time history was sampled over 360 equispaced probes  
 383 placed along a circumference that originated from the TE and had a radius of  
 384  $2c$ . Fig. 13(b) highlights the dipolar nature of the acoustic field, related to the  
 385 lift pulsation dominance. Moreover, it is evident that G1 did not adequately  
 386 predict the airfoil sound emission, whereas G3 and G4 produced almost indis-  
 387 tinguishable results. Furthermore, the comparison of our finest grid data with  
 388 the reference ones showed slight differences; in our opinion, this was because  
 389 of the peculiar LE treatment adopted by Sandberg et al. [26].

390 In this subsection, we provide an additional insight into the alteration of  
 391 the aeroacoustic field owing to airfoil suction-side heating. Specifically, we in-  
 392 tended to investigate whether the wall-heating effect, noted in laminar flows,  
 393 was still effective in the turbulent regime. Two different configurations of  
 394 wall thermal flux were analysed. In the first configuration, the dimensionless  
 395 wall temperature gradient was fixed at 1, whereas in the second, we adopted  
 396  $\partial T/\partial n|_w = 5$ . In Fig. 14(a), we show the acoustic pressure polar plot for  
 397  $\mu_0 = 5.036$ , derived from adiabatic computations. It can be clearly observed  
 398 that the sound emission was effectively reduced for both the wall thermal  
 399 fluxes. Moreover, the tonal peak frequency moved forward to higher values, as  
 400 listed in Tab. 7. Looking at Fig. 14(b) and Fig. 14(c) it is possible to observe  
 401  $p'_{rms}$  polar plots connected to the  $\mu_0$  tonal peak derived from the two thermal  
 402 fluxes considered herein. It can be clearly observed that in these cases, only  
 403 the configuration producing the  $\mu_0$  tonal peak had a significant impact on the  
 404 sound emission. Nevertheless, the effect on the overall frequencies produced  
 405 a slight  $p'_{rms}$  increase, as shown in Fig. 14(d). This is because of an oppos-  
 406 ing evidence with respect to what was observed for laminar flows. Indeed,  
 407 for the turbulent configuration studied herein, the wall heating produced lit-  
 408 tle amplification in the aerodynamic force pulsation (approximately 2% for  
 409  $\partial T/\partial n|_w = 5$ ), as shown in Tab. 7. Therefore, we can conclude that the active  
 410 thermal control of the aeroacoustic field derived from turbulent flows is not

411 as effective as that in the laminar regime, in terms of acoustic power level  
412 reduction. In particular, the wall thermal flux acts as a tonal peak frequency  
413 shifter.

## 414 5 Conclusions

415 In this study, we investigated the effectiveness of wall heating as an active  
416 control technique for the aerodynamically generated sound and/or noise for  
417 laminar and turbulent cases. In laminar flows, we investigated both single and  
418 multiple body arrangements with reference to well established literature data.  
419 By contrast, turbulent flow analyses are limited to NACA 0012 airfoil owing  
420 to their high computational resource requirements.

421 Specifically, we observed that far-field sound abatement at higher wall temper-  
422 atures was produced for laminar flows over bluff bodies. This is related to the  
423 reduction in aerodynamic force pulsations generated by the Karman vortex  
424 street, which is shed over bluff bodies in laminar flows. It is worth noting that  
425 a similar effect was obtained in several configurations involving more than one  
426 single body. For an isolated circular cylinder, we observed that localised wall  
427 heating produced the same effect for the overall wall heating. Moreover, ther-  
428 mal fluxes focused on separated flow regions is the key aspect for adequately  
429 controlling force oscillations and related sound emission.

430 In this study, we also addressed for the first time, the impact of wall heating on  
431 aeroacoustic field derived from a turbulent flow regime. Thus, we computed  
432 the flow and acoustic field developed around the NACA 0012 airfoil. First,  
433 we further validated our solver for turbulent flows against well-established  
434 literature benchmarks. Therefore, we observed that the sound emission was  
435 effectively lower for a reduced frequency with the same maximum tonal contri-  
436 bution obtained from adiabatic wall condition. However, the overall frequency  
437 contribution produced a slight  $p'_{rms}$  increase. Based on these results, we con-

438 clude that the thermal control of the aeroacoustic field derived from turbulent  
439 flows is not as effective as that in the laminar regime in terms of the acoustic  
440 power level. By contrast, the wall thermal fluxes can be considered effective  
441 as tonal peak frequency shifters. Therefore, the above discussed technique is  
442 promising for specific applications.

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## 448 **References**

- 449 [1] Sandberg, R.D. and Jones, L.E. Direct numerical simulations of airfoil  
450 self-noise. *Procedia Engineering*, 6:274–282, 2010. IUTAM Symposium  
451 on Computational Aero-Acoustics for Aircraft Noise Prediction.
- 452 [2] Lam, B. and Gan, W.S. and Shi, D. and Nishimura, M. and Elliott, S.  
453 Ten questions concerning active noise control in the built environment.  
454 *Building and Environment*, 200:107928, 2021.
- 455 [3] Inoue, O. and Hatakeyama, N. Sound generation by a two-dimensional  
456 circular cylinder in a uniform flow. *Journal of Fluid Mechanics*, 471:285–  
457 314, 2002.
- 458 [4] Mahato, B. and Ganta, N. and Bhumkar, Y.G. Control of aeroacoustic  
459 noise generation during flow past a circular cylinder using splitter plate.  
460 In *INTER-NOISE 2019 MADRID - 48th International Congress and Ex-*  
461 *hibition on Noise Control Engineering*, 2019.

- 462 [5] Chauhan, M.K. and Dutta, S. and Gandhi, B.K. Wake flow modification  
463 behind a square cylinder using control rods. *Journal of Wind Engineering*  
464 *and Industrial Aerodynamics*, 184:342–361, 2019.
- 465 [6] Bearman, P W. Vortex Shedding from Oscillating Bluff Bodies. *Ann.*  
466 *Rev. Fluid Mech.*, 16(1):195–222, 1984.
- 467 [7] Williamson, C.H.K. and Govardhan, R. Vortex-induced vibration. *Ann.*  
468 *Rev. Fluid Mech.*, 36(1):413–455, 2004.
- 469 [8] Feng, L.H. and Wang, J.J. Modification of a circular cylinder wake with  
470 synthetic jet: Vortex shedding modes and mechanism. *European Journal*  
471 *of Mechanics - B/Fluids*, 43:14–32, 2014.
- 472 [9] Ma, R. and Liu, Z. and Zhang, G. and Doolan, C.J. and Moreau, D.J.  
473 Control of Aeolian tones from a circular cylinder using forced oscillation.  
474 *Aerospace Science and Technology*, 94:105370, 2019.
- 475 [10] Ganta, N. and Mahato, B. and Bhumkar, Y.G. Analysis of sound gener-  
476 ation by flow past a circular cylinder performing rotary oscillations using  
477 direct simulation approach. *Physics of Fluids*, 31(2), 2019.
- 478 [11] Lecordier, J.C. and Hamma, L. and Paranthoen, P. The control of vor-  
479 tex shedding behind heated circular cylinders at low Reynolds numbers.  
480 *Experiments in Fluids*, 10(4):224–229, 1991.
- 481 [12] Lecordier, J.-C. and Browne, L.W.B. and Le Masson, S. and Dumouchel,  
482 F. and Paranthoen, P. Control of vortex shedding by thermal effect at low  
483 Reynolds numbers. *Experimental Thermal and Fluid Science*, 21(4):227–  
484 237, 2000.
- 485 [13] D’Alessandro, V. and Falone, M. and Ricci, R. Direct computation of  
486 aeroacoustic fields in laminar flows: Solver development and assessment  
487 of wall temperature effects on radiated sound around bluff bodies. *Com-*  
488 *puters & Fluids*, 203:104517, 2020.
- 489 [14] Sutherland, W. . The viscosity of gases and molecular force. *The London,*  
490 *Edinburgh, and Dublin Philosophical Magazine and Journal of Science*,  
491 36(223):507–531, 1893.

- 492 [15] Moshe Israeli and Steven A. Orszag. Approximation of radiation bound-  
493 ary conditions. *Journal of Computational Physics*, 41(1):115–135, 1981.
- 494 [16] Jesper Larsen and Henry Dancy. Open boundaries in short wave simula-  
495 tions – A new approach. *Coastal Engineering*, 7(3):285–297, 1983.
- 496 [17] Mani, A. Analysis and optimization of numerical sponge layers as a  
497 nonreflective boundary treatment. *Journal of Computational Physics*,  
498 231(2):704–716, 2012.
- 499 [18] Weller, H.G. and Tabor, G. and Jasak, H. and Fureby, C. A tensorial  
500 approach to computational continuum mechanics using object-oriented  
501 techniques. *Computational Physics*, 12(6):620–631, 1998.
- 502 [19] Pham, T.Q.D. and Choi, S. Numerical analysis of direct contact  
503 condensation-induced water hammering effect using OpenFOAM in re-  
504 alistic steam pipes. *International Journal of Heat and Mass Transfer*,  
505 171:121099, 2021.
- 506 [20] Barbosa, D.V.E and Santos, A.L.G. and dos Santos, E.D. and Souza, J.A.  
507 Overtopping device numerical study: Openfoam solution verification and  
508 evaluation of curved ramps performances. *International Journal of Heat  
509 and Mass Transfer*, 131:411–423, 2019.
- 510 [21] Lee, G.L. and Law, M.C. and Lee, V.C.C. Numerical modelling of liquid  
511 heating and boiling phenomena under microwave irradiation using Open-  
512 FOAM. *International Journal of Heat and Mass Transfer*, 148:119096,  
513 2020.
- 514 [22] Pirozzoli, S. Numerical methods for high-speed flows. *Annual Review of  
515 Fluid Mechanics*, 43:163–194, 2011.
- 516 [23] Kennedy, C.A. and Carpenter, M.H. and Lewis, R.M. Low-storage, ex-  
517 plicit Runge–Kutta schemes for the compressible Navier–Stokes equa-  
518 tions. *Applied Numerical Mathematics*, 35(3):177 – 219, 2000.
- 519 [24] Inoue, O. and Iwakami, W. and Hatakeyama, N. . Aeolian tones radiated  
520 from flow past two square cylinders in a side-by-side arrangement. *Physics  
521 of Fluids*, 18(4):046104, 2006.

- 522 [25] Inoue, O. and Mori, M. and Hatakeyama, N. . Aeolian tones radiated from  
523 flow past two square cylinders in tandem. *Physics of Fluids*, 18(4):046101,  
524 2006.
- 525 [26] Sandberg, R.D. and Jones, L.E. and Sandham, N.D. and Joseph, P.F.  
526 Direct numerical simulations of tonal noise generated by laminar flow  
527 past airfoils. *J. of Sound Vib.*, 320(4):838–58, 2009.
- 528 [27] Ricciardi, T.R. and Arias-Ramirez, W. and Wolf, W.R. On secondary  
529 tones arising in trailing-edge noise at moderate Reynolds numbers. *Eu-*  
530 *ropean Journal of Mechanics - B/Fluids*, 79:54–66, 2020.
- 531 [28] Jones, L. E. and Sandberg, R. D. and Sandham, N. D. Direct numerical  
532 simulations of forced and unforced separation bubbles on an airfoil at  
533 incidence. *J. Fluid Mech.*, 602:175–207, 2008.

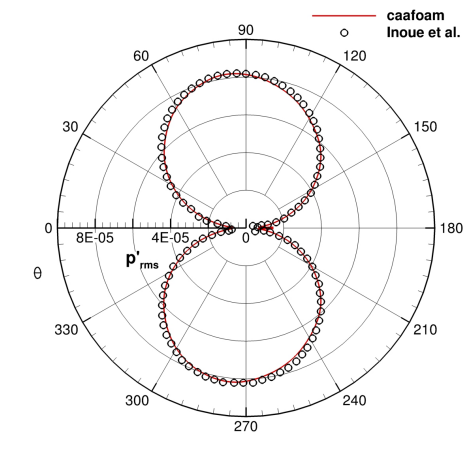


Figure 1. Flow past a circular cylinder. Solver validation.

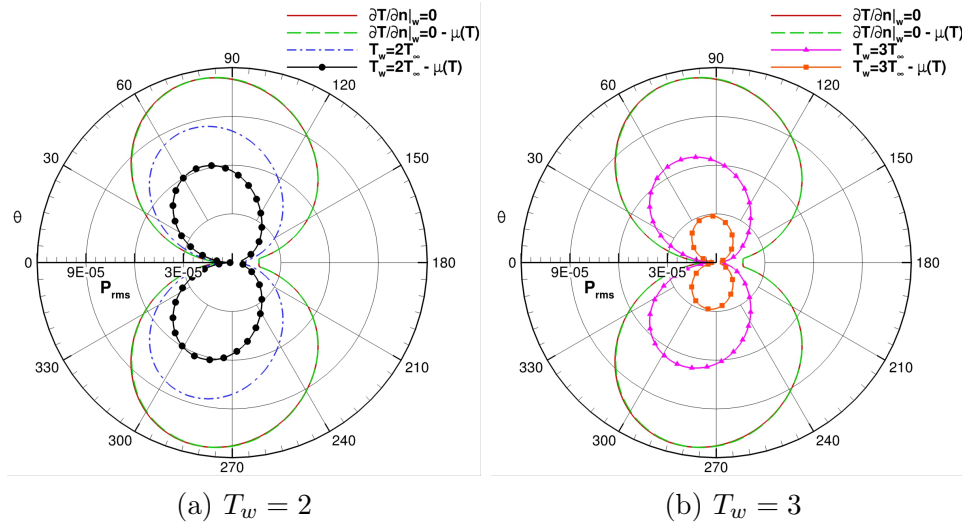


Figure 2. Flow past a circular cylinder. Fixed wall temperature effect,  $r/D = 40$ .



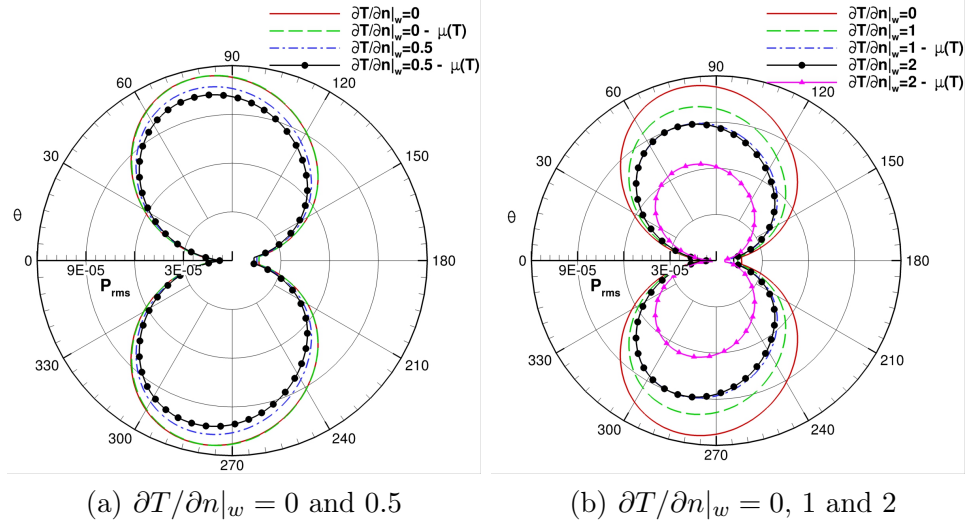


Figure 3. Flow past a circular cylinder. Fixed wall temperature gradient effect,  $r/D = 40$ .

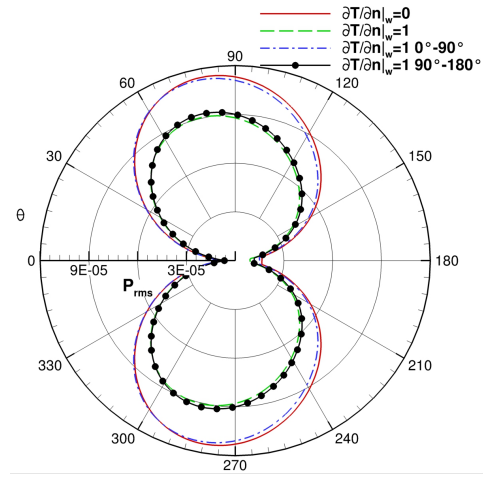


Figure 4. Flow past a circular cylinder. C1 case,  $r/D = 40$ .

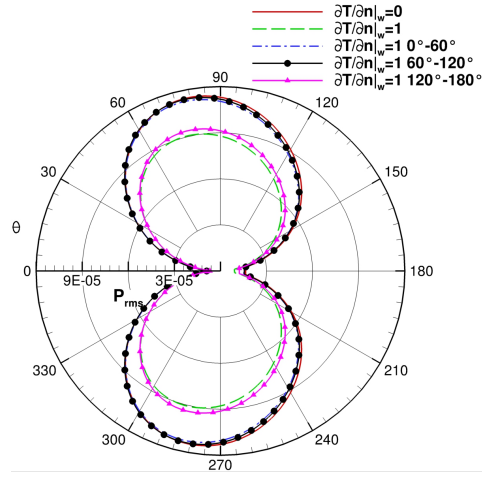


Figure 5. Flow past a circular cylinder. C2 case,  $r/D = 40$ .

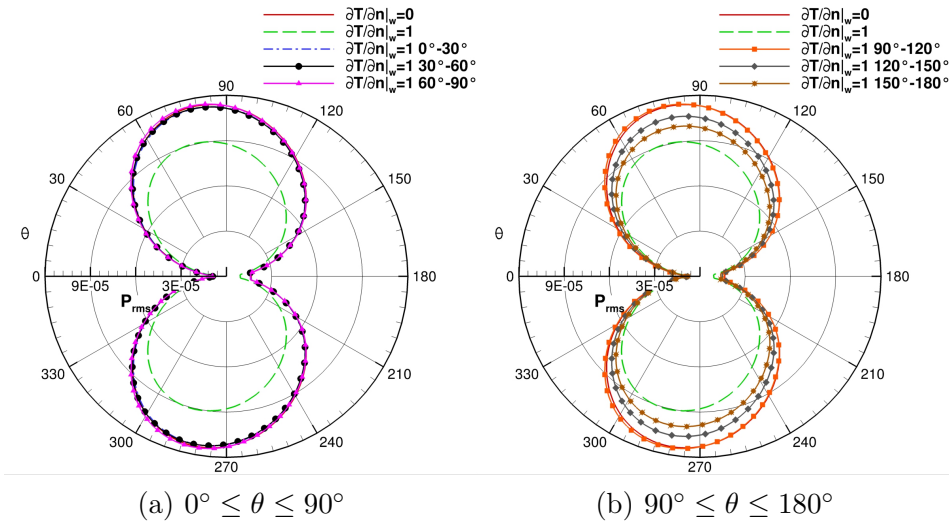


Figure 6. Flow past a circular cylinder. C3 case,  $r/D = 40$ .

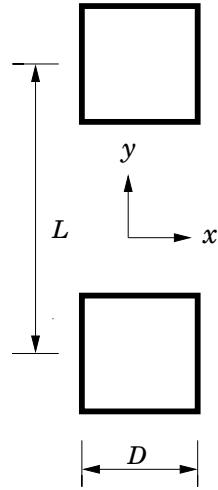
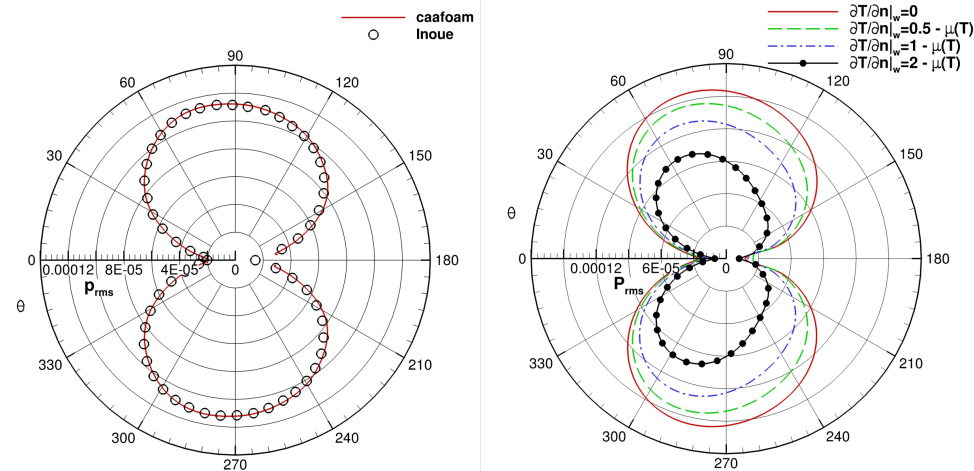


Figure 7. Cylinders arranged side by side.



(a) Solver validation

(b) Wall heating effect

Figure 8. Cylinders arranged side by side. Acoustic pressure results.

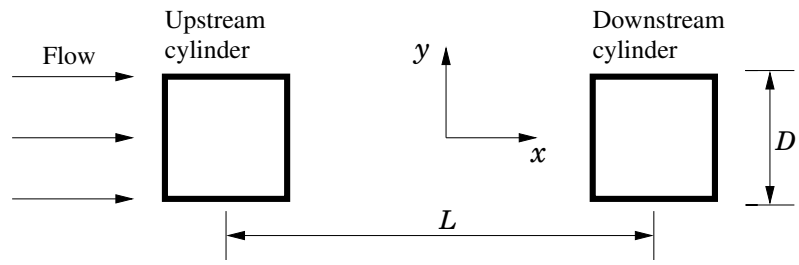
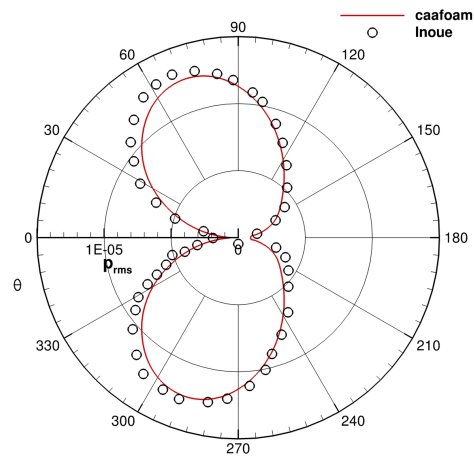
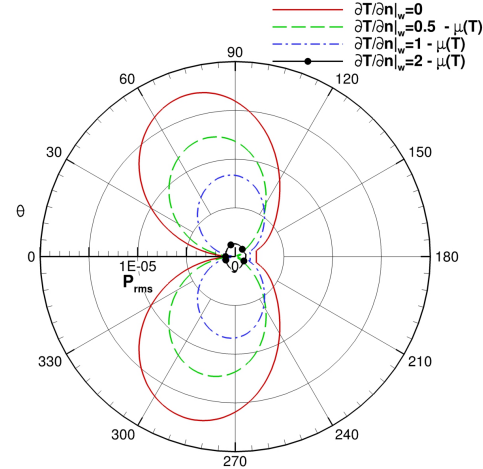


Figure 9. Square cylinders in tandem configuration.



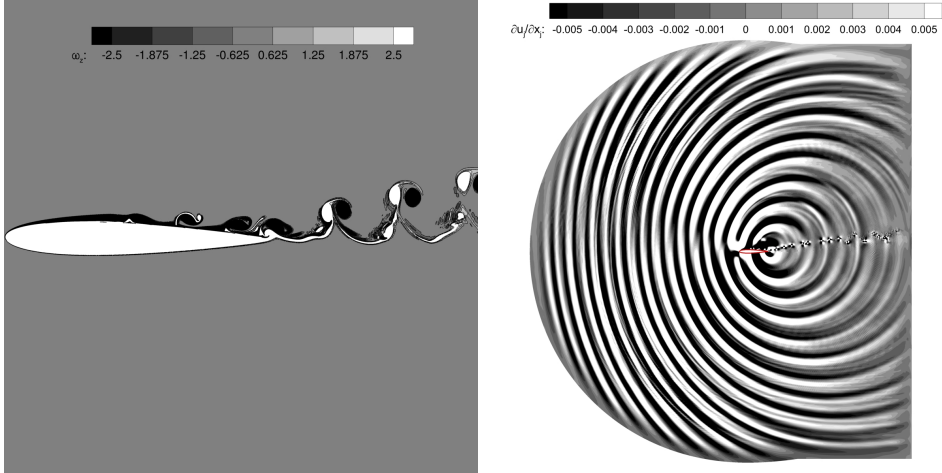
(a) Solver validation,  $r/D = 75$



(b) Wall heating effect

Figure 10. Square cylinders in tandem configuration. Acoustic pressure results.





(a) Spanwise vorticity

(b) Dilatation rate

Figure 11. Flow and acoustic field representation, NACA 0012 airfoil. Grid G4.

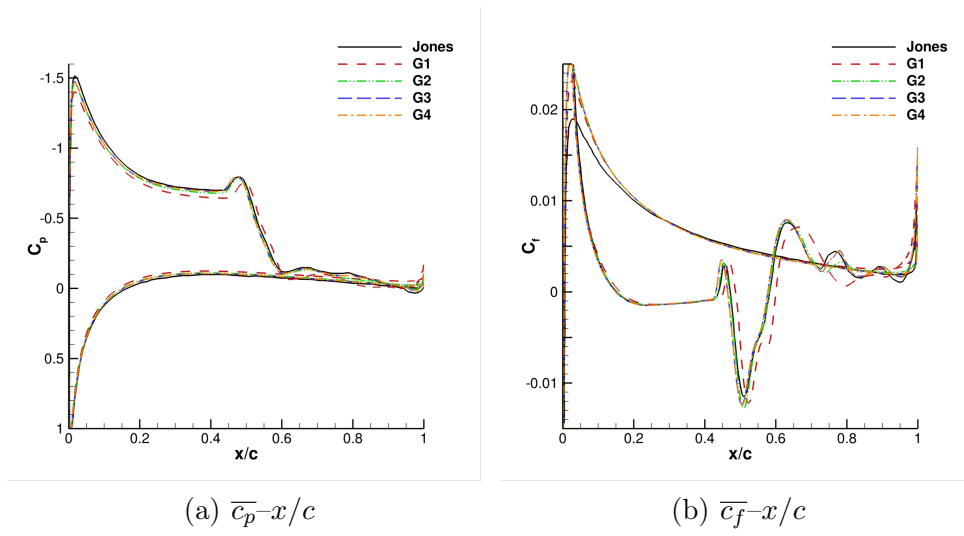
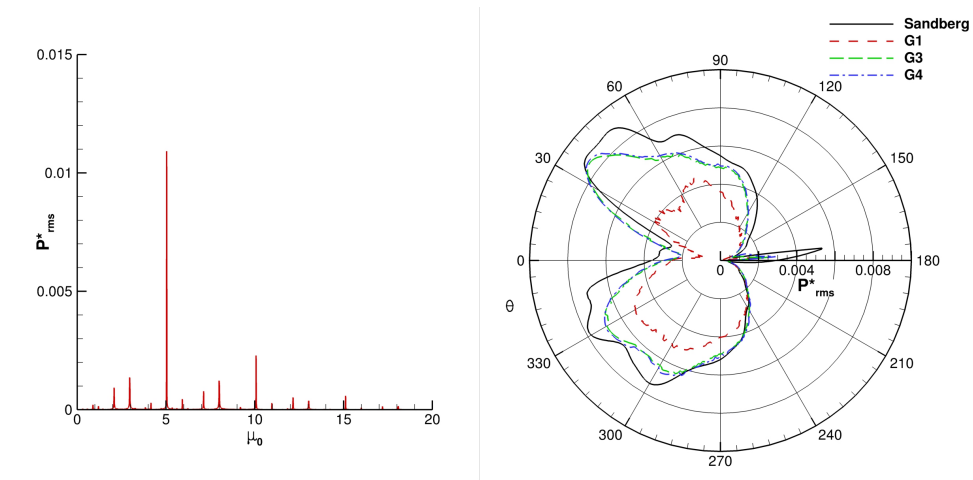


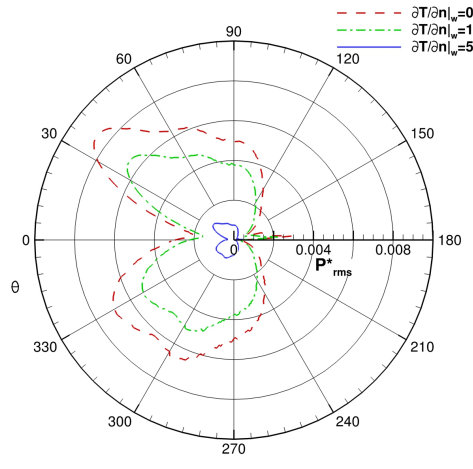
Figure 12. Time-averaged pressure and skin friction coefficient distribution



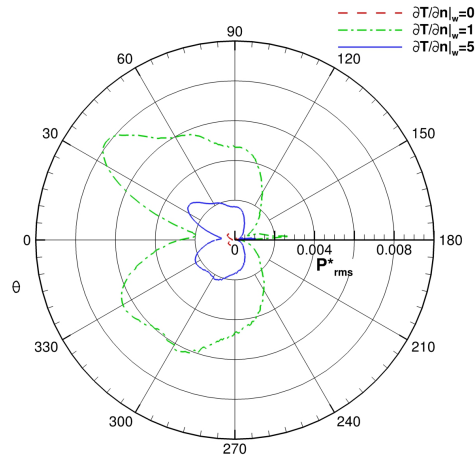
(a) Frequency spectrum, Grid G4.

(b) Polar plot  $\mu_0 = 5.036$

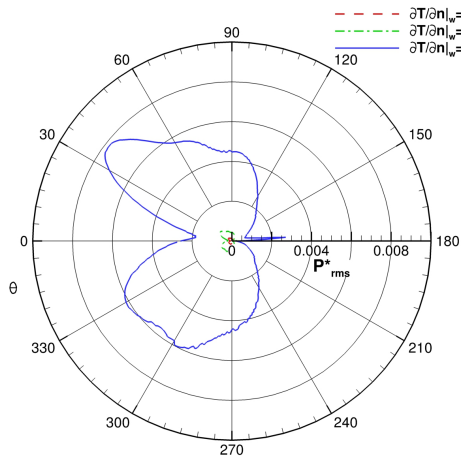
Figure 13. Dimensionless acoustic pressure data. NACA 0012 airfoil.



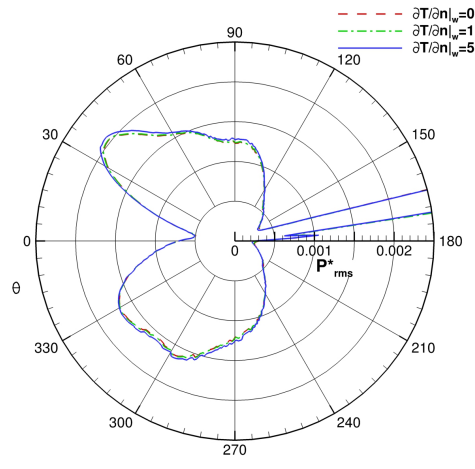
(a)  $\mu_0 = 5.036$



(b)  $\mu_0 = 5.044$



(c)  $\mu_0 = 5.061$



(d) Overall frequencies contribution

Figure 14. Thermal effect on the acoustic pressure, NACA 0012, Grid G4.

Table 1

Circular Cylinder -  $Re = 150$ ,  $M_\infty = 0.2$ ,  $\mu = cost$  - Force coefficients.

Case	$\Delta C_L$	$C_{L,rms}$	$\Delta C_D \cdot 10^2$	$\overline{C_D}$	$St$
$\partial T / \partial n _w = 0$	0.52	0.37	2.56	1.333	0.182
$\partial T / \partial n _w = 0.5$	0.49	0.35	2.45	1.333	0.182
$\partial T / \partial n _w = 1$	0.46	0.33	2.25	1.327	0.181
$\partial T / \partial n _w = 2$	0.41	0.29	1.96	1.323	0.180
$T_w = 2T_\infty$	0.36	0.25	1.61	1.335	0.177
$T_w = 3T_\infty$	0.25	0.18	0.96	1.327	0.170

Table 2

Circular Cylinder -  $Re = 150$ ,  $M_\infty = 0.2$ ,  $\mu = \mu(T)$  - Force coefficients.

Case	$\Delta C_L$	$C_{L,rms}$	$\Delta C_D \cdot 10^2$	$\overline{C_D}$	$St$
$\partial T / \partial n _w = 0$	0.52	0.36	2.56	1.333	0.182
$\partial T / \partial n _w = 0.5$	0.47	0.33	2.21	1.329	0.180
$\partial T / \partial n _w = 1$	0.41	0.29	1.84	1.325	0.178
$\partial T / \partial n _w = 2$	0.31	0.22	1.14	1.315	0.173
$T_w = 2T_\infty$	0.24	0.17	0.67	1.382	0.161
$T_w = 3T_\infty$	0.08	0.06	0.07	1.402	0.139

Table 3

Square Cylinder Side by Side,  $L/D = 3$ , in phase -  $Re = 150$ ,  $M_\infty = 0.2$ ,  $\mu = \mu(T)$   
 - Force coefficients.

Case	$\Delta C_L$	$\overline{C_L}$	$\Delta C_D$	$\overline{C_D}$	$St$
$\partial T / \partial n _w = 0$	0.41	$\pm 0.075$	0.114	1.592	0.155
$\partial T / \partial n _w = 0.5$	0.39	$\pm 0.096$	0.101	1.589	0.151
$\partial T / \partial n _w = 1$	0.35	$\pm 0.119$	0.087	1.589	0.149
$\partial T / \partial n _w = 2$	0.26	$\pm 0.163$	0.061	1.591	0.146

Table 4

Square cylinders in tandem configuration,  $L/D = 2$ ,  $Re = 150$ ,  $M_\infty = 0.2$ ,  $\mu = \mu(T)$ 

- Force coefficients.

Upstream cylinder					
Case	$\Delta C_L \cdot 10^2$	$\overline{C_L} \cdot 10^4$	$\Delta C_D \cdot 10^4$	$\overline{C_D}$	$St$
$\partial T / \partial n _w = 0$	1.951	0	1.05	1.28	0.134
$\partial T / \partial n _w = 0.5$	1.118	0.48	0.47	1.27	0.128
$\partial T / \partial n _w = 1$	0.631	1.10	0.15	1.26	0.122
$\partial T / \partial n _w = 2$	0.043	0.07	2.90	1.20	0.105
Downstream cylinder					
Case	$\Delta C_L \cdot 10^2$	$\overline{C_L} \cdot 10^4$	$\Delta C_D \cdot 10^4$	$\overline{C_D}$	$St$
$\partial T / \partial n _w = 0$	5.30	0	7.65	-0.196	0.134
$\partial T / \partial n _w = 0.5$	3.80	2.42	3.21	-0.158	0.128
$\partial T / \partial n _w = 1$	2.59	3.79	1.16	-0.114	0.122
$\partial T / \partial n _w = 2$	0.26	7.40	0.87	-0.010	0.105



Table 5

Computational domain discretisation.

Grid	$N_{\text{foil}}$	$N_{\text{radial}}$	$N_{\text{wake}}$	$N_{\text{tot}}$
G1	1084	690	755	1789860
G2	2000	690	755	2421900
G3	4000	690	755	3801900
G4	4000	800	1000	4800000

Table 6  
 Time-averaged lift and drag coefficients.

	G1	G2	G3	G4	Jones [28]
$\overline{C_L}$	0.439	0.473	0.484	0.484	0.499
$\overline{C_D} \cdot 10^2$	3.13	3.16	3.17	3.17	3.07

Table 7

NACA 0012,  $\text{Re} = 5 \cdot 10^4$ ,  $M_\infty = 0.4$ . Wall heating impact

	$\partial T/\partial n _w = 0$	$\partial T/\partial n _w = 1$	$\partial T/\partial n _w = 5$
$\overline{C_L}$	0.484	0.484	0.484
$\overline{C_D} \cdot 10^2$	3.17	3.17	3.20
$C_{LRMS} \cdot 10^2$	1.87	1.88	1.91
$C_{DRMS} \cdot 10^3$	2.02	2.05	2.16
$\mu_0$	5.036	5.044	5.061