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Floating particles transport through the free surface vortex technique: A novel numerical study to assess the interaction among different scales of vortex structures

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14 Abstract: Fat, oil and grease (FOG) floating particles in the sump of sewage pumping stations will accumulate 15 together to form rigid layers, resulting in failure for pump device. To overcome this, the free surface vortex (FSV) 16 technique has been considered and applied to transport floating particles toward the submerged suction pump inlet. 17 This paper investigates the potential of vortices as a means of downward motion of FOG. The entrainment capacity 18 of FSV is investigated by numerical simulations using a coupled level-set and volume-of-fluid method. Two 19 coherent structures are decomposed by proper orthogonal decomposition: FSV represented by the first two orders 20 with high energy content and spiral vortex bands represented by low energy and high order models. The extracted 21 ridges of the finite-time Lyapunov exponent (FTLE) delineate different regions of the flow field and effectively 22 capture the evolution of Lagrangian coherent structures. The floating particles in the sump are first caught by the 23 dividing line formed by the FTLE ridges, mixed in the entrainment zone, and then merged into the vortex. The enstrophy production term dominates the development of vorticity. Subject to the influence of flow velocity 24 25 gradients, both radial and tangential vortices undergo a transition into axial vortices. This transformation enhances 26 the vortex's capacity to entrain particles within the vortex core area, leading to their rapid inward spiraling towards 27 the vortex center and eventual expulsion due to the vortex's entrainment effect.

Keywords: Free surface vortex; Lagrangian coherent structures; particle dynamics; orthogonal decomposition;
 sewage pumping station.

30 1 Introduction

31 Sewage pumping stations are a major component of sewage services, collecting sewage from urban pipelines 32 via sumps and lifting sewage to higher water levels to assist normal gravity flow in reaching its final destination, 33 a sewage treatment plant. Sewage pumping stations play a crucial role in ensuring urban sanitation and 34 environmental protection. In urban areas across most countries, these stations often encounter challenges 35 associated with the accumulation of floating fat and other debris, collectively referred to as "floating debris" in 36 their sumps. While large floating debris typically gets stacked and filtered out by trash racks before entering the 37 sump, smaller and more pliable floating debris like fat, oil, and grease (FOG) particles might accumulate above the pump intake. Over time, these FOG floating particles undergo a series of chemical and physical transformations 38 39 until it becomes a rigid layer that covers the entire sump surface.¹ The presence of such rigid layers within urban 40 drainage systems blocks drainage and leads to rainwater and sewage overflow, which can damage sewage system 41 and pose environmental and health risks.^{2, 3} As a result, the timely removal of this rigid layer from the sump is 42 crucial for both their correct operation and reliability as well. Nevertheless, this kind of intervention is not only 43 labor-intensive, hazardous, and unhygienic but also cost-consuming. Duinmeijer⁴ proposed the creation of FSV 44 within the sump to overcome the criticalities previously mentioned, which are a means of transport for floating 45 particles from the water surface down to the suction pipe. Since vortices have a strong suction capacity for 46 particles.5

47 To evaluate the transport capacity of FSV, it is necessary to understand the three-dimensional flow field information of the vortices. Based on Navier-Stokes governing equations, different vortex theoretical models have 48 49 been derived to characterize and describe the tangential velocity, the water surface profile, and other parameters such as the Rankine and Burgers vortex models.^{6,7} Starting from these models, the simplified vortex theoretical 50 51 models have been further improved and assessed through experimental data and then analyzed deeply with 52 computational fluid dynamics.⁸ It is all known that the presence of multiple phases and the associated mass 53 transport, including the particle movement and air entrainment, heighten the operational risks faced by pumping 54 stations. Scientists and technicians have focused their studies on establishing relationships between boundary 55 conditions, operational parameters, and the occurrence of FSV to provide a better prediction of such a physical 56 phenomenon. They also investigated the mechanisms underlying the formation of FSV, the rate of air entrainment, 57 and their impact on pump devices. In the investigation of the entrainment of floating particles by FSV, Naderi et al.9 and Duinmeijer et al.10 used high-speed digital cameras to visually track and calculate the trajectories, as well 58 59 as the three-dimensional velocity components, of particles within FSV. Notably, Duinmeijer and Clemens² observed that the motion trajectories of particles driven by FSV exhibit conspicuous chaotic behavior and are 60 61 highly sensitive to the initial conditions of these particles.

62 While most of the aforementioned studies have analyzed the motion of individual or multiple particles 63 propelled by FSV, there is a paucity of research delving into the motion of particle swarm and exploring the potential coupling effects among coherent structures induced by these FSVs. At present, the discussions of FSV 64 65 flow predominantly rely on the Eulerian perspective, which is effective but cannot provide the transport and mixing process of the coherent structures.¹¹⁻¹³ The Lagrangian system of flow structure identification method has gradually 66 developed. It views the flow field as a dynamic system comprising discrete particles and leverages disparities in 67 68 particle trajectories to identify Lagrangian coherent structures (LCS) within the flow field. Due to its inherently swarm-based nature, which transcends the characteristics of individual particle trajectories, LCS offer precise 69 70 material transport evaluation and provides powerful opportunities to predict, and even influence, large-scale flow characteristics and mixing events.^{14, 15} This paper aims to provide insights into the transport mechanism of FOG 71 72 particles and the dynamic characteristics of the flow structure, which will aid in making informed decisions 73 regarding operations and management, such as employing FSV for floating particles transport or undertaking 74 manual salvage operations.

75 While LCS have demonstrated their utility in various geophysical and biomedical applications, their 76 applicability within the context of FSV flows remains unexplored. In this paper, we employ the coupled level-set 77 and volume-of-fluid (CLSVOF) method in conjunction with large-eddy simulation (LES) methods to reproduce 78 the two-phase vortex flow phenomena in the sewage sump. Through proper orthogonal decomposition (POD), the 79 intricate interaction among different-scale vortex structures is unveiled. The entrainment process of particles under 80 the force of FSV and the LCS transport mechanism are further analyzed, which are the key to understanding the 81 hydrodynamic mixing characteristics in the FSV system. The main content of this paper has been structured as 82 follows: Section 2 presents the geometric parameters and the numerical methods; in Section 3, two analysis 83 methods, namely POD and the finite-time Lyapunov exponent (FTLE), are introduced; in Section 4, the analysis 84 of the spiral vortex band, vortex structures, and particle trajectories constitutes the primary research content; lastly, 85 Section 5 gives the main conclusions derived from the research.

86 2 Numerical model

87 2.1 Governing equations

Urban sewage pumping stations are composed of submerged-type pumps so that a free surface, which is the interface between water and air, is present above them. When dealing with more than two fluids, numerical simulations of water flow in pumps become a challenging task to deal with, especially when floating particles are grouped on the sump free surface.¹⁶ Therefore, LES combined with CLSVOF to accurately capture the water-air interface, particularly in regions where high curvature deformation occurs, and simulate the coherent structures within the sump. The continuity and momentum equations of LES combined with CLSVOF are the following:

$$\frac{\partial \tilde{u}_i}{\partial x_i} = 0, \tag{1}$$

95
$$\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \frac{\partial \tilde{u}_i}{\partial x_i} = -\frac{\partial \tilde{p}}{\rho \partial x_i} + v \frac{\partial^2 \tilde{u}_i}{\partial x_i \partial x_j} + \frac{\partial \tau_{ij}}{\partial x_i} + \frac{\sigma \kappa \delta_s n_i}{\rho} - g \delta_{i3}.$$
(2)

Here, *p* is the pressure; u_i stands for the fluids velocity vector; the tilde (~) denotes the filtered quantities in LES governing equations; σ is the surface tension coefficient; δ_{ij} is the Kronecker delta symbol (If i = j, $\delta_{ij} = 1$, otherwise $\delta_{ij} = 0$); κ is local mean curvature of the immiscible fluids interface; ρ and v are the fluids mixture density and dynamic viscosity, respectively; *g* represents the effect of gravity and is the acceleration of gravity; The subgridscale stress τ_{ij} resulting from filtering is calculated by the Boussinesq hypothesis:¹⁷

$$\tau_{ij} = 2\nu_{SGS}\tilde{S}_{ij} - \frac{1}{3}\tau_{kk}\delta_{ij}, \qquad (3)$$

102
$$\tilde{S}_{ij} = \frac{1}{2} \left(\frac{\partial \tilde{u}_i}{\partial x_i} + \frac{\partial \tilde{u}_j}{\partial x_i} \right), \tag{4}$$

103 where τ_{kk} is the isotropic part of τ_{ij} and v_{SGS} is the sub-grid eddy viscosity. The wall-modeled LES (WMLES) 104 method is adopted in the simulation. As opposed to the traditional LES method, WMLES does not need a grid 105 refinement within the boundary layer, thus saving computational efforts.^{18, 19} Currently, this method is being used 106 in numerical simulations of two-phase flow ²⁰. In the WMLES method, v_{SGS} is calculated by:

107 $v_{SGS} = \min\left[(0.41d_w)^2, (0.2\Delta)^2\right] \cdot \sqrt{2\tilde{S}_{ij}\tilde{S}_{ij}} \cdot \left\{1 - \exp\left[-(y^+ / 25)^3\right]\right\},\tag{5}$

where d_w is the normal distance from the gird node to the wall and y^+ is the dimensionless wall distance for a wallbounded flow. When calculating the filter size, a wall damping function is introduced in the WMLES method, written as equation (6). Thus, this method is well suited for anisotropic grids reducing the requirement for grids near the wall and improving calculation efficiency:

112

$$\Delta = \min(\max(0.15d_w, 0.15h_{\max}, h_{wn}), h_{\max}) .$$
(6)

Here, h_{max} is the longest edge of a hexahedral cell (for other cell types, this concept is extended accordingly). h_{wn} is the grid spacing in the wall-normal direction.

The CLSVOF method has been well applied in simulating vortex phenomena in the sump or pipe.^{21, 22} This approach offers an accurate representation of the air-water interface while maintaining mass conservation.²³ The CLSVOF method tracks the interface of immiscible fluids by establishing a convection equation, from which it derives a volume fraction function *F* and a signed distance ϕ . The signed distance ϕ determines ρ and v of the immiscible fluids in equation (2):

120
$$\phi(x,t) = \begin{cases} +|d| & x \in \text{water} \\ 0 & x \in \text{interface} , \\ -|d| & x \in \text{air} \end{cases}$$
(7)

$$\rho = \rho_w H(\phi) + \rho_a [1 - H(\phi)], \qquad (8)$$

122
$$v = v_w H(\phi) + v_a [1 - H(\phi)],$$
 (9)

where *d* is the distance from the interface. Positive and negative values of ϕ represent water and air phases respectively. $H(\phi)$ stands for the smoothed Heaviside function.^{24, 25} All velocities are filtered in the subsequent section, while the tilde (~) is omitted for simplicity.

126 2.2 Setup of the simulation

Large-scale water supply and drainage pumping stations commonly employ a horizontal inflow configuration. ²⁶ As shown in Fig. 1, this layout simplified an inflow system with a horizontal pipe as a water inlet,^{27, 28} where D= 0.4 m is the pipe mouth diameter. The submergence *hs* and the suspended height of the pipe mouth are 1.5D and 1.0D, respectively. To minimize the interference of domain inlet boundary on the flow near the pipe mouth, the maximum edge length for the sump is set to 12.0D. The coordinates origin is established at the pipe mouth center. The main flow direction coincides with the positive direction of the *x*-axis.





Fig.1. Sketch of the sump model in a drainage pumping station.

The calculation conditions settings are shown in Table 1, where Q is the discharge. $Fr = u_D / \sqrt{gD}$, 135 $Ds = Q/g^{0.5}hs^{2.5}$, $Re = u_D D/v$, and $We = \rho u_D D/\sigma$. Ds is the discharge-to-submergence ratio, quantifying the 136 137 relationship between Q and hs.²⁰ Taking Case M3 as an illustrative example, the corresponding boundary 138 conditions have been defined as follows: the upper section of the sump is defined as an opening surface with an 139 atmospheric pressure. The walls of the sump and intake pipe have been configured as non-slip walls. The pipeline 140 outlet employs a pressure outlet that adjusts in real-time with variations in the flow rate. When the discharge 141 overcomes the expected value, the outlet pressure increases, and conversely, it decreases to maintain a steady 142 discharge of $0.785 u_D D^2$ throughout the computation process. The inlet of the sump adopts the air-water two-phase flow inlet. Pure water inlet ($\phi > 0$) is below z = 1.5D with a pressure distribution conforming to the hydrostatic 143 pressure. Above z = 1.5D, identical settings equal to the upper section have been applied.^{29, 30} The LES simulation 144

employed a time step of $6.41 \times 10^{-3} D/u_D$, with up to 20 iterations per step. The governing equation discretization is detailed in our previous studies.^{20, 31} The simulations of FSV were executed utilizing FLUENT 18.0 software, requiring an extensive computational time of approximately 210,000 CPU hours to complete.

Table 1 Parameters for FSV flow simulation cases.						
Case Name	$Q/u_D D^2$	hs/D	Ds	Fr	Re	We
M1	0.392	1.50	0.21	0.75	5.88×10 ⁵	1.82×10^{4}
M2	0.785	1.75	0.29	1.49	1.18×10^{6}	4.86×10 ⁴
M3	0.785	1.50	0.43	1.49	1.18×10^{6}	4.86×10 ⁴
M4	0.785	1.25	0.67	1.49	1.18×10^{6}	4.86×10 ⁴
M5	1.375	1.50	0.75	2.62	2.06×10^{6}	1.49×10 ⁵

149 **2.3 Validation of numerical results**

150 ICEM has been used to mesh the entire computational domain, where the vortex generation region is refined. 151 Grid independence analysis has been performed using the widely accepted GCI criterion.^{32, 33} In FSV flow, vertical 152 vorticity ω_z is the important characteristic parameter and is chosen as the evaluation parameter for the grid 153 convergence analysis. Three sets of grid schemes, ranging from fine to coarse (G1 = 17,456,622; G2 = 8,645,974; 154 G3 = 4,224,656) are designed for grid error analyses. The convergence index GCI²¹_{fine} of ω_z has fallen below 3%, 155 signifying that the refinement of Grids G1 and G2 adequately meets the required convergence criterion.^{31, 34} Grid 156 G2 has subsequently been selected for the simulations due to its computational efficiency, as depicted in Fig. 2.



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Fig. 3 presents the comparison of the air-water interface between the experimental result from Möller *et al.*³⁵ and the simulation result. The representation of the free surface is achieved using F = 0.5. In general agreement with the results of experiment, numerical calculation effectively captures the overall air-water interface shape as it develops from the water surface to the pipe mouth. The upper part of FSV forms a funnel shape structure, with its central part resembling an elongated air tube, while the tail transitions into a pronounced hook-shaped pattern. The simulation employed a higher flow rate compared to the experimental setup, resulting in the formation of a more pronounced vortex and an increased diameter of the air tube.



167 Fig. 3. Comparison of free surfaces: (a) experimental picture from Möller *et al.*³⁵ and (b) F = 0.5 iso-surface of 168 Case M3.

169 Fig. 4(a) presents the comparison of tangential velocities distribution of FSV between the simulation, experiment, and the theoretical results. Burgers vortex model has been selected as the theoretical model, given its 170 well-verified accuracy in representing tangential velocity according to previous experiments.³⁶ The graph 171 172 demonstrates that the calculation result is consistent with Burgers theoretical model. Tangential velocity V_{θ} 173 increases approximately linearly for radius as $r/r_c < 1$, and then slowly decreases. r_c denotes the radius of the vortex 174 core with maximum V_{θ} . Considering the primary focus on particle transport and mixing mechanisms, it is imperative to compare the differences in particle trajectories between the experiment and simulation results, as 175 plotted in Fig. 4(b). It illustrates that the particle motion trajectories, which are obtained from the simulation results 176 177 closely correspond to the measured trajectories from the experiment, while some deviation exists due to the 178 different arrangement of the intake pipe in the experiment that is situated at the bottom, unlike in the simulation 179 case.





results.

- 183 **3** Coherent structure analysis
- 184 3.1 POD analysis

166

185 Proper orthogonal decomposition (POD) is a data processing technique commonly employed for analyzing 186 large volumes of data obtained from experiments or numerical simulations of unsteady flow fields. POD 187 decomposes these physical quantities into a series of orthogonal basis functions, referred to as "POD modes", that are optimal in a least-squares sense. Each mode is associated with eigenvalues that represent its energy content.
Due to the ability to establish correlations between coherent structures and the energy they encompass, POD has
been used to identify structures at different energy levels within turbulent flows. With respect to the motion of
particles within the flow field, various flow structures (coherence structures) serve as carriers to determine particle
trajectories.

193 Currently, the most widely used POD method, based on the snapshot method introduced by Sirovich³⁷, is a 194 mathematical improvement over direct POD that addresses the challenge posed by large spatial point datasets 195 where the spatial correlation matrix cannot be efficiently computed due to the sheer number of spatial points. The 196 snapshot POD method is particularly well-suited for Particle Image Velocimetry (PIV) experiments and numerical 197 simulations, where the count of spatial points *m* substantially exceeds that of sampling points *n*.

The central idea of the snapshot POD method is to express the pulsation of a certain physical quantity as the product of a few POD bases φ_j and time modal coefficients $a_j(t)$ through the orthogonal decomposition:

200
$$X' = \sum_{j} a_{j}(t)\varphi_{j}, \qquad (10)$$

$$X' = X - \overline{X} . \tag{11}$$

Here, $\mathbf{X} = \begin{bmatrix} \mathbf{x}(t_1) & \mathbf{x}(t_2) & \dots & \mathbf{x}(t_m) \end{bmatrix} \in \square^{n \times m}$ donates instantaneous physical field at *m* moments and $\overline{\mathbf{X}}$ is it's the average part. The eigenvector $\boldsymbol{\psi}$ and its corresponding eigenvalue λ of the constructed covariance matrix $\mathbf{X}^T \mathbf{X}$ can be solved:

205 $\boldsymbol{X}^{\prime T} \boldsymbol{X}^{\prime} \boldsymbol{\psi}_{i} = \lambda_{i} \boldsymbol{\psi}_{i}, \qquad (12)$

where the symbol T indicates matrix transposition. The mode of the snapshot POD method can be solved as:

 $\boldsymbol{\varphi}_{j} = \frac{1}{\sqrt{\lambda_{j}}} \boldsymbol{X}' \boldsymbol{\psi}_{j} , \qquad (13)$

208 and time modal coefficient can be written as:

207

209 $a_{j}(t) = \left\langle \boldsymbol{X}'(t), \boldsymbol{\varphi}_{j} \right\rangle.$ (14)

POD modes are organized in descending order based on their associated eigenvalues. The energy fraction of a specific POD mode, relative to the total resolved energy, is proportional to its corresponding eigenvalue.³⁸ Typically, the first *k*-order modes, whose energy is close to the overall kinetic energy of the flow field, are selected for analysis as follows:

214
$$\sum_{j=1}^{k} \lambda_j / \sum_{j=1}^{m} \lambda_j \approx 1.$$
 (15)

215 3.2 Lagrangian approach

216 This paper focuses on the dynamic process of FOG floating particles entrainment by vortices, a phenomenon 217 predominantly governed by vortex structures. Most of the vortex feature extraction approaches are based on the 218 Euler framework for analyzing vortices in unsteady flow fields, but these methods often lack a strict physical interpretation.¹⁴ In contrast, Lagrangian approaches exploit properties of fluid particle trajectories to identify 219 220 coherent structures.^{14, 39} On the contrary, Lagrangian approaches offer objectivity, incorporate flow history 221 information, and provide a clear physical interpretation. The finite-time Lyapunov exponent (FTLE) serves as a 222 widely used method for identifying Lagrangian coherent structures (LCS) within velocity fields that are spatially and temporally discretized across domains with finite spatial and temporal dimensions. The "ridge" structure 223 224 formed by the maxima in the FTLE field effectively reveals hidden coherent structures. Importantly, LCS does not 225 necessitate artificial threshold value settings, unlike vortex criteria based on the Euler framework that heavily 226 relies on threshold values. f-FTLE and b-FTLE can be obtained by integrating in the forward and backward time directions, respectively. Between them, the ridge of b-FTLE reveals the flow structures.^{40, 41} Ridges in the b-FTLE 227 228 field can visually attract materials or unstable manifolds. In other words, along these ridges, fluid particles 229 experience elongation when their trajectories are traced backward in time. To illustrate the entrainment within the 230 vortex, the FTLE related to attracting material lines has been computed as depicted in Fig. 7.

A two-dimensional flow field represented by a transient velocity field v(x,t), is known over a finite time

232 interval $[t_0, t_1]$. The movement of fluid particles can be described by the following differential equation:¹⁵

 $\dot{x} = v(x,t), \tag{16}$

where trajectories of fluid particles are represented by $x(t; t_0, x_0)$, t_0 stands for initial time and x_0 referring to initial positions. The displacement field $x(t; t_0, x_0)$ maps the movement from the initial positions x_0 to the current positions x over the time interval $[t_0, t]$. At position x_0 , the right Cauchy–Green strain tensor $C_{t_0}^t$ is adopted to capture the stretch induced by the velocity gradient, which is written as:

238
$$C_{t_0}^t(x_0) = \left(\frac{\partial x(t;t_0,x_0)}{\partial x_0}\right)^T \left(\frac{\partial x(t;t_0,x_0)}{\partial x_0}\right).$$
(17)

239 $C_{t_0}^t$ has two positive eigenvalues. The larger of the two eigenvalues λ_2 represents the maximum extension of the 240 particle, and its corresponding eigenvector indicates the direction of the deformation. Following Haller³⁹, the FTLE 241 for a period of time [t_0 , t_1] is defined as:

242
$$FTLE_{t_0}^{t_1}(x_0) = \frac{1}{|t_1 - t_0|} \log \sqrt{\lambda_2 \left[C_{t_0}^{t_1}(x_0) \right]}$$
(18)

243 4 Results and analyses

244 4.1 Spiral vortex band

Fig. 5 shows the vertical vorticity distribution on the horizontal plane z = 0.9hs under various operating 245 conditions at different discharges and submergences in the sump. Plane z = 0.9hs is selected as the characteristic 246 plane because the vortex behavior on this particular plane exhibits a relatively stable pattern and can also well 247 248 reflect the flow structure beneath the surface.^{24, 35} Notably, the velocity vectors in Figs. 5(a)-(d) depict the 249 counterclockwise rotation of all the vortices, while in Fig. 5(e) the vortex rotates clockwise. Provided that the inlet/outlet boundaries, as well as the sump geometric model, are entirely symmetrical, the FSV rotation direction 250 251 is random. For ease of discussion and comparison, the vertical vorticity values for Case M5 in Fig. 5(e) have been 252 inverted. In all subplots, the maximum value of the vertical vorticity is focused at the vortex center, while in a 253 specific region outside the center, the vorticity approaches zero. The velocity vector diagrams reveal the presence 254 of a spiral vortex band surrounding the FSV in regions where the water flow convergence or significant flow 255 velocity gradient changes occur. These bands originate either at the side wall or at the confluence points of water 256 flows, spiral in alignment with the rotational direction of FSV, and ultimately converge into it. There are counter-257 rotating vortices in the vortex band opposite to the vorticity of FSV. As Ds increases, both the extent and intensity 258 of the spiral vortex bands increase as well.



Fig. 5. Distribution of vertical vorticity around FSV: (a) M1, Ds = 0.21; (b)M2, Ds = 0.29; (c) M3, Ds = 0.43; (d) M4, Ds = 0.67 and (e) M5, Ds = 0.75.

262 4.2 POD analyses of FSV flied

263 Following the analysis of the vorticity distribution on the characteristic planes under various flow parameter 264 conditions reported in Section 4.1, it has been determined that there are consistent flow phenomena among them. 265 As a result, Case M3 has been selected as a representative case. To analyze various coherent structures and their transport regions in the suction vortex flow, the transient vertical vorticity of the characteristic plane is selected as 266 the physical quantity for the POD method, and the decomposition results are shown in Fig. 6. Precisely, Fig. 6(a) 267 268 presents the time-averaged vertical vorticity with a legend that is consistent with the one of Fig. 5. The POD modal 269 eigenvalues calculated with different numbers of snapshots are plotted in Fig. 6(b). POD results are sensitive to 270 the number of snapshots. Generally, eigenvalues exhibit rapid decay as the modal order increases, and then the 271 decay becomes more gradual when the mode order exceeds 12. Comparing 800 and 1,000 snapshots, eigenvalues 272 from both sets are consistent, indicating that the increase in the number of snapshots does not significantly alter 273 the decomposition results. Therefore, POD results obtained from 1,000 snapshots have been chosen for further 274 analysis. The first two modes have the largest eigenvalues and carry the most energy accounting for 32.9% and 275 17.2% of the total energy, respectively. As the mode order increases, the energy carried by the third and fourth 276 modes decays by 10.1% and 7.1%, respectively. The energy carried by higher-order modes, such as the 9th and 277 16th modes, is even lower and essentially negligible.

278 To further explore the relationship between POD modes and vortex structures of various spatial scales within 279 the flow field, the first four high-energy modes were plotted, as well as the 9th and 16th modes in Fig. 6(c). In the 280 first mode, it can be noticed that there is a pair of counter-rotating vortices at the position of FSV, which is 281 approximately distributed in a symmetrical manner along the spanwise direction. In the second mode, the vortex 282 pair rotates almost 90°, parallel to the main flow direction. These findings are consistent with those from POD analyses performed on trailing vortices from wings and inlet vortices from engines.^{38, 42, 43} They argued that the 283 284 first two modes are strongly correlated with the displacement of the vortex; however, when pioneers explained the 285 physical meaning of higher-order modes, they tended to downplay the effect of these modes due to their lower 286 energy content, as depicted in Fig. 6(b). Nevertheless, even though the energies associated with higher-order modes 287 are lower compared to the previous modes, they represent the patterns of small-scale coherent structures near the 288 FSV in the sump. Moving from the first mode to the fourth mode, it is clear that the absolute value of ω_z at the 289 location where the spiral vortex band is situated in Fig. 5 gradually increases with the increasing order of modes. 290 This trend is especially pronounced in the high-order mode (16th mode) where the vortex distribution within the 291 spiral vortex bands closely resembles the results of POD decomposition observed in a von Kármán vortex street. 292 In these regions, small-scale vortices form spiral vortex bands and gradually converge toward the center under the 293 rotational influence of FSV. Consequently, each order mode in the POD results effectively represents differentscale coherent structures that act as carriers for floating particles in the flow field.



Fig. 6. POD analysis of FSV flow: (a) time-average vortical vorticity flowfield; (b) relative energy fluctuations captured by the POD modes, and (c) first four dominant POD modes, 9th and 16th POD modes.

298 4.3 Lagrangian analysis

299 *4.3.1 FTLE fields*

300 While vortex bands are discernible in Fig. 5, it is crucial to notice that the selection of the threshold in the 301 Euler method, which employs vorticity for vortex identification, plays a pivotal role in displaying the vortex 302 morphology. This choice heavily relies on the sensitivity and experience of the researcher regarding vortex-related 303 issues. Additionally, as it is clear in Figure 5, despite the author's efforts to optimize the vorticity display range, 304 both the vortex motion patterns and boundaries of the vortex bands appear somewhat blurred. Although the POD 305 decomposition can effectively decompose vortices of different scales (as shown in Fig. 6), the calculation process 306 is relatively complex and depicts their instantaneous evolution. To this end, it is imperative to employ the FTLE 307 method to capture the LCS within the FSV flow field. This approach allows the examination of the overall 308 geometric structure of the vortex flow and provides a precise quantification of the flow characteristics of vortices 309 at various scales, as well as a thorough analysis of the nearby particle transport and mixing processes.

Fig. 7 shows instantaneous FTLE fields on the characteristic plane. The dividing line formed by the upper and lower ridges surrounding the vortex center divides the flow field into two parts: the upstream main flow (Region A) and the reflected flow from the back wall (Region B). The material flux is very small at the clear dividing line formed by the flows on both sides, where the material and energy located in Region A will not cross the dividing line into Region B. This finding aligns with the research conducted by Shadden *et al.*⁴⁴ that analyzed observational data of coastal flows through LCS and identified separation areas in ocean currents, namely inner

316 and outer circulation. It is worth highlighting that particles within the water tend to converge towards nearby FTLE 317 ridges and are often captured by them. Comparing the results of the POD analysis in Fig. 6, the ridges of the FTLE 318 field clearly delineate the boundaries of energetic regions, which shows that the FTLE ridges can describe the energy transport pattern carried by the coherent structure in the FSV field. Figs. 7(a-e) show the vortex structure 319 320 rolled up in the main flow direction. With the entrainment of FSV, it gradually enters the center. When the elliptical 321 LCS (marked with a black circle in Fig. 7(a)) approaches the vortex, the flow velocity increases and the structure 322 is stretched to become flatter, as depicted in Fig. 7(c). As the LCS progress towards the FSV center, it undergoes 323 further stretching, eventually forming a long and narrow spiral line that merges into the FSV center, as seen in Fig. 324 7(e). In Figs. 7(a-c), the coherent structure remains relatively stable, whereas in Figs. 7(d-e) its shape is influenced 325 by the FSV, thus leading to the gradual disappearance of the LCS structure. Similarly, Figs. 7(e-h) show the 326 evolution of LCS in the backflow from the back wall, which is consistent with the LCS evolution in Region A. In 327 summary, FTLE clearly shows the characteristics of the formation and evolution of the LCS due to the roll-up of 328 FSV existing in the pumping station sump, which can provide insights into the transport mechanisms of coherent 329 structures.



- 331

Fig. 7. Instantaneous FTLE flow fields.

332 4.3.2 Transport process based on trajectories

333 From the evolution of the aforementioned LCS, the coherent structures undergo significant stretching,

resulting in significant alterations in the shape and intensity of the vortex. Enstrophy $\omega^2/2$ can represent the vortex 334

intensity. To examine the dynamics of the vortex, the enstrophy transport equation is utilized, expressed as:⁴⁵ 335

$$\frac{\mathbf{D}\,\omega^2/2}{\mathbf{D}t} = \omega_i \omega_j \frac{\partial u_i}{\partial x_i} + v \nabla^2 \omega^2 / 2 - v \frac{\partial \omega_i}{\partial x_i} \frac{\partial \omega_i}{\partial x_i} \,. \tag{19}$$

On the right hand of the equation, the first term represents the effects of the production term, due to the vortex stretching and tilting. The second term accounts for the effects of viscous diffusion, and the third term represents the viscous dissipation of the fluids.

340 From the Lagrangian perspective, the motion trajectories of particles at different positions in the system are 341 captured and plotted in Fig. 8(a). Green points donate the initial locations of particles. The trajectories of particles illustrate how floating particles, represented by particles with good water flow followability, are entrained by FSV 342 343 within the sump. Particles from the inflow and those from the backflow mix together in the entrainment zone. 344 Comparing the FTLE field in Fig. 7, it is evident that the entrainment zone is actually a strip region with the dividing line as the bone line. Sultana et al.⁴⁶ showed that more than 85% of the FOG particles were between 373-345 2280 µm in size. Consequently, in this study, the behavior of FOG particles in the sump has been represented with 346 347 fluid particles. While there may be deviations, this approach effectively models the dynamic process of the vortex entrainment of FOG particles, as tested in previous researches.^{4, 47, 48} When FSV entrains FOG particles within the 348 349 sump, particles from different areas initially converge along the dividing line formed by the FTLE ridge, mixing 350 within the entrainment zone. Subsequently, these particles follow the FTLE ridges, spiraling into the vortex at the 351 center of the sump and ultimately being drawn into the suction pipe along with the movement of FSV.

352 In Fig. 8(a), two trajectories, each marked with a different color, are selected to compare the variations in 353 each term of the equation (18) along distinct pathways. The horizontal axes T^* in Figs. 8(c) and (d) represent the 354 non-dimensional time, which has been normalized by the time required for each particle to move to the center of 355 the vortex. In Fig. 8(c), it is observable that, throughout much of the particle entrainment process, each term of the 356 enstrophy transport equation remains relatively constant. Once the particle enters the vortex core region, as 357 depicted in Fig. 8(b), the production term begins to rise rapidly, while the other two do not exhibit significant changes. This distinction indicates that only the vortex core region, representing the rotational motion of a rigid 358 359 body, contributes to the propagation of the vortex. The stretching and tilting effects of the vortex represented by 360 the production term are the primary factors contributing to the increased intensity of the vortex. The budgets and 361 the mechanisms described next are found to be the same along the two trajectories, so only the yellow one is further 362 analyzed.



Fig. 8. Trajectories analysis based on Lagrangian perspective: (a) trajectories of the particles entrained into FSV;
 (b) the tangential velocity contour; (c) enstrophy budget of the trajectory of Particle A (yellow) and (d) enstrophy
 budget of the trajectory of Particle B (blue).

From the Lagrangian perspective, the variation in the three terms of the enstrophy transport equation is analyzed, concluding that the dominant term is the production one. Given the rotational symmetry of the vortex flow, it becomes challenging to concisely analyze each term of the equation within the Cartesian coordinate system. Consequently, FSV from the Cartesian system is translated into the cylindrical coordinate system. This conversion facilitates a more effective analysis of both primary and secondary components of each term. The expanded form of the production term in the cylindrical coordinate system includes:

373
$$P_{\omega} = \begin{bmatrix} P_{\omega 1} \\ P_{\omega 2} \\ P_{\omega 3} \end{bmatrix} = \begin{bmatrix} \omega_{r} \omega_{r} \frac{\partial V_{r}}{\partial r} & \omega_{r} \omega_{\theta} \left(\frac{\partial V_{r}}{r \partial \theta} - \frac{V_{\theta}}{r} \right) & \omega_{r} \omega_{z} \frac{\partial V_{r}}{\partial z} \\ \omega_{\theta} \omega_{r} \frac{\partial V_{\theta}}{\partial r} & \omega_{\theta} \omega_{\theta} \left(\frac{\partial V_{\theta}}{r \partial \theta} + \frac{V_{r}}{r} \right) & \omega_{\theta} \omega_{z} \frac{\partial V_{\theta}}{\partial z} \\ \omega_{z} \omega_{r} \frac{\partial V_{z}}{\partial r} & \omega_{z} \omega_{\theta} \frac{\partial V_{z}}{r \partial \theta} & \omega_{z} \omega_{z} \frac{\partial V_{z}}{\partial z} \end{bmatrix},$$
(20)

where the subscripts 1-3 of P_{ω} represent the radial *r*, tangential θ , and axial *z* direction components in the cylindrical coordinate system, respectively. The transformations between the variables of cylindrical and Cartesian 376 coordinate systems read as:

377
$$\begin{bmatrix} V_r \\ V_{\theta} \\ V_z \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}, \qquad (21)$$

$$\theta = \tan^{-1} \left(\frac{y - y_0}{x - x_0} \right). \tag{22}$$

379 Here, (x_0, y_0) is the FSV coordinate. Fig. 9(a) illustrates the variation in the production term across the three 380 directional components along the particle trajectory. The axial component of the production term $(P_{\omega 3})$ is particularly significant and exhibits a high degree of similarity with the overall production term P_{ω} . The budget of 381 382 the other two direction components is opposite and tends to cancel each other. This phenomenon can be also found 383 in the stable laminar rotating thermal plumes, where the vortex transport equation is employed in the cylindrical coordinate system.⁴⁹ Moreover, the budgets of the tilting $(\omega_z \omega_r \frac{\partial V_z}{\partial r})$ and $\omega_z \omega_\theta \frac{\partial V_z}{r \partial \theta}$, i.e., $P_{\omega_{3,1}}$ and $P_{\omega_{3,2}}$) and 384 stretching terms ($\omega_z \omega_z \frac{\partial V_z}{\partial \tau}$, i.e., P_{ω_3}) of P_{ω_3} are plotted in Fig. 10(b), revealing that the stretching term plays a 385 386 dominant role in maintaining the high-intensity entrainment of the vortex. Although the other two terms (tilting 387 terms) fluctuated during the transportation, they eventually converged to zero. These two terms are tilted by radial

and tangential velocity gradients $\left(\frac{\partial V_z}{\partial r}\right)$ and $\frac{\partial V_z}{\partial \theta}$ during the transportation process and shifted from ω_r and ω_{θ} to ω_z .^{20,49}





393 Removing the communal terms ω_z in $P_{\omega 3}$, the physical meaning of three items ($\omega_z \omega_r \frac{\partial V_z}{\partial r}$, $\omega_z \omega_\theta \frac{\partial V_z}{r \partial \theta}$ and

394 $\omega_z \omega_z \frac{\partial V_z}{\partial z}$) can be clarified: $\omega_r \frac{\partial V_z}{\partial r}$ and $\omega_\theta \frac{\partial V_z}{r \partial \theta}$ represent the tilting effect of radial and tangential velocity

395 gradients on radial vorticity and tangential vorticity respectively. $\omega_z \frac{\partial V_z}{\partial z}$ represents the stretching effect of the

396 axial velocity gradient on the axial vorticity. Fig. 10(a) shows the process in which the radial and tangential vortices are distorted by the velocity gradients $\left(\frac{\partial V_z}{\partial r}\right)$ and $\frac{\partial V_z}{r\partial \theta}$ and tilt toward the axial vortex. Red, green, and blue 397 columns represent radial, tangential, and axial vortices, respectively. The dashed and solid columns represent the 398 399 early and late stages of the vortices, respectively. The vortex evolution in Fig. 10(a) shows that, under the action 400 of the velocity gradient, ω_r and ω_{θ} will eventually transform into ω_z . Fig. 10(b) shows vortex structures from the 401 simulation results. The flow structure near the FSV is visualized by using the Q-criterion and dyed by ω^2 , which demonstrates how radial and tangential vortices spiral and merge into the FSV center due to the influence of the 402 403 velocity gradient tilting terms. These results do not only provide strong support for the previous discussion 404 regarding the transformation of vortices with different directions but also exhibit a consistent evolution with the



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LCS observed in Fig. 7.

Fig. 10. (a) schematics of distortion effect from the velocity gradients and (b) vortex structures (iso-surface of $QD^2/u_D^2 = 5.5$).

409 **5** Conclusions

The LES turbulence model with the CLSVOF method has been used to simulate the FSV phenomenon in the sewage pumping station. Based on these comprehensive flow fields, the transport and mixing characteristics of floating particles are studied when there is a FSV at the sump. The main results are the following:

(1) The spiral vortex band induced by FSV is studied for the first time and decomposed by the POD method.
Two coherent structures have been identified: FSV represented by the first two high-energy orders, exceeding half
of the total resolved turbulent energy, and the spiral vortex band represented by the low-energy higher orders,
which consists of numerous small-scale vortices with different rotation directions.

417 (2) The LCS approach together with particle tracking is applied to reveal the evolution of the LCS and particle
418 transport in the sump. LCS can be well captured by the FTLE method. As the LCS flows towards the FSV center,

419 it undergoes significant elongation by forming an elongated spiral shape before ultimately converging at the vortex 420 center. Particles at different positions in the sump are first caught by the dividing line formed by FTLE ridges, 421 mixed in the entrainment zone, then move along the FTLE ridge pathways and spiral into the vortex center of the 422 sump. Eventually, they are drawn into the intake pipe and discharged in conjunction with the movement of the 423 vortex.

424 (3) The dynamic spiral motion of particles is analyzed using the enstrophy transport equation in cylindrical 425 coordinates to fully understand the entrainment mechanism of FSV on particles. It can be found that the dominant 426 term responsible for the vortex entrainment of particles is the production term, which increases as a result of the 427 stretching of the axially moving flow. The vorticity in the other two directions is converted into axial vorticity due 428 to the inclination of the velocity gradient, further enhancing the entrainment effect of FSV on the particles in the 429 vortex core region and accelerating the particle motion towards the pipe mouth.

430 The analytical methods used in this study have proven to be effective tools for predicting transport capacity 431 and understanding the mechanisms of the FSV. The research results can provide a reference for the renovation 432 design of existing sewage pumping stations and the operation of various urban drainage pumping stations. 433 Furthermore, the design specifications for pumping stations can be supplemented and improved regarding the 434 accumulation of floating debris in the sump. When utilizing FSV for FOG removal, the velocity gradients in the 435 sump should be enhanced, for example by adjusting valves to lower the submergence or by activating additional units to increase the flow rate. The properties of FOG particles (e.g., composition and shape) influence their 436 437 transport and mixing processes induced by FSV, a topic that warrants further exploration in future studies.

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444 **References**

- 445 1. A. M. Moreno-Rodenas, A. Duinmeijer, andF. Clemens, "Deep-learning based monitoring of FOG layer
 446 dynamics in wastewater pumping stations," Water Res. 202, 13 (2021).
- 447 2. A. Duinmeijer, andF. Clemens, "An experimental study on the motion of buoyant particles in the free-surface
 448 vortex flow," Journal of Hydraulic Research 1 (2021).
- 449 3. H. H. Yusuf, F. Roddick, V. Jegatheesan, L. Gao, and B. K. Pramanik, "Tackling fat, oil, and grease (FOG) build-
- up in sewers: Insights into deposit formation and sustainable in-sewer management techniques," Sci Total
 Environ **904**, 18 (2023).

- 4. S. P. A. Duinmeijer, "On the free-surface vortex driven motion of buoyant particles," PhD thesis, Delft
 University of Technology, Delft, The Netherlands, 2020.
- 454 5. G. E. Hecker, Fundamentals of vortex intake flow (Routledge, 2017).
- 455 6. W. J. M. Rankine, A manual of applied mechanics (Griffin, London, 1858).
- 456 7. J. M. Burgers, *A mathematical model illustrating the theory of turbulence* (1948).
- 457 8. A. Škerlavaj, L. Škerget, J. Ravnik, and A. Lipej, "Predicting free-surface vortices with single-phase
 458 simulations," Engineering Applications of Computational Fluid Mechanics 8, 193 (2014).
- 9. V. Naderi, J. Yang, C. Lin, M. H. Kuo, and C. F. Hu, *There-dimensional velocity measurement on free surface of air-core intake vortex under critical submergence* (Egmond aan Zee, The Netherlands, 2016).
- 10. S. P. A. Duinmeijer, A. M. Moreno-Rodenas, M. Lepot, C. van Nieuwenhuizen, I. Meyer, andF. Clemens, "A
 simple measuring set-up for the experimental determination of the dynamics of a large particle in the 3D
 velocity field around a free surface vortex," Flow Measurement And Instrumentation 65, 52 (2019).
- 464 11. H. Y. Cheng, X. R. Bai, X. P. Long, B. Ji, X. X. Peng, and M. Farhat, "Large eddy simulation of the tip-leakage
 465 cavitating flow with an insight on how cavitation influences vorticity and turbulence," Applied Mathematical
 466 Modelling 77, 788 (2020).
- 467 12. X. L. Hou, J. P. Yuan, Y. X. Fu, R. Lu, J. L. Shi, and P. Y. Zhang, "A study on the dynamic characteristics of
 468 surface suction vortices in an open inlet pool," Physics of Fluids 35, 16 (2023).
- 469 13. M. Hashid, andT. I. Eldho, "Numerical simulation of air-core surface vortex at critical submergence for dual
 470 horizontal intakes," Journal of Fluids Engineering 145, 15 (2023).
- 471 14. G. Haller, "Lagrangian Coherent Structures," Annual Review of Fluid Mechanics 47, 137 (2015).
- 472 15. A. Hadjighasem, M. Farazmand, D. Blazevski, G. Froyland, andG. Haller, "A critical comparison of
 473 Lagrangian methods for coherent structure detection," Chaos 27, (2017).
- 474 16. A. Schneider, D. Conrad, and M. Böhle, "Lattice Boltzmann Simulation of the Flow Field in Pump Intakes—A
 475 New Approach," Journal of Fluids Engineering 137, (2015).
- 476 17. S. B. Pope, *Turbulent Flows* (Cambridge University Press, Cambridge, 2000).
- 477 18. M. L. Shur, P. R. Spalart, M. K. Strelets, and A. K. Travin, "A hybrid RANS-LES approach with delayed-DES
 478 and wall-modelled LES capabilities," International Journal of Heat and Fluid Flow 29, 1638 (2008).
- 479 19. X. Sun, W. X. Li, Q. Y. Huang, J. Zhang, andC. C. Sun, "Large eddy simulations of wind loads on an external
 480 floating-roof tank," Engineering Applications of Computational Fluid Mechanics 14, 422 (2020).
- 20. Z. Li, H. Chen, H. Xu, J. Feng, K. Kan, T. Li, andL. Shen, "Effects of discharge-to-submergence ratio on
 evolution of air-core vortex," Ocean Engineering 281, 114830 (2023).
- 483 21. X. Huang, Q. Guo, B. Qiu, and X. Feng, "Prediction of air-entrained vortex in pump sump: influence of
 484 turbulence models and interface-tracking methods," Journal of Hydraulic Engineering 146, 04020010 (2020).
- 22. X. S. Wang, J. Zhang, Y. H. Chen, and Z. L. Kuai, "Numerical study of rising Taylor bubbles driven by buoyancy
 and additional pressure," International Journal of Multiphase Flow 159, 12 (2023).
- 487 23. M. Sussman, P. Smereka, and S. Osher, "A level set approach for computing solutions to incompressible two488 phase flow," Journal of Computational Physics 114, 146 (1994).
- 489 24. K. Kan, Y. Xu, Z. Li, H. Xu, H. Chen, D. Zi, Q. Gao, and L. Shen, "Numerical study of instability mechanism
 490 in the air-core vortex formation process," Engineering Applications of Computational Fluid Mechanics 17,
- 491 18 (2023).

- 492 25. M. Sussman, and E. G. Puckett, "A coupled level set and volume-of-fluid method for computing 3D and
 493 axisymmetric incompressible two-phase flows," Journal of Computational Physics 162, 301 (2000).
- 494 26. H. Tse, "Challenges for pumping station design in water industries: An overview of impacts from climate
 495 change and energy crisis," Water Res. 253, 21 (2024).
- 27. X. Song, Y. Luo, andZ. Wang, "Numerical prediction of the influence of free surface vortex air-entrainment on
 pump unit performance," Ocean Engineering 256, (2022).
- 28. K. Kan, Y. Zheng, H. Chen, D. Zhou, J. Dai, M. Binama, and A. Yu, "Numerical simulation of transient flow
 in a shaft extension tubular pump unit during runaway process caused by power failure," Renewable Energy
 154, 1153 (2020).
- 501 29. H. Chen, D. Zhou, K. Kan, J. Guo, Y. Zheng, M. Binama, Z. Xu, and J. Feng, "Transient characteristics during
 502 the co-closing guide vanes and runner blades of a bulb turbine in load rejection process," Renewable Energy
 503 165, 28 (2021).
- 30. K. Kan, Z. Yang, P. Lyu, Y. Zheng, and L. Shen, "Numerical study of turbulent flow past a rotating axial-flow
 pump based on a level-set immersed boundary method," Renewable Energy 168, 960 (2021).
- 31. Z. Li, H. Xu, J. Feng, H. Chen, K. Kan, T. Li, andL. Shen, "Fluctuation characteristics induced by energetic
 coherent structures in air-core vortex: The most complex vortex in the tidal power station intake system,"
 Energy 288, 129778 (2024).
- 32. G. Xia, W. You, S. Manickam, J. Y. Yoon, X. Xuan, andX. Sun, "Numerical simulation of cavitation-vortex
 interaction mechanism in an advanced rotational hydrodynamic cavitation reactor," Ultrasonics
 Sonochemistry 105, 106849 (2024).
- 33. I. B. Celik, U. Ghia, P. J. Roache, C. J. Freitas, H. Coleman, and P. E. Raad, "Procedure for estimation and
 reporting of uncertainty due to discretization in CFD applications," Journal of Fluids Engineering 130, (2008).
- 34. H. Y. Zhang, J. L. Liu, J. X. Wu, W. X. Jiao, L. Cheng, and M. B. Yuan, "Research on optimization of the bulb
 form of the bulb tubular pump device for a low-head agricultural irrigation pumping station," AgricultureBasel 13, 18 (2023).
- 517 35. G. Möller, M. Detert, and R. M. Boes, "Vortex-induced air entrainment rates at intakes," Journal of Hydraulic
 518 Engineering 141, (2015).
- 36. A. Duinmeijer, G. Oldenziel, and F. Clemens, "Experimental study on the 3D-flow field of a free-surface vortex
 using stereo PIV," Journal of Hydraulic Research 58, 105 (2019).
- 37. L. Sirovich, "Turbulence and the dynamics of coherent structures. Parts I–III," Quarterly of Applied
 Mathematics 45, 561 (1987).
- 38. D. Zi, L. Shen, F. Wang, B. Wang, andZ. Yao, "Characteristics and mechanisms of air-core vortex meandering
 in a free-surface intake flow," International Journal of Multiphase Flow 152, 104070 (2022).
- 525 39. G. Haller, "Lagrangian coherent structures from approximate velocity data," Physics of Fluids 14, 1851 (2002).
- 40. R. Sampath, M. Mathur, and S. R. Chakravarthy, "Lagrangian coherent structures during combustion instability
 in a premixed-flame backward-step combustor," Phys. Rev. E 94, 062209 (2016).
- 41. J. D. Eldredge, and K. Chong, "Fluid transport and coherent structures of translating and flapping wings," Chaos
 20, (2010).
- 42. Z. Wang, and I. Gursul, "Unsteady characteristics of inlet vortices," Experiments in Fluids 53, 1015 (2012).
- 43. C. Del Pino, J. M. Lopez-Alonso, L. Parras, and R. Fernandez-Feria, "Dynamics of the wing-tip vortex in the

- 532 near field of a NACA 0012 aerofoil," Aeronaut. J. **115**, 229 (2011).
- 44. S. C. Shadden, F. Lekien, and J. E. Marsden, "Definition and properties of Lagrangian coherent structures from
 finite-time Lyapunov exponents in two-dimensional aperiodic flows," Physica D: Nonlinear Phenomena 212,
 271 (2005).
- 45. T. Watanabe, Y. Sakai, K. Nagata, Y. Ito, and T. Hayase, "Wavelet analysis of coherent vorticity near the
 turbulent/non-turbulent interface in a turbulent planar jet," Physics of Fluids 26, (2014).
- 46. N. Sultana, F. Roddick, B. Jefferson, L. Gao, D. Bergmann, J. Papalois, M. Guo, K. Tzimourtas, and B. K.
 Pramanik, "Effectiveness of grease interceptors in food service establishments for controlling fat, oil and
 grease deposition in the sewer system," Sci Total Environ 912, 12 (2024).
- 47. M. M. Neamtu-Halic, D. Krug, G. Haller, and M. Holzner, "Lagrangian coherent structures and entrainment
 near the turbulent/non-turbulent interface of a gravity current," Journal of Fluid Mechanics 877, 824 (2019).
- 48. C. C. Tseng, and H. A. Hu, "Flow Dynamics of a Pitching Foil by Eulerian and Lagrangian Viewpoints," Aiaa
 J. 54, 712 (2016).
- 545 49. F. C. Martins, J. M. C. Pereira, and J. C. F. Pereira, "Vorticity transport in laminar steady rotating plumes,"
- 546 Physics of Fluids **32**, (2020).

547