Timescale Methods in Economics: Wavelet Analysis of Business Cycle Fluctuations

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Received 2 Feb. 2022; Revised 18 Nov. 2022

Abstract Business cycles are defined as fluctuations of economic activity between 2 and 8 years. Since market economies are complex and evolving systems subject to internal changes and to exogenous shocks, the statistical properties of economic fluctuations are likely to change over time. Frequency-based phenomena with time-varying features are better studied using time-frequency methods. Wavelet methods are preferable to Fourier analysis because of their optimal time-frequency resolution properties and ability to address the nonstationary features of economic fluctuations. The usefulness of wavelet techniques for business cycle analysis is illustrated by the application of discrete and continuous wavelet tools to certain aspects of the business cycle: output volatility moderation and the cyclical behavior of monetary aggregates.

Keywords Business cycles • Coincident indicator • Comovements • Volatility • Wavelet transform

JEL Codes C14 • E01 • E32

AMS Subject Classification Primary: 91B82, 42C40 • Secondary: 65T60, 62M10

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1 Introduction

The idea that timescales or time horizons matter in economics is anything but new. Nearly a century ago, Tinbergen (1933) was aware that timescales, or planning horizons, were an essential aspect of agents' decisions in economic and financial markets (Le Baron 2006). Consider, for example, traders operating in the market for securities: the fundamentalists, long-term traders like central Government and pension funds, may have a very long view and rely on market fundamentals in their trading activities, and concentrate their attention on long-run variables and systematic news. The chartists, short-term traders like day traders, intra-day traders, and hedge funds, may operate with a time horizon of only weeks, days, or even hours and rely on idiosyncratic news for their trading activities. What fundamentalists deem to be variable, the chartists deem constant. Equally, firms behave in distinctly different ways when responding to short-run changes in market conditions and long-run structural changes. In the first case, the firm's reaction may consist of merely altering the length of the working day; in the latter, the firm's strategic decisions may involve installing new equipment or introducing new technologies.

Monetary and fiscal policymakers also operate on a wide range of timescales. The medium-term orientation of monetary policy, aiming at price and financial stability, allows central banks to respond flexibly to economic shocks according to their origin (demand or supply) and nature (temporary or permanent), thus avoiding excessive short-term volatility in the real economy. Government policymakers, on the back of long-term debt sustainability concerns, actively pursue both short-term macroeconomic stabilization objectives, as they did in the wake of the recent financial and pandemic crises, and long-term sustainable and inclusive growth.

Different timescales are also explicitly considered in current macroeconomic textbooks and macroeconomic modeling. The workings of several runs – the short, medium, and long term – are separately explored and studied, as are the effects of demand- and supply-side policies in the most widely used framework of current macroeconomic textbooks (e.g., Blanchard 2021). Moreover, the implications of different time horizons for the specification of structural macroeconomic models are explicitly considered by Solow in his reflections on the time-horizon consistency problem of macroeconomic modeling: "At short term scales, I think, something sort of Keynesian is a good approximation, and surely better than anything straight neoclassical. At very long scales, the interesting questions are best studied in a neoclassical framework [. . .] At the five to ten years' time scale, we have to piece things together as best as we can and look for a hybrid model that will do the job" (Solow 2000, p. 156).

The concepts of *short-run* and of *long-run*, although central for modeling economic and financial decisions, have received relatively little attention in economics and finance in comparison to other disciplines such as geophysics, engineering, physics, mathematics, signal analysis, and statistics. However, after the early empirical results reported by Ramsey and his coauthors (e.g., Ramsey et al. 1995; Ramsey and Zhang 1996; Ramsey and Lampart 1998a, b; Ramsey 1999), which showed that separation by timescale decomposition can greatly assist a deeper understanding of economic relationships that operate simultaneously at multiple timescales, wavelet analysis is rapidly gaining popularity in economics and finance (e.g., Gallegati and Semmler 2014).

In this chapter we examine the main typical feature of industrialized market-oriented economies, which is the alternation between periods of expansion and contraction in overall economic activity. In this respect, there is no need to assume anything about the underlying cause of the business cycle (impulse-response function vs. a deterministic approach) beforehand (Delli Gatti et al. 2005); rather one should recognize that the different *causae causantes* (supply of and demand for durable-, nondurable-, or investment goods, as well different macroeconomic policies and shifts of factors' revenues) systematically reveal their effects on very different timescales. Consequently, in order to understand the "stylized facts" of the business cycle, one must fully understand what is going on at a disaggregated timescale level.

2 Business Cycle Fluctuations

The business cycle has long been the central focus of macroeconomic research, and one of the most puzzling phenomena in free-market economies. Although the existence of cyclical fluctuations in the general level of economic activity was well established by the late nineteenth century (Juglar 1862; Tugan-Baranovskii 1894; Wicksell 1898), it was the great depression of 1929 that gave the greatest impulse to studying the nature, causes, and consequences of cyclical fluctuations (Mitchell 1927; Frisch and Waugh 1933; Mitchell and Burns 1938; Schumpeter 1939; Burns and Mitchell 1946; Mitchell 1951; Haberler 1958). However, the first classification scheme of economic cycles was proposed by Schumpeter (1939), and he distinguished cyclical fluctuations according to the effects of different economic variables. Schumpeter's classification, which is indicative of the presence of multiple timescales in economics, divides cyclical fluctuations into the following: (i) long wave cycles with a characteristic period of 40–60 years known as Kondratieff waves (generally associated, according to his interpretation, with major technological innovations); (ii) intermediate cycles of between 15 and 25 years (Kuznets cycles) associated with the building (construction) sector cycle and population (demographic) forces; (iii) major cycles of between 7 and 10 years (Juglar cycles) associated with business investment in plant and machinery; and (iv) short cycles of between 3 and 4 years (Kitchin cycles) generally related to fluctuations in inventory investment.

Although Schumpeter's contribution remains the most comprehensive pre-World War II work concerning the theoretical definition of economic cycles, the most influential contributions, which remain relevant for the empirical studies of cyclical movements, are the pioneering studies of Burns and Mitchell, who introduced the concept of the business cycle (Mitchell 1927; Mitchell and Burns 1938; Burns and Mitchell 1946). Their working definition states that: "Business cycles are a type of fluctuation found in the aggregate economic activity of nations that organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions and revivals which merge into expansion phase of the next cycle; the sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitude approximating their own" (Burns and Mitchell 1946, p. 3).

The traditional or classical view of the business cycle developed by Burns and Mitchell is based on the idea that the business cycle refers to recurrent, but not periodic, fluctuations in the aggregate time series of many economic activities, not only in the GDP.[4] This unobserved aggregate phenomenon is characterized by distinct cyclical phases, expansion and contractions, and common movements between economic variables. The chronology of the business cycle, based on the identification of its turning points (e.g., business cycle peaks and troughs), enables the detection of recessionary and expansionary phases which, by definition, are recurring periods of either decline or growth in the overall level of economic activity. In particular, according to the methodology adopted by the NBER (National Bureau of Economic Research), the recession is a period of significant decline in economic activity diffused across many sectors of the economy and lasting at least 6 months. A recession begins just after the economy reaches a peak of activity and ends as it reaches its trough, while between the trough and the following peak the economy is in an expansionary phase. In the classical approach to business cycle analysis, the main interest lies in the identification of the main characteristics of business cycles, which include the frequency (number), duration (the time passing from peak to trough and vice versa), and amplitude (the drop or increase in the overall level of economic activity) of the various cyclical phases and the full cycles, as well as the asymmetric behavior of different phases. Therefore, in the context of the classical business cycle, the analysis of economic fluctuations is conducted in the context of time series analysis (Box 1).

Box 1 NBER Business Cycle Chronology for the United States (1960–2021)

The NBER's Business Cycle Dating Committee maintains a chronology of US business cycles based on the classical definition of the business cycle proposed by Mitchell and Burns (1938). The chronology identifies the dates of peaks and troughs using economy-wide measures of economic activity. A recession is the period between a peak of economic activity and its subsequent trough, or lowest point, where the decline in the level of economic activity is spread across the economy and lasts more than a few months. Between the trough and the peak, the economy is in an expansion. Expansion is the normal state of the economy, so expansions are generally longer than recessions. However, the time that it takes for the economy to return to its previous peak level of activity, or its previous trend path, may be quite extended, because a recession may involve a significant decline in the level of economic activity. The determination of peaks and troughs is based on a range of economy-wide measures of real economic activity published by federal statistics agencies. These measures include real personal income less transfers, nonfarm payroll employment, employment as measured by the household survey, real personal consumption expenditures, wholesale-retail sales adjusted for price changes, and industrial production. The committee's approach to determining the dates of turning points is retrospective. In making its peak and trough announcements, it waits until sufficient data are available to avoid the need for major revisions to the business cycle chronology. When determining the date of a peak in activity, it waits until it is confident that a recession has occurred. As a result, the committee tends to wait to identify a peak until several months have passed since it occurred. Similarly, in determining the date of a trough, the committee waits until it is confident that an expansion is underway, that is, several months after the rough has been passed. The official dates of US business cycle chronology since 1960 are reported below.

(continued)

Box 1 (continued)

NBER's US business cycle reference dates

Peaks	Troughs	Contraction	Expansion	Cycle (T-T)	Cycle (P-P)
April 1960	February 1961	10	24	34	32
December 1969	November 1970	11	106	117	116
November 1973	March 1975	16	36	52	47
January 1990	July 1990	6	58	64	74
July 1981	November 1982	16	12	28	18
July 1990	March 1991	8	92	100	108
March 2001	November 2001	8	120	128	128
December 2007	June 2009	18	73	91	81
February 2020	April 2020	2	128	130	146
Average in months bv (9 cycles)		10.5	72.1	83.2	83.3

Source: https://www.nber.org/research/data/us-business-cycle-expansions-and-contractions

Figure 1 plots the US real Gross Domestic Product (GDP) over the period 1870–2018. The shaded regions correspond to NBER recessions, determined, according to the classical definition of business cycle, as the period between a peak of the level of economic activity and its subsequent trough. The most striking feature of US economic history since 1870 is the sustained growth in real GDP: it grew during this period at an average annual rate of 4.5%. Other notable features are the strong reduction in the level of economic activity after the 1929 financial crisis and in the period after World War II; the strong reduction of short-run macroeconomic fluctuations; and the reduction in the frequency of recessions in the post-World War II period. Based on the pattern of the real GDP in Fig. 1, three distinct periods may be detected: the pre-World War I period (characterized by high growth and high volatility), the interwar period (of very high volatility), and the post-World War II period (demonstrating high growth and low volatility). This separation is also more evident in Fig. 2 where the growth rate of the real GDP is plotted. In addition to the findings emerging from Fig. 1, Fig. 2 shows the almost

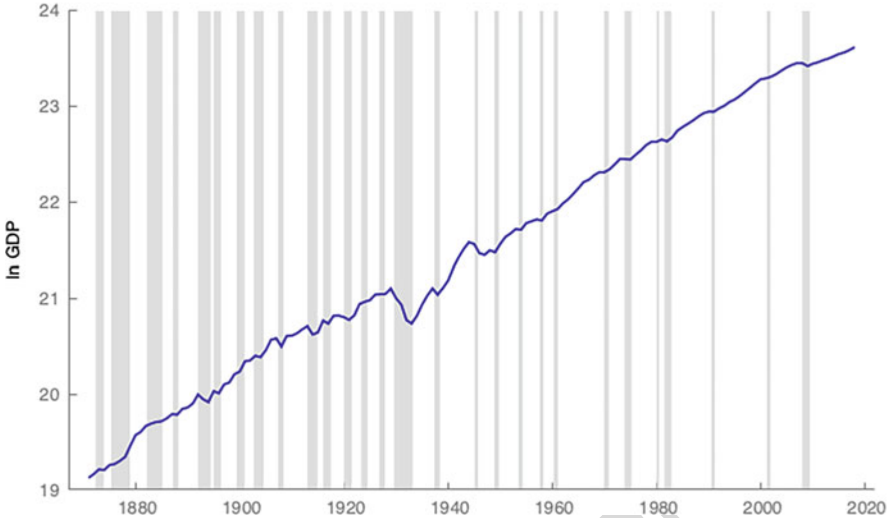

Figure 1 US real GDP (1870–2018) and NBER recessions (shaded regions). Note: Author's own calculation using Maddison Project Database (2020) data

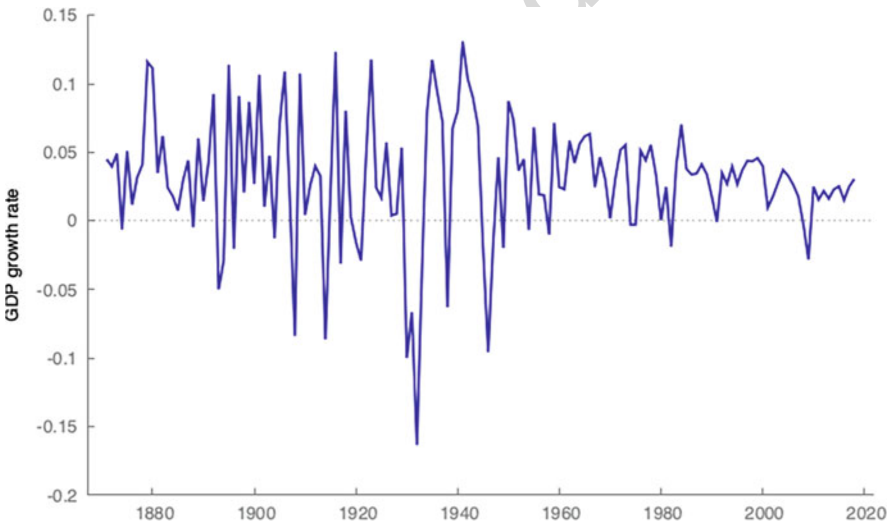

Figure 2 US GDP annual growth rates (1871–2018). Note: Author's own calculation using Maddison Project Database (2020) data

complete absence of large negative values in the GDP growth rate in the post-World War II period,[1] a result of the economic boom experienced by the Western countries, especially in the 1950s and 1960s.

[1] The only exceptions are the early 1990s recession, when the growth rate decreased by 2%, and the Great Recession in 2009.

These changes in the pattern of economic fluctuations reflect the tendency of market economies to be complex, evolutionary, dynamic systems. The shift of output and employment first from agriculture to the industrial sector, and then from the industrial to the services sector; the stabilization role of macroeconomic policy; and the globalization and financialization of the economy are the most important, but not the only, structural changes to affect the economic system of industrialized countries during the second half of the twentieth century and contribute to the increasing mildness of economic fluctuations. This moderation, and especially the reduction in the number of absolute and sustained declines in the level of aggregate economic activity experienced by many developed countries since the end of World War II, made the detection of expansionary and recessionary phases problematic. Indeed, in the traditional approach to business cycle analysis proposed by Burns and Mitchell (1946) and identified with the NBER's research methodology, expansions and recessions are defined, respectively, as periods of absolute increases and declines in the overall level of economic activity. However, during periods of fast and stable growth, such as those experienced by major advanced countries in the 1950s and the 1960s, the economy is characterized by slowdowns, rather than absolute declines in overall economic activity. Therefore, based on Burns and Mitchell's definition of cyclical phases, recessions became a rare exception because they tend to be obscured by the trend component. Given the striking reduction in the number of contractions of economic activity levels during periods of high growth trends, several NBER researchers proposed identifying business cycles with cyclical deviations from a long-term trend (Mintz 1969, 1972; Klein and Moore 1985). The analysis based on detrended data, referred to as a growth or deviation cycle, characterizes the modern approach to business cycles. According to the growth cycle definition, a recession is a low-growth phase where output is below its trend, while an expansion is a high-growth phase where output is above its trend. Business cycles are "the repeated fluctuations about trend and the regularities observed in the co-movements among different aggregative time series which are common to all decentralised market economies, and therefore, with respect to the qualitative behaviour of co-movements among time series, they are all alike" (Lucas 1977, p. 217).

Both the classical and modern definitions of business cycles share the view that the business cycle is the tendency of economic variables to possess persistent cycles of approximately constant amplitude and somewhat irregular periodicity from one cycle to the next. Based on the dates for troughs and peaks established by the NBER over the period 1854–1990, business cycles have been generally identified with fluctuations from 1½ or 2 to 8 years

(Stock and Watson 1999).[2] Therefore, business cycles may be described as fluctuations in aggregate economic activity within a specific frequency range, 2–8 years, whose statistical properties are not constant over time but change considerably, as market economies are complex and evolving systems subject to internal changes and exogenous shocks. To handle such a class of processes, that is, frequency-based phenomena with time-varying features, time-frequency methods are called for.

3 Methodology: Fourier Versus Wavelet Methods

The trend-cycle decomposition represents the main difficulty in measuring the business cycle using the modern approach. Detrending and smoothing may be pursued using time domain methods that allow researchers and economists to extract the trend component from a time series, and then to obtain the cyclical component residually. An alternative approach is to formulate the detrending and smoothing problem in the frequency domain.[3] The transition from the time domain to the frequency domain is done by a mathematical operation known as Fourier transform. The Fourier transform is given by a pair of functions, the transform pair, represented by forward and inverse Fourier transform (FFT and IFT). The Fourier transform decomposes a signal or a function into a sum of harmonic components of different frequencies via a linear combination of Fourier basic functions (sines and cosines). Thus, the Fourier transform is a frequency domain representation of

[2] Whereas some scholars generally believed that business activity was subject to cycles of 7–10 years (see Schumpeter 1939; Hansen 1951), the strong empirical evidence for the United States provided by Burns and Mitchell (1946) and later by NBER and CIBCR studies for several developed countries (France, Germany, and the United Kingdom) was supportive of the main business cycle being a relatively short or “minor” cycle, generally varying from 3 to 4 years (i.e., a Kitchin cycle). Indeed, the observed timing of fluctuations of the early business cycles in the NBER chronology conformed well to the cycle length popularized as the Kitchin cycle by Schumpeter (1939). Kitchin (1923) distinguishes between minor cycles averaging 3 and a half years and major cycles (or trade cycles) which are an aggregation of two or three minor cycles. Similar results in terms of average duration for economic fluctuations are provided by Hansen (1951), who identified 27 minor cycles between 1837 and 1937 for the United States. All of these results were supportive of the rejection of the notion of a regular pattern of economic activity of roughly 7–10 years’ duration (Matthews 1959).

[3] Time domain analysis tells us everything about the value of a signal at a specific location, but little about its frequency content. Frequency domain analysis tells us everything about the frequency content of a signal, but not when these frequencies occur.

a signal or a function containing the same information of the original function but summarized as a function of frequency. As such, it may be interpreted as a decomposition of a signal on a frequency-by-frequency basis.

Both methods have some drawbacks: pure time domain analysis averages the relationships over the entire frequency range (frequency domain aggregation), while frequency domain analysis averages over the time domain and thus stationarity becomes a critical assumption. However, the assumption of stationarity is often violated for many practical signals. In order to overcome the main limitation of the Fourier transform, which is its inability to deal with nonstationary signals, a modified time-dependent version of it has been developed, that is, the Gabor transform or short-time (or time-variable) Fourier transform (STFT) (see Gabor 1946). Time-frequency analysis represents a compromise between them, as it provides an estimate of the frequency structure of a signal locally at a given time point. The short-time Fourier transform uses a fixed-width window function with respect to frequency. The original signal is partitioned into sufficiently small segments such that these portions of the nonstationary signal can be assumed to be stationary over the duration of the window function. The choice of the window length is based on the trade-off between the desired frequency resolution, which depends inversely on the duration of the window function, and the assumption of short-time stationarity. Once the window function is determined, both the time and the frequency resolutions become fixed for all frequencies and times, respectively. Hence, the short-time Fourier transform does not allow any change in resolutions in terms of time or frequency.

An alternative to the short-time Fourier transform for the analysis of nonstationary signals is represented by the wavelet transform. Wavelet analysis can overcome the main problems evidenced by the Fourier transform (and the short-time Fourier transform) because its frequency resolution is not uniform over the entire frequency range. The term "wavelets" literally means small waves and is used because the transforms have finite length (i.e., they are compactly supported) and oscillatory behavior. They are particular types of basic functions used to decompose a function (t) in more elementary functions that include information about (t). These basis window functions used in transforming the signal are derived from a single prototype function, called the mother wavelet, through its scaled (dilated or compressed) and translated versions. Scaling refers to the level of detail in the analysis of the signal, and translation refers to the location of the window. Thus, in contrast to the fixed time-frequency partition of the short-time Fourier transform, wavelet multiresolution analysis adaptively partitions the time-frequency plane using short windows at high frequencies and long windows at low frequencies. Figure 3 compares the time-frequency resolution properties of

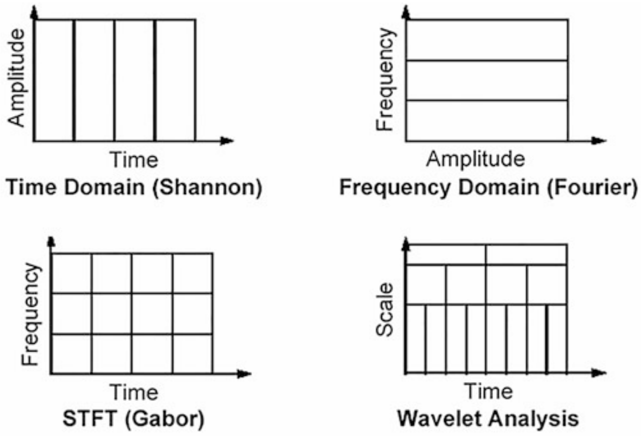

Figure 3 The set of possible domains in the time-frequency plane

the short-time Fourier transform (lower left panel) and the wavelet transform (lower right panel), along with, in the upper panels, the time domain (Shannon) and frequency domain (Fourier) methods. Unlike the short-time Fourier transform, which has a fixed partition of the time-frequency plane and thus constant resolution at all times and frequencies, the wavelet transform, through the optimal partition of the time-frequency plane, provides good frequency resolution (and poor time resolution) at low frequencies and good time resolution (and poor frequency resolution) at high frequencies.

In economics, wavelet analysis originated as an alternative analytic method to classical Fourier methods in signal analysis.[4] Although the wavelet transform was introduced in the early twentieth century, when Alfred Haar proposed the simplest orthonormal wavelet basis function with compact support (Haar 1910), wavelet applications first became widespread in the early 1980s for the analysis of seismic signals (Morlet et al. 1982a, b). Since then, wavelet applications have rapidly grown in the areas of signal processing and data compression in many other disciplines like engineering, physics, mathematics, statistics, and economics and finance.[5]

[4] Frequency domain methods introduced in the early 1960s (e.g., Hannan 1963; Nerlove 1964; Granger and Hatanaka 1964) were used to uncover key characteristics of economic time series such as the typical spectral shape of an economic variable (Granger 1966), and to run regression analysis in the frequency domain, for example, band spectrum regressions (Engle 1974).

[5] After the initial studies undertaken by Ramsey and his coauthors, applications in economics and finance have increased significantly in recent years (Gallegati and Semmler 2014).

Wavelets are mathematical functions that transform a signal into a mathematically equivalent representation using a set of orthogonal basis functions, named wavelets, which are defined by a compactly supported, localized wavelet function that is dilated (or compressed) and shifted to provide a flexible timescale window.[6] Thus, in contrast to Fourier analysis, whose basis functions are defined globally over the whole computational domain, wavelet analysis can handle a variety of nonstationary and complex signals and attain an optimal trade-off between time and frequency resolution levels. Moreover, the wavelet method provides a "model-free" approach to frequency extraction problems, as it allows simultaneous estimation of different unobserved components without making any explicit assumption about the characteristics of the data generating process.

Wavelets are so-called because they have finite length (compactly supported) and oscillatory behavior. The wavelet transform is designed based on some desired properties associated with that function and the admissibility and regularity conditions. According to the admissible condition, the wavelet must oscillate to have its mean value equal to zero:

$$\int_{-\infty}^{\infty} \psi(t)dt = 0$$

According to the regularity condition, the wavelet has exponential decay so that the wavelet must oscillate and is localized in the sense that it decreases rapidly to zero as t tends to infinity:

$$\int_{-\infty}^{\infty} \psi(t)^2 dt = 1$$

All wavelet functions used in the transformation of the signal are generated from a basis wavelet function $\psi(t)$, through scaling and translation, as follows:

$$\psi_{j,k}(t) = 2^{-j/2}\, \psi\left(\frac{t - 2^j k}{2^j}\right)$$

where $\psi_{j,k}$ is a daughter wavelet function, defined as the scaled and translated version of the mother wavelet $\psi(t)$. The scale factor 2^j is the dilation factor and controls the length of the wavelet. Large scales (low frequencies) dilate

[6] Useful introductions to wavelets for economists are provided in Gencay et al. (2002), Crowley (2007), and Aguiar-Conraria and Soares (2014).

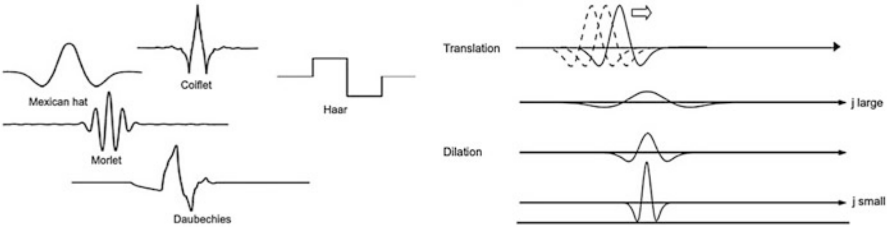

Figure 4 Types of wavelet families (left panel), and translation and dilation effects (right panel)

the signal and provide global information about the signal, while small scales (high frequencies) compress the signal and provide detailed information hidden in the signal.[7] The translation parameter 2^jk is called the location parameter and indicates where the wavelet is centered along the signal (and also indicates the nonzero portion of each wavelet basis vector).[8]

Figure 4 shows several commonly used base wavelets (left panel) and the effects of translation and dilation on the wavelet functions (right panel). Among the types of wavelet families, the Mexican Hat and Morlet wavelets are used for performing the continuous wavelet transform (CWT), while the Haar, Daubechies, and Coiflet wavelets are used for the discrete wavelet transform (DWT).

The projections of the time series onto the basis generated by the chosen family of mother wavelets represent the wavelet detail coefficients, $d_{j,k}$, given by the following integral:

$$d_{j,k}(t) = \int \psi_{j,k}(t) f(t) dt$$

where $j = 1, 2, \ldots, J$ is the number of scales (or multiresolution components) and k ranges from 1 to the number of coefficients in the specified component.

A complete reconstruction of the signal requires a scaling function, $\varphi(t)$, also known as the "father wavelet," that represents the smoothest components of the signal and is defined as:

$$\varphi_{J,k}(t) = 2^{-J/2} \varphi\left(\frac{t - 2^J k}{2^J}\right)$$

[7] Dividing by $1/\sqrt{2^j}$ ensures that the energy of the wavelet family will remain the same at different scales.

[8] The translation parameter is matched to the scale parameter in the sense that as the wavelet basis functions broaden, their translation steps become correspondingly larger.

While the mother wavelet integrates to 0 and captures all deviations from the trend, the father wavelet integrates to 1 and reconstructs the smooth part of the signal. The projection of the signal onto father wavelets represents the wavelet scaling coefficients, $s_{J,k}$:

$$s_{J,k}(t) = \int \varphi_{J,k}(t) f(t) dt$$

Therefore, while the wavelet detail coefficients $d_{J,k}, \ldots, d_{2,k}, d_{1,k}$ represent weighted "differences" at each scale and progressively finer scale deviations from the smooth behavior (capturing high-frequency oscillations), the smooth coefficients $s_{J,k}$ represent averaging at each scale, and correspond to the smooth behavior of the data at the coarsest scale 2^J (capturing low-frequency oscillations).

Given the wavelet detail and scaling coefficients, $d_{j,k}$, and $s_{J,k}$, from the functions:

$$S_{J,k} = \sum_k s_{J,k} \varphi_{J,k}$$

$$D_{j,k} = \sum_k d_{j,k} \psi_{j,k}$$

we may obtain what are called the *smooth* signal, $S_{J,k}$, and the *detail* signals, $D_{j,k}$, respectively. The sequence of terms $S_J, D_J, \ldots, D_j, \ldots, D_1$ for $j = 1, 2, \ldots, J$ provide the approximation to the signal $f(t)$ as:

$$f(t) \approx S_J + D_J + D_{J-1} + \cdots + D_2 + D_1$$

where S_J contains the "smooth component" of the signal, and the D_j, $j = 1, 2, \ldots, J$, detail signal components at ever increasing levels of detail. If, to use an analogy, S_J provides the large-scale road map, then D_1 shows the potholes. The previous equation provides the multiresolution decomposition of a signal where each component is associated to a specific frequency band and has a corresponding frequency domain interpretation.

4 Wavelet Applications for Business Cycle Analysis: Breaks in Volatility and Comovements

Recent theoretical and empirical business cycle research has focused on the main statistical properties of business cycle fluctuations to look for regularities in terms of persistence, volatility, asymmetry, and synchronization, as measured by autocorrelation, standard deviation, and cross-correlations, respectively. In what follows, we apply wavelet analysis to some "stylized facts" widely analyzed in the literature: the change in the volatility pattern of output fluctuations and the cyclical behavior of monetary aggregates.

4.1 Changes in Output Volatility

The decline in the US output volatility after World War II has been extensively documented in the literature, especially for the period after the mid-1980s (e.g., Kim and Nelson 1999; McConnell and Perez Quiros 2000; Stock and Watson 2002). For instance, Blanchard and Simon (2001) showed that since 1984, the standard deviation of quarterly growth in the United States declined by half while that of inflation declined by two thirds. This "Great Moderation," as defined by Stock and Watson (2002), has fostered the debate on the nature and the causes of this reduction in output fluctuations, especially as it is common to most industrialized countries. Controversy remains as to whether this moderation is due to the absence of large adverse shocks after mid-1980s (McConnell and Perez Quiros 2000) or is the result of a decline which started in the late 1950s and was temporarily interrupted only in the 1970s and early 1980s (Blanchard and Simon 2001). Several explanations have been proposed for the causes of this volatility moderation: improvements in monetary policy ("good policies"), and a fortuitous reduction in exogenous disturbances ("good luck").[9]

Time-frequency decomposition methods may be useful for detecting the sources of the changes in volatility by determining whether the reduction in volatility is associated with specific frequencies or periods. For example, Ahmed et al. (2002) and Fritsche and Kuzin (2005) apply a frequency domain approach to investigate the sources of the decline in US output volatility, using the spectrum of GDP growth to decompose its variance by frequency. Since each explanation can be associated with a specific pattern for the shift of the spectrum, it is possible to develop some insights into the nature of the volatility decline by determining the specific frequency range at which the downward shift in the spectrum occurs. The same goal may be

[9] Better business practices ("good practices"), such as better inventory management techniques, more sophisticated financial markets, or expanding international trade flows, are also considered potential explanations for the decline in output volatility. However, since the decline in variance reflecting better business practices is expected to occur primarily at relatively high frequencies, we are forced to exclude this explanation from our analysis.

pursued using wavelet analysis, with the advantage that the assumption of covariance-stationarity for output growth is not needed anymore. Thus, wavelet analysis may be a suitable instrument to detect the relative importance of the various explanations for the moderation of volatility. Indeed, once such explanations have been associated with specific timescales, an informal test of the different explanations for the "moderation" of volatility may be carried out. In particular:

- Improvements in (monetary and fiscal) policy management, in the form of a more aggressive response to inflation and output changes, are likely to be associated with business cycle timescales.
- Exogenous shocks are likely to be associated with different scales depending on their impact (temporary or permanent) and on their nature (demand or supply-side). For example, temporary shocks like strikes and other temporary influences in aggregate production are likely to be associated with short-term scales; on the other hand, the effects of permanent shocks, like permanent increases in oil prices or productivity shocks, are likely to be associated with all scales. This is because demand-side effects (such as those affecting consumers' spending) are likely to be associated with short-term scales, while supply-side effects (such as those affecting production and capital investment decisions) are likely to be associated with longer scales.

The application of the DWT is normally based on a dyadic arrangement of scales and locations (i.e., integer powers of two), a solution that reduces the original N data points into two series of length $N/2$: one of these contains the smoothed information[22] and the other contains the detail information. By keeping the details and performing an additional transform of the smoothed series, we can produce two series of length $N/4$, with smoothed and detail information, and so on. If the original time series was some power of 2, $N = 2^J$, then the number of coefficients at the end would total N, and would contain all the information in the original time series, organized according to scale and location, the number of coefficients at each scale being:

$$N = N/2^J + N/2^J + N/2^{J-1} + \cdots + N/4 + N/2$$

Any function $f(t)$ in $L^2(R)$ can be represented by the following wavelet series expansion:

$$f(t) = \Sigma_k\, S_{J,k}\varphi_{J,k}(t) + \Sigma_k\, d_{J,k}\psi_{J,k}(t) + \Sigma_k\, d_{J-1,k}\psi_{J-1,k}(t) + \cdots + \Sigma_k\, d_{1,k}\psi_{1,k}(t)$$

with $N/2^J$ $S_{J,k}$ coefficients, $N/2^J$ $d_{J,k}$ coefficients, $N/2^{J-1}$ $d_{J-1,k}$ coefficients … and $N/2$ $d_{1,k}$ coefficients.

However, because of the practical limitations of the DWT, wavelet analysis is generally performed by applying the Maximal Overlap Discrete Wavelet Transform (MODWT), a compromise between the continuous wavelet transform (CWT), with its continuous variations in scale, and the DWT, which uses only a limited number of translated and dilated versions of the mother wavelet to decompose the original signal. Unlike the DWT, the MODWT can be applied to any sample size, not only to datasets of length multiple of 2^J, and returns at each scale a number of wavelet and scaling coefficients equal to the length of the original series.[15] Although the MODWT is highly redundant, so that transformations at each scale are not orthogonal, the offsetting gain is that the transform is phase invariant, a very useful property in analyzing transformations (Percival and Walden 2000). In sum, the MODWT (i) is translation invariant, as shifts in the signal do not change the pattern of coefficients, (ii) can be applied to data sets of length not divisible by 2^J, and (iii) returns at each scale a number of coefficients equal to the length of the original series.

Figure 5 shows the results from the application of the MODWT,[10] which is the most widely used filter in economic applications. Since the level of the transform defines the effective scale of the corresponding wavelet coefficients, for all families of Daubechies (1992) compactly supported wavelets the level j wavelet coefficients are associated with changes at scale 2^{j-1}. Since scale 2^{j-1} corresponds to frequencies in the interval $[1/2^{j+1}, 1/2^j]$, using annual data scale 1 wavelet coefficients are associated with 2–4 years, while scales 2–4 are associated with periods of 4–8, 8–16 and 16–32 years, respectively. From the frequency interpretation of wavelet detail levels, we infer that the detail level components D_1 and D_2, capturing fluctuations between 2–4 years and 4–8 years, respectively, correspond to the business cycle frequency range used in the modern definition of business cycles (Stock and Watson 1999) (Box 2).[11]

[10] The boundary method used in the wavelet decomposition is the reflection boundary condition. In this method, an extension is made by pasting a reflected (time-reversed) version of the original signal of length T at its end and then applying a periodic boundary condition on the first T elements of the reconstructed signal (and omitting the remaining T elements).

[11] The 4-level decomposition extracts the smooth component S_4 (not shown in Table 1), which captures oscillations with a period longer than 32 years that correspond to the very low-frequency components of the signal.

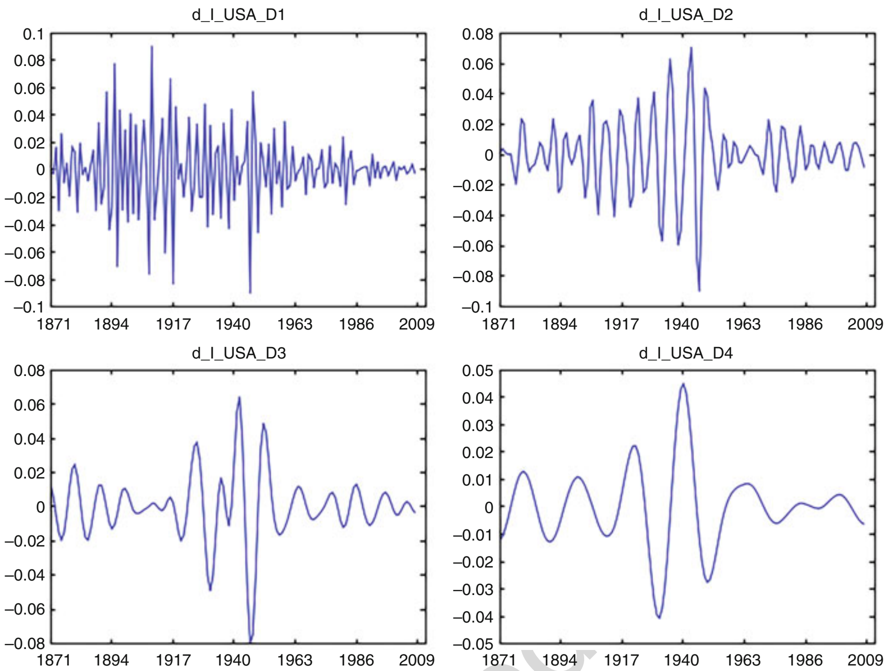

Figure 5 MODWT 4-level multiresolution decomposition for the US real GDP

Box 2 Wavelet Versus Band-Pass Filtering Methods

The wavelet method provides a systematic way of performing band-pass filtering (Proietti 2011). As the wavelet filter belongs to the high-pass filter with passband given by the frequency interval $[1/2^{j+1}, 1/2^{j}]$ for scales $1 < j < J$, inverting the frequency range to produce a period of time means that wavelet coefficients associated with scale $j = 2^{j-1}$ are associated with periods $[2^{j}, 2^{j+1}]$. Table 1 provides the frequency domain interpretation of the timescale components extracted through the DWT or MODWT. The sum of the D_1 and the D_2 components, representing fluctuations between 2 and 8 years, provides an estimate of the business cycle component. Band-pass filtering methods have been widely used to extract the unobserved cyclical component corresponding to fluctuations in the 1.5(2) to 8 years frequency band. To avoid the stationarity problems of filtering in the frequency domain, band-pass filters such as Baxter and King (1999) and Christiano and Fitzgerald (2003) are performed in the time domain, although their desired properties are formulated in the frequency domain (Stier 1989; Metz 1992). They differ in the assumptions about the spectral density of the variables and the symmetry of the weights of the filter.

(continued)

Box 2 (continued)

Table 1 Frequency interpretation of (MO)DWT multiresolution decomposition analysis with annual data

Detail level, D_j	Years
D_1	2–4
D_2	4–8
D_3	8–16
D_4	16–32

Regarding the first assumption, the approximation by Baxter and King assumes independent and identically distributed variables, whereas Christiano and Fitzgerald assume a random walk. As to the latter, Baxter and King develop an approximate band-pass filter with symmetric weights on leads and lags in order to avoid the filter introducing phase shift in the cycles of filtered series, whereas Christiano and Fitzgerald's asymmetric filter has the advantage of avoiding the loss of observations at the beginning and end of the sample. Therefore, since a random walk puts more weight on lower frequencies (independent and identically distributed variables weight all frequencies equally), the filter by Baxter and King approximates the ideal band-pass filter for shorter business cycles with higher accuracy than the filter created by Christiano and Fitzgerald. Equally, the filter by Christiano and Fitzgerald is expected to approximate the ideal band-pass filter for cycles with long durations better than the filter by Baxter and King. In sum, the optimizing criteria adopted by band-pass filter approximations implicitly define the specific class of model for which the approximating filter is optimal. By contrast, the wavelet method allows the simultaneous estimation of different unobserved components without making any explicit assumption about the characteristic of the data generating process. It can thus be considered a "model-free" approach to frequency extraction problems, as they are optimal under any time series representation of the process. This feature is critical for long-term historical data when complexity and nonlinearities are intrinsic features of these datasets. The secular movements in macroeconomic time series are likely to display short-lived transient components like abrupt changes, jumps, and volatility clustering, typical of war episodes or crisis episodes; and to exhibit structural changes in the trend function because of policy changes or paradigm shifts (e.g., Clementi et al. 2015; Gallegati et al. 2017).

A simple frequency-based measure of economic volatility may be represented by the standard deviation of each timescale's components of the growth of real GDP. Figure 5 provides striking, though informal, evidence on the occurrence of several breaks in the volatility pattern of output growth. The timescale components D_3 and D_4, which represent fluctuations beyond the business cycle frequency range (i.e., between 8 and 32 years), display similar patterns. Apart from the sharp increase in the period between the two World Wars, which is common to both components, the decline in output volatility between the pre- and post-World War period is evident for the longest timescale component only, a finding that is compatible with the structural change explanation associated with economic development. In contrast, the volatility pattern at business cycle timescale D_1 and D_2 displays interesting differences. Several breaks in volatility are evident at the higher business cycle frequency range (2–4 years): these occur in the early 1890s, before World War I and at the end of World War II, with volatility first increasing (from 0.2 to 0.4) and then reducing (from 0.4 to 0.3). The post-World War II period is characterized by a constantly slowing declining volatility trend (from 0.3 to 0.05).[12] At the lower business cycle frequency range D_2 (4–8 years), several breaks are obvious. The first, at the end of World War II, occurs after a long period in which volatility was progressively increasing; the other two coincide with the oil shocks period. Sharp increases in oil prices occurred in response to the Arab-Israeli war in 1973, the Iranian revolution in 1978, and the Iran-Iraq war in 1980. These oil price shocks represent examples of what Hamilton (2003) identified as major oil supply disruptions, the only that matter for macroeconomic stability.

To summarize, based on the volatility pattern displayed in the upper panels of Fig. 3, it appears that the drop in the volatility of GDP growth occurred primarily at the higher business cycle frequencies of 2–4 years. At lower business cycle frequencies, 4–8 years, the decline in volatility after the mid-1980s is only relative to the volatility of the oil shocks period. Indeed, the periods before and after the oil shocks period of 1970 to the early 1980s are characterized by close values of output variability. As a result, the stabilization described by the literature is the effect or consequence of the reduction of short-term business cycle fluctuations. We take our evidence to be supportive of both the “good policy” and “good luck” hypotheses, although it does not completely rule out other explanations. Improved policy has played a leading role in explaining the fall in the volatility of shorter-term output fluctuations, with the decline in the variance of structural (exogenous) shocks hitting the economy mainly influencing the volatility of lower business cycle frequencies.

[12]The only exception is the temporary increase of output volatility during the oil price shocks of the 1970s.

4.2 The Cyclical Behavior of Money

A key feature of the modern business cycle approach concerns the analysis of business cycle regularities as "co-movements of the deviations from trend in different aggregative time series" that are common to all decentralized market economies with no restriction "to particular countries or time period." As a result, business cycles are all alike with respect to the qualitative behavior of comovements among series (Lucas 1977, p. 217). The patterns of comovements over time in the cyclical components of macroeconomic variables with those of real output are of primary interest to business cycle researchers. For instance, the performance of Real Business Cycle models[13] is based on the qualitative comparison of model properties, in terms of standard deviation, correlation, and serial correlations of simulated data, against the corresponding statistical features of macroeconomic aggregates. Among the key stylized facts considered (i.e., volatility, persistence and comovements), comovements represent the most significant statistical feature to examine (Box 3).

Box 3 Business Cycle Stylized Facts

A key issue of the modern business cycle approach concerns the analysis of the statistical properties of a certain set of important macroeconomic time series. Business cycle empirical regularities, the so-called stylized facts, are identified in terms of the volatility and persistence of the cyclical component of output, and the cyclical behavior of macroeconomic time series. Output persistence can be measured in the time domain by the autocorrelation function (ACF), which computes the correlation of GDP with its own past previous time periods, while a simple measure of volatility may be represented by the standard deviation of the cyclical component of output. Finally, the cyclical behavior of macroeconomic variables with output may be measured through cross-correlation analysis, which can provide information both on the direction and the timing of a variable's movement relative to aggregate economic activity. From the sign of the contemporaneous correlation coefficient, we can define a variable as procyclical (same direction), a-cyclical (no clear pattern), or countercyclical (opposite direction), and according to the timing of the maximum correlation coefficient we can determine whether that variable is leading, synchronous, or lagging the cycle. Specifically, according to the value

(continued)

[13] Real Business Cycle models are a class of macroeconomic models that attribute fluctuations in business cycles to real shocks such as technological or productivity shocks, rather than to nominal or monetary shocks.

Box 3 (continued)

of the contemporaneous correlation with output, we define a series as "procyclical" when its value is strongly positive; "a-cyclical" when it is around zero; and "countercyclical" when it is strongly negative. Moreover, we say that a series, x, is leading, is synchronous or is lagging the cycle if the largest value of its cross-correlation with output (in absolute value) is in entries $x(t-i)$, $x(t)$ or $x(t+i)$, respectively.

The main stylized facts for developed countries (as found, e.g., in Backus and Kehoe 1992) show that cyclical fluctuations of macroeconomic variables are highly persistent (first-order autocorrelation are generally greater than 0.80 and mostly greater than 0.90). In terms of volatility, it is possible to distinguish between variables far less volatile than output (such as consumption, labor market variables, money supply, and long-term interest rate) and variables much more variable than output (such as investments, short-term interest rate, and, especially, stock prices), for example, Gallegati and Gallegati (2003). The set of stylized facts reported in the literature about the cyclical behavior of macroeconomic variables is presented in Table 2.

Table 2 Cyclical behavior of macroeconomic variables: stylized facts

Variable	Direction	Timing
Demand components		
Consumption	Pro-cyclical	Coincident
Investment	Pro-cyclical	Coincident
Government spending	A-cyclical	Uncertain
Net exports	Counter-cyclical	Leading
Labor market		
Employment	Pro-cyclical	Coincident
Unemployment	Counter-cyclical	Lagging
Labor productivity	Pro-cyclical	Coincident
Real wage	A-cyclical	Uncertain
Monetary and financial aggregates		
Money growth	Pro-cyclical	Leading
Inflation rate	Pro-cyclical	Lagging
Stock prices	Pro-cyclical	Leading

The cyclical behavior of monetary variables has been widely investigated in the macroeconomic literature as monetary factors are considered an important element of business cycle fluctuations (Friedman and Schwartz

1963; Friedman 1968). The observed positive comovements between nominal money stocks and real activity over the business cycle have been one of the main challenges to monetary economics.[14] However, in recent decades the role of money has reduced significantly in macroeconomic modeling and in the conduct of monetary policy. Monetary analysis, after the monetary targeting experience of the 1970s, now occupies a secondary role with respect to economic analysis in central banking policy-making strategy, with interest rate rules replacing monetary aggregates in the conduct of monetary policy. Therefore, monetary factors are now largely ignored in today's consensus macroeconomic equilibrium models where real factors are the key to economic fluctuations, for example, Real Business Cycle and Dynamic Stochastic General Equilibrium models.[15]

The DWT uses only a limited number of translated and dilated versions of the mother wavelet to decompose the original signal. Its continuous analogs, the continuous wavelet transform (CWT), operates on smooth continuous functions, and decomposes a signal on all scales, producing information in a two-dimensional format where each wavelet coefficient is represented by a pair of data, designing time, or location, and scale (Gencay et al. 2002). As a results, the CWT, in contrast to the DWT, is a highly redundant transform.

If we denote with s the scale levels and with u the translation parameter in the continuous case, the CWT of a signal $x(t)$, $W_x(s,u)$, is defined as:

$$W_x(s,u) = \int_{-\infty}^{\infty} f(t)\psi^*_{s,u}(t)dt$$

where the superscript * indicates the complex conjugate. The set of CWT wavelet coefficients, each representing the amplitude of the wavelet function at a specific position and scale, is obtained by projecting $x(t)$ onto the family of "wavelet daughters" $\psi_{(s,u)}$ defined as:

$$\psi_{s,u}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-u}{s}\right)$$

[14] Nominal money growth tends to fall in the late stages of an expansionary phase as banks become increasingly restrained in their ability to create deposits by the availability of reserves. Real money balances would typically decline before an economic downturn, as the increase in prices usually picks up late in the cycle.

[15] Dynamic Stochastic General Equilibrium models represent the current macroeconomic framework adopted by Central Banks for their economic and monetary policy analyses.

where the scale and translation parameters that characterize the "mother wavelet" $\psi(t)$, s, and u respectively, are allowed to vary continuously.[16]

The squared (absolute) values of the wavelet coefficients, denoted as $|W_x(s,u)|^2$ and referred to as the wavelet power spectrum, depict the local variance of $x(t)$ and can be interpreted as the energy density of the signal in the time-frequency plane. The values of the power spectrum $|W_x(s,u)|^2$ are visualized by using contour plots where the color of each point measures the amount of signal energy contained at a specific scale and location. $|W_x(s,u)|^2$ represents the wavelet power and can be interpreted as the energy density of the signal in the time-frequency plane of a signal $x(t)$ with respect to the wavelet function ψ is a function $W_x(s,u)$.

Let $W_x(s,u)$ and $W_y(s,u)$ be the continuous wavelet transform of the signals $x(t)$ and $y(t)$, so that their wavelet cross-spectrum is given by:

$$W_{xy}(s,u) = S\left(W_x^*(s,u)W_y(s,u)\right)$$

where S is a smoothing operator in time and scale in order that the coherence estimator is not simply unity (see Torrence and Webster 1999). The wavelet cross-spectrum depicts the local covariance of two time series at each scale, while frequency can identify regions where two time series have high common power. Frequency is also analogous to the local covariance at each timescale (see Hudgins et al. 1993). However, being the product of two non-normalized wavelet spectra, the cross-wavelet spectrum can identify significant regions between two time series, although there is no significant correlation between them.

The wavelet coherence, defined as the modulus of the wavelet cross-spectrum normalized by the wavelet power spectrum of each signal, as seen in the following equation:

$$WC_{xy}(s,u) = \frac{|S\left(W_x^*(s,u)W_y(s,u)\right)|^2}{S(|W_x(s,u)|) \cdot S\left(|W_y(s,u)|\right)}$$

can overcome this problem by detecting regions in the time-frequency space where the examined time series comove, but do not necessarily have a high common power. The squared wavelet coherence coefficient $WC_{xy}(s,u)$ can be

[16] The continuous and discrete scale parameters s and j are linked by the following relationship: $s = s_0^j$, so that when the computation is done octave by octave, that is, $s^0 = 2$, the scaling parameter in the discrete case is $s = 2^j$. When the Nyquist sampling rule is used the link between the translation parameters u and k is: $u = k\, u_0\, s_0^j$, so that when $u_0 = 1$ and $s^0 = 2$ the translation parameter in the discrete case is $u = k\, 2^j$.

considered a direct measure of the local correlation between two time series at each scale, and it is analogous to the squared correlation coefficient in linear regression. Hence, it can be used to assess how the degree of association between two series changes across frequencies and over time. Moreover, from the imaginary and real parts of the cross-wavelet transform, we can get information about the relative position of the two series through the phase difference, defined as:

$$\phi_{xy} = \tan^{-1}\left(\frac{\Im\left[W_{xy}(s,u)\right]}{\Re\left[W_{xy}(s,u)\right]}\right)$$

To explore the pattern of comovements between monetary aggregates and the business cycles, we apply the main tools of the continuous wavelet transform (CWT), which are wavelet coherence and phase. In order to apply the CWT, we need to choose a wavelet family filter.[17] The most widely used type of mother wavelet is the Morlet wavelet introduced by Goupillaud et al. (1984), which has optimal joint time-frequency concentration in the Heisenberg sense. The Morlet wavelet is defined as a plane wave modulated by a Gaussian envelope, thus:

$$\psi(t) = \pi^{-1/4} e^{-t^2/2} e^{i\omega_0 t}$$

where t is dimensionless time, ω_0 is the dimensionless wavelet central frequency, and the $\pi^{-1/4}$ term is a normalization factor that ensures that the wavelet has unit energy. The parameter ω_0 controls the time-frequency resolution trade-off. The choice $\omega_0 = 6$ provides a good balance between time localization and frequency localization and simplifies the interpretation of the wavelet analysis. It does so because the wavelet scale, s, is inversely related to the frequency, $f \approx 1/s$ (Grinsted et al. 2004).

Figure 6 shows the plots of the wavelet-squared coherency between real money supply (M2) and the coincident economic indicator (TCB-CEI).[18]

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[17] All practical implementations of the CWT use a near-continuous discretization.

[18] The series real M2 is an inflation-adjusted version of the M2 money supply, which includes currency, demand deposits, other checkable deposits, travelers' checks, savings deposits, small denomination time deposits, and balances in money market mutual funds. The TCB-CEI is the composite coincident indicator developed by the Conference Board for measuring and dating business cycles in the aggregate economy using Employees on Non-Agricultural Payrolls, Index of Industrial Production, Real Personal Income less Transfer Payments, and Real Manufacturing and Trade Sales as individual components (*The Conference Board* 2000). Monthly data from the TCB dataset are used for the period 1959:1–2017:4.

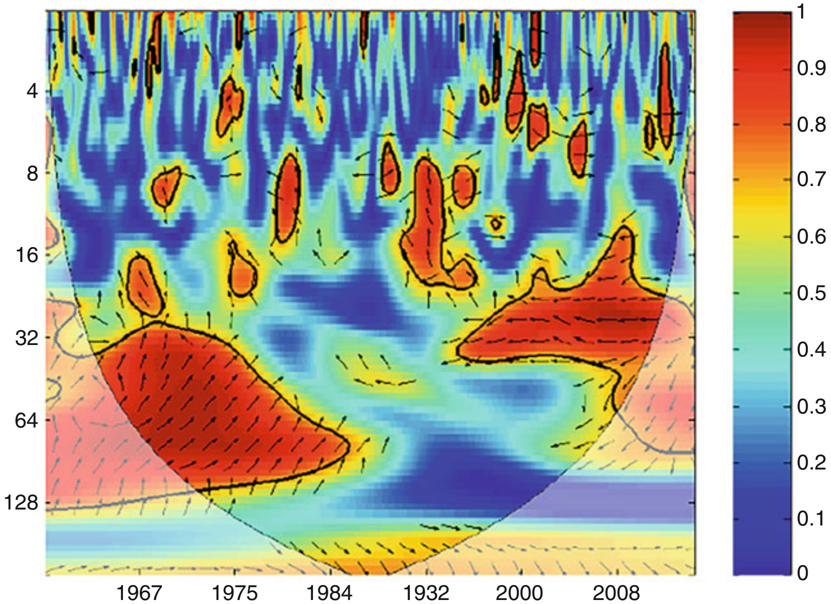

Figure 6 Wavelet coherence and phase between the TCB-CEI and real M2 (1945–2015). Note: Squared wavelet coherence between TCB-CEI and real M2 in Fig. 4 was calculated using the wavelet coherence Matlab package presented in Grinsted et al. (2004) (available from http://www.pol.ac.uk/home/research/waveletcoherence/). Time is recorded on the horizontal axis and the vertical axis gives us the periods (and the corresponding scales of the wavelet transform with higher scales in the bottom part of the figure). Warmer regions (red) indicate high correlation, and colder regions (blue) indicate low correlation. The bold black line indicates regions with significant coherence at the 95% level. Arrows indicate the phase difference: right arrows indicate series are in-phase (positive relationship), left arrows indicate series are in anti-phase (negative relationship), and an arrow pointing right upward (left downward) means that M2 is leading (lagging). Thin solid lines indicate the cone of influence, which is the region influenced by edge effects; results there must be viewed with caution

The outcome of the wavelet coherence analysis takes the form of a heatmap, which allows a straightforward interpretation of the results. Since the magnitude of each squared coherence coefficient is indicated by the color scale, the warmer the color, the higher the coherence power between the two series at that location in the time-frequency plane. Therefore, wavelet coherence maps can easily identify low- and high-coherence power regions in the time-frequency plane, by which we mean areas where the degree of association between two time series is weak or strong. The color code for power ranges from dark blue (low coherency) to yellow (high coherency). Regions with warmer (colder) colors correspond to areas of higher (lower) power, that is, squared coherence coefficients of large (small) modulus, with regions of high coherency between two time series corresponding to areas of strong local correlation.[19]

The wavelet phase is superimposed in regions characterized by high coherency and indicated by arrows. The phase information is graphically coded by *arrow* orientation: a right (left) *arrow* means that two variables are in *phase* (anti-*phase*). A right arrow pointing up (down) means that the first variable is leading (lagging) the latter. A left arrow pointing up (down) means that the first variable is lagging (leading) the latter. In Fig. 6, if the right arrow points up (down) it means that the TCB-CEI is lagging (leading) real M2, while if the left arrow points down (up) means that the TCB-CEI is lagging (leading) real M2.

Since the theoretical distribution of the wavelet-squared coherence is not known, the statistical significance of the coherence is tested using Monte Carlo simulations. The cumulative areawise test developed by Schulte (2016) is applied to reduce the number of spurious results from the pointwise significance test (see Torrence and Compo 1998). A black contour line delimiting regions of 5% cumulative areawise significance against the null hypothesis of an AR process of the first order (AR1). A thin black line marks the cone of influence, which is the region where edge effects become significant at different scales. Areas of high coherence occurring outside the cone of influence should be interpreted with caution, as they result from a significant contribution of zero padding at the beginning and the end of the time series.

Figure 6 shows two high coherency regions: the first is concentrated at scales corresponding to business cycle frequencies and is located between the early 1950s and the early 1980s; the latter is concentrated at scales corresponding to periods between 16 and 40 months occurring since the mid-1990s. However, the wavelet phase difference in the two high coherence regions indicates a striking change in the timing relationship between real M2 and economic activity. Until early 1980s, real M2 had a positive leading relationship with output at business cycle frequencies,[20] while from the mid-1990s onward monetary growth appears to be inversely related to

[19] Wavelet coherence coefficient values range between 0 and 1, so that values close to zero indicate weak correlation at a given frequency, while values close to one imply strong correlation between the two series considered.

[20] Nominal money growth tends to fall in the late stages of an expansionary phase as banks become increasingly restrained in their ability to create deposits by the availability of reserves. Real money balances would typically decline before an economic downturn, as the increase in prices usually picks up late in the cycle (Levanon et al. 2011).

current economic activity with the relationship shifted toward higher frequencies (16–32 months).[21] In sum, the wavelet coherency plot provides clear evidence on the breaks in the relationship between monetary aggregates represented by real M2 and economic activity.[22]

The breakdown in the procyclical relationship between real M2 and output has been attributed to several factors that occurred in the 1980s, namely, the change in the US monetary policy regime and the structural reforms in the banking and financial sectors. The changes in the goals and the strategy of the US Federal Reserve (Fed) in the conduct of monetary policy in the 1980s determined the abandonment of targeting monetary aggregates in favor of interest rate targeting, weakening the positive link between real M2 and economic activity. In the late 1970s, the period of restrictive regulation of financial markets created under the Banking Act of 1933 became difficult to prolong after the developments in the financial markets. Financial innovation, such as the development of securitization, and financial market deregulation, in particular the creation of interest-bearing checking accounts and money market mutual funds, contributed to the transition toward a market-based financial system. The effect of these changes was to increase the public's preference for transaction balances relative to income, which is at least partially responsible for the structural shift in money-output correlations from positive to negative.

Although the comovements of most macroeconomic variables are surprisingly strong and stable across countries and over time, this regularity may have some important exceptions. In the US economy, the cyclical behavior of money has undergone a major break, which may be easily detected using timescale methods, with the relationship between general economic activity and real monetary aggregates weakening and becoming unstable since the early 1980s. The financial innovations, the deregulation of the banking and financial sectors in the 1980s, and the change of monetary policy regime (from monetary to interest-rate targeting) all contributed to reducing the role of monetary indicators in the macroeconomy.

[21] The pattern evidenced in Fig. 4 is consistent with the recent revision of the TCB-LEI that replaces M2 with an index of financial conditions because, as noted in Levanon et al. (2010), starting from the early 1990s real money supply M2 has ceased to be a useful leading indicator.

[22] Friedman and Kuttner (1992) were among the first to document that by the early 1990s the relationship between M2 and GDP had weakened.

5 Conclusion

A fundamental benefit of wavelet analysis, in contrast to Fourier series or splines, is that, in general, wavelet analysis is more robust in a messy world than are the other techniques. By a messy world, we mean one in which randomly occurring sizable shocks distort the dynamical system, the parameterization of approximating models needs to be changed over time, and distributions relevant in one time period are not statistically similar in another. In sum, wavelet analysis, by allowing researchers to be less committed to a particular class of model, can overcome most of the methodological difficulties faced by previous methods. Moreover, because of its ability to localize the nonstationary structure (which depends on time), wavelets are well suited for the analysis of time series resulting from complex, nonlinear processes, as is the case for the secular movements in the level of economic activity that exhibit structural changes in the trend function as well as in volatility and comovement patterns.

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