Optimal control of inventory level for perishable goods with uncertain decay factor and uncertain forecast information: a new robust MPC approach

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ABSTRACT
This paper deals with the inventory control in supply chains under the following assumptions: 1) perishable goods with uncertain deteriorating factor, 2) a future uncertain customer demand that, over a limited prediction horizon, belongs to a known compact set. The problem is to define a smooth control policy maximizing the fulfilled customer demand, and minimizing the inventory level. This problem is here solved through a new Robust Model Predictive Control (RMPC) approach. This implies solving a min-max optimization problem with hard constraints on the control effort (i.e. the sequence of replenishment orders). To drastically reduce the numerical complexity of this problem, the control signal is sought in the space of B-spline functions, which are known to be universal approximators admitting a parsimonious parametric representation. This allows us: 1) to reduce the number of both decision variables and constraints involved in the optimization procedure, 2) to reformulate the numerically demanding minimization of the worst case cost functional as a simpler Weighted Constrained Robust Least Squares (WCRLS) estimation problem. The WCRLS algorithm can be efficiently solved using interior point methods. A rigorous analysis of stability and feasibility conditions is provided.

KEYWORDS
Supply chain, Optimal inventory management, Model predictive control, Min-Max optimization.

1. Introduction

The widely acknowledged importance of MPC in inventory management problems is mainly due to the constraint handling capability and to the receding horizon nature of the control law (Rossiter, 2004). The first feature allows limiting the inventory level and the replenishment orders, the second one allows a proper incorporation of the demand forecast into the control problem and compensates for the negative effects of possible time delays. As a consequence, an extensive research on the application of MPC to different aspects of inventory control problem in supply chain systems has been carrying out for many years. Just to cite a few contributions we mention the following ones: an adaptive MPC scheme for the simultaneous identification and control of production-inventory systems has been proposed in (Aggelogiannaki, Doganis, & Sarimveis, 2008), the same authors consider a similar problem in (Doganis, Aggelion-

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naki & Sarimveis, 2008) using a neural network time series forecasting method, the stock replenishment policy defined in (Alessandri, Gaggero, & Tonelli, 2011) deals with the uncertainty affecting the future customer demand using a worst case approach. A comparison between MPC and Internal Model Control strategies is made in (Schwartz &Rivera, 2010), the case of multiple supply sources is considered in (Xie, Wang, & Yang, 2021).

In the case of multi echelon SC, a self-adaptive MPC is applied in (Fu et al., 2016) using centralized and decentralized control schemes. To reduce the numerical complexity of classical centralized MPC schemes, (Schildback & Morari, 2016) propose a novel centralized scenario based MPC strategy.

More recently, distributed MPC schemes have been proposed using non cooperative (Fu et al., 2019) and cooperative (Fu et al., 2020; Kohler et al., 2021) strategies.

A thorough list of MPC based techniques, dealing with different aspects of the inventory problem in supply chain, can be found in (Dotoli et al. , 2019; Ivanov et al., 2018; Sarimveis et al., 2008).

All the previous papers do not consider the problem implied by the presence of perishable goods in the inventory system. On the other hand, if the effect of stock deterioration is not taken into account, a serious degradation of the supply chain system is observed. This is especially true for highly perishable products like food, blood, chemical materials, medicines, etc.

The inventory level control problem for deteriorating stocks is much less developed with respect to that of nonperishable goods. In the framework of MPC, (Gaggero & Tonelli , 2015) propose a method based on a graph representation of distribution chains, (Hipolito et al., 2022) define a centralized MPC scheme based on a suitably defined extended discrete state-space representation including perishable goods, also the MPC proposed in (Taparia, Janardhanan &Gupta, 2020) is based on a discrete state-space representation of the supply chain dynamics. Perishability of goods is taken into account through a decay parameter $\rho \in (0, 1]$: for $\rho = 1$ the case of non perishable goods is recovered.

We also mention that different control techniques for inventory systems with deteriorating stock have been proposed outside the MPC framework. For example (Ignaciuk , 2012) proposes a generalized Smith predictor, (Ignaciuk & Bartoszewicz, 2012) frames the problem in the context of linear quadratic optimal control, the non linear saturated control strategy proposed in (Ignaciuk , 2013) reduces the output overshoot observed in the Smith predictor approach following abrupt changes in the customer demand, (Ignaciuk , 2015) defines an appropriately modified base-stock type policy to compensate the effects of goods decay.

All the above mentioned papers dealing with perishable goods compute the control input (i.e.the replenishment policy) under the assumption of an exact knowledge of the decaying factor. In (Ignaciuk , 2015) the effect of an uncertain decay factor is only evaluated "a posteriori" through simulations.

Unfortunately, the assumption of an exactly known decay factor is rarely satisfied in practice due to unstable and variable storage conditions. This calls for robust control techniques where uncertainties are directly taken into account in the replenishment policy optimization.

Thorough surveys on inventory management for perishable products are reported in (Chaudary, Kulshrestha, & Routroy, 2018; Li & Mawhinney, 2010).

Based on the foregoing considerations, the purpose of this paper is to propose a Robust MPC (RMPC) approach for the optimal inventory control under the "a priori" assumption of perishable goods with an uncertain decay factor belonging to a given

compact interval. To the best of the author's knowledge this problem has not yet been considered in the literature.
As for the customer forecast information, we only assume that at any time instant $k \in Z^+$ and over an M-steps prediction horizon, the future customer demand is arbitrarily time varying inside a given compact set. This underlying assumption is based on very practical experience based considerations, it is general enough to include almost all real situations regardless the sources of uncertainty on the future customer demand. No assumption is here made on the statistics of the demand generation process. Though many forecasting methods based on time series analysis have been proposed (see e.g. (Box et al., 2016; Montgomery, Jennings & Kulachi, 2015)), it has been observed that they are not able to capture several statistical phenomena underlying the nature of the demand generation process (Lafont et al., 2015). Moreover, they often result in numerically demanding algorithms requiring tools like neural networks, see e.g. Kochak &Sharma (2015) and references therein, vector regression analysis, see e.g. Levis & Papageorgiu (2005) and references therein, big-data analytics, see e.g. Seyedan & Makafery (2020) and references therein.

Coherently with the assumptions on the uncertainties, we develop a RMPC approach based on a min-max optimization procedure: the control law is obtained minimizing the worst case of a quadratic cost functional, which is computed by maximizing with respect to all the possible decay factor values.

Another significant novelty of our approach is the parametrization of the control input $u(k)$, $k \in Z^+$, as a B-spline function. This drastically decreases the number of decision variables involved in the optimization procedure because B-splines admit a parsimonious parametric representation (De Boor, 1978).
Improving the numerical efficiency of MPC through a parametric representation of the control law has been proposed in (Wang, 2004), where Laguerre functions are used. This approach has been applied to inventory systems in (Taparia al., 2020).
Our preference for B-splines has a twofold motivation: 1) B-splines are smooth functions that can be used as universal approximators of curves which exhibit different shapes over different time-intervals, 2) B-splines admit a parsimonious parametric representation given by a time varying, linear, convex combination of some parameters named "control points" (De Boor, 1978).

Property 1 allows us to obtain a smooth replenishment order signal $u(k)$, $k \in Z^+$. Property 2 allow us to transfer any hard constraint on $u(k)$ to its control points and to reformulate the constrained minimization of the cost functional with respect to $u(k)$ as a WCRLS estimation problem with only constraints on the unknowns (the control points defining the admissible B-spline function $u(k)$). The WCRLS problem can be efficiently solved using interior point methods (Lobo, Vandenberghe, Boyd, & Lébret, 1998). Finally, as shown in the theorem of Section 5, Property 2 allows us to rigorously prove both stability and feasibility of the proposed control law without any further assumption. This is a very important difference with respect to (Wang, 2004) where terminal state constraints are imposed and recursive feasibility is "a priori" assumed to hold.
With reference to this last point we stress that although, stability and feasibility are recognized to be fundamental issues of MPC approach, see e.g. (Kouvaritakis & Cannon, 2016; Lofberg, 2012; Rossiter, 2004) and references therein, most applications of MPC to supply chain do not rigorously face these topics. This paper fills this gap.

The paper is organized in the following way. Some mathematical preliminaries on B-splines and the Robust Least Squares problem are recalled in Section 2. The system model is described in Section 3. The RMPC problem is formally stated in Section 4 and solved in Section 5, where it is reformulated as a WCRLS estimation problem. Numerical results and concluding remarks are reported in Sections 6 and 7 respectively.

2. Mathematical background

A scalar B-spline curve is defined as a linear combination of B-splines basis functions and control points:

$$s(t) = \sum_{i=1}^{\ell} c_i B_{i,d}(t), \qquad t \in [\hat{t}_1, \hat{t}_{\ell+d+1}] \subseteq R, \tag{1}$$

where the c_i's are real numbers representing the control points of $s(t)$, the integer d is the degree of the B-spline, the $(\hat{t}_i)_{i=1}^{\ell+d+1}$ are the non decreasing knot points and the $B_{i,d}(t)$ are the uniformly bounded B-spline basis functions which can be computed by the Cox-de Boor recursion formula

$$B_{i,d}(t) = \frac{t-\hat{t}_i}{\hat{t}_{i+d}-\hat{t}_i} B_{i,d-1}(t) + \frac{\hat{t}_{i+1+d}-t}{\hat{t}_{i+1+d}-\hat{t}_{i+1}} B_{i+1,d-1}(t), \;\; d \geq 1, \tag{2}$$

with $B_{i,0}(t) = 1$ if $\hat{t}_i \leq t < \hat{t}_{i+1}$, otherwise 0.
In (2) possible division by zero are resolved by the convention that "anything divided by zero is zero".
An equivalent representation of $s(t)$ in (1) is

$$s(t) = \mathbf{B}_d(t)\mathbf{c}, \qquad t \in [\hat{t}_1, \hat{t}_{\ell+d+1}] \subseteq R, \tag{3}$$

where $\mathbf{c} \stackrel{\triangle}{=} [c_1, \cdots, c_\ell]^T$ and $\mathbf{B}_d(t) \stackrel{\triangle}{=} [B_{1,d}(t), \cdots, B_{\ell,d}(t)]$.

Convex hull property. Any value assumed by $s(t)$, $\forall t \in [\hat{t}_j, \hat{t}_{j+1}]$, $j > d$, lies in the convex hull of its $d+1$ control points $c_{j-d}, \cdots, c_j$. △

Smoothness property. Suppose that $\hat{t}_i < \hat{t}_{i+1} = \cdots = \hat{t}_{i+m} < \hat{t}_{i+m+1}$, with $1 \leq m \leq d+1$ then the B-spline function $s(t)$ has continuous derivative up to order $d-m$ at knot $\hat{t}_{i+1}$. This property implies that the spline smoothness can be changed using multiple knot points. It is common choice to set $m = d+1$ multiple knot points for the initial and the last knot points and to evenly distribute the other ones. In this way (1) assumes the first and the final control points as initial and final values. △

Remark 1. From (3) it is apparent that, once the degree d and the knot points $\hat{t}_i$ have been fixed, the scalar B spline function $s(t)$, $t \in [\hat{t}_1, \hat{t}_{\ell+d+1}]$, is completely determined by the corresponding vector $\mathbf{c}$ of ℓ control points. △

2.1. The robust least squares problem (Lobo et al., 1998)

Given an overdetermined set of linear equations $Df \approx g$, with $D \in \mathbb{R}^{r\times m}$, $g \in \mathbb{R}^r$, subject to unknown but bounded errors: $\|\delta D\|_s \leq \beta$ and $\|\delta g\|_s \leq \xi$, the robust least squares estimate $\hat{f} \in \mathbb{R}^m$ is the value of f minimizing

$$\min_f \max_{\|\delta D\|_s \leq \beta,\, \|\delta g\|_s \leq \xi} \|(D+\delta D)f - (g+\delta g)\|, \tag{4}$$

where $\|\cdot\|_s$ denotes the spectral norm.
Problem (4) is equivalent to minimizing a sum of Euclidean norms

$$\min_f \; \|D\,f - g\| + \beta\|f\| + \xi \tag{5}$$

Possible constraints on f of the kind

$$\underline{f} \leq f \leq \bar{f} \tag{6}$$

can be taken into account by imposing all the scalar linear inequalities deriving from the above vector constraint.

3. The system model

We consider a single echelon supply chain where the operations of stock updating of a given product are periodically performed at equally separated time instants kT, $k \in Z^+$, T is the review period.

We assume: **A1)** each non null replenishment order placed at the supplier is realized with a time delay $T_d = nT$, where $n \in Z^+$. The goods arrive at the distribution centre new and deteriorate while kept in stock; **A2)** the perishability rate of the stocked goods inside each review period is $\alpha \in [\alpha^-, \alpha^+] \stackrel{\triangle}{=} \Lambda_\alpha \subset (0,1)$; **A3)** the operations of inventory replenishment and goods delivery are executed simultaneously at the beginning of each review period; **A4)** The demand is a nonnegative uniformly bounded function $w(k), k \in Z^+$. More specifically we assume that at any time instant k, and limitedly to an M-steps prediction horizon $P_k \stackrel{\triangle}{=} [k+1, k+M]$, the unknown future customer demand $w(k+\ell)$, $\ell = 1, \cdots, M$, $k \in Z^+$ fluctuates within a compact set W_k limited below and above by two known boundary trajectories: $w^-(k+\ell)$ and $w^+(k+\ell)$, $\ell = 1, \cdots, M$. The minimum value of $w^-(k+\ell)$ and the maximum value of $w^+(k+\ell)$, $\ell = 1, \cdots, M$, are denoted by w_k^- and w_k^+ respectively.

Figure 1 shows a typical example of a customer demand over a fixed P_k.

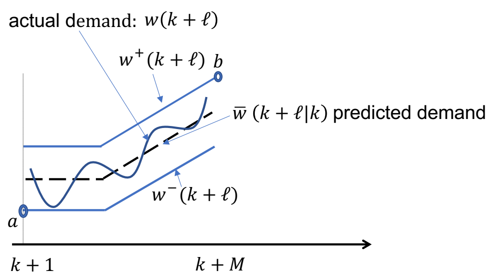

Figure 1. Example of a set W_k with known time varying boundaries trajectories: $w^-(k+\ell)$ and $w^+(k+\ell)$, $\ell = 1, \cdots, M$. The solid and dashed trajectories are the actual and predicted customer demand respectively. Points a and b denote w_k^- and w_k^+, respectively.

The above considerations imply that the stock level dynamics is described by the following uncertain equation

$$y(k+1) = \rho(y(k) + u(k-n) - h(k)) \tag{7}$$

where:

- $y(k)$ is the on hand stock level, i.e. the amount of goods left in stock after satisfying the demand at the beginning of the $k-1$ review period; $u(k-n)$ is the replenishment order placed at time $k-n$ and realized at time k. The sum $y(k)+u(k-n) \stackrel{\triangle}{=} y_1(k)$ represents the effective amount of goods available for sale at the beginning of k-th review period,
- $h(k)$ is the fulfilled customer demand and is given by

$$h(k) \stackrel{\triangle}{=} \min\{w(k), y_1(k)\} \tag{8}$$

- $\rho = 1-\alpha \in [\rho^-, \rho^+] \stackrel{\triangle}{=} \Lambda_\rho \subset (0,1)$ is the decay factor.

4. Problem setup

With reference to the uncertain supply chain model described in Section 3, the control problem we consider is to define an optimal replenishment order policy $u(k)$ conciliating the three following conflicting Control Requirements: CR1) the satisfied customer demand should be maximized, CR2) the warehouse storage should be minimized, CR3) the replenishment order policy $u(k)$ should be as smooth as possible.

The antagonism of CR1, CR2 and CR3 calls for an optimum criterion. Owing to the presence of uncertainties, we formulate this control problem in the framework of the RMPC. This requires to repeatedly solve a Min-Max Constrained Optimization Problem (MMCOP) over a future N steps control horizon $H_k \stackrel{\triangle}{=} [k, k+N-1]$, (for some $N \leq M$), and, according to the receding horizon control, to only apply the first sample of the computed optimal control sequence $[u(k), \cdots, u(k+N-1)], k \in Z^+$. The min-max formulation of the optimization problem allows us to minimize at each k the worst case of the cost functional which is computed as the maximum with respect to all the possible values of $\rho = 1-\alpha$.

The counterpart of this powerful approach is the numerical complexity of the algorithm (Scokaert & Mayne, 1998). As explained in Section 5, this drawback is drastically reduced through a WCRLS formulation of the MMCOP problem.

On the basis of CR1, CR2 and CR3, the MMCOP is formally defined as follows:

$$\min_{[u(k),\cdots,u(k+N-1)]} \max_{\rho\in[\rho^-,\rho^+]} J_k, \tag{9}$$

subject to: $\quad u_k^- \leq u(k+i) \leq u_k^+, \;\; i = 0, \cdots N-1, \;\; k \in Z^+, \quad (10)$

where:

$$J_k = \sum_{i=1}^{N} e^T(k+n+i|k) q_i e(k+n+i|k) + \sum_{i=1}^{N-1} \lambda_i \Delta u^2(k+i) \tag{11}$$

$$e(k+n+i|k) \stackrel{\triangle}{=} w^+(k+n+i) - y(k+n+i|k), \tag{12}$$

$$\Delta u(k+i) \stackrel{\triangle}{=} u(k+i) - u(k+i-1) \tag{13}$$

$$y(k+n+i|k) = \rho^{n+i}y(k) + \sum_{\ell=0}^{n-1}\rho^{n+i-\ell}u(k+\ell-n) + \sum_{\ell=0}^{i-1}\rho^{i-\ell}u(k+\ell|k) - \sum_{\ell=0}^{n+i-1}\rho^{n+i-\ell}h(k+\ell|k). \tag{14}$$

Remark 2. Some considerations on the cost functional J_k are now in order.

1 Note that by (12), the number M of future steps over which $w^+(k+\ell)$, $\ell = 1, \cdots, M$ must be known is inferiorly limited by $M = N + n$.

2 In (12), the tracking error has been defined with respect to a desired inventory level given by $w^+(k+n+i)$, $i = 1, \cdots, N$. Taking into account CR1, CR2, and the interval type uncertainty on $w(k+\ell), \ell = 1, \cdots, M$, this is the most appropriate choice: keeping the actual inventory level as near as possible to the possible maximum level of the customer demand maximizes the amount of fulfilled demand over each shifted prediction horizon $[k+n+1, k+n+N] \subseteq P_k$ and prevents unnecessarily larger stock levels.

3 the predicted stock level $y(k+n+i|k)$ in (14) is affected both by the uncertainties on ρ and $h(k+\ell)$. How to minimize the maximum effect of these uncertainties are explained at point 6 of this remark and in Section 5 respectively.

4 the term $\sum_{i=1}^{N-1}\lambda_i \Delta u^2(k+i)$ has been introduced to take into account CR3: penalizing large deviations on the control variables smoothes the control effort, thus reducing the unvoidable costs related to the order quantity changes.

5 The terms q_i, $i = 1, \cdots, N$, and λ_i, $i = 1, \cdots, N-1$, are positive coefficients introduced to progressively decrease the weight of future predictions.

6 the future values of $h(k+\ell|k)$ in (14) are estimated conforming to (8) assuming: **A5)** a predicted demand $\bar{w}(k+\ell|k)$, coinciding with the middle trajectory between $w^-(k+\ell)$ an $w^+(k+\ell)$, $\ell = 1, \cdots, n+i-1$ (see dashed line in Fig. 1), **A6)** $y(k+n+i) + u(k+i) \geq w(k+n+i)$, $i = 0, \cdots, N-1$.
A5) minimizes the maximum ℓ_2 norm of the approximation error between the true and predicted demand over each $[k+1, k+n+i-1]$, **A6)** is justified because the control sequence minimizes the maximum weighted ℓ_2 norm of the distance between the on-hand stock and the maximum customer demand.
As a consequence of **A5,A6)** and (8), the term $h(k+\ell|k)$ in (14) is replaced by $w(k)$ for $\ell = 0$ and by $\bar{h}(k+\ell|k) + \delta h(k+\ell|k)$ for $\ell \neq 0$, where $\bar{h}(k+\ell|k) = \bar{w}(k+\ell|k)$ and $\delta h(k+\ell|k)$ is the approximation error with minimum maximum l_2 norm over each H_k.

4.1. Determining the hard constraints on the control effort

The hard constraints on $u(k+i)$ imposed by (10) are determined on the basis of CR1 and CR2, taking into account the opportunity of limiting the amplitude of the interval $[u_k^-, u_k^+]$ to reduce the bullwhip effect (Moussaoui, Abbou & Loiseau, 2017). More precisely we search for an interval $[u_k^-, u_k^+]$ whose amplitude is the smallest one among all those that guarantee a fully acceptable degree of satisfaction of any possible customer demand fulfilling the assumptions **A1)-A4)** of Section 3.

Owing to the uncertainty on the future customer demand and on the perishability factor, we estimate u_k^- and u_k^+ with reference to two possible, limit situations compatible with the considered supply chain dynamics.

Consider the plant equations (7)-(8) and the following scenario:

- $h(k+n+i) = w(k+n+i)$, $i = 0, \cdots, N-1$, according to Point 6 of Remark 2;
- $w(k+n+i)$, $i = 0, \cdots, N-1$, is a constant signal with value $\tilde{w}_k \in [w_k^-, w_k^+]$. The two mentioned limit scenarios are $\tilde{w}_k = w_k^-$ and $\tilde{w}_k = w_k^+$;
- each control horizon $H_k = [k, k+N-1]$ is long enough to allow $y(k+n+i), i = 1, \cdots, N$, to practically attain the steady-state value $\tilde{y}_k$ under the forcing action of a constant $u(k+i) = \tilde{u}_k, i = 0, \cdots, N-1$.

Note that the existence of an output steady-state response is assured by the asymptotic stability of (7) (consequence of $\rho < 1$) and it is practically attained for a sufficiently large N such that ρ^N is significantly smaller than $\rho^0 = 1$. The problem we now consider is: for a given $\tilde{w}_k \in [w_k^-, w_k^+]$ it is required to find the corresponding constant control input $\tilde{u}_k$ over each H_k, such that $\tilde{y}_k \geq \tilde{w}_k$, $\forall \rho \in [\rho^-, \rho^+]$.
Using classical z-transform methods and applying the final value theorem (Kuo, 1980) we have

$$\tilde{y}_k = [W_{u,y}(z)]_{z=1}\tilde{u}_k - [W_{w,y}(z)]_{z=1}\tilde{w}_k, \tag{15}$$

where $W_{u,y}(z) = \frac{\rho}{z^n(z-\rho)}$ is the transfer function between the $\mathcal{Z}$ transforms of $u(k-n)$ and $y(k+1)$, $k \in Z^+$, and $W_{w,y}(z) = \frac{\rho}{(z-\rho)}$ is the transfer function between the $\mathcal{Z}$ transforms of $h(k) = w(k)$ and $y(k+1)$, $k \in Z^+$.
If ρ were exactly known, then, choosing $\tilde{u}_k = \frac{\tilde{w}_k}{\rho}$, equation (15) would readily imply $\tilde{y}_k = \tilde{w}_k, \forall \tilde{w}_k \in [w_k^-, w_k^+]$. As ρ is uncertain, the minimum $\tilde{u}_k$ guaranteeing $\tilde{y}_k \geq \tilde{w}_k$, $\forall \rho \in [\rho^-, \rho^+]$ is $u_k = \frac{\tilde{w}_k}{\rho^-}$.
In conclusion, over each H_k we choose u_k^- according to the limit scenario 1: $\tilde{w}_k = w_k^-$ and u_k^+ according to the limit scenario 2: $\tilde{w}_k = w_k^+$, obtaining

$$u_k^- \triangleq \frac{w_k^-}{\rho^-} \leq u(k+i) \leq \frac{w_k^+}{\rho^-} \triangleq u_k^+, \quad k \in Z^+, i = 0, \cdots, N-1. \tag{16}$$

5. Robust estimation of the optimal control policy

In this section we reformulate the MMCOP as a WCRLS estimation problem. The purpose is to drastically reduce the numerical complexity of the algorithm solving the MMCOP.

For any fixed k, the functional (9) is minimized assuming that the control sequence $[u(k), \cdots, u(k+N-1)]$, is given by the sampled version (with sampling period coinciding with the review period T) of a B-spline function. According to (3) one has

$$u(j) \triangleq \mathbf{B}_d(j)\mathbf{c}_k, \quad j = k, k+1, \cdots, k+N-1, \tag{17}$$

and the parameter vector $\mathbf{c}_k \triangleq [\mathbf{c}_{k,1}, \cdots, \mathbf{c}_{k,\ell}]^T$ defining $u(j)$ is computed as the solution of the WCRLS estimation problem defined beneath.
As $\rho \in [\rho^- \rho^+]$, an equivalent representation of ρ is

$$\rho = \bar{\rho} + \delta\rho, \qquad \bar{\rho} \triangleq (\rho^- + \rho^+)/2 \tag{18}$$

where $\bar{\rho}$ is the nominal value and $\delta\rho$ is the perturbation with respect to $\bar{\rho}$ satisfying $|\delta\rho| \leq (\rho^+ - \rho^-)/2$.
From (18) it follows that

$$\rho^k = (\bar{\rho} + \delta\rho)^k = \bar{\rho}^k + \Delta\rho_k \tag{19}$$

where $\Delta\rho_k \stackrel{\triangle}{=} (\bar{\rho} + \delta\rho)^k - \bar{\rho}^k$ is the sum of all terms containing $\delta\rho$ in the explicit expression of $(\bar{\rho} + \delta\rho)^k$. Exploiting (19) one has that the term $\rho^{n+1}y(k)$ of (14) can be rewritten as

$$\rho^{n+1}y(k) = (\bar{\rho}^{n+1} + \Delta\rho_{n+1})y(k) \tag{20}$$

Analogously, for the remaining terms of (14), one has

$$\sum_{\ell=0}^{n-1} \rho^{n+i-\ell}u(k+\ell-n) = \sum_{\ell=0}^{n-1}(\bar{\rho}^{n+i-\ell} + \Delta\rho_{n+i-\ell})u(k+\ell-n) \tag{21}$$

$$\sum_{\ell=0}^{i-1} \rho^{i-\ell}u(k+\ell) = \sum_{\ell=0}^{i-1}(\bar{\rho}^{i-\ell} + \Delta\rho_{i-\ell})\mathbf{B}_d(k+\ell)\mathbf{c}_k \tag{22}$$

and

$$\sum_{\ell=0}^{n+i-1} \rho^{n+i-\ell}h(k+\ell|k) = \sum_{\ell=0}^{n+i-1} (\bar{\rho}^{n+i-\ell} + \Delta\rho_{n+i-\ell})h(k+\ell|k)$$

By (20)-(23), an equivalent representation of the predicted tracking error given by (12) is

$$e(k+n+i|k) = (b_{k,i} + \delta b_{k,i}) - (D_{k,i} + \delta D_{k,i})\mathbf{c}_k$$

where

$$b_{k,i} \stackrel{\triangle}{=} w^+(k+n+i) - \bar{\rho}^{n+1}y(k) - \sum_{\ell=0}^{n-1} \bar{\rho}^{n+i-\ell}u(k+\ell-n) + \sum_{\ell=0}^{n+i-1} \bar{\rho}^{n+i-\ell}\bar{h}(k+\ell|k) \tag{23}$$

$$\begin{aligned}\delta b_{k,i} \stackrel{\triangle}{=} & -\Delta\rho_{n-1}y(k) - \sum_{\ell=0}^{n-1} \Delta\rho_{n+i-\ell}u(k+\ell-n) + \\ & \sum_{\ell=0}^{n+i-1} \bar{\rho}^{n+i-\ell}\delta h(k+\ell|k) + \sum_{\ell=0}^{n+i-1} \Delta\rho_{n+i-\ell}h(k+\ell|k)\end{aligned} \tag{24}$$

$$D_{k,i} \stackrel{\triangle}{=} \sum_{\ell=0}^{i-1} \bar{\rho}^{i-\ell} \mathbf{B}_d(k+\ell) \tag{25}$$

$$\delta D_{k,i} \stackrel{\triangle}{=} \sum_{\ell=0}^{i-1} \Delta\rho_{i-\ell} \mathbf{B}_d(k+\ell) \tag{26}$$

Equations (23)-(24) have been obtained expressing $h(k+\ell|k)$ as $h(k+\ell|k) = \bar{h}(k+\ell|k) + \delta h(k+\ell|k)$. For $\ell = 0$ one has $\bar{h}(k|k) = w(k)$ and $\delta h(k|k) = 0$. According to point 6 of Remark 2, $\bar{h}(k+\ell|k) = \bar{w}(k+\ell|k)$ is the nominal term and $\delta h(k+\ell|k)$ is the corresponding approximation error. The way $\bar{w}(k+\ell|k)$ is defined implies that $\delta h(k+\ell|k)$ has the minimum maximum l_2 norm over each H_k.
Similarly, the term $\Delta u(k+i)$ in the functional (9) can be rewritten as $\Delta u(k+i) = b_{u_{k,i}} - D_{u_{k,i}}\mathbf{c}_k$ where $b_{u_{k,i}} = 0$ and $D_{u_{k,i}} = -(\mathbf{B}_d(k+i) - \mathbf{B}_d(k))$.
Define the following vectors and matrices

$$\underline{e}_k = \begin{bmatrix} q_1^{1/2} e(k+n+1|k) \\ \vdots \\ q_{N-1}^{1/2} e(k+n+N-1|k) \\ \lambda_1^{1/2} \Delta u(k+1) \\ \vdots \\ \lambda_{N-1}^{1/2} \Delta u(k+N-1) \end{bmatrix}, \underline{D}_k = \begin{bmatrix} q_1^{1/2} D_{k,1} \\ \vdots \\ q_{N-1}^{1/2} D_{k,N-1} \\ \lambda_1^{1/2} D_{u_{k,1}} \\ \vdots \\ \lambda_{N-1}^{1/2} D_{u_{k,N-1}} \end{bmatrix}$$

$$\underline{b}_k = \begin{bmatrix} q_1^{1/2} b_{k,1} \\ \vdots \\ q_{N-1}^{1/2} b_{k,N-1} \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \underline{\delta b}_k = \begin{bmatrix} q_1^{1/2} \delta b_{k,1} \\ \vdots \\ q_{N-1}^{1/2} \delta b_{k,N-1} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\underline{\delta D}_k = \begin{bmatrix} q_i^{1/2} \delta D_{k,1} \\ \vdots \\ q_{N-1}^{1/2} \delta D_{k,N-1} \\ \lambda_1^{1/2} \delta D_{u_{k,1}} \\ \vdots \\ \lambda_{N-1}^{1/2} \delta D_{u_{k,N-1}} \end{bmatrix} \tag{27}$$

Exploiting the above defined vectors and matrices, allows us to reformulate the constrained min-max optimization problem (9)-(11) as the following WCRLS estimation problem:

$$\min_{\mathbf{c}_k} \max_{\|\delta \underline{D}_k\|_s \le \beta_k \, \|\delta \underline{b}_k)\|_s \le \xi_k} J_k \tag{28}$$

where

$$J_k = \|(\underline{b}_k + \delta \underline{b}_k) - (\underline{D}_k + \delta \underline{D}_k)\mathbf{c}_k\|^2 \tag{29}$$

$$\text{subject to } \; u_k^- \le \mathbf{c}_{k,i} \le u_k^+, i = 1, \cdots, \ell. \tag{30}$$

Constraints (30) derive from (17) and the convex hull property of B splines.

It is seen that (28)-(30) define a problem of the kind (4)-(6). Hence, according to Section 2.1, at any k the WCRLS estimation problem (28)-(30) can be reformulated as

$$\min_{\mathbf{c}_k} \|\underline{b}_k - \underline{D}_k \, \mathbf{c}_k\| + \beta_k \|\mathbf{c}_k\| + \xi_k \tag{31}$$

where the components of $\mathbf{c}_k$ must satisfy (30).

Remark 3. As for the numerical calculation of β_k and ξ_k, the following considerations hold:

1 As the term ξ_k of (31) is independent of $\mathbf{c}_k$, it cannot be minimized. Hence it can be removed from the objective function. This implies that in (31) only the upper bound β_k on $\|\delta \underline{D}_k\|_s$ needs to be determined at each k.
2 The way the B-spline basis functions are defined by the Cox de Boor formula (2) implies that $\mathbf{B}_d(\tau) = \mathbf{B}_d(\tau + N)$, $\forall \tau \in H_k$, $k \in Z^+$. Hence, by (26) and (27) one has that $\beta_k \stackrel{\triangle}{=} \beta$, $\forall k = 0, 1, \cdots$ and moreover β is easily determined putting $\rho = \rho^+$.

Feasibility and stability properties of the proposed control strategy can be now formally stated in the following theorem.
Theorem The control input $u(k)$ computed as the solution of the WCRLS estimation problem (28)-(30), guarantees the recursive feasibility and the internal asymptotic stability of the proposed RMPC control strategy.

Proof. Recursive feasibility is a consequence of parametrizing $u(k)$ as in (17), namely as the convex combination of the elements of the vector $\mathbf{c}_k$. This vector is computed as the solution of an optimization problem where the box-constraints (30) are imposed on the components of the same vector $\mathbf{c}_k$ with respect to which the functional J_k is minimized. Hence, constraints (30) (and therefore (10)) are surely feasible. Internal asymptotic stability is a direct consequence of the internal asymptotic stability of the supply chain model (implied by $\rho < 1$) and of the uniform boundedness of $u(k)$ resulting from (17) and (30). □

6. Numerical results

The performance of the RMPC strategy has been tested by a numerical simulation and compared with the Order Up To (OUT) control policy, see e.g. Sarimveis et al.

Table 1. Model parameters

time delay	*perishability factor*	*decay factor*	*review period*
$n = 5$	$\alpha \in [\alpha^-, \alpha^+] = [0.1, 0.14]$	$\rho = 1 - \alpha \in [\rho^-, \rho^+] = [0.86, 0.9]$	$T = 1$ day

Table 2. Parameters of the control algorithm

B-spline degree d	*number of control points* ℓ	*length of the prediction horizon* $N = M - n$	*scalar weights in (11)* q_i	λ_i
3	6	12	$e^{-0.1\,(i-1)}$	$e^{-1\,(i-1)}$

(2008) adapted to the plant model (7) and with the Dead-Time Compensation Mechanism (DTCM) proposed in (Ignaciuk , 2013).

The model parameters are reported in Table 1. The uncertainty interval $[\rho^-, \rho^+] = [0.86,\ 0.9]$ is centered on the value $\rho = 0.886$ (the same chosen in (Ignaciuk , 2013)). At each k, the future customer demand $w(k)$, is known to belong to a compact set W_k, with $M = 17$, like the example shown in figure 1. Figure 2 shows the actual customer demand over the whole simulation period (800 samples) enclosed in the contiguous positioning of all the W_k's. [1] The parameters of the control

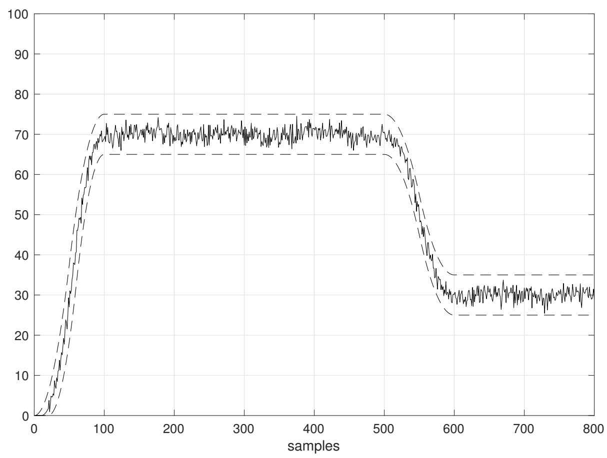

Figure 2. The actual customer demand $w(k)$ (solid line), the upper $w^+(k)$ and the lower $w^-(k)$ boundaries (dashed lines)

algorithm are reported in Table 2. The value $N = 12$ has been chosen to guarantee that, even for $\rho = \rho^+ = 0.9$, a significant amount of transient modes is included in the prediction horizon ($0.9^{12} = 0.28$) (Rossiter, 2004). According to point 2 of Remark 3, the upper bound $\beta = 0.6559$ has been found. The simulation has been performed choosing $\rho = 0.885$ and a desired inventory level $\tilde{y}(k) = w^+(k)$, where $w^+(k)$ is the upper dashed line of figure 2. The simulation has been stopped at time $k = 800$. The generated orders $u(k)$ are shown in figure 3. This figure shows the actual control effort (solid line) and the constraints curves computed as in (16). The effective amount of goods $y_1(k)$, available for sale at the beginning of each k-th review period, is reported in figure 4. This figure shows that $y_1(k) > w(k)$, $\forall k \in Z^+$, hence, as shown in figure 5, the customer demand is always satisfied i.e. $h(k) \equiv w(k)$.

Figure 6 compares the desired inventory level $\tilde{y}(k) = w^+(k)$ with the amount of goods

[1]The customer demand has been generated as the sum of bounded white noise with two S-shaped curve membership functions obtained through the smf function of Matlab.

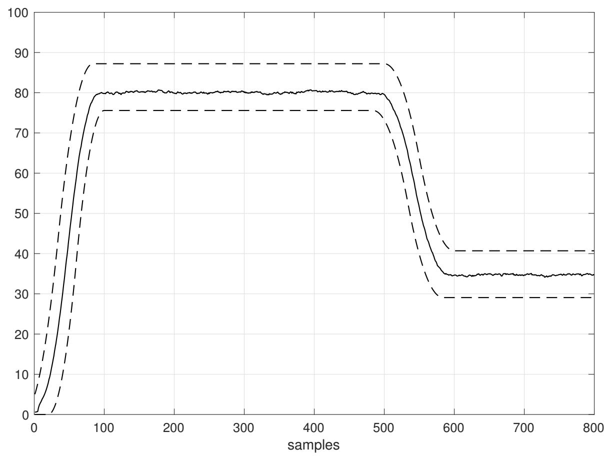

Figure 3. (RMPC with $\alpha \in [0.1, 0.14]$) The generated order $u(k)$ (solid line) and the constraints u_k^- and u_k^+ (dashed lines).

left in stock $y(k)$ after satisfying the demand for the $k-1$ review period.

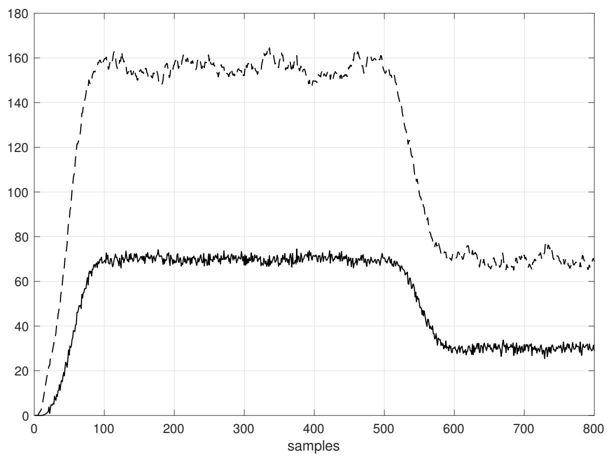

Figure 4. (RMPC with $\alpha \in [0.1, 0.14]$) The effective amount of goods $y_1(k)$ (dashed line) available for sale and the actual demand $w(k)$ (solid line).

Assume now to shift the interval $[\alpha^-, \alpha^+]$ from $[0.1, 0.14]$ to $[0.2, 0.24]$ (the width of the intervals is the same) and to run the simulation with $\rho = 0.785$, while all the other parameters are kept identical. The value $\rho = 0.785$ corresponds to a deviation of 0.005 from the central value of the range (similarly to $\rho = 0.885$ used in the scenario $[\alpha^-, \alpha^+] = [0.1, 0.14]$). The performed simulations give fairly similar results in terms of amount of goods available for sale and of amount of goods left in stock (compare figures 4 and 6 with 7 and 8 respectively). According to figure 7, also in this case, the customer demand is fully satisfied. The substantial difference between the two scenarios is in the replenishment policy (compare figure 3 with 9). The greater the degree of perishability, the greater the range of variability of $u(\cdot)$ (as indicated in (16)) to allow the replenishment policy to compensate for a faster decaying goods.

With reference to the scenario $\alpha \in [0.1, 0.14]$, the performance of the proposed

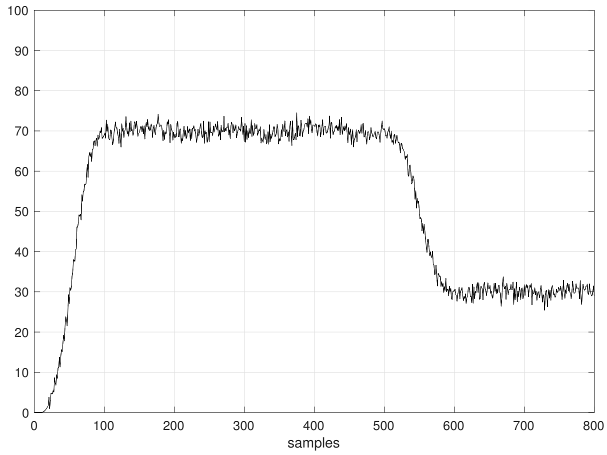

Figure 5. (RMPC with $\alpha \in [0.1, 0.14]$) The fulfilled demand $h(k) = \min\{w(k), y_1(k)\}$ coinciding with the actual demand $w(k)$.

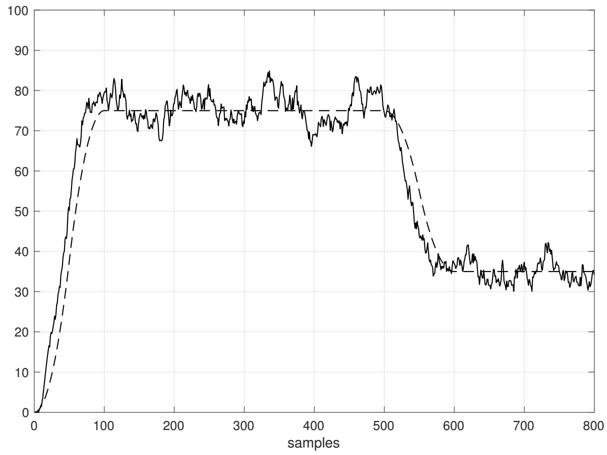

Figure 6. (RMPC with $\alpha \in [0.1, 0.14]$) The on hand stock level $y(k)$ (solid line) and the desired inventory level $\tilde{y}(k)$ (dashed line)

RMPC has been compared with the following version of the OUT replenishment policy adapted to take into account the presence of perishable goods and of a time delay in the plant model (7)

$$u(k) = (\tilde{y} - \rho^{n+1} y(k) - \sum_{\ell=2}^{n+1} \rho^{\ell} u(k - \ell + 1))/\rho \tag{32}$$

where the fixed desired inventory level $\tilde{y}$ has been computed as

$$\tilde{y} = w^{+} \sum_{j=0}^{n} \rho^{j} \tag{33}$$

and w^{+} is the maximum value of the customer demand over all the simulation period to guarantee a strictly positive on-hand stock level $y(k)$ for $k \geq n - 1$ (Ignaciuk , 2013). The OUT replenishment policy (32) has been applied to the customer demand

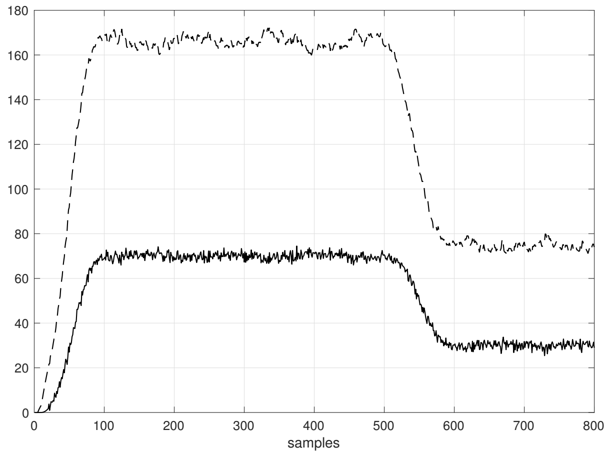

Figure 7. (RMPC with $\alpha \in [0.2, 0.24]$) The effective amount of goods $y_1(k)$ (dashed line) available for sale and the actual demand $w(k)$ (solid line)

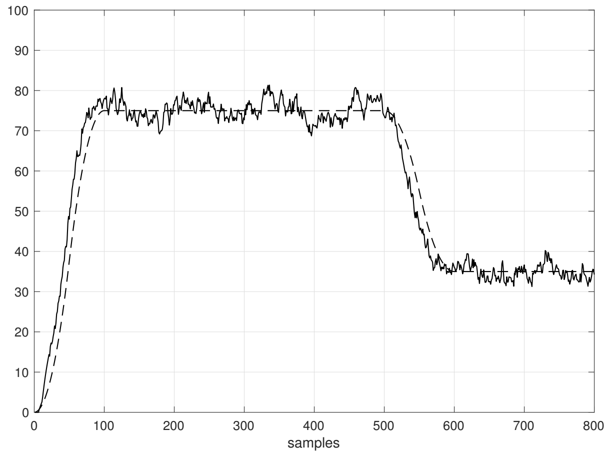

Figure 8. (RMPC with $\alpha \in [0.2, 0.24]$) The on hand stock level $y(k)$ (solid line) and the desired inventory level $\tilde{y}(k)$ (dashed line)

reported in figure 2 to which the value $w^+ = 75$ corresponds. Assuming $\rho = \bar{\rho} = 0.88$, condition (33) gives $\tilde{y} = 335$. The orders $u(k)$ generated with $\rho = \bar{\rho} = 0.88$ and the on hand stock level $y(k)$ (generated with $\rho = 0.885$) are reported in figures 10 and 11 respectively. Also the OUT replenishment policy guarantees a full demand satisfaction. The relative plot is identical to that shown in figure 5.

A second comparison has been performed with the DTCM proposed in (Ignaciuk , 2013) (eqns. (33), (34)). In the present case (where $n = 5$ and $\delta n = 0$) the DTCM has been applied choosing: $\rho = \bar{\rho} = 0.88$, $u_{\max} = d_{\max} = w^+ = 75$ (according to (45)) and $y_{ref} = u_{\max} \sum_{j=0}^{n} \bar{\rho}^j = 335$ (according to (46)). The orders $u(k)$ generated with $\rho = \bar{\rho} = 0.88$ and the on hand stock level $y(k)$ (generated with $\rho = 0.885$) are reported in figures 12 and 13 respectively. Also the DTCM guarantees a full demand satisfaction.

6.1. Discussion

A comparison between the reported simulations highlight the following:

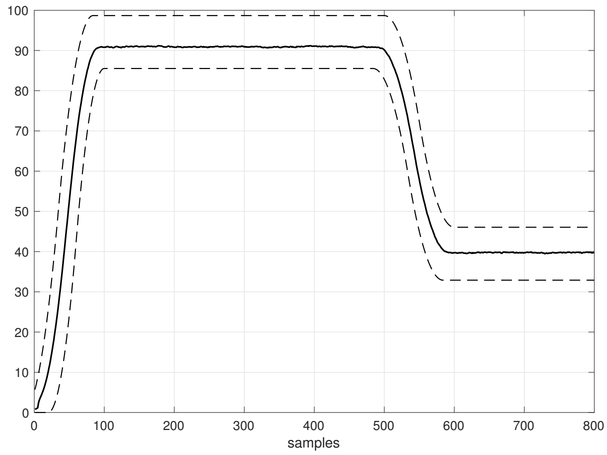

Figure 9. (RMPC with $\alpha \in [0.2, 0.24]$) The generated order $u(k)$ (solid line) and the constraints u_k^- and u_k^+ (dashed lines).

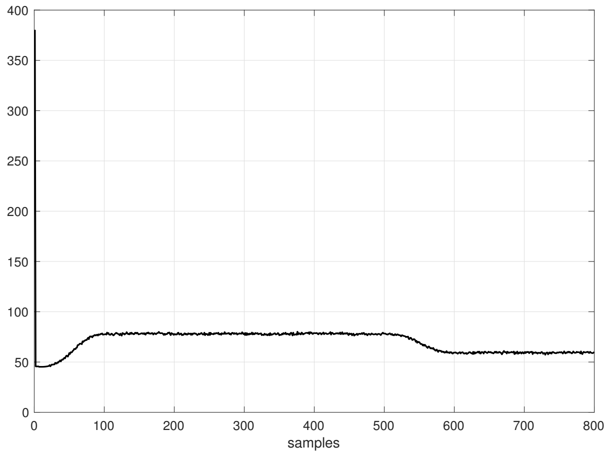

Figure 10. (OUT) The generated order $u(k)$

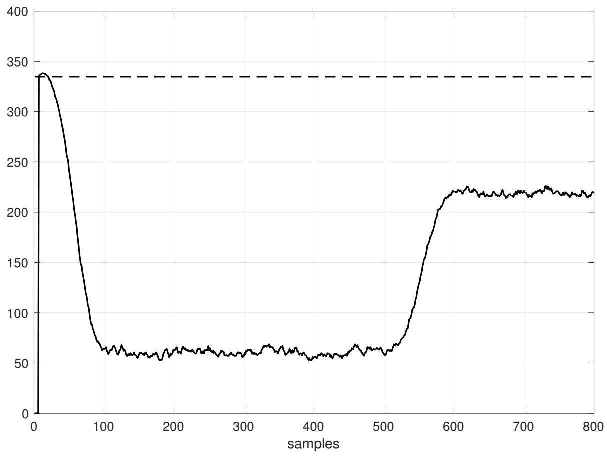

Figure 11. (OUT) The on hand stock level $y(k)$ (solid line) and the desired inventory level $\tilde{y}$ (dashed line).

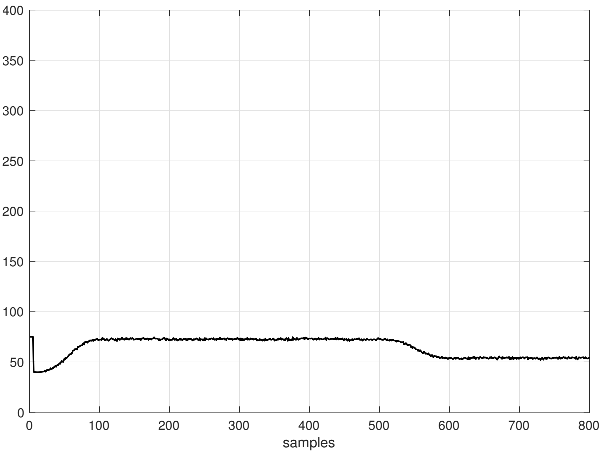

Figure 12. (DTCM) The generated order $u(k)$

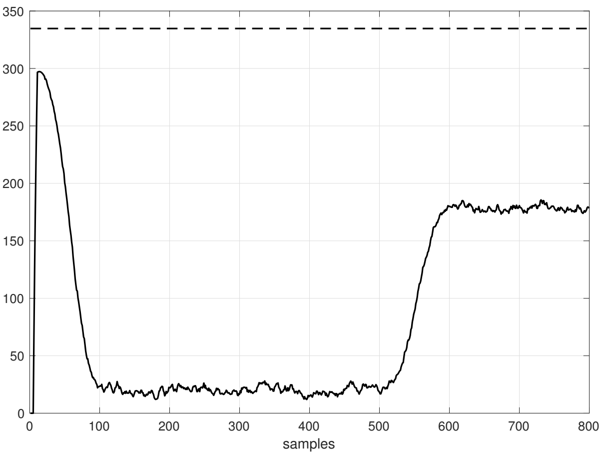

Figure 13. (DTCM) The on hand stock level $y(k)$ (solid line) and the desired constant reference level $\tilde{y}$ (dashed line).

- All the three methods fully satisfy the customer demand, but the proposed RMPC approach requires a very smaller warehouse occupancy with respect to OUT and DTCM. This is visually evidenced by figures 6, 11 and 13 and numerically quantified by the sum of stored goods at each kT, $k = 1, \cdots, 800$. See the entries of row 1 of Table 3. Row 2 of the same table shows the respective average stocks of goods in warehouse. The reduction of warehouse occupancy is a consequence of tracking a time varying inventory level which is adapted at any k on the basis of the current $w^+(k)$. On the contrary, both OUT and DTCM define a constant desired inventory level, which is "a priori" computed using a conservative formula requiring the "a priori" knowledge of the maximum value w^+ of the customer demand over an indefinitely long future time interval. Moreover, as w^+ is never exactly known, it is often over-estimated.
- Figures 14-16 and the entries of row 2 of Table 3 show that the RMPC policy provides a smoother control signal with respect to OUT and DTCM strategies.

This is a consequence of: 1) introducing the term $\sum_{i=1}^{N-1} \lambda_i \Delta u^2(k+i)$ in the cost functional (11), 2) expressing $u(k)$ as a linear combination of smooth functions like

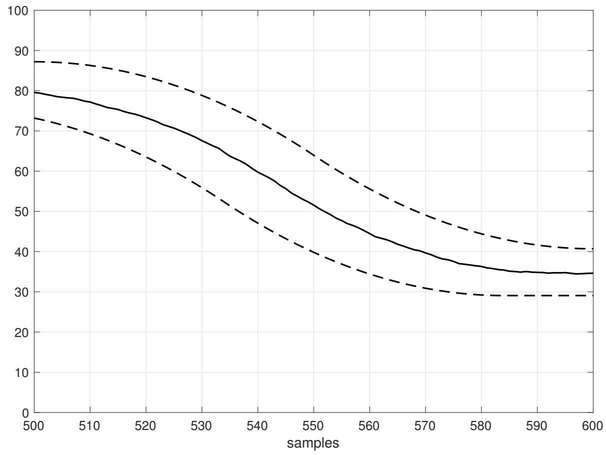

Figure 14. (RMPC) The zoomed generated order u(k) (solid line) between the two boundaries trajectories u_k^- and u_k^+ (dashed lines)

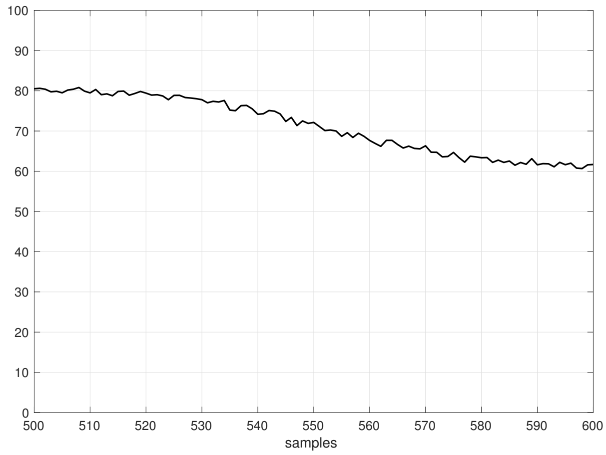

Figure 15. (OUT) The zoomed generated order u(k) .

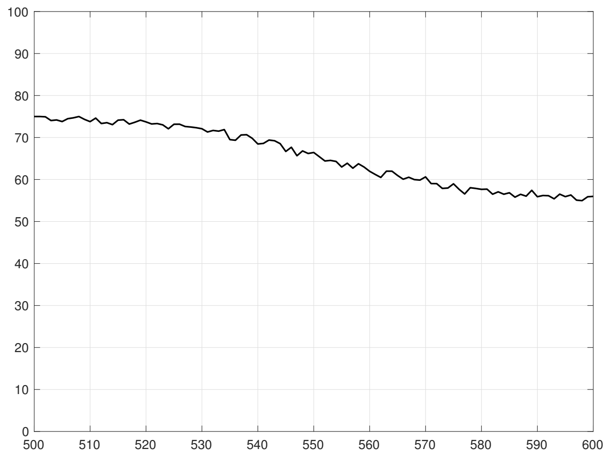

Figure 16. (DTCM) The zoomed generated order u(k)

Table 3. The total sum of the inventory stock $\sum_{k=1}^{800} y(k)$, the average stock of goods $\frac{\sum_{k=1}^{800} y(k)}{800}$ and the sum of the point-wise changes of the orders $\sum_{k=1}^{799}(u(k+1)-u(k))$.

	RMPC	OUT	DTCM
$\sum_{k=1}^{800} y(k)$	4.6908e4	1.1510e5	8.0696e4
$\frac{\sum_{k=1}^{800} y(k)}{800}$	58.635	143.875	100.870
$\sum_{k=1}^{799}(u(k+1)-u(k))$	187	478	943

polynomial B-splines.
The improved smoothness deriving from RMPC yields less order quantity changes with respect to OUT and DTCT. As evidenced in (Song, Li & Garcia, 2009), frequency and amplitude of control changes are elements for measuring the bullwhip effect.

7. Conclusions

The main novelties we have proposed in this paper are: 1) the supply chain dynamics is characterized by perishable goods with uncertain perishability factor, 2) the proposed RMPC approach provides a B-splines parametrization of the replenishment order. The assumption on the supply chain dynamics generalizes many existing modeling approaches. The B-splines parametrization allowed us to reformulate the conceptually and numerically demanding min-max optimization problem implied by the RMPC as a simpler WCRLS estimation problem. The method we propose also allowed us to define a time-varying inventory level conciliating the opposite control requirements CR1 and CR2. CR3 is addressed penalizing the difference between control moves and also parametrizing the control moves as polynomial B-Spline functions. A rigorous proof of feasibility and stability of the RMPC strategy has been also provided. The numerical test confirmed the validity of the approach: it is actually able to reduce the inventory level without affecting customer service quality and without incurring in an excessive control effort.

8. Data availability statement

The authors declare that the data supporting the findings of this study have been generated by simulation. Data are available from the corresponding author [V.O.] on request.

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