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*Original*

Combined Wind and Sea Excitations in the Nonlinear Dynamics of a Simple Model of FOWT Under Internal Resonance Conditions / Settimi, V., Lenci, S. - 675:(2025), pp. 785-794. [10.1007/978-3-031-96106-9\_80]

*Availability:*

This version is available at: 11566/364552 since: 2026-10-14T08:55:09Z

*Publisher:*

Springer Science and Business Media Deutschland GmbH

*Published*

DOI:10.1007/978-3-031-96106-9\_80

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# Combined wind and sea excitations in the nonlinear dynamics of a simple model of FOWT under internal resonance conditions\*

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**Abstract.** The nonlinear dynamics of a simplified nonlinear model of a floating offshore wind turbine under wind and sea excitations is investigated. The motion of sea waves is represented as the heave displacement of the turbine tower, leading to a parametric excitation of both the along-wind and cross-wind displacements. The study explores the occurrence of a 1:1 internal resonance between along-wind and cross-wind oscillations using an asymptotic approach grounded in the multiple scale method. The findings highlight a rich dynamical scenario, which is significantly influenced by the sea excitation. In particular, a detrimental contribution to the system stability can be pointed out for low sea amplitude values, while a stabilizing effect is displayed for moderate to high values of the sea excitation. A behavior chart is developed to illustrate the model response in relation to the forcing parameters.

**Keywords:** Wind turbine towers · Nonlinear dynamics · Coupled nonlinear vibrations · Sea excitation.

## 1 Introduction

Among the various renewable energy sources, offshore wind stands out for its capacity to generate significant amounts of electricity [1]. The advent of floating structures allows wind turbines to be installed farther from the coastline. Utilizing floating wind turbines in deeper waters presents several advantages, such as access to stronger and more consistent winds, minimized visual and environmental impacts, and flexible noise regulations [2].

The development of a Floating Offshore Wind Turbine (FOWT) necessitates a multidisciplinary approach to assess the interrelated components, including hydrodynamics, aerodynamics, structural dynamics, and control systems, which adds to the complexity of these offshore structures [3]. Consequently, the design and analysis of FOWT rely on fully coupled dynamic response simulation

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\* Supported by the European Union - Next Generation EU, in the framework of the project PRIN 2022 M4-C2-I.1.1, 2022W7SKTL, NonlinEar Phenomena in floatIng offshore wind tUrbiNEs (NEPTUNE).

tools. However, assessing the dynamic performance of FOWTs through numerical simulations can be a time-consuming process with substantial computational demands, primarily due to the need for multiple load scenarios [4,5]. A solution is to create reduced dynamic models that utilize streamlined dynamic equations to capture the key physical phenomena, associated with the dynamics of both the wind turbine and its platform, significantly lowering computational costs. Additionally, these models can be easily enhanced to incorporate new elements and load configurations.

In this respect, in some recent papers by the authors [6,7] a 3 dof reduced nonlinear model of the wind turbine tower is formulated as a function of the two horizontal translations and the rotation around the vertical axis of the wind turbine tower. The presence of a time-dependent harmonic wind excitation is accounted for [6], with also the contemporary existence of the sea excitation modelled as heave displacement [7]. The possible occurrence of a 1:1 internal resonance in a FOWT is investigated, motivated by the fact that the symmetrical cross-section of the tower naturally induces the closeness of two natural frequencies. An approximate asymptotic response is obtained through the multiple scale method. As expected, the results show the arise of coupled bimodal oscillations involving the cross-wind and along-wind displacements, which display a rich dynamical behavior. In particular, the sea excitation proves to have a significant effect on the dynamical response of the turbine. However, the proposed results [7] consider only the presence of the sea excitation into the equation governing the response in the along-wind direction, while neglecting the contributions into the equation of the cross-wind direction which naturally ensue from the model formulation.

Moving from these considerations, the present work aims at completing the description of the dynamics of the proposed turbine model under wind and sea excitation [7], by contemplating the presence of the sea excitation in both equations governing the two along-wind and cross-wind displacements. The asymptotic multiple scale procedure is then enriched with the objective to get a more refined description of the possible dynamical scenarios.

## 2 Mechanical model and asymptotic treatment

The reference model for the wind turbine is presented in [7]. The nacelle-rotor blades system is modelled as a concentrated mass applied at the relevant center of mass at the top of the tower, whose distributed mass is neglected. As concerns the excitation force, a time-dependent wind force  $\bar{F}$  is applied to the center of mass along the  $x$  direction. Moreover, the heave displacement  $\bar{G}$  is applied to the base of the turbine system in order to model the sea excitation. By applying the general formula for the 3-dimensional rigid motion, and by imposing the stationarity of the Lagrangian function under the hypothesis of small yaw rotations  $\alpha$ , the equations of motion of the turbine along  $x$  and  $y$  directions, and

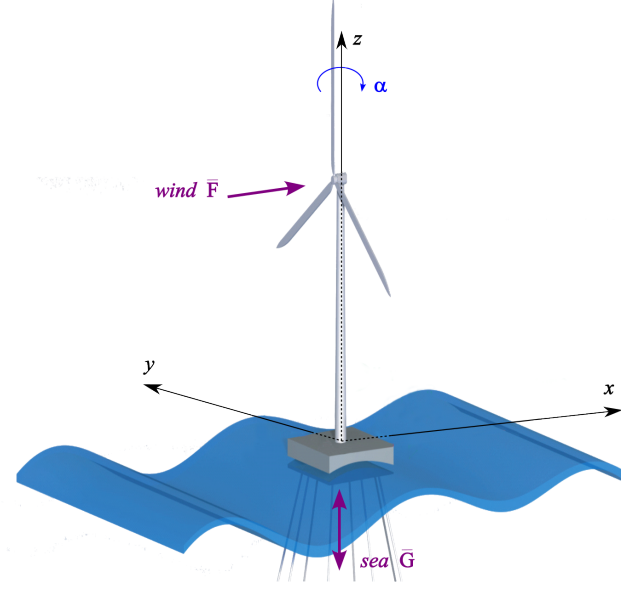


Fig. 1: FOWT model under wind and sea excitations.

as a function of the yaw rotation read [7]:

$$\begin{aligned}
 \ddot{x} + 2c_x\dot{x} + (1 + \ddot{G})x + k_3(x^3 + xy^2) - \dot{\alpha}^2 - \alpha\ddot{\alpha} + \left(1 - \frac{\alpha^2}{2}\right)\ddot{G} &= F, \\
 \ddot{y} + 2c_y\dot{y} + (1 + \ddot{G})y + k_3(y^3 + x^2y) - \alpha\dot{\alpha}^2 + \left(1 - \frac{\alpha^2}{2}\right)\ddot{\alpha} + \left(\alpha - \frac{\alpha^3}{6}\right)\ddot{G} &= 0, \\
 (1 + I_p)\ddot{\alpha} + 2c_\alpha\dot{\alpha} + k_\alpha\alpha + \left(\frac{\alpha^3}{6} - \alpha\right)\ddot{x} + \left(1 - \frac{\alpha^2}{2}\right)\ddot{y} &= \left(\frac{\alpha^3}{6} - \alpha\right)F.
 \end{aligned} \tag{1}$$

With respect to [7], here the presence of the sea excitation also into the motion of the cross-wind  $y$  displacement is accounted for. As a consequence, the presence of the heave displacement due to the sea excitation results in the appearance of a parametric time-dependent excitation  $G$  in the equations governing both the along-wind and cross-wind displacements. A further coupling term related to the torsion angle  $\alpha$  is also present in both the  $x$  and  $y$  equations of motion.

In the present work, interest is devoted to investigate the system behavior around primary external resonance with the forcing excitations, as it is well known that such condition represents a dynamically rich and potentially dangerous scenario. Moreover, the internal 1:1 resonance condition between along-wind and cross-wind oscillations is imposed to ascertain the arise of possible bimodal responses. To analytically investigate this region, the asymptotic multiple scale approach is applied, following the scheme proposed in [7]. Here, the so-called

frequency detuning method [8,9] is followed, based on the application of the detuning to the system natural frequencies, rather than to the excitation frequency, as commonly done. To follow this strategy, the linear part of the FOWT model must be uncoupled in order to make the natural frequencies appear into the system equations. The resulting equations in matrix form read [7]

$$\ddot{\Xi} + \mathbf{C}\dot{\Xi} + (\mathbf{A} + \mathbf{G}(t))\Xi + \mathbf{n}^*(\Xi, \dot{\Xi}, \ddot{\Xi}, F(t)) + \mathbf{F}(t) = \mathbf{0}, \quad (2)$$

where  $\Xi = [\xi_1, \xi_2, \xi_3]$  is the vector of the transformed variables,  $\mathbf{C}$  and  $\mathbf{A}$  are the damping and stiffness matrices, respectively, while  $\mathbf{F}$  and  $\mathbf{G}$  are forcing terms including only wind and sea excitation terms. Their expressions are reported in Appendix B of [7]. The vector  $\mathbf{n}^* = [n_1^*, n_2^*, n_3^*]$  includes all the nonlinear quadratic and cubic terms, and with respect to the  $\mathbf{n}$  vector in [7], it is enriched by some  $g$ -dependent terms in  $n_2$  and  $n_3$  (while  $n_1^* = n_1$ ):

$$\begin{aligned} n_2^* = & n_2 - \frac{1}{6} g v_1 \Omega_g^2 (\xi_2^3 (-v_2^3) - 3\xi_3 \xi_2^2 v_2^2 v_4 + 6\xi_2 v_1 - 3\xi_2 v_2 (\xi_3^2 v_4^2 - 2) \\ & + \xi_3 (-\xi_3^2 v_4^3 + 6v_4 + 6v_3)) \cos(t\Omega_g), \\ n_3^* = & n_3 - \frac{1}{6} g v_1 \Omega_g^2 (\xi_2^3 (-v_2^3) - 3\xi_3 \xi_2^2 v_2^2 v_4 + 6\xi_2 v_1 - 3\xi_2 v_2 (\xi_3^2 v_4^2 - 2) \\ & + \xi_3 (-\xi_3^2 v_4^3 + 6v_4 + 6v_3)) \cos(t\Omega_g). \end{aligned} \quad (3)$$

It is worth remembering that it is possible to move back to the original variables representing the nondimensional displacements along  $x$  and  $y$  axes, and the yaw rotation  $\alpha$  by applying the linear transformation:  $x(t) = \xi_1(t)$ ,  $y(t) = v_1 \xi_2(t) + v_3 \xi_3(t)$ ,  $\alpha(t) = v_2 \xi_2(t) + v_4 \xi_3(t)$ , with  $v_j, j = 1, \dots, 4$  reported in [7].

The asymptotic multiple scale approach is developed around the 1:1 internal resonance condition between the  $\xi_1$  and  $\xi_2$  variables. Accordingly, three time scale are introduced, and variables and parameters are scaled as follows [7]:

$$\begin{aligned} \xi_1 &= \epsilon \hat{\xi}_1, \quad \xi_2 = \epsilon \hat{\xi}_2, \quad \xi_3 = \epsilon^2 \hat{\xi}_3, \\ c_i &= \epsilon^2 \hat{c}_i, i = x, y, \alpha, \quad F = \epsilon^3 \hat{f} \cos(\Omega_f t), \quad G = \epsilon^2 \hat{g} \cos(\Omega_g t), \end{aligned} \quad (4)$$

where the  $\xi_3$  variable is scaled according to its passive role of non-resonant dragged variable; sea and wind are modelled as harmonic excitations, and  $\Omega_g = 2\Omega_f$ . To study the response around primary external resonance, an external detuning parameter  $\sigma_e$  is added to the  $\xi_1$  natural frequency,

$$\omega_1^2 = \Omega_f^2 - \epsilon^2 \sigma_e, \quad (5)$$

while an internal detuning  $\sigma_i$  is introduced in order to ensure that 1:1 internal resonance occurs between the  $\xi_1$  and  $\xi_2$  variables,

$$\omega_2^2 = \omega_1^2 + \epsilon^2 \sigma_i = \Omega_f^2 + \epsilon^2 (\sigma_i - \sigma_e). \quad (6)$$

The resulting perturbation equations at  $\epsilon$  and  $\epsilon^2$  orders are the same as [7], while the third order is enriched by some  $g$ -dependent terms inside the  $\xi_{23}$  and  $\xi_{33}$  equations:

Order  $\epsilon^3$

$$\begin{aligned} D_0^2 \xi_{13} + \Omega_f^2 \xi_{13} &= \text{rhs of } (22)_1 \text{ of [7]}, \\ D_0^2 \xi_{23} + \Omega_f^2 \xi_{23} &= \text{rhs of } (22)_2 \text{ of [7]} + a_{21} g \xi_{21} (e^{2i\Omega_f T_0} + e^{-2i\Omega_f T_0}), \\ D_0^2 \xi_{33} + a_1 \xi_{33} &= \text{rhs of } (22)_3 \text{ of [7]} + a_{22} g \xi_{21} (e^{2i\Omega_f T_0} + e^{-2i\Omega_f T_0}), \end{aligned} \quad (7)$$

where  $a_{21} = 2v_1 \Omega_f^2 (v_1 + v_2)$  and  $a_{22} = 2v_3 \Omega_f^2 (v_1 + v_2)$ . This implies that at the third order, the  $g$ -dependent terms inside the solvability conditions (Eqs. (26) of [7]) and the particular solutions (Eqs. (27) of [7]) are modified by the enrichment of the following coefficients:

$$\begin{aligned} c_9 &= -\frac{8a_2 \Omega_f^2 + 3a_{21}}{6\Omega_f}, \\ c_{34} &= -\frac{a_2}{3} - \frac{a_{21}}{8\Omega_f^2}, \quad c_{37} = \frac{8a_3 \Omega_f^2 + 3a_{22}}{3(a_1 - 9\Omega_f^2)}, \quad c_{44} = \frac{8a_3 \Omega_f^2 + 3a_{22}}{3a_1 - 3\Omega_f^2}, \end{aligned} \quad (8)$$

where the new presence of the contributions related to  $a_{21}$  and  $a_{22}$  coefficients can be highlighted. According to the usual reconstitution procedure, the Cartesian form of the Amplitude Modulation Equations (AMEs) is [7]

$$\begin{aligned} \dot{p}_1 &= p_1 \left( -\frac{1}{2} c_5 p_2 q_2 + c_7 \right) + q_1 \left( \frac{c_5 - c_6}{4} p_2^2 - \frac{c_5 + c_6}{4} q_2^2 - c_2 + c_3 g \right) \\ &\quad - \frac{1}{4} c_4 p_1^2 q_1 - \frac{1}{4} c_4 q_1^3, \\ \dot{q}_1 &= q_1 \left( \frac{1}{2} c_5 p_2 q_2 + c_7 \right) + p_1 \left( \frac{c_5 + c_6}{4} p_2^2 + \frac{c_6 - c_5}{4} q_2^2 + c_2 + c_3 g \right) \\ &\quad + \frac{1}{4} c_4 p_1 q_1^2 + \frac{1}{4} c_4 p_1^3 + 2c_1 f, \\ \dot{p}_2 &= p_2 \left( -\frac{1}{2} c_{11} p_1 q_1 + c_{13} \right) + q_2 \left( \frac{c_{11} - c_{12}}{4} p_1^2 - \frac{c_{11} + c_{12}}{4} q_1^2 - c_8 + c_9 g \right) \\ &\quad - \frac{1}{4} c_{10} p_2^2 q_2 - \frac{1}{4} c_{10} q_2^3, \\ \dot{q}_2 &= q_2 \left( \frac{1}{2} c_{11} p_1 q_1 + c_{13} \right) + p_2 \left( \frac{c_{11} + c_{12}}{4} p_1^2 + \frac{c_{12} - c_{11}}{4} q_1^2 + c_8 + c_9 g \right) \\ &\quad + \frac{1}{4} c_{10} p_2 q_2^2 + \frac{1}{4} c_{10} p_2^3, \end{aligned} \quad (9)$$

while the trigonometric form of the reconstructed solutions is [7]

$$\begin{aligned}
\xi_1(t) &= p_1 \cos(\Omega_f t) - q_1 \sin(\Omega_f t) + c_{12}^c \cos(2\Omega_f t) + c_{12}^s \sin(2\Omega_f t) \\
&\quad + c_{13}^c \cos(3\Omega_f t) + c_{13}^s \sin(3\Omega_f t), \\
\xi_2(t) &= c_{20}^c + p_2 \cos(\Omega_f t) - q_2 \sin(\Omega_f t) + c_{22}^c \cos(2\Omega_f t) + c_{22}^s \sin(2\Omega_f t) \\
&\quad + c_{23}^c \cos(3\Omega_f t) + c_{23}^s \sin(3\Omega_f t), \\
\xi_3(t) &= c_{30}^c + c_{31}^c \cos(\Omega_f t) + c_{31}^s \sin(\Omega_f t) + c_{32}^c \cos(2\Omega_f t) + c_{32}^s \sin(2\Omega_f t) \\
&\quad + c_{33}^c \cos(3\Omega_f t) + c_{33}^s \sin(3\Omega_f t).
\end{aligned} \tag{10}$$

Definition of the coefficients can be found in [7]. From a mathematical viewpoint, considering the sea excitation also into the equation of the cross-wind displacement implies the modification of the terms  $c_9 g q_2$  and  $c_9 g p_2$  inside the  $\dot{p}_2$  and  $\dot{q}_2$  expressions of the Cartesian AMEs, respectively. As concerns the stationary solutions, the  $\xi_2$  expression sees the enrichment of the order-3 superharmonic terms ( $\sin(\cos)(3\Omega_f t)$ ), while in the  $\xi_3$  expression the modified terms are the harmonic and the order-3 superharmonic ones ( $\sin(\cos)(\Omega_f t)$  and  $\sin(\cos)(3\Omega_f t)$ ). All the three responses show the presence of harmonic and superharmonic components of order 2 and 3; moreover, the  $\xi_2$  and  $\xi_3$  solutions have also constant terms responsible for the shift of the oscillation with respect to the rest position.

### 3 Numerical results

The reference turbine for the analysis of the dynamical behavior of the FOWT model under wind and sea excitation is the NREL 5-MW wind turbine [10], whose mechanical properties are reported in Appendix D of [7]. The resulting nondimensional coefficients of the dynamical system (1) are  $I_p = 2.08846$ ,  $k_\alpha = 1725.97485$ . Moreover, the numerical values of the cubic stiffness and the viscous linear dampings are  $k_3 = 1$ ,  $c_x = c_y = c_\alpha = 0.05$ .

The resulting values of the natural frequencies are  $\omega_1 = 1$ ,  $\omega_2 = 0.99942$ ,  $\omega_3 = 826.915$ , while the normalized eigenvectors are  $(1, 0, 0)$ ,  $(0, v_1, v_2) = (0, 0.99942, 5.79748 \times 10^{-4})$ ,  $(0, v_3, v_4) = (0, -0.692808, 0.69197)$ . The internal detuning  $\sigma_i$  governing the internal 1:1 resonance between  $x$  and  $y$  displacements is fixed to be  $\sigma_i = 2.89916 \times 10^{-4}$ .

It is interesting to note that for the selected numerical values, and with  $\Omega_f = 1$ , considering the sea excitation also in the equation of the cross-wind displacement  $y$  implies that the coefficients (8) related to the sea excitation assume values of different order of magnitude with respect to the case presented in [7], as shown in Table 1, a fact that justifies the present analysis. From a phenomenological point of view, this entails a significant effect of the sea excitation in governing the dynamical response of the system, as displayed in Figure 2 (to be compared with Figure 3 of [7]). The chart is realized in the detuning-amplitude plane  $\sigma_e$ - $g$ , for a fixed value of the wind excitation  $f = 0.1$ , and identifies various coloured regions characterized by different kinds of dynamical response. In particular, the gray regions correspond to monomodal (i.e., with

Table 1: Comparison of the coefficient values between the present model with the sea excitation  $g$  into the  $y$  equation (1) and the model without  $g$  into the  $y$  equation (1) presented in [7].

|          | With $g$ (present)       | Without $g$ [7]          |
|----------|--------------------------|--------------------------|
| $c_9$    | -0.99942                 | $-4.48143 \cdot 10^{-7}$ |
| $c_{34}$ | -0.249855                | $-1.12036 \cdot 10^{-7}$ |
| $c_{37}$ | $-1.69277 \cdot 10^{-3}$ | $1.30794 \cdot 10^{-6}$  |
| $c_{44}$ | $-1.67638 \cdot 10^{-3}$ | $1.29527 \cdot 10^{-6}$  |

$\xi_1 \neq 0$  and  $\xi_2 = \xi_3 = 0$ ) period-1 responses, the blue regions represent bimodal (i.e., with  $\xi_1 \neq 0$ ,  $\xi_2 \neq 0$ ,  $\xi_3 \neq 0$ ) period-1 responses, and the green regions identify bimodal quasiperiodic responses.

For low values of the sea excitation amplitude ( $g = 0.005$ ), a sample dynamical response of the system is displayed in the bifurcation diagrams of Figure 3. The results are obtained integrating equations (9) by means of the continuation software AUTO [11]. In the figure, the responses in terms of  $\xi_1$ ,  $\xi_2$ ,  $\xi_3$  variables are shown around the resonance condition. To be able to describe the behavior

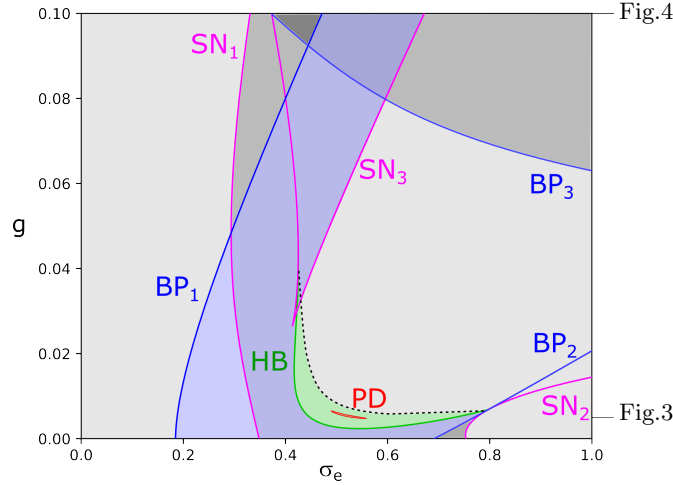


Fig. 2: For  $\Omega_f = 1$  and  $f = 0.1$ , behavior chart around the resonance region ( $\sigma_e$ - $g$  plane). Light gray region: monomodal periodic response; blue region: bimodal periodic response; green region: bimodal quasiperiodic response; red region: bimodal quasiperiodic period-doubled response. The superposition of the coloured regions (i.e., the darker blue and darker gray areas) implies the contemporary presence of different responses. Blue curve: branch point bifurcation threshold; magenta curve: saddle-node bifurcation threshold; green curve: Hopf bifurcation threshold; red curve: period doubling bifurcation threshold; dashed black curve: crisis.

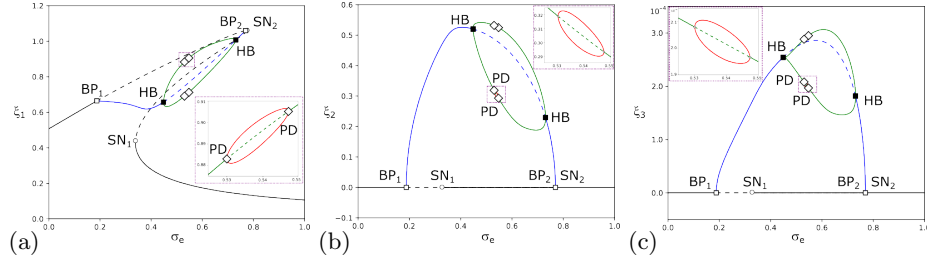


Fig. 3: Response curve in terms of  $\xi_1$  (a),  $\xi_2$  (b) and  $\xi_3$  (c) for  $\Omega_f = 1$ ,  $f = 0.1$  and  $g = 0.005$ . Black curve: monomodal periodic response along  $\xi_1$ ; Blue curve: bimodal response; Green curve: bimodal quasiperiodic response; Red curve: bimodal quasiperiodic period-doubled response. Circle: saddle-node bifurcation. Square: branch point bifurcation. Black square: Hopf bifurcation. White diamond: period doubling bifurcation. Dashed curves: unstable responses.

of the dragged variable  $\xi_3$ , the maximum value of the reconstructed solutions is plotted as a function of the external detuning  $\sigma_e$ , considering the contributions—in terms of amplitudes—of all harmonic and superharmonic components [7]:

$$\begin{aligned}
 \xi_1 &= \sqrt{p_1^2 + q_1^2} + \sqrt{(c_{12}^c)^2 + (c_{12}^s)^2} + \sqrt{(c_{13}^c)^2 + (c_{13}^s)^2}, \\
 \xi_2 &= c_{20}^c + \sqrt{p_2^2 + q_2^2} + \sqrt{(c_{22}^c)^2 + (c_{22}^s)^2} + \sqrt{(c_{23}^c)^2 + (c_{23}^s)^2}, \\
 \xi_3 &= c_{30}^c + \sqrt{(c_{31}^c)^2 + (c_{31}^s)^2} + \sqrt{(c_{32}^c)^2 + (c_{32}^s)^2} + \sqrt{(c_{33}^c)^2 + (c_{33}^s)^2}.
 \end{aligned} \tag{11}$$

In the figure, the resonant monomodal response has an unstable region delimited by the  $BP_1$  and  $BP_2$  branch point (pitchfork) bifurcations. Inside the unstable region, a bimodal solution arises, which moves from periodic to quasiperiodic oscillations due to the presence of the HB Hopf bifurcation. Eventually, inside the narrow red region delimited by the PD period-doubling curve, a bimodal quasiperiodic period-doubled response can be detected. For a very narrow range of frequency values, between the  $BP_2$  and  $SN_2$  thresholds, the resonant monomodal solution regains stability just near the resonance peak. As concerns the nonresonant (low-amplitude) monomodal response, it coexists with the bimodal responses for a quite large range of frequencies, and becomes unstable through the  $SN_1$  saddle-node bifurcation. It is worth noting that for  $g \lesssim 0.05$ , the light blue region in Figure 2 corresponds to the existence of the sole bimodal solution as stable response for the system, for a frequency range which increases as the sea excitation decreases.

Looking at the amplitude of the responses in Figure 3, it can be observed that the  $\xi_1$  and  $\xi_2$  solutions display comparable oscillations, while  $\xi_3$  has three order of magnitude lower values. Moving to the actual (nondimensional) turbine displacements, it results that  $x = \xi_1$ ,  $y = 0.99942 \xi_2 - 0.692808 \xi_3$  and  $\alpha = 5.79748 \times 10^{-4} \xi_2 + 0.69197 \xi_3$ . This means that the yaw rotation is represented by  $\xi_3$  (with a value reduction of 30%), while the  $y$  displacement can be

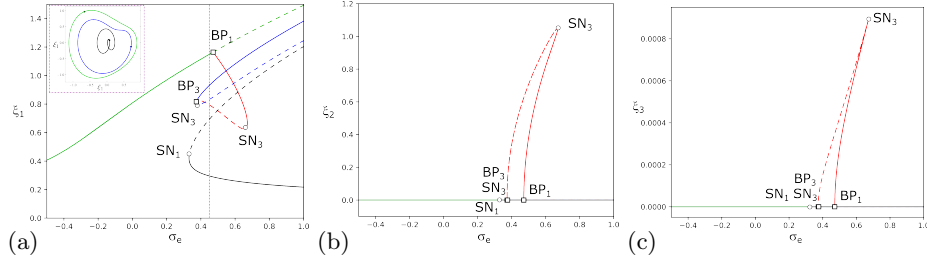


Fig. 4: Response curve in terms of  $\xi_1$  (a),  $\xi_2$  (b) and  $\xi_3$  (c) for  $\Omega_f = 1$ ,  $f = 0.1$  and  $g = 0.005$ . In (a), phase portraits and Poincaré maps in the  $(\xi_1, \dot{\xi}_1)$  plane of the monomodal solutions. Black curve: monomodal nonresonant periodic response along  $\xi_1$ ; Green curve: monomodal resonant response; Red curve: monomodal high-amplitude response; Blue curve: bimodal response. Circle: saddle-node bifurcation. Square: branch point bifurcation. Dashed curves: unstable responses.

practically assimilated to the behavior of  $\xi_2$ . Thus, in the bimodal responses the tower oscillates in both  $x$  and  $y$  directions with comparable amplitudes, while undergoing a significantly smaller yaw rotation.

The described scenario is quite similar to that already presented in [7], while some interesting differences can be highlighted for higher values of the sea excitation where the bimodal blue oscillation undergoes a strong reduction of its existence domain. For  $g \gtrsim 0.03$ , in fact, it becomes unstable due to the saddle-node bifurcation threshold  $SN_3$ , which remains almost parallel to the  $BP_1$  curve as the amplitude increases, thus limiting the existence of bimodal oscillations in a fairly narrow range of frequencies.

Moreover, for sea amplitude values similar to the wind ones (i.e., for  $g \cong 0.1$ ), a further monomodal periodic response arises from the  $BP_3$  pitchfork bifurcation, as shown in Figure 4. Such solution remains stable for the whole considered frequency range, coexisting with the other periodic responses. As a consequence, in the dark gray triangular region at the top of Figure 2, three monomodal solutions are simultaneously stable, corresponding to a low-amplitude nonresonant oscillation (black solution in Figure 4) and two high-amplitude oscillations representing the resonant response (green solution in Figure 4) and the newly born response connected to the bimodal branch (blue solution in Figure 4).

## 4 Conclusions

The dynamical response of a reduced order model of FOWT is investigated by considering the simultaneous presence of wind and sea excitations. In particular, for the latter all contributions to both the along-wind and cross-wind displacements are here accounted for. The asymptotic multiple scale method is applied to analytically investigate the response of the system around the primary resonance under 1:1 internal resonance conditions between the displacements. From a practical point of view, the described behavior highlights that for low amplitude

values, the sea excitation furnishes a detrimental contribution to the stability of the dynamical response of the turbine tower. In fact, a rich dynamical behavior is detected, bimodal solutions arise, becoming also the sole stable solutions for some frequency values, and involved quasiperiodic solutions eventually appear. In an operational perspective, it is worth underlining that the bimodal oscillations are very dangerous in terms of fatigue and life time of the tower. In contrast, for moderate to high-amplitude values, the sea excitation provide a stabilizing contribution to the dynamical response of the turbine, which is characterized by monomodal periodic responses coexisting with a bimodal periodic oscillation, which is, however, stable for a limited frequency range.

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