



Beneficial effects of TMDs with embedded vibro-impacting mass on fully isolated rigid block-like structures

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ABSTRACT

This study investigates the nonlinear seismic response of rigid block-like structures supported by a dual horizontal-vertical base-isolation system and equipped with a Tuned Mass Damper (TMD) incorporating an embedded Vibro-Impacting Mass (VIM). The proposed configuration combines three strongly interacting mechanisms—base isolation, rocking dynamics, and vibro-impact interactions—and explores a protection strategy that has not yet been systematically investigated for isolated rigid block-like structures. A unified Lagrangian formulation is developed to consistently describe both full-contact and rocking phases, together with impact events occurring at the block base and within the TMD-VIM device. The nonlinear hysteretic behavior of the horizontal isolation system is represented through a Bouc–Wen model, whereas the vertical isolation system and the TMD restoring forces are described by Kelvin–Voigt viscoelastic formulations. The resulting framework enables the analysis of the coupled isolation–rocking–impact dynamics under strong horizontal and vertical seismic excitations. A comprehensive parametric investigation is performed by varying the horizontal and vertical isolation periods, the TMD dynamic properties, and the mass ratio and clearance of the embedded vibro-impacting mass. Rocking and displacement maps are constructed to identify the boundaries separating full-contact, rocking, and overturning regimes. The results demonstrate that the integration of a vibro-impacting TMD into a dual horizontal–vertical seismic isolation system significantly enhances the seismic performance of the protected structure by modifying the stability boundaries that govern its response. In particular, the embedded VIM enlarges the full-contact region, converts rocking responses into full-contact motion over significant portions of the parameter space, and prevents overturning in configurations that would otherwise be unstable. The effectiveness of the proposed device is shown to depend strongly on the distribution of mass between the TMD and the VIM, with the best performance achieved when the primary TMD mass remains dominant. The findings provide new insight into the nonlinear mechanisms governing the interaction among seismic isolation, rocking motion, and vibro-impact phenomena, and demonstrate the potential of TMD–VIM systems as an effective strategy for enhancing the seismic stability of isolated rigid block-like structures.

1. Introduction

Passive vibration-control systems are widely employed to improve the seismic performance of structures and equipment by reducing vibration amplitudes and limiting the transmission of dynamic loads. Among passive vibration-control strategies, Tuned Mass Dampers (TMDs) represent one of the most established and widely adopted solutions for mitigating structural vibrations under dynamic loading. Originally formalized by Den Hartog [1], the classical TMD consists of a secondary mass elastically connected to the primary structure and tuned to a

target structural frequency. When properly designed, the device mitigates structural vibrations by transferring a portion of the vibrational energy from the main system to the attached mass.

The effectiveness of conventional TMDs is strongly dependent on the mass ratio between the auxiliary device and the protected structure. In practical seismic applications, limitations associated with space, weight, and installation constraints often restrict the admissible auxiliary mass, consequently reducing the achievable mitigation performance. To overcome this limitation, several enhanced configurations

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have been proposed. Among these, inerter-based configurations such as Tuned Mass Damper Inerters (TMDIs) have been proposed to increase the effective inertia of the auxiliary system without requiring a proportional increase in physical mass [2–8]. Hybrid configurations combining TMDIs with seismic isolation systems have also been investigated in the literature [9–11].

Recent research has also explored alternative passive vibration-control concepts based on negative-stiffness mechanisms, inertial amplification devices, and hybrid auxiliary oscillators. These include the KDamper and related configurations, which exploit the interaction between positive and negative stiffness elements to improve vibration mitigation while reducing the auxiliary mass typically required by conventional absorber systems [12–19]. Hybrid extensions incorporating inerter elements have also been proposed as additional passive control strategies for structural and seismic engineering applications [20].

In parallel, semi-active and adaptive vibration-control systems have also been investigated in the literature [21]. In the context of tuned mass dampers, semi-active strategies allow the stiffness and/or damping properties of the auxiliary device to be adjusted during the response, with the aim of improving performance under variable structural and excitation conditions [22–26]. More recent developments have examined advanced control frameworks, including applications to nonlinear tall buildings subjected to mainshock–aftershock sequences and phase-control algorithms for variable-damping semi-active TMD configurations [27,28].

Alongside tuned vibration absorbers and adaptive control devices, increasing attention has recently been devoted to strongly nonlinear vibration-control systems based on impact interactions. Among these, Vibro-Impacting Masses (VIMs) consist of secondary masses moving within prescribed clearances and interacting with mechanical stops through impacts. Whenever the relative displacement exceeds the available gap, impact events occur, generating impulsive forces that modify the system response. Owing to the intrinsic nonlinearity of the impact mechanism, VIM devices can influence structural dynamics over a broad range of operating conditions without requiring precise tuning. Applications of vibro-impact devices to frame structures subjected to seismic excitation have been investigated in several studies [29–32]. In most configurations, the impacting mass is directly connected to the protected structure; although effective, this arrangement may be limited by the load-carrying capacity of the supporting structural members.

The application of vibration-control devices to rigid block-like structures is a relatively recent development. These systems represent a particularly challenging class of structures because their seismic response is governed by strongly nonlinear phenomena, including uplift, rocking, impacts, and possible overturning. Various control strategies have been proposed for such systems, including running masses [33–35] and pendulum dampers [36,37], which can be regarded as particular realizations of tuned mass damping concepts. More recently, inerter-based protective devices have been proposed for rocking bodies [38], while hybrid hysteretic mass damper–inerter configurations have also been investigated [39].

Only a limited number of studies have investigated hybrid devices combining a Tuned Mass Damper and an embedded Vibro-Impacting Mass (TMD–VIM) for seismic protection purposes [40,41]. These studies highlighted the potential of combining a tuned mass damper with an embedded impacting mass for seismic protection applications. However, the available investigations have focused on frame-type structures, leaving other classes of systems largely unexplored.

When rigid block-like structures are protected through seismic isolation systems, their response becomes particularly challenging under simultaneous horizontal and vertical ground motions. In this context, combined horizontal–vertical base-isolation systems have been proposed to mitigate the effects of both excitation components and to improve the seismic performance of rigid bodies subjected to strong ground motions [42,43]. The interaction among isolation mechanisms, rocking motion, impact phenomena, and auxiliary vibration-control

devices generates a highly nonlinear dynamic response, whose governing mechanisms are not yet fully understood.

The analysis of horizontally and vertically isolated rigid block-like structures also requires an appropriate representation of the nonlinear behavior of the isolation devices. In this context, hysteretic constitutive models are frequently adopted to reproduce the response of seismic isolation systems. Among them, the Bouc–Wen model is widely employed in seismic engineering applications because of its ability to represent a broad range of hysteretic responses within a relatively compact mathematical framework [44–47].

Despite the growing literature on vibration-control devices, vibro-impact systems, seismic isolation, and rocking rigid bodies, the role of TMD–VIM devices in the seismic protection of horizontally and vertically isolated rigid block-like structures remains largely unexplored. In particular, it is still unclear how the interaction among isolation mechanisms, rocking motion, and internal vibro-impact phenomena modifies the stability boundaries separating full-contact, rocking, and overturning regimes.

To address this research gap, the present study develops a comprehensive mechanical framework for investigating a rigid block-like structure supported by a dual horizontal–vertical isolation system and equipped with a Tuned Mass Damper incorporating an embedded Vibro-Impacting Mass (TMD–VIM), with particular emphasis on the underlying nonlinear interaction mechanisms governing the system response. The principal novel features and contributions of this work can be summarized as follows:

- A unified Lagrangian framework is developed to describe the coupled dynamics of horizontally and vertically isolated rigid block-like structures equipped with a TMD–VIM device, consistently accounting for full-contact motion, rocking phases, base impacts, and internal vibro-impact interactions.
- The nonlinear interaction among horizontal–vertical seismic isolation, rocking dynamics, vibro-impact phenomena, and hysteretic isolation behavior is investigated within a single mechanical model based on a Bouc–Wen representation of the horizontal isolation system.
- A systematic investigation of TMD–VIM devices applied to seismically excited rigid block-like structures is carried out, addressing a problem that has not yet been explored in the literature.
- Extensive rocking and displacement maps are constructed to characterize the influence of the TMD–VIM parameters on the stability boundaries separating full-contact, rocking, and overturning regimes.
- The study identifies the mechanical conditions under which the embedded vibro-impacting mass enhances seismic performance, suppresses rocking motion, enlarges the full-contact region, prevents overturning, and reduces the displacement demand of the isolation system.

The remainder of the paper is organized as follows. [Section 2](#) presents the mechanical model and the governing equations of motion. [Section 3](#) describes the adopted seismic inputs and the parameters considered in the numerical investigation. [Section 4](#) briefly discusses the role of the dual horizontal–vertical isolation system. [Section 5](#) examines the influence of the TMD and the TMD–VIM device on the rocking behavior and displacement response of the isolated rigid block-like structures. Finally, the main conclusions and perspectives for future developments are summarized in [Section 6](#).

2. Mechanical model

This section presents the mechanical model adopted to describe the coupled dynamics of the isolated rigid block-like structure and the TMD–VIM device. The kinematic assumptions governing the admissible motion phases are first introduced, followed by the derivation

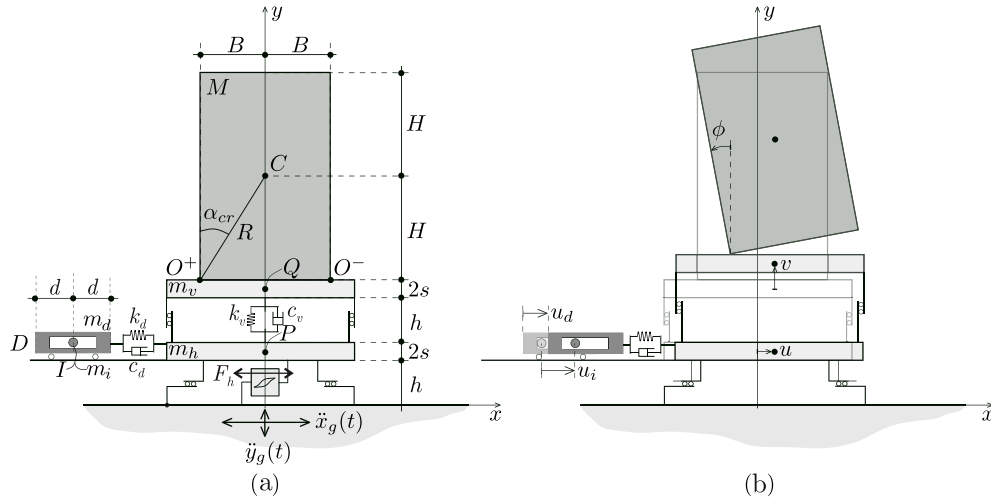


Fig. 1. Mechanical model of the isolated rigid block equipped with the TMD-VIM device: (a) geometrical configuration and main system components; (b) rocking motion about the left base vertex O^+ and positive directions of the Lagrangian coordinates.

of the governing equations through a unified Lagrangian formulation. Particular attention is devoted to the transition conditions between full-contact and rocking phases, as well as to the impact laws governing both base impacts and internal vibro-impact interactions. Subsequently, the constitutive models adopted for the isolation system and the TMD-VIM device are described. Finally, the numerical implementation and the solution procedure adopted to integrate the governing equations are presented.

The mechanical system consists of a symmetric rigid block with mass M , base $2B$, height $2H$, and a vertical position of the mass center C equal to H . The depth L (i.e., the dimension orthogonal to the x - y plane) depends on the specific rigid block considered in the analyses. It is, however, always assumed that the base dimension $2B$ is smaller than the depth L , so that rocking occurs around the end vertices of the base $2B$, O^+ and O^- , i.e., within the x - y plane (see Fig. 1(a)).

The rigid block is supported on an isolated base consisting of two plates capable of horizontal and vertical displacements. These plates are connected to the ground and to each other through dissipative devices, and are separated from the ground and from each other by a distance h , with a thickness equal to $2s$, where throughout the paper $h = 0.1$ m and $s = 0.01$ m are assumed. The mass of each plate is given by $m_h = m_v = 3B \times L \times 2s \times \rho_p$, with $\rho_p = 7850$ kg/m³. A noteworthy aspect is that the plates are designed to be slightly larger ($3B$) than the base dimension of the rigid block-like structure ($2B$), in order to facilitate the positioning of the rigid block on the isolated base. This configuration acts as a dual horizontal-vertical isolation system for the structure.

The horizontally movable component of the base-isolated support is coupled to a TMD with mass m_d , which incorporates a VIM of mass m_i embedded within the moving mass of the TMD. The impacting mass is free to slide with negligible friction and can collide with the inner boundaries of its guiding track. The clearance of the VIM is denoted by d (see Fig. 1(a)). The combined device consisting of the TMD and the internal VIM is referred to throughout the paper as the TMD-VIM device.

Assuming that the rigid block is sufficiently slender, i.e., that the slenderness parameter $\lambda = H/B$ is sufficiently large, the possible motion phases of the system can be reasonably restricted to the following two:

- the *full-contact* phase, in which the rigid block remains in continuous contact with the base and no uplift or rocking motion occurs;
- the *rocking* phase, in which the rigid block undergoes rocking motion and oscillates alternately about one of the two base corners $O^\pm$. When the rocking pivot switches from one corner to the

other, an impact with the support takes place, resulting in a loss of mechanical energy.

During the full-contact phase, the system is described by four Lagrangian parameters u , v , u_d , and u_i , representing the horizontal and vertical displacements of the isolated base, the displacement of the TMD, and the displacement of the VIM, respectively. Accordingly, four equations of motion are required to describe the full-contact phase. During the rocking phase, in addition to the four Lagrangian parameters listed above, the rocking angle ϕ is introduced, and therefore five equations of motion are needed to describe the motion of the system. The positive directions of the Lagrangian parameters are shown in Fig. 1(b).

The mechanical system is analyzed under seismic excitation, represented by imposed ground displacements in the horizontal and vertical directions (i.e., the x - and y -directions), denoted as $x_g = x_g(t)$ and $y_g = y_g(t)$, respectively, where t denotes time.

2.1. Equation of motion

The equations of motion for the two phases described above are derived using a Lagrangian formulation, which involves the expression of the kinetic and potential energies of the system, as well as the work of non-conservative forces.

2.1.1. Full-contact phase

In the full-contact phase, the displacements of the centers of mass of the components forming the system can be expressed as functions of the four Lagrangian coordinates u , v , u_d , and u_i . In particular, denoting by C the center of mass of the rigid block, by P and Q the centers of mass of the two plates constituting the isolated base, and by D and I the centers of mass of the TMD and the VIM, respectively, the displacements of these points are given by:

$$\mathbf{u}_C^{FC} = \begin{pmatrix} x_g + u \\ y_g + v \\ 0 \end{pmatrix}, \quad \mathbf{u}_P = \begin{pmatrix} x_g + u \\ y_g \\ 0 \end{pmatrix}, \quad \mathbf{u}_Q = \begin{pmatrix} x_g + u \\ y_g + v \\ 0 \end{pmatrix}, \quad \mathbf{u}_D = \begin{pmatrix} x_g + u_d \\ y_g \\ 0 \end{pmatrix}, \quad (1)$$

$$\mathbf{u}_I = \begin{pmatrix} u_i \\ y_g \\ 0 \end{pmatrix}.$$

Thus, the kinetic and potential energies are defined as:

$$T^{FC} = \frac{1}{2} m_h \dot{\mathbf{u}}_P \cdot \dot{\mathbf{u}}_P + \frac{1}{2} m_v \dot{\mathbf{u}}_Q \cdot \dot{\mathbf{u}}_Q + \frac{1}{2} m_d \dot{\mathbf{u}}_D \cdot \dot{\mathbf{u}}_D + \frac{1}{2} m_i \dot{\mathbf{u}}_I \cdot \dot{\mathbf{u}}_I + \frac{1}{2} M \dot{\mathbf{u}}_C^{FC} \cdot \dot{\mathbf{u}}_C^{FC}, \quad (2a)$$

$$U^{FC} = Mg \mathbf{u}_C^{FC} \cdot \mathbf{j} + m_v g \mathbf{u}_Q \cdot \mathbf{j}, \quad (2b)$$

where $\mathbf{j} = (0, 1, 0)^T$ is the canonical unit vector along the y -axis. Consequently, the Lagrangian is defined as $L^{FC} := T^{FC} - U^{FC}$.

The virtual work of the non-conservative forces is expressed as:

$$\delta W = F_h \delta u + F_v \delta v + F_d (\delta u - \delta u_d), \quad (3)$$

where F_h , F_v , and F_d denote the horizontal and vertical restoring forces of the isolated base, and the restoring force of the TMD device, respectively.

The Euler-Lagrange equations can thus be written as:

$$\frac{d}{dt} \left(\frac{\partial L^{FC}}{\partial \dot{u}} \right) - \frac{\partial L^{FC}}{\partial u} = F_h + F_d, \quad (4a)$$

$$\frac{d}{dt} \left(\frac{\partial L^{FC}}{\partial \dot{v}} \right) - \frac{\partial L^{FC}}{\partial v} = F_v, \quad (4b)$$

$$\frac{d}{dt} \left(\frac{\partial L^{FC}}{\partial \dot{u}_d} \right) - \frac{\partial L^{FC}}{\partial u_d} = -F_d, \quad (4c)$$

$$\frac{d}{dt} \left(\frac{\partial L^{FC}}{\partial \dot{u}_i} \right) - \frac{\partial L^{FC}}{\partial u_i} = 0. \quad (4d)$$

and by using the expressions introduced in Eqs. (1) and (2) read:

$$(M + m_h + m_v)(\ddot{u} + \ddot{x}_g) = F_h + F_d, \quad (5a)$$

$$(M + m_v)(\ddot{v} + \ddot{y}_g + g) = F_v, \quad (5b)$$

$$m_d(\ddot{u}_d + \ddot{x}_g) = -F_d, \quad (5c)$$

$$m_i \ddot{u}_i = 0. \quad (5d)$$

2.1.2. Rocking phase

During the rocking phases about the corners $O^\pm$, the displacements of the points P and Q , as well as D and I , remain the same as defined in Eq. (1), whereas the displacement of the point C changes and is expressed as:

$$\mathbf{u}_C^\pm = \begin{pmatrix} \mp B + u + x_g \\ -H + v + y_g \\ 0 \end{pmatrix} + \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} \pm B \\ H \\ 0 \end{pmatrix}. \quad (6)$$

The superscript “ $\pm$ ” denotes the kinematic description of the rocking motion with respect to the base vertex O^+ or O^- . When present in the equations, the double sign $\pm$ or $\mp$ should be interpreted such that the upper sign corresponds to the motion about the left vertex O^+ , and the lower sign to the right vertex O^- , respectively.

Consequently, the kinetic and potential energies become:

$$T^\pm = \frac{1}{2} m_h \dot{\mathbf{u}}_P \cdot \dot{\mathbf{u}}_P + \frac{1}{2} m_v \dot{\mathbf{u}}_Q \cdot \dot{\mathbf{u}}_Q + \frac{1}{2} m_d \dot{\mathbf{u}}_D \cdot \dot{\mathbf{u}}_D + \frac{1}{2} m_i \dot{\mathbf{u}}_I \cdot \dot{\mathbf{u}}_I + \frac{1}{2} M \dot{\mathbf{u}}_C^\pm \cdot \dot{\mathbf{u}}_C^\pm + \frac{1}{2} J_C \dot{\phi}^2, \quad (7a)$$

$$U^\pm = Mg \mathbf{u}_C^\pm \cdot \mathbf{j} + m_v g \mathbf{u}_Q \cdot \mathbf{j}, \quad (7b)$$

where J_C is the mass moment of inertia of the rigid block with respect to its center of mass C , i.e., $J_C = \frac{M}{3}(B^2 + H^2)$. The corresponding Lagrangian is defined as $L^\pm := T^\pm - U^\pm$, while the virtual work of the non-conservative forces is expressed as in Eq. (3).

The Euler-Lagrange equations are then written as:

$$\frac{d}{dt} \left(\frac{\partial L^\pm}{\partial \dot{u}} \right) - \frac{\partial L^\pm}{\partial u} = F_h + F_d, \quad (8a)$$

$$\frac{d}{dt} \left(\frac{\partial L^\pm}{\partial \dot{v}} \right) - \frac{\partial L^\pm}{\partial v} = F_v, \quad (8b)$$

$$\frac{d}{dt} \left(\frac{\partial L^\pm}{\partial \dot{\phi}} \right) - \frac{\partial L^\pm}{\partial \phi} = 0, \quad (8c)$$

$$\frac{d}{dt} \left(\frac{\partial L^\pm}{\partial \dot{u}_d} \right) - \frac{\partial L^\pm}{\partial u_d} = -F_d, \quad (8d)$$

$$\frac{d}{dt} \left(\frac{\partial L^\pm}{\partial \dot{u}_i} \right) - \frac{\partial L^\pm}{\partial u_i} = 0. \quad (8e)$$

Substituting the expressions previously introduced, these equations become:

$$(M + m_h + m_v)(\ddot{x}_g + \ddot{u}) \mp M(B \cos \phi \mp H \sin \phi) \dot{\phi}^2 - M(H \cos \phi \pm B \sin \phi) \ddot{\phi} = F_h + F_d, \quad (9a)$$

$$(M + m_v)(g + \ddot{y}_g + \ddot{v}) - M(H \cos \phi \pm B \sin \phi) \dot{\phi}^2 \pm M(B \cos \phi \mp H \sin \phi) \ddot{\phi} = F_v, \quad (9b)$$

$$[J_C + M(B^2 + H^2)] \ddot{\phi} - M \cos \phi [H(\ddot{x}_g + \ddot{u}) \mp B(g + \ddot{y}_g + \ddot{v})] - M \sin \phi [\pm B(\ddot{x}_g + \ddot{u}) + H(g + \ddot{y}_g + \ddot{v})] = 0, \quad (9c)$$

$$m_d(\ddot{u}_d + \ddot{x}_g) = -F_d, \quad (9d)$$

$$m_i \ddot{u}_i = 0. \quad (9e)$$

Note that the last equations of Eq. (5) and Eq. (9) are written under the assumption that the impacting mass m_i does not interact directly with the ground or the rest of the system, due to negligible friction.

2.2. Uplift, impact, full-contact, and overturning conditions

In this section, all the conditions occurring during the full-contact and rocking phases that are necessary to fully describe the system's motion are derived.

2.2.1. Uplift conditions

Starting from the rest position, the transition from the full-contact phase to the rocking phase is governed by the uplift conditions. These are determined by requiring that the overturning moments induced by the horizontal acceleration of the isolated base and the horizontal seismic excitation are greater than the restoring moments provided by gravity and the vertical acceleration of the isolated base, including the vertical seismic excitation, all evaluated about the poles $O^\pm$.

These conditions, expressed as inequalities, are written as

$$\pm(\ddot{u} + \ddot{x}_g) > \frac{1}{\lambda}(\ddot{v} + \ddot{y}_g + g), \quad (10)$$

where λ is the slenderness parameter introduced previously. If the inequality is satisfied with the plus sign, rocking initiates around O^+ ; otherwise, it initiates around O^- (Fig. 2).

2.2.2. Impact conditions for rigid block

The impact conditions occur when the rocking angle ϕ becomes zero, which can happen during rocking about either O^+ or O^- . Assuming that no bouncing occurs and that the impact is instantaneous, the post-impact velocities can be determined by enforcing the conservation of horizontal, vertical, and angular momentum. These conservation equations are

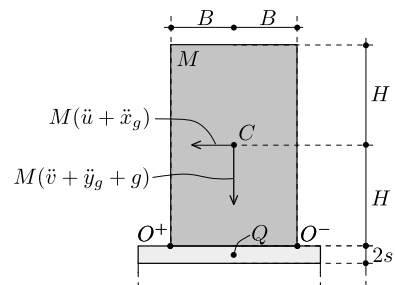


Fig. 2. Forces acting on the rigid block during the full-contact state.

expressed as:

$$[(m_h \dot{\mathbf{u}}_P + m_v \dot{\mathbf{u}}_Q + M \dot{\mathbf{u}}_C^\mp \cdot \mathbf{i})]_b = [(m_h \dot{\mathbf{u}}_P + m_v \dot{\mathbf{u}}_Q + M \dot{\mathbf{u}}_C^\pm \cdot \mathbf{i})]_a, \quad (11a)$$

$$[(m_v \dot{\mathbf{u}}_Q + M \dot{\mathbf{u}}_C^\mp) \cdot \mathbf{j}]_b = [(m_v \dot{\mathbf{u}}_Q + M \dot{\mathbf{u}}_C^\pm) \cdot \mathbf{j}]_a, \quad (11b)$$

$$[J_C \phi + M (\mathbf{R}^\pm \times \dot{\mathbf{u}}_C^\mp) \cdot \mathbf{k}]_b = [J_C \phi + M (\mathbf{R}^\pm \times \dot{\mathbf{u}}_C^\pm) \cdot \mathbf{k}]_a. \quad (11c)$$

where $\mathbf{i} = (1, 0, 0)^T$ is the canonical unit vector along the x -axis, $\mathbf{k} = (0, 0, 1)^T$ is the canonical unit vector along the z -axis and:

- the terms on the left-hand side of the equalities ($[\cdot]_b$) correspond to the pre-impact condition, while those on the right-hand side ($[\cdot]_a$) correspond to the post-impact condition;
- the vectors $\mathbf{R}^\pm = (\pm B, H, 0)^T$ are the position vectors of the mass center C with respect to the rocking vertices, where the plus sign is taken if the impact sequence is $O^- \rightarrow O^+$, and the minus sign otherwise.

In explicit form, if the pre-to-post impact sequence is $O^\mp \rightarrow O^\pm$, and denoting post-impact velocities by the subscript “a” and pre-impact velocities by the subscript “b”, the post-impact velocities read:

$$\dot{u}_a = \dot{u}_b - r_u \phi_b, \quad (12a)$$

$$\dot{v}_a = \dot{v}_b \mp r_v \phi_b, \quad (12b)$$

$$\phi_a = r_\phi \phi_b. \quad (12c)$$

where r_u , r_v , and r_ϕ are the *restitution coefficients*, defined as:

$$r_u = \frac{2B^2 H M^2 m_v}{\Delta}, \quad (13a)$$

$$r_v = \frac{2B H^2 M^2 (m_h + m_v)}{\Delta} + \frac{2B M J_C (M + m_h + m_v)}{\Delta}, \quad (13b)$$

$$r_\phi = \frac{H^2 M (M + m_v)(m_h + m_v)}{\Delta} - \frac{B^2 M m_v (M + m_h + m_v)}{\Delta} + \frac{J_C (M + m_v)(M + m_h + m_v)}{\Delta}. \quad (13c)$$

where:

$$\Delta = H^2 M (M + m_v)(m_h + m_v) + B^2 M m_v (M + m_h + m_v) + J_C (M + m_v) \times (M + m_h + m_v). \quad (14)$$

Note that the restitution coefficients in Eq. (13) are identical for both impact sequences $O^\mp \rightarrow O^\pm$, owing to the symmetry of the system with respect to the y -axis.

2.2.3. Impact conditions for the VIM

Since the aim of this study is to provide a preliminary assessment of the proposed protection system in mitigating seismic effects on horizontally and vertically isolated rigid block-like structures, without addressing the detailed design of the system, an idealized model of the impacting mass was adopted. Several more refined models are available in the literature, including nonlinear elastic (Hertz) and viscoelastic formulations [48–50].

The key distinction between an idealized and a refined model lies in the treatment of impact duration. In the idealized model, the impact is assumed to occur instantaneously, whereas refined models provide various methods to estimate the actual contact time. For sufficiently stiff materials, this duration is extremely short, typically much less than one second. Consequently, the idealized model is considered adequate for a preliminary investigation, as also confirmed by the results reported in [51].

If the VIM is constrained by end-stops, a discontinuity in its motion is introduced. Let the VIM have two end-stops located at $u_d - u_i = \pm d$, resulting in two possible behaviors:

- $|u_d - u_i| < d$: the VIM moves freely within the TMD.

- $|u_d - u_i| = d$: the VIM impacts the end-stops inside the TMD. Under the rigid impact assumption, the post-impact velocity can be determined using a coefficient of restitution r_i , which depends solely on the material properties of the impacting masses (i.e., the TMD and the VIM) and quantifies the fraction of kinetic energy preserved after the impact.

Denoting the post-impact velocities by the subscript “a” and pre-impact velocities by the subscript “b”, the impact condition of the VIM on the end-stops within the TMD can be written as:

$$(\dot{u}_d - \dot{u}_i)_a = -r_i (\dot{u}_d - \dot{u}_i)_b, \quad (15)$$

where r_i is the coefficient of restitution. This condition must be enforced together with the conservation of horizontal momentum, which gives

$$(m_d \dot{u}_d + m_i \dot{u}_i)_a = (m_d \dot{u}_d + m_i \dot{u}_i)_b. \quad (16)$$

Solving the system composed of Eqs. (15) and (16), the post-impact velocities can be expressed as functions of the pre-impact velocities:

$$(\dot{u}_d)_a = \left[\frac{m_d - r_i m_i}{m_d + m_i} \dot{u}_d + \frac{m_i (1 + r_i)}{m_d + m_i} \dot{u}_i \right]_b, \quad (17a)$$

$$(\dot{u}_i)_a = \left[\frac{m_d (1 + r_i)}{m_d + m_i} \dot{u}_d + \frac{m_i - r_i m_d}{m_d + m_i} \dot{u}_i \right]_b. \quad (17b)$$

In this paper, the restitution coefficient r_i is assumed to be constant and equal to $r_i = 0.7$. Preliminary analyses show that the system response is not significantly affected by this parameter. Nevertheless, investigating the influence of the restitution coefficient could be of interest in future studies.

2.2.4. Full-contact and overturning conditions

The full-contact phase strictly occurs when the rigid block-like structure remains in continuous contact with the isolated support, without uplifting, throughout the entire duration of the excitation. However, the analysis of numerous time-histories of the rocking angle ϕ , obtained by varying the system's governing parameters, reveals a non-negligible number of cases in which, following uplift, the maximum amplitude of ϕ remains very small. Furthermore, the duration of the rocking motion associated with such small rotations is extremely short, as the body tends to rapidly return to the full-contact phase. Therefore, a conventional condition is adopted for the full-contact phase, which also includes cases in which uplift occurs but, due to the small amplitude and short duration of the resulting rocking motion, the response manifests as a tremor rather than a genuine rocking motion; this conventional full-contact phase is introduced in order to present the subsequent numerical results.

It is therefore conventionally assumed that the rigid block-like structure remains in a full-contact condition if the following requirement is satisfied throughout the entire motion:

$$\max |\phi| \leq 0.03 \alpha_{cr}. \quad (18)$$

This condition implies that the rocking angle never exceeds 3% of the critical angle α_{cr} . As an illustrative example, the largest critical angle among the two rigid block-like structures examined in this study is $\alpha_{cr} = 16.7^\circ$, which corresponds to a very small threshold for the rocking angle, namely $0.03 \alpha_{cr} = 0.5^\circ$.

Finally, it is assumed that the overturning condition conventionally occurs during the rocking motion when the center of mass of the rigid body lies on the vertical projection of one of the base poles $O^\pm$. Specifically, when:

$$\phi = \pm \alpha_{cr}, \quad (19)$$

where $\alpha_{cr} = \arctan(B/H)$ (see Fig. 1(a)).

2.3. Constitutive laws of the isolation devices and TMD device

This section addresses the constitutive laws governing the devices integrated into the isolation system, as well as those of the tuned mass damper.

2.3.1. Isolation devices

The horizontal and vertical devices embedded in the isolation base exhibit distinct mechanical responses. In particular, the restoring forces associated with the horizontal isolation system are modeled using a hysteretic Bouc–Wen formulation [44], while those of the vertical isolation system are represented by a linear viscoelastic Kelvin–Voigt model [45]. Based on these assumptions, the corresponding restoring forces F_h and F_v of the horizontal and vertical isolation systems, respectively, are expressed as:

$$F_h = -[\psi k_h u + k_h u_y(1 - \psi)z + c_h \dot{u}], \quad (20a)$$

$$F_v = -(k_v v + c_v \dot{v}). \quad (20b)$$

In Eq. (20)(a), the parameter ψ is defined as the ratio between the post-yield stiffness k_{hp} and the initial (pre-yield) horizontal stiffness k_h , so that $k_{hp} = \psi k_h$. The quantity u_y denotes the yield displacement. The special case $\psi = 0$ describes an ideal elasto-plastic behavior of the horizontal connecting elements, whereas $\psi = 1$ corresponds to a fully elastic response. In addition, $z = z(t)$ represents the hysteretic internal variable that controls the post-yield behavior and is determined by integrating the following nonlinear first-order ordinary differential equation:

$$\dot{z} = \frac{\dot{u}}{u_y} [A_h - |z|^n (\beta_h + \gamma_h \text{sign}(\dot{u} \cdot z))]. \quad (21)$$

In Eq. (20)(b), k_v denotes the vertical elastic stiffness, whereas in Eq. (20) the coefficients c_h and c_v refer to the associated horizontal and vertical viscous damping, respectively.

It is well established that the parameters of the Bouc–Wen model exhibit functional redundancy. To remove this redundancy, the values $A_h = 1$ and $\beta_h = \gamma_h = 0.5$ are selected. With these assumptions, the Bouc–Wen formulation reduces to a strain-softening model with a transparent physical meaning for the mechanical parameters. Under such constraints, the dimensionless hysteretic variable z varies within the interval between -1 and 1 ([46,47]). Finally, the exponential parameter n governs the smoothness of the transition from the elastic branch to the post-elastic branch of the hysteretic response.

The selected parameter values $A_h = 1$ and $\beta_h = \gamma_h = 0.5$, together with the values of $n = 2$, are consistent with commonly adopted simplified Bouc–Wen formulations ensuring a stable and physically interpretable hysteretic response [46,47,52]. Moreover, the choice $\psi = 0.25$ reflects a realistic ratio between post-yield and pre-yield stiffness, consistent with typical ranges adopted for seismic isolation devices and bilinear hysteretic models in structural applications [53]. Finally, the yield displacement is defined as $u_y = \alpha_y(M + m_h + m_v)PGA_h/k_h$, with $\alpha_y = 0.1$. This definition assumes that u_y corresponds to a fraction α_y of the horizontal displacement of the isolated system under the static application of an acceleration equal to the horizontal Peak Ground Acceleration (PGA_h) of the earthquake.

2.3.2. TMD device

The restoring forces of the impacting TMD are modeled using the linear viscoelastic Kelvin–Voigt formulation. According to this model, the restoring force is expressed as:

$$F_d = -[k_d(u - u_d) + c_d(\dot{u} - \dot{u}_d)], \quad (22)$$

where k_d is the elastic stiffness and c_d is the viscous damping coefficient of the TMD.

2.4. Numerical implementation and solution procedure

The equations of motion governing both the full-contact and rocking phases, together with the uplift, impact, and overturning conditions

described above, were solved numerically through an implementation developed in *Mathematica* [54], using the built-in differential equation solvers and event-detection routines. To independently verify the numerical implementation, the same governing equations and transition conditions were also implemented in a dedicated *Python* code, where the time evolution was computed using a Runge–Kutta integration scheme coupled with appropriate checks for uplift, impact, and overturning events. Consistent results were obtained from the two independent implementations.

3. Parametric analysis

This section describes the parameters considered in the numerical investigation and the methodology adopted to assess the seismic response of the proposed system. The selected earthquake records, the characteristics of the reference rigid block-like structures, and the properties of the isolation system and TMD–VIM device are first introduced. Subsequently, the response measures and graphical tools employed throughout the paper, namely rocking and displacement maps, are defined.

An extensive parametric analysis is carried out by varying several parameters that characterize the system. More specifically, the variables considered include the external excitation, represented by two different seismic records; the properties of two server cabinet blocks; the dynamic characteristics of the horizontal and vertical isolation system; and the parameters defining the TMD–VIM device.

The results of the parametric analysis are presented through two types of maps. The first type, called *rocking maps*, are contour plots of the rocking index $\zeta = \max|\phi|/\alpha_{cr}$ within a given parameter space, where the isolated rigid block is subjected to seismic excitation from a specific earthquake record. The rocking maps are divided into three distinct regions:

- (i) A dark gray region where $\zeta = 1$, indicating direct overturning of the block;
- (ii) A light gray region where $\zeta \leq 0.03$, corresponding to a full-contact state (see, Section 2.2.4);
- (iii) An intermediate region with a gradient of gray shading, representing rocking motion without overturning. The contour levels indicate the maximum value of the rocking index ζ (i.e., the maximum rocking angle) reached during the motion.

In these graphs, the overturning region (dark gray) and the rocking region (shaded gray) are always separated by a thick solid curve, called overturning boundary, while the rocking region and the full-contact region (light gray) are divided by a thick dashed curve, called rocking boundary.

The second type, referred to as *displacement maps*, shows contour plots of either the maximum absolute horizontal or vertical displacements of the isolated base, i.e., $\max|u|$ or $\max|v|$, within the same parameter space, under seismic excitation from an earthquake record. Depending on the direction of displacement represented, these maps are called horizontal displacement maps or vertical displacement maps. A gray-scale gradient from light to dark is used to indicate increasing displacement magnitudes, from lower to higher values.

In the following, the parameters treated as variables are described in detail. All other quantities not reported in this section are considered fixed, and their values are provided throughout the paper whenever explicitly referenced.

3.1. Seismic excitation

A set of three earthquake records, including both horizontal and vertical ground motions, is employed in the seismic analyses. The records were selected from strong-motion earthquakes. The aim of this selection is to investigate the ability of the TMD–VIM to enhance the performance of base-isolated rigid block-like structures against overturning.

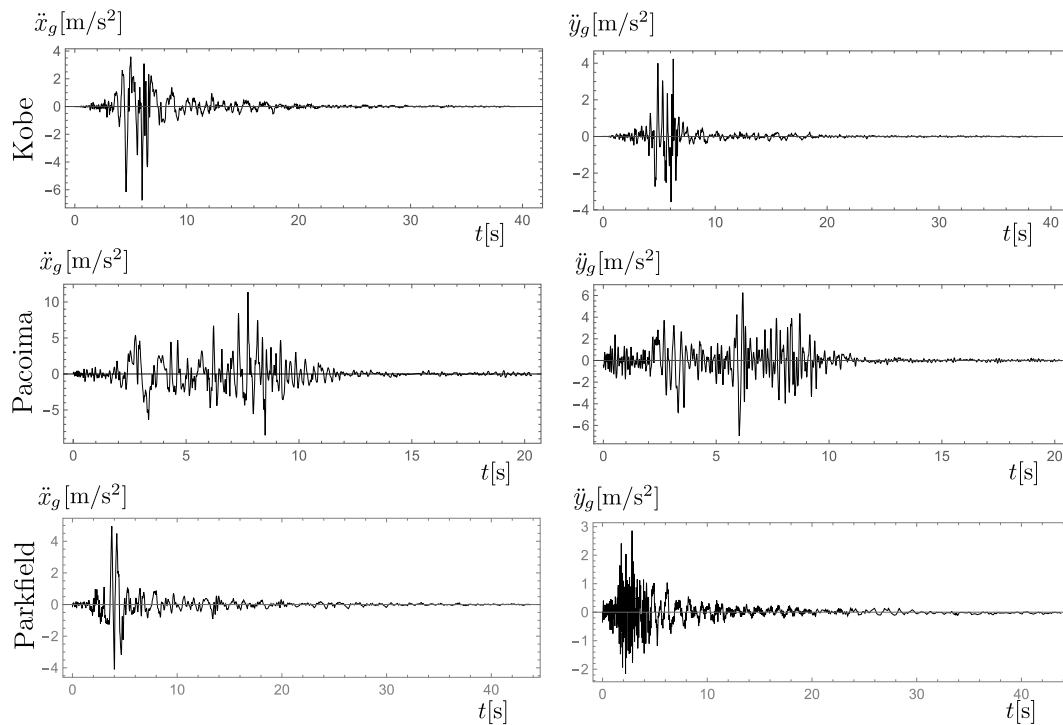


Fig. 3. Horizontal ($\ddot{x}_g$) and vertical ($\ddot{y}_g$) acceleration time histories of the Kobe, Pacoima and Parkfield earthquake records adopted in the analyses.

In fact, as shown in the previous study [42], a rigid block resting on a dual horizontal–vertical isolation system that experiences uplift under a strong-motion earthquake tends to overturn rapidly, due to the very limited possibility of developing rocking motion.

Fig. 3 shows the time-histories of the horizontal and vertical components of the chosen records. The earthquake records considered are:

- (a) Kobe, 1995 Japan earthquake, Takarazuka station, 0 deg, ground level, position of the station: 34.8090N, 135.3440W;
- (b) Pacoima, 1971 San Fernando, California earthquake, station no. 279, comp S16E, position of the station: 34.335N, 118.3967W;
- (c) Parkfield, 1966 California earthquake, station no. 013, comp N65E, position of the station: 35.726N, 120.287W.

For reference, each record is identified by the underlined name provided in the list above.

3.2. Assumptions and reference cabinets

The numerical investigation presented in this study considers two objects selected from a set of reference server cabinets representative of commercially available 19-inch rack systems. The adopted geometric dimensions and mass properties are consistent with international standards, manufacturers' technical documentation, and widely accepted engineering practice.

The selected cabinets comply with the 19-inch rack standard defined by IEC [55] and EIA [56] specifications. The minimum base dimension is taken as the rack width $2B = 0.60$ m, while the depth L is equal to 1.00 m or 1.20 m, corresponding to standard and deep-rack configurations, respectively. The cabinet heights are $2H = 2.00$ m or $2H = 2.45$ m.

Since the base dimensions of the two cabinets are identical, slenderness represents the primary parameter governing the assessment of rocking response and overturning stability.

Table 1

Geometrical and mechanical properties of the reference server cabinets considered in this study.

Cabinet	H [m]	B [m]	L [m]	λ [-]	Mass [kg]
42U	1.00	0.30	1.00	3.33	620
52U	1.225	0.30	1.20	4.08	880

The following two cabinet typologies are considered:

- Standard racks (**42U**): representative of typical enterprise data-center installations.
- High racks (**52U**): increasingly adopted in high-density facilities.

The main characteristics of the two selected cabinets are reported in Table 1. The total masses reported in Table 1 are obtained by adopting equivalent volumetric mass densities of 517 kg/m^3 for the 42U standard rack and 499 kg/m^3 for the 52U deep-rack configuration, estimated by combining the self-weight of the steel enclosure with that of a fully populated configuration of IT equipment.

3.3. Base isolation

The parameters treated as variables are the horizontal T_h and vertical T_v isolation periods. They are defined as:

$$T_h = 2\pi \sqrt{\frac{M + m_h + m_v}{k_h}}, \quad (23a)$$

$$T_v = 2\pi \sqrt{\frac{M + m_v}{k_v}}. \quad (23b)$$

It should be noted that changing T_h or T_v implies a corresponding change in the stiffnesses k_h and k_v , as the system masses remain constant.

The viscous damping coefficients c_h and c_v (see Eq. (20)) are expressed as:

$$c_h = 2(M + m_h + m_v) \xi_h \omega_h, \quad (24a)$$

$$c_v = 2(M + m_v) \xi_v \omega_v, \quad (24b)$$

where

$$\omega_h = \sqrt{\frac{k_h}{M + m_h + m_v}}, \quad (25a)$$

$$\omega_v = \sqrt{\frac{k_v}{M + m_v}}, \quad (25b)$$

are the conventional natural frequencies of the isolation system, and ξ_h , ξ_v are the corresponding damping ratios. In the following, $\xi_h = 0.05$ and $\xi_v = 0.10$ are assumed.

3.4. Tuned mass damper and vibro-impact mass

The proposed TMD–VIM device combines the vibration-mitigation capability of a conventional tuned mass damper with the additional nonlinear effects generated by an embedded vibro-impacting mass. The purpose of the VIM is to exploit vibro-impact interactions capable of modifying the dynamic response of the isolated rigid block-like structure and, in particular, the transitions between full-contact, rocking, and overturning regimes. The effectiveness of the device depends on both the dynamic properties of the TMD and the characteristics of the embedded impacting mass. For this reason, the parameters treated as variables are the period T_d and the mass m_d of the TMD, as well as the clearance d and the mass m_i of the VIM. These parameters are defined as follows:

- Period T_d of the TMD:

$$T_d = 2\pi \sqrt{\frac{m_d}{k_d}}. \quad (26)$$

The viscous damping coefficient c_d (see Eq. (22)) of the TMD is defined as:

$$c_d = 2 m_d \xi_d \omega_d, \quad (27)$$

where $\omega_d = \sqrt{k_d/m_d}$ is the natural frequency of the TMD and $\xi_d = 0.05$ is the corresponding fixed damping ratio.

- Total mass m_{tot} of the TMD–VIM, mass m_d of the TMD, and VIM mass m_i are defined as:

$$m_{tot} = \mu(M + m_h + m_v), \quad (28a)$$

$$m_d = \mu_d m_{tot}, \quad (28b)$$

$$m_i = \mu_i m_{tot}. \quad (28c)$$

The variation of m_{tot} , m_d , and m_i is performed by adjusting the nondimensional coefficients μ , μ_d , and μ_i , respectively. Note that $\mu_d + \mu_i = 1$.

- Clearance d of the vibro-impacting mass:

The VIM clearance is treated as a variable within a suitable dimensional range, expressed in meters.

4. Brief remarks on the role of combined horizontal and vertical isolation

Before investigating the contribution of the TMD and the embedded vibro-impacting mass, it is useful to establish a reference response for the system equipped solely with the dual horizontal–vertical isolation system. This preliminary discussion highlights the influence of the horizontal and vertical isolation periods on the occurrence of full-contact, rocking, and overturning responses, providing the baseline against which the effectiveness of the proposed vibration-control devices will subsequently be assessed.

The response of the dual isolation system has been previously investigated in [42,43], although for rigid blocks with different geometrical

and mechanical characteristics. The main trends identified in those studies are confirmed by the present analyses and are therefore briefly summarized here.

Fig. 4 shows the rocking maps in the T_h – T_v parameter plane for the two cabinets, 42U and 52U (maps in the first and second rows, respectively), subjected to the two earthquake records, Kobe and Pacoima (maps in the first and second columns, respectively). The main observations are:

- The boundary between the rocking and overturning regions (thick solid line) is predominantly vertical, particularly for the Kobe earthquake, which has the smaller vertical Peak Ground Acceleration (PGA_v). This indicates that the vertical isolation period T_v has a limited influence compared to the horizontal isolation period T_h .
- The boundary between the rocking and full-contact regions (thick dashed line) is strongly influenced by the vertical isolation period T_v , especially for values below 0.75 s. In this range of T_v , there exist multiple intervals where the rigid block never attains a full-contact state during the motion.
- For $T_v > 0.75$ s, the boundary between the rocking and full-contact regions is predominantly vertical, suggesting that this boundary is only weakly dependent on the vertical isolation period T_v . The rocking region can be regarded as a safety buffer between the stable full-contact state and the overturning region. When $T_v > 0.75$ s, the rocking region becomes very narrow, indicating that the gap between the full-contact and overturning conditions is small.
- As can be observed in the zoomed-in views near the origin of the rocking maps (green dashed box), the rigid block-like structure overturns for all values of T_h when T_v is very small. This behavior is caused by the vertical seismic components, which make the vertical isolation system essential to prevent overturning of the block. Therefore, under strong vertical excitation, a vertical isolation system, even with a sufficiently small period T_v , is necessary to ensure the safety of the system against overturning.

The rocking maps suggest that maximizing the full-contact region requires relatively large values of the vertical isolation period T_v ($T_h > 0.75$ s). However, increasing T_v (i.e., reducing the vertical stiffness k_v) leads to a larger vertical displacement of the isolated base, which may exceed acceptable limits.

Fig. 5 presents the vertical displacement maps in the T_h – T_v parameter plane for the two cabinets, 42U and 52U (shown in the two rows), subjected to the Kobe and Pacoima earthquake records (shown in the two columns). As indicated by the contour levels, increasing the vertical isolation period T_v leads to a rapid growth of the vertical displacement of the isolated base, which reaches values on the order of one meter as T_v approaches 2.0 s. Such large vertical displacements are mainly attributable to the vertical static settlement induced by the considerable weight of the two selected cabinets, together with the reduction of the vertical stiffness k_v as the vertical isolation period T_v increases.

By assuming that the acceptable vertical displacement is limited to 0.10 m, the displacement maps indicate that the vertical isolation period T_v should not exceed 0.5 s. The horizontal red dashed lines in the rocking maps of Fig. 4 pass through $T_v = 0.25$ s in the maps referring to the Kobe earthquake (left column), whereas they pass through $T_v = 0.50$ s in the maps referring to the Pacoima earthquake (right column). These values of T_v are selected because, in their vicinity, the rocking maps exhibit sufficiently large full-contact regions. The ability to adopt relatively small values of T_v while still achieving an extended full-contact region is one of the key findings reported in [42,43]. In the vicinity of $T_v = 0.25$ s and $T_v = 0.50$ s, the rocking maps exhibit sufficiently large full-contact regions while simultaneously ensuring sufficiently small vertical displacements of the isolated base (see Fig. 5). Finally, for subsequent analyses, the values of the horizontal isolation period T_h at which the

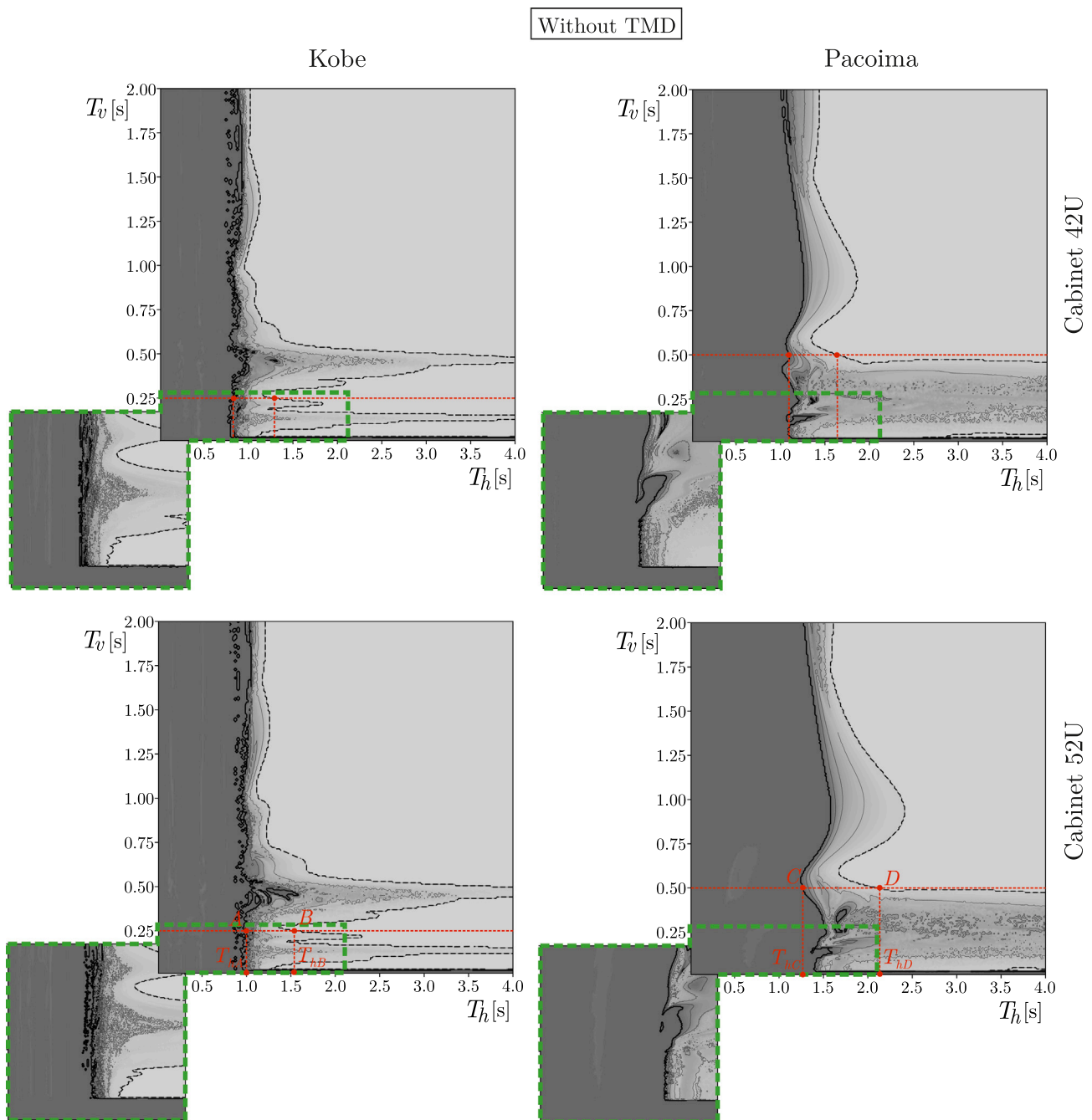


Fig. 4. Rocking maps in the $T_h - T_v$ plane for the 42U and 52U cabinets subjected to the Kobe and Pacoima earthquakes, considering the dual horizontal-vertical isolation system only (without TMD). Light-gray regions correspond to full-contact response, shaded regions to rocking motion, and dark-gray regions to overturning. Thick dashed and solid curves denote the full-contact/rocking and rocking/overturning boundaries, respectively. Green dashed boxes indicate enlarged views near the origin. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

horizontal red dashed line intersects the two boundaries (thick solid and thick dashed curves) are identified. In the maps shown in Fig. 4, these values are highlighted by vertical red dashed lines and are specifically denoted as T_{hA} , T_{hB} , T_{hC} , and T_{hD} in the maps referring to the 52U server cabinet (lower maps).

Following the previous considerations regarding the vertical isolation period T_v , fixed values of T_v are consistently adopted throughout the numerical simulations in this paper. In particular, for excitation by the Kobe and Parkfield earthquakes, $T_v = 0.25$ s is used, whereas for the Pacoima earthquake, $T_v = 0.50$ s is employed.

5. The role of the TMD and the VIM

This section investigates the influence of the TMD and the TMD-VIM device on the seismic response of the isolated rigid block-like structures. The analysis is carried out progressively, first examining the effectiveness of a conventional TMD and subsequently assessing the additional contribution provided by the embedded vibro-impacting mass. Particular attention is devoted to the modification of the rocking and overturning boundaries and to the reduction of the displacement demand of the isolation system.

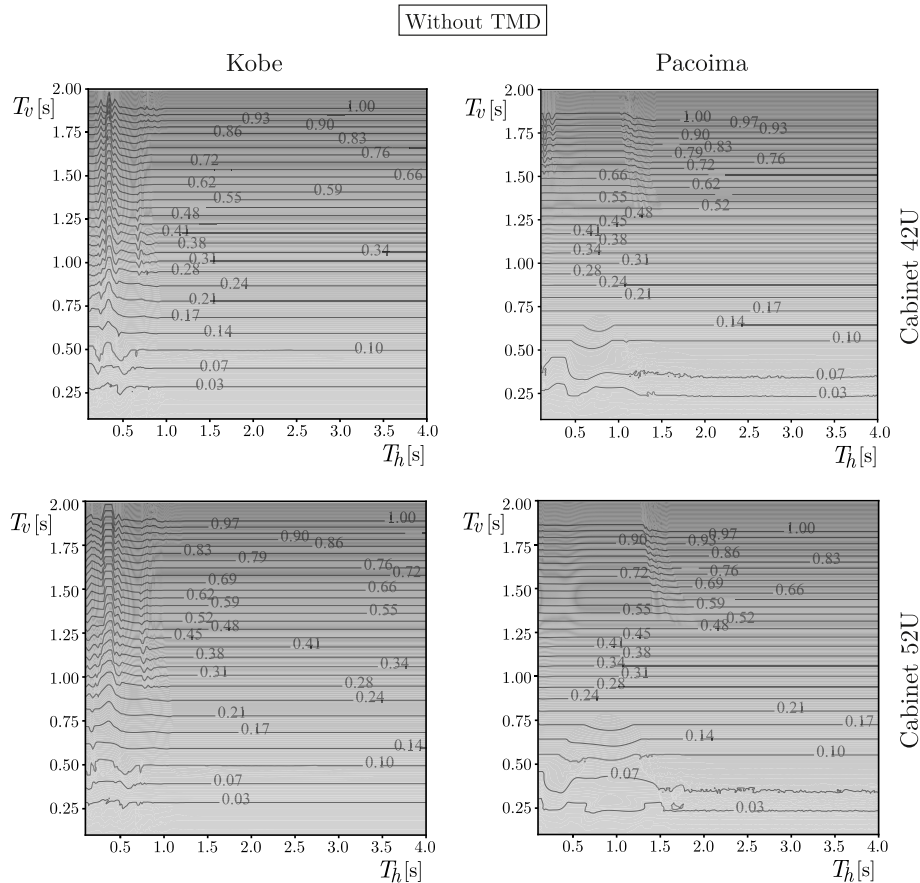


Fig. 5. Vertical displacement maps in the $T_h - T_v$ plane for the 42U and 52U cabinets subjected to the Kobe and Pacoima earthquakes, considering the dual horizontal-vertical isolation system only (without TMD). Contours represent the maximum vertical displacement of the isolated base.

The effectiveness of the proposed devices is assessed by examining their ability to enlarge the full-contact region and reduce the overturning region with respect to the isolation-only configuration. In the rocking maps presented throughout this section, an improvement is achieved when both the rocking–full-contact boundary and the rocking–overturning boundary move towards smaller values of the horizontal isolation period T_h relative to the corresponding boundaries of the isolation-only system (see Fig. 4). This indicates that smaller horizontal isolation periods can be adopted without compromising seismic safety, resulting in a reduction of the horizontal displacement demand of the isolated base.

In the following, results obtained for the case with only base isolation are labeled as *without TMD*; results obtained using only the TMD (without VIM) are labeled as *with TMD*; whereas results referring to the presence of both TMD and VIM are labeled as *with TMD–VIM*.

5.1. The effect on the rocking motion of the block

The first part of the investigation focuses on the influence of the TMD and the TMD–VIM device on the rocking response of the isolated rigid block. The objective is to assess whether the proposed devices can enlarge the full-contact region, delay the onset of rocking, and reduce the likelihood of overturning. The analysis is performed through rocking maps constructed in different parameter spaces.

The analysis begins with the TMD-only configuration, which serves as a reference for evaluating the additional contribution provided by the embedded vibro-impacting mass. The mass of the TMD, m_d , is equal to the total mass of the device, m_{tot} (i.e., $m_i = 0$; see Eq. (28)). Fig. 6 shows the rocking maps in the $T_h - T_d$ parameter plane, obtained by exciting the 52U server cabinet with the Kobe earthquake (left column)

and the Pacoima earthquake (right column). The maps in the three rows correspond to different values of the coefficient μ , which is directly related to the total mass of the device (see Eq. (28)(a)). For the case considered, μ takes the values 0.1, 0.2, and 0.3 from the top row to the bottom row, respectively. The vertical red dashed lines correspond to the values of T_{hA} , T_{hB} , T_{hC} , and T_{hD} , which, for a fixed value of T_v ($T_v = 0.25$ s for the Kobe earthquake and $T_v = 0.50$ s for the Pacoima earthquake), represent the boundaries of the rocking regions (T_{hA} and T_{hC}) and of the full-contact regions (T_{hB} and T_{hD}) of the system without the TMD. These values of T_h have already been reported and discussed in Fig. 4.

The vertical red lines at T_{hB} and T_{hD} , which in the isolation-only configuration represent the boundaries between the rocking and the full-contact regions, are almost entirely located within the new full-contact regions when the TMD-only configuration is considered. Only for the largest investigated TMD periods T_d , and only in the case of the Pacoima earthquake (right column), do the vertical red lines fall within the rocking regions. In general, it can be noted that, over nearly the entire range of the investigated T_d values, the boundary between the rocking and full-contact regions shifts to the left, effectively enlarging the full-contact region. This implies that smaller horizontal periods of the isolated base can be adopted if the system is equipped with a suitable TMD, while still ensuring that the rigid block remains in full contact.

The vertical red lines at T_{hA} and T_{hC} , which in the isolation-only configuration represent the boundaries between the overturning and the rocking regions, are almost entirely located within the new rocking regions when the TMD-only configuration is considered. In particular, for the Kobe earthquake (left column), these vertical red lines are nearly tangent to the overturning boundary, whereas for sufficiently large values

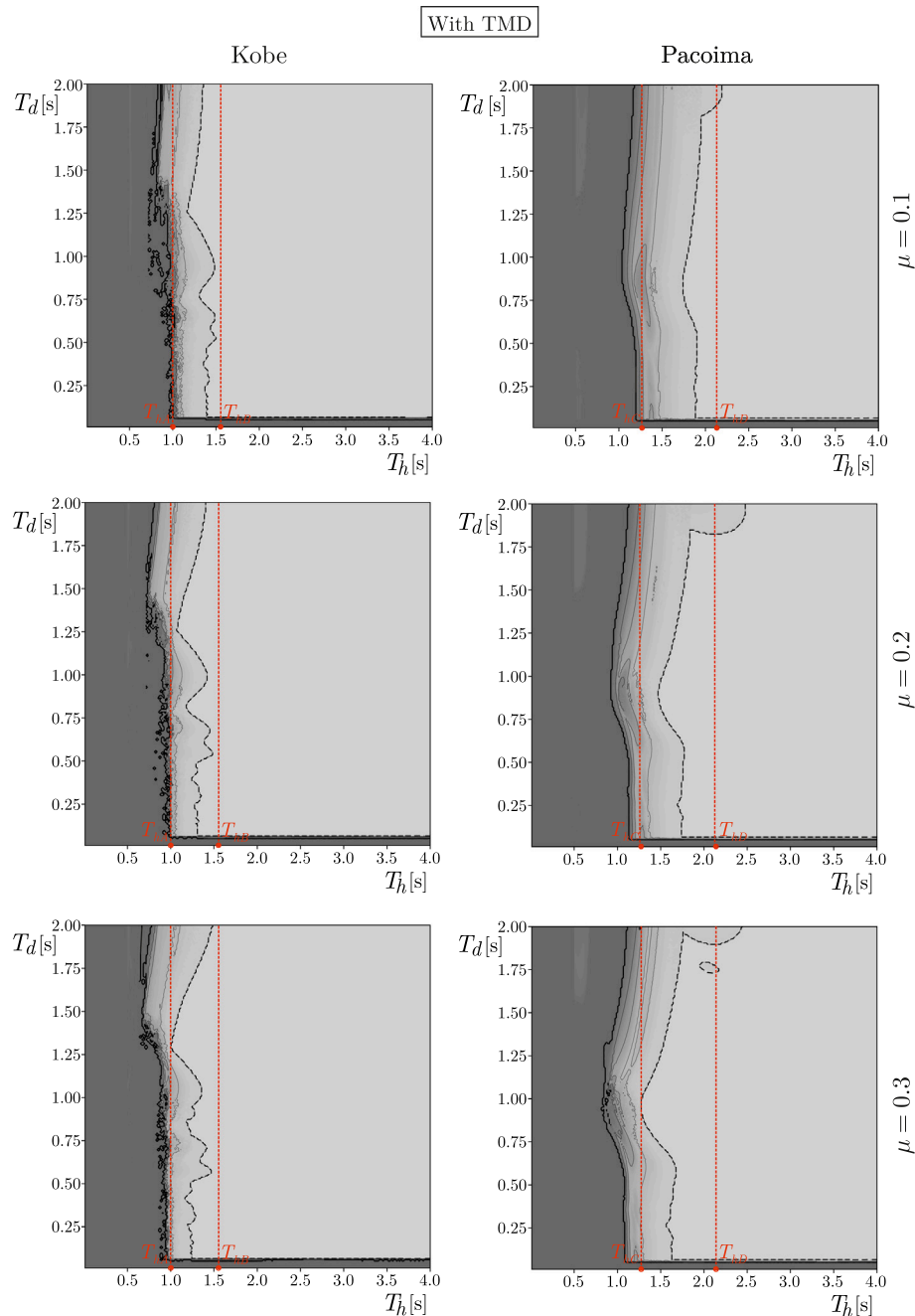


Fig. 6. Rocking maps in the $T_h - T_d$ plane for the 52U cabinet subjected to the Kobe ($T_v = 0.25$ s) and Pacoima ($T_v = 0.50$ s) earthquakes, considering the TMD-only configuration ($m_i = 0$). The three rows correspond to increasing values of the total mass ratio μ . Vertical red dashed lines indicate the reference values T_{hA} , T_{hB} , T_{hC} , and T_{hD} identified in Fig. 4. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

of T_d they lie well inside the rocking region, i.e., at a significant distance from the overturning boundary. In the case of the Pacoima earthquake, the corresponding vertical red lines are always located within the rocking regions. Also in this case, the results indicate that the boundary between the overturning and rocking regions shifts to the left. This implies that smaller horizontal periods of the isolated base can be adopted, if a suitable choice of TMD is made, while still ensuring that the rigid block does not overturn.

Finally, the rocking maps in Fig. 6 show a slight dependence on the parameter μ (i.e., on the total mass m_{tot} of the device), with a modest deterioration of the response as μ increases. Therefore, in what follows, the value $\mu = 0.2$ is consistently adopted.

At this stage, it is useful to assess the beneficial effects arising from the simultaneous presence of the TMD and the VIM. The results of this analysis are presented in Fig. 7. The results shown herein refer to the 52U cabinet subjected to the Kobe earthquake (maps in the left column) and the Pacoima earthquake (maps in the right column). In particular, Fig. 7(a) reproduces the rocking maps in the $T_h - T_d$ parameter plane for the TMD-only configuration, already shown and discussed in Fig. 6 (middle row, $\mu = 0.2$), for the sake of clarity and ease of comparison. In these maps, the red dashed lines correspond to values of the horizontal period T_h for which, irrespective of the TMD period T_d , the system response, without VIM, falls within the rocking regions.

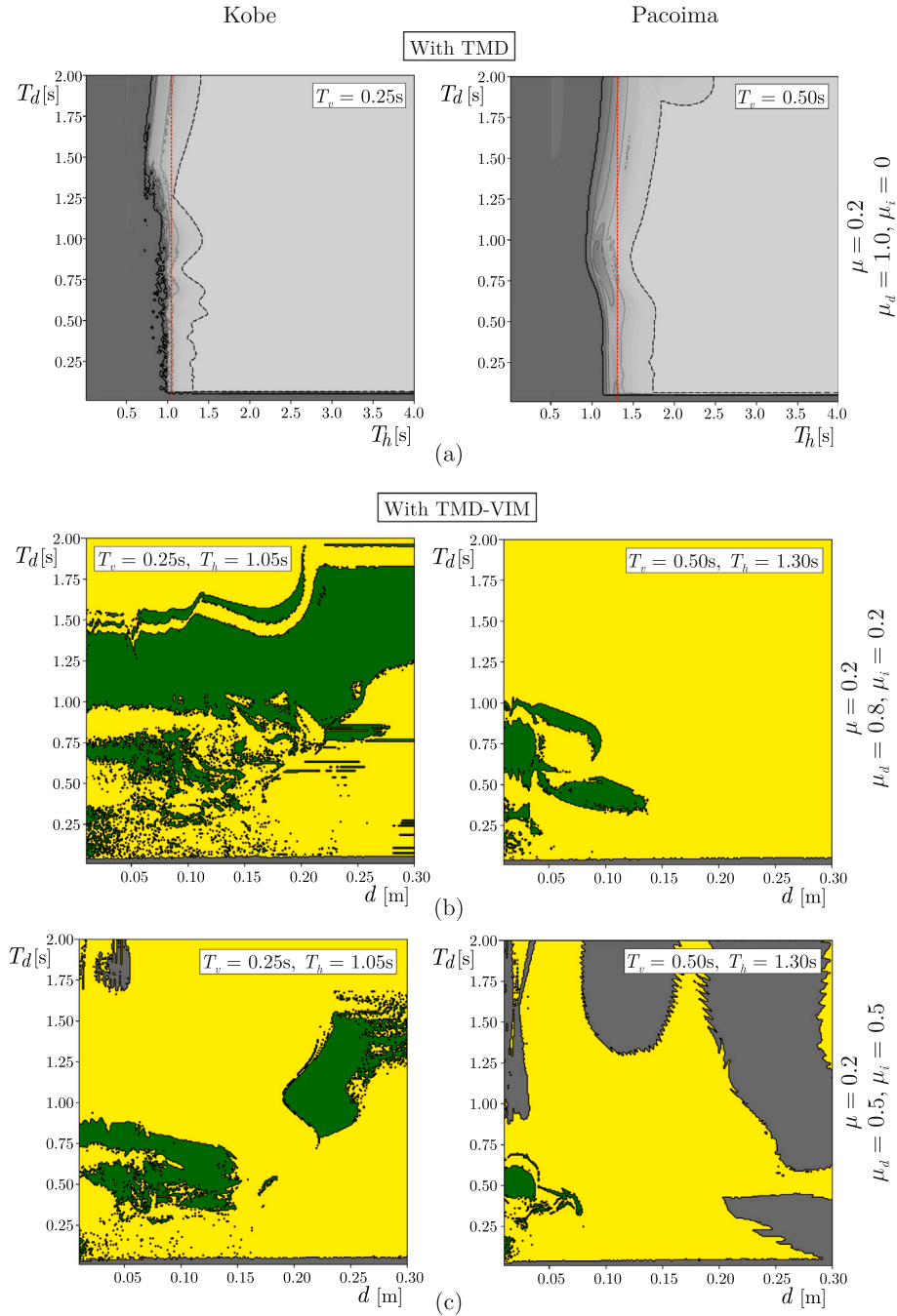


Fig. 7. Rocking maps for the 52U cabinet subjected to the Kobe ($T_v = 0.25$ s) and Pacoima ($T_v = 0.50$ s) earthquakes and equipped with devices having different mass distributions: (a) $T_h - T_d$ maps for the TMD-only configuration ($\mu = 0.2, \mu_d = 1.0, \mu_i = 0$); (b) $d - T_d$ maps for the TMD-VIM configuration ($\mu = 0.2, \mu_d = 0.8, \mu_i = 0.2, T_h = 1.05$ s for Kobe and $T_h = 1.30$ s for Pacoima earthquakes); (c) $d - T_d$ maps for the TMD-VIM configuration ($\mu = 0.2, \mu_d = 0.5, \mu_i = 0.5, T_h = 1.05$ s for Kobe and $T_h = 1.30$ s for Pacoima earthquakes). In panels (b) and (c), green, yellow, and dark-gray regions identify full-contact, rocking, and overturning responses, respectively. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

To assess the simultaneous presence of the TMD and the VIM, rocking maps in the $d - T_d$ parameter plane are obtained for different values of the mass ratios characterizing the TMD-VIM system. In particular, the maps in Fig. 7(b) refer to $\mu_d = 0.8$ and $\mu_i = 0.2$, whereas those in Fig. 7(c) correspond to $\mu_d = 0.5$ and $\mu_i = 0.5$. These maps are derived by keeping both the horizontal and vertical periods, T_h and T_v , fixed. For the case considered, the adopted values of T_h coincide with those indicated by the red dashed lines in Fig. 7(a), which correspond to system responses always lying within the rocking region, regardless of the value of T_d . The

aim of this analysis is to verify whether there exist pairs of parameters (d, T_d) for which the system is able to exhibit a full-contact response. The rocking maps shown in Fig. 7(b),c clarify this aspect. Three distinct regions, colored in green, yellow, and dark gray, characterize these maps. Specifically, for pairs of parameters (d, T_d) selected within the green regions, the system exhibits a full-contact condition. Therefore, the simultaneous use of the TMD and the VIM enables the system to operate in a full-contact state, in contrast to the TMD-only configuration, in which the system undergoes rocking motion. The extent of the green

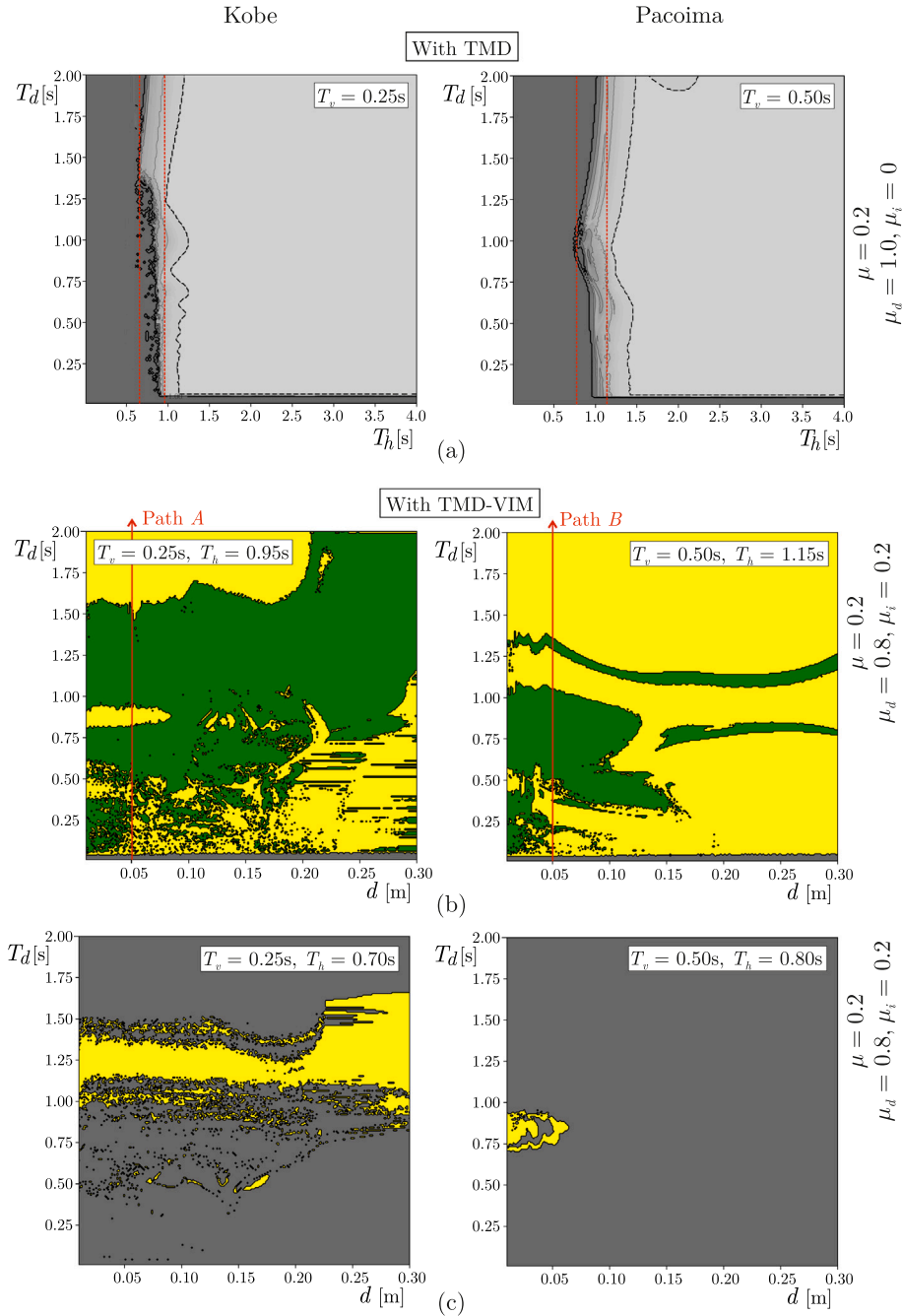


Fig. 8. Rocking maps for the 42U cabinet subjected to the Kobe ($T_v = 0.25$ s) and Pacoima ($T_v = 0.50$ s) earthquakes and equipped with devices having different characteristics: (a) $T_h - T_d$ maps for the TMD-only configuration ($\mu = 0.2$, $\mu_d = 1.0$, $\mu_i = 0$); (b) $d - T_d$ maps, with TMD-VIM, obtained for the values of T_h identified by the right red dashed lines in panel (a) ($\mu = 0.2$, $\mu_d = 0.8$, $\mu_i = 0.2$); (c) $d - T_d$ maps, with TMD-VIM, obtained for the values of T_h identified by the left red dashed lines in panel (a) ($\mu = 0.2$, $\mu_d = 0.8$, $\mu_i = 0.2$). Green, yellow, and dark-gray regions identify full-contact, rocking, and overturning responses, respectively. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

regions depends on the input earthquake, as evidenced by their reduced size when comparing the left column (Kobe earthquake) with the right column (Pacoima earthquake). Moreover, the size of the green regions decreases as the mass of the VIM increases relative to the mass of the TMD, as observed by comparing Fig. 7(b) and Fig. 7(c).

The yellow regions correspond to rocking motion of the rigid block-like structure; therefore, within these regions, no advantage is achieved by adopting the combined TMD-VIM configuration with respect to the TMD-only configuration. Finally, the dark-gray regions correspond to

overturning of the rigid block. Hence, within these regions, the combined use of the TMD and the VIM is detrimental compared to the TMD-only configuration.

These dark-gray regions are almost absent in the maps shown in Fig. 7(b), whereas they become significant in the maps of Fig. 7(c). This observation further confirms that the system performs more effectively when the mass of the TMD is sufficiently larger than that of the VIM.

Based on the results obtained and discussed above, the mass ratios $\mu_d = 0.8$ and $\mu_i = 0.2$ are consistently adopted in what follows.

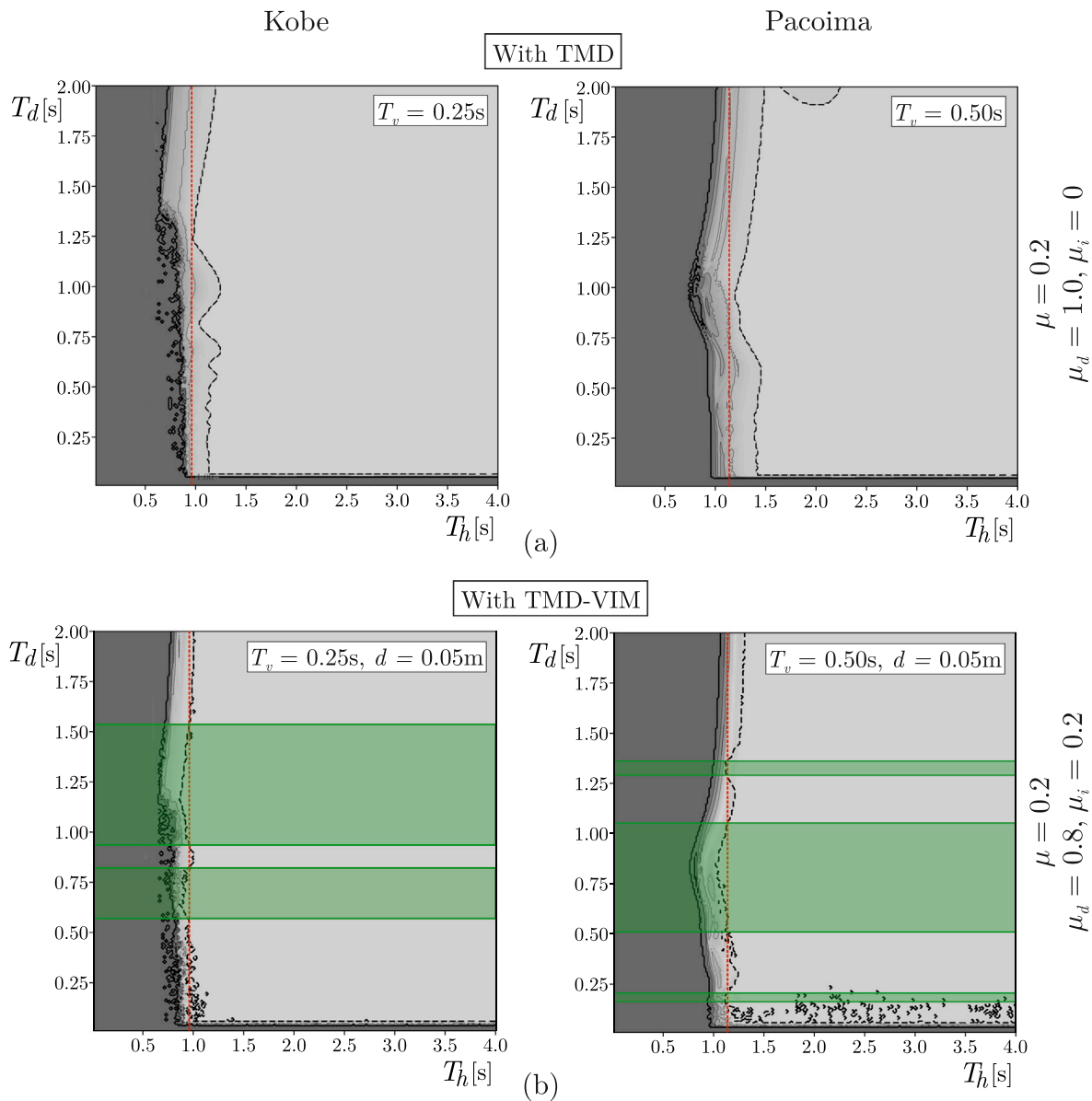


Fig. 9. Rocking maps in the $T_h - T_d$ plane for the 42U cabinet subjected to the Kobe and Pacoima earthquakes: (a) TMD-only configuration; (b) TMD-VIM configuration with fixed clearance $d = 0.05$ m (Path A for Kobe and Path B for Pacoima earthquakes, see Fig. 8(b)). Green bands indicate ranges of T_d for which the response along the selected paths changes from rocking to full-contact. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Fig. 8 shows the rocking maps referring to the 42U cabinet subjected to the Kobe earthquake (maps in the left column) and the Pacoima earthquake (maps in the right column). This figure is intended to highlight an additional aspect related to the combined use of the TMD and the VIM. More specifically, Fig. 8(a) reproduces, for the first time, the rocking maps in the $T_h - T_d$ parameter plane for the TMD-only configuration. In these maps, the right red dashed lines correspond to values of the horizontal period T_h for which, irrespective of the TMD period T_d , the system response always lies within the rocking regions. Conversely, the left red dashed lines correspond to values of the horizontal period T_h for which, regardless of the TMD period T_d , the system response falls within the overturning regions.

Fig. 8(b),(c) show the rocking maps in the $d - T_d$ parameter plane, obtained by fixing the horizontal isolation period T_h at the value corresponding to the right red dashed line (Fig. 8(b)) and to the left red dashed line (Fig. 8(c)), respectively. By starting from the rocking maps

referring to the left red dashed line, it is evident that several pairs of parameters (d, T_d) lead to a full-contact condition (green regions), even though, in the TMD-only configuration, the system undergoes rocking motion. This confirms that the combined TMD-VIM configuration is effective in ensuring the full-contact state of the rigid block under the Kobe earthquake, whereas its effectiveness is reduced under the Pacoima earthquake, as evidenced by the larger extent of the green regions in the former case. Furthermore, by analyzing the maps in Fig. 8(b) and comparing them with those in Fig. 7(b), it can be observed that the TMD-VIM system performs better for less tall cabinets. Indeed, the green regions corresponding to the cabinet 42U are larger than those associated with the taller cabinet 52U.

Finally, the maps shown in Fig. 8(c) demonstrate that the combined TMD and VIM configuration is effective in preventing overturning of the rigid block. In these maps, the yellow regions correspond to rocking motion rather than overturning, as would occur in the TMD-only

configuration. Moreover, by comparing the extent of the yellow regions, it is evident that the TMD–VIM system performs better under the Kobe earthquake (left map) than under the Pacoima earthquake (right map).

As a final analysis, it is of interest to examine how the TMD–VIM system modifies the boundaries between the full-contact and rocking regions, as well as between the rocking and overturning regions. Fig. 9 presents the results of this investigation, conducted on the 42U cabinet under the Kobe earthquake (maps in the left column) and the Pacoima earthquake (maps in the right column). In particular, Fig. 9(a) shows the rocking maps in the T_h – T_d parameter plane of the system with the TMD-only configuration. These maps are identical to those in Fig. 8(a) and are reproduced here for clarity. The red dashed lines, similarly to the right ones in Fig. 8(a), correspond to values of the horizontal period T_h for which, irrespective of the TMD period T_d , the system response always lies within the rocking regions.

The rocking maps in Fig. 9(b) are obtained by fixing the clearance of the VIM at $d = 0.05$ m, corresponding to the red dashed lines shown in the maps of Fig. 8(b).

These maps allow for the evaluation of how the boundaries between the rocking and full-contact regions are modified with respect to those shown in Fig. 8(a). As can be observed, along the red dashed lines corresponding to the same horizontal period T_h in both Fig. 8(a) and Fig. 8(b), the system undergoes either rocking motion or full-contact conditions, depending on whether it lies to the left or to the right of the boundary (black dashed line), respectively. In the green shaded intervals of the maps, since the red dashed line lies within the full-contact region, the rigid block remains in perfect contact with the isolated base throughout the motion. Therefore, by suitably selecting the clearance d of the VIM, the boundary between the rocking and full-contact regions, which for a fixed T_h would otherwise lie entirely within the rocking region, can be shifted to the left over a larger range of T_d values, allowing the system to regain a full-contact state.

It should be emphasized that the green intervals of the TMD period T_d in the maps of Fig. 9(b) correspond to the ranges where the red lines (Path A and B) intersect the green regions in the maps of Fig. 8(b).

Finally, the combined effects of TMD and VIM are examined for the Parkfield earthquake, focusing on the 42U cabinet (Fig. 10). Fig. 10(a) shows rocking maps in the T_h – T_d plane for the TMD-only case; red dashed lines indicate values of T_h for which the response, without VIM, always lies in the rocking region regardless of T_d . To assess the combined system, rocking maps in the d – T_d plane are computed for fixed $T_v = 0.25$ s and $T_h = 0.50$ s, and for mass ratios $\mu = 0.2$, $\mu_d = 0.8$, and $\mu_i = 0.2$. The goal is to identify parameter pairs (d, T_d) enabling full-contact response. As shown in Fig. 10(b), full-contact occurs within the green regions, demonstrating that the TMD–VIM combination can suppress rocking. However, this region is significantly smaller than for the other earthquakes (see Figs. 7 and 8). This suggests that the TMD–VIM system is more effective for high-PGA events (e.g., Kobe and Pacoima), while for lower-PGA earthquakes the TMD-only configuration with dual base isolation is already sufficiently effective.

5.2. The effect on the horizontal motion of the isolated base

In addition to rocking mitigation, the effectiveness of the proposed devices is evaluated in terms of their ability to reduce the horizontal displacement demand of the isolation system. This aspect is particularly important because excessive base displacements may limit the practical applicability of highly flexible isolation systems. The following analyses therefore compare the displacement performance of the isolation-only, TMD-only, and TMD–VIM configurations.

Beyond its influence on rocking mitigation, the TMD–VIM system can also reduce the horizontal displacement demand of the isolation system. First, it should be noted that a fundamental indirect effect in reducing the amplitude of horizontal displacement is achieved through the leftward shift of the boundaries between the rocking and full-contact regions and between the rocking and overturning regions, as discussed

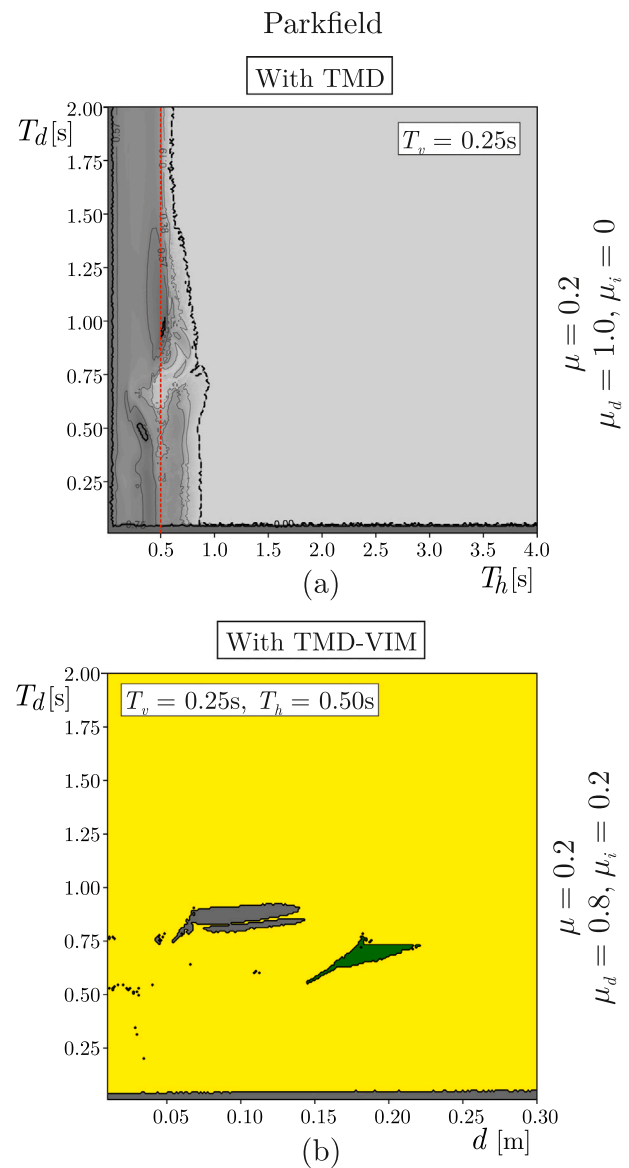


Fig. 10. Rocking maps of the cabinet 42U, under Parkfield ($T_v = 0.25$ s) earthquake and coupled with devices having different characteristics: (a) parameter's plane $T_h - T_d$, with TMD ($\mu = 0.2$, $\mu_d = 1.0$, $\mu_i = 0$); (b) parameter's plane $d - T_d$, with TMD–VIM ($\mu = 0.2$, $\mu_d = 0.8$, $\mu_i = 0.2$, $T_h = 0.50$ s).

above. Indeed, a leftward shift of these boundaries allows the horizontal period T_h of the isolated base to be reduced while still ensuring that the rigid block does not overturn or maintain full-contact. This reduction in T_h corresponds to an increase in the horizontal stiffness of the isolated base, which, in turn, leads to a decrease in the amplitude of its horizontal displacement.

The results of the investigation conducted to evaluate the direct effects of the TMD–VIM system on reducing the horizontal displacement of the isolated base are presented in Fig. 11. The results shown herein refer to the 52U cabinet subjected to the Kobe earthquake (maps in the left column) and the Pacoima earthquake (maps in the right column). Specifically, Fig. 11(a) shows the horizontal displacement maps in the T_h – T_v parameter plane for the system with only the isolation (no TMD and VIM). The red lines indicate the boundaries between the full-contact and rocking regions (dashed red lines) and between the rocking and overturning regions (solid red lines), as already shown in the rocking maps of the second row of Fig. 4.

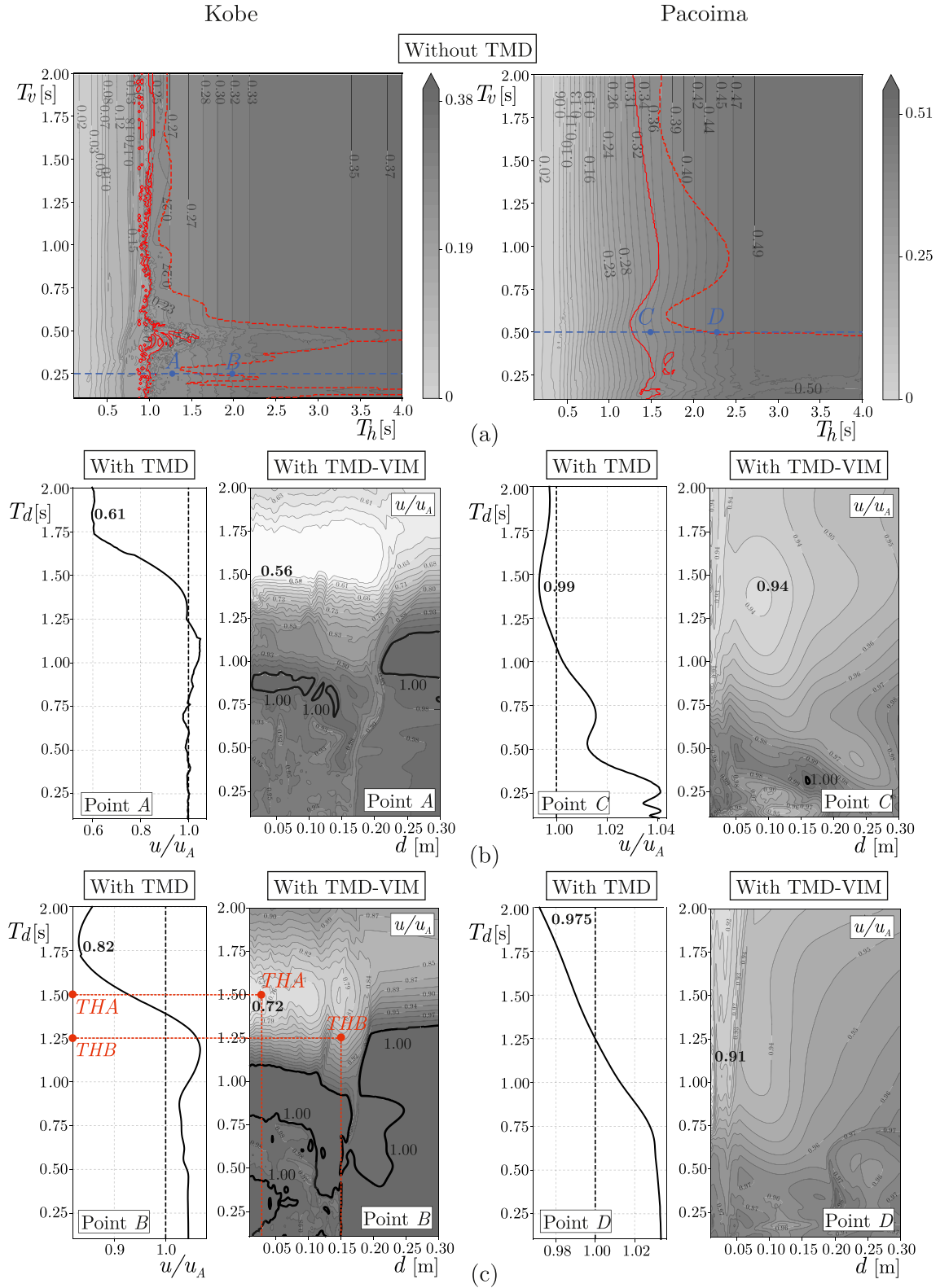


Fig. 11. Horizontal displacement response of the isolated base supporting the 52U cabinet under the Kobe and Pacoima earthquakes: (a) displacement maps in the $T_h - T_v$ plane for the isolation-only configuration (without TMD); (b) normalized maximum horizontal displacement for points A ($T_h = 1.25\text{s}$, $T_v = 0.25\text{s}$) and C ($T_h = 1.50\text{s}$, $T_v = 0.50\text{s}$); (c) normalized maximum horizontal displacement for points B ($T_h = 2.00\text{s}$, $T_v = 0.25\text{s}$) and D ($T_h = 2.25\text{s}$, $T_v = 0.50\text{s}$). In panels (b) and (c), the left plots refer to the TMD-only configuration ($\mu = 0.2$, $\mu_d = 1.0$, $\mu_i = 0$), whereas the right plots refer to the TMD-VIM configuration ($\mu = 0.2$, $\mu_d = 0.8$, $\mu_i = 0.2$). Values smaller than unity indicate a reduction of the maximum horizontal displacement with respect to the corresponding isolation-only configuration.

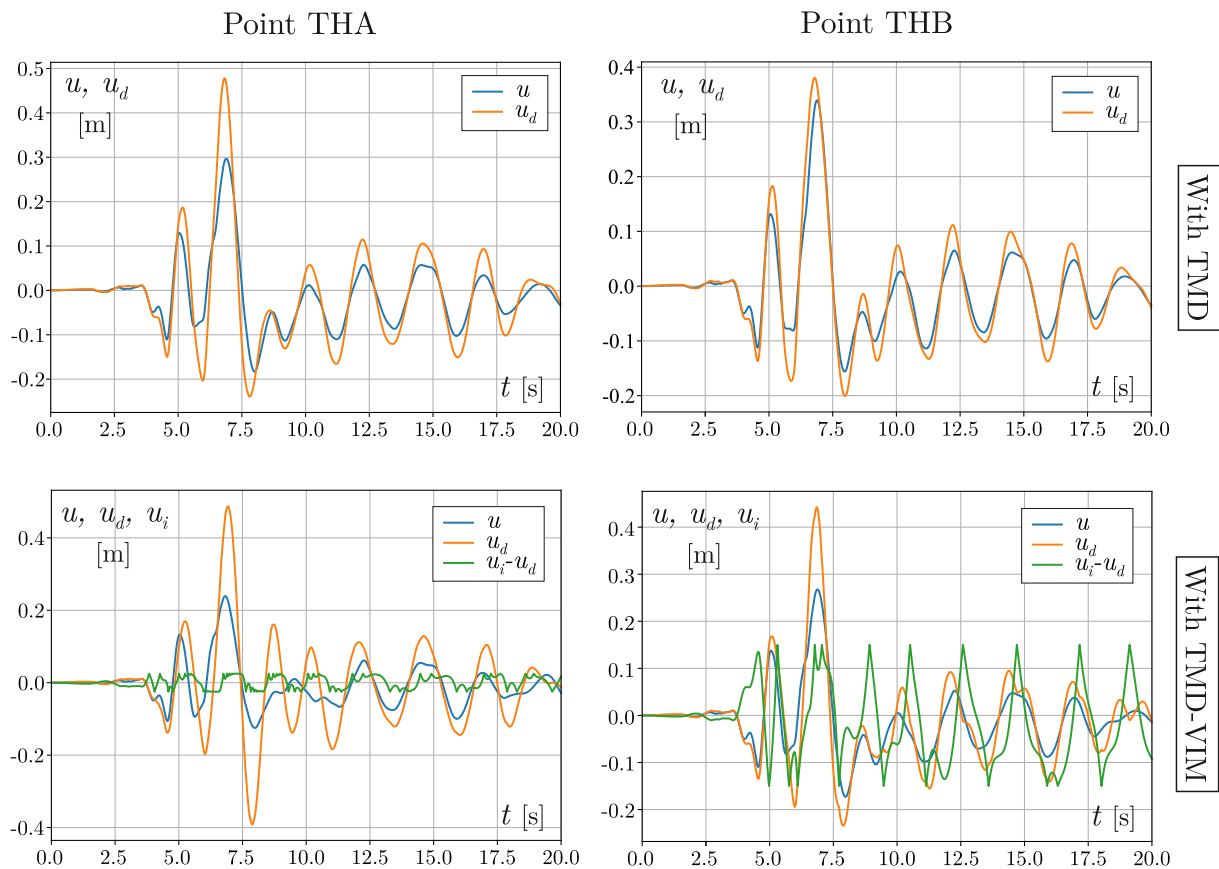


Fig. 12. Time-histories of the horizontal displacements u and u_d (for both the “with TMD” and “with TMD-VIM” configurations), together with $u_i - u_d$ (for the “with TMD-VIM” configuration only), evaluated at point *THA* ($T_h = 2.00\text{s}$, $T_v = 0.25\text{s}$, $T_d = 1.50\text{s}$, $d = 0.025\text{m}$); and at point *THB* ($T_h = 2.00\text{s}$, $T_v = 0.25\text{s}$, $T_d = 1.25\text{s}$, $d = 0.15\text{m}$).

The aim of this analysis is to evaluate whether the TMD-only or the more complete TMD-VIM configuration is effective in reducing the horizontal displacement. To perform this analysis, the periods T_v and T_h are selected corresponding to the points labeled *A*, *B*, *C*, and *D*. In these analyses, as consistently done throughout the paper, $T_v = 0.25\text{ s}$ is assigned to points *A* and *B*, which lie within the displacement map corresponding to the Kobe earthquake, while $T_v = 0.50\text{ s}$ is assigned to points *C* and *D*, located within the displacement map corresponding to the Pacoima earthquake. Moreover, points *A* and *C* are chosen inside the rocking regions of the system with only the dual isolation system (i.e., points located between the red solid and dashed boundaries), whereas points *B* and *D* are selected inside the full-contact boundaries of the system with only the dual isolation system (i.e., points located to the right of the red dashed boundary).

Fig. 11(b),c show two sets of plots, one on the left (Kobe earthquake) and the other on the right (Pacoima earthquake). These plots report the nondimensional maximum horizontal displacement, defined as the ratio between the maximum displacement obtained for the system equipped with the TMD or the TMD-VIM and the corresponding maximum displacement at points *A*, *B*, *C*, and *D* observed in the system with only the dual isolation system. Therefore, when this ratio is less than unity, the TMD or the TMD-VIM configuration is effective in reducing the horizontal displacement of the isolated base compared to the system with only dual isolation. Conversely, when the ratio exceeds unity, the TMD or TMD-VIM configuration has a detrimental effect on the system.

Each group of plots consists of two subplots. The left subplot shows the nondimensional maximum horizontal displacement of the system with the TMD-only configuration as a function of the TMD period T_d , hereafter referred to, for brevity, as the displacement curve. The right subplot presents the corresponding nondimensional maximum

horizontal displacement map in the d - T_d parameter plane, hereafter referred to as the displacement map, and refers to the TMD-VIM configuration.

The graphs in Fig. 11(b) refer to points *A* and *C*, both located inside the rocking regions of the system equipped only with the dual isolation system (see Fig. 11(a)). By first analyzing the displacement curves, it is worth noting that when the curve lies below unity (represented here by the dashed horizontal line), the TMD-only configuration reduces the horizontal displacement with respect to the system equipped only with the dual isolation system. However, there exist ranges of the TMD period T_d for which the curve exceeds unity, thereby indicating a detrimental effect on the system. For both displacement curves, the minimum values of the nondimensional displacement are also reported (0.61 for the left curve and 0.99 for the right curve). The displacement maps, on the other hand, refer to the TMD-VIM configuration and are therefore plotted in the d - T_d parameter plane. The adopted gray-scale highlights smaller values of the nondimensional displacement in lighter regions and larger values in darker regions. The solid lines represent the boundary beyond which the TMD-VIM configuration becomes detrimental (i.e., where the nondimensional displacement exceeds unity). No contour lines are shown in the regions where the nondimensional displacement is greater than unity. There exist parameter regions in which the TMD-VIM system performs effectively. In these regions, the minimum value of the nondimensional displacement is also highlighted in bold. Comparison of the displacement curves and maps indicates that the TMD-VIM configuration slightly outperforms the TMD-only configuration in reducing the horizontal displacement, although the improvement is relatively modest. Also with regard to the horizontal displacements, the effectiveness of the TMD-only and TMD-VIM configurations is earthquake-dependent. In particular, better performance in

reducing the horizontal base displacements is observed under the Kobe earthquake compared to the Pacoima earthquake.

Finally, the graphs in Fig. 11(c) refer to points *B* and *D*, both located inside the full-contact regions of the system equipped only with the dual isolation system (see Fig. 11(a)). Also in this case, when the displacement curves lie below unity, the TMD-only configuration reduces the horizontal displacement with respect to the system equipped only with the dual isolation system. For both displacement curves, the minimum values of the nondimensional displacement are reported (0.82 for the left curve and 0.975 for the right curve). In the displacement maps, there also exist parameter regions in which the TMD–VIM system performs effectively. Once again, the TMD–VIM configuration slightly outperforms the TMD-only configuration in reducing the horizontal displacement, although the improvement remains relatively modest. It is further confirmed that the effectiveness of both the TMD-only and the TMD–VIM configurations is earthquake-dependent. In particular, better performance in reducing the horizontal base displacements is observed under the Kobe earthquake compared to the Pacoima earthquake. Overall, both the TMD-only and TMD–VIM configurations are more effective in reducing the horizontal base displacement for systems that, when equipped only with the dual isolation system, operate within the rocking regions rather than within the full-contact regions (compare curves and maps of Fig. 11(b) and Fig. 11(c)).

Further insight can be gained by examining the time histories of the displacements u , u_d , and $u_i - u_d$. The latter represents the relative displacement of the VIM with respect to the TMD displacement. Fig. 12 shows these time-histories. Specifically, the graphs in the first row refer to the TMD-only configuration, whereas those in the second row correspond to the TMD–VIM configuration. Moreover, the graphs in the left column correspond to the values of T_d and d at point *THA*, while those in the right column correspond to the values of T_d and d at point *THB*. Both points are indicated in the left plots of Fig. 11(c). The two points are selected according to the following criteria: (i) at point *THA*, both the TMD-only and the TMD–VIM configurations are beneficial; (ii) at point *THB*, only the TMD–VIM configuration is beneficial, as the TMD-only configuration yields a nondimensional displacement exceeding unity.

Starting from graphs in the first row, which refer to the TMD-only configuration, at point *THA* (left graph) the horizontal motion of the isolated base, u , and that of the TMD, u_d , are slightly out of phase, particularly during the initial stages of motion. This results in a modest reduction in the maximum horizontal displacement of the base compared to the case without TMD. Conversely, at point *THB* (right graph), the motions of the base and the TMD are nearly perfectly in phase, leading to an increase in the maximum base displacement relative to the case without TMD.

The graphs in the second row, corresponding to the TMD–VIM configuration, show that at point *THA* (left graph) the motions of the TMD, u_d , and the VIM, $u_i - u_d$, are nearly always in phase with each other, while remaining predominantly out of phase, especially during the initial stage of the response, with respect to the horizontal motion of the isolated base, u . Consequently, both the TMD and the VIM contribute to reducing the base displacement. In contrast, the time histories at point *THB* (right graph) show that the motion of the TMD is almost always in phase with the base motion, whereas the motion of the VIM is predominantly out of phase with the base throughout most of the response. Therefore, in this case, the reduction in base displacement is mainly attributable to the VIM.

As a final remark, it is interesting to note that the relative motion of the VIM with respect to the TMD, $u_i - u_d$, is bounded by the selected clearance d (i.e., $|u_i - u_d| \leq d$, see Section 2.2.3).

6. Conclusions

The study has presented a comprehensive mechanical analysis of a fully horizontally and vertically isolated rigid block-like structure coupled with a tuned mass damper incorporating an embedded

vibro-impacting mass. A Lagrangian formulation was employed to describe both full-contact and rocking phases. Base impacts and internal vibro-impact events were rigorously modeled and described. The transition conditions between motion regimes were derived explicitly, and the nonlinear hysteretic behavior of the horizontal isolation system was consistently integrated with viscoelastic representations of vertical isolation and TMD restoring forces.

A broad parametric campaign was performed by varying isolation periods, TMD dynamic properties, and vibro-impact parameters. Rocking and displacement maps were constructed to track the evolution of regime boundaries and to quantify how the addition of the TMD and the TMD–VIM device modifies the dynamical response under strong-motion seismic excitation.

The principal mechanical findings and original contributions of the study can be summarized as follows:

- Dual horizontal–vertical isolation exhibits a pronounced sensitivity of the rocking–overturning boundary to the horizontal isolation period, whereas the vertical isolation period primarily governs the rocking–full-contact transition for small values.
- Introducing a conventional TMD induces a systematic leftward shift of both the rocking and overturning boundaries in the isolation-period parameter space, allowing smaller horizontal periods to be adopted without triggering overturning.
- Embedding a vibro-impacting mass enables the conversion of rocking responses into full-contact motion for parameter combinations where the TMD-only configuration remains ineffective.
- The effectiveness of the TMD–VIM configuration depends critically on the mass distribution within the device; improved stabilization is achieved when the primary TMD mass remains dominant with respect to the impacting mass.
- The vibro-impact mechanism is capable of preventing overturning in regions where both the isolation-only and TMD-only configurations would lead to instability, demonstrating a beneficial modification of the system response.
- The beneficial effects are excitation-dependent, with stronger vertical components reducing the extent of the recoverable full-contact region, thus emphasizing the importance of coupled horizontal–vertical dynamics.

Overall, the results demonstrate that vibro-impact coupling represents a mechanically grounded strategy for altering the stability landscape of isolated rigid bodies, offering enhanced resistance to rocking amplification and overturning under severe seismic excitation.

The findings discussed above provide insight into the mechanical potential of the proposed TMD–VIM strategy and the mechanisms through which vibro-impact interactions modify the response of isolated rigid block-like structures. Nevertheless, the conclusions presented herein should be interpreted within the scope of the investigated configurations and seismic inputs. In particular, the study is intended to clarify the governing mechanical mechanisms of the proposed system rather than to provide an exhaustive assessment of record-to-record variability or large-scale seismic performance under a broad range of earthquake scenarios.

Building upon these findings, future research will focus on extending the present investigation to a larger set of earthquake records, including near-fault pulse-like motions and suites of ground motions representative of different seismic scenarios. Such analyses will enable the evaluation of record-to-record variability, provide a broader assessment of the robustness of the proposed vibration-control strategy, and support the development of systematic optimization frameworks for TMD–VIM systems under prescribed auxiliary-mass constraints. In addition, the experimental calibration of the hysteretic isolation model against data from actual devices (e.g., lead-rubber bearings or wire rope isolators) represents a relevant direction for future work, in order to complement the literature-based selection of the Bouc–Wen parameters adopted

herein. Ultimately, these developments will contribute to the identification of optimal design configurations, enable rigorous comparisons with alternative vibration-control strategies, and support the formulation of design-oriented recommendations for TMD–VIM-protected isolated rigid block-like structures.

CRedit authorship contribution statement

Rajni Kant Kumar: Writing – original draft, Investigation, Data curation. **Lorenzo Amoroso:** Writing – original draft, Validation, Investigation, Formal analysis. **Valeria Settini:** Writing – review & editing, Supervision, Methodology, Funding acquisition. **Manuel Ferretti:** Writing – review & editing, Supervision, Methodology, Funding acquisition, Conceptualization. **Angelo Di Egidio:** Writing – review & editing, Supervision, Methodology, Investigation, Conceptualization.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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