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Engaging Mathematics Teachers as Designers of Digital Educational Resources to Foster Their Awareness in Counsel

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Abstract. The paper describes a project concerning the design and implementation of an educational path devoted to in-service lower secondary school teachers (ISTs). The aim of the project is to increase the ISTs' technological skills as well as their awareness, by involving them in designing digital educational resources for their students. Moreover, we are interested in the participants' perceived impact of the educational path on their professional development. Our analysis, carried out according to a case study methodology, allowed us to highlight a change of perspective and the increase of the awareness of most of the ISTs, who shifted from the occasional use of simple GeoGebra files (mainly created by others) to the potential plan to autonomously producing GeoGebra applets with different educational goals. Moreover, most ISTs perceived an improvement in their professional development in terms of technological and didactical skills, fostered by their engagement and communication within the cooperative activities carried out in remote learning.

Keywords: teachers as designers, teachers' awareness, digital resources.

1 Introduction

Recent studies in mathematics education have focused on the teachers' engagement in the design of didactical resources and, in particular, tasks [1], [2]. According to [3], the partnership between teachers and researchers in resource design processes could significantly improve teaching and learning practices in mathematics, under the condition that the principles of design are shared and the effective integration between the task design and the pedagogical philosophy is taken into account. It has been recognized that working with tasks and, in particular, being engaged in the design process of digital resources should foster the teachers' professional development, in terms of both mathematical content knowledge and pedagogical knowledge [4]. Moreover, collaborative didactical design should favor the teachers' metacognitive reflection on their own didactical practice, as well as their understanding of methodological aspects and benefits of cooperative learning by direct experience [5]. Indeed, through working on mathematical tasks, teachers could "*engage in mathematical thinking* […]*, reflect on the experience of doing mathematics-related tasks* […]*, consider implications for teaching*" [6, pp. 207-208]. When the design

process of tasks or resources envisages the use of digital tools, some further considerations are needed. In this case, according to [3], it is necessary to take into account *what* to design, i.e. the object of the design (a task/a sequence of tasks, an explorative/self-assessment/remedial activity, …) and possibly the students' difficulties that it should allow to overcome; *what tools* are necessary (manipulative artifacts, virtual artifacts, specific technological tools, …); and *under what conditions* the design process could be carried out, i.e. how the resource should be used to promote the students' learning (individual use in the classroom, working in groups, …). Moreover, also the constraints and affordances that the technology offers with respect to traditional environments should be carefully considered.

This paper sets in that stream of studies and should contribute to investigating the perceived effects of the engagement in the design resource process for the teacher's professional development in terms of metacognitive practices [7, 8]. More in detail, the educational path is designed with the goal of improving the participants' *awareness in counsel* (i.e. the awareness needed to be a "real teacher" [8], p. 243) about the potentialities of the integration of technological tools and resources in their teaching to foster students' learning.

2 Theoretical framework

The main component of our theoretical framework is the notion of *awareness* as described by Mason [8]. According to the author, "*to be a real teacher involves the refinement and development of a complex of awarenesses on three levels*". Mason identifies the following levels of awareness:

a. the *awareness-in-action,* which consists in the power of acting in the world, with a focus of attention that is typically implicit; for what concerns mathematics, it consists in the actions of counting, ordering, classifying, relating, and so on;
b. the *awareness-in-discipline,* which is the awareness of awareness-in-action and arises when the focus of attention of the individual becomes explicit, while he is acting in the world. For example, "*mathematics as a discipline arises when we become aware of awarenesses-in-action such as those constituting counting, ordering, classifying and relating and start to formalize these in the language of algebra and geometry*" [8, p. 258];
c. the *awareness-in-counsel,* which is the awareness of awareness-in-discipline, i.e. the awareness needed to be sensitive to what others require in order to build their own awarenesses-in- action and-in-discipline.

In particular, the awareness-in-discipline is the awareness of the expert in a specific subject, for example, the awareness of the mathematician. But teachers need something more: it is just becoming aware of the awarenesses-in-discipline that enables the teachers to teach effectively their subject.

Indeed, the teacher has a crucial role in making the students orient their attention like an expert in the discipline. This can be pursued by creating conditions and learning environments in which students could experience suitable *shifts of attention* [8], allowing them to focus on some aspects rather than others and to become aware of specific mathematical facts.

In this sense, the educational path that we devoted to our ISTs aimed at developing their awareness-in-counsel about the didactical potentialities of technological tools, specifically in provoking *shifts of attention* in their students.

3 Research context and methodology

The educational program involved 114 ISTs of lower secondary schools of Marche (Italy) and was carried out through the conference platform Zoom. According to the constraints imposed by the Regional School Office of Marche, the educational path lasted two months, from May to July 2021. The participants were exposed to 4 plenary lectures of two hours and half and to 4 laboratory sessions of two hours. Moreover, the course also envisaged 12 hours of autonomous practice by the ISTs aimed at allowing them to create their final project work.

The plenary lectures have been devoted to give an overview about the following issues: students' difficulties and misconceptions in Mathematics; the potential of the software GeoGebra and its community; Euclidean geometry of the plane and three-dimensional space with GeoGebra. During these lectures, the opportunities offered by the software GeoGebra and its functionalities (drag and drop, check boxes, input fields, …) to support the students' learning have been explored. Indeed, some GeoGebra applets with different educational goals (visualization, exploration, self-assessment, …) have been shown to the ISTs.

The laboratory sessions have been conducted by faculty members, among them the authors of this paper. The ISTs worked in small groups of 4-5 people and interacted through the breakout rooms of Zoom. In the first two laboratory sessions, the ISTs were asked to construct their first GeoGebra applet by following some accurate guiding instructions, so that they gradually become familiar with the software. The instructions allowed them to construct either a mathematical object (e.g., the construction of an equilateral triangle with "ruler and compass") or a didactical part aimed at inducing a meaningful exploration and learning of the mathematical object itself. In the subsequent meetings, the ISTs have been invited to discuss the students' typical difficulties and misconceptions about a specific mathematical proposed topic or object and, consequently, to design and implement GeoGebra applets aimed to support the overcoming of these hardships. The tutors were available to support the ISTs during and between the sessions. The ISTs jointly wrote a design sheet of a specific applet and implemented it as a final project work. It consisted in an applet where the geometrical object and its construction were provided, but the ISTs should autonomously design and implement the educational structure. The geometrical objects on which the ISTs could choose to focus on are: a parallelogram that can be continuously transformed into a rectangle; a triangle with highlighted heights, axes, bisectors, and medians; a prism that continuously changes in its plan development; a solid composed by a cube surmounted by a cylinder. Finally, each IST individually carried out a metacognitive analysis of the resource created by his/her group, guided by some stimulus questions.

In the first laboratory meetings, the ISTs' activity has been strongly scaffolded, while in the subsequent meetings, the scaffolding gradually faded [9, 10], mainly for what concerns the educational part of the applet (guiding questions, feedback, remarks, theoretical notions, …), so that the ISTs were involved both in decision-making

processes and in metacognitive reflections on the use of the GeoGebra potentialities for teaching and learning the topic at stake.

In this study, within the research framework concerning the different levels of awareness needed to be a "real teacher" [8] and the necessary shifts of attention to be promoted in order to foster students' learning, we address the following explicit research questions: (1) Does the designed educational path improve the participants' awareness in counsel? (2) What is the ISTs' perception of the educational path for their professional development?

We used a case study methodology [11] and different kinds of data have been collected: the design sheet written collectively by the groups of ISTs; the GeoGebra applet created by the groups of ISTs as a final project work; the sheet containing the individual metacognitive analysis of the created GeoGebra applet and its educational potentialities; the individual answers of the ISTs to a final questionnaire.

The three researchers separately carried out the analysis of the productions of each group. They compared the design and the metacognitive reflection with the produced applet, identified and coded the excerpts of the ISTs' individual answers to the questionnaire referred to the perceived impact of the educational path. Finally, the main emerging themes have been discussed to reach an agreement and answer to the research questions.

4 Results

In this section, in order to give an answer to the first research question about the increasing of the awareness-in-counsel of the ISTs involved in the educational path, we analyse the GeoGebra applet produced by two groups of ISTs, comparing it with the corresponding (a-priori) design sheet and the (a-posteriori) metacognitive reflection. In tune with [3], for each group, we will follow the evolution of the ISTs' awareness, focusing on *what* they designed, i.e. the object of the design and the students' difficulties that it should allow to overcome; *why* the resource has been designed, i.e. the cognitive processes on which the design should have an impact; *what tools* were used to reach the didactical aim; and, finally, *under what conditions* the resource should be used to foster students' learning, i.e. what should be the use of the resource within the learning path of the students.

We chose to focus on Group 1 and Group 2 since they chose two different mathematical object to work on and their evolution is representative of that displayed by the other groups of ISTs.

Group 1 chose to work on the planar representation of a prism, while Group 2 chose to work on the continuous transformation of a parallelogram into a rectangle.

In Table 1 we synthesized the a-priori design and the a-posteriori metacognitive analysis carried on by Group 1 about their applet.

Table 1. Group 1's a-priori design and metacognitive analysis in terms of *what* is the object of the digital task, *why* it is designed, *what tools* have been used, and *under what conditions* the resource should be used.

Group 1	DESIGN	METACOGNITIVE ANALYSIS
WHAT?	a file allowing the visualization of surfaces	a sequence of tasks that can be faced one by one thanks to a slider and allows one to visualize a polyhedron as a set of polygons on different planes
WHY?	to foster visualization, exploration, self-assessment	to foster, behind the visualization, the exploration, the construction of mathematical meanings, and self-assessment
WHAT TOOLS?	sliders to change the number of sides of the prism and their length input fields to count the number of vertices, sides, faces input fields to compute the total surface and the volume of the prism	sliders, allowing the exploration of the prism and the continuous transformation of the prism in its planar representation input fields, allowing to give immediate and facilitative feedback to make the students autonomously find the solution. check boxes to verify the correctness of an answer
UNDER WHAT CONDITIONS?	in the classroom, with students grouped in pairs, since it is an explorative file	individually, at home or in the classroom for the exploration, then collective discussion to favor the connection of pieces of knowledge

During the design phase, Group 1 generically declared they wanted to design a resource addressing the "*students' difficulty of visualization of solids and identification of their elements (vertices, sides, faces)*". Their first idea was quite vague for what concern *what to design*: they wanted to produce a file to foster "*visualization, exploration and self-assessment*", without further explanations. They had in mind to use, as *tools*, "*sliders to change the number of sides and their length, and input fields, to compute the total surface and the volume of the prism*". Moreover, for what concerns the *use of the resource*, the ISTs suggested that it should be exploited by pairs of students or individually, at home or in the classroom, since the applet should be mainly explorative.

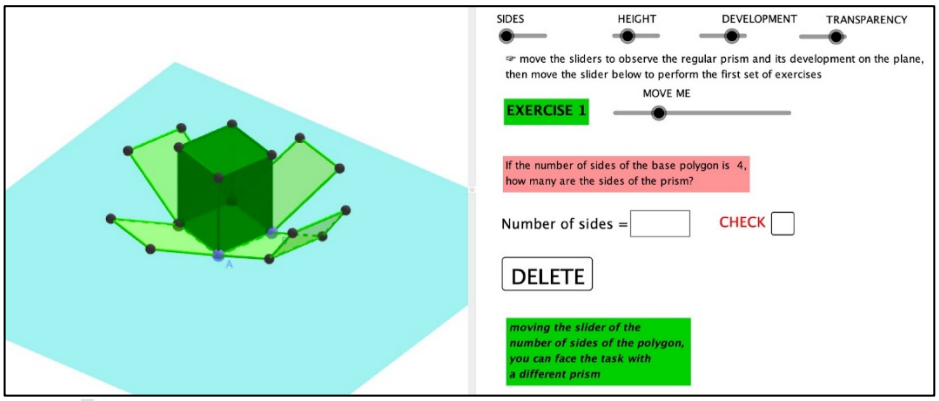

Fig. 1. A screenshot of the GeoGebra applet created by Group 1 concerning the planar representation of a prism.

After the design and implementation process, the awareness of the ISTs of the didactical potential of the resource seems to be increased. The joint analysis of the applet they created, and their metacognitive reflections allowed us to highlight a careful design, with specific and often conscious choices aimed at guiding the users to develop specific awareness-in-action and awareness-in-discipline. For example, the applet includes some remarks such as "*moving the sliders* […] *you can face the task with a different prism*" (Figure 1). Commenting on this design choice, an IST of Group 1 declared that "*the remark suggests focusing on what is invariant when some data change*", so explicitly promoting a shift of attention [8] in the students' minds. Another IST of Group 1 addressed cognitive, affective, and metacognitive levels of learning, saying that "*the students are invited to face the tasks by changing the initial parameters, in order to obtain general rules, so learning by her/his own mistakes. This should favor a greater engagement of the student and, at the same time, his reflection on his own learning process*". Moreover, the same IST discussed the structure of the resource as a sequence of tasks that progressively should bring the students to achieve the identified didactical goal, i.e. discover the formula for the surface of a prism:

> "*A slider has been introduced in order to give the students the possibility of having a sequence of tasks of increasing level of difficulties*; […] *students start with the explorative phase, then they pass to the operative one. The first task concerns* […]. *For each request, feedback is envisaged, and, in case of a wrong answer, some suggestions are provided to obtain the result. The second task consists of true/false question, and it is preliminary to the third task, where, through some guiding questions, the students "discover" the formula for calculating the lateral and total surfaces of a prism*".

About the *why* issue, concerning the specific goal of the resource, the ISTs were more precise after the design and implementation process. One of them said that "*the applet allows to explore a set of regular prisms so that the students grasp the mutual relations between the different elements. The applet envisages also self-assessment with some remarks guiding the student in searching a general method to analyze such a solid*". Another IST declared that "*the applet is aimed to enable the students to correlate in a stable way (i.e, by a formula), the number of sides, vertices and faces of a prism*". Moreover, in the metacognitive a-posteriori analysis of the produced applet, the ISTs have been able to associate each digital tool to a specific didactical goal. One of them

declared that "*the use of the functionalities offered by GeoGebra, such as the sliders and the planar representation is crucial to foster the development of the students' abstract thinking*". This addresses a fruitful feature of Mathematics as a discipline and displays the IST's awareness-in-counsel about the didactical potential of the applet to favour the students' shift of attention towards a crucial epistemological aspect. Another IST said that "*the possibility to visualize the planar representation of the prism, together with the sliders and some guiding questions with immediate feedback of the correctness of the answers, should make the student reflect on possible mistakes and improve his/her capability to learn*". It is worthwhile to notice that an IST declared that the applet determines that "a relationship between the image and who observe it […] the opportunity to modify the parameters allows to better understand the figure", so highlighting the opportunities of an individualized teaching/learning of Mathematics through careful use of technology, and in particular digital tasks [12, 13].

Finally, also the idea about the *conditions* under which the resource should be used evolves and becomes more precise and aware: in the a-posteriori analysis, some ISTs of Group 1 suggested to integrate the individual use of the resource with a collective discussion. For example, it arises in the following excerpt: "*after the use of the applet by each student or pairs of students, a collective discussion in the classroom will be crucial to make remarks and awareness arise by the students about the connection with previously acquired pieces of knowledge*".

Group 2 chose to work on the continuous transformation of a parallelogram into a rectangle through the translation of a triangle.

In Table 2 we synthesized the a-priori design and the a-posteriori metacognitive analysis carried on by Group 2 about their applet.

Table 1. Group 2's a-priori design and metacognitive analysis in terms of *what* is the object of the digital task, *why* it is designed, *what tools* have been used, and *under what conditions* the resource should be used.

Group 1	DESIGN	METACOGNITIVE ANALYSIS
WHAT?	a file supporting the idea of equidecomposability	a file to make the students understand where the formula for the area of the parallelogram comes from
WHY?	to foster visualization, exploration, remediation	to foster the awareness of the relation between the measure of the sides and the areas to deepen the relationship between the equivalence and the congruence of polygons to discover the formula of the area of a parallelogram to enable the students to understand that the mathematical formulas come from reasoning

		to connect pieces of knowledge
WHAT TOOLS?	buttons, input fields, sliders drag and drop automatic computation of areas to support the students' understanding	sliders, to visualize the translation automatic computation, to verify the equivalence of figures check boxes to enable students to visualize the process in different steps
UNDER WHAT CONDITIONS?	in the classroom, individually or in groups	integration between manual artifacts and virtual artifacts

During the design phase, Group 2 declared they wanted to produce a file concerning the idea of equidecomposability of planar figures. The group did not identify a specific students' difficulty to be addressed, but they said that the goal of the resource was to foster "*visualization, remediation and exploration*" (*what to design*). About the *technological tools* to be used in the implementation of the applet, the ISTs provided a list containing "*buttons, sliders, input fields*", without explaining their educational function within the applet. They declared that also "*drag and drop functionality and the automatic computation of areas*" should be used, in order "*to support the students' understanding*". Moreover, for what concerns the *use of the resource*, the ISTs suggested that it should be exploited in the classroom, with students working individually or in groups.

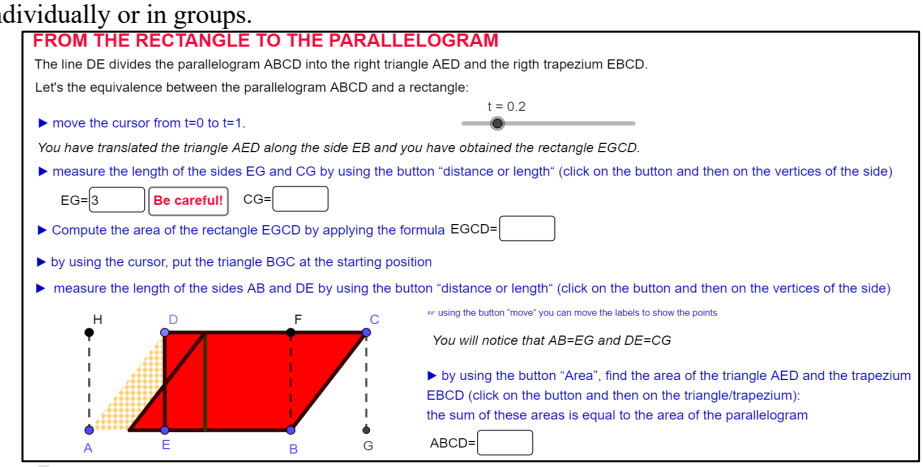

Fig. 2. A screenshot of the GeoGebra applet created by Group 2 concerning the area of planar representation of a prism.

The applet produced by Group 2 is characterized by a design that strongly scaffolds the students' exploration of the continuous transformation of the parallelogram into the rectangle. The ISTs inserted some instructions guiding the students in using some tools, such as the drag and drop function when moving the cursor, or the button allowing to see the measure of areas or length of sides (blue writings in Figure 2). These instructions are supported by remarks aimed at supporting the students' noticing and monitoring of

what the applet allows them to observe. For example, "you have translated the triangle AED along the side EB and you have obtained the rectangle EGCD" or "you will notice that AB=EG and DE=CG" (black writings in Figure 2). Commenting on this aspect in his metacognitive analysis of the applet, an IST of Group 2 said that

> "*the resource aims to bring the student to an aware and meaningful learning through an experimental approach guided by some suggestions. The interaction with the software GeoGebra, the visualization and virtual manipulation of figures, the use of specific tools for computing lengths and areas induce the student to deepen the meanings and discover new pieces of knowledge".*

Focusing on the experimental approach, another IST declared that "*the applet allows the concretization of the knowledge*".

In the a-posteriori analysis of the resource, the ISTs of Group 2 motivated the object of the design (*what to design*) saying that "*the concept of the surface of a planar figure is a critical aspect for many students*". The constructed resource is described as "*a file supporting the students' understanding of the fact that the formulae come from reasoning*". In this sense, the digital resource conveys not only the learning of mathematical contents but also the cultural and epistemological value of the discipline, hence promoting the development of students' awareness-in-discipline. About the *why* question, the ISTs identified different goals of the applet in their a-posteriori analysis: it should foster the students' awareness of the relation between the measure of the sides and the areas; it should allow deepening that congruent figures are equivalent, while equivalent figures could be not congruent; it should favor the connection between pieces of knowledge and allows self-assessment.

Moreover, in the a-posteriori metacognitive reflection on the produced resource, the ISTs have been able to associate each technological tool they used to a specific educational aim. This constitutes a significant improvement with respect to the degree of awareness displayed in the design phase. For example, an IST declared that "*the sliders allow to highlight the crucial steps, providing the student with a clear logical sequence*". Another IST commented about the links between the technological tools and her own individual ability to use them in a significant way. She declared that

> "*the choice of some functionalities depends on my ability in using the software. At my level of expertise, I have been able to use sliders, check boxes, and input fields which allowed me to give meaning to the activity, which turned out to be richer with respect to an activity in a traditional environment*".

Finally, the ISTs of Group 2 chose to mainly focus on the *conditions* under which the resource they created could be used, also connecting them to the individual preferences of learning: "*Since each student has a preferred learning channel, the integration of methods and tools (cardboards, videos, software ,…) is useful to impact different kinds of intelligence*". Another IST of Group 2 suggested different ways to use the resource:

> "*It is possible to give the resource to the students, allowing them to interact with it: in this case, they will answer the questions and fill in the blanks (individually or in pairs). Then, a subsequent collective discussion in the classroom will be suitable to compare the different answers provided by the students. In another way, the construction of the file could be proposed to the students (providing them with the protocol of construction) as an in-depth activity devoted to the most interested students. They could be the tutor of their classmates in difficulty*".

In tune with this idea, also another IST suggested proposing the activity into two different phases, envisaging the devolution of the construction of the resource to the students, so engaging them more deeply:

> "*during the first phase the students in small groups construct their parallelogram, following a protocol of construction; during the second phase, a collective reflection guided by the teacher is carried out in the classroom, so that the students could grasp that the parallelogram has the same area but different perimeter of the rectangle with the same basis and height*".

For what concerns the second research question, regarding the ISTs' perceived impact of the educational path on their professional development, we analyse the ISTs' answers to the final questionnaire submitted to them after the conclusion of the educational path. In that questionnaire, the questions concern (a) the previous experience with the software Geogebra, (b) the criteria that had guided the design of the Geogebra applet, (c) the difficulties encountered during the design and implementation of the final project work, (d) the kinds of the possible increased knowledge (pedagogical, mathematical, technological) after the educational path.

For what regards issue (a), about 86% of the participants did not use GeoGebra or had a basic experience with it, mainly using files created by others (Figure 3). Moreover, about 72% of the participants gave a score less than 5 in a scale from 0 to 10 to their own knowledge of the software GeoGebra. This data suggested that the participants' technological background, in particular concerning the software GeoGebra, was not sturdy.

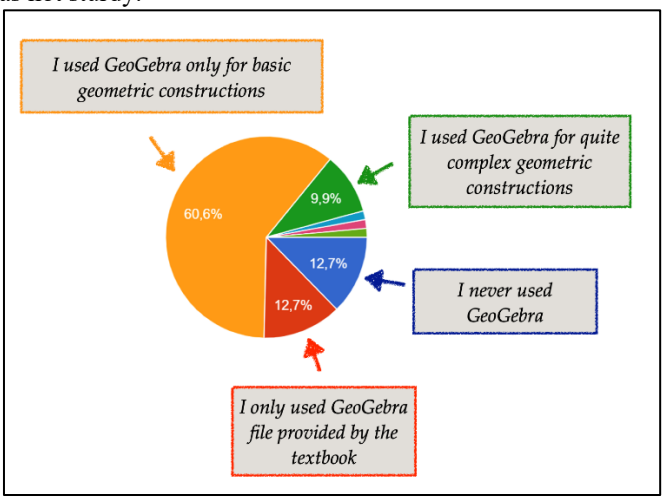

Fig. 3. The statistics of the ISTs' answers about their previous experience with GeoGebra.

The qualitative analysis of the ISTs' answers, mainly of those of kind (d), allowed us to highlight their perception of the impact of the educational path on their professional development. Many ISTs appreciated the intertwinement between technology and pedagogy: an IST declared "*I increased my technological competencies and mainly my awareness in using GeoGebra, grasping some educational potentialities of the software*"; another participant said: "*I really understood that acquiring digital competencies allow me the improvement of the teaching and the acquisition of*

knowledge by the students". Some ISTs focused on the opportunity of improving the students' learning by exploiting the digital resources they created not only at the cognitive level, but also at the metacognitive and affective ones. This arises in the following answer: "*Now I can teach interactive and dynamic lectures which will engage the students and support them in strengthening the previous knowledge, acquiring new contents, reflecting on them and self-assessing their own learning*". Linked with this theme, also the inclusion issue is mentioned in some answers, such as the following one: "*Working with GeoGebra supports geometric constructions and consequently could favor the inclusion of students with specific learning difficulties and poor motivation*".

Another recurrent issue is the perception of the increasing of awareness, which was an explicit aim of our project. This arises in many answers such as "*My knowledge and awareness increased. In the last days I used the applet concerning the important points of a triangle I created. The students understood this topic, thanks to the use of the software, and faced it with more pleasure*". Another IST declared "*I learned to use GeoGebra with more teacher's awareness. I discovered many functionalities that I did not imagine. They are essential to favor the teacher's work and her didactical choices*", referring just to the "teacher's awareness" which is the awareness-in-counsel, according to [8]. Another IST declared that

> "*Until now I considered GeoGebra only as a tool to enable students to visualize some facts. I never thought that was possible to create authentic didactical files, with guiding questions and automatic feedback, through which the students could autonomously interact in an explorative approach to a specific topic*".

It is worthwhile to notice that another theme emerges from many ISTs' answers concerns their perception of the joint working in group. Generally, this approach has been appreciated as an opportunity to shared difficulties and doubts, like in the following answers:

> "*Working in group has been very useful, since we shared ideas and doubts. I feel this supports my professional development"; "the collaboration within my group has been very positive; we discussed a lot of the students' typical mistakes and found collective solutions to the design and implementation troubles. It would be great if in all the job contexts there was the same approach*".

5 Discussion and conclusion

In this paper we presented the outcomes of a case study carried out with 114 ISTs from different lower secondary schools of Marche, in Italy. They were exposed to an educational path where they have been involved in the design and implementation of digital resources for their students, by using the software GeoGebra. We investigated 1) at what extent the awareness-in-counsel of the ISTs increased and 2) the ISTs' perceived impact of the educational path, and, in particular, of the engagement in the process of designing digital resources, on their professional development.

For what concerns the first research goal, our analysis highlighted that, standing some differences in the design, generally the ISTs displayed a change of perspective with respect to the educational potentialities of careful use of the software GeoGebra. They grasped the strong intertwinement between the mathematical content, pedagogical aspects, allowing to consider the critical element of that content for the

students' learning, and the technological resources, which can support the overcoming of the students' hardships. As a consequence, many ISTs who previously had a poor and limited experience with GeoGebra, declared their intention to autonomously produce GeoGebra applet for their teaching in future, in order to have an impact on the specific difficulties and learning needs of their students. Within the process of designing digital resources, the teachers generally identified didactical steps aimed at guiding the students in the learning process, so displaying an increased awareness-in-counsel as the sensitivity to what other need to develop their own awareness-in-action and awareness-in-discipline. In some cases, the resource has been used to convey some epistemological values of Mathematics, such as the idea of generalization or the cultural genesis of mathematical formulas.

For what concerns the second research goal, some recurrent issues emerged from the ISTs' answers to the final questionnaire they filled in after the conclusion of the educational path. Generally the ISTs perceived a positive impact of the experience on their professional development, mainly referring to a new awareness about the large educational potentialities of the software GeoGebra, allowing them to improve their students' learning also at the metacognitive and affective levels. Moreover, the ISTs appreciated the collaborative design and implementation process that they carried out working in small groups, identifying it as a model for the collaboration with their colleagues.

As a further development of the research, we planned to propose a second edition of the educational path, re-designed on the basis of the strengths and weaknesses of the first edition, according to the design-based research model. From the research point of view, we would like to deepen the investigation of the cooperative learning carried out by the ISTs during the laboratory sessions with a specific focus on the group dynamics and the possible misalignment between the participants in the design or implementation processes. In particular, we would like to carry out a joint analysis of the ISTs' screens during the work sessions and a-posteriori individual interviews.

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