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Multiscale modeling of opinion dynamics phenomena with external influence through drift term

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Abstract This paper develops a multiscale kinetic framework for modeling opinion dynamics under the action of external influences. The evolution of the opinion distribution is described through probabilistic interactions among individuals, while external influences acting on the system are represented by a drift term. From the kinetic formulation, the macroscopic evolution of the system is analyzed through the dynamics of the mean opinion and the variance. Different specifications of the drift field are considered in order to investigate how external actions may affect the collective behavior of the population.

1 Introduction

The dynamics of opinions has been widely studied during the last decades and even more prominently in recent years after a series of events such as the COVID-19 pandemic. In some situations, we have experienced a first example of *infodemic* [1], i.e., the extensive dissemination of both accurate and misleading information, potentially impacting public perception, individual decision-making, and political stability. The spreading of fake news [2–4] and the polarization effects [2, 5, 6] have been seen with their consequences. In this context, opinion dynamics is expected to play an increasingly crucial role in the coming years. This is not only due to the pervasive use of the internet and social media platforms, which have profoundly altered the mechanisms of information exchange and social interaction, but also because of the rapid development and widespread adoption of artificial intelligence (AI) systems. These technologies are progressively influencing the production, filtering, and dissemination of information, as well as the interaction between individuals and digital environments. As a consequence, understanding the mechanisms governing opinion formation, evolution, and collective behavior becomes essential in order to characterize and possibly predict emergent social phenomena in complex and interconnected societies [3, 4, 7]. For these reasons, the study and description of opinion dynamics represent an important, and actually rapidly growing, field of analysis. In particular, it is essential to understand the mechanisms underlying potentially harmful phenomena, such as the diffusion of fake news and the occurrence of polarization, and also to identify possible strategies to mitigate their effects. From this perspective, a quantitative and predictive understanding of opinion formation processes may provide useful insights for the design of effective intervention policies at the societal level. Among these, increasing attention has been devoted to debunking [8, 9] and prebunking strategies [10, 11], which aim at correcting misinformation after it has spread or at preventing its influence by strengthening the resilience of individuals to misleading content, respectively. This latter approach has recently been mathematically adapted in [12].

In the aims of this paper, hereafter we refer to the kinetic modeling approach [13, 14], even if there could be possible alternative frameworks. Specifically, these models allow for a multiscale description of the phenomenon, which is essential when the observed behavior arises from multiple interactions, as in the case of opinion dynamics [14, 15]. The phenomenon is considered at three different scales: microscopic, mesoscopic, and macroscopic. At the microscopic level, the interactions among agents, also called particles, are considered, and these interactions follow probabilistic schemes. Then, at the mesoscopic level, i.e., the statistical one characteristic of the original Boltzmann equation [16], the evolution of the system is obtained through one or more distribution functions. Finally, the appropriate moments of these distributions provide the macroscopic evolution of the system. In particular, the evolution of means and variances is obtained from the first two moments. It is worth pointing out that in order to close the system of high-order moments, the Fokker-Planck approach is usually assumed [14].

Bearing in mind the kinetic frameworks, some papers have recently been developed for modeling opinion dynamics evolution and its consequences. As already highlighted, opinion dynamics has been a phenomenon that has received even greater attention from the scientific community during the recent COVID-19 pandemic. In particular, this period has emphasized the crucial role played

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by the rapid dissemination of information, which occurs primarily through online platforms and social networks. The widespread use of these digital environments has significantly accelerated the circulation of both reliable and misleading content, amplifying their influence on individual and collective behaviors. This has further reinforced the need to understand and quantitatively describe the mechanisms governing opinion formation and evolution in modern interconnected societies. Indeed, in [17, 18], the evolution of opinion dynamics in an epidemic framework is assumed. Similarly, a kinetic model for opinion dynamics has been developed in the context of epidemic spreading, with particular focus on vaccine hesitancy [19]. These papers have followed the wide context of kinetic models in epidemic dynamics; see, among others, [20–23]. At the same time, the polarization phenomena have been assumed to be the result of such interactions, and it has been recently investigated with kinetic theory in [24–26]. In this framework, the structure of the underlying interaction network plays a crucial role in shaping the connectivity patterns among individuals and, consequently, in determining the evolution of collective opinions. In particular, the topology of social networks influences how information propagates, how clusters of like-minded individuals emerge, and how polarization phenomena are reinforced or mitigated [17, 27, 28]. Moreover, the widespread diffusion of fake news within online social networks has further amplified these effects, altering the natural flow of information and contributing to the emergence of misinformation-driven opinion patterns [29, 30]. The rapid and large-scale dissemination of misleading content may distort individual perception and reinforce polarization, thus affecting the overall dynamics of the system. Finally, the recent development of artificial intelligence technologies is expected to play an increasingly important role in these processes. AI systems are progressively involved in content generation, recommendation, and filtering, thereby influencing information exposure and interaction mechanisms at an unprecedented scale. This introduces new challenges and perspectives for the study of opinion dynamics, making the development of reliable mathematical models essential for understanding and predicting the collective behavior of complex social systems (see, among others, [31, 32], and references therein). It is also worth mentioning that the introduction of external actions in kinetic models of opinion dynamics is closely related to the well-established research line concerning control strategies for multiagent systems [33–36]. In this direction, external interventions are often introduced at the microscopic level, through controlled binary interactions, feedback controls, or model predictive control strategies, and are then upscaled to the kinetic scale. This procedure naturally leads, in suitable asymptotic regimes, to controlled Boltzmann-type equations and to mean-field or Fokker–Planck descriptions in which the action of the control may appear as an additional drift contribution. In the context of opinion dynamics, this approach has been successfully used to study consensus formation, the role of opinion leaders, and the design of strategies able to steer the collective opinion toward desired configurations.

In this context, the present paper fits naturally within the aforementioned lines of research. Specifically, we adopt a kinetic modeling framework with interacting agents [37], which has proven to be particularly effective in describing the evolution of opinion distributions arising from binary interactions among individuals. The main novelty of the proposed model lies in the introduction of a drift term, designed to capture the collective tendency of the population to shift its opinion over time. This term provides a representation of external influences and/or systematic biases affecting the agents, and acquires a specific interpretation depending on the application under consideration and on the mechanisms one aims to describe. As a consequence, the resulting model remains highly versatile and can be used to investigate a wide range of phenomena, including the emergence of polarization, as well as the effects and possible countermeasures associated with the spread of fake news. The introduction of external actions has already been considered for kinetic frameworks in [38–40], even if in different contexts. With respect to these contributions, the specific aim of the present paper is to focus on the role of drift-type external actions in opinion dynamics. In particular, we do not introduce a single prescribed external force, but we consider several drift formulations within the same kinetic framework, keeping the binary interaction structure fixed. This allows us to isolate the effect of the drift term and to compare, at the macroscopic level, how different external mechanisms may enhance or reduce polarization and opinion dispersion. From this viewpoint, the main contribution of the paper is the derivation and qualitative analysis of closed moment systems for the mean opinion and the variance under different conservative drift structures.

The contents of the paper are organized as follows. In Sect. 2, the kinetic model is introduced and presented by providing a quick overview of collisional and non-collisional terms. Then, in Sect. 3, we specialize the kinetic model introduced in the previous section in a specific case of opinion dynamics, by providing some preliminary a priori assumptions. In the subsequent subsection, we analyze, in particular from a numerical point of view, some specific cases, with a focus on the first two moments of the system, i.e., mean and variance. Final outlooks and future research perspectives are investigated in Sect. 4.

2 Multiscale mathematical model for opinion dynamics

Let us consider a population characterized by an opinion on a specific topic whose meaning depends on the particular application under consideration. For the purposes of this paper, we assume a single population composed of interacting individuals who may influence each other through pairwise interactions.

At the *microscopic level*, the opinion of the population is represented by a continuous microscopic variable u defined in a continuous real subset D_u, i.e., $u \in D_u \subseteq \mathbb{R}$. For the sake of simplicity, hereafter we assume that

$$D_u = \mathbb{R}.$$

This choice does not limit the realism of the model. Indeed, we assume that opinions close to $u = 0$ represent moderate positions. As u moves farther away from the origin, toward $+\infty$ or $-\infty$, the corresponding opinions become increasingly extreme. It reflects what typically occurs in opinion formation on several topics, such as political, economic, social, or medical issues. The microscopic behavior of the system is defined by the interaction rate

$$\eta(v,\ w) : \mathbb{R} \times \mathbb{R} \to \mathbb{R}^{+},$$

which accounts for interactions per unit of time between people with opinion v and people with opinion w, for $v,\ w \in \mathbb{R}$, and by the *transition probability*

$$C(u,\ v,\ w) : \mathbb{R} \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}^{+},$$

which models the probability that an individual with opinion v moves to opinion u, after interacting with an individual with opinion w, for $u,\ v,\ w \in \mathbb{R}$. Since $C(u, v, w)$ represents a probability, the following assumption is hereafter assumed:

$$\int_{\mathbb{R}} C(u, v, w)\, du = 1, \quad \forall v, w \in \mathbb{R}. \tag{1}$$

These quantities model the microscopic evolution of the system, as the evolution of opinion in the population follows these stochastic rules. At the kinetic level, namely, the *mesoscopic scale* of the system, we introduce the *distribution function*

$$f(t,\ u) : [0,\ T] \times \mathbb{R} \to \mathbb{R}^{+},$$

such that $f(t,\ u)du$ represents the fraction of individuals that have an opinion in the interval $[u,\ u + du]$, at time $t \geq 0$. Moreover, $\int_{\mathbb{R}} f(t,\ u)\, du$ represents the total amount of the population that we may assume constant at this first step.

Up to this point, the evolution of opinions within the population depends exclusively on internal interactions, i.e., the quantities $\eta(v,\ w)$ and $C(u, v, w)$. However, external influences cannot be neglected if we aim to reproduce more realistic phenomena, such as those we increasingly observe through social media. These actions may take into account a wide range of aspects, from manipulative mechanisms such as the dissemination of fake news to corrective interventions aimed at mitigating misinformation, thus emphasizing the versatility and broad applicability of the proposed framework. Therefore, they are not internal to the system, but they act from the outside. From a mathematical point of view, we define an operator

$$W[f](t,u) : [0,\ T] \times \mathbb{R} \to \mathbb{R} \tag{2}$$

that models this external action. We stress that, in the present paper, the operator $W[f]$ is not derived from any specific microscopic interaction. More precisely, this statement only concerns the external-action term. The binary operator $J[f]$ is still obtained from the prescribed microscopic stochastic interactions through the interaction rate $\eta(v,\ w)$ and the transition probability $C(u, v, w)$. Therefore, the model keeps a multiscale structure: the collisional part describes the effect of pairwise interactions, whereas $W[f]$ represents an effective external field acting at the kinetic level. Such a choice allows us to investigate how different conservative external mechanisms modify the macroscopic evolution of the mean opinion and of the variance, without requiring a full microscopic derivation of each external intervention. In general, it depends on time, microscopic variable u, and the current state of the system through the distribution function $f(t,\ u)$. In fact, this dependence is not merely a mathematical aspect, but is also relevant to applications. In the case of fake news manipulation, for instance, the external action is modeled according to how opinions are distributed at that specific time. In accordance with these choices, the mathematical properties of the operator W, such as its shape and regularity, may vary. It is worth noting that in this paper, the mathematical and modeling choices are always closely intertwined.

Then, bearing all the above in mind, the *kinetic equation* that provides the evolution of the system at the mesoscopic level, i.e., in terms of the distribution function, is written as

$$\begin{aligned} \partial_t f(t,u) &= \\ &= \int_{\mathbb{R}\times\mathbb{R}} \eta(v,w) C(u,v,w) f(t,v) f(t,w)\, dv dw - f(t,u) \int_{\mathbb{R}} \eta(u,w) f(t,w)\, dw + W[f](t,u) \\ &= J[f](t,u) + W[f](t,u), \end{aligned} \tag{3}$$

where

$$J[f](t,\ u) = \int_{\mathbb{R}\times\mathbb{R}} \eta(v,\ w) C(u,\ v,\ w) f(t,\ v) f(t,\ w)\, dv dw - f(t,\ u) \int_{\mathbb{R}} \eta(u,\ w) f(t,\ w)\, dw$$

is the conservative operator, due to the assumption (1), which accounts for the effect of binary stochastic interactions on the evolution of the system; according to assumption (1), we have that

$$\int_{\mathbb{R}} J[f](t,\ u)\, du = 0.$$

As we want to ensure the conservative structure of the system, i.e., no proliferative/destructive events are assumed, the quantity $\int_{\mathbb{R}} f(t, u)\,du$ has to remain constant during the evolution of the system, even under the action of $W[f](t, u)$. Therefore, hereafter we assume that

$$\int_{\mathbb{R}} W[f](t,u)\,du = 0. \tag{4}$$

Indeed, if an initial condition $f^0(u) : \mathbb{R} \to \mathbb{R}^+$ is such that

$$\int_{\mathbb{R}} f^0(u)du = 1,$$

then, under the assumptions (1) and (4), one has that

$$\int_{\mathbb{R}} f(t, u)du = 1,$$

for all $t > 0$, i.e., the system preserves the overall mass. Moreover, with this choice, we may assume that the distribution function $f(t, u)$ is a probability density, as expected.

Moreover, in order to define the initial-boundary-value problem related to equation (3), we assume for the well-posedness of the system and according to the conservative assumptions that

$$f(t, u) \xrightarrow{u\to\pm\infty} 0.$$

Then, the related problem reads

$$\begin{cases} \partial_t f(t,u) = J[f](t,u) + W[f](t,u) & [0,T] \times \mathbb{R} \\ f(0,u) = f^0(u) & u \in \mathbb{R}. \end{cases} \tag{5}$$

Before providing the specific expression of the terms involved in the kinetic equations (3), we briefly introduce the *macroscopic description* of the system. For this aim, we define the pth-order moment of the system, for $p \in \mathbb{N}$, as

$$\mathbb{E}_p[f](t) := \int_{\mathbb{R}} u^p f(t, u)\,du.$$

In general, these quantities are not necessarily well defined, since they depend on the regularity and integrability properties of $f(t, u)$. In particular, the following macroscopic quantities are obtained:

- For $p = 0$, one has the density of the system $\rho(t) := \mathbb{E}_0[f](t) = \int_{\mathbb{R}} f(t, u)\,du$. It is worth stressing that under conservative assumptions (1) and (4) this quantity is preserved, i.e.,

$$\dot{\rho} = 0;$$

- for $p = 1$, the moment of the system represents the mean value $m(t) := \mathbb{E}_1[f](t) = \int_{\mathbb{R}} uf(t, u)\,du$ of the distribution $f(t, u)$. This quantity represents the mean opinion in the population, and it will play a crucial role at the modeling level, as we will show in the following.
- For $p = 2$, the 2nd-order moment is related to the variance $v(t)$ of the system. Indeed:

$$v(t) = \int_{\mathbb{R}} (u - m(t))^2 f(t, u)\,du = \mathbb{E}_2[f](t) - m^2(t),$$

  or, equivalently,

$$\mathbb{E}_2[f](t) = v(t) + m^2(t).$$

It is worth emphasizing that the variance plays a key role in describing the evolution of opinion dynamics models. Indeed, it measures how widely opinions are dispersed around the mean, providing a meaningful indicator of the overall state of opinion within the population.

For all $p \in \mathbb{N}$, the evolution of the pth-order moment $\mathbb{E}_p[f](t)$ is obtained by multiplying both sides of equation (3) by u^p, and integrating over $\mathbb{R}$; then, one has

$$\dot{\mathbb{E}}_p[f](t) = \int_{\mathbb{R}} u^p J[f](t,u)\,du + \int_{\mathbb{R}} u^p W[f](t,u)\,du. \tag{6}$$

In general, the system of moments is not in closed form. Then, under the hypotheses assumed in this paper, the evolution of the 0th-, 1st-, and 2nd-order moment satisfies the following system:

$$\begin{cases} \dot{\rho}(t) = 0 \\ \dot{m}(t) = \int_{\mathbb{R}} u J[f](t,u)\,du + \int_{\mathbb{R}} u W[f](t,u)\,du \\ \dot{v}(t) = \dot{\mathbb{E}}_2[f](t) - 2\,m(t)\dot{m}(t) = \int_{\mathbb{R}} u^2 J[f](t,u)\,du + \int_{\mathbb{R}} u^2 W[f](t,u)\,du - 2\,m(t)\dot{m}(t). \end{cases} \tag{7}$$

Then, given the initial conditions $(1,\ m^0,\ v^0) \in (\mathbb{R}^3)^+$, the system of moments is obtained. However, once again, it is worth pointing out that the system above is not in closed form.

We now specify the form of the external operator for the purposes of this paper, as it represents the main novelty of the proposed model. In particular, we model the external action through a *drift operator* acting on the distribution function f, namely

$$W[f](t,u) = -\partial_u(a(u)\,f(t,u)), \tag{8}$$

where $a : \mathbb{R} \to \mathbb{R}$ is a prescribed drift field. Consequently, the kinetic equation (3) can be rewritten as

$$\partial_t f(t,u) = J[f](t,u) - \partial_u(a(u)\,f(t,u)). \tag{9}$$

Assuming a sufficient decay of $f(t, u)$ as $u \to \pm\infty$ (as required in (5)), one immediately gets

$$\int_{\mathbb{R}} W[f](t,u)\,du = -\int_{\mathbb{R}} \partial_u(a(u)f(t,u))\,du = 0, \tag{10}$$

hence the drift modeling (8) is consistent with the conservative requirement (4). In particular, if $\int_{\mathbb{R}} f^0(u)\,du = 1$, then $\int_{\mathbb{R}} f(t, u)\,du = 1$ for all $t \in [0, T]$. Then, recalling that $\mathbb{E}_p[f](t) = \int_{\mathbb{R}} u^p f(t,\ u)\,du$ is pth-order moment, multiplying (9) by u^p and integrating over $\mathbb{R}$ yields

$$\dot{\mathbb{E}}_p[f](t) = \int_{\mathbb{R}} u^p J[f](t,u)\,du + \int_{\mathbb{R}} u^p W[f](t,u)\,du. \tag{11}$$

Under the decay assumptions of $f(t, u)$ at infinity, one has

$$\int_{\mathbb{R}} u^p W[f](t,u)\,du = -\int_{\mathbb{R}} u^p \partial_u(a(u)f(t,u))\,du = p\int_{\mathbb{R}} u^{p-1}a(u)f(t,u)\,du. \tag{12}$$

In particular, for $p = 1$ and $p = 2$ we obtain

$$\int_{\mathbb{R}} u\,W[f](t,u)\,du = \int_{\mathbb{R}} a(u)\,f(t,u)\,du, \tag{13}$$

$$\int_{\mathbb{R}} u^2\,W[f](t,u)\,du = 2\int_{\mathbb{R}} u\,a(u)\,f(t,u)\,du \tag{14}$$

Therefore, the evolution of the mean value $m(t)$ and of the second-order moment $\mathbb{E}_2$, that is related to the variance of the system, reads

$$\dot{m}(t) = \int_{\mathbb{R}} u\,J[f](t,u)\,du + \int_{\mathbb{R}} a(u)f(t,u)\,du, \qquad \dot{\mathbb{E}}_2[f](t) = \int_{\mathbb{R}} u^2 J[f](t,u)\,du + 2\int_{\mathbb{R}} u\,a(u)f(t,u)\,du. \tag{15}$$

Since $v(t) = \mathbb{E}_2[f](t) - m^2(t)$, we also have

$$\dot{v}(t) = \dot{\mathbb{E}}_2[f](t) - 2m(t)\dot{m}(t). \tag{16}$$

Then, the system of moments reads

$$\begin{cases} \dot{m}(t) = \int_{\mathbb{R}} uJ[f](t,u)\,du + \int_{\mathbb{R}} a(u)f(t,u)\,du \\ \dot{v}(t) = \int_{\mathbb{R}} u^2 J[f](t,u)\,du - 2m(t)\dot{m}(t) + 2\int_{\mathbb{R}} ua(u)f(t,u)\,du \\ m(0) = m^0 \\ v(0) = v^0. \end{cases} \tag{17}$$

Once the drift operator $a(u)$ and the binary interaction coefficients $C(u, v, w)$ and $\eta(v,\ w)$ are assigned, the kinetic equation (9) and the corresponding system of moments (17) are completely determined.

It is worth stressing that the macroscopic description in terms of the mean value $m(t)$ and the variance $v(t)$ provides only a reduced representation of the full kinetic dynamics. Indeed, these two quantities describe the collective orientation of the population and the overall dispersion of opinions around it, but they do not allow one to reconstruct the complete distribution function $f(t, u)$.

In particular, information on higher-order moments, asymmetry, multimodality, tail behavior, or the detailed shape of the opinion distribution is not retained by the reduced system. Consequently, different kinetic distributions may share the same mean and variance while corresponding to different microscopic or mesoscopic configurations. Nevertheless, the moment description is useful for the purposes of the present work. The mean value provides a macroscopic indicator of the prevailing opinion trend, whereas the variance gives a first measure of opinion heterogeneity and possible dispersion effects. Therefore, although the reduced system cannot replace a full analysis of the kinetic equation, it captures the main macroscopic features that we aim to compare under different drift mechanisms. In this sense, the closed systems for $m(t)$ and $v(t)$ should be understood as macroscopic projections of the kinetic dynamics, suitable for detecting whether a given external action tends to promote moderation, dispersion, or polarization at the level of the first two moments. This point is particularly relevant since the variance is not only a technical quantity, but a key observable for detecting the emergence of heterogeneous, fragmented, or polarized configurations, which are typical of complex collective phenomena such as opinion dynamics.

3 Model specification and numerical simulations

In order to obtain analytical results and to provide numerical simulations for the mathematical model of the previous section, with specific applications to opinion dynamics problems, we need to assign the mathematical form of the interaction rate, $\eta(v, w)$, transition probability, $C(u, v, w)$, and drift operator $W[f](t, u)$.

First, we assume the homogeneous condition for the interaction rate, i.e.,

$$\eta(v, w) = \eta \qquad \forall v, w \in \mathbb{R}. \tag{18}$$

At this stage of the modeling procedure, with the aim of investigating the main features of the proposed model, this assumption does not seem too restrictive, even if more general formulations could be adopted for specific situations. The choices introduced in this section should be understood as prototype modeling assumptions rather than as a calibration on a specific empirical dataset. More precisely, the binary interaction operator is the part of the model directly associated with microscopic pairwise interactions, whereas the external action is prescribed at the kinetic level through the conservative drift operator (8). Hence, we do not claim that the whole modeling setting is derived from microscopic controlled interactions. Rather, our aim is to keep the binary interaction structure sufficiently simple in order to isolate, at the level of the closed moment equations, the effect of externally prescribed drift fields. In this perspective, the homogeneous interaction rate provides a reference framework in which the role of the transition probability and of the drift contribution can be analyzed without additional heterogeneity effects. According to (18), we have the following expression for the terms involved in the r.h.s. of the first two equations of system (17)

$$\int_{\mathbb{R}} uJ[f](t,u)\,du = \eta \int_{\mathbb{R}^3} uC(u,v,w)f(t,v)f(t,w)\,dv\,dw\,du - \eta m(t) \tag{19}$$

$$\int_{\mathbb{R}} u^2 J[f](t,u)\,du = \eta \int_{\mathbb{R}^3} u^2 C(u,v,w)f(t,v)f(t,w)\,dv\,dw\,du - \eta\big(m^2(t) + v(t)\big) \tag{20}$$

Then, as a first modeling approximation, the transition probability is assumed to follow a Gaussian distribution, i.e.,

$$C(u,v,w) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(u-\mu)^2}{2\sigma}}, \tag{21}$$

where the variance σ is fixed, and the mean μ is equal to

$$\mu = \frac{w + kv + m(t)}{3}; \tag{22}$$

$m(t)$ is the mean of opinion on the overall population, i.e., the 1st-order moment. The choice of the mean value μ reflects a compromise between the opinions involved in the interaction and the global state of the population, represented by the average opinion $m(t)$. This modeling assumption allows one to account for both local interaction effects and the influence of the overall opinion distribution. The Gaussian form is adopted as a first approximation of a stochastic compromise mechanism. The parameter σ accounts for the uncertainty in the post-interaction opinion update, while the mean value μ combines the opinion of the interacting agents with the average opinion of the population. The parameter k modulates the relative weight of the pre-interaction opinion v, and may be interpreted as a simple way to account for persistence or self-conviction effects. Furthermore, this choice is particularly convenient for deriving a closed macroscopic system for the mean opinion and the variance.

Then, the system (17) for mean and variance is written as

$$\begin{cases} \dot{m}(t) = \eta\left(\frac{k-1}{3}\right)m(t) + \int_{\mathbb{R}} uW(u)du \\ \dot{v}(t) = \eta\sigma + \eta v(t)\left(\frac{k^2-8}{9}\right) + \eta m^2(t)\left(\frac{k-1}{3}\right)^2 - 2m(t)\int_{\mathbb{R}} uW(u)du + \int_{\mathbb{R}} u^2 W(u)du \\ m(0) = m^0 \\ v(0) = v^0. \end{cases} \tag{23}$$

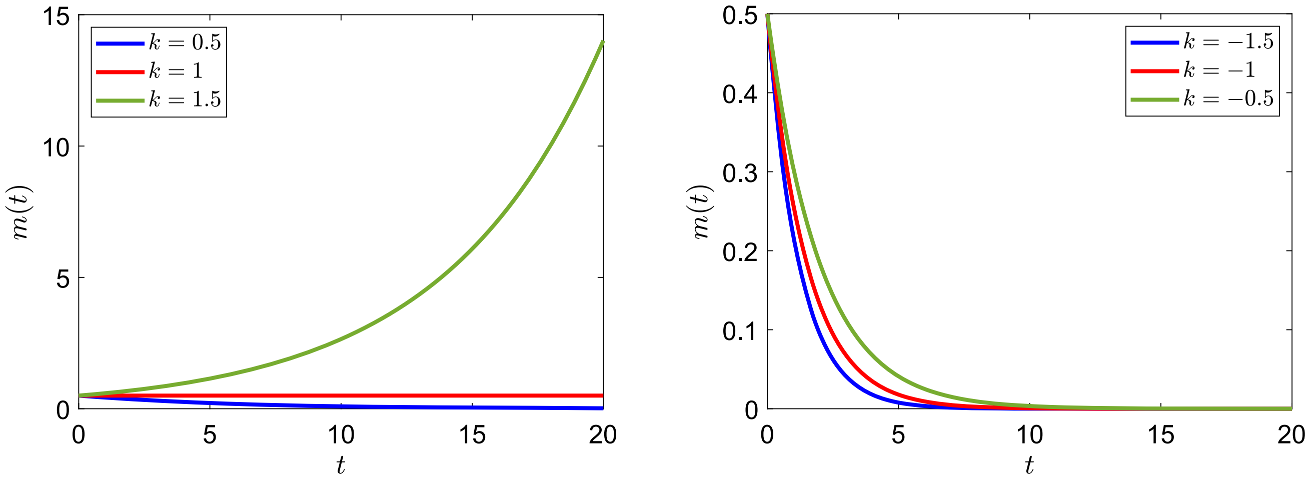

Fig. 1 Mean value $m(t)$ versus time t of the model without external perturbations with different values of the parameter k, in particular $k = 0.5,\ 1,\ 1.5$ on the left panel and $k = -1.5,\ -1,\ -0.5$ on the right one. The other parameters are: $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.5$, $v^0 = 10^{-3}$

In general, the system above is not in closed form. In the present work, we are interested in a minimal choice leading to a closed macroscopic system for the pair $(m(t), v(t))$. To this end, in the following subsections, we will suitably define the drift term, and hence the function $a(u)$, in such a way as to ensure the closure of the moment system for the mean and the variance. Then, numerical simulations will be presented in order to illustrate the behavior of the model under the different scenarios considered. For the sake of brevity, we will focus on the specific features of each scenario, avoiding the repetition of modeling aspects already discussed above.

Before introducing the different specifications of the drift operator, we first define a baseline scenario that will serve as a reference throughout the paper. This baseline configuration corresponds to the case in which no drift term is present, namely $a(u) = 0$, so that the dynamics is entirely governed by the binary interaction mechanism described above, i.e., interaction rate and transition probability. In all the scenarios considered in the following subsections, the binary collisional structure of the model remains unchanged. The different cases will therefore correspond exclusively to the introduction of specific drift terms, allowing us to isolate and assess their impact on the macroscopic evolution of the mean and the variance with respect to the same reference configuration. In particular, the baseline scenario depends on the above parameters k and σ, along with the choice of initial data m^0 and v^0, such that the macroscopic system (23) reads

$$\begin{cases} \dot{m} = \eta\left(\frac{k-1}{3}\right)m(t) \\ \dot{v} = \eta\left(\frac{k^2-8}{9}\right)v(t) + \eta\left(\frac{k-1}{3}\right)^2 m^2(t) + \eta\sigma \\ m(0) = m^0 \\ v(0) = v^0. \end{cases} \tag{24}$$

The numerical results in Figs. 1, 2, 3, 4 and 5 show that the macroscopic observables $m(t)$ and $v(t)$ may display markedly different regimes depending on the values of parameters k, σ and on the initial conditions m^0, v^0. The mean opinion $m(t)$ captures the prevailing collective orientation. When $m(t)$ relaxes toward zero, the system approaches a moderate regime, with the bulk of the population remaining close to the neutral state. Conversely, when $|m(t)|$ increases, the population progressively drifts toward more extreme states, signaling the emergence of a polarized collective trend. In parallel, the variance $v(t)$ quantifies the spread of opinions: a rapid growth of $v(t)$ indicates a widening of the distribution and hence an amplification of heterogeneity, i.e., the coexistence of increasingly distant viewpoints.

Overall, these simulations indicate that binary interactions alone may already induce substantial macroscopic changes, leading either to moderation or to polarization and fragmentation within the simulated time horizon, depending on the parameter regime. This baseline configuration, therefore, provides the natural reference framework for assessing the impact of the drift terms introduced in the following subsections.

3.1 Specification of the drift operator

We introduce different models to define the drift operator $W[f](t, u)$ and we discuss the corresponding numerical simulations. Within the general framework described above, we prescribe the function $a(u)$ in such a way as to guarantee the closure of the system (17) for the mean $m(t)$ and the variance $v(t)$, while keeping a transparent kinetic and macroscopic interpretation of the external action.

We stress that the drift fields considered below are not derived from any specific microscopic dynamics, nor in an optimal control sense. Rather, they have to be understood as prototype external-action mechanisms. From a physical modeling viewpoint, the drift

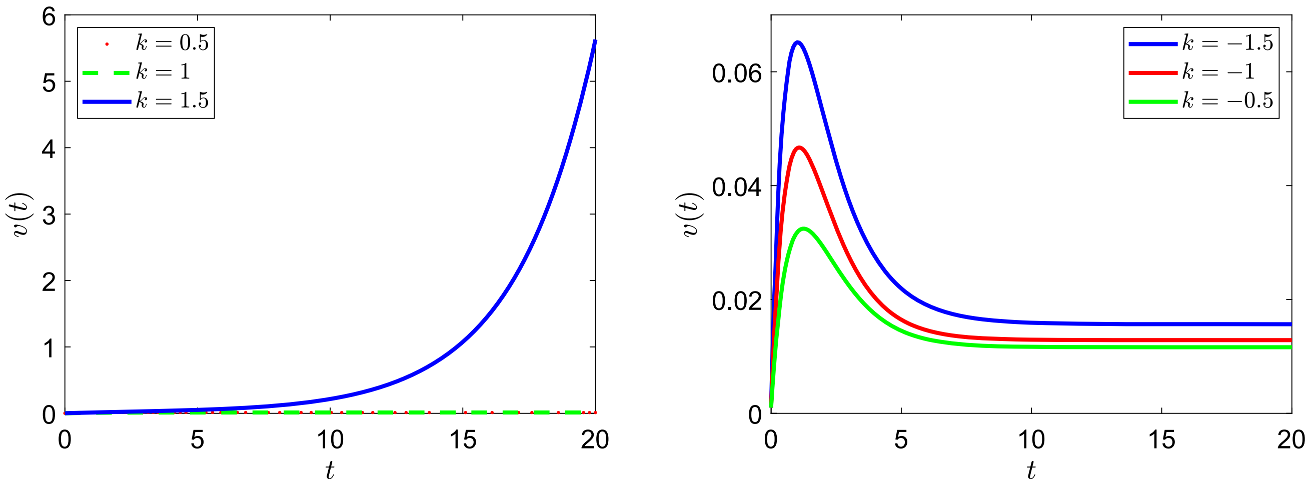

Fig. 2 Variance $v(t)$ versus time t of the model without external perturbations with different values of the parameter k, in particular $k = 0.5,\ 1,\ 1.5$ on the left panel and $k = -1.5,\ -1,\ -0.5$ on the right one. The other parameters are: $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.5$, $v^0 = 10^{-3}$

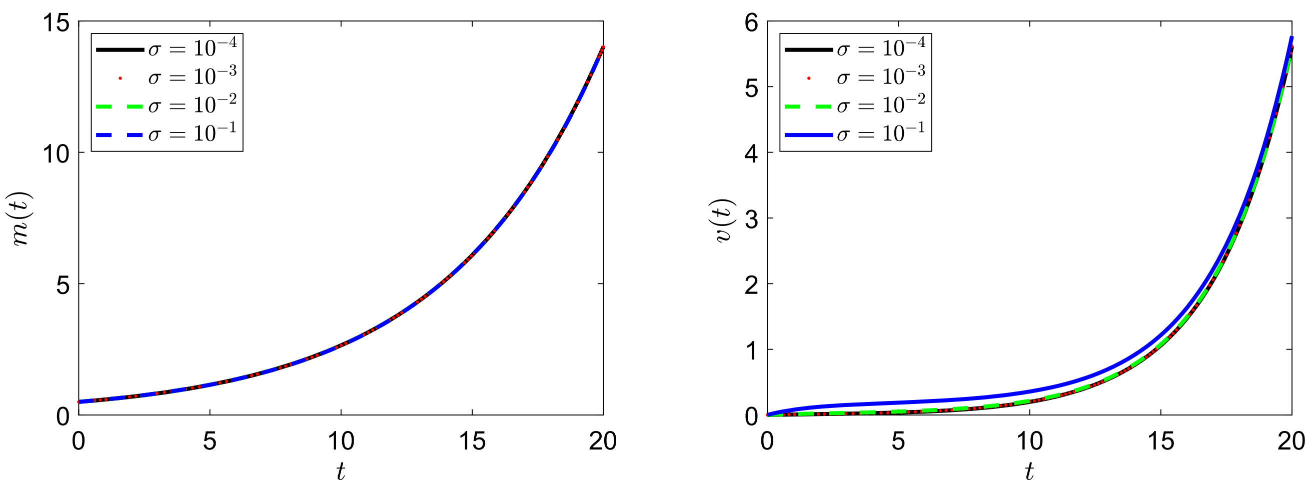

Fig. 3 Mean value $m(t)$ versus time t, on the left side, and variance $v(t)$ versus time t, on the right side, of the model without external perturbations with different values of the parameter σ. $k = 1.5$, $\eta = 1$, $m^0 = 0.5$, $v^0 = 10^{-3}$

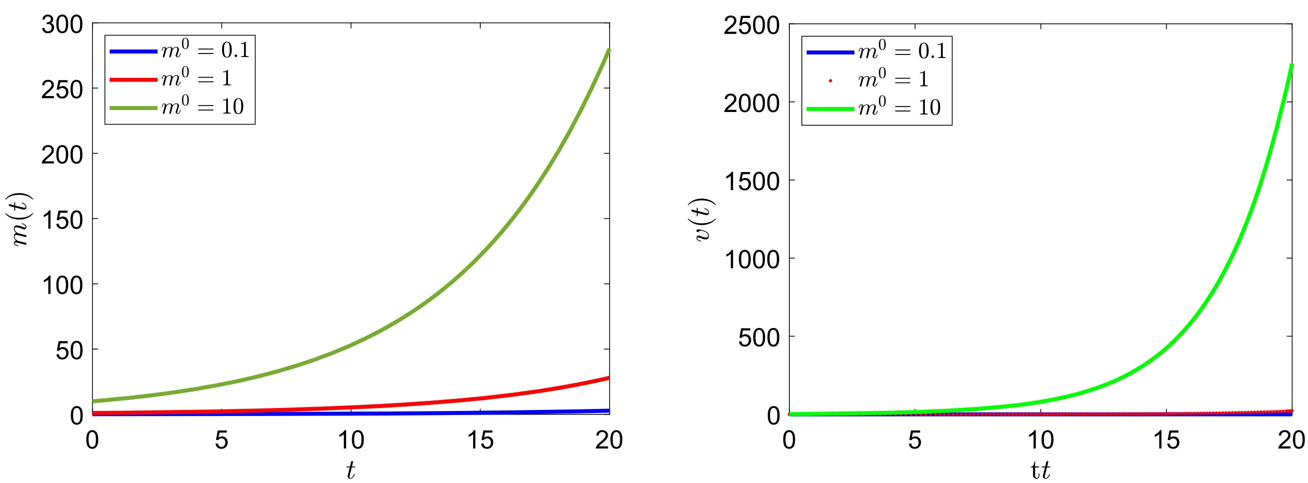

Fig. 4 Mean value $m(t)$ versus time t, on the left side, and variance $v(t)$ versus time t, on the right side, of the model without external perturbations with different values of the initial condition m^0. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $v^0 = 10^{-3}$

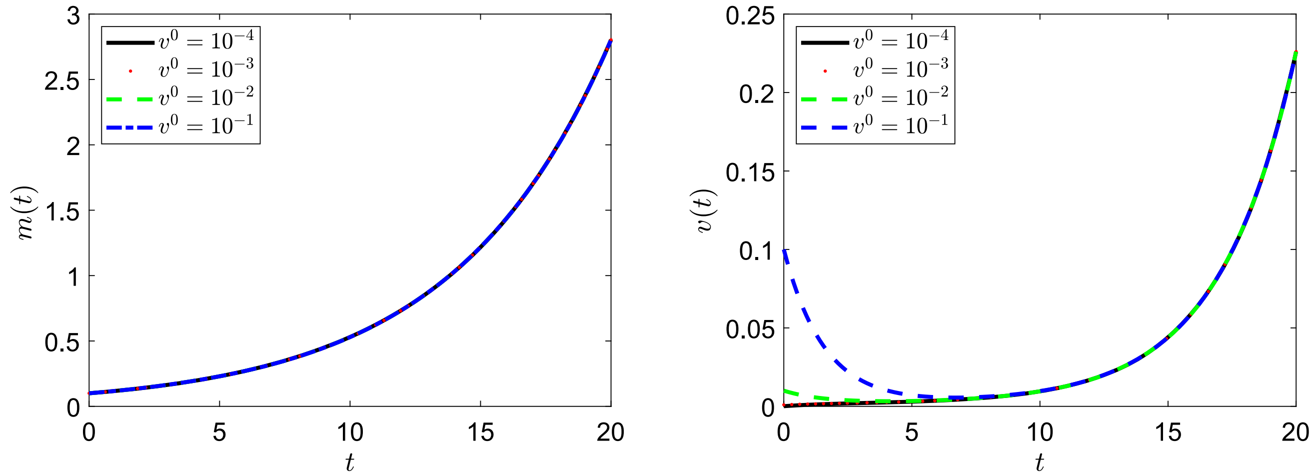

Fig. 5 Mean value $m(t)$ versus time t, on the left side, and variance $v(t)$ versus time t, on the right side, of the model without external perturbations with different values of the initial condition v^0. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$

term summarizes at the kinetic level the action of external influences which systematically modify the evolution of the opinion distribution. Their role is therefore to provide a first qualitative investigation of how different drift structures, possibly inspired by controlled or biased microscopic interactions, may affect the macroscopic evolution of the mean opinion and of the variance. In this sense, in the present analysis, instead of deriving an optimal intervention, we prescribe different conservative drift fields and compare their macroscopic effects with the baseline dynamics without external action. The specific meaning of the drift term is defined in each of the cases below. In particular, positive or destabilizing drift contributions are interpreted as external mechanisms that may reinforce extreme opinions or increase dispersion, whereas negative or contracting contributions may describe stabilizing actions that reduce heterogeneity and favor moderation. We also stress that the numerical tests are performed on the closed macroscopic systems for $m(t)$ and $v(t)$. This is consistent with the main objective of the paper, which is to analyze the impact of drift terms at the level of the first two moments. Accordingly, the use of ODE solvers follows directly from the moment closure obtained for the selected operators, and is not meant to replace a full numerical analysis of the kinetic equation.

Let us also point out that the drift mechanisms considered below are related, at least at a formal level, to controlled kinetic descriptions of opinion dynamics. In such frameworks, external actions may be introduced through feedback controls, leader effects, or penalized strategies, and then reflected at the kinetic scale through additional transport contributions; see, for instance, [33–35]. In the present paper, we do not solve an optimal control problem. Nevertheless, the drift fields below may be interpreted as phenomenological prototypes of external actions acting on the opinion distribution.

Model 1. We start by introducing an affine drift field, i.e.,

$$a(u) = \alpha + \beta u, \qquad \alpha, \beta \in \mathbb{R}. \tag{25}$$

This affine structure is also a natural way to connect the present formulation with linear feedback mechanisms toward a prescribed target opinion. For instance, in controlled opinion dynamics, a penalized action steering the population toward a target x_T typically produces a transport term which is affine in the opinion variable, up to the sign convention adopted for the drift operator. Therefore, model 1 may be regarded as a simple prototype, among those considered here, of mean-field control-inspired external actions. Under (25), one has that

$$\int_{\mathbb{R}} a(u) f(t,u)\, du = \alpha + \beta m(t), \qquad \int_{\mathbb{R}} u\, a(u) f(t,u)\, du = \alpha m(t) + \beta \mathbb{E}_2[f](t). \tag{26}$$

Hence, using (14) and (16), the drift contribution to the variance becomes

$$\begin{aligned}\int_{\mathbb{R}} u^2 W[f](t,u)\, du - 2m(t) \int_{\mathbb{R}} u W[f](t,u)\, du &= 2 \int_{\mathbb{R}} u\, a(u) f(t,u)\, du - 2m(t) \int_{\mathbb{R}} a(u) f(t,u)\, du \\ &= 2(\alpha m(t) + \beta \mathbb{E}_2[f](t)) - 2m(t)(\alpha + \beta m(t)) \\ &= 2\beta(\mathbb{E}_2[f](t) - m^2(t)) = 2\beta v(t)\end{aligned} \tag{27}$$

The drift modeling (25) yields a linear contribution on $m(t)$ and $v(t)$. Specifically, under the affine drift assumption (25), the mean value satisfies

$$\dot{m}(t) = \int_{\mathbb{R}} u\, J[f](t,u)\, du + \alpha + \beta m(t), \tag{28}$$

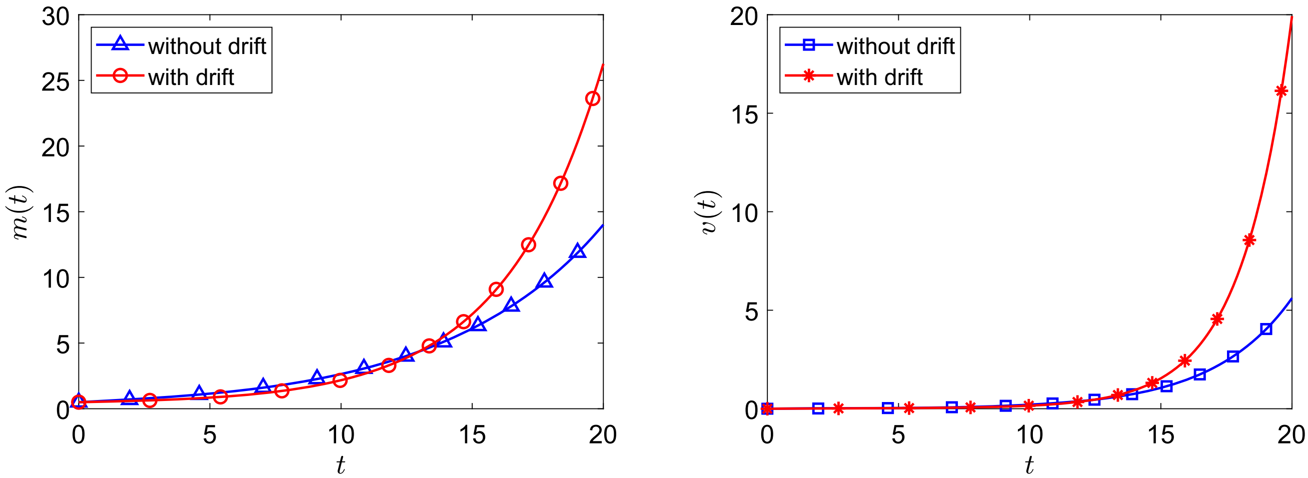

Fig. 6 Mean value $m(t)$ and variance $v(t)$ versus time t of model 1 with $\alpha = -0.1$ and $\beta = 0.1$. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$

while, using (16) together with (27), for the variance we have that

$$\dot{v}(t) = \int_{\mathbb{R}} u^2 J[f](t,u)\,du - 2m(t)\int_{\mathbb{R}} u\,J[f](t,u)\,du + 2\beta v(t). \tag{29}$$

Therefore, the system for the macroscopic variables $(m(t), v(t))$ with drift (25) is obtained from (17), and it reads

$$\begin{cases} \dot{m}(t) = \eta\left(\frac{k-1}{3}\right)m(t) + (\alpha + \beta m(t)) \\ \dot{v}(t) = 2\beta v(t) + \frac{1}{9}\eta\left[(k^2-8)v(t) + (k-1)^2 m^2(t) + 9\sigma\right] \\ m(0) = m^0 \\ v(0) = v^0. \end{cases} \tag{30}$$

From a qualitative point of view, the affine drift field allows one to distinguish different macroscopic regimes according to the sign and the strength of the linear coefficient β. When β is positive, the drift acts as an expanding transport field in the opinion variable and tends to amplify the spreading of the distribution. Conversely, negative values of β correspond to a contracting contribution, which may counterbalance the dispersion generated by the binary interactions and favor more stable configurations. The parameter α, instead, shifts the evolution of the mean opinion and therefore affects the variance indirectly through the coupling between $m(t)$ and $v(t)$. In this sense, even without deriving explicit threshold formulas, model 1 already displays a natural qualitative separation between stabilizing and destabilizing drift regimes.

Some numerical simulations are shown in Figs. 6, 7, 8 and 9. Compared with the baseline configuration without external influence, the simulations show deviations in both the mean and the variance, with the latter exhibiting either accelerated growth or marked suppression depending on the parameters. In particular, when $\beta > 0$, the variance grows significantly faster than in the baseline case, indicating a stronger spreading of the opinion distribution, whereas $\beta < 0$ leads to a stabilization and reduced dispersion. This behavior reflects the affine structure of the drift, which introduces a transport whose intensity increases with the opinion magnitude, thereby directly modifying the redistribution of individuals across the opinion space. Unlike the baseline case, where the growth of variance is solely due to interactions, here dispersion is actively driven or damped by the external field. In opinion-dynamics terms, this corresponds to an external influence that either amplifies opinion differences, promoting polarization when $\beta > 0$, or counteracts dispersion and favors convergence toward moderate states when $\beta < 0$.

Model 2. In this case, we consider an external action proportional to the mean value $m(t)$ of the system under investigation as follows

$$a(u) = \lambda m(t), \qquad \lambda \in \mathbb{R}. \tag{31}$$

The system of macroscopic variables evolution $\dot{m}(t)$ and $\dot{v}(t)$ reads as

$$\begin{cases} \dot{m}(t) = \eta\left(\frac{k-1}{3}\right)m(t) + \lambda m(t) \\ \dot{v}(t) = \frac{1}{9}\eta\left[\left(k^2-8\right)v(t) + (k-1)^2 m^2(t) + 9\sigma\right] \\ m(0) = m^0 \\ v(0) = v^0, \end{cases} \tag{32}$$

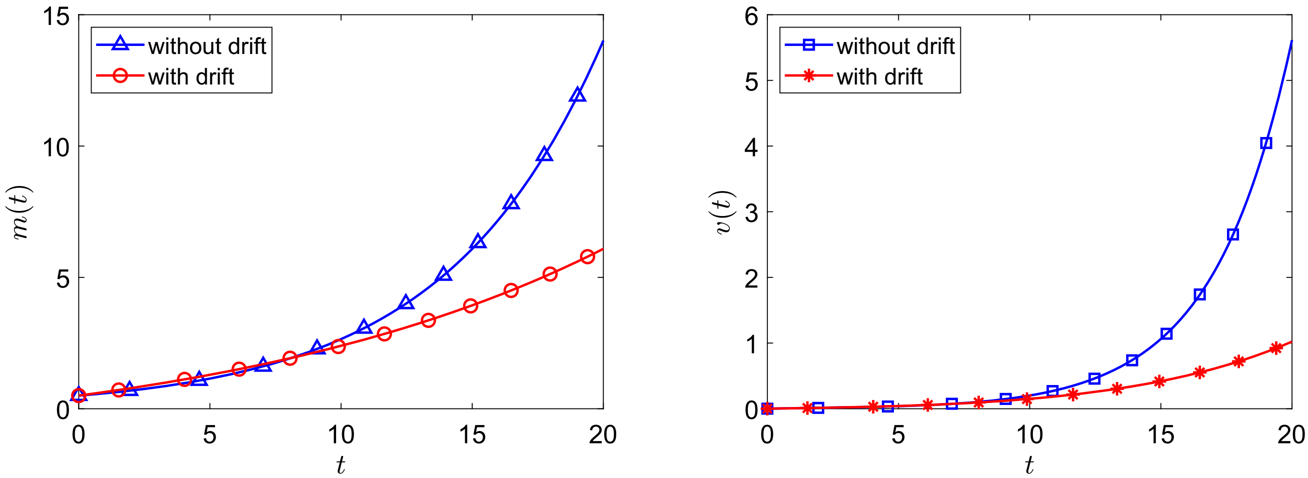

Fig. 7 Mean value $m(t)$ and variance $v(t)$ versus time t of model 1 with $\alpha = 0.1$ and $\beta = -0.1$. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$

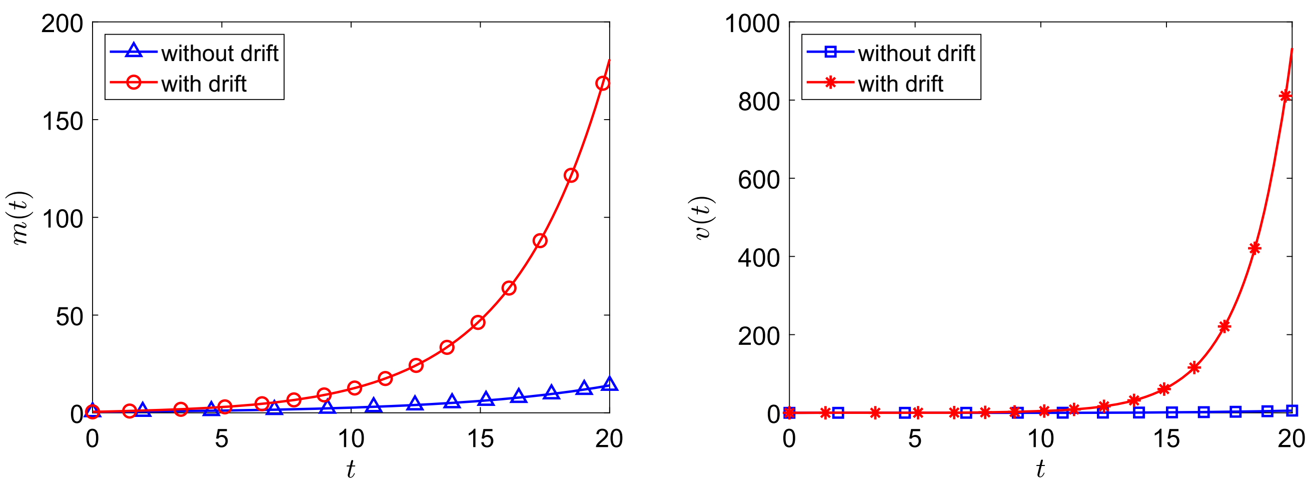

Fig. 8 Mean value $m(t)$ and variance $v(t)$ versus time t of model 1 with $\alpha = 0.1$ and $\beta = 0.1$. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$

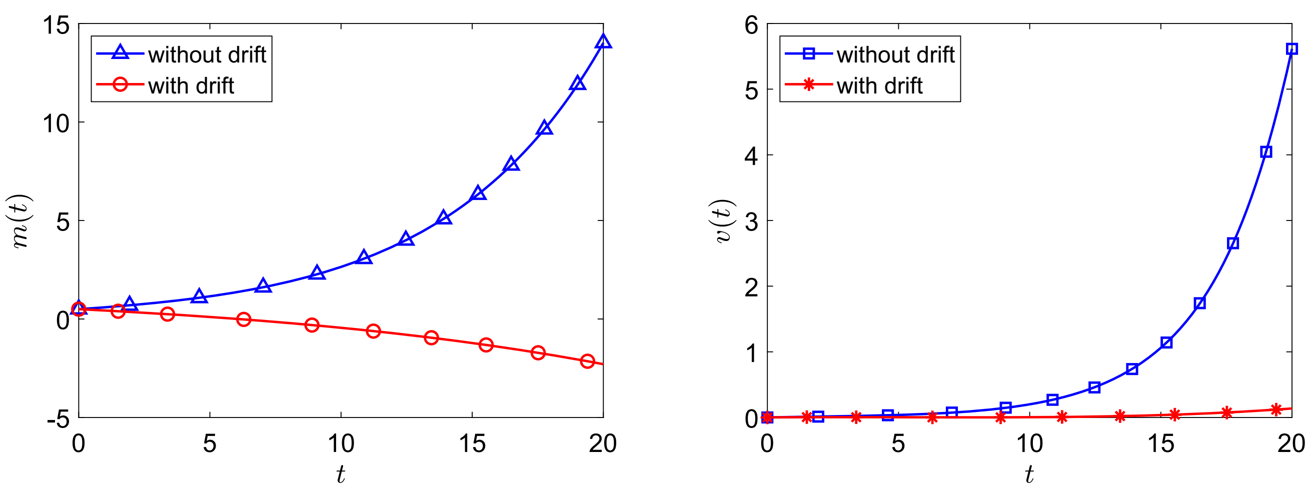

Fig. 9 Mean value $m(t)$ and variance $v(t)$ versus time t of model 1 with $\alpha = -0.1$ and $\beta = -0.1$. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$

We note that the parameter λ of the drift term does not explicitly enter the expression of $\dot{v}(t)$, influencing the variance evolution through the mean value $m(t)$. The numerical solution of system (32) is reported in Figs. 10 and 11 for different values of the parameter λ. Relative to the baseline evolution without external influence, both the mean and the variance exhibit significant differences, whose

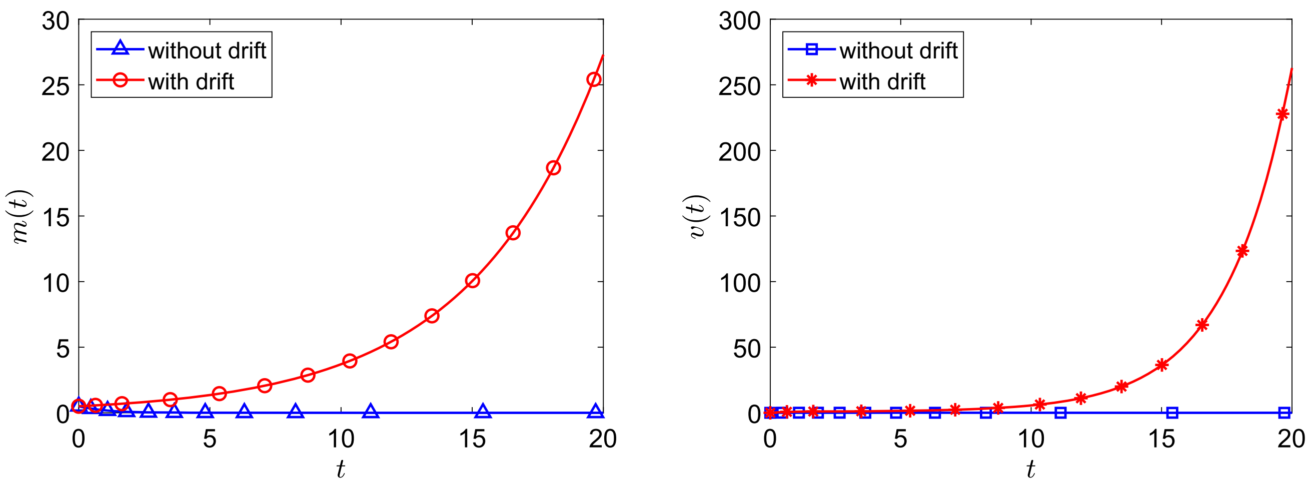

Fig. 10 Mean value $m(t)$ and variance $v(t)$ versus time t of model 2 with $\lambda = 1.2$. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$

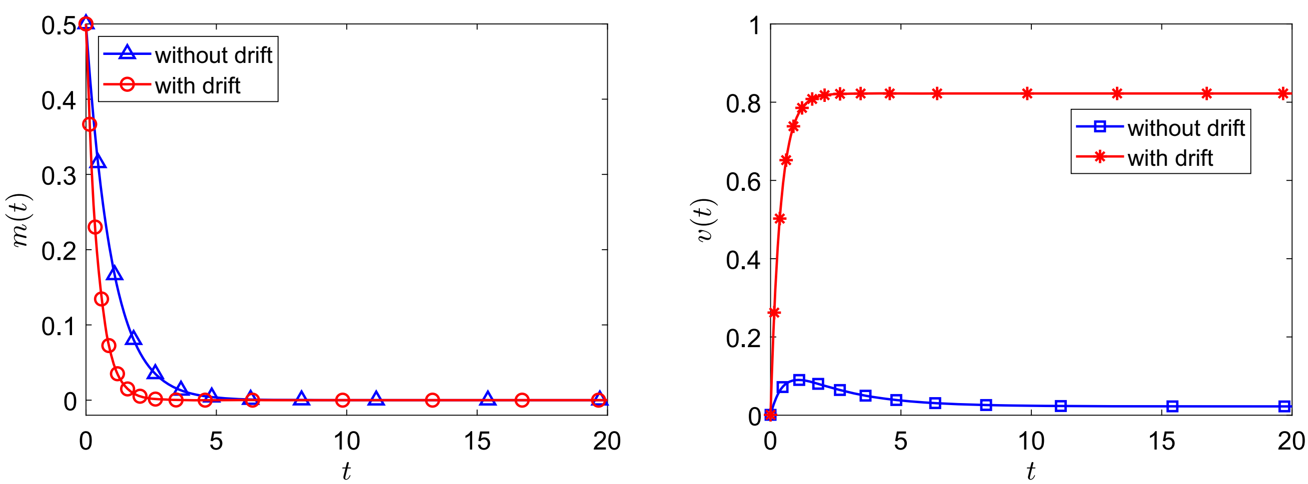

Fig. 11 Mean value $m(t)$ and variance $v(t)$ versus time t of model 2 with $\lambda = -1.2$. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$

behavior strongly depends on the sign of λ. In particular, for $\lambda > 0$, the mean and variance grow significantly faster than in the baseline case, indicating an enhanced spreading of the opinion distribution, whereas for $\lambda < 0$ the growth of both quantities is reduced and the system remains closer to moderate configurations. This behavior reflects the structure of the drift, which depends on the collective state through $m(t)$ and therefore modifies the global transport of the distribution. Although the drift does not depend explicitly on the microscopic variable u, its effect on the mean feeds back into the variance through the intrinsic coupling between macroscopic quantities, resulting in either amplification or suppression of dispersion compared with the baseline configuration. In the context of opinion-dynamics, this external influence reinforces the prevailing collective trend when $\lambda > 0$, promoting polarization and increasing heterogeneity, while for $\lambda < 0$ it counteracts the spreading of opinions and favors more moderate collective states.

Model 3. We consider the following specification of the drift term

$$a(u) = m(t) + \lambda, \qquad \lambda \in \mathbb{R}. \tag{33}$$

The system of macroscopic quantities in this case reads as

$$\begin{cases} \dot{m}(t) = \eta\left(\frac{k-1}{3}\right)m(t) + m(t) + \lambda \\ \dot{v}(t) = \frac{1}{9}\eta\left[\left(k^2 - 8\right)v(t) + (k-1)^2 m^2(t) + 9\sigma\right] \\ m(0) = m^0 \\ v(0) = v^0, \end{cases} \tag{34}$$

Similarly to model 2, parameter λ does not explicitly appear in the equation for $\dot{v}(t)$ and influences the system via the mean value $m(t)$. Some numerical simulations are shown in Figs. 12, 13, 14, 15, 16 and 17. With respect to the baseline scenario, the simulations

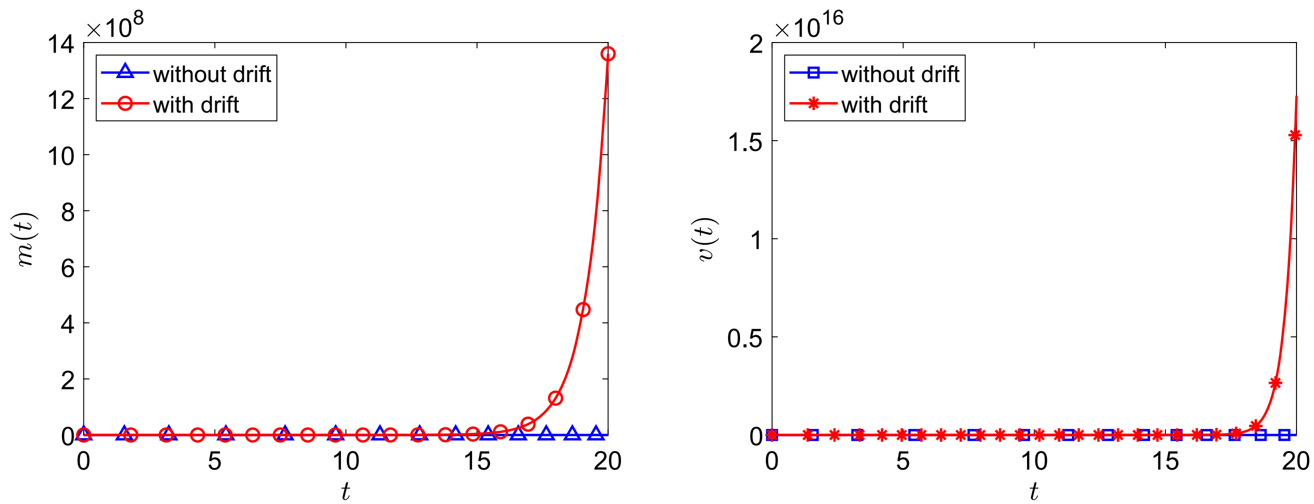

Fig. 12 Mean value $m(t)$ and variance $v(t)$ versus time t of model 3 with $\lambda = 0$. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$

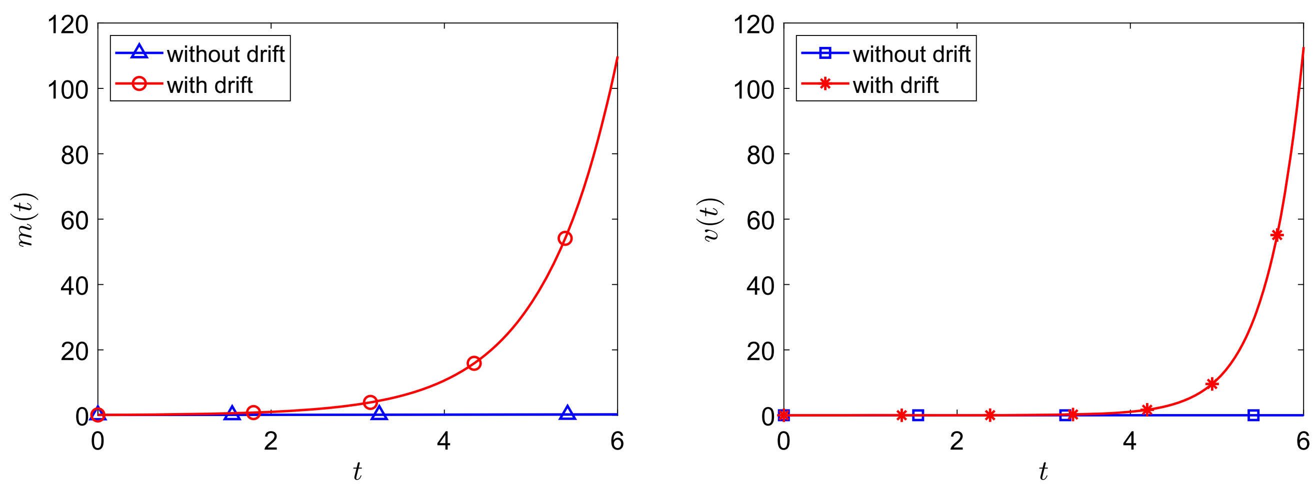

Fig. 13 Behavior of mean value $m(t)$ and variance $v(t)$ versus time t of model 3 at short times, with $\lambda = 0$. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$

show an immediate and dramatic departure of both mean and variance, with a clear blow-up within the simulated time horizon for all tested values of λ. The key feature is that $v(t)$ no longer follows the comparatively mild baseline growth; instead, it increases by many orders of magnitude together with $m(t)$, indicating a rapid spreading of the distribution. This is consistent with the analytical structure of the drift, which is uniform in u and contains a constant bias λ: the population undergoes a persistent global transport in opinion space that does not weaken near moderate states, so the system is continuously pushed away from the baseline regime. When applied to opinion-dynamics problems, this corresponds to an external influence that acts on the whole population in a sustained way, producing a rapid shift of the collective orientation and, through the coupling between moments, a strong amplification of dispersion, hence a fast transition toward extreme and highly heterogeneous configurations.

Model 4. With the following form of the drift term

$$a(u) = \lambda m(t) u, \qquad \lambda \in \mathbb{R}, \tag{35}$$

the system (17) is written as

$$\begin{cases} \dot{m}(t) = \eta\left(\frac{k-1}{3}\right)m(t) + \lambda m^2(t) \\ \dot{v}(t) = \frac{1}{9}\eta\left[\left(k^2 - 8\right)v(t) + (k-1)^2 m^2(t) + 9\sigma\right] + 2\lambda m(t)v(t) \\ m(0) = m^0 \\ v(0) = v^0. \end{cases} \tag{36}$$

The corresponding numerical solutions are shown in Figs. 18, 19 and 20. At variance with the reference model, the simulations display a sort of threshold behavior: for about $\lambda = 0.0612$ the trajectories of both $m(t)$ and especially $v(t)$ depart sharply from the

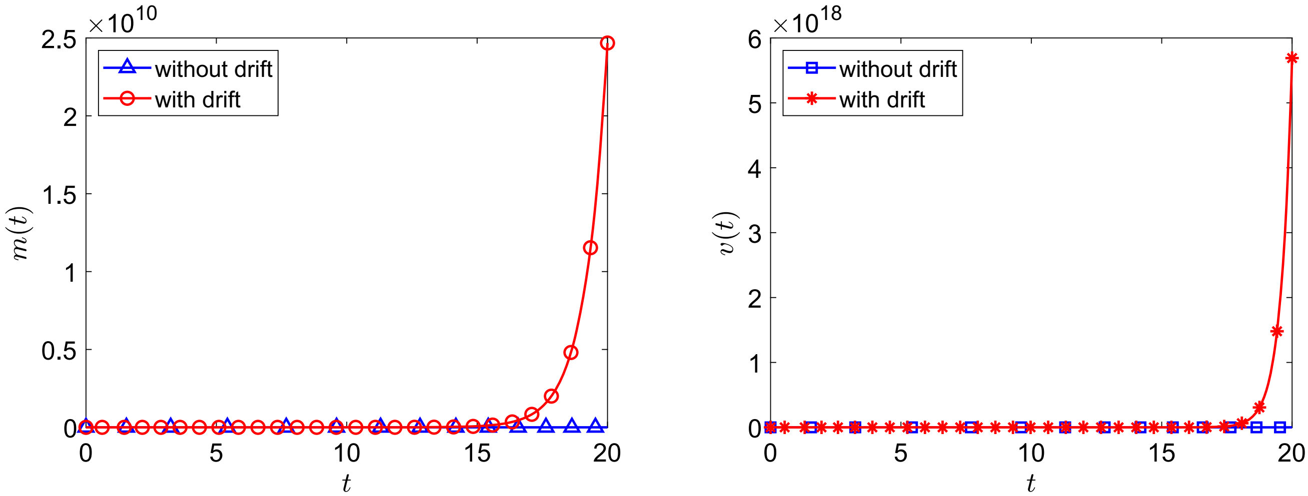

Fig. 14 Mean value $m(t)$ and variance $v(t)$ versus time t of model 3 with $\lambda = 2.\ k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$

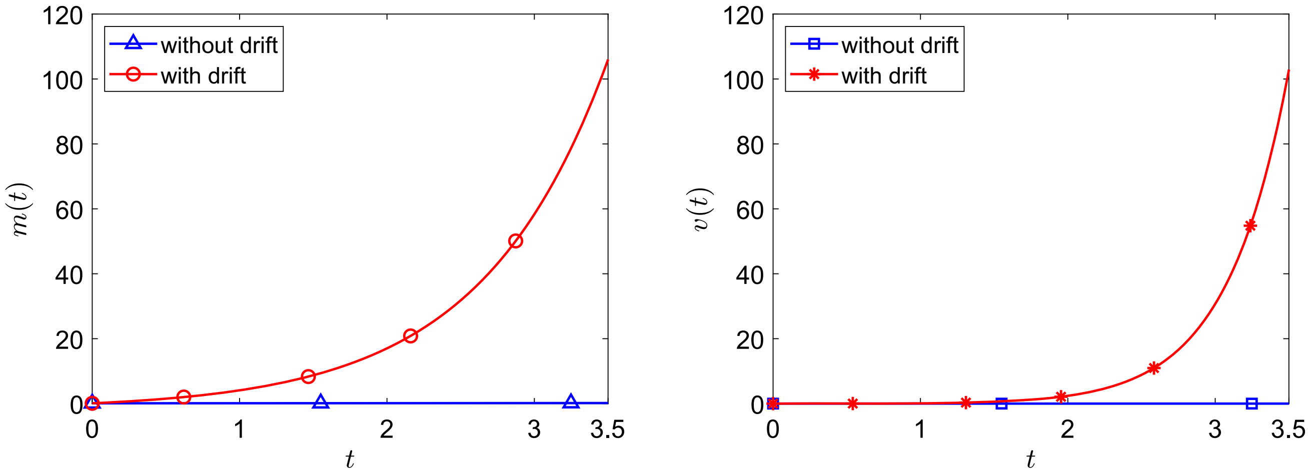

Fig. 15 Behavior of mean value $m(t)$ and variance $v(t)$ versus time t of model 3 at short times, with $\lambda = 2.\ k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$

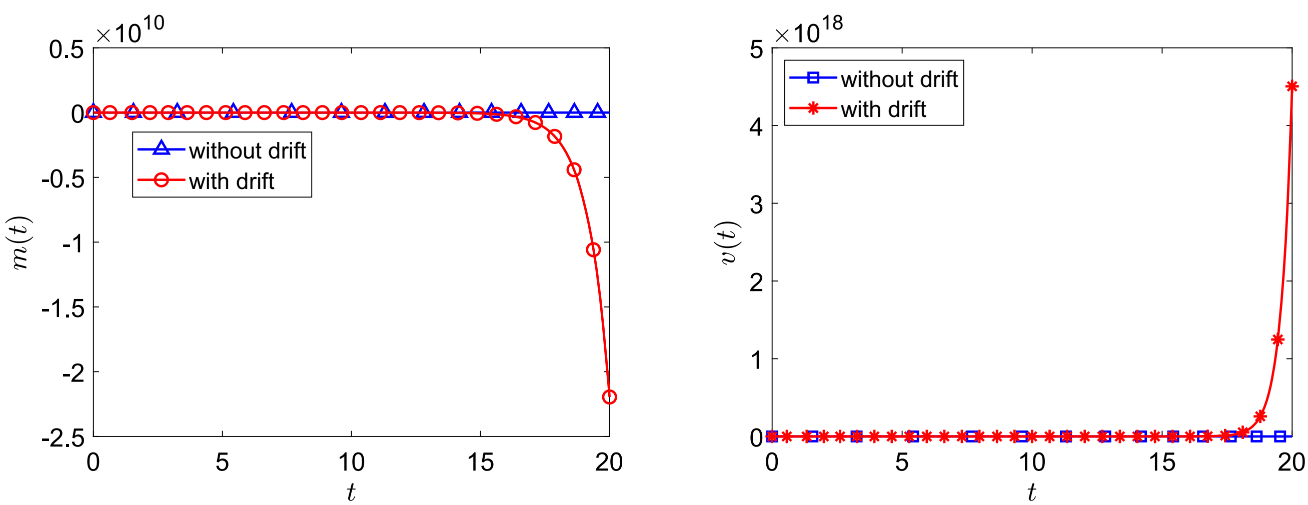

Fig. 16 Mean value $m(t)$ and variance $v(t)$ versus time t of model 3 with $\lambda = -2.\ k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$

without-force case and exhibit a rapid growth, while for $\lambda = -2$ the evolution remains bounded and the variance stays close to small values. The crucial deviation concerns the variance: once $m(t)$ is non-negligible, the multiplicative structure in u makes the

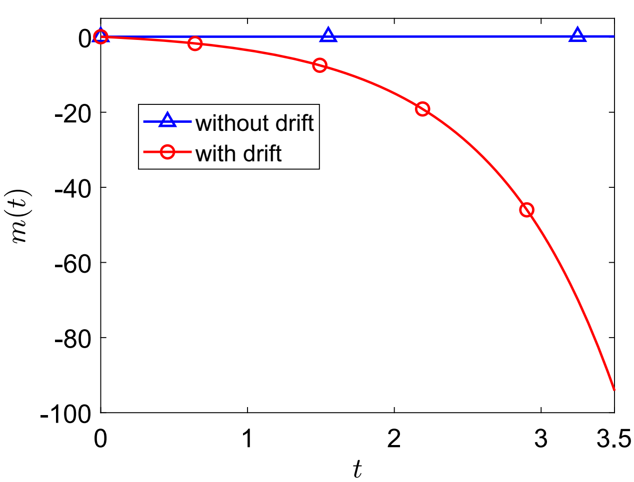

Fig. 17 Behavior of mean value $m(t)$ and variance $v(t)$ versus time t of model 3 at short times, with $\lambda = -2$. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$

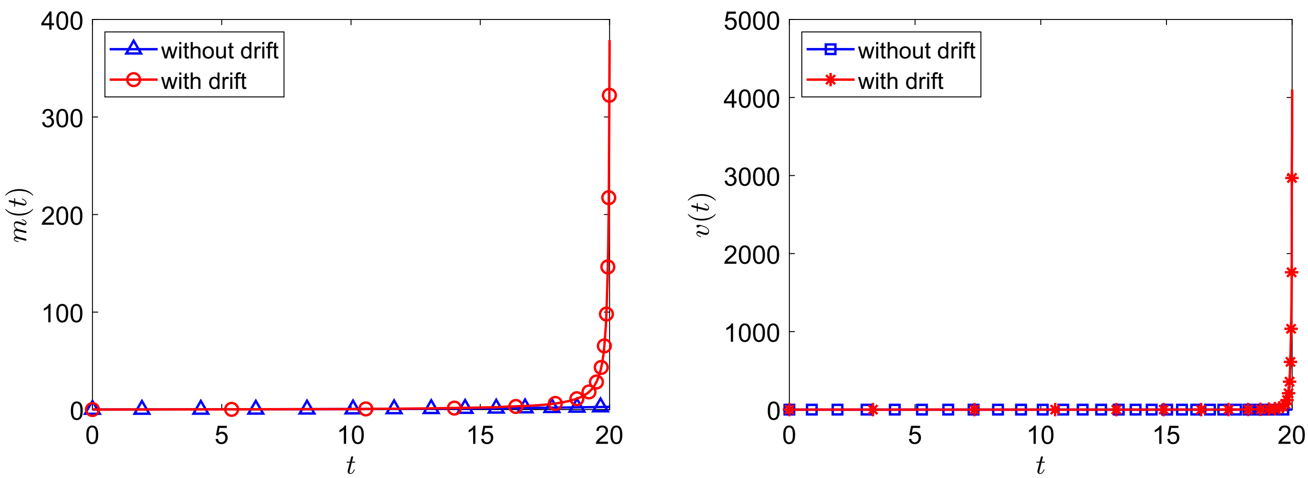

Fig. 18 Mean value $m(t)$ and variance $v(t)$ versus time t of model 4 with $\lambda = 0.0612$. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$

drift stronger for agents with larger opinion magnitude, thereby actively enhancing the spreading of the distribution and producing a much larger $v(t)$ than in the baseline configuration. This explains why the instability manifests first and foremost as an explosion of dispersion. From the point of view of opinion-dynamics, this drift models an external influence that targets extreme opinions more strongly and is reinforced by the current collective orientation; consequently, when the population starts drifting, the external action selectively pushes the tails, leading to accelerated polarization and fragmentation compared with the baseline regime.

Model 5. Let us now assume for the drift term the following form

$$a(u) = (m(t) + \lambda)u, \qquad \lambda \in \mathbb{R}. \tag{37}$$

Then, at the macroscopic level, we have the system

$$\begin{cases} \dot{m}(t) = \eta\left(\frac{k-1}{3}\right)m(t) + \lambda m(t) + m^2(t) \\ \dot{v}(t) = \frac{1}{9}\eta\left[\left(k^2 - 8\right)v(t) + (k-1)^2 m^2(t) + 9\sigma\right] + 2(\lambda + m(t))v(t) \\ m(0) = m^0 \\ v(0) = v^0. \end{cases} \tag{38}$$

The numerical solutions are reported in Figs. 21 and 22 and they show a strong reduction of both the mean and, crucially, the variance in the presence of the drift. For $\lambda = -2$ the trajectories with external influence remain close to the moderate regime, with $m(t)$ staying near zero and $v(t)$ remaining very small, while the baseline curves exhibit a much larger growth of both quantities; for $\lambda = -0.25$ the same stabilizing trend persists, although the damping is weaker. This behavior is consistent with the analytical form chosen for $a(u)$: for negative λ (and in the observed regime where $m(t)$ does not compensate it), the prefactor $(m(t) + \lambda)$ remains negative, so the

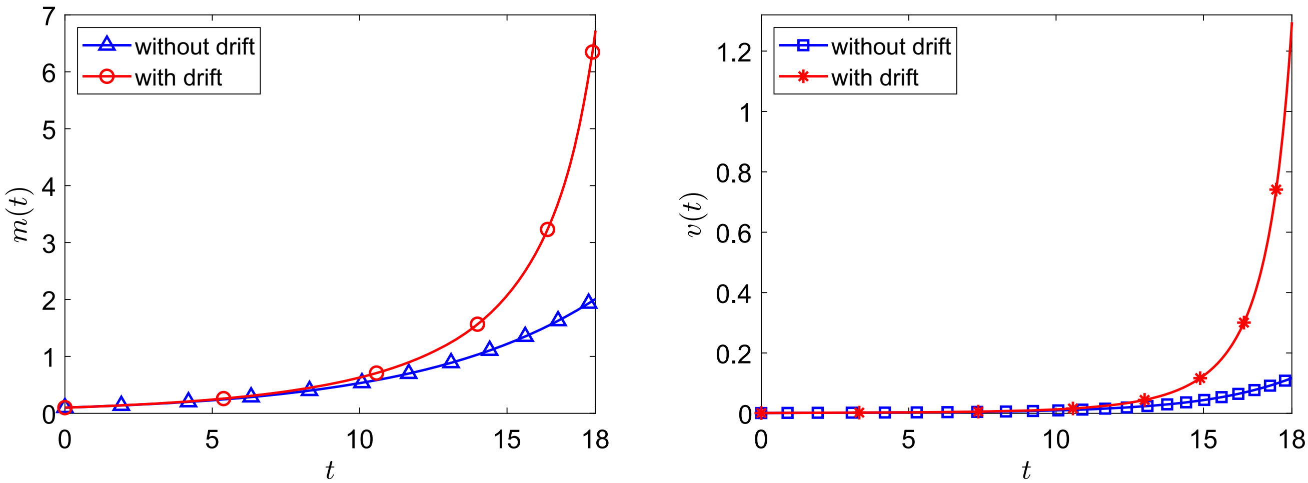

Fig. 19 Behavior of mean value $m(t)$ and variance $v(t)$ versus time t of model 4 at short times, with $\lambda = 0.0612$. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$

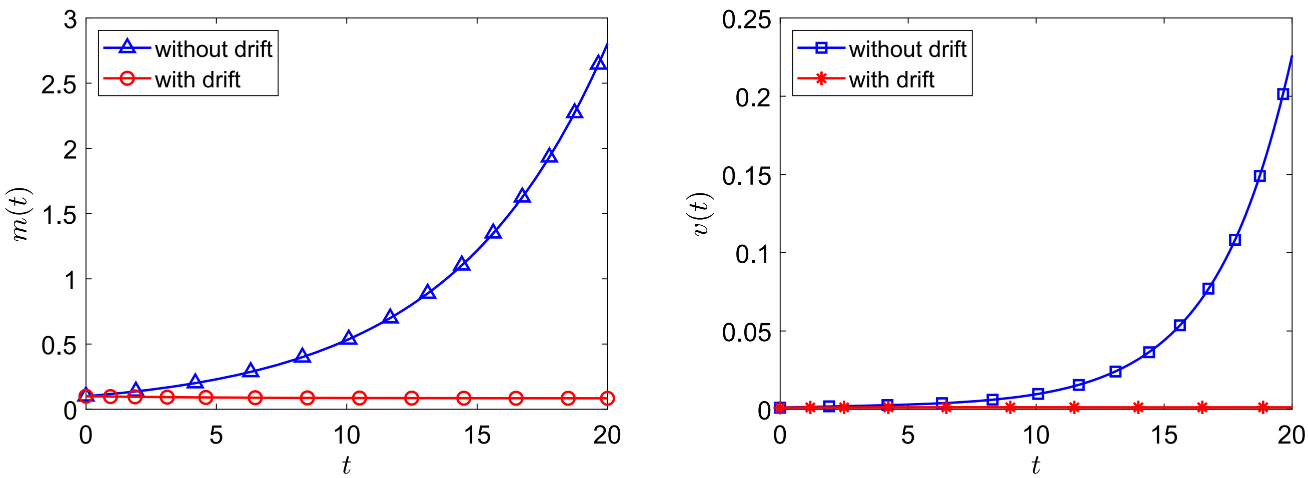

Fig. 20 Mean value $m(t)$ and variance $v(t)$ versus time t of model 4 with $\lambda = -2$. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$

drift acts as a contracting linear field in u that counteracts spreading and suppresses the growth of variance relative to the baseline. In opinion-dynamics terms, the external influence behaves as a restoring mechanism toward moderation, reducing fragmentation by actively compressing the distribution and preventing the amplification of extreme opinions that is otherwise produced by binary interactions alone.

Model 6. In this model, the drift operator is defined as

$$a(u) = u - \lambda m(t), \qquad \lambda \in \mathbb{R}, \tag{39}$$

and leads to the following system for macroscopic variables $m(t)$ and $v(t)$

$$\begin{cases} \dot{m}(t) = \eta\left(\frac{k-1}{3}\right)m(t) + (1-\lambda)m(t) \\ \dot{v}(t) = \frac{1}{9}\eta\left[\left(k^2 - 8\right)v(t) + (k-1)^2 m^2(t) + 9\sigma\right] + 2v(t) \\ m(0) = m^0 \\ v(0) = v^0. \end{cases} \tag{40}$$

Some numerical simulations are shown in Figs. 23, 24, 25, 26, and 27. Compared with the baseline evolution without external influence, the simulations show a significant deviation that is particularly evident in the variance, whose growth becomes much more rapid in the presence of the drift. This indicates that the opinion distribution spreads quickly across the state space. For high values of λ, the evolution remains bounded, but the variance still deviates from the baseline trajectory, showing that the redistribution of opinions is actively modified by the external influence. This behavior reflects the fact that the drift directly depends on the opinion variable, introducing a systematic transport that enhances deviations from moderate states when the stabilizing effect

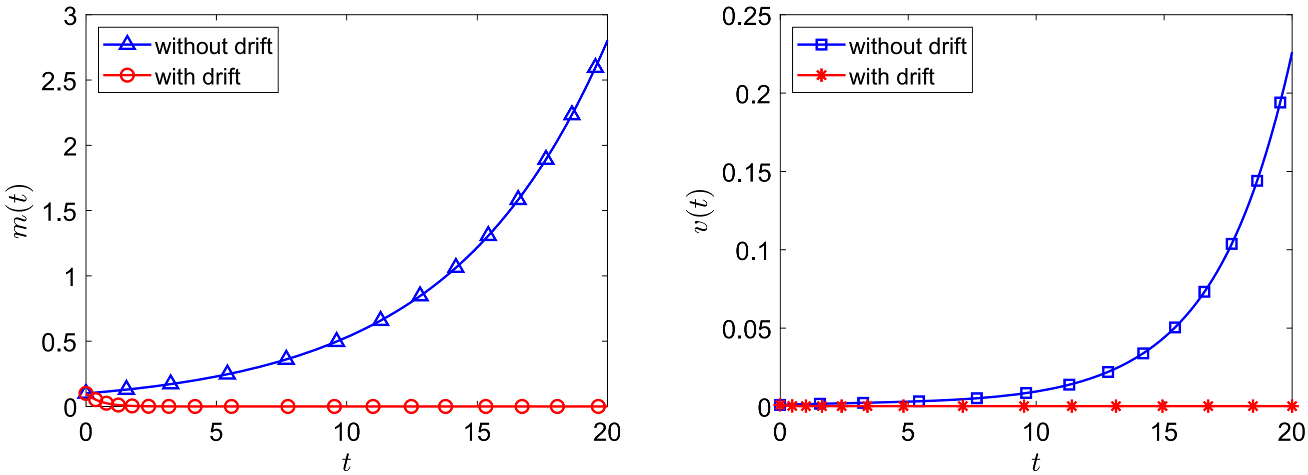

Fig. 21 Mean value $m(t)$ and variance $v(t)$ versus time t of model 5 with $\lambda = -2$. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$

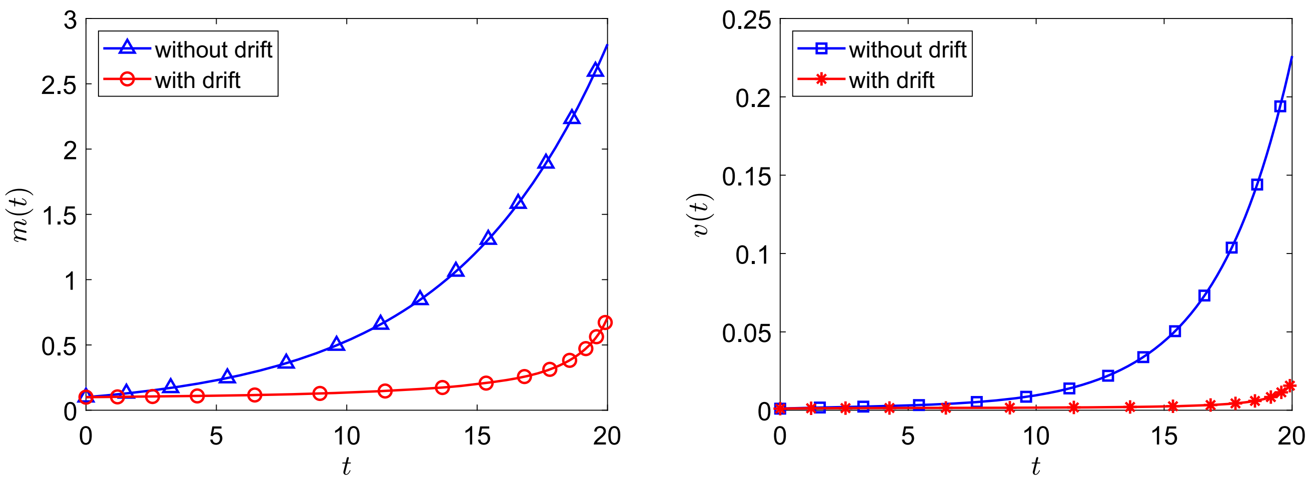

Fig. 22 Mean value $m(t)$ and variance $v(t)$ versus time t of model 5 with $\lambda = -0.25$. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$

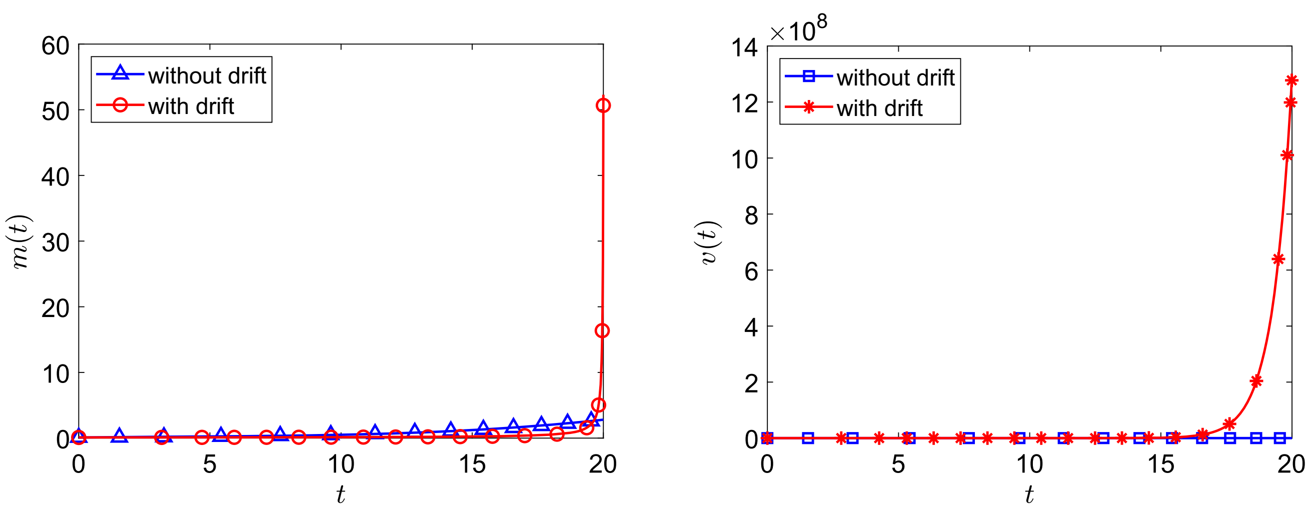

Fig. 23 Mean value $m(t)$ and variance $v(t)$ versus time t of model 6 with $\lambda = 1.2464$. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$

is insufficient. Regarding the opinion dynamics applications, this corresponds to an external influence that can either amplify or partially control opinion differences depending on its strength, but which, in destabilizing regimes, promotes rapid spreading and increased fragmentation compared with the interaction-driven baseline case.

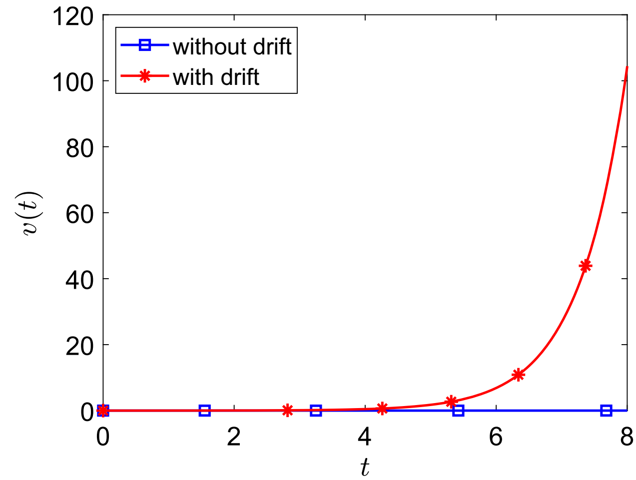

Fig. 24 Behavior of variance $v(t)$ versus time t of model 6 at short times, with $\lambda = 1.2464$. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$

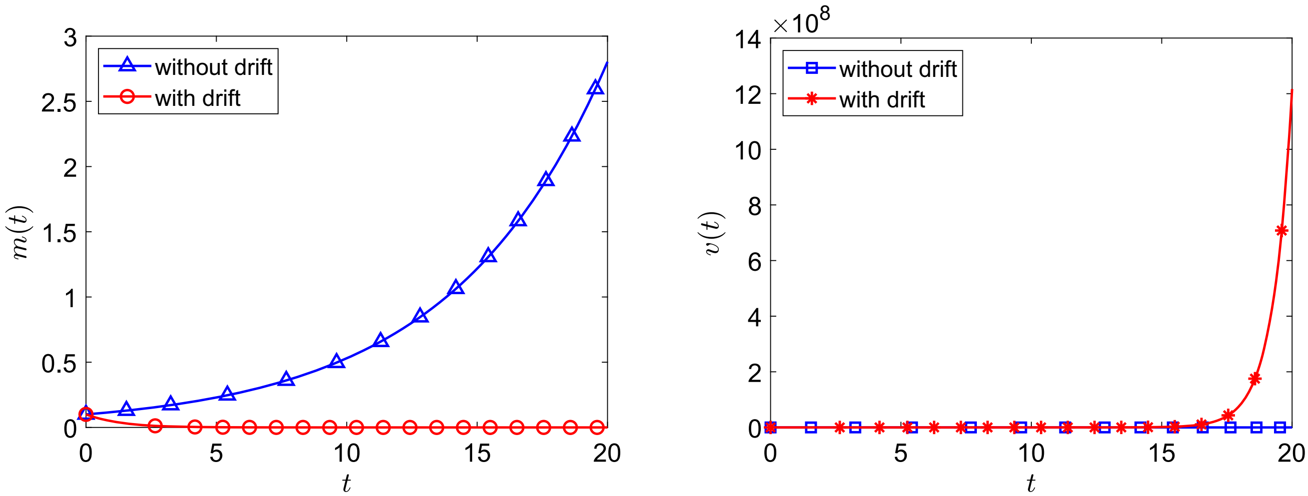

Fig. 25 Mean value $m(t)$ and variance $v(t)$ versus time t of model 6 with $\lambda = 2$. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$

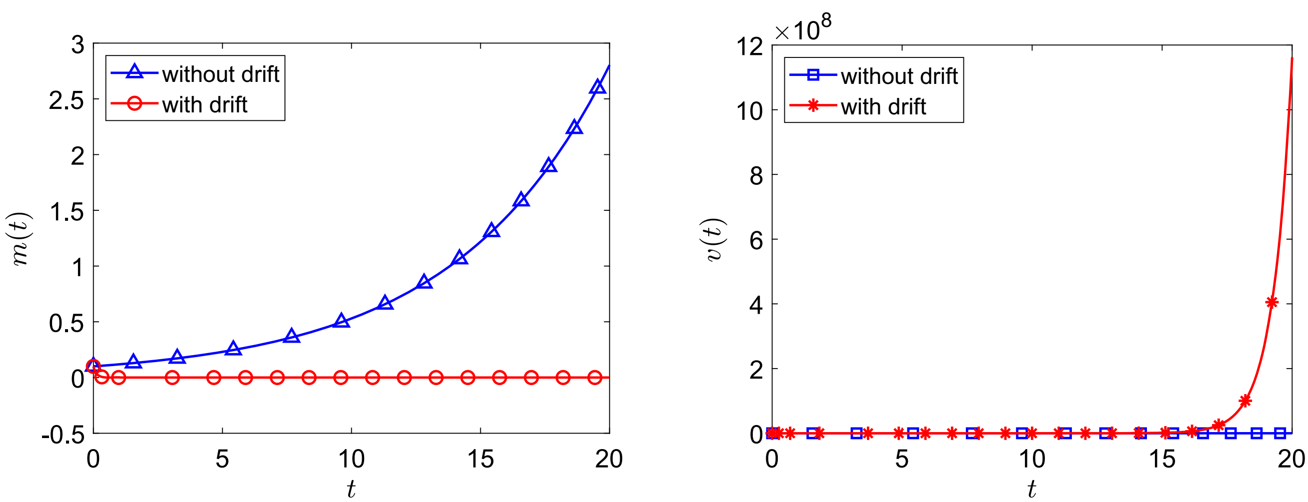

Fig. 26 Mean value $m(t)$ and variance $v(t)$ versus time t of model 6 with $\lambda = 10$. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$

Model 7. Finally, we consider the following specification of the drift term:

$$a(u) = \lambda - um(t), \qquad \lambda \in \mathbb{R}, \tag{41}$$

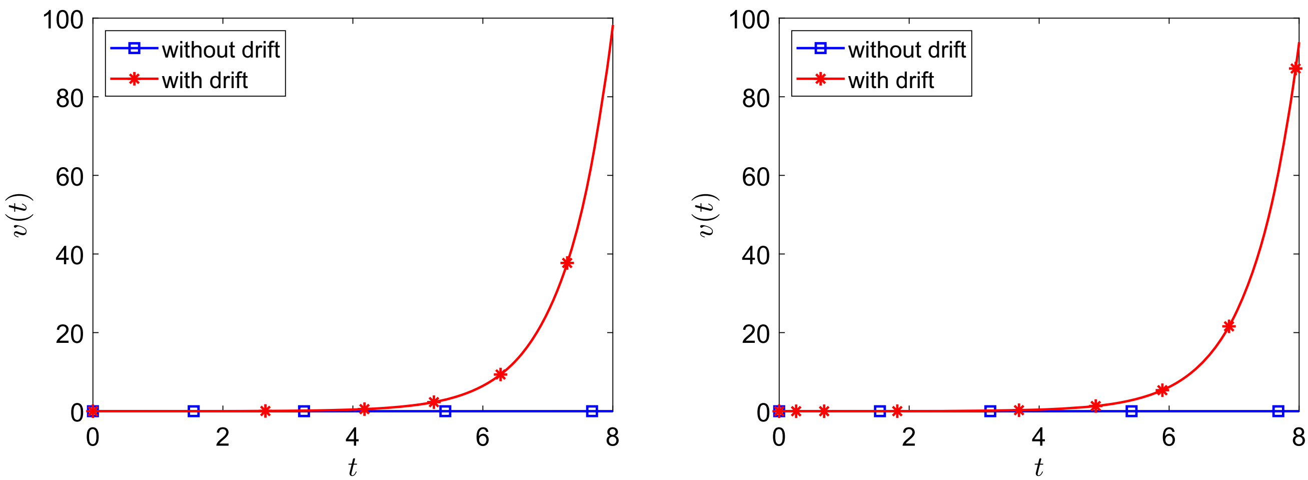

Fig. 27 Behavior of variance $v(t)$ versus time t of model 6 at short times, with $\lambda = 2$. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$ on the left panel, and with $\lambda = 10$. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$ on the right

and the associated system for $m(t)$ and $v(t)$ becomes

$$\begin{cases} \dot{m}(t) = \eta\left(\frac{k-1}{3}\right)m(t) + \left(\lambda - m^2(t)\right) \\ \dot{v}(t) = \frac{1}{9}\eta\left[\left(k^2 - 8\right)v(t) + (k-1)^2 m^2(t) + 9\sigma\right] - 2m(t)v(t) \\ m(0) = m^0 \\ v(0) = v^0. \end{cases} \tag{42}$$

The numerical results for $m(t)$ and $v(t)$ are shown in Figs. 28, 29, 30 and 31. The presence of external influences produces markedly different behaviors of the macroscopic variables depending on the value of λ, with the most significant deviations appearing in variance. For some negative values of λ, the variance grows much faster than in the baseline case and diverges for negative values of λ higher than about $\lambda = -4.5$, indicating a rapid spreading of the opinion distribution and a strong amplification of dispersion. In contrast, for positive values of λ, the variance remains clearly bounded and significantly smaller than in the baseline scenario, showing that the external influence actively limits the spreading of opinions and keeps the population closer to moderate configurations. This difference reflects the state-dependent nature of the drift, whose action may either enhance or suppress the redistribution of opinions depending on the parameter regime. In the opinion-dynamics perspective, this means that the external action can either promote fragmentation and polarization when acting in a destabilizing direction, or conversely stabilize the collective state and reduce heterogeneity, effectively preventing the emergence of extreme and highly dispersed opinion configurations compared with the only interaction-driven baseline evolution.

After the analysis of the different scenarios presented above, it is important to emphasize the role of the numerical simulations and the interpretation of regimes displaying fast growth. For some of the macroscopic systems considered above, the closed system for the first two moments has a sufficiently simple structure to be treated analytically. However, the resulting expressions may become rather lengthy and do not necessarily provide a clearer comparison among the different drift mechanisms. For this reason, we keep the numerical simulations as a unified and readable tool to compare all the scenarios considered in this section, including linear, nonlinear, and state-dependent drift terms. This choice allows us to highlight more directly the different stabilizing or destabilizing effects induced by the drift fields on the macroscopic evolution of the system. To improve the readability of the regimes characterized by very rapid growth of the macroscopic quantities, we have added additional plots on shorter time intervals. These plots show the initial stage of the dynamics before the solution undergoes strong amplification, and clarify that the high values observed at later times are produced by the drift-induced growth mechanism. In this sense, such figures should be interpreted as indicators of strongly destabilizing regimes, rather than as inconsistencies of the numerical simulations.

We conclude this section by summarizing the main qualitative indications provided by the numerical simulations and by the structure of the corresponding macroscopic systems. Since the analysis is performed on closed equations for $m(t)$ and $v(t)$, the long-time behavior is mainly governed by the effective feedback induced by the drift term on the mean opinion and on the variance. In the baseline case, the evolution is entirely driven by binary interactions, and the parameters k and σ determine whether the mean opinion remains close to moderate values or moves toward more extreme configurations, while the variance measures the corresponding dispersion of the population.

When drift terms are introduced, three main regimes can be identified. First, some drift configurations act as destabilizing mechanisms: they reinforce the prevailing opinion trend or directly amplify the variance, leading to a rapid growth of $v(t)$ and, in some cases, to blow-up within the simulated time horizon. From the modeling viewpoint, this behavior is particularly relevant

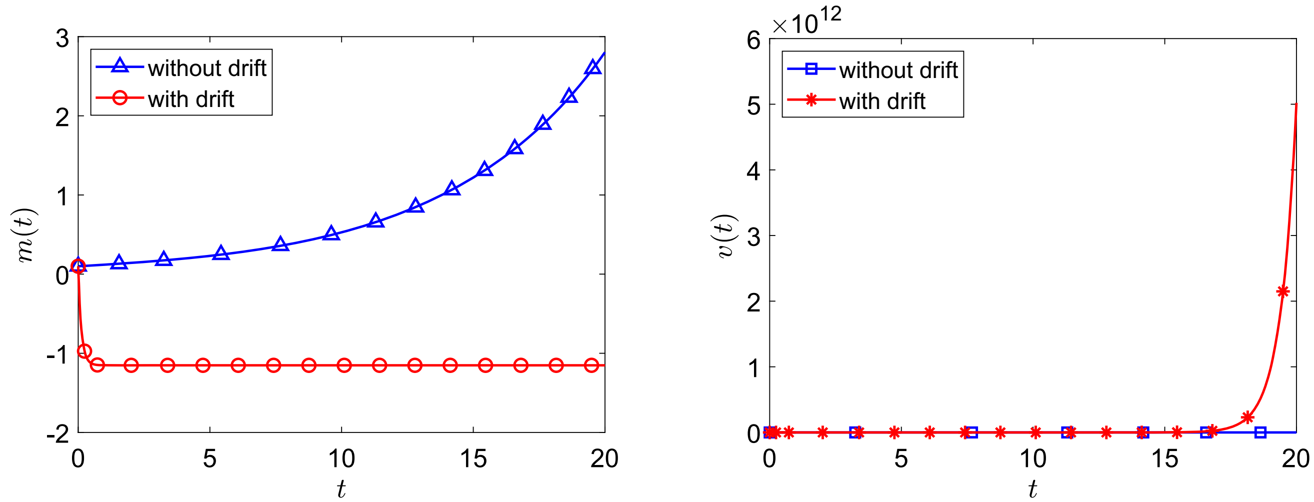

Fig. 28 Mean value $m(t)$ and variance $v(t)$ versus time t of model 7 with $\lambda = -10$. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$

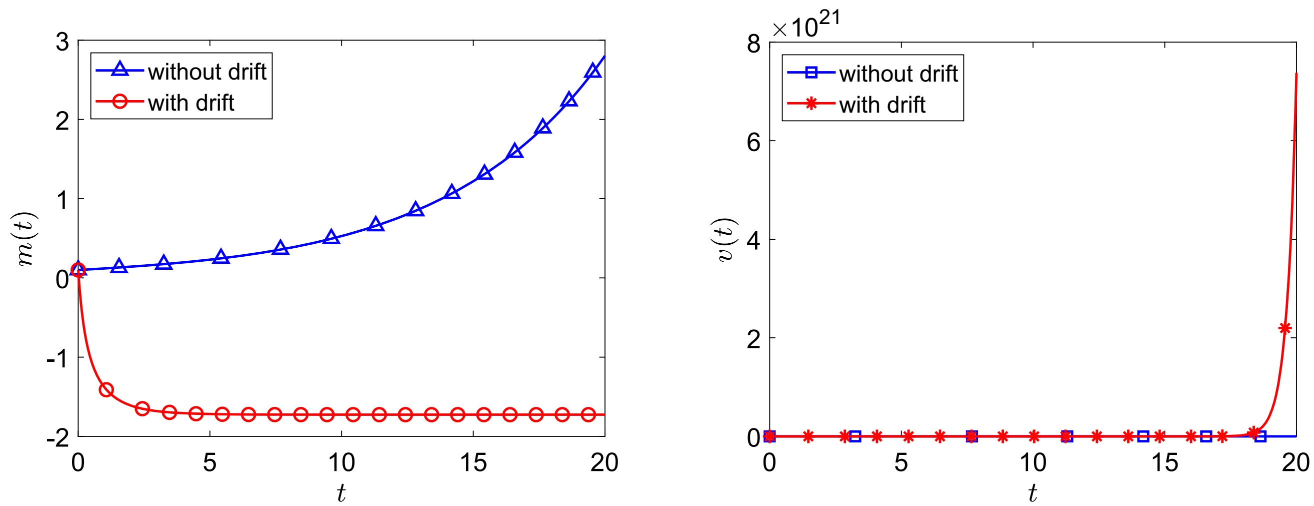

Fig. 29 Mean value $m(t)$ and variance $v(t)$ versus time t of model 7 with $\lambda = -4.5$. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$

because the growth of the variance is the macroscopic signal of increasing opinion heterogeneity. Therefore, the blow-up of $v(t)$, when observed, should not be interpreted only as a loss of boundedness of the reduced system, but also as an indication of a strongly destabilizing regime in which the population may undergo a rapid transition toward fragmentation and polarization. These regimes can be interpreted as external actions promoting polarization and fragmentation. Second, other drift terms have a contracting effect on the opinion variable and reduce the growth of the variance. In this case, the dynamics remains closer to moderate configurations, suggesting a stabilizing role of the external action. Finally, in state-dependent cases, the effect of the drift may change according to the sign and size of the parameters and to the current value of the mean opinion, producing threshold-like transitions between bounded and strongly amplified dynamics.

Therefore, the numerical simulations do not only illustrate different examples, but also show that the proposed drift formulation is able to distinguish between external mechanisms that enhance dispersion and external mechanisms that mitigate it. A rigorous characterization of equilibria, stability regions, and possible asymptotic regimes for the different moment systems remains an interesting analytical problem and will be addressed in future developments.

4 Final remarks and research perspectives

In this paper, we have developed a multiscale approach based on a kinetic framework for modeling opinion dynamics phenomena. The main novelty lies in the introduction of a drift operator, i.e., (8), which allows one to mimic external actions and/or interventions in opinion dynamics processes. The macroscopic system for the mean value and the variance (17) has been derived under general assumptions. In Sect. 3, the model has been specialized to the specific aims of this paper. Moreover, several scenarios have been

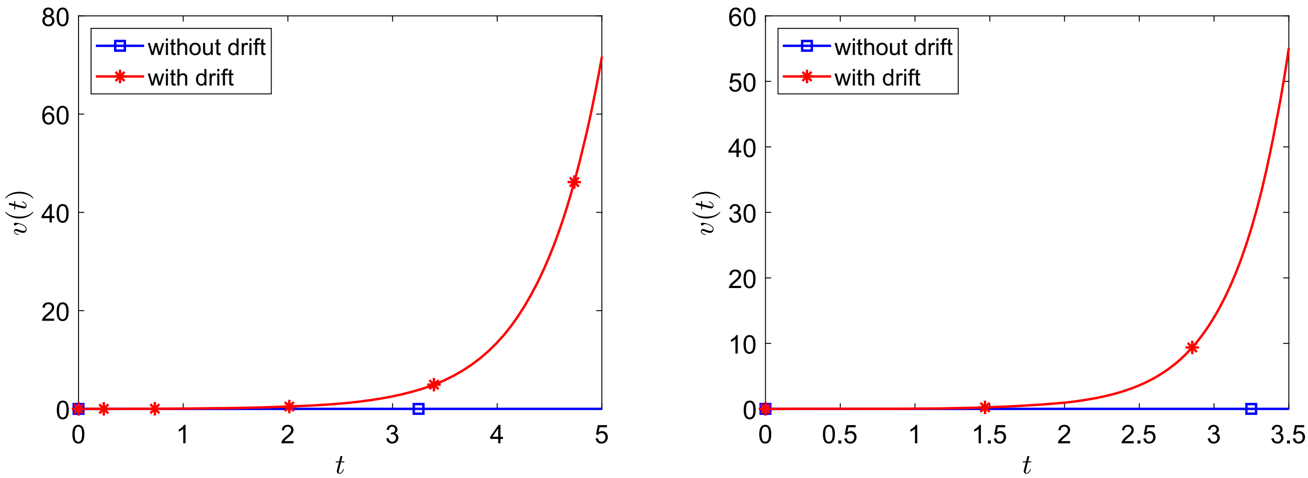

Fig. 30 Behavior of variance $v(t)$ versus time t of model 7 at short times, with $\lambda = -10$. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$ on the left panel, and with $\lambda = -4.5$. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$ on the right

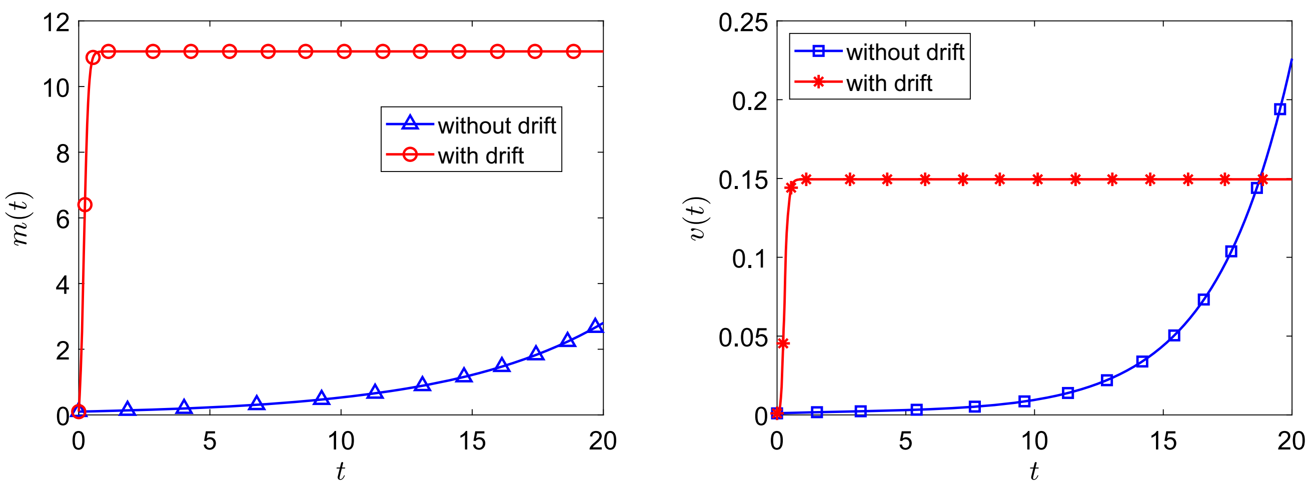

Fig. 31 Mean value $m(t)$ and variance $v(t)$ versus time t of model 7 with $\lambda = 10$. $k = 1.5$, $\sigma = 10^{-3}$, $\eta = 1$, $m^0 = 0.1$, $v^0 = 10^{-3}$

numerically investigated in Sect. 3.1, depending on the specific choice of the drift term. For each scenario, we have considered the corresponding baseline for comparison, intended as the reference configuration without external action, namely in the absence of the drift term.

The numerical simulations have revealed the emergence of qualitatively different results depending on the specific choice of the drift operator. This highlights the crucial role played by external actions in the determination of the collective dynamics of the system. In particular, certain drift configurations may reproduce the effect of negative external influences, such as misinformation, fake news, or manipulative mechanisms, which tend to drive the population toward more extreme opinion states. From a macroscopic viewpoint, this corresponds to a deviation of the mean from moderate values and, most importantly, to a significant increase in the variance, reflecting the onset of polarization and the progressive fragmentation of the population into more heterogeneous opinion groups. Conversely, alternative drift specifications may be interpreted as positive external actions, such as reliable information, educational interventions, or stabilizing social mechanisms. In these cases, the numerical results show a mitigation of both the mean and the variance, indicating a tendency of the system toward more moderate and less dispersed opinion configurations. This corresponds to a reduction of extremism and a stabilization of the collective state around more homogeneous and balanced distributions. A distinctive strength of the present work, in accordance with kinetic theory models, is precisely that the variance is treated as a dynamical quantity on the same footing as the mean, allowing us to detect dispersion, fragmentation, and polarization effects that would remain hidden in a purely mean-based macroscopic description. Overall, these findings confirm that the proposed modeling framework is able to capture, through an appropriate choice of the drift term, a wide range of realistic external influences and their macroscopic effects, providing a flexible and effective tool for describing both destabilizing and stabilizing mechanisms in opinion dynamics. We believe that this approach is particularly relevant in the current context, where artificial intelligence increasingly contributes to shaping opinion dynamics.

We are aware that this paper represents a first step toward this modeling framework, at least from a quantitative viewpoint. First, a rigorous derivation of the drift term from suitable microscopic dynamics is a natural research line beyond the current paper. It is worth stressing that the analysis developed in Sect. 3.1 is mainly qualitative. Rigorous analytical results are still lacking, particularly concerning the asymptotic behavior of the mean and the variance in relation to the specific choice of the drift term. Therefore, a natural direction for future research consists in deriving such results. In particular, the study of equilibrium states at both the mesoscopic scale (9) and the macroscopic scale (17) represents a challenging open problem. Similarly, future work will focus on comparing the proposed model directly with real data, with the aim of calibrating both the interaction parameters and the drift terms on specific empirical scenarios. This step would allow one to move from the qualitative prototype mechanisms investigated in the present paper to a quantitative description of concrete opinion-dynamics phenomena, such as the influence of mass media, misinformation campaigns, or corrective information strategies.

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Data availability The datasets generated during the current study are available from the corresponding author upon request.

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