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Measuring income polarization using Bonferroni and De Vergottini inequality indices: evidences from European countries

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Abstract
The aim of this paper is to measure to which extent income distribution is polarized across European countries by means of polarization measures based on the Bonferroni and De Vergottini indices of inequality. Different from traditional measures of polarization, the indices proposed in this paper are sensitive to progressive transfers, attaching more importance to some part of the income distribution. These indices enriches the analysis and contribute to disentangle the different faces of income polarization. In the empirical application we compare European countries over the period 2010–2019 using EU-SILC data. Results reveal significant changes in polarization over the last decades for most countries.

Keywords Bonferroni index · De Vergottini index · Gini index · Inequality · Subgroup decomposition · Polarization measurement

JEL Classification D31 · D63 · C43 · I32

1 Introduction

The recent economic crises due first to the Great Recession, then to COVID-19 pandemic and, lately, to the global energy crisis have revealed the importance of monitoring their distributive effects on inequality, poverty, vulnerability and measuring not only the inequalities among individuals but also across the groups into which population may be divided. For this reason, beyond the analysis of income inequality it is important to take into account polarization measures.

Polarization is a concept similar to but distinct from inequality. Both concern the degree of disparity present in income distribution, but in different ways. Inequality analyses pair-

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wise differences along the whole distribution, while polarization focuses on inequality both between and within groups.

Income polarization indices can be classified into *bipolarization* indices and *multipolar* indices; see Permanyer [30] and Gigliarano [24] for an extensive review. Bipolarization indices measure the extent to which an income distribution is clustered around two poles, typically the poor and the rich. Multipolar indices measure the extent to which an income distribution has an arbitrary number of antagonistic poles.

More in details, bipolarization refers to the tendency of incomes to shift away from the middle point of the distribution towards the tails, thus obtaining two groups (typically, the rich and the poor) well separated from each other but homogenous inside and at the same time a hollowed-out middle class. Maximum bipolarization is reached when population is split in two equal-sized groups, such that members in the first one is penniless, while the total income is equally shared among individuals in the second group. Of course, this can be generalized to the case of several groups. In the latter case, the income distribution is said to be (multi-)polarized if the relative frequency of observations is low in correspondence to the central value and it is high at the tails.

The main reason why the study of polarization has obtained as increasing attention over the last few decades is the strong relationship between the decline of the middle class and social instability. Indeed, the presence of groups similar in size, whose members have analogous characteristics but differ from the individuals of the other groups, can bring to an unstable political situation. Esteban and Ray [15] show that the presence of contrasting groups as well as a weak and hollowed-out middle class can lead to an unstable society and cause possible social conflicts. Similarly, Gasparini et al. [18] find that for Latin American and Caribbean countries, in the period 1989–2004, high levels of income polarization are positively correlated with a high level of social conflict.

The first systematic works on polarization measurement date back to 1994, when, in two independent papers, Wolfson [39] and Esteban and Ray [15] propose rigorous treatments of such phenomenon.

These works have given rise to two different strands of literature: the so-called *Wolfson and Foster approach* Wolfson [39, 40] and Foster and Wolfson [16, 17], and the *Esteban and Ray approach* [15]. The former focuses on the shrinking middle class, monitoring how the income distribution spreads out from its center. The latter focuses on the rise of separated income groups: polarization increases if population groups with similar size become more homogeneous inside and more separated from each other.[1]

However, as discussed in Rodrìguez and Salas [33], the two approaches have a common core. Indeed, the Wolfson index can be rewritten as a function of the difference between the Gini index between groups and the Gini index within groups; thus revealing that, also according to Wolfson's approach, polarization increases with inequality between groups and decreases with inequality within groups.

After these pioneering works, several scholars have investigates further the phenomenon of polarization. We cite, among others, Wang and Tsui [38], Gradín [25], Anderson [1], Lasso de la Vega and Urrutia [27], Silber et al. [37], Gigliarano and Mosler [22], Chakravarty and D'Ambrosio [9], Pittau and Zelli [32], Gigliarano et al. [23].

All the above cited contributions to polarization measurement are based on the Gini inequality index. Very recently, Ciommi et al. [12] propose, instead, to measure bipolarization using different inequality indices. In particular, the authors introduce bipolarization indices

[1] See, among others, Permanyer [30] and Gigliarano [24] for a broad review of the literature on polarization measurement.

based on Bonferroni and De Vergottini indices, which are characterized by different sensitivity to progressive transfers. Bonferroni-based polarization index is indeed more sensitive to income transfers occurring among poorer individuals, while De Vergottini-based polarization index is indeed more sensitive to transfers occurring among richer people.

Here we follow Ciommi et al. [12] approach and provide and empirical application of the bipolarization indices based on the Bonferroni and the De Vergottini inequality measures in the special case of two non-overlapping groups separated by median.

The aim of the paper is, therefore, to compare, using an empirical analysis, bipolarization indices characterized by different sensitivity to income transfers. The comparison has the advantage to enrich the analysis based on standard measures and contribute to disentangle the different faces of income polarization. This is particularly relevant, since recent studies encourage the simultaneous use of more than one inequality index to better capture the inequality in different parts of the income distribution and thus better understand the socio-economic reality (see, e.g., Piketty [31]).

The rest of the paper is organized as follows. Section 2 is devoted to the methodology: first we recall the inequality indices proposed by Gini, Bonferroni and De Vergottini, then we illustrate their subgroup decomposition in the special case of two non overlapping groups, and finally we review the polarization indices based on Bonferroni and De Vergottini inequality measures, as proposed in Ciommi et al. [12]. Section 3 provides an in-depth analysis of income bipolarization for European Countries in three different years, namely 2010, 2015 and 2019, using European Union Statistics on Income and Living Conditions (EU-SILC) data. Finally Sect. 4 concludes and addresses the possible extension of our approach.

2 From inequality indices to polarization measures

2.1 Notation

For a population of size n, let $y = (y_1, y_2, \ldots, y_p, \ldots, y_n)$ be a positive non-decreasing income distribution, that is $y_a \leq y_b$ for any $a \leq b$. We denote with $m(y)$ the median income and with y^U and y_L the vector of incomes y_i above and below the median, respectively. Moreover, let $\mu(y)$ be the mean of the overall population and $\mu_p(y)$ be the mean of individuals with income smaller than or equal to y_p. Similarly, μ^U and μ_L indicate the mean values of the incomes in y^U and y_L, respectively.

Also, we define a *polarization index* as follows:

Definition 1 A polarization index $P(y)$ is a continuous (real valued) function $P : \mathcal{D} \to \mathbb{R}^+$, where $\mathcal{D}$ is the set of all possible income distributions for a population of n individuals.

Income bipolarization indices are based on two crucial axioms: the *Increased Spread* axiom and the *Increased Bipolarity* axiom. The former has been firstly formulated by Chakravarty and Majumder [11] and states that bipolarization should increase if we transfer income from an individual below the median to an individual above the median income.

Axiom 1 (Increased Spread) *Let $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$ be two income distributions such that $m(x) = m(y) = m$. Consider the following scenarios:*

(i) There exists $j \in \{1, \cdots, k\}$ such that $x_j < y_j < m$ and $x_i = y_i$ for all $i \neq j$;
(ii) There exists $l \in \{1, \cdots, k\}$ such that $m < y_l < x_l$ and $x_i = y_i$ for all $i \neq l$;

If either (i), (ii) or both hold, then $P(x) > P(y)$.

The axiom states that a rank-preserving increase in the incomes above the median, or a rank-preserving decrease in incomes below the median, extends the gap between the two groups and hence increases the degree of bipolarization.

The second important axiom is the *Increased Bipolarity* axiom, which states that polarization should increase if a progressive transfer between individuals belonging to the same income group takes place. In other words, the axiom refers to the case where the incomes below or above the median become closer to each other, so that inequality within the groups decreases, leading to an increase in bipolarization; see, among others, Permanyer [30].

Axiom 2 (Increased Bipolarity) *Let* $x = (x_1, \ldots, x_n)$ *and* $y = (y_1, \ldots, y_n)$ *be two income distributions such that* $m(y) = m(x) = m$. *Consider the following scenarios:*

(i) x has been obtained from y by a progressive transfer of income from richer person b to poorer person a with $y_b < m$*;*

(ii) x has been obtained from y by a progressive transfer of income from richer person d to poorer person c with $y_c > m$*.*

If either (i), (ii) or both holds, then $P(x) > P(y)$.

2.2 Gini, Bonferroni and De Vergottini

The Gini index [19] is the most common statistical measure employed in the socio-economic sciences for measuring concentration in the distribution of income or wealth. The Gini index has several formulations;[2] one of the formulations in the discrete case is:

$$G(y) = 1 - \frac{2}{n(n+1)\mu(y)} \sum_{i=1}^{n} \sum_{j=1}^{i} y_j, \tag{1}$$

with $G(y) \in \left[0, \frac{n-1}{n+1}\right]$.

In 1930, Bonferroni [7] proposes an index of inequality that is more sensitive than the Gini index to lower values in the income distribution, assigning more weight to income transfers occurring among the poor (see Nygard and Sandstrom [28]). Recently, several scholars have investigated the main features of Bonferroni index and proposed interesting applications in social and economic contexts (see, among others, Bàrcena-Martìn and Silber [4–6]; Chakravarty and Muliere [10], Chakravarty [8], Dong et al. [14]).

The Bonferroni index corresponds to the area between the line of perfect equality (horizontal line at height 1) and the Bonferroni curve $B(p) = \mathcal{L}(p)/p$, where $\mathcal{L}(p)$ represents the Lorenz Curve; see Giorgi and Crescenzi [21].

Following Nygard and Sandstrom [28] and Bàrcena-Martìn and Imedio [3], the formulation of the Bonferroni index $B(y)$ in the discrete case can be written as:

$$B(y) = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{\mu(y) - \mu_i(y)}{\mu(y)} \right) = 1 - \frac{1}{n\mu(y)} \sum_{i=1}^{n} \mu_i(y) = 1 - \frac{1}{n\mu(y)} \sum_{i=1}^{n} \frac{1}{i} \sum_{j=1}^{i} y_j, \tag{2}$$

with $B(y) \in \left[0, \frac{n-1}{n}\right]$.

In 1950 De Vergottini [13] proposes an index of inequality, which is more sensitive than the Gini index to the right tail of the income distribution, i.e., it is more affected by income

[2] See Giorgi [20] and Yitzhaki [41].

changes occurring among the rich. The De Vergottini index corresponds to the area between the De Vergottini curve $V(p) = (1 - \mathcal{L}(p))/(1 - p)$ and the line of perfect equality and it can be interpreted as a weighted average of the relative differences between the mean of the population and the partial means of the i-th richest group (see Tarsitano [35]). Its discrete formulation is:

$$V(y) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{M_i(y) - \mu(y)}{\mu(y)}\right) = \frac{1}{n\mu(y)}\sum_{i=1}^{n} M_i(y) - 1, \tag{3}$$

where $M_i(y) = \frac{1}{(n-i+1)}\sum_{j=i}^{n} y_j$. We have that $V(y) \in \left[0, \left(\sum_{j=1}^{n}\frac{1}{n-j+1}\right) - 1\right]$.[3]

The three above-mentioned indices are, thus, characterized by a different sensibility to income transfers. They can be written as a weighted mean of incomes, where the weight associated with an individual's income depends on his position in the income distribution and increases with the individual's rank in the distribution.

According to Bàrcena-Martìn and Imedio [3], the Gini index $G(y)$ can be written as:

$$G(y) = \frac{1}{n\mu(y)}\sum_{i=1}^{n}\gamma_i y_i, \quad \text{with } \gamma_i = \left(\frac{2i-1}{n}\right) - 1, \quad \gamma_{i+1} = \gamma_i + \frac{2}{n}, \quad \sum_{i=1}^{n}\gamma_i = 0 \tag{4}$$

Tarsitano [34] formulates the Bonferroni index $B(y)$ as a linear combination of incomes with weights depending on the individual ranks, as follows:

$$B(y) = \frac{1}{n\mu(y)}\sum_{i=1}^{n} w_i y_i \quad \text{with } w_i = 1 - \sum_{j=i}^{n}\frac{1}{j}, \quad w_{i+1} = w_i + \frac{1}{i}, \quad \sum_{i=1}^{n} w_i = 0. \tag{5}$$

Finally, for the De Vergottini index, we have

$$V(y) = \frac{1}{n\mu(y)}\sum_{i=1}^{n}\xi_i t_i, \quad \text{with } \xi_i = \sum_{j=1}^{i}\frac{1}{n-j+1} - 1 \quad \xi_{i+1} = \xi_i + \frac{1}{n-i} \quad \sum_{i=1}^{n}\xi_i = 0. \tag{6}$$

The three weights w_i, γ_i, ξ_i have a different behaviour: in the Gini index the weight sequence increases constantly w.r.t. the individual rank, with an absolute increment of $2/n$, whereas both in Bonferroni and in De Vergottini indices weights grow at a non-constant rate (the absolute increment is decreasing and equal to $1/i$ in Bonferroni, it is increasing and equal to $1/(n-i)$ in De Vergottini).

Therefore, the Bonferroni index is more sensitive to transfers that occur at the lower tail of the income distribution, while De Vergottini index is more sensitive to variations occurring among the richest incomes.

2.3 Decomposition as a tool for defining bipolarization indices: the case of two non-overlapping groups

Subgroup decomposition is a fundamental step in the definition of polarization indices. For instance, the Wolfson bipolarizarion measure splits the population into two groups divided by

[3] Note that the De Vergottini index does not have a unit upper bound. The maximum inequality corresponds to the income profile in which only one individual holds the total income, i.e. $y_i = n\mu(y)$ and $y_j = 0$ for $j = 1, \ldots, n, j \neq i$. The upper bound of I_V can be written as: $V^{MAX} = \sum_{j=2}^{n}\frac{1}{j}$. In this way it is easy to see that V^{MAX} only depends on the population size.

the median income and can be written as function of the *between-group* inequality component and the *within-group* inequality component of the Gini index.

Ciommi et al. [12] consider the decomposition used by Lambert and Aronson [26] for the Gini index and by Bàrcena-Martìn and Silber [6] for the Bonferroni index and extend the approach to De Vergottini concentration index. In the special case of two non-overlapping groups divided by the median, Ciommi et al. [12] consider these decompositions to define new polarization indices based on the Bonferroni and on the De Vergottini concentration indices.

We now review briefly the subgroup decompositions of Gini, Bonferroni and De Vergottini inequality indices, which lead to the new transfer-sensitive polarization indices proposed in Ciommi et al. [12].

We suppose that the two groups are divided by the median income $m(y)$.

The Gini index can be written in terms of two components: the *between group* component and the *within group* component. We use subscript B for the *between* component, subscript W for the *within* component. In general, the between group component represents the inequality level of a theoretical distribution, in which each individual income is replaced by the mean of the group. Whereas, the within group component is a weighted sum of the inequality within each subgroup. Thus, the subgroup decomposition of the Gini index in case of two groups divided by the median can be written as:

$$G = G_B + G_W \tag{7}$$

where, if n is an even number, the between-group component is

$$G_B = \frac{1}{4\mu(y)} \left(\mu^U(y) - \mu^L(y)\right)$$

and the within-group component corresponds to

$$G_W = \frac{1}{2\mu(y)} \left[\frac{1}{2}\mu^L(y) - \frac{1}{n/2(n/2+1)} \sum_{i=1}^{n/2} \sum_{j=1}^{i} y_j\right] + \frac{1}{2\mu(y)} \left[\frac{1}{2}\mu^U(y) - \frac{1}{n/2(n/2+1)} \sum_{i=1}^{n/2} \sum_{j=1}^{i} y_j\right].$$

Differently, for Bonferroni and De Vergottini inequality indices, the decomposition accounts for three terms: the *between* component, the *within* component and a *residual* term, which we denote with the subscript R.

The residual term depends on the *re-ranking* in the calculation of Bonferroni and De Vergottini indices, which occurs since the two indices, differently from the Gini index, are not replication invariant (see Barcena-Martin and Silber [5]).

Thus, the subgroup decomposition of the Bonferroni index in case of two groups divided by the median can be written as:

$$B = B_B + B_W + B_R \tag{8}$$

where, if n is an even number, the *between* component is given by

$$B_B = \frac{1}{2\mu(y)} \left(\mu^U(y) - \mu^L(y)\right) \left(\sum_{j=1}^{n/2} \frac{1}{n/2+j}\right),$$

while the *within* component and the *residual* term are, respectively, defined as follows:

$$B_W = \frac{1}{2n\mu(y)}\left[\frac{n}{2}\mu^L(y) - \sum_{i=1}^{n/2}\mu_i^L(y)\right] + \frac{1}{2n\mu(x)}\left[\frac{n}{2}\mu^U(y) - \sum_{i=1}^{n/2}\mu_i^U(y)\right],$$

and

$$B_R = \frac{1}{2n\mu(y)}\left[\frac{n}{2}\mu^L(y) - \sum_{i=1}^{n/2}\mu_i^L(y)\right] + \frac{1}{2n\mu(y)}\left[\mu^U(y)\sum_{i=1}^{n/2}\left(\frac{(i-n)/2}{(i+n)/2}\right) - \left(\frac{(i-n)/2}{(i+n)/2}\right)\mu_i^U(y)\right].$$

From the previous definition we observe that the effect of the residual component is stronger the closer the observation is to the median.

Finally, the subgroup decomposition of the De Vergottini index in case of two groups divided by the median can be written as:

$$V = V_B + V_W + V_R \tag{9}$$

where, if n is an even number, the *between* component corresponds to

$$V_B = \frac{1}{2\mu(y)}\left(\mu^U(y) - \mu^L(y)\right)\left(\sum_{i=1}^{n/2}\frac{1}{n-i+1}\right),$$

while the *within* component and the *residual* component can be written, respectively, as

$$V_W = \frac{1}{2n\mu(y)}\left[\sum_{i=1}^{n/2}M_i(y^L) - \frac{n}{2}\mu^L(y)\right] + \frac{1}{2n\mu(y)}\left[\sum_{i=1}^{n/2}M_i(y^U) - \frac{n}{2}\mu^U(y)\right]$$

and

$$V_R = \frac{1}{2n\mu(y)}\left[\sum_{i=1}^{n/2}\left(\frac{i}{i-n}\right)M_i(y^L) - \sum_{i=1}^{n/2}\left(\frac{i}{i-n}\right)\mu^L(y)\right] + \frac{1}{2n\mu(y)}\left[\sum_{i=1}^{n/2}M_i(y^U) - \frac{n}{2}\mu^U(y)\right].$$

2.4 The bipolarization index

Foster and Wolfson [16] propose the following bipolarization measure:

$$P^{FW} = \frac{2\mu(y)}{m(y)}\left[1 - 2L(0.5) - G\right] = \frac{2\mu(y)}{m(y)}\left[G_B - G_W\right] \tag{10}$$

where $L(q)$ is the value of the Lorenz curve at the q-th quantile of y and G is Gini index defined as twice the area between the equidistribution line (p) and the Lorenz curve $L(p)$, while G_B and G_W denote the Gini index between and within groups.

Following Foster and Wolfson's approach [16, 17], a measure of bipolarization is an increasing function of the inequality between groups and decreasing function of the within groups. Ciommi et al. [12] propose bipolarization measures P^B, based on the Bonferroni index, and P^V, based on the De Vergottini index, defined as follows:

$$P^B = \frac{2\mu(y)}{m(y)} \left[B_B - (B_W + B_R)\right] \tag{11}$$

$$P^V = \frac{2\mu(y)}{m(y)} \left[V_B - (V_W + V_R)\right]. \tag{12}$$

The two polarization measures proposed are increasing functions of the between-group inequality, and decreasing with respect to both the within-group and the residual components. Different from Foster and Wolfson's measure, which is based on the Gini index, the polarization measures based on Bonferroni and De Vergottini contain a residual term representing the role played by the individuals' rank. The new polarization measures are characterized by different sensitivity to transfers. In particular, measure P^B is more sensitive to income transfers involving poorer individuals, while P^V to transfers occurring among richer people. The new indices are always coherent with the *Increased Bipolarity axiom (IB)*, while the *Increased Spread axiom (IS)* is satisfied under some regularity conditions.[4]

3 An empirical illustration

The empirical application is based on the EU-SILC (European Union Statistics on Income and Living Conditions) dataset, which collects comparable microdata on households income and living conditions for the 27 EU member states plus Norway and Switzerland.[5]

We have analysed the level of income bipolarization in the 29 countries under analysis comparing the measures illustrated above and referring to the years 2010, 2015 and 2019.[6]

As income variable we considered the equivalent household disposable income, defined as the sum of the personal income components of all household members plus the family income components, net of income tax and social contributions, using the modified OECD scale. Negative incomes have been excluded from the analysis. In addition, we applied a trimming procedure by deleting, for each country, the top and bottom 2% of weighted household disposable income.

We estimated the Gini index of inequality $G(y)$, the Bonferroni index of inequality $B(y)$ and the De Vergottini inequality index $V(y)$ using the following simple weighted estimators:

$$\widehat{G}(y) = 1 - \frac{2}{n(n-1)\mu(y)} \sum_{i=1}^{n-1} \sum_{j=1}^{i} w_j y_j, \tag{13}$$

$$\widehat{B}(y) = 1 - \frac{1}{(n-1)\mu(y)} \sum_{i=1}^{n-1} \frac{1}{i} \sum_{j=1}^{i} w_j y_j, \tag{14}$$

4 See Ciommi et al. [12] Proposition 3 and Proposition 4, respectively.

5 In details, the 29 countries considered here are: Austria (AT), Belgium (BE), Bulgaria (BG), Croatia (HR), Cyprus (CY), Czech Republic (CZ), Denmark (DK), Estonia (EE), Germany (DE), Greece (EL), Finland (FI), France (FR), Hungary (HU), Ireland (IE), Italy (IT), Latvia (LV), Lithuania (LT), Luxembourg (LU), Malta (MT), Netherlands (NL), Norway (NO), Poland (PL), Portugal (PT), Romania (RO), Slovakia (SK), Slovenia (SI), Spain (ES), Sweden (SE), Switzerland (CH)

6 We note that in EU-SILC the reference period of income refers to the calendar year before the year in which the survey took place.

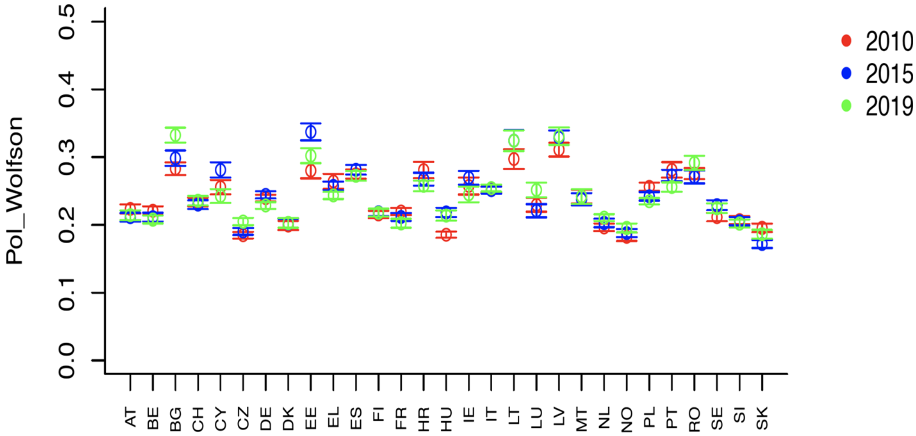

Fig. 1 Foster and Wolfson polarization index and 95% confidence intervals, for European countries and years 2010, 2015 and 2019. *Source*: Our elaboration on EU-SILC dataset

$$\widehat{V}(y) = \frac{1}{\left(1 + n\sum_{s=2}^{n-1}\frac{1}{s}\right)\mu(y)}\sum_{i=1}^{n-1}\frac{1}{i}\sum_{j=n-i+1}^{n} w_j y_j - 1 \tag{15}$$

where the sums are up to $n-1$ to ensure the accuracy of the indices and w_j are the sample weights.[7]

Sampling error and confidence intervals for polarization indices based on Bonferroni and De Vergottini measures have been estimated using bootstrap techniques, based on 100 replications of samples with size 1000.[8] Estimates related to Foster and Wolfson polarization measure have been obtained using DASP - Distributive Analysis Stata Package [2].

Figure 1 depicts bipolarization estimates, as well as their confidence intervals, according to Foster and Wolfson index. The corresponding numbers are available in Table 4 of the Appendix. The picture reveals that the most polarized countries in Europe are the Baltic countries (Estonia, Lithuania, Latvia) and Bulgaria, while the least polarized countries are in the Scandinavian region (Norway, Finland) and in eastern Europe (Hungary, Czech Republic, Slovakia). A similar pattern occurs also for inequality, as measured by the Gini index and shown in Table 1 in the Appendix: the most polarized countries, indeed, exhibit the highest between-group inequality levels. Over the decade under consideration, some countries registered a significant increase in polarization (in particular, Lithuania, Latvia, Bulgaria, Romania, Hungary), mainly due to a significant increase in the between-group component as shown in Table 1, while other countries remained quite stable. In none of the European countries polarization has significantly decreased.

When estimating polarization with the measure based on Bonferroni index (Fig. 2; Table 5), we observe that the countries with the highest and the lowest levels of polarization are basically the same detected by Foster and Wolfson index (in Fig. 1; Table 4). However, changes over time are more pronounced with the Bonferroni-based index, revealing that

7 These weights are obtained starting from the inverse of the inclusion probability of the family, corrected for the overall non-response rate.

8 In the implemented bootstrap procedure, sampling weights have been taken into account. In particular, we have replicated each sample unit according to its individual sample weight and then performed the bootstrap procedure with 100 replications of samples with size 1000. The R code for implementing the new polarization measures is available upon request.

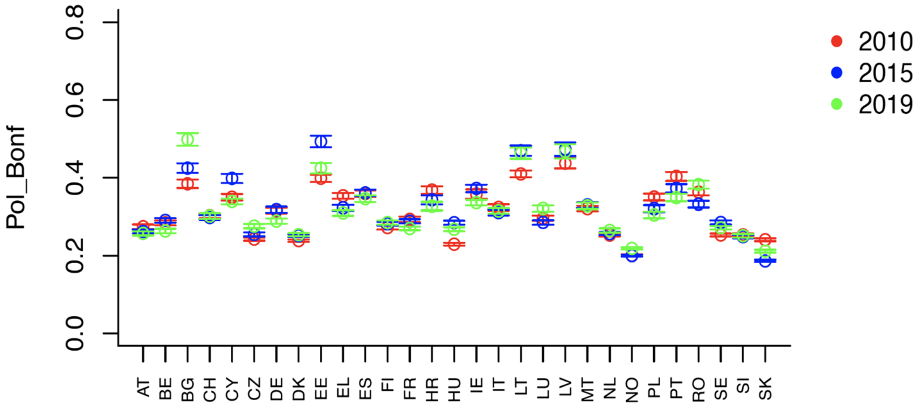

Fig. 2 Bonferroni-based polarization index and 95% bootstrap confidence intervals, for European countries and years 2010, 2015 and 2019. *Source*: Our elaboration on EU-SILC dataset

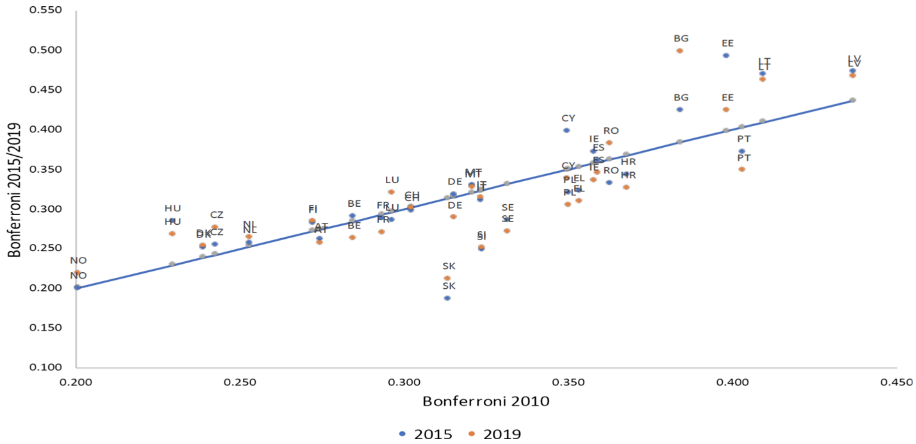

Fig. 3 Bonferroni-based polarization across European countries, comparison over time. *Source*: Our elaboration on EU-SILC dataset

significant changes have occurred mainly in the bottom part of the income distribution. This is also confirmed by Fig. 3, which plots the polarization levels of each country in 2015 and in 2019 compared to the level at the beginning of the period of analysis, the year 2010. We note, in particular, that the increases registered in Bulgaria, Lithuania and Latvia are more pronounced when we attach higher weight to lower incomes. According to the Bonferroni-based index some countries registered a reduction in polarization from 2010 to 2015 or 2019 (in particular, Slovakia), mainly due to a decrease in the between-group inequality component, as shown in Table 2.

Moving now to analysing polarization pattern based on De Vergottini measure (see Figs. 4, 5 as well as Table 6 in the Appendix) we note a quite different trend overtime if compared to the previous polarization indices. When we attach higher importance to changes occurring at the top incomes, polarization seems to increase mainly in the Baltic countries (Latvia, Lithuania, Estonia), in Slovakia and also in some of the southern European countries (such as Spain and Greece). On the contrary, polarization decreases mainly in Bulgaria (due to a

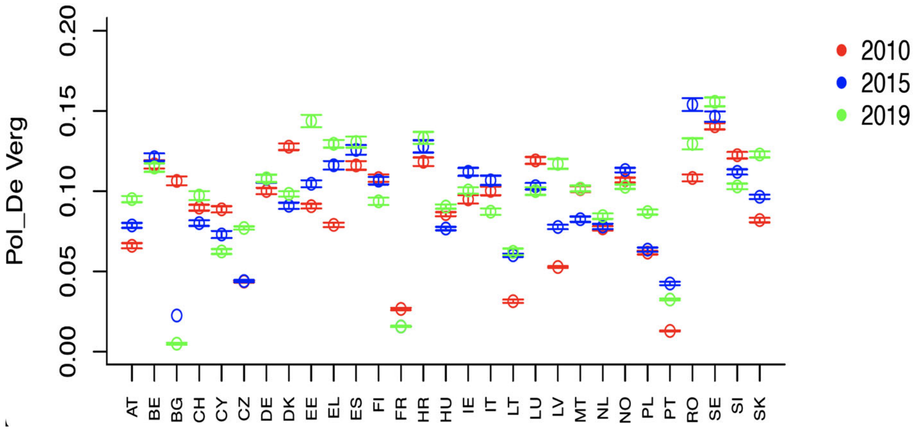

Fig. 4 De Vergottini-based polarization index and 95% bootstrap confidence intervals, for European countries and years 2010, 2015 and 2019. *Source*: Our elaboration on EU-SILC dataset

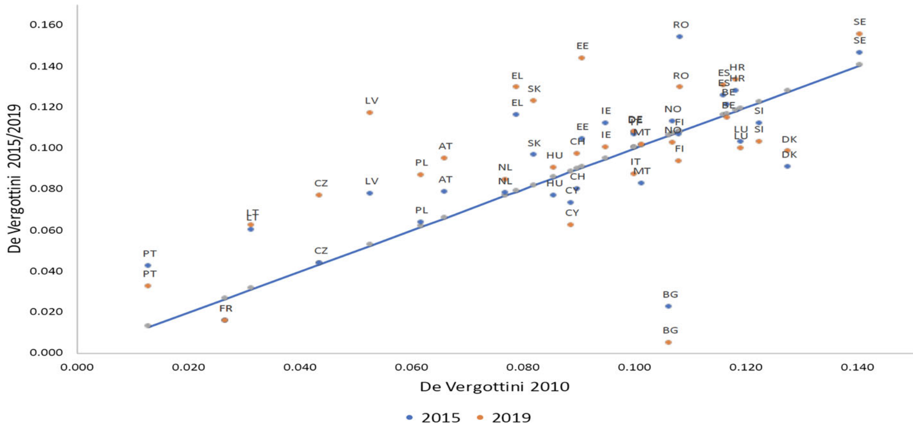

Fig. 5 De Vergottini-based polarization across European countries, comparison over time. *Source*: Our elaboration on EU-SILC dataset

reduction in the between-group component, as shown in Table 3), Cyprus and Denmark (as a consequence of an increase in the within-group inequality).

We now compare the rankings of the European countries provided by the three polarization measures (see Figs. 6, 7). The countries in the top positions of the ranking exhibit lower polarization levels. For most countries the rankings provided by the three polarization indices are very similar, while in a bunch of countries the rankings strongly reverse if we change polarization measure. In particular, the Scandinavian countries (Norway, Sweden, Finland) and some northern countries (Belgium, Denmark, Luxembourg) are among the least polarized countries according to the Foster and Wolfson and to the Bonferroni-based measures, that is when we attach more weight to bottom or middle incomes, while they move to the group of the highest polarized countries, when we attach more weight to top incomes following the De Vergottini approach. These results apparently seem in contradiction, but actually reveal that polarization in these countries is mainly due to inequalities in top incomes rather than in inequalities among bottom incomes. This is also confirmed by Table 3, according to

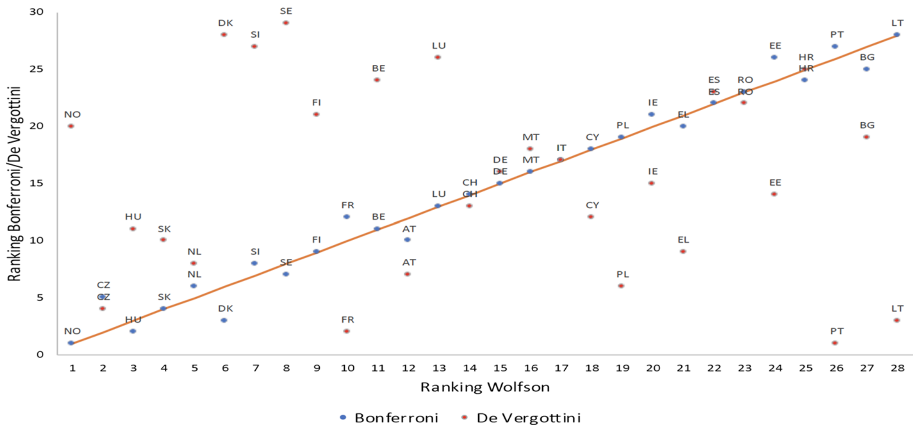

Fig. 6 Ranking comparison of the European countries, year 2010. *Source*: Our elaboration on EU-SILC dataset *Note*: Top positions in the ranking correspond to lower polarization levels

Fig. 7 Ranking comparison of the European countries, year 2019. *Source*: Our elaboration on EU-SILC dataset *Note*: Top positions in the ranking correspond to lower polarization levels

which the above-mentioned countries reveal high levels of De Vergottini inequality measure. On the contrary, the Baltic countries as well as some southern countries (Greece, Portugal, Cyprus) are classified among the least polarized countries according to De Vergottini-based measure but among the most polarized according to Bonferroni-based and Foster and Wolfson measures. Here again, the reason of this discrepancy is mainly due to the fact that in these countries the within group inequality is higher among the poorer than among the richer.

In particular, if we focus on year 2019, Fig. 7 shows that Bulgaria, Latvia, Lithuania and Portugal are at the bottom of the ranking provided by the Bonferroni-based polarization measure (i.e. high polarization) but at the same time at the top of of the ranking provided by De Vergottini (i.e. low polarization); this is due to the fact that these countries exhibit the highest level of inequality within the group of the poor. On the contrary, countries that are at the top of the Bonferroni ranking (such as Slovakia, Slovenia and Norway) fall instead to the bottom part of the De Vergottini ranking, since they registered the highest levels of inequality in the group of the rich, to which De Vergottini-based measure attaches more importance.

In conclusion, the bipolarization indices that we propose in this empirical application provide more insights than the traditional analysis of income polarization, by introducing the flexibility of attaching different weights to the different part of income distribution.

4 Concluding remarks

In this paper we have analyzed how income polarization has changed over the last decade across European countries by means of new Bonferroni-based and De Vergottini-based polarization measures proposed in Ciommi et al. [12]. Different from traditional measures of polarization, the indices proposed in this paper are sensitive to progressive transfers, attaching more importance to some part of the income distribution. In particular, Bonferroni-based polarization index is more sensitive to income transfers involving poorer individuals, while De Vergottini-based measure to transfers occurring among richer people.

In the empirical application we compared European countries over the period 2010–2019 using EU-SILC data. Results revealed significant changes in polarization over the last decades for most countries. The empirical application showed how the new indices enrich the analysis based on standard measures of income polarization and how they contribute to disentangle the different faces of income inequality and polarization. Here we have focused on the case of two groups separated by the median. Future research may explore two additional directions: extending the new indices for (i) the presence of more than two groups, and (ii) groups that may also overlap. These extensions may accommodate researchers who assume that the population is made up of more than two groups, even if the groups are formed by characteristics other than income—a situation that generally introduces some overlap between the groups.

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Declarations

Conflict of interest The authors have no conflicting interests to disclose.

Appendix

See Tables 1, 2, 3, 4, 5 and 6.

Table 1 Gini inequality index and its subgroup decomposition, for European countries and years 2010, 2015 and 2019. *Source*: Our elaboration on EU-SILC dataset

Country	2010			2015			2019		
	Ov	Be	Wi	Ov	Be	Wi	Ov	Be	Wi
AT	0.270	0.183	0.087	0.250	0.171	0.078	0.255	0.175	0.080
BE	0.246	0.173	0.072	0.246	0.174	0.072	0.234	0.165	0.070
BG	0.311	0.221	0.091	0.326	0.229	0.096	0.342	0.243	0.100
CH	0.271	0.187	0.084	0.264	0.178	0.086	0.268	0.184	0.084
CY	0.290	0.204	0.086	0.301	0.210	0.091	0.282	0.198	0.084
CZ	0.213	0.149	0.064	0.218	0.152	0.066	0.228	0.163	0.065
DE	0.260	0.177	0.083	0.265	0.183	0.082	0.254	0.176	0.077
DK	0.209	0.129	0.080	0.225	0.132	0.093	0.216	0.141	0.075
EE	0.276	0.193	0.082	0.314	0.222	0.092	0.299	0.215	0.084
EL	0.294	0.209	0.085	0.291	0.203	0.088	0.260	0.186	0.074
ES	0.301	0.210	0.091	0.307	0.212	0.095	0.297	0.207	0.090
FI	0.257	0.168	0.088	0.248	0.158	0.090	0.248	0.151	0.098
FR	0.266	0.178	0.089	0.256	0.173	0.082	0.295	0.212	0.083
HR	0.317	0.226	0.091	0.306	0.218	0.088	0.236	0.168	0.068
HU	0.221	0.155	0.066	0.238	0.168	0.070	0.236	0.168	0.068
IE	0.276	0.197	0.079	0.282	0.203	0.080	0.270	0.191	0.079
IT	0.282	0.194	0.089	0.291	0.194	0.097	0.291	0.194	0.096
LT	0.318	0.209	0.109	0.323	0.220	0.103	0.331	0.227	0.104
LU	0.263	0.186	0.077	0.251	0.177	0.074	0.282	0.196	0.086
LV	0.328	0.231	0.097	0.334	0.240	0.094	0.348	0.253	0.095
MT	0.266	0.189	0.077	0.260	0.183	0.076	0.263	0.188	0.075
NL	0.213	0.132	0.081	0.222	0.136	0.087	0.233	0.159	0.074
NO	0.209	0.138	0.071	0.215	0.140	0.075	0.225	0.144	0.081
PL	0.286	0.201	0.085	0.280	0.197	0.083	0.269	0.191	0.078
PT	0.324	0.228	0.096	0.316	0.220	0.096	0.309	0.215	0.094
RO	0.296	0.208	0.088	0.295	0.208	0.087	0.307	0.216	0.091
SE	0.217	0.143	0.074	0.233	0.153	0.080	0.236	0.157	0.078
SI	0.224	0.151	0.073	0.228	0.156	0.072	0.222	0.151	0.070
SK	0.222	0.153	0.069	0.203	0.141	0.062	0.208	0.148	0.060

Ov indicates overall inequality, while Be and Wi are the between-group and within-group components, respectively. Estimates have been calculated using `ineqdecgini` Stata Package

Table 2 Bonferroni inequality index and its subgroup decomposition, for European countries and years 2010, 2015 and 2019. *Source*: Our elaboration on EU-SILC dataset

Country	2010				2015				2019			
	Ov	Be	Wi	Re	Ov	Be	Wi	Re	Ov	Be	Wi	Re
AT	0.365	0.245	0.110	0.009	0.341	0.231	0.102	0.008	0.346	0.233	0.103	0.010
BE	0.325	0.228	0.093	0.003	0.324	0.229	0.093	0.002	0.311	0.218	0.088	0.005
BG	0.410	0.290	0.121	−0.001	0.432	0.307	0.132	−0.007	0.449	0.326	0.139	−0.016
CH	0.358	0.248	0.106	0.004	0.356	0.246	0.106	0.004	0.363	0.251	0.107	0.006
CY	0.374	0.265	0.110	−0.002	0.386	0.279	0.116	−0.009	0.353	0.252	0.105	−0.004
CZ	0.296	0.203	0.091	0.001	0.300	0.208	0.092	0.000	0.308	0.217	0.091	0.000
DE	0.362	0.253	0.107	0.002	0.371	0.258	0.109	0.004	0.356	0.245	0.104	0.007
DK	0.321	0.216	0.093	0.012	0.333	0.225	0.099	0.009	0.328	0.223	0.097	0.008
EE	0.388	0.278	0.119	−0.009	0.428	0.316	0.128	−0.015	0.409	0.299	0.117	−0.006
EL	0.384	0.271	0.114	−0.001	0.400	0.274	0.119	0.007	0.367	0.254	0.107	0.006
ES	0.405	0.282	0.120	0.002	0.423	0.292	0.126	0.005	0.406	0.281	0.120	0.005
FI	0.322	0.224	0.092	0.005	0.322	0.226	0.094	0.002	0.320	0.225	0.094	0.001
FR	0.344	0.238	0.107	0.000	0.336	0.233	0.104	−0.001	0.330	0.226	0.103	0.001
HR	0.417	0.291	0.124	0.003	0.403	0.280	0.118	0.005	0.393	0.272	0.114	0.007
HU	0.297	0.202	0.088	0.007	0.344	0.236	0.104	0.003	0.342	0.233	0.102	0.007
IE	0.361	0.259	0.108	−0.005	0.368	0.267	0.107	−0.005	0.343	0.247	0.100	−0.004
IT	0.381	0.264	0.113	0.004	0.390	0.266	0.115	0.008	0.403	0.273	0.121	0.009
LT	0.435	0.303	0.138	−0.006	0.435	0.314	0.134	−0.013	0.443	0.318	0.136	−0.010
LU	0.333	0.235	0.097	0.002	0.338	0.235	0.099	0.004	0.388	0.267	0.116	0.005
LV	0.433	0.307	0.135	−0.009	0.436	0.317	0.131	−0.012	0.450	0.326	0.133	−0.008
MT	0.355	0.250	0.106	0.000	0.344	0.247	0.101	−0.004	0.350	0.249	0.102	−0.002
NL	0.307	0.212	0.093	0.003	0.322	0.220	0.097	0.005	0.330	0.226	0.098	0.005
NO	0.314	0.205	0.092	0.017	0.333	0.215	0.098	0.020	0.334	0.219	0.099	0.016

Table 2 continued

Country	2010				2015				2019			
	Ov	Be	Wi	Re	Ov	Be	Wi	Re	Ov	Be	Wi	Re
PL	0.383	0.269	0.116	−0.002	0.378	0.261	0.115	0.002	0.365	0.252	0.109	0.004
PT	0.407	0.289	0.126	−0.009	0.411	0.287	0.127	−0.003	0.390	0.272	0.121	−0.002
RO	0.410	0.286	0.122	0.002	0.424	0.289	0.124	0.011	0.432	0.301	0.128	0.003
SE	0.332	0.226	0.094	0.011	0.342	0.238	0.097	0.007	0.343	0.236	0.097	0.011
SI	0.324	0.221	0.094	0.009	0.333	0.226	0.097	0.011	0.320	0.219	0.093	0.008
SK	0.314	0.212	0.096	0.006	0.298	0.193	0.090	0.015	0.302	0.201	0.088	0.013

Ov indicates overall inequality, while Be, Wi and Re are the between-group, within-group and residual components, respectively

Table 3 De Vergottini inequality index and its subgroup decomposition, for European countries and years 2010, 2015 and 2019. *Source*: Our elaboration on EU-SILC dataset

Country	2010				2015				2019			
	Ov	Be	Wi	Re	Ov	Be	Wi	Re	Ov	Be	Wi	Re
AT	0.461	0.245	0.136	0.080	0.425	0.231	0.124	0.071	0.344	0.201	0.096	0.047
BE	0.402	0.228	0.111	0.063	0.402	0.229	0.111	0.062	0.381	0.218	0.106	0.058
BG	0.533	0.290	0.151	0.092	0.604	0.307	0.177	0.120	0.389	0.219	0.113	0.057
CH	0.456	0.248	0.131	0.077	0.456	0.246	0.132	0.078	0.389	0.219	0.111	0.060
CY	0.491	0.265	0.140	0.086	0.526	0.279	0.148	0.099	0.398	0.236	0.109	0.053
CZ	0.386	0.203	0.112	0.071	0.396	0.208	0.114	0.073	0.398	0.217	0.112	0.069
DE	0.460	0.253	0.131	0.076	0.467	0.258	0.133	0.076	0.400	0.223	0.114	0.064
DK	0.373	0.216	0.104	0.053	0.407	0.225	0.116	0.066	0.408	0.225	0.115	0.068
EE	0.518	0.278	0.143	0.097	0.588	0.316	0.161	0.112	0.413	0.226	0.118	0.069
EL	0.506	0.271	0.145	0.091	0.494	0.274	0.141	0.080	0.421	0.233	0.122	0.067
ES	0.513	0.282	0.145	0.086	0.528	0.292	0.149	0.087	0.424	0.233	0.121	0.070
FI	0.398	0.224	0.112	0.062	0.403	0.226	0.112	0.065	0.440	0.245	0.126	0.070
FR	0.464	0.238	0.137	0.089	0.459	0.233	0.137	0.089	0.446	0.226	0.133	0.086
HR	0.529	0.291	0.150	0.088	0.501	0.280	0.142	0.080	0.449	0.247	0.125	0.077
HU	0.364	0.202	0.104	0.058	0.438	0.236	0.125	0.076	0.449	0.254	0.125	0.070
IE	0.476	0.259	0.132	0.085	0.483	0.267	0.133	0.084	0.453	0.249	0.127	0.076
IT	0.482	0.264	0.137	0.081	0.483	0.266	0.139	0.079	0.457	0.251	0.131	0.075
LT	0.593	0.303	0.172	0.118	0.604	0.314	0.171	0.119	0.465	0.252	0.133	0.079
LU	0.414	0.235	0.115	0.065	0.423	0.235	0.119	0.068	0.477	0.252	0.137	0.088
LV	0.593	0.307	0.169	0.116	0.601	0.317	0.168	0.116	0.482	0.272	0.136	0.074
MT	0.454	0.250	0.127	0.077	0.456	0.247	0.129	0.081	0.488	0.267	0.139	0.082
NL	0.388	0.212	0.110	0.066	0.404	0.220	0.116	0.069	0.503	0.281	0.141	0.081

Table 3 continued

Country	2010				2015				2019			
	Ov	Be	Wi	Re	Ov	Be	Wi	Re	Ov	Be	Wi	Re
NO	0.359	0.205	0.104	0.050	0.375	0.215	0.109	0.051	0.507	0.273	0.148	0.086
PL	0.510	0.269	0.147	0.094	0.494	0.261	0.144	0.089	0.529	0.272	0.155	0.102
PT	0.573	0.289	0.167	0.116	0.555	0.287	0.162	0.106	0.534	0.299	0.146	0.089
RO	0.523	0.286	0.149	0.089	0.507	0.289	0.143	0.074	0.545	0.301	0.153	0.090
SE	0.385	0.226	0.107	0.053	0.407	0.238	0.111	0.058	0.601	0.326	0.168	0.107
SI	0.385	0.221	0.108	0.056	0.398	0.226	0.114	0.059	0.610	0.318	0.173	0.119
SK	0.386	0.212	0.110	0.064	0.341	0.193	0.098	0.050	0.651	0.326	0.189	0.135

Ov indicates overall inequality, while Be, Wi and Re are the between-group, within-group and residual components, respectively

Table 4 Foster and Wolfson polarization index and and 95% confidence intervals, for European countries and years 2010, 2015 and 2019. *Source*: Our elaboration on EU-SILC dataset

Country	2010			2015			2019		
	P	LB 95%	UB 95%	P	LB 95%	UB 95%	P	LB 95%	UB 95%
AT	0.223	0.217	0.230	0.211	0.205	0.218	0.214	0.207	0.221
BE	0.221	0.214	0.227	0.211	0.205	0.218	0.208	0.202	0.214
BG	0.283	0.273	0.292	0.298	0.287	0.310	0.332	0.321	0.343
CH	0.234	0.227	0.240	0.230	0.223	0.236	0.235	0.228	0.242
CY	0.256	0.245	0.266	0.281	0.270	0.292	0.242	0.232	0.253
CZ	0.185	0.180	0.190	0.191	0.185	0.196	0.205	0.200	0.210
DE	0.239	0.234	0.244	0.244	0.239	0.249	0.228	0.223	0.233
DK	0.199	0.193	0.206	0.202	0.196	0.209	0.203	0.196	0.210
EE	0.280	0.269	0.291	0.337	0.325	0.350	0.302	0.291	0.313
EL	0.264	0.253	0.275	0.257	0.251	0.264	0.243	0.238	0.249
ES	0.275	0.268	0.281	0.281	0.274	0.288	0.272	0.265	0.279
FI	0.215	0.210	0.221	0.218	0.213	0.224	0.218	0.212	0.224
FR	0.220	0.214	0.225	0.211	0.206	0.217	0.202	0.196	0.209
HR	0.281	0.269	0.293	0.267	0.258	0.277	0.257	0.249	0.265
HU	0.186	0.181	0.190	0.218	0.211	0.225	0.213	0.206	0.221
IE	0.257	0.245	0.270	0.269	0.259	0.280	0.245	0.233	0.256
IT	0.252	0.248	0.257	0.251	0.246	0.256	0.254	0.249	0.259
LT	0.297	0.282	0.312	0.324	0.309	0.340	0.324	0.309	0.339
LU	0.229	0.219	0.240	0.221	0.211	0.230	0.251	0.240	0.262
LV	0.311	0.301	0.321	0.329	0.318	0.339	0.331	0.318	0.344
MT	0.242	0.232	0.252	0.238	0.229	0.247	0.242	0.231	0.253
NL	0.196	0.191	0.202	0.203	0.197	0.209	0.210	0.205	0.216
NO	0.183	0.176	0.189	0.188	0.182	0.194	0.195	0.189	0.202
PL	0.256	0.250	0.262	0.242	0.236	0.248	0.235	0.230	0.239
PT	0.281	0.270	0.292	0.273	0.265	0.281	0.256	0.249	0.263
RO	0.275	0.267	0.284	0.271	0.261	0.280	0.291	0.281	0.302
SE	0.211	0.205	0.217	0.229	0.222	0.236	0.224	0.217	0.232
SI	0.207	0.201	0.213	0.205	0.199	0.211	0.202	0.196	0.208
SK	0.196	0.189	0.202	0.172	0.166	0.178	0.186	0.180	0.193

P indicates the point estimate, while LB 95% and UB 95% are, respectively, the lower and upper bounds of a 95% confidence interval. Estimates have been calculated using DASP: Distributive Analysis Stata Package; see [2]

Table 5 Bonferroni-based polarization index and and 95% bootstrap confidence intervals, for European countries and years 2010, 2015 and 2019. *Source*: Our elaboration on EU-SILC dataset

Country	2010			2015			2019		
	P	LB 95%	UB 95%	P	LB 95%	UB 95%	P	LB 95%	UB 95%
AT	0.275	0.269	0.280	0.262	0.257	0.267	0.257	0.252	0.262
BE	0.285	0.278	0.291	0.291	0.285	0.297	0.264	0.258	0.269
BG	0.384	0.374	0.395	0.425	0.412	0.437	0.498	0.482	0.515
CH	0.302	0.297	0.308	0.298	0.293	0.304	0.302	0.294	0.309
CY	0.350	0.342	0.358	0.398	0.386	0.410	0.339	0.331	0.346
CZ	0.243	0.239	0.247	0.255	0.250	0.260	0.276	0.271	0.281
DE	0.315	0.308	0.322	0.318	0.310	0.326	0.289	0.282	0.296
DK	0.239	0.236	0.242	0.252	0.248	0.256	0.254	0.250	0.258
EE	0.398	0.389	0.408	0.493	0.478	0.508	0.424	0.411	0.438
EL	0.353	0.346	0.361	0.323	0.315	0.330	0.310	0.303	0.317
ES	0.359	0.351	0.368	0.361	0.352	0.369	0.346	0.337	0.355
FI	0.272	0.267	0.277	0.282	0.277	0.287	0.285	0.280	0.290
FR	0.293	0.287	0.300	0.289	0.283	0.294	0.270	0.265	0.275
HR	0.368	0.359	0.378	0.343	0.331	0.355	0.327	0.316	0.338
HU	0.230	0.226	0.233	0.285	0.280	0.290	0.268	0.263	0.273
IE	0.358	0.346	0.370	0.372	0.363	0.382	0.336	0.329	0.343
IT	0.323	0.315	0.332	0.311	0.303	0.318	0.315	0.308	0.322
LT	0.410	0.401	0.418	0.469	0.456	0.482	0.463	0.448	0.477
LU	0.296	0.290	0.303	0.286	0.280	0.291	0.321	0.313	0.329
LV	0.437	0.424	0.450	0.473	0.456	0.491	0.467	0.449	0.486
MT	0.321	0.314	0.328	0.330	0.323	0.337	0.328	0.320	0.336
NL	0.253	0.250	0.257	0.257	0.253	0.261	0.265	0.259	0.270
NO	0.201	0.198	0.203	0.200	0.198	0.202	0.219	0.216	0.221
PL	0.350	0.341	0.359	0.321	0.312	0.330	0.305	0.296	0.313
PT	0.403	0.392	0.415	0.372	0.362	0.383	0.349	0.339	0.358
RO	0.363	0.354	0.371	0.332	0.323	0.341	0.382	0.372	0.392
SE	0.253	0.250	0.257	0.285	0.280	0.291	0.272	0.268	0.276
SI	0.254	0.250	0.257	0.249	0.245	0.253	0.251	0.247	0.256
SK	0.241	0.237	0.245	0.187	0.184	0.189	0.212	0.208	0.215

P indicates the point estimate, while LB 95% and UB 95% are, respectively, the lower and upper bounds of a bootstrap 95% confidence interval

Table 6 De Vergottini-based polarization index and and 95% bootstrap confidence intervals, for European countries and years 2010, 2015 and 2019. *Source*: Our elaboration on EU-SILC dataset

Country	2010			2015			2019		
	P	LB 95%	UB 95%	P	LB 95%	UB 95%	P	LB 95%	UB 95%
AT	0.066	0.064	0.068	0.079	0.077	0.080	0.123	0.121	0.125
BE	0.117	0.114	0.119	0.121	0.119	0.124	0.115	0.112	0.117
BG	0.106	0.104	0.109	0.023	0.023	0.023	0.103	0.101	0.104
CH	0.090	0.088	0.092	0.080	0.078	0.082	0.103	0.101	0.105
CY	0.089	0.087	0.091	0.073	0.071	0.075	0.156	0.153	0.158
CZ	0.044	0.043	0.044	0.044	0.043	0.045	0.077	0.076	0.078
DE	0.100	0.098	0.102	0.108	0.105	0.110	0.098	0.097	0.100
DK	0.128	0.126	0.130	0.091	0.089	0.093	0.094	0.091	0.096
EE	0.091	0.089	0.092	0.105	0.102	0.107	0.084	0.083	0.086
EL	0.079	0.078	0.080	0.116	0.113	0.119	0.095	0.093	0.097
ES	0.116	0.113	0.119	0.126	0.123	0.129	0.090	0.089	0.092
FI	0.108	0.106	0.110	0.107	0.104	0.109	0.108	0.106	0.110
FR	0.027	0.026	0.027	0.016	0.015	0.016	0.016	0.015	0.016
HR	0.118	0.116	0.121	0.128	0.124	0.132	0.100	0.098	0.102
HU	0.086	0.084	0.087	0.077	0.076	0.078	0.129	0.127	0.132
IE	0.095	0.092	0.098	0.112	0.110	0.114	0.102	0.100	0.104
IT	0.100	0.097	0.103	0.107	0.104	0.110	0.097	0.094	0.100
LT	0.031	0.030	0.032	0.060	0.059	0.061	0.087	0.085	0.089
LU	0.119	0.117	0.121	0.103	0.101	0.105	0.062	0.061	0.064
LV	0.053	0.052	0.053	0.078	0.076	0.079	0.133	0.129	0.137
MT	0.101	0.100	0.103	0.083	0.081	0.084	0.100	0.098	0.102
NL	0.077	0.076	0.078	0.078	0.076	0.080	0.131	0.127	0.134
NO	0.107	0.105	0.108	0.113	0.112	0.115	0.087	0.085	0.089
PL	0.062	0.061	0.063	0.064	0.062	0.065	0.032	0.032	0.033
PT	0.013	0.013	0.013	0.042	0.041	0.043	0.144	0.140	0.148
RO	0.108	0.106	0.110	0.154	0.150	0.158	0.129	0.126	0.133
SE	0.140	0.138	0.142	0.146	0.143	0.150	0.117	0.114	0.120
SI	0.122	0.120	0.124	0.112	0.110	0.114	0.062	0.060	0.064
SK	0.082	0.081	0.083	0.096	0.095	0.098	0.005	0.004	0.006

P indicates the point estimate, while LB 95% and UB 95% are, respectively, the lower and upper bounds of a bootstrap 95% confidence interval

References

1. Anderson, G.: Toward an empirical analysis of polarization. J. Econom. **122**, 1–26 (2004)
2. Araar, A., Duclos, J.-Y.: DASP: Distributive analysis stata package—user manual DASP version 3.03. Universitè Laval, PEP and World Bank (2022)
3. Bàrcena-Martìn, E., Imedio, L.J.: The Bonferroni, Gini and De Vergottini indices. Inequality, welfare and deprivation in the European Union in 2000. Res. Econ. Inequality **16**, 231–257 (2008)
4. Bàrcena-Martìn, E., Silber, J.: On the concepts of Bonferroni segregation index and curve. Riv. Ital. Econ. Demogr. Stat. **65**(2), 57–74 (2011)
5. Bàrcena-Martìn, E., Silber, J.: On the generalization and decomposition of the Bonferroni index. Soc. Choice Welf. **41**, 763–787 (2013)

6. Bàrcena-Martìn, E., Silber, J.: The Bonferroni index and the measurement of distributional change. Metron **75**(1), 1–16 (2017)
7. Bonferroni, C.E.: Elementi di Statistica Generale. Seber, Firenze (1930)
8. Chakravarty, S.R.: A deprivation-based axiomatic characterization of the absolute Bonferroni index of inequality. J. Econ. Theory **5**(3), 339–351 (2007)
9. Chakravarty, S.R., D'Ambrosio, C.: Polarization orderings of income distributions. Rev. Income Wealth **56**, 47–64 (2010)
10. Chakravarty, S.R., Muliere, P.: Welfare indicators: a review and new perspectives. 2. Measurement of poverty. Metron **62**(2), 247–281 (2004)
11. Chakravarty, S.R., Majumder, A.: Inequality, polarization and welfare: Theory and applications. Austral. Econ. Pap. **40**, 1–13 (2001)
12. Ciommi, M., Gigliarano, C., Giorgi, G.M.: Bonferroni and De Vergottini are back: new subgroup decompositions and bipolarization measures. Fuzzy Sets Syst. **433**, 22–53 (2022)
13. De Vergottini, M.: Methods of measuring the concentration of wealth. Statistica **X**(4), 445–454 (1950)
14. Dong, Z., Tillé, Y., Giorgi, G.M., Guandalini, A.: Linearization and variance estimation of the Bonferroni inequality index. J. R. Stat. Soc. Ser. A **184**(3), 1008–1029 (2021)
15. Esteban, J., Ray, D.: On the measurement of polarization. Econometrica **62**(4), 819–851 (1994)
16. Foster, J.E., Wolfson, M.C.: Polarization and the decline of the Middle Class: Canada and the US, OPHI Working Paper, No. 31. University of Oxford (1992)
17. Foster, J.E., Wolfson, M.C.: Polarization and the decline of the Middle Class: Canada and the US. J. Econ. Inequal. **8**, 247–273 (2010)
18. Gasparini, L., Horenstein, E., Molina, E., Olivieri, S.: Income polarization in Latin America: patterns and links with institutions and conflict. Oxf. Dev. Stud. **36**(4), 463–84 (2008)
19. Gini, C.: Variabilitá e Mutabilitá: Contributo Allo Studio Delle Distribuzioni e Relazioni Statistiche, Vol. III (part II). Bologna: Cuppini (1912)
20. Giorgi, G.M.: Il rapporto di concentrazione di Gini: genesi, evoluzione ed una bibliografia commentata. Libreria Editrice Ticci, Siena (1992)
21. Giorgi, G.M., Crescenzi, M.: A look at the Bonferroni inequality measure in a reliability framework. Statistica **61**(4), 571–583 (2001)
22. Gigliarano, C., Mosler, K.: Constructing indices of multivariate polarization. J. Econ. Inequal. **7**, 435–460 (2009)
23. Gigliarano, C., Nowak, D., Mosler, K.: A polarization index for overlapping groups. Rev. Income Wealth **65**(4), 712–735 (2019)
24. Gigliarano, C.: Income and social polarization: empirical findings. In: C. D'Ambrosio (ed.), Handbook of Research on Economic and Social Well-Being, pp. 460–479. Edward Elgar Publishing (2018)
25. Gradín, C.: Polarization by sub-populations in Spain, 1973–91. Rev. Income Wealth **46**, 457–474 (2000)
26. Lambert, P.J., Aronson, R.J.: Inequality decomposition analysis and the Gini coefficient revisited. Econ. J. **103**(420), 1221–1227 (1993)
27. Lasso de la Vega, C., Urrutia, A.: An alternative formulation of the Esteban–Gradin–Ray extended measure of polarization. J. Income Distrib. **15**, 42–54 (2006)
28. Nygard, F., Sandstrom, A.: Measuring income inequality. Acta Universitatis Stockholmiensis, Almqvist & Wiksell International Stockholm (1981)
29. Okamoto, M.: Decomposition of Gini and multivariate Gini indices. J. Econ. Inequal. **7**(2), 153–177 (2009)
30. Permanyer, I.: Income and social polarization: theoretical approaches. In: C. D'Ambrosio (ed.), Handbook of Research on Economic and Social Well-Being, pp. 434–459. Edward Elgar Publishing (2018)
31. Piketty, T.: About capital in the twenty-first century. Am. Econ. Rev. **105**(5), 48–53 (2015)
32. Pittau, M.G., Zelli, R.: Mixture models, convergence clubs and polarization. Rev. Income Wealth **56**, 102–121 (2010)
33. Rodrìguez, J.G., Salas, R.: Extended bi-polarization and inequality measures. Res. Econ. Inequal. **9**, 69–83 (2003)
34. Tarsitano, A.: Methods of measuring the concentration of wealth. In: Dagum, C., Zenga, M. (eds.) Income and Wealth Distribution, Inequality and Poverty, pp. 228–242. Springer-Verlag, Berlin (1990)
35. Tarsitano, A.: A new class of inequality measures based on a ratio of L-statistics. Metron **62**(1–2), 137–160 (2004)
36. Shorrocks, A.F.: The class of additively decomposable iInequality measures. Econometrica **48**, 613–625 (1980)
37. Silber, J., Deutsch, J., Hanoka, M.: On the link between the concepts of kurtosis and bipolarization. Econ. Bull. **4**, 1–6 (2007)

38. Wang, Y., Tsui, K.: Polarization orderings and new classes of polarization indices. J. Public Econ. Theory **2**, 349–363 (2000)
39. Wolfson, M.C.: When inequality diverge. Am. Econ. Rev. **84**(2), 353–358 (1994)
40. Wolfson, M.C.: Divergent inequalities: theory and empirical results. Rev. Income Wealth **43**(4), 401–421 (1997)
41. Yitzhaki, S.: More than a dozen alternative ways of spelling Gini. Res. Econ. Inequal. **8**, 13–30 (1998)