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IFAC PapersOnLine 59-12 (2025) 43–48

Model Matching Problems for Max-plus Linear Systems with Polytopic Uncertainty in the Dynamics ⋆

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Abstract: This work deals with the problem of finding a control input that forces the output of a max-plus linear system, affected by polytopic uncertainty in the dynamics, to match the output of a given model. Model matching is a fundamental problem in control theory and its solution provides a powerful and viable control strategy in many situations. Polytopic uncertainty arises naturally in modeling real plants whenever their parameters are only known to belong to given intervals of values. The approach to the model matching problems adopted herein leverages structural notions derived from the geometric approach to systems with coefficients in a field and exploits interpretations grounded on max-plus algebraic theory. Novel notions, such as robust controlled invariance and robust feedback controlled invariance, are specifically defined for uncertain max-plus linear systems and used to derive constructive, sufficient, solvability conditions for the stated problems. The methodological discussion ends with an illustrative example aimed to show feasibility and effectiveness of the proposed approach.

Keywords: structural properties, model matching, uncertain systems, max-plus linear systems.

1. INTRODUCTION

This work investigates the solvability of model matching problems for max-plus linear systems affected by polytopic uncertainty in the dynamics—i.e., linear systems defined over the max-plus algebra $\mathbb{R}_{\max}$, where the entries of the dynamic matrix are unknown, except for taking their values in given intervals of the set of real numbers. Uncertainty arises naturally in mathematical modelling of real-world systems and, in many cases, it can be conveniently represented in polytopic form. Indeed, uncertainty may affect not only the dynamics, but also the input and output channels. Nonetheless, to avoid cumbersome notations and ensure a concise presentation, only uncertainty in the dynamics is considered herein.

As to the model uncertainty which may arise when real systems are mathematically described by max-plus linear systems, a consolidated approach hinges on interval max-plus linear systems. For instance, some recent works investigate reachability, optimal state-feedback control, and set-membership estimation for this class of uncertain max-plus linear systems (Mufid et al., 2022; Yin et al., 2024; Espindola-Winck et al., 2025). On appropriate conditions, interval max-plus linear systems can be recast as max-plus linear systems affected by polytopic uncertainty. The latter formalization is preferred herein, since it is simpler to handle with the structural methods that will be adopted to solve the model matching problems considered in this work — i.e., the methods derived from the geometric approach to linear systems introduced by Basile and Marro (1992) and Wonham (1985).

Although numerous variations have been developed, the fundamental idea behind model matching is to find a control input such that the output of a given system replicates that of a specified model. This problem holds a pivotal role in systems and control theory, both conceptually and in practical applications. Indeed, reformulating a general control problem as a model matching problem has proven to be an effective strategy in many practical scenarios (Ichikawa, 1985). For this reason, it has been extensively studied for several classes of dynamical systems, in particular with structural methods (Morse, 1973; Malabre and Kučera, 1984; Marro and Zattoni, 2005; Perdon et al., 2016b,a; Zattoni et al., 2024).

In max-plus linear systems, where model matching typically expresses the aim of imposing a specified production

⋆ The work of E. Zattoni, V. Bartolucci, and D. Scaradozzi was supported by the project "MAXFISH: Multi agents systems and max-plus algebra theoretical frameworks for a robot-fish shoal modelling and control" 20225RYMJE, funded by MUR Progetti di Ricerca di Rilevante Interesse Nazionale (PRIN) Bando 2022.

10.1016/j.ifacol.2025.09.565

schedule on a manufacturing plant, the problem was initially addressed using timed event graph-based approaches (Libeaut and Loiseau, 1996; Cottenceau et al., 2001; Maia et al., 2005). Meanwhile, some fundamental notions of the structural methods were extended to max-plus linear systems (Gaubert and Katz, 2004; Katz, 2007; Di Loreto et al., 2010; Hardouin et al., 2011). Building on those and further extended concepts, the mentioned structural methods have recently led to solvability conditions applicable in the absence of uncertainties (Animobono et al., 2022, 2023, 2024).

In this framework, the main results of this work consist of sufficient conditions for the solution, either in open- or in closed-loop, of the problem of matching the output of a given max-plus linear system with polytopic uncertainty in the dynamics with the output of an exactly-known max-plus linear model. Such conditions are constructive, as their proofs provide procedures to synthesize the control laws. The solvability conditions hinge on the novel notions of robust invariant subsemimodule and robust controlled invariant subsemimodule of the semimodule of the states of a max-plus linear system with uncertainty in the dynamic matrix. Moreover, the algorithmic procedures exploit a characterization of those semimodules that efficiently deals with the presence of the polytopic uncertainty as well as a characterization of the maximal controlled invariant semimodule contained in a given semimodule.

Notation. The symbols $\mathbb{N}$ and $\mathbb{R}$ denote the set of nonnegative integers and the set of real numbers, respectively. The max-plus algebra, $\mathbb{R}_{\max}$, is defined over the set $\mathbb{R}\cup\{-\infty\}$ by the operations of tropical sum, $\oplus$, and tropical product, $\otimes$, respectively defined by $a\oplus b=\max\{a,b\}$ for any $a,b\in\mathbb{R}_{\max}$ and by $a\otimes b=a+b$ if $a,b\in\mathbb{R}$ and $(-\infty)\otimes a=a\otimes(-\infty)=-\infty$ for all $a\in\mathbb{R}_{\max}$. The neutral elements for $\oplus$ and $\otimes$ are denoted by ϵ and e, respectively, with $\epsilon=-\infty$ and $e=0\in\mathbb{R}$. The symbol $\otimes$ will be usually omitted in the expressions which involve it, if no confusion arises. Since $\otimes$ distributes over $\oplus$, $\mathbb{R}_{\max}$ is a semiring. The free semimodule over $\mathbb{R}_{\max}$ of dimension n, with $n\in\mathbb{N}$, is denoted by $\mathbb{R}^n_{\max}$. If no confusion arises, the origin of $\mathbb{R}^n_{\max}$, that is the n-dimensional vector whose components are all equal to ϵ, is simply denoted by ϵ. For any subsemimodule $\mathcal{V}\subseteq\mathbb{R}^n_{\max}$, the symbol $\mathcal{V}^{[N]}$, with $N\in\mathbb{N}$, denotes the subsemimodule of $\mathbb{R}^{n\times N}_{\max}$ which consists of all vectors $[v_1^\top,\ldots,v_N^\top]^\top$, where $v_i\in\mathcal{V}$, with $i=1,\ldots,N$. If V is a matrix whose columns form a basis of $\mathcal{V}$, a basis of $\mathcal{V}^{[N]}$ is given by the columns of the block diagonal matrix

$$V^{[N]}=\mathrm{diag}\,\{\underbrace{V,\ldots,V}_{N\text{ times}}\}.$$

Given a set of $p\times m$ matrices A_i, with $i\in\mathcal{I}$ and $\mathcal{I}=\{1,\ldots,N\}$, the notation $\mathrm{co}_{i\in\mathcal{I}}(A_i)$ stands for the $(Np)\times m$ matrix obtained by vertically stacking the matrices A_i, with $i\in\mathcal{I}$, $\mathrm{co}_{i\in\mathcal{I}}(A_i)=\left[A_1^\top\ldots A_N^\top\right]^\top$. Moreover, the matrix $\mathrm{co}_{i\in\mathcal{I}}(A_i)$ defines the linear map, denoted by the same symbol, $\mathrm{co}_{i\in\mathcal{I}}(A_i):\mathbb{R}^m_{\max}\mapsto\mathbb{R}^{Np}_{\max}$, $v\mapsto[(A_1v)^\top,\ldots,(A_Nv)^\top]^\top$. Finally, the symbol $\mathrm{co}_{i\in\mathcal{I}}(B)$, with a constant matrix B and $\mathcal{I}=\{1,\ldots,N\}$, denotes the vertical concatenation of N copies of B, i.e.

$$\mathrm{co}_{i\in\mathcal{I}}(B)=[\underbrace{B^\top\ldots B^\top}_{N\text{ times}}]^\top.$$

2. MAX-PLUS LINEAR SYSTEMS WITH POLYTOPIC UNCERTAINTY IN THE DYNAMICS

A max-plus linear system is a dynamical object whose event-driven time evolution is described by a set of equations of the form

$$\Sigma\equiv\begin{cases} x(k+1)\,=\,Ax(k)\oplus Bu(k+1)\\ \quad\;\; y(k)\,=\,Cx(k)\end{cases}\tag{1}$$

where $k\in\mathbb{N}$ is the index of occurrence of the events, $x(.):\mathbb{N}\to\mathcal{X}=\mathbb{R}^n_{\max}$, $u(.):\mathbb{N}\to\mathcal{U}=\mathbb{R}^m_{\max}$, and $y(.):\mathbb{N}\to\mathcal{Y}=\mathbb{R}^p_{\max}$ are the daters of the internal events, input events, and output events, respectively; $A\in\mathbb{R}^{n\times n}_{\max}$, $B\in\mathbb{R}^{n\times m}_{\max}$, and $C\in\mathbb{R}^{p\times n}_{\max}$ are coefficient matrices of appropriate dimensions. The components of x, u, and y correspond to n types of internal events, m types of input events, and p types of output events, respectively, so that the meaning of the vector $x(k)=[x_1(k),...,x_n(k)]^\top\in\mathbb{R}^n_{\max}$ is that the k-th internal event of type i occurs at time $x_i(k)$, and similarly for $u(k)$ and $y(k)$. To have a physical meaning, daters must be nondecreasing. To assure this for any initial condition, it is assumed that $A\geq I_n$, where the inequality is taken component wise and I_n denotes the $n\times n$ identity matrix with all the diagonal elements equal to e and all the off-diagonal elements equal to ϵ.

When max-plus linear systems are used to model real dynamical systems, like production processes, the matrix entries may be affected by uncertainty. To deal with this situation, limiting our attention to the case in which uncertainty affects only the dynamic matrix, we consider the class of *max-plus linear system with polytopic uncertainty in the dynamics*, whose elements are defined by a set of equations of the form

$$\Sigma\equiv\begin{cases} x(k+1)\,=\,A(\mu)x(k)\oplus Bu(k+1)\\ \quad\;\; y(k)\,=\,Cx(k)\end{cases}\tag{2}$$

in which (the other symbols having the same meaning as in (1)) the dynamic matrix $A(\mu)$ is defined by

$$A(\mu)=\bigoplus\nolimits_{i\in\mathcal{I}}\mu_iA_i\tag{3}$$

where $\mathcal{I}=\{1,\ldots,N\}$ is a set of indices; $\mu=[\mu_1,\ldots,\mu_N]^\top\in\mathbb{R}^N_{\max}$ is a vector of independent parameters taking value in $\mathbb{R}_{\max}$ such that

$$\bigoplus\nolimits_{i\in\mathcal{I}}\mu_i=e;\tag{4}$$

$\{A_1,\ldots,A_N\}=\{A_i\}_{i\in\mathcal{I}}$ is a family of $n\times n$ matrices with entries in $\mathbb{R}_{\max}$, satisfying $A_i\geq I_n$ for all $i\in\mathcal{I}$. Note that condition (4) implies $\mu_i\leq 0$ for all $i\in\mathcal{I}$, with the equality holding at least for one of the components μ_i. Hence, (3) can be seen as a tropical (i.e., over $\mathbb{R}_{\max}$) convex combination of the matrices A_i.

Remark 1. In particular, the representation (2) can be used when uncertainty implies that the entries of the dynamic matrix are only known to take value in given intervals, or, in other words, when the dynamic matrix is an interval matrix in the sense of (Espindola-Winck et al., 2025). In fact, given a polytope $P_A\subseteq\mathbb{R}^{n\times n}_{\max}$, which consists of matrices whose entries take value in given intervals and which is defined by

$$P_A=\big\{A\in\mathbb{R}^{n\times n}_{\max},\text{ such that }a^{min}_{hk}\leq a_{hk}\leq a^{max}_{hk},\text{ with } a^{min}_{hk},a^{max}_{hk}\in\mathbb{R}_{\max},\text{ for }h=1,...,n;k=1,...,n\big\}\,,$$

each element of P_A can be expressed as a tropical convex combination of a specific set of n^2+1 vertices of P_A. In fact, let A_0 be the $n \times n$ matrix belonging to P_A with entries $a_{hk} = a_{hk}^{min}$ for all $h = 1, ..., n$ and all $k = 1, ..., n$, and let A_{hk} be the $n \times n$ matrix belonging to P_A with $a_{hk} = a_{hk}^{\max}$ and all other entries equal to their minimum value. To simplify the notation, let the $n^2 + 1$ matrices thus introduced be renamed as $A_1, \ldots, A_N$, with $N = n^2 + 1$. Let $\mathcal{I} = \{1, \ldots, N\}$. Then, any element $A \in P_A$, whose entries are denoted by a_{hk} for $h = 1, \ldots, n$ and $k = 1, \ldots, n$, can be written as

$$A = eA_0 \oplus \left(\bigoplus_{h=1,\ldots,n;k=1,\ldots,n} (a_{hk} - a_{hk}^{max}) A_{hk} \right) = \bigoplus_{i \in \mathcal{I}} \mu_i A_i$$

with $\bigoplus_{i \in \mathcal{I}} \mu_i = e$. A system whose dynamic matrix is an interval matrix will be considered in Section 6.

3. STATEMENT OF THE PROBLEMS

A possible strategy to control the performance of a max-plus linear system with polytopic uncertainty in the dynamics is forcing its output to match that of a given model. This gives rise to the Model Matching Problem (MMP) and the Feedback Model Matching Problem (FMMP), which are formally stated as follows.

Problem 1. (Model Matching Problem). Let us consider a max-plus linear system with polytopic uncertainty in the dynamics

$$\Sigma_P \equiv \begin{cases} x_P(k+1) = A_P(\mu) x_P(k) \oplus B_P u_P(k+1) \\ y_P(k) = C_P x_P(k) \end{cases} \quad (5)$$

of the form (2), called the *plant*, with $x_P \in \mathbb{R}_{\max}^{n_P}$; $u_P \in \mathbb{R}_{\max}^{m_P}$; $y_P \in \mathbb{R}_{\max}^{p}$; $A_P(\mu) = \bigoplus_{i \in \mathcal{I}} A_{P_i}$ with $A_{P_i} \in \mathbb{R}_{\max}^{n_P \times n_P}$; $B_P \in \mathbb{R}_{\max}^{n_P \times m_P}$ and $C_P \in \mathbb{R}_{\max}^{p \times n_P}$, and a max-plus linear system

$$\Sigma_M \equiv \begin{cases} x_M(k+1) = A_M x_M(k) \oplus B_M u_M(k+1) \\ y_M(k) = C_M x_M(k) \end{cases} \quad (6)$$

of the form (1) (without polytopic uncertainty), called the *model*, with $x_M \in \mathbb{R}_{\max}^{n_M}$, $u_M \in \mathbb{R}_{\max}^{m_M}$, $y_M \in \mathbb{R}_{\max}^{p}$, $A_M \in \mathbb{R}_{\max}^{n_M \times n_M}$, $B_M \in \mathbb{R}_{\max}^{n_M \times m_M}$ and $C_M \in \mathbb{R}_{\max}^{p \times n_M}$. Note that, $\dim y_P = \dim y_M$. Then, the Model Matching Problem (MMP) consists in finding, for any possible nondecreasing input sequences $\{u_M(k+1)\}_{k \in \mathbb{N}}$ of the model, a nondecreasing control input sequence $\{u_P(k+1)\}_{k \in \mathbb{N}}$ for the plant, such that, for all $\mu = [\mu_1, \ldots, \mu_N]^\top$ with $\bigoplus_{i \in \mathcal{I}} \mu_i = e$ and for initial conditions $x_P(0) = \epsilon$ and $x_M(0) = \epsilon$, the output $\{y_P(k+1)\}_{k \in \mathbb{N}}$ of this latter equals the output $\{y_M(k+1)\}_{k \in \mathbb{N}}$ of the model, i.e. $y_P(k+1) = y_M(k+1)$ for all $k \in \mathbb{N}$.

In practice, the above formulation applies to the situation in which the control objective is forcing, by a suitable control input, a given plant affected by uncertainty to produce according to a desired schedule. No uncertainty is present in the model since its output is used to describe the desired production schedule. A more demanding formulation of the above problem is the following.

Problem 2. (Feedback Model Matching Problem). Given a plant Σ_P and a model Σ_M of the form (5) and (6), respectively, the Feedback Model Matching Problem (FMMP) consists in finding, for all possible nondecreasing input sequences $\{u_M(k+1)\}_{k \in \mathbb{N}}$ of the model, two matrices $F \in \mathbb{R}_{\max}^{m_P \times (n_P + n_M)}$ and $G \in \mathbb{R}_{\max}^{m_P \times m_M}$ such that, for all $\mu = [\mu_1, \ldots, \mu_N]^\top$ with $\bigoplus_{i \in \mathcal{I}} \mu_i = e$ and for initial conditions $x_P(0) = \epsilon$ and $x_M(0) = \epsilon$, the control input sequence $\{u_P(k+1)\}_{k \in \mathbb{N}}$ defined by

$$u_P(k+1) = F \begin{pmatrix} x_P(k) \\ x_M(k) \end{pmatrix} \oplus G\, u_M(k+1) \oplus u_P(k) \text{ for } k \geq 0, \quad (7)$$

with $u_P(0) = \epsilon$, is a solution of the corresponding MMP.

Remark 2. Since it is not required that the entries of F and G are greater than or equal to 0, a solution to the FMMP may be given by an anticipative feedback, whose implementation requires to know in advance part of the input sequence $\{u_M(.)\}$ (Animobono et al., 2023, Rem. 2).

To reformulate the above problems so as to facilitate their study, given the plant Σ_P and the model Σ_M described by (5) and (6), let us consider the joint internal event dater $x_E(.) = \begin{pmatrix} x_P(.) \\ x_M(.) \end{pmatrix} : \mathbb{N} \to \mathcal{X}_E = \mathbb{R}_{\max}^{(n_P+n_M)}$ and its dynamics

$$x_E(k+1) = A_E(\mu) x_E(k) \oplus B_1 u_P(k+1) \oplus B_2 u_M(k+1) \quad (8)$$

with $A_E(\mu) = \begin{pmatrix} A_P(\mu) & \epsilon \\ \epsilon & A_M \end{pmatrix}$, $B_1 = \begin{pmatrix} B_P \\ \epsilon \end{pmatrix}$, $B_2 = \begin{pmatrix} \epsilon \\ B_M \end{pmatrix}$, and $x_E(0) = \epsilon$. Then, Problem 1 can be expressed as that of finding, for any possible nondecreasing sequence $\{u_M(k+1)\}_{k \in \mathbb{N}}$, a non decreasing sequence $\{u_P(k+1)\}_{k \in \mathbb{N}}$ that, for all $\mu = [\mu_1, \ldots, \mu_N]^\top$ with $\bigoplus_{i \in \mathcal{I}} \mu_i = e$, forces $x_E(k)$ to evolve inside the *output equalizer* subsemimodule $\mathcal{K} \subseteq \mathcal{X}_E = \mathbb{R}_{\max}^{(n_p+n_M)}$ defined by

$$\mathcal{K} = \left\{ x_E = \begin{pmatrix} x_P \\ x_M \end{pmatrix} \in \mathbb{R}_{\max}^{(n_p+n_M)} \text{ with } C_P x_P = C_M x_M \right\}. \quad (9)$$

The same can obviously be done for Problem 2, asking in addition that the sequence $\{u_P(k+1)\}_{k \in \mathbb{N}}$ be of the form (7) for suitable matrices F and G.

In the framework of max-plus systems, control problems which consist of forcing the evolution of the internal event dater inside a given subsemimodule were considered and dealt with by employing a structural geometric approach in (Katz, 2007; Maia et al., 2011; Animobono et al., 2022, 2023, 2024). To apply the same approach to the MMP and FMMP for a plant affected by polytopic uncertainty in the dynamics, a suitable and novel extension of that approach will be developed in the next sections.

4. STRUCTURAL NOTIONS

To introduce the basic notions of a structural geometric approach that applies to the case of max-plus linear systems with politopic uncertainty in the dynamics, let us state the following definitions and related results.

Definition 1. Given a max-plus linear system with polytopic uncertainty in the dynamics Σ of the form (2), a subsemimodule $\mathcal{V} \subseteq \mathcal{X}$ is said to be a *robust invariant subsemimodule* for Σ if and only if

$$A(\mu)\mathcal{V} = \left(\bigoplus_{i \in \mathcal{I}} \mu_i A_i\right) \mathcal{V} \subseteq \mathcal{V}$$

for all $\mu = [\mu_1, \ldots, \mu_N]^\top$ with $\bigoplus_{i \in \mathcal{I}} \mu_i = e$.

Proposition 1. Given a max-plus linear system with polytopic uncertainty in the dynamics Σ of the form (2), a subsemimodule $\mathcal{V} \subseteq \mathcal{X}$ is a robust invariant subsemimodule for Σ if and only if $A_i\mathcal{V} \subseteq \mathcal{V}$ for all $i \in \mathcal{I} = \{1, \ldots, N\}$.

Definition 2. Given a max-plus linear system with polytopic uncertainty in the dynamics Σ of the form (2), a subsemimodule $\mathcal{V} \subseteq \mathcal{X}$ is said to be a *robust controlled invariant subsemimodule*, or a *robust* (A, B)*-invariant subsemimodule*, for Σ if and only if for all $x \in \mathcal{V}$ there exists $u \in \mathbb{R}^m_{\max}$ such that

$$A(\mu)x \oplus Bu = \left(\textstyle\bigoplus_{i \in \mathcal{I}} \mu_i A_i\right) x \oplus Bu \subseteq \mathcal{V}$$

for all $\mu = [\mu_1, \ldots, \mu_N]^\top$ with $\bigoplus_{i \in \mathcal{I}} \mu_i = e$.

Proposition 2. Given a max-plus linear system with polytopic uncertainty in the dynamics Σ of the form (2), a subsemimodule $\mathcal{V} \subseteq \mathbb{R}^n_{\max}$ is a robust controlled invariant subsemimodule for Σ if and only if, for all $x \in \mathcal{V}$, there exists $u \in \mathbb{R}^m_{\max}$, such that $A_i x \oplus Bu \subseteq \mathcal{V}$ for all $i \in \mathcal{I} = \{1, \ldots, N\}$.

Proposition 3. Given a max-plus linear system with polytopic uncertainty in the dynamics Σ of the form (2) and a subsemimodule of its state semimodule $\mathcal{K} \subseteq \mathcal{X}$, the set of all robust controlled invariant subsemimodules for Σ contained in $\mathcal{K}$ has a maximum element, denoted by $\mathcal{V}^\star(\mathcal{K})$.

The maximum element $\mathcal{V}^\star(\mathcal{K})$ coincides with the limit $\mathcal{V}_\infty$ of the sequence of semimodules $\{\mathcal{V}_k\}_{k \in \mathbb{N}}$ defined by

$$\begin{aligned} \mathcal{V}_0 &= \mathcal{K} \\ \mathcal{V}_k &= \mathcal{V}_{k-1} \cap (\mathrm{co}_{i \in \mathcal{I}}(A_i))^{-1} \left(\mathcal{V}^{[N]}_{k-1} \ominus \mathrm{Im}\, \mathrm{co}_{i \in \mathcal{I}}(B)\right) \end{aligned} \tag{10}$$

where

$$\mathcal{V}^{[N]} \ominus \mathrm{Im}\, \mathrm{co}_{i \in \mathcal{I}}(B) = \big\{x \in \mathbb{R}^{n \times N}_{\max}, \text{ for which there exists } u \in \mathbb{R}^m_{\max} \text{ such that } x \oplus (\mathrm{co}_{i \in \mathcal{I}}(B))u \in \mathcal{V}^{[N]}\big\}$$

and $(\mathrm{co}_{i \in \mathcal{I}}(A_i))^{-1}(\mathcal{Y})$ denotes the inverse image of $\mathcal{Y}$ with respect to the linear map defined by $\mathrm{co}_{i \in \mathcal{I}}(A_i)$.

Note that the sequence $\{\mathcal{V}_k\}_{k \in \mathbb{N}}$ does not necessarily converge in a finite number of steps. Thus, (10) does not provide a general algorithm for computing $\mathcal{V}^\star(\mathcal{K})$. However, if $\mathcal{V}_{\bar{k}+1} = \mathcal{V}_{\bar{k}}$ for some $\bar{k} \in \mathbb{N}$, then $\mathcal{V}_\infty = \mathcal{V}_{\bar{k}} = \mathcal{V}^\star(\mathcal{K})$. From this viewpoint, the situation differs from that encountered in computing maximum controlled invariant submodules for systems with coefficients in a field (Basile and Marro, 1992; Wonham, 1985), while it is similar to that of systems with coefficients in a ring (Conte and Perdon, 1994).

Remark 3. Given two finitely generated semimodules $\mathcal{Z}$ and $\mathcal{Y}$ and a matrix A of suitable dimensions, the semimodules $\mathcal{Y} \ominus \mathcal{Z}$, $A^{-1}(\mathcal{Y})$, and $\mathcal{Y} \cap \mathcal{Z}$ are all finitely generated (see Gaubert, 1998, Corollary 86). Hence, if $\mathcal{K}$ is finitely generated, so are the semimodules $\mathcal{V}_k$ for all $k \in \mathbb{N}$. As explained in (Katz, 2007, Remark 1), their generators can be obtained as the set of solutions of appropriate systems of max-plus linear equations, that can be solved through the (Scicos, 2007) software containing the Max-plus Toolbox (McGettrick et al., 2009).

Definition 3. Given a max-plus linear system with polytopic uncertainty in the dynamics Σ of the form (2), a subsemimodule $\mathcal{V} \subseteq \mathcal{X}$ is said to be a *robust feedback controlled invariant subsemimodule*, or a *robust* (A, B)*-invariant subsemimodule* of feedback type, for Σ if there exists a matrix $F \in \mathbb{R}^{m \times n}_{\max}$ such that $(A(\mu) \oplus BF)\mathcal{V} = \left(\left(\bigoplus_{i \in \mathcal{I}} \mu_i A_i\right) \oplus BF\right)\mathcal{V} \subseteq \mathcal{V}$, for all $\mu = [\mu_1, \ldots, \mu_N]^\top$ with $\bigoplus_{i \in \mathcal{I}} \mu_i = e$.

Proposition 4. Given a max-plus linear system with polytopic uncertainty in the dynamics Σ of the form (2), a subsemimodule $\mathcal{V} \subseteq \mathcal{X}$ is a robust feedback controlled invariant subsemimodule for Σ if there exists a matrix $F \in \mathbb{R}^{m \times n}_{\max}$ such that $(A_i \oplus BF)\mathcal{V} \subseteq \mathcal{V}$, for all $i \in \mathcal{I} = \{1, \ldots, N\}$.

It is known that feedback controlled invariance is equivalent to controlled invariance for systems with coefficients in a field (Basile and Marro, 1992; Wonham, 1985). However, this is not true for systems with coefficients in a ring (Hautus, 1982) or in a semiring (Katz, 2007), nor in the present case, although, clearly, robust feedback controlled invariance implies robust controlled invariance.

5. SOLUTION OF THE PROBLEMS

The structural geometric notions introduced above are used in this section to provide a sufficient solvability condition to the MMP and FMMP and to construct solutions if any exist. For the MMP, we have the following theorem.

Theorem 1. Let Σ_P be a plant of the form (5) and let Σ_M be a model of the form (6). The related MMP is solvable if for any $u_M \in \mathbb{R}^{m_M}_{\max}$ there exists $u_P \in \mathbb{R}^{m_P}_{\max}$ such that $B_1 u_P \oplus B_2 u_M$ belongs to $\mathcal{V}^*(\mathcal{K})$, where $\mathcal{V}^*(\mathcal{K}) \subseteq \mathcal{X}_E$ is the maximum robust $(A_E(\mu), B_1)$-invariant semimodule contained in the output equalizer semimodule $\mathcal{K} \subseteq \mathcal{X}_E$ defined by (9).

Proof. By robust $(A_E(\mu), B_1)$-invariance of $\mathcal{V}^*(\mathcal{K})$, we have that for any $x_E(k) \in \mathcal{V}^*(\mathcal{K})$ there exists $u_P \in \mathbb{R}^{m_P}_{\max}$ such that $A_E(\mu)x_E(k) \oplus B_1 u_P \subseteq \mathcal{V}^*(\mathcal{K})$ for all $\mu = [\mu_1, \ldots, \mu_N]^\top$ with $\bigoplus_{i \in \mathcal{I}} \mu_i = e$. Moreover, by hypothesis, given $u_M \in \mathbb{R}^{m_M}_{\max}$, there exists $u_P \in \mathbb{R}^{m_P}_{\max}$ such that $B_1 u_P \oplus B_2 u_M \subseteq \mathcal{V}^*(\mathcal{K})$. Then, given any nondecreasing sequence $\{u_M(k+1)\}_{k \in \mathbb{N}}$, it is possible to construct a sequence $\{u_P(k+1)\}_{k \in \mathbb{N}}$ recursively defined by

$$u_P(k+1) = \begin{cases} u_2(1) \text{ for } k = 0 \\ u_1(k+1) \oplus u_2(k+1) \oplus u_P(k) \text{ for } k > 0 \end{cases} \tag{11}$$

as follows. Start by taking $u_2(1)$ such that $B_1 u_2(1) \oplus B_2 u_M(1) \in \mathcal{V}^*(\mathcal{K})$ and set $u_P(1) = u_2(1)$. Then, compute $x_E(1)$ by means of (8), $x_E(0)$, $u_P(1)$ and $u_M(1)$, and take $u_1(2)$ and $u_2(2)$ in such a way that $A_E(\mu)x_E(1) \oplus B_1 u_1(2) \in \mathcal{V}^*(\mathcal{K})$ for all $\mu = [\mu_1, \ldots, \mu_N]^\top$ with $\bigoplus_{i \in \mathcal{I}} \mu_i = e$ and $B_1 u_2(2) \oplus B_M u_M(2) \in \mathcal{V}^*(\mathcal{K})$. Set $u_P(2) = u_1(2) \oplus u_2(2) \oplus u_P(1)$ and iterate the procedure increasing by 1 the index k at each step. The sequence $\{u_P(k+1)\}_{k \in \mathbb{N}}$ is non decreasing, thanks to the presence of $u_P(k)$ in the second equation of (11). Moreover, using $\{u_P(k+1)\}_{k \in \mathbb{N}}$ and $\{u_M(k+1)\}_{k \in \mathbb{N}}$ in (8), we get

$$\begin{aligned} x_E(1) &= A_E(\mu)x_E(0) \oplus B_1 u_P(1) \oplus B_2 u_M(1) \\ &= A_E(\mu)x_E(0) \oplus B_1 u_2(1) \oplus B_2 u_M(1) \text{ for } k = 0, \end{aligned} \tag{12}$$

$$\begin{aligned} x_E(k+1) &= A_E(\mu)x_E(k) \oplus B_1 u_P(k+1) \oplus B_2 u_M(k+1) \\ &= A_E(\mu)x_E(k) \oplus B_1 u_1(k+1) \oplus B_1 u_2(k+1) \\ &\quad \oplus B_1 u_P(k) \oplus B_2 u_M(k+1) \text{ for } k > 0. \end{aligned} \tag{13}$$

Since $A_E(\mu)x_E(0) = A_E(\mu) \otimes \epsilon = \epsilon$ belongs to $\mathcal{V}^*(\mathcal{K})$, by (12) and by the definition of $u_2(1)$, we have that also

$x_E(1)$ belongs to $\mathcal{V}^*(\mathcal{K})$ for all $\mu = [\mu_1, \ldots, \mu_N]^\top$ with $\bigoplus_{i \in \mathcal{I}} \mu_i = e$. In the right-hand member of (13), the term $B_1 u_P(k)$ is irrelevant, since $x_E(k) \geq B_1 u_P(k)$. Hence, thanks to the assumption $A_{E_i} \geq I_{n_P+n_M}$ for all $i \in \mathcal{I}$, also $A_E(\mu) x_E(k) \geq B_1 u_P(k)$ for all $\mu = [\mu_1, \ldots, \mu_N]^\top$ with $\bigoplus_{i \in \mathcal{I}} \mu_i = e$. Moreover, by the definition of $u_1(k+1)$, the term $A_E(\mu) x_E(k) \oplus B_1 u_1(k+1)$ belongs to $\mathcal{V}^*(\mathcal{K})$ for all $\mu = [\mu_1, \ldots, \mu_N]^\top$ with $\bigoplus_{i \in \mathcal{I}} \mu_i = e$ if the same holds for $x_E(k)$ and, by the definition of $u_2(k+1)$, the term $B_1 u_2(k+1) \oplus B_2 u_M(k+1)$ belongs to $\mathcal{V}^*(\mathcal{K})$. Then, by induction, we have that also $x_E(k+1)$ for $k > 0$ belongs to $\mathcal{V}^*(\mathcal{K})$ for all $\mu = [\mu_1, \ldots, \mu_N]^\top$ with $\bigoplus_{i \in \mathcal{I}} \mu_i = e$. Since $\mathcal{V}^*(\mathcal{K})$ is contained in the output equalizer semimodule $\mathcal{K}$, it follows that the output $\{y_P(k+1)\}_{k \in \mathbb{N}}$ of the plant generated by the input $\{u_P(k+1)\}_{k \in \mathbb{N}}$ defined by (11) is equal to the output $\{y_M(k+1)\}_{k \in \mathbb{N}}$ of the model generated by $\{u_M(k+1)\}_{k \in \mathbb{N}}$ for all $\mu = [\mu_1, \ldots, \mu_N]^\top$ with $\bigoplus_{i \in \mathcal{I}} \mu_i = e$ and the MMP is solved. □

As to the solvability of the FMMP, we can state the following theorem.

Theorem 2. Given a plant Σ_P of the form (5) and a model Σ_M of the form (6), the related FMMP is solvable if there exists a robust (A_E, B_1)-invariant subsemimodule of feedback type $\mathcal{V} \subseteq \mathcal{X}_E$ contained in the output equalizer subsemimodule $\mathcal{K} \subseteq \mathcal{X}_E$, defined by (9), with the property that for any $u_M \in \mathbb{R}_{\max}^{m_M}$ there exists $u_P \in \mathbb{R}_{\max}^{m_P}$ such that $B_1 u_P \oplus B_2 u_M$ belongs to $\mathcal{V}$.

Proof. Let $\mathcal{V} \subseteq \mathcal{K}$ be a robust $(A_E(\mu), B_1)$-invariant subsemimodule of feedback type for which the theorem condition holds. By hypothesis, there exist a matrix F such that for each $x_E(k) \in \mathcal{V}$, $(A_E(\mu) \oplus B_1 F) x_E(k)$ belongs to $\mathcal{V}$ for all $\mu = [\mu_1 \ldots \mu_N]^\top$ with $\bigoplus_{i \in \mathcal{I}} \mu_i = e$ and an $m_P \times m_M$ matrix G such that the columns of the matrix $\begin{pmatrix} \epsilon \\ B_M \end{pmatrix} I_{m_M} \oplus \begin{pmatrix} B_P \\ \epsilon \end{pmatrix} G = \begin{pmatrix} B_P G \\ B_M \end{pmatrix}$ belong to $\mathcal{V}$. Then, given any nondecreasing sequence $\{u_M(k+1)\}_{k \in \mathbb{N}}$, one can construct a sequence $\{u_P(k+1)\}_{k \in \mathbb{N}}$ defined by

$$u_P(k+1) = F x_E(k) \oplus G u_M(k+1) \oplus u_P(k), \qquad (14)$$

with $u_P(0) = \epsilon$, as follows. Start by taking $u_P(1) = F x_E(0) \oplus G u_M(1) \oplus u_P(0) = G u_M(1)$. Then, compute $x_E(1)$ by means of (8), $x_E(0)$, $u_P(1)$ and $u_M(1)$, take $u_P(2) = F x_E(1) \oplus G u_M(2) \oplus u_P(1)$ and iterate the procedure increasing by 1 the index k at each step. Following lines similar to those of the proof of Theorem 1, we get that $x_E(.)$ evolves inside $\mathcal{V}$ for all $\mu = [\mu_1 \ldots \mu_N]^\top$ with $\bigoplus_{i \in \mathcal{I}} \mu_i = e$. Since $\mathcal{V}$ is contained in the output equalizer semimodule $\mathcal{K}$, it follows that the output $\{y_P(k+1)\}_{k \in \mathbb{N}}$ of the plant generated by the input $\{u_P(k+1)\}_{k \in \mathbb{N}}$ defined by (14) is equal to the output $\{y_M(k+1)\}_{k \in \mathbb{N}}$ of the model generated by $\{u_M(k+1)\}_{k \in \mathbb{N}}$ for all $\mu = [\mu_1 \ldots \mu_N]^\top$ with $\bigoplus_{i \in \mathcal{I}} \mu_i = e$ and the FMMP is solved. □

Remark 4. To check the condition of Theorem 1 and to implement (11), an approach akin to (Animobono et al., 2023, Remark 7) can be followed, by solving bilateral systems of max-plus linear equations which involve $A_{E_1}, \ldots, A_{E_N}, B_1, B_2$ and the generators of $\mathcal{V}^*(\mathcal{K})$. Similar considerations hold for Theorem 2 and the construction of F and G in (14). Computational methods and challenges are described and discussed in (Butkovič and Hegedüs, 1984; Gaubert, 1998; Katz, 2007).

6. AN ILLUSTRATIVE EXAMPLE

Let Σ_P be a plant of the form (5), with $\mathcal{I} = \{1, 2\}$ and

$$A_{P1} = \begin{pmatrix} e & e \\ 2 & e \end{pmatrix}, \; A_{P2} = \begin{pmatrix} e & e \\ 3 & e \end{pmatrix}, \; B_P = \begin{pmatrix} 2 \\ e \end{pmatrix}, \; C_p = (e \;\, e).$$

Let Σ_M be a model of the form (6), described by

$$\Sigma_M \equiv \begin{cases} x_M(k+1) = 3\, x_M(k) \oplus 4\, u_M(k+1) \\ y_M(k) = x_M(k) \\ x_M(0) = \epsilon \end{cases}$$

The entry a_{21} of the dynamic matrix of the plant is assumed to be unknown, except for varying in the interval $[2, 3]$. This means that the only available information about the time between the k-th internal event of type 1, i.e. $x_1(k)$, and the $(k+1)$-th internal event of type 2, i.e. $x_2(k+1)$, is that it is not smaller than 2 and not greater than 3 units. A similar example, where, however, the plant dynamic matrix was assumed to be perfectly known, was developed in (Animobono et al., 2023).

The dynamics of the joint internal event dater $x_E(.) = \begin{pmatrix} x_P(.) \\ x_M(.) \end{pmatrix}$ has the form (8) with $\mathcal{I} = \{1, 2\}$ and

$$A_{E_1} = \begin{pmatrix} e & e & \epsilon \\ 2 & e & \epsilon \\ \epsilon & \epsilon & 3 \end{pmatrix}, \; A_{E_2} = \begin{pmatrix} e & e & \epsilon \\ 3 & e & \epsilon \\ \epsilon & \epsilon & 3 \end{pmatrix}, \; B_1 = \begin{pmatrix} 2 \\ e \\ \epsilon \end{pmatrix}, \; B_2 = \begin{pmatrix} \epsilon \\ \epsilon \\ 4 \end{pmatrix}.$$

Then, the output equalizer subsemimodule $\mathcal{K}$ is given by

$$\mathcal{K} = \operatorname{Im} \begin{pmatrix} e & \epsilon \\ \epsilon & e \\ e & e \end{pmatrix} = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3_{max} \text{ with } x_1 \oplus x_2 = x_3 \right\}.$$

To search for the maximum robust $(A_E(\mu), B_1)$-invariant semimodule $\mathcal{V}^*(\mathcal{K})$ contained in $\mathcal{K}$, we construct the sequence of semimodules defined by (10), which converges in two steps, with $\mathcal{V}^\star(\mathcal{K})$ that coincides with $\mathcal{K}$—i.e., $\mathcal{V}_0 = \mathcal{V}_1 = \mathcal{V}^*(\mathcal{K}) = \mathcal{K}$. By Proposition 4, $\mathcal{V}^*(\mathcal{K})$ is of feedback type, since $(A_{E_1} \oplus B_1 F)\mathcal{V}^*(\mathcal{K}) \subseteq \mathcal{K}$ and $(A_{E_2} \oplus B_1 F)\mathcal{V}^*(\mathcal{K}) \subseteq \mathcal{K}$ for $F = (\,1 \;\, 1 \;\, e\,)$. The condition of Theorem 2 is satisfied and, in particular, for any $u_M \in \mathbb{R}_{\max}^{m_M}$, letting $u_p = G u_M$ with $G = 2$, $B_1 u_P \oplus B_2 u_M \in \mathcal{V}^*(\mathcal{K})$, or, in other terms, the columns of the matrix $\begin{pmatrix} \epsilon \\ B_M \end{pmatrix} I_{m_M} \oplus \begin{pmatrix} B_P \\ \epsilon \end{pmatrix} G = \begin{pmatrix} B_P G \\ B_M \end{pmatrix}$ belong to $\mathcal{V}^*(\mathcal{K})$. Hence the FMMP and, consequently, the MMP are solvable. A solution of both, for a nondecreasing input sequence $\{u_M(k+1)\}_{k \in \mathbb{N}}$, is given by the nondecreasing input sequence $\{u_p(k+1)\}_{k \in \mathbb{N}}$ defined, as in (14), by

$$\begin{aligned} u_P(k+1) &= F x_E(k) \oplus G u_M(k+1) \oplus u_P(k) = \\ &= (\,1 \;\, 1 \;\, e\,)\, x_E(k) \oplus 2 u_M(k+1) \oplus u_P(k). \end{aligned}$$

7. CONCLUSIONS

By exploiting a structural approach based on newly introduced notions, sufficient conditions for the solvability of the model matching problem and of the feedback model matching problem for max-plus linear systems with polytopic uncertainty in the dynamics have been found. Although space limitations prevent us from investigating herein the more complex case in which the uncertainty also affects the input and output matrices, we can anticipate that our approach still applies to that case and the complete discussion will be the object of a forthcoming work.

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