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Business and financial cycle across regimes: Does financial stress matter?

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Identifying the business and financial cycles and their regimes is crucial for understanding the economy's state and financial market dynamics. Using a bivariate four-regime Markov-switching framework, we assess the accuracy of a model that identifies the business cycle by combining financial and real variables and compare it to a model that only uses real variables. We find that a model that incorporates information from a financial stress index outperforms a model based solely on real variables, not only during a financial crisis, but also, and more interestingly, during economic downturns. The empirical evidence provides insights for policymakers and investors to improve economic policy formulation, investment strategies, strategic decision-making, and proactive risk management. The results are robust to various model specifications.

1. Introduction

Financial factors have been important drivers of economic fluctuations since at least the Great Depression. More recently, the 2008 global financial crisis reminded us of the financial sector's importance to the macroeconomy, a lesson many had forgotten in the decades since the previous global crisis (Borio, 2014; Danielsson et al., 2023).

The financial crisis of 2008–2009 demonstrated how financial market developments can spill over into the real economy, emphasizing the importance of including the financial sector in empirical modeling (Berger et al., 2022; Ma & Zhang, 2016). The financial cycle literature has clearly shown how the financial system can amplify business cycle fluctuations and when the impact can be significant (Balcilar et al., 2015; Burns & Mitchell, 1946; Diebold & Rudebusch, 1996; Filardo, 1994; Harding & Pagan, 2002; Hodrick & Prescott, 1997; Kontolemis, 2001; Mitchell, 1927; Yan & Huang, 2020).

Despite the topic's importance for economic analysis and policymaking, the majority of the empirical literature has relied solely on variables that summarize the profile of the real economy (employment, GDP, leading economic indicators, and consumer and industrial price indexes). In contrast, some empirical studies have explicitly included financial variables in determining the business cycle and the role of specific market conditions (Claessens et al., 2012; Yan & Huang, 2020). Furthermore, the lack of financial variables has been especially noticeable in studies that have focused on labeling regimes in multi-regime approaches (Hamilton, 1989), resulting in incorrect interpretations of the entire cycle, particularly during downturns or adverse financial conditions.

Recent economic and financial events demonstrate the limitations of empirical modeling. During the 1990 Gulf War, models based

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List of abbreviations:

CLI = Composite leading indicator
CPI Acceleration = Consumer price index acceleration
FS = Financial stress
KCFSI = Kansas City Fed Financial Stress Index
R-F Model = Real–financial model
R-R Model = Real–real model
RMSE = Root mean square error

solely on real variables only partially recognized the war's effects, indicating that the months following September 1990 were economic growth, despite the MSCI USA index falling nearly 6% by September 1990. Furthermore, the models predicted a recovery only in February 1991, even though the MSCI USA index had recovered six months earlier. Similarly, during the Lehman Brothers crisis, real-indicator models predicted stagflation by July 2009, while the MSCI USA index returned to positive territory as early as March 2009, almost five months before the recovery. More recently, the financial market correction at the end of 2018 resulted in significant negative returns in the MSCI USA index in the following months, a fact that real-variable models almost completely ignored. Finally, these models find it difficult to handle the current challenges confronting the entire economic and financial system as a result of the recent COVID-19 pandemic crisis and the conflicts in Ukraine and the Middle East.[1] In each of these cases, the lack of a financial component in the model resulted in a substantial underestimation of the evidence and significant divergence between predicted and actual trends across cycles and regimes.

In this case, including a financial stress index is not unnecessary because several factors suggest that financial components should be considered when determining the business cycle. Claessens et al. (2012) demonstrated that the financial cycle is important in determining real recessions and recoveries, whereas Borio and Zhu (2012) argued that it is crucial to reexamine the importance of the financial cycle for macroeconomic dynamics. Nolan and Thoenissen (2009), Mandelman (2010), and Liu et al. (2011) included shocks from the financial sector and suggested that these shocks have a significant impact as sources of business cycle volatility. According to Ma and Zhang (2016), the financial cycle is critical to the business cycle, with a financial shock emerging as the primary driver of macroeconomic fluctuations, particularly during periods of financial instability. Finally, Yan and Huang (2020) demonstrated that the financial cycle is inextricably linked to the business cycle, with the financial cycle driving real interest rates and serving as a major source of volatility in both financial and real dynamics.

This article aims to identify business cycle regimes by integrating the modeling strategy with a variable that explicitly includes a financial stress index. We chose a stress indicator for our analysis because we wanted to study the moments of disruption in the economy's financial profile: for this purpose, we needed an indicator that was specifically designed and capable of detecting these disruptive phases early on. We followed the steps below to capture the dynamics of financial markets and their impact on regimes. First, we conduct the main analysis and develop a four-regime benchmark model of the business cycle based on two real variables, the Composite Leading Indicator (CLI)[2] and Consumer Price Index (CPI) Acceleration.[3] This model is known as the "real–real model" (henceforth R-R model). Unlike Gupta et al. (2014), who identified regimes using a simple empirical counting of cases where the regimes are considered to be realized,[4] we used a Markov-switching (MS) model (Hamilton, 1989, 1990, 1994), which is a well-known paradigm widely used to study regimes across the business cycle (Guidolin & Timmermann, 2006; Ang & Timmermann, 2012; Balcilar et al., 2015; Boudt et al., 2017; Cotter & Salvador, 2022; Hammerschmid & Lohre, 2018; Hwang & Kim, 2021; Sarafrazi et al., 2015; Stafylas et al., 2023; Zhang et al., 2020). Second, to examine the content of the financial stress index data, we replaced the CPI Acceleration with the Kansas City Fed Financial Stress Index (KCFSI). The KCFSI is a stress indicator for the US financial system developed to detect early warning signs of a financial crisis. It summarizes information about specific events that may result in critical market outcomes affecting the real economy and potential economic and financial stress.

Like the CLI, the index is a comprehensive and all-encompassing indicator that goes beyond simply describing a single segment of the financial market. Like the CLI, this stress indicator includes several variables that are summarized using a Principal Component Analysis (PCA) to maximize explanatory power with respect to the total variation observed in the variables. Finally, the availability of data dating back to 1990 extends the KCFSI's validity period to 33 years, which increases its reliability and usability in empirical analyses. The KCFSI was chosen as one of several potential financial indicators after carefully considering the index's composition,

[1] To confirm this, the NBER index, which is based on real variables, does not show a recession as of May 2020.

[2] CLI is a forward-looking economic indicator that summarizes current real activity conditions and identifies potential turnarounds or regime changes.

[3] The CPI is an economic indicator that measures price growth in the economy. Its purpose is to identify periods of inflation or deflation. In our analysis, we focused on CPI Acceleration, which is the difference between rolling 3-month and 36-month variations in the price index.

[4] Gupta et al. (2014) prioritized an approach for evaluating the macro-framework based on two main dimensions: the business cycle trend and the inflation rate.

properties, and construction methods. Appendix A contains details about the index selection process.[5] This model, known as the "real-financial model" (hereafter referred to as the R-F model), is intended to redraw the business cycle dynamics by incorporating information about financial market stress conditions. Third, to determine the best configurations of the R-R and R-F models, we ran several estimations with various initial values and statistical metrics, ensuring that the results were consistent with the dynamics observed over the last few decades. Finally, to compare the R-R and R-F models, we evaluated the simulation results' performance and accuracy using the RMSE across different time windows covering both specific periods and the entire sample period. To emphasize the comparison and the difference in relative performance, we conducted these tests during financial crises and NBER recessions, when the R-R model is expected to be more accurate. The data set included monthly observations for all variables from February 1990 to February 2024, for a total of 409 data points per variable. The empirical section will provide more details.

As a preview of our findings, we found that a model with a financial component significantly improves accuracy in identifying cycles and regimes. Although the R-R model does not perform poorly in describing the dynamics of real economic activity, it frequently overlooks negative events that affect financial markets, a disadvantage offset only partially by its accuracy in identifying periods of economic growth. In contrast, the R-F model better explains real and financial dynamics. Indeed, we found that the R-F model's identified regimes are more closely aligned with the overall real and financial cycle. Furthermore, during periods of recession and difficult financial market conditions, the differences between the models become more pronounced, with the R-R model suffering the most. Interestingly, the results confirm that the R-F model performs better not only in periods of financial constraints, as expected given the nature of the indicators used, but also in periods of pure economic crisis, as listed in the NBER index.

This paper contributes to a growing body of literature on the interaction of the financial sector and the real economy in identifying cycles (Bernanke et al., 1996; Bernanke & Gertler, 1989; Kiyotaki & Moore, 1997; Zabavnik & Verbič, 2021). Despite the importance of existing studies, there are still significant gaps in incorporating financial variables into business cycle fluctuations models. Specifically, it remains difficult to analyze both business and financial cycles at the same time and effectively integrate financial variables. Scholars have pursued various lines of inquiry, attempting to address the relationship between real activity and financial market conditions (Guiso et al., 2004; Yan & Huang, 2020), the implementation of conventional and unconventional monetary policies (Bernanke & Blinder, 1992; Borio & Zhu, 2012), and the emergence/influence of financial crises (Brunnermeier, 2009; Christiano et al., 2014; Claessens et al., 2012). Our paper adds to the third stream of research, which focuses on the role of financial crises and their consequences for business cycle fluctuations, by directly addressing the impact of financial crises and financial stress on business cycle modeling. Indeed, the literature on financial stress has primarily focused on summarizing financial market dynamics using stress indicators, but it has not fully explored the effects of financial stress on the real economy or integrated these indicators into models for business cycle fluctuations and regime identification (Stona et al., 2018; Saliminezhad & Bahramian, 2021; Pang et al., 2021). The extant research frequently overlooks critical financial market events by failing to consider specific regimes within the financial cycle as well as the business cycle (Yan & Huang, 2020).

Furthermore, our research contributes to those studies by considering a multi-regime framework. By investigating the relationship between real and financial fluctuations, we contribute to studies that explore the impact of financial market performance in various regimes on global activity (Ghysels et al., 2014; BenSaïda, 2015). In this area, some studies have recognized the importance of considering multiple regimes within both business and financial cycles (Ang & Timmermann, 2012; Boudt et al., 2017; Hammerschmid & Lohre, 2018), but most have failed to incorporate this into their analyses. This limitation emphasizes the need for a multi-regime framework that accurately captures developments in both financial markets and the real economy. Other studies have examined only two regimes' real and financial aspects (Sarafrazi et al., 2015; Hammerschmid & Lohre, 2018). By broadening the analysis to include multiple regimes within both business and financial cycles, we allow for a more nuanced assessment of concurrent changes in both domains, improving the accuracy and depth of our analysis in comparison to previous studies. Furthermore, by using four regimes rather than two, we capture a broader range of events in real and financial contexts, resulting in a more robust framework for identifying business-financial interactions.

The remainder of the article is structured as follows. Section 2 provides a brief theoretical background. Section 3 describes the data and methods, and Section 4 presents the empirical analysis. Section 5 covers the key implications for stakeholders. Finally, Section 6 concludes.

2. Related literature

In this section, we review the literature on two key themes: (i) the relationship between real and financial activity, which received a lot of attention following the 2008–2009 financial crisis, and (ii) the increased use of the financial stress index as an immediate indicator of the overall health of financial markets. Additionally, we identify the major existing gaps.

2.1. The relationship between real activity and finance

Researchers' interest in the relationship between the real economy and financial markets has increased significantly since the early 1990s (Bernanke & Blinder, 1992; Frankel & Rose, 1996; Guiso et al., 2004; Kaminsky & Reinhart, 1999).

[5] The list includes the Federal Reserve Board of St. Louis Financial Stress Index (STLFSI), the Philadelphia Federal Reserve's Aboura-Diebold-Scotti Business Condition Index (ADS), the Chicago Federal Reserve's National Financial Conditions Index (NFCI), and the IMF Financial Instability Index (FSI).

Zabavnik and Verbič (2021) identified three major areas of potential research. The first examined the relationship between financial development and real growth. Guiso et al. (2004) and Yan and Huang (2020) emphasized the positive impact that a growing financial system can have on the real economy, whereas Alfaro et al. (2004) showed that economic growth is only possible when a certain level of financial market development is reached. Similarly, Arcand et al. (2015) and Ouyang and Li (2018) found that an overdeveloped financial market may benefit the real economy. The second line of inquiry has sought to examine the relationship between real activity and the financial crisis. Frankel and Rose (1996) and Kaminsky and Reinhart (1999) investigated the relationship between the banking and currency crises and their impact on the real economy. Brunnermeier (2009) and Gerali et al. (2010) focused on the Lehman Brothers crisis and its economic consequences, whereas Christiano et al. (2014) examined the impact of crises on GDP and other macroeconomic variables. Claessens et al. (2009, 2012), who studied the interactions between business and financial cycles in several countries, examined both the "up" and "down" phases, made an important contribution in this area. They showed that interactions between business and financial cycles are critical in determining recession and recovery phases. Indeed, recessions caused by financial disruptions are more severe and long-lasting, whereas recoveries caused by rapid credit and house price growth are usually stronger. Finally, the third strand of research has focused on the relationship between real activity and the effects of conventional and unconventional monetary policy. This line of studies includes the early contributions of Bernanke and Blinder (1992), who examined how monetary policy affects the economy, and Bernanke and Gertler (1995), who focused on the credit transmission channel of monetary policy. Later, Borio and Zhu (2012) studied the relationship between monetary policy and risk perception, and Borio (2014) studied the importance of finance in the business cycle to adjust macroeconomic policy.

2.2. *Financial stress*

A growing body of research examines the role of financial stress (FS) in determining and defining the business cycle. Several institutions, including the European Central Bank, the Federal Reserve, the International Monetary Fund (IMF), the Organization for Economic Cooperation and Development (OECD), and the Bank for International Settlements, are developing metrics to measure financial stress in various countries and assess financial stability (Aboura & Van Roye, 2017).

For high levels of FS, Semmler and Chen (2014) found that shocks have large and persistent effects on the real economy,[6] whereas low levels of stress have no persistent effects or no effects at all. Dovern and Van Roye (2014) analyzed how movements in the FS index affect global economic activity and found that: (i) the comovement of economic activity and the FS index increases during financial turmoil; (ii) the risk of spillovers from financial stress increases with the openness of the economy; and (iii) the effects of a movement in the FS index can be transmitted to other economies.[7] Aboura and Van Roye (2017) developed an FS indicator to proxy financial stability in France, defining financial stress as a combination of uncertainty and risk perception.[8] They demonstrated that a high level of FS corresponds to a decrease in real activity, whereas low values of FS do not show strong variations in the economy. Following this line of inquiry, Stona et al. (2018) developed the Brazil Financial Stress Index to investigate the interactions between real activity, inflation, and monetary policy with macro financial variables.[9] Yao et al. (2020) focused on emerging economies and measured the dynamics of financial stress in the Chinese financial market. Meanwhile, Ilesanmi and Tewari (2020) constructed a financial stress index for South Africa. They studied the effects on real-world activities to prevent instabilities and warning periods, and they demonstrated the detrimental impact of FS on economic growth. In Luxembourg, Bahramian et al. (2022) examined the causal relationship between FS and economic conditions. They found a bidirectional causal relationship between the variables, but only in the medium-long term. Cipollini and Mikaliunaite (2021) used a similar method and investigated the causal relationship between daily FS in Lithuania and monthly industrial production growth, confirming a negative effect of FS on the real economy. Similarly, Salimi-nezhad and Bahramian (2021) studied the relationship between FS and real activity in the United Kingdom and Germany and found a negative causal relationship between FS and the real economy using the financial stress index (Composite Indicator of Systematic Stress-CISS) as a thermometer of financial sector activities. Finally, Pang et al. (2021) explored the impact of FS on oil market volatility and confirmed that an FS index allows for better oil price prediction.

2.3. *Gaps and contributions*

Despite the interest in studying the relationship between the business and financial cycles, there are some gaps in the current literature. In fact, most empirical studies have primarily focused on models that include only variables reflecting the real economy, such as GDP, employment, and price indexes, while ignoring the inclusion of financial variables (Claessens et al., 2012; Yan & Huang, 2020). Only a few studies have included the financial aspect of the business cycle evaluation. For example, Claessens et al. (2012)

[6] To study the empirical evidence of the impact of financial stress on macro dynamics, the authors specifically used the IMF Financial Instability Index (FSI).

[7] The authors chose a set of financial variables that represent stress in various segments of the financial market. Then, to obtain Financial Stress Indicators (FSI), they summarized well-known indicators in the literature that may represent financial stress in specific market segments.

[8] To find a comprehensive assessment of financial stress, the authors measured financial stress in the banking sector and capital markets before identifying financial stress in the foreign exchange market.

[9] The authors primarily used the Federal Reserve Board of St. Louis Financial Stress Index (STLFSI), the Kansas City Financial Stress Index (KCFSI), the Aboura-Diebold-Scotti Business Condition Index (ADS) of the FED of Philadelphia, and the National Financial Conditions Index (NFCI) of the FED of Chicago to develop an index of financial sector instability in Brazil, as well as other indexes from the literature.

found links between GDP fluctuations and financial cycles (credit, house prices, and equity prices) from 1960:1 to 2010:4, emphasizing the importance of financial market dynamics for the overall economy. Similarly, Borio (2014) examined the key empirical features of the financial cycle and their implications for economic fluctuations and policy decisions. Finally, Yan and Huang (2020) demonstrated that the traditional methods of analyzing the financial cycle are ineffective,[10] and they studied the relationship between the financial cycle and the business cycle using wavelet analysis, the VAR model, and the extended IS equation.

However, it remains difficult to analyze the business and financial cycles concurrently, and research has not included the financial side when identifying different economic dynamics. In particular, these studies do not use two indicators (one real and one financial) to identify a business-financial cycle in the same model: a model that considers both the real and financial aspects. Furthermore, the lack of well-defined regimes in the literature can lead to inaccuracies, particularly during times of financial distress (Hamilton, 1989). Because these regimes influence not only the development of financial markets (Ang & Timmermann, 2012; Boudt et al., 2017; Hammerschmid & Lohre, 2018), but also the real economy, a multi-regime framework is required. This method allows us to identify the regimes that define a business-financial cycle. Finally, this paper's contribution is the use of four regimes rather than two (Sarafrazi et al., 2015; Hammerschmid & Lohre, 2018), but it differs from Guidolin and Timmermann (2006) in that it uses four regimes to summarize the entire range of possible events in a real-world financial context given two indicators.[11]

In terms of the financial stress variable, the existing literature focuses primarily on the synthesis of financial market dynamics using financial stress indicators, but it fails to fully explore the effects of financial stress on the real economy by combining real variables and financial stress in a single model. In particular, there is little emphasis on incorporating financial stress indicators into models that examine business cycle fluctuations and identify regimes. Extensive research frequently overlooks critical events in financial markets by failing to consider specific regimes within the financial and business cycles.

This article closes the gap by incorporating financial stress indicators into business cycle fluctuations. Given the effects of financial dynamics on the real economy, particularly during times of stress, we provide a more comprehensive understanding of the relationship between financial market dynamics and real economic activity. We broaden the scope of the analysis to include more regimes, allowing for a more focused examination of the current real financial landscape and the critical periods that have characterized recent years - two aspects that have been overlooked in previous studies.

3. Data and methods

3.1. Variables and data

We used the following variables to define the R-R and R-F models: the OECD CLI and the CPI Acceleration for the R-R model and the OECD CLI and the KCFSI for the R-F model (Hakkio & Keeton, 2009).

The CLI is a forward-looking economic indicator that summarizes the current state of real activity and detects signs of a possible turnaround or regime shift. The index is designed to respond quickly to fluctuations in economic activity while remaining sensitive to short-term movements in the real economy. The CPI is an economic indicator that measures the growth of prices in the economy. Changes in the CPI are used to assess price changes associated with the cost of living and are frequently used to detect periods of inflation or deflation. In our analysis, we looked at CPI Acceleration, which is the difference between the rolling 3-month and 36-month variations in the price index.

In the R-F model, we replaced the CPI Acceleration with the KCFSI, which measures financial system stress. The Kansas City Federal Reserve provides the index KCFSI. This index was selected after carefully considering the various financial stress indices available in the empirical literature. Table 1 presents the descriptive statistics for these three indicators.

Finally, we included the MSCI USA index, which measures the performance of the US equity market's large- and mid-cap segments. We used this index to select and contrast the R-R and R-F models.

As for the sources, CLI and CPI from the St Louis Fed (FRED), KCFSI from the Kansas City FED, and the MSCI USA index from the MSCI website. In terms of variable returns, CLI and CPI were expressed as percentage returns, whereas KCFSI returns were calculated using simple monthly differences. The data set included monthly observations for all variables from February 1990 to February 2024, for 409 data points per variable.

3.2. The model

3.2.1. Business regimes and the Markov-switching model

We used the MS model (Hamilton, 1989, 1990, 1994) to identify the various phases of the business and business-financial cycles. Unlike previous research (Alexander & Kaeck, 2008; Balcilar et al., 2015; Hammerschmid & Lohre, 2018; Sarafrazi et al., 2015), we developed a four-regime bivariate MS framework to account for both increasing and decreasing performance of the selected indicators. Appendix D includes a discussion of MS and multi-regime modeling.

[10] (i) the analysis of turning points, (ii) frequency-based filter methods, and (iii) spectral analysis.

[11] To account for the joint distribution of bond and stock returns, the authors use a four-state model with regimes labeled as crash, slow growth, bull, and recovery.

Table 1
Descriptive statistics.

	KCFSI	CPI Acceleration	CLI
Average Return (annual.)	−0.021	−0.009	0.009
Stand. Deviation (annual.)	1.216	0.596	1.469
Minimum Return	−1.464	−1.042	−4.388
Maximum Return	2.933	0.503	5.748
Skewness Coefficient	2.824	−1.155	3.520
Excess Kurtosis	28.072	10.740	114.490
Percentage of Positive Returns	46.078	50.245	50.245
Percentage of Negative Returns	53.922	49.755	49.755
1% Percentile	−0.701	−0.412	−0.754
5% Percentile	−0.446	−0.240	−0.288
10% Percentile	−0.283	−0.174	−0.193
90% Percentile	0.258	0.165	0.171
95% Percentile	0.475	0.272	0.297
99% Percentile	0.982	0.434	0.517

Table 1 shows the descriptive statistics for the three variables used to determine the business cycle and business-financial cycle, namely the KCFSI, the CPI Acceleration, and the CLI. Source: Our own elaboration.

Once $\boldsymbol{y}_t$ and $\boldsymbol{c}_{s_t}$ had been defined as $(2 \cdot 1)$ vectors containing respectively the returns and the constants of the couple of variables (CLI-CPI in the R-R model or CLI-KCFSI in the R-F model respectively),[12] the MS model employed in the analysis was expressed as follows:

$$\boldsymbol{y}_t = \boldsymbol{c}_{s_t} + \boldsymbol{\varepsilon}_t \tag{1}$$

where $\boldsymbol{\varepsilon}_t$ is a $(2 \cdot 1)$ vector representing the error terms and is independent of S_t. The vector $\boldsymbol{\varepsilon}_t \sim NID(0, \boldsymbol{\Sigma}_{s_t})$ is an exogenous noise, normally and independently distributed with zero mean and variance $\boldsymbol{\Sigma}_{s_t}$. In particular, S_t is a stochastic exogenous process with s_t varying from 1 to 4. The four hypothesized regimes are assumed to follow a Markov chain with transition probabilities $p_{i,j} = P(s_t = j \mid s_{t-1} = i)$, where the generic element $p_{i,j}$ is the transition probability of $s_t = j$ given that $s_{t-1} = i$. The Markov chain is therefore described by the following transition matrix, which defines the probabilities of s_t:

$$\boldsymbol{P} = \begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \\ p_{41} & p_{42} & p_{43} & p_{44} \end{pmatrix} \tag{2}$$

We estimate all the parameters of interest (32 in total). The parameters were stacked in the transposed vector $\boldsymbol{\theta} = \left(\boldsymbol{c}_{s_t}, \boldsymbol{\sigma}^2_{s_t}, p_{i,j}\right)'$ and estimated within a quasi-maximum likelihood framework. Defining $\boldsymbol{Y}_t = \{y_t, y_{t-1}, \dots\}$ as the vector which contains all observed values of the time series until the time t, we first focused on estimation of $\boldsymbol{\xi}_{t|T,}$ which is a $(4 \cdot 1)$ vector containing the smooth probabilities of being in one of the four regimes, considering the entire information sample up to period T, as defined by Hamilton (1994, p. 679). For $s_t = i$ the component of $\boldsymbol{\xi}_{t|T}$ is given by $P(s_t = i \mid \boldsymbol{Y}_T; \theta)$ and the conditional expectation of $\boldsymbol{\xi}_{t+1|T}$ is obtained by pre-multiplying $\boldsymbol{\xi}_{t|T}$ by P. Thus, Equation (3) describes the likelihood of transitioning between regimes.

$$E\left(\boldsymbol{\xi}_{t+1|T} \middle| \boldsymbol{\xi}_{t|T}\right) = \boldsymbol{P}^* \boldsymbol{\xi}_{t|T} \tag{3}$$

Under normality, the conditional density of $\boldsymbol{y}_t$ for each s_t is represented by the vector $f\left(\boldsymbol{y}_t \middle| s_t = i, \boldsymbol{Y}_{t-1}; \boldsymbol{\theta}\right)$ whose typical elements are

$$f(\boldsymbol{y}_t | s_t = i, \boldsymbol{Y}_{t-1}; \boldsymbol{\theta}) = \frac{1}{\sqrt{2\pi\sigma_i^2}} exp\left\{\frac{-(\boldsymbol{y}_t - \boldsymbol{c}_i)^2}{2\sigma_i^2}\right\} \tag{4}$$

Furthermore, given the probability $P(s_t = i \mid \boldsymbol{Y}_{t-1}; \boldsymbol{\theta})$, the joint density function of $\boldsymbol{Y}_t$ conditioned to $\boldsymbol{Y}_{t-1}$ is:

$$f(\boldsymbol{Y}_t | \boldsymbol{Y}_{t-1}; \boldsymbol{\theta}) = \sum_{i=1}^{4} P(s_t = i | \boldsymbol{Y}_{t-1}; \boldsymbol{\theta})^* f(\boldsymbol{y}_t | s_t = i, \boldsymbol{Y}_{t-1}; \boldsymbol{\theta}) \tag{5}$$

After iterating all equations for $t = 0, \dots, T$ we obtained a recursive procedure known in the literature as "Hamilton's filter" (Hamilton, 1994). Given an initial value $P(s_k = i \mid \boldsymbol{Y}_{k-1}; \boldsymbol{\theta})$ and iterating through time, we obtained the conditional densities $f\left(\boldsymbol{y}_t \middle| \boldsymbol{Y}_{t-1}; \boldsymbol{\theta}\right)$ and finally the log-likelihood function as

[12] Because the two variables in pairs are independent of one another, they can be treated separately. Within the model, we use them together to investigate the structure of the time-varying correlation between them.

$$L_T(\boldsymbol{\theta}) = \frac{1}{T}\sum_{t=1}^{T} ln\, f(\boldsymbol{y}_t|\boldsymbol{Y}_{t-1};\boldsymbol{\theta}) \tag{6}$$

Finally, to assess the behavior of the two R-R and R-F models, we used the MSCI USA index. We used the estimated smoothed likelihood to construct a historical series of the prevailing regimes in each month, taking the Markov chain estimated in the two models as exogenous. Then, for each R-R and R-F model, we used a dummy random walk model to estimate the returns of the MSCI USA index. The equation that describes the index dynamic is a regime-dependent dummy random walk, and the vector includes the parameters of interest $\boldsymbol{\pi} = (\mu_{s_t}, \nu_{s_t})'$:

$$r_{m,t} = \mu_{s_t} + \nu_{s_t} {}^* u_t \tag{7}$$

where $r_{m,t}$ is a scalar containing the return of the market index. μ_{s_t} and ν_{s_t} are two scalars[13] representing the index's average return and the volatility in the specific regime s_t. Finally, u_t is an error term following a standardized normal distribution and differs from $\boldsymbol{\varepsilon}_t$ in Equation (1).

3.2.2. Identification of regimes and likelihood function estimation

Identifying regimes in macro-financial MS models is useful for analyzing the behavior of indicators during specific stages of the business or business-financial cycle. However, in the estimation process, one must first determine how to classify regimes and define the procedures for selecting the appropriate number of regimes.[14] Specifically, one must deal with potential identification issues common in MS models (so-called *label-switching*) and the risk of having multiple maxima in the log-likelihood function profile, i.e., obtaining different solutions in each run of the maximization procedure. We addressed this issue by imposing appropriate restrictions on the intercept signs. Although the resulting regime configuration under this approach appears somewhat restricted, the introduction of drift constraints is necessary in practice to improve their interpretability while reducing the risk of convergence on a local maximum.

The strategy of imposing constraints on the drift term in Equation (1) may not always be effective because it only reduces the parameter space that can be explored. Furthermore, we knew that the proposed restrictions may be insufficient to ensure complete statistical identification. To address this issue, we employed the Simulated Annealing algorithm (Kirkpatrick et al., 1983), which is likely to explore the parametric space more efficiently, lowering the risk of becoming stuck on a local maximum of the likelihood function.[15] The estimation results were generated by MS Lab software, an *ad hoc* Windows application developed in C# language.

We also delved into another important aspect of MS models: the model selection process. We ran multiple estimation runs for each model class to determine the best configuration of each R-R and R-F model. We also used a set of statistical metrics (Root Mean Square Error (RMSE), information criteria, and statistical significance of model parameters) to evaluate model accuracy. Furthermore, our approach should have been as consistent as possible with the history and dynamics of the last decades, particularly when focusing on the specific critical events previously mentioned. Appendix B for the R-R model and Appendix C for the R-F model contain details on the methodologies used and the resulting models.

3.2.3. Regimes identification in the R-R and R-F models

We used a four-regime approach to identify the various regimes of the business and financial cycles. Our decision was based on synthesizing the entire event space, considering both rising and falling dynamics for the two indicators.

For the R-R model, we followed Gupta et al. (2014),[16] and defined four regimes with the following characteristics.

- Twin Growth: rising growth (i.e., positive CLI) and rising inflation (i.e., positive CPI Acceleration). Real activity is growing but also prices show acceleration;
- Slowing Down: slowing growth (i.e., negative CLI) and falling inflation (i.e., negative CPI Acceleration). Real activity is experiencing a temporary slowdown but also prices dynamic is not positive;

[13] They differ from previous constants and assume four different values, according to *s-t*. There are four results for each parameter that depend on the prevailing regime in each month, based on the identified Markovian chain.

[14] In this framework, standard convergence theorems for obtaining test statistics, such as the Lagrange multiplier or the Likelihood Ratio, are not applicable. Errors are also not normally distributed (Hamilton, 1994, p. 698).

[15] The most used algorithm in maximum likelihood estimation is the BFGS (Broyden, 1970; Fletcher, 1970; Goldfarb, 1970; Shanno, 1970). However, under general smoothness conditions for the function to be maximized, this algorithm only guarantees convergence to a local maximum, a feature that leaves it vulnerable to the problem of multiple maxima. Furthermore, BFGS requires an approximation of the Hessian matrix in each iteration. In addition to being a computationally difficult task, this can result in numerical instability during the estimation procedure. Instead, the SA algorithm can more efficiently explore larger parameter regions without the need for numerical calculations of the gradient or the Hessian matrix. Furthermore, after infinite iterations, it converges asymptotically to a global optimum. Although this may not be true for a finite number of iterations, this algorithm is very effective at exploring the parameter space, especially when dealing with several parameters.

[16] Gupta et al. (2014) classified the business cycle into four well-defined regimes: (i) Heating Up: rising growth and rising inflation; (ii) Slow Growth: slowing growth and falling inflation; (iii) Stagflation: slowing growth and rising inflation; (iv) Goldilocks: rising growth and falling inflation.

- Stagflation: slowing growth (i.e., negative CLI) and rising inflation (i.e., positive CPI Acceleration). While real activity is experiencing a slowdown, the prices dynamic is accelerating;
- Goldilocks: rising growth (i.e., positive CLI) and falling inflation (i.e., negative CPI Acceleration). Real activity exhibits signs of growth and recovery while prices acceleration is negative.

These four regimes are especially effective at isolating specific economic situations, even over short periods, and accurately identifying relevant real and financial events. Even if remote identification issues persist, this approach allows for assessing the impact of specific events that, although occurring infrequently during the study period, may significantly affect the business cycle.[17] For these reasons, a four-regime approach provides the flexibility required to estimate the R-R model over a long period. Fig. 1 summarizes the R-R model's regime characteristics.

In terms of the R-F model, we distinguished four regimes based on conditions of "high/low stress" in financial markets and "growth/decline" in real activity. Unlike the R-R model, the four-regime R-F model is expected to respond more quickly to specific financial conditions and anticipate changes in the financial scenario. The R-F model identifies four regimes within the business-financial cycle.

- Heating Up: rising growth (i.e., positive CLI) and rising financial stress (i.e., positive KCFSI). Real activity is growing but financial markets show early signals of stress and reversal trend;
- Cooling Down: slowing growth (i.e., negative CLI) and falling financial stress (i.e., negative KCFSI). Real activity is slowing down, but financial markets are likely to expand as financial stress is falling;
- Hard Times: slowing growth (i.e., negative CLI) and rising financial stress (i.e., positive KCFSI). Real activity is slowing down together with financial markets because the crises in this regime have a negative impact on both the real and the financial side;
- Golden Times: rising growth (i.e., positive CLI) and falling financial stress (i.e., negative KCFSI). Both real activity and financial markets are in a recovery phase and are rising, while financial stress is decreasing.

Fig. 2 summarizes the R-F model's regime characteristics.

3.3. Preliminary analysis

In this section, we briefly show the temporal evolution of variables in the data set to highlight some stylized facts. Although the overall analysis shows that the annualized average returns for the three series (CPI, CLI, and KCFSI) in percentage are close to zero (Table 1), interesting insights emerge when we consider specific periods and events: the 1990 Gulf War, the Lehman Brothers crisis, the late 2018 financial market correction, the COVID-19 pandemic crisis,[18] and the effects of the wars in Ukraine and the Middle East on the overall system. All of these events demonstrate different business cycle dynamics when financial stress variables are combined with real variables.

The first event was the Gulf War in 1990. Analysis of the CLI and CPI Acceleration values throughout 1990 suggests no significant shocks were affecting the real side. In fact, both indexes were still showing positive percentage growth in August and September 1990, indicating economic expansion, and it was not until the early months of 1991 that these indicators began to reflect the negative effects of the war. On the contrary, using the financial stress index indicates significant fluctuations that began as early as August 1990, when the KCFSI increased rapidly.

The Lehman Brothers crisis, during which the KCFSI reached its highest value and largest percentage growth (+2.933% in October 2008), has been the most significant event in terms of significance and impact in recent decades.[19] The impact on the real economy was negative, but with a slight lag. Indeed, both the CPI Acceleration and the CLI fell significantly during the crisis period, but the former only reached its historical low (−1.042%) in December 2008, while the latter fell to −0.847% in October 2009, a year after the crisis's peak.[20] Thus, the crisis had a profound and immediate impact on financial markets, causing unprecedented financial stress that was not experienced again. The real economy was also severely affected, but with a delay, and only began to recover and grow in early 2010.

The financial market correction at the end of 2018 characterized the third notable period. Although this short event primarily affected the financial market, it cannot be completely ignored, even in purely economic models. Following the 2008–2009 crisis, a correction such as the one observed at the end of 2018 caused financial uncertainty, temporarily stifling economic growth.

[17] Technically, the evidence from information criteria (IC) and the statistical significance of the estimated parameters suggest that a four-regime model is not over-identified.

[18] The Lehman Brothers crisis began in the financial markets and eventually had an impact on real-world activities. The effects were immediately visible on the financial markets before spreading to the rest of the economy. Instead, the COVID-19 pandemic crisis was exogenous, affecting both financial markets and economic activity.

[19] The KCFSI also recorded the maximum value (105.399) in November 2008.

[20] In fact, the crisis had already been partially resolved, and financial stress had been largely absorbed. To confirm this, the KCFSI fell by −1.464% as early as May 2009.

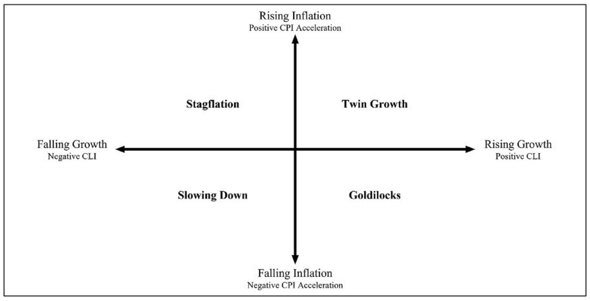

Fig. 1. Schematic classification of economic regimes using CLI and CPI Acceleration.
Fig. 1 depicts the schematic classification of the four regimes identified in the R-R model, beginning with the two indices CLI and CPI Acceleration. Source: Our own elaboration.

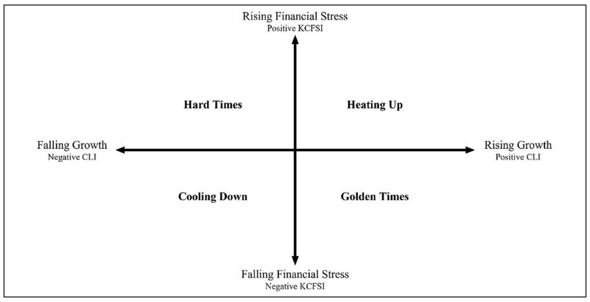

Fig. 2. Schematic classification of economic-financial regimes using CLI and KCFSI.
Starting with the two indices CLI and KCFSI, Fig. 2 shows the schematic classification of the four regimes identified in the R-F model. Source: Our own elaboration.

Fourth, the COVID-19 pandemic crisis was an important event. Based on the observed data, it appears that the indicators did not directly capture the negative impact of the pandemic and subsequent lockdown measures until March 2020. Indeed, during the first months of the pandemic crisis, both CLI and CPI Acceleration remained within the range of values observed in previous months.[21] In contrast, in April 2021, the CLI reached its lowest real and percentage value since 1990 and recorded its sharpest monthly decline, while the CPI Acceleration increased slightly throughout 2021. The KCFSI, on the other hand, which had been hovering around zero for more than a year, reflected the climate of uncertainty at both the economic and financial levels, rising to +2.765% in March 2020, the index's second highest value since 2008.

We conclude with a focus on the current scenario, which includes ongoing conflicts in Ukraine and the Middle East.[22] According to CPI and CLI values, almost all of 2022 was characterized by periods of high real-side criticality, with the CPI Acceleration reaching an all-time high of +0.503% in June 2022. On the other hand, the CLI decreased throughout the period. Although critical, this situation must also be analyzed financially, as the stress index shows peaks, including one in the middle of the year and one near the end of the period, but then immediately decreases in value. Given these figures, 2022 was therefore a worrying period in real terms, but it only partially affected the financial markets. Finally, 2023, despite having a declining CPI and a rising CLI, does not fully correspond to a positive period. Indeed, the ongoing conflict in the Middle East and its effect are already evident from the peak of financial stress in October 2023.

In summary, these examples help to contextualize the business cycle analysis and demonstrate the interconnectedness of the business and financial cycles. First, including a financial stress indicator allows for incorporating events such as financial market corrections and insights from recent challenges that are likely to affect both real and financial dynamics. Second, it allows for a more accurate assessment of important events such as the Lehman Brothers crisis, which affected both the financial sector and the real economy: in this context, identifying the financial cycle on time is critical importance. Finally, the results suggest that the KCFSI indicator can better capture the immediate consequences of events such as the 1990 Gulf War, the COVID-19 pandemic, or the Middle Eastern conflict. Overall, examining these stylized facts emphasizes finance's critical role in detecting extreme real and financial events early. In this regard, empirical evidence shows that financial stress has a significant value in accurately identifying specific phenomena

[21] Actually, the CLI indicator showed an improvement in March, rising from −0.136% in January 2020 to −0.107% in February 2020, and finally −0.085% in March 2020. Meanwhile, the CPI Acceleration changed from slightly positive (0.044%) in January 2020 to negative values in February and March 2020 (−0.027% and −0.163%, respectively).

[22] To confirm this, the NBER index, based on real variables, does not show any recession as of May 2020.

and supports its inclusion in formal models. Based on these stylized facts, the following section evaluates and analyzes the results of the R-R and R-F models in the MS framework.

4. Empirical analysis

Sections 4.1 and 4.2 contain the empirical results for the R-R and R-F models. The regimes were determined using the framework shown in Section 3.2.3, and the parameter values were obtained by estimating Equations (1)–(7). These values, which represented the optimal configuration for each model, were determined through simulations, the details of which are presented in Appendices B and C. Our analysis included parameter estimates, a transition matrix that drove the dynamics of the regimes, and empirical evidence from the MSCI USA index. In Section 4.3, we compare the accuracy of the two models. Because our analysis focused on model accuracy over specific historical periods and under varying real-world financial conditions, we used RMSE as a statistical measure.[23]

4.1. Results for the R-R model

Table 2 shows the regime transition matrix based on the R-R model's optimal configuration.

Inspection of the diagonal elements show significant persistence of the four regimes. The Twin Growth regime is the most persistent (88.020%) and shows a tendency to be followed by Slowing Down (10.461%) and Goldilocks (7.717%). Following that, slowing periods are typically followed by periods of further deterioration (9.576%) or continued growth. Stagflation is very likely to enter Slowing Down (0.773%) before returning to growth, either as Twin Growth or Goldilocks. Finally, in Goldilocks, the probability of entering a recession is low, but there is a residual probability (5.031%) of transitioning to a Slowing Down state. Overall, the global business cycle dynamics revealed by the transition matrix can be summarized as follows.[24] Assuming the business cycle is in Twin Growth, the economy is likely to remain in this state for nearly eight months, with a slight increase in real activity and inflation. Subsequently, the economy typically experiences a brief period of slowing, lasting about six months, during which real activity declines and prices fall. Although these are the most common regimes, two more unstable situations have opposing characteristics. The first situation corresponds to the Goldilocks regime, which usually precedes or follows Twin Growth, or represents a reversal of the trend of real activity after a slowdown period. Thus, the Goldilocks regime consists of periods of strong economic growth followed by average price declines lasting nearly six months. The second scenario is Stagflation, which is unlikely to lead to real growth but rather to a regime in which real activity falls and CPI Acceleration slows. This regime has a shorter average duration and only appears three times in our sample period: after the Lehman Brothers crisis,[25] after the COVID-19 pandemic crisis,[26] and for nearly a year from the end of 2021 to the first half of 2022 due to unprecedented inflationary surges.[27] However, it is important to isolate these specific negative events. Fig. 3 shows the dynamics of the business cycle and the alternation of the four identified regimes.

Fig. 3 depicts the economic system alternating between two regimes (Twin Growth and Slowing Down), with longer periods of moderate growth and shorter periods of slowdown. Stagflation and Goldilocks interrupt the other two regimes to highlight particularly negative or favorable periods, respectively. Although the first two regimes dominated the cycle, the other regimes appear to capture specific economic scenarios whose main features are summarized by the drifts and variance/covariance matrix of the CLI and CPI Acceleration reported in Table 3, which were estimated using Equation (1). First, inflation tends to accelerate significantly only in Stagflation (0.149), whereas it increases less pronouncedly in Twin Growth (0.038). The increase in the CPI Acceleration reflects the rise in inflation in recent years. Stagflation also refers to the primary decline in real activity (−0.434), which is characterized by its severity (confirmed by the results of the drifts and volatility of the CLI and CPI Acceleration indices). In contrast, Goldilocks is distinguished by moderate economic growth (0.104), which is higher than Twin Growth (0.053), as well as a slowdown in price dynamics (−0.044). This is also the regime of moderate volatility for both CPI Acceleration and CLI. Last, Slowing Down includes periods of slowing in both real activity (−0.130) and the inflation process (−0.033). In absolute terms, these slowdowns are lower than those observed in Stagflation and Goldilocks for the two indices, respectively.[28]

To assess the R-R model's ability to capture the dynamics of financial market indices, we examined the impact of regime changes on the performance of the MSCI USA index using Equation (7). Table 4 shows the results for the index's expected returns and variance in the four R-R regimes.

[23] Because of the nature of the comparison, we were unable to use Diebold and Mariano's (1995) test to assess the difference in forecast accuracy between competing models.

[24] From the transition matrix, we derive the expected average durations of each regime of the cycle, through the formula $d_i = \frac{1}{p_{ii}}$ for $i = 1, \ldots 4$, where p_{ii} are the diagonal elements of the transition matrix.

[25] After about 12 months of Slowing Down (from July 2008 to June 2009), the cycle entered Stagflation, which lasted about 7 months (from July 2009 to January 2010), before returning to Slowing Down. In fact, the effects of the Lehman Brothers crisis were transmitted to real activity with a temporal lag, resulting in a period of slowdown followed by a real recession, as consumption collapsed within months.

[26] Following a long period of almost continuous real growth, the cycle began to slow in March 2020 and remained so until July 2020. Following 7 months of Twin Growth, the cycle was expected to enter Stagflation for 3 months due to a sharp decrease in real activity and consumption.

[27] After a short period of 2 months of Goldilocks, the cycle returned to Stagflation, not due to COVID-19, but due to an increase in inflation that affected the end of 2021, particularly the first seven months of 2022.

[28] For example, a decrease in the CLI index for real activity indicates the start of a regime more negative than Twin Growth or Goldilocks, but not so much as to suggest a recession as Stagflation.

Table 2
Transition matrix for the R-R model.

Transition matrix	Twin Growth		Slowing Down		Stagflation		Goldilocks		Average Regime Duration (Months)
Twin Growth	0.880	(4.196)	0.105	(0.600)	0.037	(0.874)	0.077	(4.463)	8.347
Slowing Down	0.038	(0.964)	0.837	(4.475)	0.096	(0.968)	0.064	(3.778)	6.148
Stagflation	0.004	(0.920)	0.008	(1.184)	0.828	(3.706)	0.007	(3.922)	5.807
Goldilocks	0.078		0.050		0.039		0.852		6.756

Table 2 shows the R-R model's transition matrix, which is based on CPI Acceleration and CLI. In the last column, we show the average regime length in months. Shown in brackets are the t-statistics (absolute value). Source: Our own elaboration.

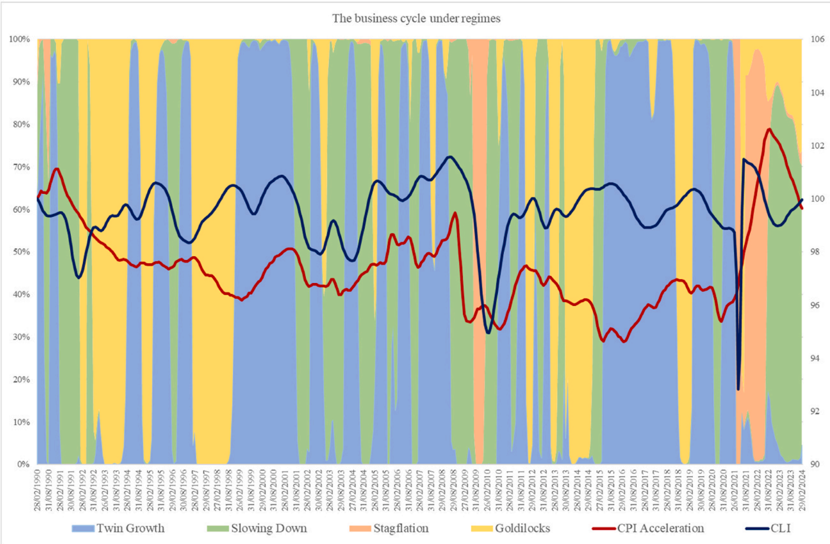

Fig. 3. Evidence on the business cycle from the R-R model
Fig. 3 depicts the evolution of the regimes estimated by the R-R model using the CLI and CPI Acceleration time series from February 1990 to February 2024. Source: Our own elaboration.

Table 3
Estimation of intercept parameters and variance-covariance of CPI Acceleration and CLI (R-R model).

Parameter	Twin Growth		Slowing Down		Stagflation		Goldilocks	
CPI Acceleration	0.0382	(5.4752)	−0.0329	(3.5960)	0.1486	(2.9836)	−0.0442	(4.6878)
CLI	0.0532	(4.8834)	−0.1296	(8.3655)	−0.4343	(3.8338)	0.1037	(8.7967)
Variance CPI Acceleration	0.0228	(10.7477)	0.0230	(11.7867)	0.0517	(9.5537)	0.0261	(9.9243)
Covariance CLI/CPI Acceleration	0.0080	(6.9909)	0.0204	(1.0637)	−0.0310	(0.5217)	−0.0159	(1.2228)
Variance CLI	0.3524	(2.8021)	0.2878	(2.7202)	1.0187	(2.0299)	0.2437	(2.5379)

Table 3 shows the results for the estimated drifts and variances-covariances (net of drifts) of the CPI Acceleration and the CLI for each R-R regime. Parentheses show the parameters' t-statistics (in absolute value). Source: Our own elaboration.

Table 4
Estimation of the parameters of the R-R model for the MSCI USA index.

Parameter	Twin Growth		Slowing Down		Stagflation		Goldilocks	
MSCI USA index Alpha	0.615	(1.892)	0.695	(3.092)	1.115	(2.617)	2.180	(2.428)
Variance MSCI USA index	4.044	(17.220)	4.583	(14.918)	3.687	(13.117)	3.152	(6.171)
Sharpe ratio	0.306		0.325		0.580		1.228	

Table 4 summarizes the estimated drifts, variances (net of the drifts), and Sharpe ratios of the MSCI USA index for each regime in the R-R model. Parentheses show the parameters' t-statistics (in absolute value). Source: Our own elaboration.

Overall, the business cycle shows a prevalence of moderate growth periods, which tends to rule out a slowdown scenario and implies a low risk of recession, as reflected in the MSCI USA index performance. Goldilocks appears to be the optimal regime for financial markets to generate additional returns: The MSCI USA index has the highest expected return (2.180%) and the lowest volatility, making this the best period of the business cycle for investment, regardless of duration. However, during the Twin Growth regime, volatility and a significant decrease in expected returns are observed. Regardless of the favorable economic conditions, an increase in CPI Acceleration has a negative impact on the stock market.[29] This idea is confirmed when considering periods of high inflation, when expectations of future economic growth are insufficient to guarantee high-risk premiums. Based on these findings, financial markets are likely more influenced by inflation dynamics than real growth.[30] As further evidence, in Slowing Down - a regime characterized by low growth and falling inflation - the expected return of the MSCI USA index is slightly higher than Twin Growth, but volatility peaks. Finally, despite the sharpest slowdown in real activity and acceleration of financial asset prices, the MSCI USA index's expected return is positive during Stagflation, suggesting a mismatch between real and financial dynamics.

The facts observed in the events discussed in Section 3.3 serve as evidence of the R-R model's main flaw in light of these results. First, the model classifies the months following the Gulf War in 1990 as a period of increasing economic activity, known as Twin Growth, while only capturing the period of economic distress (Slowing Down) beginning in February 1991. Second, while the R-R model correctly identifies the crises that result from the collapse of Lehman Brothers and the COVID-19 pandemic as periods of economic slowdown, the severity of these crises is not recognized until a later occurrence. In the first event, the period from July 2009 to January 2010 is known as stagflation, whereas in the second event, the economic downturn is not recognized until a year later, in March 2021. Third, the periods beginning in September 2008 and March 2020 are likely to fall within the Slowing-Down regime. Unfortunately, the R-R model completely ignores the late-2018 financial market correction and assigns it to the current Goldilocks regime, which should imply the expectation of favorable economic and financial conditions. Finally, the R-R model labels the entire period beginning in August 2021 as months of stagnation and slowing, ignoring any potential positive financial signals. To summarize, financial markets appear to be coordinated with the business cycle dynamics only in the case of Goldilocks and, to a lesser extent, Twin Growth. In contrast, when the business cycle enters Stagflation, financial markets may have already discounted the negative effects of the economic recession in the two most likely pre-Stagflation regimes, that is, Slowing Down and Twin Growth. In fact, the risk premium of the MSCI USA index fell consistently in these two regimes. Overall, the R-R model captures the various stages of the business cycle fairly well and can be considered a good framework. However, the model's lack of a financial component resulted in a significant underestimation of events that affected the system as a whole.

4.2. *Results for the R-F model*

This section examines whether the R-F model has a greater effect than the R-R model in identifying regimes. Based on the transition matrix (Table 5), we find that all regimes are fairly persistent, except for the Hard Times regime, which has a short average duration.

Specifically, the Heating Up regime tends to be very persistent (85.272%), and it is found to enter Cooling Down (9.200%) or Golden Times (7.377%). The Golden Times averages six months of stability before returning to Heating Up (7.640%) or Hard Times (10.651%). Indeed, following a particularly favorable period for both the economy and financial markets, the entire economic-financial system can either continue to grow in a more contained manner (Heating Up) or devolve into Hard Times if the real growth is not based on fundamentals (for example, speculative, financial, or technological bubbles). Hard Times appears to have a high likelihood of persisting (71.090%) or transitioning to Cooling Down (3.487%).

The transition matrix shows the various stages of the business-financial cycle, beginning with Heating Up, which is accompanied by a slight increase in financial stress. In this regime, it is reasonable to expect the economy to enter a more optimistic phase in which real activity continues to grow (Golden Times) or to transition into a Cooling Down regime in which the opposite conditions occur. In the latter case, Heating Up is followed by a period of contraction in economic activity (about six months on average), but this does not affect the overall good performance of financial markets, which show no signs of high stress. However, in some cases, the business-financial cycle deteriorates even further, resulting in a Hard Times regime that lasts an average of three months and puts the entire system to the test due to extremely adverse economic and financial conditions. Fig. 4 shows the business and financial cycle dynamics.

Overall, the R-F model oscillates between regimes in which both financial stress and real activity increase and both variables decline. In the first two regimes, Heating Up and Cooling Down, economic activity is only partially synchronized with financial markets, as real-side growth (declines) coincides with increased (decreased) stress in financial markets. In contrast, Golden Times and Hard Times appear to be coordinated as both increase (decrease): that is, Hard Times, which are shorter and have a much lower frequency, tend to signal extremely negative phases affecting both financial markets and real economic activity. Unlike Stagflation in the R-R model, Hard Times refers to events with increased financial stress accompanied by economic contraction, excluding pure economic crises (which would realistically fall under Cooling Down) and bearish phases in financial markets (Heating Up). The Hard Times regime includes events such as the Twin Towers attack (September–November 2001), the Lehman crisis (August–October 2008 and then May–September 2009), the COVID-19 pandemic crisis (March 2020 and then March–May 2021), and severe financial market contractions (January–July 2022).[31] An important factor is that these events have primarily occurred since the 2000s, indicating a stronger correlation between real activity and financial dynamics over time. Furthermore, they are distinguished by their severity, as

[29] Since the early 2000s, some studies (e.g., Boyd et al., 2001; Huybens & Smith, 1999) have shown that inflation has a negative effect on equity performance.

[30] Economic growth can influence financial markets, specifically the stock market and dividends. Indeed, the positive expectations for real growth are likely to lead to an increase in earnings and dividends, causing the stock market to rise. Rising inflation, or the prospect of future price increases, erode risk premia, limiting the positive effects of economic development or negatively influencing the stock market.

[31] There are some residual months in Hard Times, in addition to these ones already mentioned.

Table 5
Transition matrix for the R-F model.

Transition matrix	Heating Up		Cooling Down		Hard Times		Golden Times		Average duration of the regime (months)
Heating Up	0.853	(3.762)	0.092	(0.717)	0.072	(0.811)	0.074	(2.820)	6.790
Cooling Down	0.066	(0.661)	0.826	(2.985)	0.111	(1.036)	0.083	(2.071)	5.734
Hard Times	0.005	(1.125)	0.035	(0.739)	0.711	(4.944)	0.017	(2.964)	3.459
Golden Times	0.076		0.048		0.107		0.826		5.743

Table 5 shows the R-F model's transition matrix, which is based on KCFSI and CLI. In the last column, we show the average regime length in months. Shown in brackets are the t-statistics (absolute value). Source: Our own elaboration.

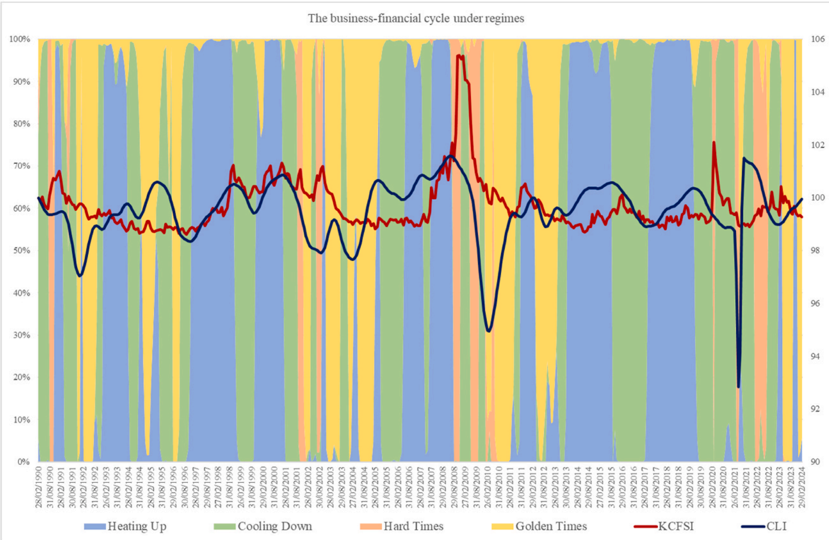

Fig. 4. Evidence on the business-financial cycle from the R-F model.
Fig. 4 shows the succession of regimes estimated by the R-F model using the CLI and KCFSI time series from February 1990 to February 2024. Source: Our own elaboration.

confirmed by an analysis of the drift and variance/covariance of the KCFSI and CLI (Table 6), which was calculated using Equation (1).

Inspection of the KCFSI and CLI drift estimates reveals that the lowest levels of financial stress are found in Golden Times (−0.026), accompanied by low volatility and simultaneous economic upturns (0.095). This regime is most likely associated with periods when financial markets and real economic activity are coordinated and increasing in momentum. Given the two sides, real and financial, it therefore corresponds to the best scenario possible. The terms Heating Up and Cooling Down refer to periods that occurred frequently beginning in 1990. Heating Up refers to periods of high real growth and low volatility, which are associated with a slight increase in financial stress, most likely due to the impact of real growth on expectations, prices, and financial markets. Cooling Down represents the inverse situation, i.e., declining real activity, expanding financial markets, and decreasing financial stress. In both regimes, the two indicators increase (decrease) simultaneously, but without reaching extreme peaks. Finally, the Hard Times require special emphasis. This regime not only considers crises that affect both financial markets and real economic activity, but it also isolates some particularly severe downturns and volatility. In this regime, financial stress is at its peak (0.123), while real activity is ats its lowest (−0.205). Furthermore, it is the most volatile regime for both indicators, with the CLI suffering the greatest losses and the KCFSI experiencing the greatest shocks, owing primarily to the Lehman collapse and the COVID-19 pandemic crisis.

Given these results, the final factor to consider is the behavior of the MSCI USA index, as obtained from Equation (7) when applied to the R-F framework. We want to look at the consistency between macroeconomic regimes and the MSCI USA index within these regimes (Table 7).

Based on the expected return and volatility of the MSCI USA index, Golden Times appears to be the best option for equity investments. In this regime, the MSCI USA index has the highest expected return (1.416) and the lowest volatility (3.533), indicating that these are the best economic and financial conditions. In contrast, the index exhibits the worst dynamics in Hard Times, at least in terms of risk-return profile. Because this regime contains the most important economic and financial crises, the index's return is negative (−1.324), and the index's high volatility (7.126) shows the overall high uncertainty. The other two regimes fall somewhere between these two extremes. Real economic activity falls as the economy cools, while the stock market performs well on average due to negative financial stress. Even if volatility increases slightly, these conditions should encourage the market index to perform well. In fact, the decline in real activity does not appear to have affected the financial market, which in this regime shows the second-highest return/risk ratio, after Golden Times. Finally, in Heating Up, real growth is accompanied by an increase in financial stress, which penalizes the

Table 6
Estimation of intercept parameters and variance-covariance of KCFSI and CLI (R-F model).

Parameter	Heating Up		Cooling Down		Hard Times		Golden Times	
KCFSI	0.067	(2.028)	−0.055	(6.030)	0.123	(1.250)	−0.026	(1.958)
CLI	0.058	(11.466)	−0.127	(2.177)	−0.205	(11.827)	0.095	(5.945)
Variance KCFSI	0.236	(9.300)	0.240	(8.498)	0.916	(9.443)	0.123	(6.701)
Covariance CLI/KCFSI	0.033	(2.010)	0.058	(8.756)	0.066	(9.269)	0.037	(2.497)
Variance CLI	0.209	(0.834)	0.453	(0.951)	0.663	(1.772)	0.547	(0.411)

Table 6 shows the results for the estimated drifts and variance-covariances (net of drifts) of the KCFSI and CLI for each R-F model regime. Parentheses show the parameters' t-statistics (in absolute value). Source: Our own elaboration.

Table 7
Estimation of the parameters of the R-F model for the MSCI USA index.

Parameter	Heating Up		Cooling Down		Hard Times		Golden Times	
MSCI USA index Alpha	0.652	(2.616)	0.971	(3.156)	−1.324	(1.702)	1.416	(2.876)
MSCI USA variable index	4.097	(18.257)	4.038	(16.304)	7.126	(6.266)	3.533	(9.842)
Sharpe ratio	0.322		0.483		−0.496		0.753	

Table 7 shows the results for the estimated drifts, variances (net of the drifts), and Sharpe ratios of the MSCI USA index for each regime in the R-F model. Parentheses show the parameters' t-statistics (in absolute value). Source: Our own elaboration.

MSCI USA index in terms of expected return and volatility, further lowering its ratio (0.322).

Considering these results, we can now conclude by examining the classification of critical events discussed in Section 3.3. The 1990 Gulf War belongs to Hard Times and Heating Up, because after a first nod from the financial stress, the impact of the crisis took on a real nature, falling in Heating Up. This fact effectively eliminates the possibility of a generalized recession and clarifies the timing of its impact. The R-F model finds that September most likely falls within the economic and financial crisis regime and thus qualifies as difficult times when the focus is shifted to the Lehman Brothers crisis. A similar scenario emerges for the COVID-19 crisis, with March 2020 falling within the Hard Times regime. The R-F model, which is consistent with empirical evidence and the observed impact on the real economy, also shows a nearly one-year slowdown in the economy. Furthermore, this model correctly assigns the final months of 2018 to Heating Up, indicating that the financial correction primarily affected the financial sector rather than real economic growth. Finally, the R-F model allows for better timing in identifying recent year dynamics and distinguishing between worse periods (the first half of 2022) and negative periods, but only for real activity and financial markets. To summarize, the R-F model can distinguish between real, financial, and real financial crises and appears to better describe the MSCI USA index's dynamics and downward phases.

4.3. *Comparison of accuracy of modeling approaches*

After describing the features of the R-R and R-F models and the main differences between them, we now highlight the time periods when the R-F model shows local improvements over the R-R model. To do this, we conduct a model comparison. While previous descriptive evidence has shown that the R-F model is more efficient in capturing the dynamics of the MSCI USA index than the R-R model, further testing is required to determine whether the modeling approaches perform differently across phases of real and financial dynamics.

To that end, we assessed the R-R and R-F models' accuracy over various time periods and economic and financial scenarios. To do so, we examined the dynamics of the MSCI USA index and used the RMSE as a statistical measure to estimate the error between the MSCI USA index's historical series and the two MSCI USA index series estimated in the R-R and R-F models, respectively, using Equation (7). We first looked at the MSCI USA index's entire time series, from February 1990 to February 2024. Second, to focus on particularly critical periods, we chose various time windows, including periods when the index lost or gained more than 10%. Third, we filtered the time series using four different percentile values to account for the MSCI USA index's loss ranges. We examined the percentiles at the probability levels of 10%, 5%, 2.5%, and 1% and identified MSCI USA index returns that fell below the specific percentile. Fourth, we used the results of Iqbal et al. (2020), who identified 18 financial crises from 2002 to 2020, and limited the period of analysis to the intervals in which the specific crises occurred.[32] Finally, we focused only on real recession periods as identified by the NBER Business Cycle Index.[33] Table 8 summarizes the evaluation results for the R-R and R-F models. In addition to the RMSE

[32] In addition, we included the global energy crisis (January 2022) and central bank monetary tightening (May 2023) in these periods to take into account more recent events. The former, caused by a combination of factors such as increased energy demand, a shortage of supplies, and sanctions imposed on Russia, resulted in an increase in energy prices, which had a negative impact on the entire economy. The crisis is still ongoing, with governments seeking alternative solutions and implementing energy-efficient measures to mitigate its impact. The latter has sparked fears of a global recession and a drop in stock markets.

[33] To quote the NBER: "The NBER's Business Cycle Dating Committee maintains a chronology of US business cycles. The chronology identifies the dates of peaks and troughs that frame economic recessions and expansions. A recession is the period between a peak of economic activity and its subsequent trough, or lowest point. Between trough and peak, the economy is in an expansion …."

Table 8
RMSE from the R-R model and R-F model.

	R-R model	R-F model	Difference (%)
Total (n = 409)	4.429	4.425	0.076%
Loss < −10% (n = 8)	14.418	13.649	5.637%
Gain > +10% (n = 7)	11.232	11.008	2.033%
Percentile: α = 10.0% (n = 41)	9.375	9.169	2.243%
Percentile: α = 5.0% (n = 21)	11.511	11.164	3.106%
Percentile: α = 2.5% (n = 11)	13.391	12.948	3.421%
Percentile: α = 1.0% (n = 5)	15.825	14.880	6.347%
Filtered data by crisis (n = 41)	6.878	6.671	3.101%
NBER recession index (n = 36)	7.753	7.578	2.309%

Table 8 summarizes the evaluation results for the R-R and R-F models. The first and second columns show the results of the Root Mean Square Error (RMSE), which is defined as the difference between the MSCI USA index estimated from the two frameworks and the MSCI USA index time series, respectively. The results are evaluated over time and under various economic and financial conditions. The third column shows the percentage differences in the RMSE of the R-R and R-F models. Source: Our own elaboration.

values, the last column of Table 8 shows the percentage difference in RMSE values between the R-R and R-F models.

These results provide insights into the models' tracking quality relative to the MSCI USA index across regimes, where the sequence of estimated regimes is considered exogenous. The RMSE in Table 8 shows that the R-F model better captures the index's global dynamics overall, but the difference is small when the entire sample is considered (0.076%). As previously stated, the R-R model performs well in this scenario and can also interpret the dynamics of financial markets under certain regimes, thereby eliminating the possibility of model selection in the strict sense.

When selected data samples and periods of unfavorable economic conditions are considered, larger differences emerge, resulting in a local improvement of the R-F model. First, when the MSCI USA index falls by more than 10%, the R-F model significantly outperforms the R-R model. Second, even with strongly positive market index returns, the financial stress variable does not negatively affect or penalize the model, as the R-F model outperforms the R-R model (by 2.033%). Third, when the percentiles of the index return distribution are considered, we find that the R-F model tracks the MSCI USA index particularly well, with a difference of 2.243% for $\boldsymbol{\alpha} =$ 10%. Finally, as the percentile increases and the comparison is made with a reduced sample that focuses solely on negative values, the performance gap between the models widens (up to 6.347%).[34]

Overall, the percentage differences between the RMSE of the two models in Table 8's last two rows demonstrate the benefit of including a measure of financial stress. Indeed, by conditioning the MSCI USA index on specific financial crises, we can better capture its dynamics (3.101%). Although this result appears to be more predictable when considering financial downturns, the R-F model shows better performance in terms of lower RMSE even when we examine periods of real macroeconomic slowdown identified by the NBER index (2.309%). Considering this evidence, we can confirm that the R-R model, even if it is based solely on real variables, overlooks some crises that have their roots in real activity and are better captured by a model that includes both real and financial components.

In conclusion, these results appear to support three major conclusions. First, when considering the entire time series, the R-F model tends to identify the overall dynamics better, though the difference in replicating the MSCI USA index between the R-F and R-R models is less than 1%. Second, implementing the KCFSI enables faster detection of both negative and turning points in financial markets. Third, the R-F model identifies regimes more accurately than the R-R model, particularly during stock market downturns. Furthermore, these benefits become more significant as the losses increase. Finally, and most importantly, the benefits of the R-F model are greater during financial crises and real economy recessions.

5. Discussion and implications

This section discusses some implications of using an R-R model rather than an R-F model now that the results of each model and the comparison made through the market index have been analyzed. To begin with, a model based solely on real variables generally describes history well but suffers during particularly extreme periods, failing to capture sudden changes in the scenario, abrupt deterioration of economic conditions, or real impacts resulting from financial shocks. As a result, even if such a model remains a viable option for stakeholders, its use in investment decisions implies underestimating a portion of recent history, particularly in terms of financial aspects. Indeed, while not immediately obvious, information about financial dynamics is critical for making decisions in the current context, so an R-R model can undoubtedly be improved.

More specifically, if one only wants to study the real part of the cycle, an R-R model may be sufficient until 2008 (as it was for the technological bubble of the 2000s or the attack on the Twin Towers in September 2001), but since then financial dynamics have come to exert a significant effect, albeit indirectly, on real activity. As proof of this, it has been observed that the R-R model does not perform well in the most recent period because it ignores a significant portion of history, including recent years characterized by Stagflation and

[34] As the value of α gets smaller, the area subtended to the left tail of the distribution diminishes. If $\alpha = 10\%$ we filter 41 values that are less than -4.603%; if $\alpha = 1\%$ we consider only 5 values less than -12.269%.

Slowing Down regimes, without taking into account the positive aspects of 2023, particularly on the financial front. These aspects cannot be overlooked because of the close relationship between real and financial events. Furthermore, using an R-R model allows for accurate identification of some real phenomena, albeit with slightly delayed timing when compared to a mixed R-F model. On the other hand, R-F can effectively describe trends with fewer difficulties and more efficient timing. Although inflation trends and recent shocks challenge the R-R model, the R-F model emerges as a viable option for stakeholders. Finally, an R-R model fails to accurately capture some specific extreme events involving both real and financial aspects, whereas an R-F framework is capable of addressing such scenarios more effectively.

Following these comments, policymakers, investors, corporations, and stakeholders in general can profitably take action. First, regulatory agencies, such as central banks and financial regulators, can benefit from the insights provided by both models. The dual model approach (R-R and R-F models) presented in this paper offers a more comprehensive framework for policymakers seeking to jointly manage both real economic activity and financial stability. A purely real-based model, while useful for understanding historical trends, often fails to anticipate sudden economic shifts triggered by financial stress. This is especially relevant in today's interconnected global economy, where financial markets have become a major driver of economic fluctuations. The R-F model, which includes financial stress indicators, provides a more complete picture of the overall economy, particularly during times of crisis. Policymakers can use this data to develop targeted monetary and fiscal policies to mitigate the negative effects of financial downturns. Similarly, the R-F model's ability to identify real and financial crises could help informing economic policy. Indeed, policymakers can prioritize measures to stabilize financial markets and boost economic activity during economic downturns, as indicated by the R-F model's regime classifications. Furthermore, policymakers could also implement proactive measures, such as adjusting interest rates or introducing fiscal stimulus, based on the R-F model's regime classifications, to mitigate the negative effects of a crisis before it spills over into the real economy. By monitoring financial stress indicators alongside traditional risk metrics, government authorities can identify emerging threats and take pre-emptive measures to reduce their impact. This proactive risk management approach helps to safeguard against adverse market movements and minimizes vulnerabilities. In contrast, the R-R model could be more useful during periods of economic expansion, when real variables like GDP and employment are more reliable indicators of economic health.

Second, investors and stakeholders, including institutional investors and individual traders, can use the insights provided by both models to guide their investment decisions. The R-R model, which focuses on real economic variables, can assist investors in understanding broader economic trends and identifying long-term investment opportunities. Meanwhile, the R-F model can generate early warning signals of market downturns, allowing investors to adjust their portfolios based on financial risk. Therefore, the R-F model's ability to predict financial stress is proven to be particularly valuable for investors and stakeholders. Indeed, financial markets are highly sensitive to changes in investor sentiment, and sudden downturns can have outsized impacts on portfolios. Meanwhile, assessing the overall health of the economy is more straightforward and effective with an R-R model. As a result, combining insights from both models can improve risk management strategies and can help investors make more informed decisions. During times of high financial stress, risk managers can use the R-F model's regime classifications to suggest more efficient portfolio allocations and implement hedging strategies to protect against downside risks, increasing investments in safe-haven assets or implementing hedging strategies. During periods of economic expansion, investors can use the R-R model's regime classifications to capitalize on growth opportunities while minimizing risk exposure. Indeed, the R-R model could be more suitable for identifying long-term investment opportunities during periods of economic stability. As a result, stakeholders' preferences for one model over the other may vary depending on the time horizon of interest, and they can effectively achieve their goals by combining the two models.

Third, corporations, especially those in sectors sensitive to both economic cycles and financial market volatility, can use the data provided by both models to inform their strategic decisions. Companies may prioritize expansion and investment initiatives during periods of economic growth, utilizing insights from the R-R model to identify market opportunities. During periods of economic downturn or financial stress, companies may focus on cost-cutting measures and risk mitigation strategies, which are made more effective by the R-F model's indicators. This helps companies in mitigating the impact of financial market disruptions on their operations. The insights provided by each model may affect different sectors of the economy in different ways. For example, sectors that rely heavily on consumer spending may benefit from the R-R model's focus on real economic variables, whereas sectors that are more exposed to financial market fluctuations, such as those in the financial services industry, may find the R-F model's insights particularly relevant to navigate financial market risks more effectively.

The broader implication of this research is that combining real and financial variables in business cycle modeling provides a more nuanced understanding of the economy. Overall, while the R-R model sheds light on real economic dynamics, the R-F model improves understanding of systemic risks and vulnerabilities in the overall system by separating the causes into real and financial factors. This difference is particularly relevant in times of financial crises, such as the global financial crisis of 2008 or the COVID-19 pandemic, where real economic downturns were closely linked to financial stress. By providing a clearer picture of the interplay between financial markets and real economic activity, the findings of this paper can help to enhance the design of policies aimed at promoting economic stability. Policymakers, investors, and all stakeholders can tailor interventions more effectively by combining insights from both models, addressing the economy's real and financial aspects.

6. Conclusions

In this paper, we analyze the role of real and financial variables in identifying the business cycle, the business-financial cycle, and their regimes. Our goal has been to find a modeling framework that can better capture the full dynamics of recent financial and economic events. Leveraging the literature on the relationship between financial markets and real activity as a guide, we examined the dynamics of the R-R and R-F model regimes using a bivariate MS model with four modes. To that end, we will first describe the main features of the two models, beginning with Gupta et al.'s (2014) framework. We then compared the R-R and R-F models to demonstrate the benefits of including a financial stress variable in the cycle definition.

This paper adds to the growing and expanding literature on the business and financial cycles in several ways. First, it estimated a four-regime purely real economy business cycle model, known as the R-R model, using Hamilton's (1989) framework. This model works well during the growth and recovery phases, but it ignores some negative financial factors that significantly impact real dynamics. Second, the regimes of the R-F model have greater explanatory power and are more effective in distinguishing between purely real crises, purely financial crises, and economic and financial crises. Third, the financial variable is critical in assigning appropriate weight to financial-type events. Finally, as shown by the results for the selected periods based on the NBER index, the R-F also shows local improvements in capturing real crises.

The empirical evidence suggests the following conclusions. Although the two models' regimes are not directly comparable, the R-F model outperforms the R-R model in identifying past dynamics over recent decades. Furthermore, the benefits of the R-F model become clearer when considering periods of real downturn or financial contraction. Although these conditions have only occurred infrequently, the greater precision of the R-F model provides significant advantages over the R-R model because the KCFSI captures both critical situations and turning points in financial markets and the real economy.

On the practical side, this paper has significant actionable implications. Policymakers could consider using both the R-R and R-F models to develop comprehensive economic policies. Furthermore, investors can use insights from both models to improve their diversification and risk management strategies. Finally, corporations can use the R-R and R-F models to improve their strategic decision-making. In any case, our actionable recommendations for stakeholders across domains emphasize the importance of integrating insights from both the R-R and R-F models and the R-F model's greater informative content, particularly in light of recent events and the current economic-financial context.

Our results support the hypothesis that combining a real model with a financial variable, in this case, a financial stress index, produces better results in terms of model accuracy, stock market tracking, and, most importantly, the interpretability of macro-financial dynamics over the last two decades. This approach focuses on extreme events in which financial conditions can significantly affect real economic performance. Based on these findings, we conclude that combining real and financial variables better describes the complexity of modern economic and financial systems and emphasizes the importance of financial markets.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper. The authors declare that they have no conflicts of interest to disclose.

Data availability

Data will be made available on request.

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Appendix A. the Kansas City Fed Financial Stress Index

Appendix A contains a detailed description of the Kansas City Fed Financial Stress Index (KCFSI). This monthly stress test aims to identify warning signs of a financial crisis, summarize information about critical events, and assess the financial system's health. The KCFSI construct has a mean of zero and a standard deviation of one. As a result, a positive KCSFI value indicates that financial stress exceeds the long-term average, whereas a negative value indicates that financial stress is less than the long-term average.

More specifically, the KCFSI is a composite index based on five major stress indicators: increased uncertainty about asset fundamental value; increased uncertainty about other investors' behavior; increased information asymmetry; decreased willingness to hold risky assets (flight to quality); and decreased willingness to hold illiquid assets (flight to liquidity). It consists of 11 variables reflecting stress in the US financial system that fall into two broad categories (average yield spreads and measures based on the actual or expected behavior of asset prices) and are summarized using Principal Component Analysis. Table A.1 shows the variables used in the KCFSI.

Table A.1
Components of the KCFSI.

Variable	Aspects of financial stress represented
Yield spreads	
DTCC GCF Treasury Repo/3-month Treasury (REPO) spread	Flight to quality, flight to liquidity, increased asymmetry of information
2-year swap spread	Flight to liquidity, flight to quality
Off-the-run/on-the run 10-year Treasury spread	Flight to liquidity
Aaa/10-year Treasury spread	Flight to liquidity
Baa/Aaa spread	Flight to quality, increased asymmetry of information
High-yield bond/Baa spread	Flight to quality, flight to liquidity, increased asymmetry of information
Consumer ABS/5-year Treasury spread	Flight to quality, increased asymmetry of information
Behavior of asset prices	
Correlation between stock and Treasury returns	Flight to quality
Implied volatility of overall stock prices (VIX)	Uncertainty about fundamentals and behavior of other investors
Idiosyncratic volatility of bank stock prices	Uncertainty about fundamentals and behavior of other investors
Cross-sectional dispersion of bank stock returns	Increased asymmetry of information

Table A.1 lists the 11 variables chosen as KCFSI components, as well as the financial stress characteristics represented by them. These components are synthesized using Principal Component Analysis (PCA). Source: Hakkio and Keeton (2009) and the Federal Reserve Bank of Kansas City. (https://www.kansascityfed.org/).

Given these variables, we chose the KCFSI for our analysis after careful and extensive comparison with other indicators of financial stress, such as the Federal Reserve Board of St. Louis Financial Stress Index (STLFSI), the Aboura-Diebold-Scotti Business Condition Index (ADS) of the FED of Philadelphia, the National Financial Conditions Index of the FED of Chicago, the IMF Financial Instability Index (FSI), and others.

The reasons we chose the KCFSI indicator were numerous and supported by a variety of factors. For starters, it employs a diverse set of financial and macroeconomic indicators to effectively capture the dynamics of financial markets and pressures. More specifically, the KCFSI is a comprehensive indicator that encompasses more than just one segment of the financial market, such as the equity or bond markets. As a result, this indicator is more likely to be consistent with the CLI, which is made up of a variety of variables that collectively reflect the state of the real economy. Among the many possible stress indicators, it is critical to select the index that best encompasses and represents the complexity of the financial market. Furthermore, the KCFSI is a synthetic indicator that provides a comprehensive assessment of financial stress by considering factors other than market volatility. A third notable feature of the KCFSI is the use of Principal Component Analysis (PCA) as a synthesis methodology. This method optimizes the coefficient selection for the 11 variables to maximize the index's explanatory power in terms of the total variation observed in the variables. Specifically, PCA allows for the reduction of variables while minimizing information loss and maintaining data quality. Furthermore, this methodology requires regular updates to the KCFSI, making it easier to track the evolution of financial conditions over time. Finally, the KCFSI benefits from a sufficiently long period of monthly data. Index construction, data selection, and reprocessing techniques all have an impact on the estimation process for financial market stress indicators. Thus, the availability of extensive historical series helps to improve the indicator's reliability and allows for robust stress level analysis.

Appendix B. R-R Model Selection

In Appendix B, we describe the methodology used to evaluate the R-R models and present the results of the top 20 best simulations based on accuracy. To check for accuracy, we used the MSCI USA index as a common benchmark for the R-R models. After identifying the characteristics of the four regimes, based on Equations (1)–(6) and the values of μ_{s_t} and σ_{s_t} in each s_t for the MSCI USA index in each model, as deductible from Equation (7), we derived a simulated time series for the returns of the MSCI USA index based on each R-R model.

We first defined the value of our variable of interest, that is, the original MSCI USA index, at time t as y_t and the expected time series from the R-R model of the MSCI USA index as $E(r_t)$. Then, we calculated the MSCI's time-varying expected returns using the equation below:

$$E(r_t) = \sum_{i=1}^{k} r_{s_t,i} {}^{*} p_i \tag{1}$$

where $r_{s_{t,i}}$ corresponds to the calculated return up to the preceding period, based on the prevailing regimes, while p_i is the smoothing probability for each regime.[35] The estimation is obtained as $y_t - E(r_t)$. Thus, given the time series and multiple expected time series $E1(r_t)$,$E2(r_t)$… $EN(r_t)$ from the R-R model, where N is the number of simulations, we can compute the RMSE as a comparison measure.

After excluding runs where the Simulated Annealing algorithm failed to converge, we obtained over a hundred estimations for the R-R model. Table B.1 shows the results for the top 20 most accurate R-R models based on RMSE.

Table B.1Top 20 most accurate R-R models, based on RMSE.

Model	RMSE	AKAIKE Information Criterion	SCHWARZ Information Criterion	HANNAN-QUINN Information Criterion
	(1)	(2)	(3)	(4)
R-R Model 1	4.429	−1506.287	−1348.539	−1481.328
R-R Model 2	4.431	−1504.585	−1347.116	−1480.880
R-R Model 3	4.431	−1490.283	−1335.008	−1471.551
R-R Model 4	4.436	−1505.582	−1348.174	−1480.613
R-R Model 5	4.438	−1553.154	−1397.343	−1431.510
R-R Model 6	4.444	−1547.390	−1402.687	−1502.246
R-R Model 7	4.452	−1566.426	−1412.002	−1443.519
R-R Model 8	4.473	−1502.304	−1345.874	−1478.544
R-R Model 9	4.475	−1595.457	−1442.616	−1571.059
R-R Model 10	4.476	−1386.234	−1227.168	−1356.444
R-R Model 11	4.483	−1506.453	−1348.623	−1481.344
R-R Model 12	4.485	−1573.234	−1418.119	−1505.792
R-R Model 13	4.510	−1505.709	−1348.868	−1480.782
R-R Model 14	4.511	−1474.511	−1315.453	−1445.614
R-R Model 15	4.526	−1573.010	−1417.757	−1511.243
R-R Model 16	4.537	−1497.869	−1336.203	−1475.415
R-R Model 17	4.551	−1536.246	−1377.791	−1510.042
R-R Model 18	4.555	−1503.935	−1347.364	−1480.327
R-R Model 19	4.586	−1486.805	−1326.778	−1457.862
R-R Model 20	4.588	−1495.671	−1337.020	−1466.056

Table B.1 reports the results for the top 20 most accurate R-R models, according to RMSE. In columns 2 to 4 the values of the information criteria are reported. Source: our own elaboration.

Given the available evidence, the best R-R model has an RMSE of 4.429. To better understand the significance of this result, we show the frequency distribution of all R-R model RMSEs (Table B.2). In fact, the model with the lowest RMSE does not appear to be an outlier; rather, it belongs to the first class with the highest frequency.

Table B.2
Frequency distribution of the RMSE for R-R models.

Classes	Class 1	Class 2	Class 3	Class 4	Class 5
Min	4.429	4.461	4.493	4.524	4.556
Max	4.461	4.493	4.524	4.556	4.588
Frequency	0.350	0.250	0.100	0.200	0.100

Table B.2 reports the distribution of the RMSE for the R-R models, considering five classes. For each class, the maxima, minima values and frequency are shown. Source: our own elaboration.

Appendix C. R-F Model Selection

Appendix C contains the results of a similar analysis to the one reported in Appendix B. In this case, we aimed to identify the best R-F model using the same methodology and process, which relied on Equations (1)–(7) and used the RMSE as an accuracy measure. As a result, in this case, we report the RMSE results for the first 20 most accurate R-F models. The results show that the lowest RMSE value is 4.425 (see Table C.1).

[35] The smoothing probabilities, indicated with $P(s_t = i \mid Y_T; \theta)$, are calculated monthly and consider the entire information sample, including values up to T.

Table C.1
Top 20 most accurate R-F models, based on RMSE.

Model	RMSE	AKAIKE Information Criterion	SCHWARZ Information Criterion	HANNAN-QUINN Information Criterion
	(1)	(2)	(3)	(4)
R-F Model 1	4.425	−1052.722	−894.975	−1027.764
R-F Model 2	4.458	−1031.411	−873.229	−1006.454
R-F Model 3	4.488	−1048.905	−891.724	−1023.948
R-F Model 4	4.489	−991.626	−834.977	−966.711
R-F Model 5	4.513	−939.816	−783.230	−914.899
R-F Model 6	4.518	−998.036	−841.483	−981.719
R-F Model 7	4.519	−1049.531	−894.784	−1024.294
R-F Model 8	4.523	−1008.014	−857.037	−997.274
R-F Model 9	4.527	−1056.993	−905.246	−1045.229
R-F Model 10	4.540	−881.920	−724.199	−855.812
R-F Model 11	4.586	−1056.693	−905.194	−1044.703
R-F Model 12	4.589	−1024.894	−866.448	−997.181
R-F Model 13	4.755	−903.545	−745.824	−879.539
R-F Model 14	4.809	−888.749	−730.027	−854.264
R-F Model 15	5.103	−923.985	−766.264	−899.497
R-F Model 16	5.104	−1033.610	−874.863	−1006.101
R-F Model 17	5.256	−990.531	−831.783	−964.515
R-F Model 18	5.395	−1002.301	−844.555	−976.788
R-F Model 19	5.840	−1004.183	−846.436	−978.669
R-F Model 20	5.887	−920.801	−768.055	−900.288

Table C.1 reports the results for the top 20 most accurate R-F models, according to RMSE. In columns 2 to 4 the values of the information criteria are reported. Source: our own elaboration.

Furthermore, in this case, the RMSE of the R-F model falls within a broad range of values and does not correspond to a specific or anomalous model. Table B.2 shows the frequency distribution of all R-F model RMSEs, which confirms that the best RMSE is in the first and largest class.

Table C.2
Frequency distribution of the RMSE for R-F models.

Classes	Class 1	Class 2	Class 3	Class 4	Class 5
Min	4.425	4.717	5.010	5.302	5.595
Max	4.717	5.010	5.302	5.595	5.887
Frequency	0.600	0.100	0.150	0.050	0.100

Table C.2 reports the distribution of the RMSE for the R-R models, considering five classes. For each class, the maxima, minima values and frequency are shown. Source: our own elaboration.

Appendix D. Multi-regimes Approach in Finance

The MS model (Hamilton, 1989, 1990) is a popular nonlinear model used to analyze the dynamic of a time series when multiple regimes drive the data generation process. Following the Hamilton approach, this model family has been increasingly used to study the statistical properties of financial and economic series, including structural breaks and regime changes.

Goodwin (1993) analyze the business cycle dynamics in eight develop market economies. Similarly, Kim and Nelson (1999) looked for structural changes in the volatility of US GDP growth, while Kontolemis (2001) identified two business cycle regimes, corresponding to recovering and declining phases.

Following the Lehman Brothers crisis, there was a surge of interest in the use of these models in finance. Alexander and Kaeck (2008) found that credit default swap spreads were extremely sensitive to stock volatility and even more sensitive to stock returns when financial markets were not stressful. Dionne et al. (2011) examined the impact of macroeconomic factors, primarily inflation, and regime changes on yield spreads, whereas Çevik et al. (2012) show that a time-varying transition probability MS framework was better able to explain US stock returns.[36] Then, Balcilar et al. (2015) studied the relationship between the West Texas Intermediate spot crude oil price and the S&P 500 index, showing that nonlinear models can handle nonstationary series with greater effect.[37] More recently, Hwang and Kim (2021) explored the effect of oil price shocks on US stock returns and found asymmetries in the response of

[36] To quote Çevik et al. (2012): “the MS models can be further classified into two categories, those that involve fixed transition probabilities between the regimes and those where the regime transition probabilities depend on other variables. The latter is called the time-varying transition probability MS model.”

[37] Specifically, in both high- and low-volatility regimes, an increase in the oil price has a negative (non-significant) effect on the stock price. Furthermore, in contrast to the NBER business cycle, the high-volatility regime is typically associated with recessions.

US stock returns to disaggregated shocks throughout the business cycle, particularly during depression phases.

Sarafrazi et al. (2015) used a regime-changing approach to investigate the relationship between financial and economic variables, studying stock market volatility and financial stress and identifying low- and high-volatility regimes. Finally, Hammerschmid and Lohre (2018) explore various economic variables (e.g., inflation, GDP, unemployment, etc.) and show that identifying two economic regimes for each of these variables is useful for analyzing time-varying risk premiums.

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