

Enhancing Noisy Inventory Data Accuracy in Perishable Supply Chains Using a Fast and Robust Observer Algorithm

Valentina Orsini ^a

Department of Information Engineering, Università Politecnica delle Marche, UNIVPM, Ancona, Italy

Keywords: Perishable Supply Chains, Inventory Inaccuracy, Robust Inventory Estimation.

Abstract: Reliable inventory data is an essential prerequisite for an effective management of supply chains. The purpose of this paper is to propose an efficient, cost-effective alternative to the widely used conventional methods based on high-cost sensor technologies. The proposed new approach relies on integrating a physical sensor with a custom-built estimation algorithm designed to derive accurate inventory data from inherently noisy sensor readings. The two key advantages are: an accurate, low-cost inventory estimation despite measurement noise, the capacity to regulate the convergence speed of the inventory estimation error toward zero.

1 INTRODUCTION

Inventory record inaccuracy poses a significant challenge to the formulation of effective replenishment strategies, particularly within supply chains handling perishable goods (Grunow and Piramuthu, 2013; Dolgui et al., 2018).

Employing high-precision sensors is a widely adopted approach for achieving accurate inventory tracking (Bertolini et al., 2013; Delen et al., 2011; Piramuthu and Zhou, 2013; Sarac et al., 2010). However, despite its hardware-centric nature, this solution is not without limitations. Challenges include the absence of universal standards, significant financial costs, susceptibility to signal interference, rapid technological obsolescence, and cybersecurity vulnerabilities (Kaur et al., 2011; Mahmoud et al., 2019).

Furthermore, even the most advanced sensors inevitably introduce a non-negligible, albeit minimal, measurement error (Goyal et al., 2009).

In this paper, we introduce an efficient, cost-effective alternative. Our approach employs a hybrid sensing architecture that integrates a physical sensor with a state observer. The observer refines the raw measurements from the physical sensor (potentially low-cost barcode or passive Radio Frequency Identification devices (Chawla and Ha, 2007; Deepali et al., 2024)) yielding a much more precise inventory estimate. We refine and extend recent findings (Ietto and Orsini, 2025b) in which the rate at which the

estimation error converges to zero is governed by the decay rate parameter $\rho \in (0, 1)$ of the goods. This behavior may prove unsatisfactory when ρ approaches unity. To accelerate the convergence of the estimate toward the true inventory value, we propose an observer whose gain is dynamically adjusted over time.


The effectiveness of this approach is demonstrated through its application to a perishable goods supply chain operating under an Order-Up-To (OUT) inventory policy.

2 UNCERTAIN SUPPLY CHAIN MODEL

2.1 Inventory Level Equation

We consider a periodically reviewed single-product supply chain composed of a retailer and a manufacturer in series. The model is defined by the following assumptions:

- A1)** At the beginning of each review period $[kT, (k+1)T)$, $k \in \mathbb{Z}^+$, the retailer updates inventory, receives shipments, dispatches goods, and places a replenishment order.
- A2)** The manufacturer ensures complete fulfillment of every order after a consistent lead time of L periods.

^a  <https://orcid.org/0000-0003-4965-5262>

A3) Goods deteriorate only while stored in the warehouse.

Assumptions A1-A3 align with those employed in our earlier contributions ((Ietto and Orsini, 2024a; Ietto and Orsini, 2024b; Ietto and Orsini, 2025a)). In line with the subject matter of this paper, we here introduce the following assumptions:

A4) The deterioration rate of the perishable stored between two consecutive review period is $\delta \in (0, 1)$.

A5) The inventory measurements $y(k)$ are collected at the beginning of each period and are affected by sensor noise $v(k)$, modeled as $v(k) = \gamma_v(k)x(k)$, for some $\gamma_v(k) \in [-\gamma, +\gamma]$ and $\gamma \in (0, 1)$. The parameter γ reflects the sensor's relative accuracy and is assumed to be known.

Assumption A5 is justified by the observation that, as inventory levels increase, the likelihood of cumulative measurement inaccuracies also rises, thereby rendering the measurement noise a variable directly proportional to the magnitude of the quantity being assessed.

The supply chain dynamics that can be modeled based on the aforementioned assumptions is:

$$x(k+1) = \rho [x(k) + u(k-L) - h(k)], \quad (1)$$

$$x(0) \in [x_0^-, x_0^+] \quad (2)$$

$$y(k) = x(k) + v(k) \quad (3)$$

where:

- $x(k+1)$: inventory level at the end of period k ;
- $[x_0^-, x_0^+]$ is the given interval containing the unknown initial inventory level;
- $\rho = 1 - \delta$: decay factor due to deterioration;
- $u(k-L)$: replenishment order placed L periods earlier;
- $h(k)$: fulfilled customer demand, defined as:

$$h(k) = d(k) - z(k) \quad (4)$$

where $d(k)$ is the customer demand and $z(k) \in [0, d(k)]$ is the unmet demand. Since $h(k)$ reflects actual deliveries, it is known at each k ;

- $y(k)$: measured inventory level;
- $v(k)$: measurement noise.

3 THE FAST ROBUST OBSERVER

For the specified SC models given in (1)-(2) the proposed estimator provides an inventory estimate that

exhibits fast convergence to the true value, independent of both the origin and intensity of measurement noise $v(k)$. This behavior marks a significant distinction from Kalman and H_∞ filters.

Defining the following extended state vector:

$$\bar{x}(k) = [x(k) \ v(k)]^T$$

and taking into account A5, the supply chain dynamics (1)-(4), can be expressed in the following compact state-space form:

$$\begin{aligned} \bar{x}(k+1) &= \begin{bmatrix} \rho & 0 \\ \gamma_v(k+1)\rho & 0 \end{bmatrix} \bar{x}(k) + \begin{bmatrix} \rho \\ \gamma_v(k+1)\rho \end{bmatrix} \eta(k) \\ &= A(k+1)\bar{x}(k) + B(k+1)\eta(k) \end{aligned} \quad (5)$$

$$y(k) = [1 \ 1] \bar{x}(k) = C\bar{x}(k) \quad (6)$$

where $\eta(k) \triangleq u(k-L) - h(k)$.

An estimate $\hat{x}(k) \triangleq [\hat{x}(k) \ \hat{v}(k)]^T$ of $\bar{x}(k)$ converging to $\bar{x}(k)$ can be obtained through the following observer

$$\zeta(k+1) = G(\alpha(k))\bar{A}\hat{x}(k) + G(\alpha(k))\bar{B}\eta(k) \quad (7)$$

$$\hat{\hat{x}}(k) = \zeta(k) - F(\alpha(k))y(k) \quad (8)$$

where:

- \bar{A} and \bar{B} are nominal matrices defined as:

$$\bar{A} \triangleq \begin{bmatrix} \rho & 0 \\ 0 & 0 \end{bmatrix}, \bar{B} \triangleq \begin{bmatrix} \rho \\ 0 \end{bmatrix}$$

- $G(\alpha(k))$ and $F(\alpha(k))$ are the design matrices.

The robust observer is initialized with $\hat{\hat{x}}(0) = \begin{bmatrix} (x_0^- + x_0^+)/2 \\ 0 \end{bmatrix}$.

Let $\hat{e}(k)$ be the estimation error defined as

$$\hat{e}(k) \triangleq \bar{x}(k) - \hat{\hat{x}}(k) \triangleq \begin{bmatrix} \hat{e}_x(k) \\ \hat{e}_v(k) \end{bmatrix} = \begin{bmatrix} x(k) - \hat{x}(k) \\ v(k) - \hat{v}(k) \end{bmatrix}$$

By (6) and (8), the estimation error $\hat{e}(k+1)$ is given by

$$\begin{aligned} \hat{e}(k+1) &= \bar{x}(k+1) - \zeta(k+1) + F(\alpha(k))y(k+1) \\ &= \bar{x}(k+1) - \zeta(k+1) + F(\alpha(k))C\bar{x}(k+1) \\ &= (I + F(\alpha(k))C)\bar{x}(k+1) - \zeta(k+1) \\ &= G(\alpha(k))\bar{x}(k+1) - \zeta(k+1) \end{aligned} \quad (9)$$

provided that the two design matrices $F(\alpha(k))$ and $G(\alpha(k))$ can be determined so as to fulfill the following condition

$$(I + F(\alpha(k))C) = G(\alpha(k)) \quad (10)$$

It is easy to see that condition(10) is satisfied choosing

$$G(\alpha(k)) = \begin{bmatrix} 1 - \alpha(k) & -\alpha(k) \\ -1 & 0 \end{bmatrix}, F(\alpha(k)) = \begin{bmatrix} -\alpha(k) \\ -1 \end{bmatrix}$$

where $\alpha(k)$ is a scalar parameter whose value controls the convergence to zero of the estimation error. By (5) and (7), simple calculations show that

$$\begin{aligned}\hat{e}(k+1) &= G(\alpha(k))A(k+1)\hat{e}(k) \\ &+ G(\alpha(k))(A(k+1) - \bar{A})\hat{x}(k) \\ &+ G(\alpha(k))(B(k+1) - \bar{B})\eta(k) \\ &= \begin{bmatrix} \rho - \alpha(k)(1 + \gamma_v(k+1)) & 0 \\ -\rho & 0 \end{bmatrix} \hat{e}(k) \\ &+ \begin{bmatrix} -\alpha(k)\gamma_v(k+1)\rho & 0 \\ 0 & 0 \end{bmatrix} \hat{x}(k) \\ &+ \begin{bmatrix} -\alpha(k)\gamma_v(k+1)\rho \\ 0 \end{bmatrix} \eta(k) \quad (11)\end{aligned}$$

and in component-wise representation:

$$\begin{aligned}\hat{e}_x(k+1) &= (\rho - \alpha(k)(1 + \gamma_v(k+1))) \hat{e}_x(k) \\ &- \alpha(k)\gamma_v(k+1)\rho (\hat{x}(k) + \eta(k)) \quad (12)\end{aligned}$$

$$\hat{e}_v(k+1) = -\rho \hat{e}_x(k) \quad (13)$$

Remark

Equations (12) and (13) imply that setting $\alpha(k) = 0, \forall k \geq 0$, the estimation errors $|\hat{e}_x(k)|$ and $|\hat{e}_v(k)|$ asymptotically converge to zero, with the same convergence rate ρ . More precisely, assuming $|\hat{e}_x(\bar{k})| \leq M(\bar{k})$ for some known $M(\bar{k}) \geq 0$, the following inequalities hold:

$$|\hat{e}_x(\bar{k}+1)| \leq M(\bar{k}+1), \quad |\hat{e}_v(\bar{k}+1)| \leq M(\bar{k}+1)$$

with

$$M(\bar{k}+1) \triangleq \rho M(\bar{k}) \quad (14)$$

Namely the sequences $\{|\hat{e}_x(k)|\}$ and $\{|\hat{e}_v(k)|\}, k \geq 0$ are upper bounded by a sequence $\{M(k)\}$ monotonically converging to zero as:

$$M(k+1) \triangleq \rho^{k+1}M(0) \quad (15)$$

where $M(0)$ is a known upper bound of $|\hat{e}_x(0)|$. This rate of convergence may be unsatisfactory for slowly perishable goods ($\rho \simeq 1$). In this case it is convenient to enhance the convergence rate choosing $\alpha(k)$ as stated in the following theorem.

Theorem

The sequence of estimation error $\{|\hat{e}_x(k)|\}, k \geq 0$, produced by the observer (7),(8) converge to zero at a given time varying rate $\tilde{\rho}(k) \in (0, \rho]$ if the parameter $\alpha(k)$ can be chosen so as to satisfy the two following conditions:

$$C_1 : 0 \leq \rho - \alpha(k)(1 + \gamma) \leq \rho$$

$$C_2 : (\rho - \tilde{\rho}(k))M(k) \leq \alpha(k)[(1 - \gamma)M(k) - \gamma\rho|\hat{x}(k) + \eta(k)|].$$

Proof

Suppose that at time k , the estimation error $\hat{e}_x(k)$ satisfies

$$|\hat{e}_x(k)| \leq M(k)$$

for some known $M(k)$.

We show that, under the stated conditions, the error term $|\hat{e}_x(k+1)|$ is bounded by $M(k+1)$ where:

$$|\hat{e}_x(k+1)| \leq M(k+1) \triangleq \tilde{\rho}(k)M(k) \quad (16)$$

From the error dynamics (12), it is seen that

$$0 \leq \rho - \alpha(k)(1 + \gamma_v(k+1)) \leq \rho, \forall \gamma_v(k+1) \in [-\gamma, +\gamma]$$

if condition C_1 is satisfied.

Equation (12) imply:

$$\begin{aligned}|\hat{e}_x(k+1)| &\leq (\rho - \alpha(k)(1 - \gamma))M(k) \\ &+ \alpha(k)\gamma\rho|\hat{x}(k) + \eta(k)| \quad (17)\end{aligned}$$

To ensure that the error contracts at the rate $\tilde{\rho}(k) \leq \rho$, as specified in (16), we impose:

$$|\hat{e}_x(k+1)| \leq \tilde{\rho}(k)M(k) \quad (18)$$

Fulfillment of (18) follows directly from inequality (17), imposing:

$$\begin{aligned}(\rho - \alpha(k)(1 - \gamma))M(k) + \alpha(k)\gamma\rho|\hat{x}(k) + \eta(k)| \\ \leq \tilde{\rho}(k)M(k) \quad (19)\end{aligned}$$

that is equivalent to condition C_2 .

From assumption $|\hat{e}_x(0)| \leq M(0)$, and iterating inequality (18), we obtain that $|\hat{e}_x(k+1)|$ decays to zero at a rate determined by the sequence $\{\tilde{\rho}(k)\}$, more precisely

$$|\hat{e}_x(k+1)| \leq \prod_{j=0}^k \tilde{\rho}(j)M(0)$$

and by (13) it directly follows that

$$|\hat{e}_v(k+1)| \leq \rho \prod_{j=0}^{k-1} \tilde{\rho}(j)M(0)$$

3.1 On the Existence of an $\alpha(k)$ Satisfying C_1 and C_2

The possibility of accelerating the errors convergence to zero depends on the existence of an $\alpha(k)$ satisfying both C_1 and C_2 . In this section, we examine the scenarios that may arise and outline the appropriate action.

- Scenario 1: Since, by C_1 , $\alpha(k) \geq 0$ and by definition $(\rho - \tilde{\rho}(k))M(k) \geq 0$, condition C_2 requires:

$$(1 - \gamma)M(k) - \gamma\rho|\hat{x}(k) + \eta(k)| > 0 \quad (20)$$

so, if for some values of k it happens that

$$(1 - \gamma)M(k) - \gamma\rho|\hat{x}(k) + \eta(k)| \leq 0$$

the only feasible solution that satisfies both C_1 and C_2 is to set $\alpha(k) = 0$. At these time instants we have that $\tilde{\rho}(k) = \rho$.

- Scenario 2: Assume that (20) is satisfied. Condition C_1 requires that $\alpha(k)$ belongs to the interval:

$$0 \leq \alpha(k) \leq \frac{\rho}{1 + \gamma}$$

and, to satisfy C_2 , $\alpha(k)$ must also fulfill:

$$\alpha(k) \geq \frac{(\rho - \tilde{\rho}(k))M(k)}{(1 - \gamma)M(k) - \gamma\rho|\hat{x}(k) + \eta(k)|} \triangleq L_{\tilde{\rho}(k)}$$

To ensure both conditions are met, the following inequality must hold:

$$L_{\tilde{\rho}(k)} \leq \frac{\rho}{1 + \gamma}$$

Within Scenario 2, two cases can be distinguished:

- 2a (Immediate Feasibility)

If

$$(1 - \gamma)M(k) - \gamma\rho|\hat{x}(k) + \eta(k)| > 0$$

and

$$L_{\tilde{\rho}(k)} \leq \frac{\rho}{1 + \gamma}$$

then a feasible interval for $\alpha(k)$ exists

$$\alpha(k) \in \left[L_{\tilde{\rho}(k)}, \frac{\rho}{1 + \gamma} \right].$$

Setting

$$\alpha(k) = \frac{\rho}{1 + \gamma} \quad (21)$$

minimizes the scalar $\rho - \alpha(k)(1 + \gamma_v(k + 1))$ describing the dynamics of the estimation error $\hat{e}_x(k)$ in equation (12).

- 2b (Feasibility after modification)

If

$$L_{\tilde{\rho}(k)} > \frac{\rho}{1 + \gamma},$$

then $\tilde{\rho}(k)$ must be increased (approaching ρ) until the new decreasing value of $L_{\tilde{\rho}(k)}$ satisfies $L_{\tilde{\rho}(k)} \leq \frac{\rho}{1 + \gamma}$ and $\alpha(k)$ is again chosen as in (21).

4 APPLICATION TO THE OUT POLICY

The guiding principle of the OUT policy is to issue a purchase order that adjusts the current inventory level to reach a new reference value, which is determined based on the forecasted customer demand. The calculation of the purchase order is performed through a model inversion approach.

4.1 Assumption on the Future Customer Demand

The demand forecast used in the OUT policy is based on a practical assumption grounded in real-world observations:

- A6) The demand is uniformly bounded and for each k it is known that the future demand $d(k + j)$ is bounded above by a given value $d^+(k + j)$, for $j = 1, \dots, L + 1$.

This assumption is agnostic to the nature of uncertainty, whether deterministic or stochastic, and only requires boundedness and known variation over short horizons.

4.2 The Proposed OUT Policy

We propose a replenishment policy $u(k)$ that utilizes the current inventory estimate $\hat{x}(k)$, generated by the fast robust observer. The inventory target is defined to ensure complete satisfaction of customer demand while maintaining a safety stock buffer.

The predicted inventory level at the end of the $(k + L)$ -th period is given by:

$$x(k + L + 1|k) = \rho^{L+1}x(k) + \sum_{l=0}^{L-1} \rho^{L+1-l}u(k + l - L) + \rho u(k) - \rho^{L+1}h(k) - \sum_{l=1}^L \rho^{L+1-l}h(k + l|k) \quad (22)$$

To account for perishability, lead time delays, and demand uncertainty, we adopt a modified version of the OUT policy. In this formulation, the predicted inventory level $x(k + L + 1|k)$ is set equal to the maximum expected demand $d^+(k + L + 1)$, as defined in assumption A6.

The replenishment order $u(k)$ is computed by solving equation (22) with respect to $u(k)$, substituting $x(k + L + 1|k)$ and $x(k)$ with $d^+(k + L + 1)$ and $\hat{x}(k)$, respectively. Furthermore, following a precautionary worst-case approach, the demand forecasts are assumed as $h(k + l|k) = d(k + l) = d^+(k + l)$ for all $l = 1, \dots, L$. Let

$$\tilde{y}(k + L + 1) \triangleq d^+(k + L + 1) + \sum_{\ell=1}^L \rho^{L+1-\ell}d^+(k + \ell) + \rho^{L+1}h(k)$$

$$p(k) \triangleq \tilde{y}(k + L + 1) - \rho^{L+1}\hat{x}(k) - \sum_{\ell=0}^{L-1} \rho^{L+1-\ell}u(k + \ell - L)$$

the replenishment order is then computed as:

$$u(k) = \max \left(0, \frac{p(k)}{\rho} \right) \quad (23)$$

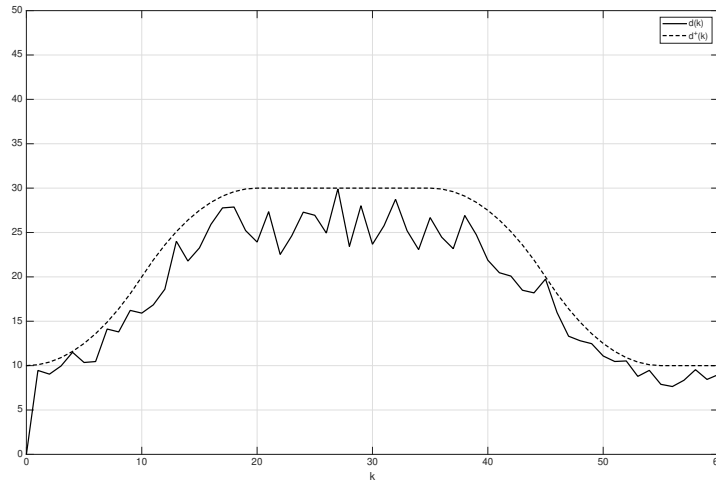


Figure 1: The actual unknown customer demand $d(k)$ (solid line). The dashed line represents the known boundary trajectory $d^+(k)$.

This formulation introduces a dual adaptation mechanism: the order quantity $u(k)$ dynamically adjusts based on both the worst-case forecasted demand and the current estimated inventory level. This ensures resilience in the face of uncertainty and supports proactive inventory control.

5 NUMERICAL SIMULATIONS

We consider a periodically reviewed perishable SC where the dynamics of inventory level (1)-(4) is characterized by a decay factor $\rho = 0.9$, a lead time $L = 2$, an unknown initial inventory level $x(0) \in [x_0^-, x_0^+] = [0, 20]$. In accordance with A6, at each $k \in \mathbb{Z}^+$, customer demand over the prediction horizon $L + 1 = 3$ is bounded above by the known trajectory $d^+(k + j)$ for $j = 1, \dots, L + 1 = 3$. Figure 1 shows the actual demand $d(k)$ alongside its upper bound $d^+(k)$.

In the performed simulations, the noise $v(k)$ is modeled as $v(k) = \gamma_v(k)x(k)$, where $\gamma_v(k)$ is a zero mean unknown random sequence taking values on $[-\gamma, +\gamma]$ for some $\gamma \in (0, 1)$. The parameter γ , assumed to be known, reflects the relative accuracy of the sensor.

Three simulations (Sim 1, Sim 2 and Sim 3) are conducted. They are initialized with $x(0) = 11$ and stopped at time $k = 60$.

In Sim 1, a high-precision (and costly) sensor is used for inventory measurement assuming $\gamma = 0.02$, corresponding to 98% accuracy. The replenishment order $u(k)$ is computed by replacing $x(k)$ in equation (22) with the sensor measurement $y(k)$.

In Sim 2, it is employed a significantly more affordable sensor with an accuracy of approximately 80% (i.e., $\gamma = 0.2$). The noisy measurements from

this sensor are refined using a fast robust observer, initialized as follows:

$$\hat{x}(0) = \begin{bmatrix} \frac{x_0^- + x_0^+}{2} \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}.$$

This yields an initial estimation error bounded by

$$|\hat{e}_x(0)| \leq M(0), \quad \text{where} \quad M(0) = \frac{x_0^+ - x_0^-}{2} = 10.$$

The parameter $\tilde{\rho}(k)$ is set to 0.1 for all $k \in \mathbb{Z}^+$. The replenishment order $u(k)$ is then computed by substituting $x(k)$ in equation (22) with the observer estimate $\hat{x}(k)$.

Figure 2 depicts the actual evolution of the inventory, along with the measurements obtained from a high-precision, high-cost sensor.

Figure 3 shows the effectiveness of the robust observer, which processes the noisy measurements of the inventory provided by an economic sensor and yields a highly accurate estimate $\hat{x}(k)$. This estimate exhibits rapid convergence toward the true inventory value $x(k)$, thereby demonstrating the observer's reliability and precision.

A comparative analysis of the two figures demonstrates that the robust observer attains a higher level of accuracy than the high-precision sensor. To substantiate this observation with quantitative metrics, the following performance indexes

$$P_{y(k)} = \frac{\sum_{k=1}^{60} |y(k) - x(k)|}{\sum_{k=1}^{60} x(k)} \times 100 \quad (\text{Sim 1})$$

$$P_{\hat{x}(k)} = \frac{\sum_{k=1}^{60} |\hat{x}(k) - x(k)|}{\sum_{k=1}^{60} x(k)} \times 100 \quad (\text{Sim 2})$$

are computed, yielding values of 2 and 0.56, respectively.

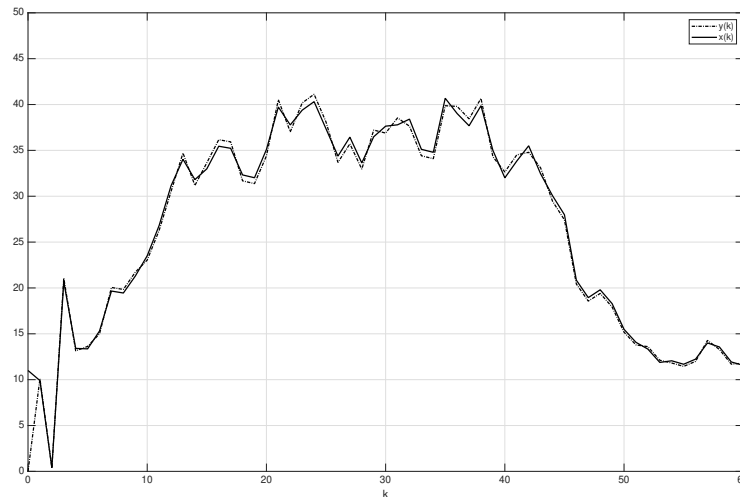


Figure 2: Sim 1. The actual inventory level $x(k)$ and the measurements $y(k)$ obtained from a highly precise and costly sensor.

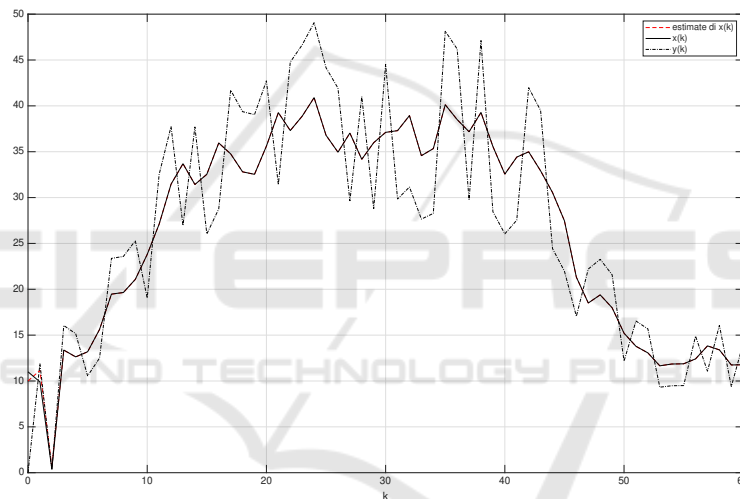


Figure 3: Sim 2. The actual inventory level $x(k)$, the measurements $y(k)$ obtained from a low-cost sensor, and the estimates $\hat{x}(k)$ produced by the robust and fast observer.

Figure 4 compares the two replenishment policies derived respectively from the high-precision sensor measurements (Sim 1) and from the estimates generated by the robust observer using data from the low-cost sensor (Sim 2). Although both approaches achieve a similar fill rate and fully satisfy customer demand, they differ substantially in the economic cost associated with the sensing infrastructure required to obtain high-quality data and, consequently, to sustain an efficient replenishment strategy.

Sim 3 is performed to highlight the role of inventory accuracy in defining a replenishment policy. The restocking signal $u(k)$ is computed replacing $x(k)$ with the raw measurement from the low-cost sensor. Figure 5 shows that the resulting reorder pattern is far more irregular than in Figure 3. This reveals how a

large amplitude of the noise in $y(k)$ propagates into the ordering decisions. The increased variability is a clear manifestation of the bullwhip effect, amplified by the poor quality of the inventory signal.

6 CONCLUSIONS

This paper proposes a cost-effective and accurate solution for inventory estimation in perishable supply chains by integrating a low-cost sensor with a fast and robust observer.

The key findings can be summarized as follows:

- 1 Accuracy Enhancement: The robust observer significantly improves the accuracy of inventory estimates derived from noisy, low-cost sensor mea-

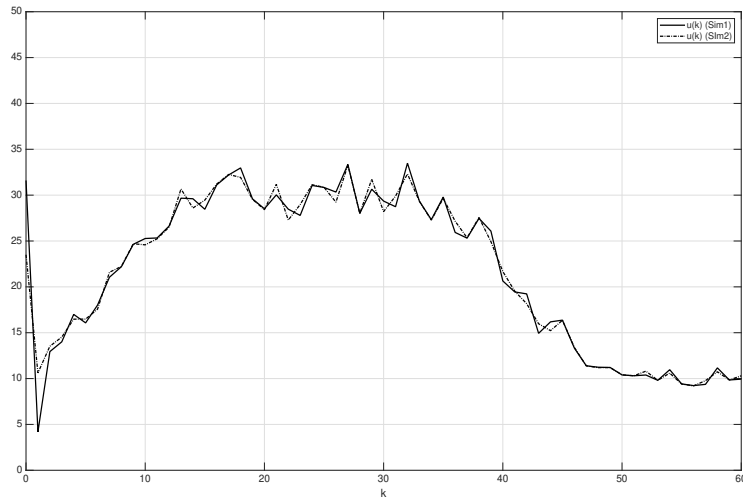


Figure 4: The two replenishment policies $u(k)$, derived respectively from the measurements provided by the high-precision sensor (Sim 1) and from the estimates generated by the robust observer (Sim 2).

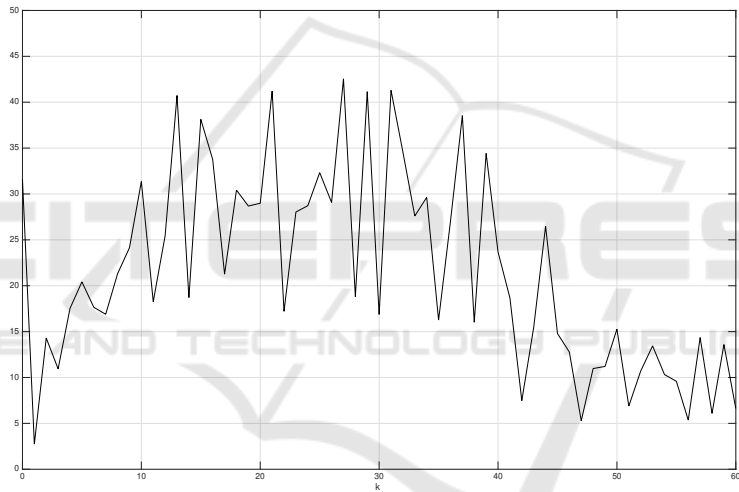


Figure 5: Sim 3. The replenishment policy $u(k)$ obtained from the raw measurements provided by the low cost sensor.

surements. Simulation results show that the estimation error is substantially reduced, with a performance index of 0.56 compared to 2.0 for the high-precision sensor.

- 2 **Economic Advantage:** The proposed approach offers a viable alternative to expensive sensing technologies, enabling reliable inventory tracking without the financial burden associated with high-end sensors.
- 3 **Policy Effectiveness:** When applied to an OUT replenishment policy, the observer-based estimates support the generation of replenishment orders that fully satisfy customer demand, demonstrating the practical applicability of the method.
- 4 **Rapid Convergence:** The novelty of a time varying gain observer enables dynamic tuning of the

convergence rate, allowing the inventory estimate to quickly align with the true value, even under substantial measurement noise and in supply chains involving goods with low deterioration rate ($\delta \simeq 0$ i.e. $\rho \simeq 1$).

7 FUTURE WORK

The results on the enhanced convergence of the observer establishes a foundation for deployment in complex operational settings, with a particular focus on the accurate detection and classification of anomalies within multi-echelon supply chain networks such as inventory shrinkage, data registration discrepancies, and item misplacement.

REFERENCES

- Bertolini, M., Bottani, E., Rizzi, A., Volpi, A., and Renzi, P. (2013). Shrinkage reduction in perishable food supply chain by means of an rfid-based fifo management policy. *International Journal of RF Technologies*, 5:123–136.
- Chawla, V. and Ha, D. (2007). An overview of passive rfid. *IEEE Communications Magazine*, 45:11–17.
- Deepali, C., P.Monika, and Dharmendra, S. (2024). Integrating barcode technology into warehouse management systems for enhanced efficiency and inventory accuracy. *Journal of Computer Science and Research*, 2:8–14.
- Delen, D., Sharda, R., and Hardgrave, B. (2011). The promise of rfid-based sensors in the perishables supply chain. *IEEE Wireless Communications*, 18:82–88.
- Dolgui, A., Tiwari, M. K., Sinjana, Y., Kumar, S. K., and Son, Y. J. (2018). Optimizing integrated inventory policy for perishable items in a multi-stage supply chain. *International Journal of Production Research*, 56:1–24.
- Goyal, S., Aloysius, J., and Hardgrave, B. (2009). Using rfid to improve inventory accuracy. In *Americas Conference on Information Systems AMCIS. AISeL*.
- Grunow, M. and Piramuthu, S. (2013). Rfid in highly perishable food supply chains-remaining shelf life to supplant expiry date? *International Journal of Production Economics*, 146:717–727.
- Letto, B. and Orsini, V. (2024a). Resilient and robust management policy for multi-stage supply chains with perishable goods and inaccurate forecast information: A distributed model predictive control approach. *Optimal Control Applications and Methods*, 45:2383–2414.
- Letto, B. and Orsini, V. (2024b). Supervisory multi-model control for supply chains with large uncertainty on the perishability rate. *IFAC-PapersOnLine*, 58:421–426.
- Letto, B. and Orsini, V. (2025a). Optimal inventory level control in the case of perishable goods with unknown time-varying decay factor: A data-driven supervised robust model predictive control approach. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 55:6597–6610.
- Letto, B. and Orsini, V. (2025b). A robust observer based algorithm to improve the accuracy of inventory measures in perishable supply chains. *IFAC PapersOnLine*, 59:346–351.
- Kaur, M., Sandhu, M., Mohan, N., and Sandhu, P. (2011). Rfid technology principles, advantages, limitations & its applications. *International Journal of Computer and Electrical Engineering*, 3:151–157.
- Mahmoud, M., Hamdan, M., and Baroudi, U. (2019). Modeling and control of cyber-physical systems subject to cyber attacks: A survey of recent advances and challenges. *Neurocomputing*, 338:101–115.
- Piramuthu, S. and Zhou, W. (2013). Rfid and perishable inventory management with shelf-space and freshness dependent demand. *International Journal of Production Economics*, 144:635–640.
- Sarac, A., Absi, N., and Dauzere-Peres, S. (2010). A literature review on the impact of rfid technologies on supply chain management. *International Journal of Production Economics*, 128:77–95.