

An improved bilevel programming approach for assortment and cut of defective stocks

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Abstract: We consider the problem of simultaneously (a) finding a limited assortment of stock items and (b) cutting them with minimum trim loss to fulfill a known demand of different parts. We develop a framework to address the problem when stock sheets can be affected by faults, providing and testing a new bilevel model for both defect occurrence and robust optimization of the production process. The model generalizes an existing bilevel approach and overcomes some of its drawbacks, returning more credible scenarios in terms of defect distributions.

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Keywords: Uncertainty, Optimization, Manufacturing Systems, Multi-level Programming, Operations Research.

1. INTRODUCTION

We consider the problem of simultaneously (a) finding a limited assortment of large stock plates and (b) cutting them with minimum trim loss to meet a given demand of different parts. We develop a bilevel programming approach to tackle this problem when stock plates can be affected by faults, providing and testing mathematical models for both defect occurrence and robust optimization of the production process.

This approach extends deterministic optimization models originally developed by Arbib and Marinelli (2007, 2009) for assortment problems that arise in glass production for the automotive sector.

Two possible points of view can be adopted to face the reduced process efficiency associated with the occurrence of faults. One possibility is to maintain the original demand constraint and, in a perspective of material usage minimization, to seek an extra production that ensures its fulfillment. In a complementary view, one instead tries and minimizes the expected production losses that can be incurred given a certain amount of raw material. The former approach is more sensible, for instance, in just-in-time production, where demand fulfillment is a priority. The latter is instead suitable when safety stocks are available to cope with misproduction.

1.1 Scope of the paper

Our models and methods are inspired by an assessed optimization model for stock assortment and cut in a real manufacturing plant. The process, a standard in the production of automotive glass parts, is divided into two main phases (see Fig. 1). In Phase 1, large rectangular glass sheets of various sizes and types are first manufactured by widening/narrowing a ribbon of molten glass, and then sent to warehouse: we will refer to them as the *large items* (or *plates*), with heights (lengths) roughly ranging between 2 and 3 mt (4-6 mt). In Phase 2, small rectangular sheets (the *small items*) are cut from the large ones taken from the warehouse, and are then sent to downstream departments for bending and refinishing. Cuts are of *guillotine* type, that is, edge-to-edge.

A mathematical model to optimize such a process, discussed in Arbib and Marinelli (2007, 2009), refers to a plant operated by Pilkington Italia and presently owned by Nippon Sheet Glass Co. Ltd. The purpose is to simultaneously control the assortment of large items produced in the first phase and of the trim loss generated in the second. The word *assortment* indicates the number of different large sizes that are stored in the warehouse at any time: this number must be limited to a certain amount p to both reduce holding cost and set-ups, and ease handling operations. To give an idea of the problem scale, the production data set includes about 50 distinct small sizes and over 6500 different choices of large sizes: as the desired

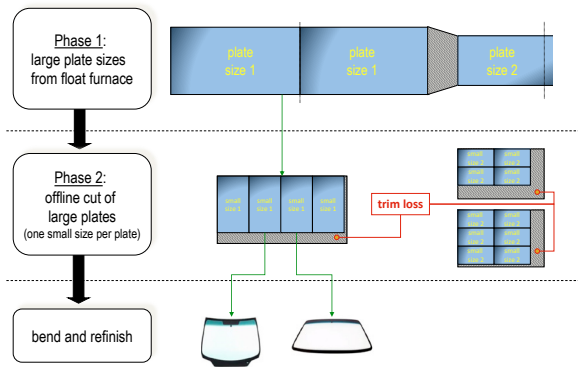


Fig. 1. Scheme of an automotive glass production process.

assortment level p ranges around 20, over 10^{57} different assortment combinations can possibly be considered.

In Arbib and Marinelli (2009), the stock assortment and cutting problem is modeled as a 0-1 linear program and addressed by column generation. In this model, perfect data knowledge is assumed. However, defects such as bubbles can occasionally appear in the glass ribbon as it emerges from the furnace. These defects render certain areas of large items unusable, thus potentially reducing yield, and this is a challenge not considered in the referenced research.

1.2 Handling defect occurrence

Different reactions to the presence of defects (or, more generally, of forbidden areas) can be implemented depending on available technology, production line equipment and work organization. In the literature on this type of problems, the presence of defects is addressed since the earliest contributions (see e.g. Hahn (1968); Sculli (1981); Sarker (1988); Sweeney and Haessler (1990)) and covers multiple fields of application, such as

- Lumber and furniture: Ghodsi and Sassani (2005); Wenshu et al. (2015)
- Shoes and textile: Sarker (1988); Ozdamar (2000)
- Paper: Aboudi and Barcia (1998)

Most papers assume defects known in advance and given as input data, see e.g. Ghodsi and Sassani (2005); Wenshu et al. (2015); Ozdamar (2000); Aboudi and Barcia (1998); Carnieri et al. (1993); Neidlein et al. (2009); Afsharian et al. (2014); Wenshu et al. (2015); Sweeney and Haessler (1990); Sculli (1981); Sarker (1988); Rönqvist and Åstrand (1998); Durak and Tüzün (2017). Consequently, the main focus is on the design of defect-free cutting patterns in a deterministic setting.

In this paper, we instead treat defect occurrence as a random process and address the problem of managing its uncertainty. Our approach combines worst-case analysis, typical of adversarial models, with certain stochastic assumptions. Specifically, while the total number of defects within the planning horizon is treated as a model parameter, the adversary only decides *how many* defects are assigned to each large item and not *where* those defects are placed. The yield reduction is then represented as an

expected value, leveraging the probability that a defect cannot be mitigated through cut reconfiguration.

This framework is motivated by both problem-specific and practical considerations. From a theoretical standpoint, uncertainty management is essential because large item sizes are not predefined, being part of the assortment decision. Both the assortment and the cutting patterns must be computed before producing the large items, and hence before defects become visible. In the plant, at present, defects are detected at the end of Phase 1. However, they are not considered before the cutting stage, and faulty small items are simply discarded.

2. AN UPDATED BILEVEL MODEL

Referring to the two-phase glass cutting process described in Fig. 1, let S denote the set of small sizes required and let d^i be the demand of size $i \in S$ that is necessary to fulfil in a given planning horizon. The rules adopted in the plant and the technological constraints of the downstream cutting machines greatly simplify the cutting patterns, that are limited to those obtaining small items of the same size from each single large item. In particular:

- the small items of a pattern have all the same orientation: of the two possible orientations, we always select the most productive;
- the first guillotine cut is always performed horizontally, i.e., along the width of the large item, and trim loss is always located on the bottom-right side of the large item.

In Arbib and Marinelli (2007, 2009), 0-1 linear programming models are proposed to select an assortment of $\leq p$ different large sizes from the set L of all possible size choices, and to decide how many plates of the selected sizes must be cut to fulfil demand. A very effective approximated version of the problem, provided in Arbib and Marinelli (2007), consists of a special p median problem that restricts the total demand for each item i to be produced in a single lot $j = (i, k)$, that is, by cutting plates of a single size k . On this basis, a bilevel programming model is developed in Arbib et al. (2022a) to find optimal recourse actions to point-shaped defects occurring in Phase I of the process.

Generally speaking, in a bilevel model one has two decision makers D_1 (the *leader*) and D_2 (the *follower*) with contrasting goals. Specifically, we use this paradigm in a way close to two-stage robust optimization. As illustrated, e.g., in Mattia et al. (2024) for a different application, this approach has a key feature in the fact that the uncertainty set partly depends on the leader's choice: that is, different first-level solutions must be protected against different realizations of the uncertainty. In fact, in our case, D_1 aims at the best usage of material to meet demand while its adversary, D_2 , tries to impair it as much as possible by distributing f defects in the plates used by D_1 . But different assortment and pattern selections correspond to different probabilities that small items are hit by defects, and so relate uncertainty to endogenous conditions.

As in Arbib et al. (2022a), for simplicity of notation, we illustrate the model referring to the p -median approximation of the assortment problem faced by D_1 . Let $J = S \times$

L be the set of all possible lots. The leader's decision variables are:

- $x_j \in \{0, 1\}$, $j \in J$: for $j = (i, k)$, these variables get value 1 if and only if the whole requirement d^i is cut from plates of size k .
- $u_k \in \{0, 1\}$, $k \in L$: these variables get value 1 if and only if plate size k is part of the assortment adopted.

In the following, we will denote by J^i (by J_k) the set of all lots that involve the production of items of size i (in which items are cut from plates of size k). We now introduce a bilevel model in which the feasible solutions to the leader's optimization problem are those maintaining the total requirement of raw material under a prescribed supply limit r .

2.1 Leader's model

Let A_k be the area of a plate of size $k \in L$. For a generic production lot $j = (i, k)$, let also n_k^i be the largest number of items of size i that fit on a plate of size k . Then $d_j = \lceil \frac{d^i}{n_k^i} \rceil$ is the minimum number of plates of size k that should be used to meet the demand for items of size i , and $r_j = d_j A_k$ is the total volume of the production lot. For sufficiently large r , the following system has a solution that represents a set of production lots that fulfill the whole demand and use an assortment of only p plate sizes in L :

$$\begin{aligned}
 [D_1] \quad & \sum_{j \in J^i} x_j = 1 & i \in S & \quad (1) \\
 & \sum_{j \in J} r_j x_j \leq r \\
 & x_j \leq u_k & k \in L, j \in J_k \\
 & \sum_{k \in L} u_k \leq p \\
 & x_j \geq 0 & j \in J \\
 & u_k \geq 0 \text{ and integer} & k \in L
 \end{aligned}$$

Since D_1 decides to produce the whole demand of small items, all the feasible solutions to (1) have the same total value

$$\gamma = \sum_{i \in S} v^i a^i d^i$$

where a^i and v^i are, respectively, the area and the economic value per square meter attributed to each item of size $i \in S$ (we assume that overproduction has no economic value).

2.2 Follower's model

Let us now detail the structure of the follower's optimization model. The decision of D_2 is encoded by variables $\eta_j^t \in [0, 1]$ that associate a specific production lot j with a number t of defects chosen in $T = \{0, 1, \dots, f\}$, with f denoting the expected total amount of defects. Precisely, for $j = (i, k)$ and $t \in T$,

- η_j^t = probability that a plate of size k , used to produce items of size i , is affected by t defects.

D_2 then wishes to find a probability distribution that, if used to realize glass defects on plates, maximizes (the expected value of) the economic damage done to D_1 .

Let us describe the building blocks of the model. Let $\tilde{x}$ be a basic feasible solution (BFS) to (1). We expect that the defects that occur on lot $j = (i, k)$ are distributed in a way more or less closely proportional to the volume r_j of the raw material involved within that lot. Specifically, we suppose that, for some parameter $\alpha \geq 0$, a fraction

$$f_j = \frac{(1 + \alpha)r_j f}{\sum_{j \in J} r_j \tilde{x}_j} \sim (1 + \alpha) \frac{r_j}{r} f \quad (2)$$

of the whole lot j will potentially be affected by faults. (We have $\sum_j f_j \geq f$, with $\sum_j f_j = f$ for $\alpha = 0$.)

Referring to lot $j = (i, k)$, let π_{jq}^t denote the probability of q faulty items from a single plate affected by $t \geq q$ defects. The total value of the items produced in lot $j = (i, k)$ is $v^i a^i n_k^i d_j$, hence

$$\tilde{v}_j = v^i a^i d_j \sum_{q=0}^t q \pi_{jq}^t \quad (3)$$

is the expected value of the items lost when each plate involved in the production is affected by t defects.

According to Arbib et al. (2022b), let ϕ_k^t be the probability that a single plate of size k has t defects, given a total of f defects, and hence

$$\phi_k = \sum_{t=1}^f \phi_k^t \quad (4)$$

is the probability that a plate of size k is faulty.

The largest return of the adversarial choice is obtained by maximizing the expected value of D_1 loss. Let $\tilde{J}$ be the set of lots used by D_1 , i.e., the lots j for which $\tilde{x}_j = 1$. With the positions above:

$$[D_2] \quad \max_{\eta} \sum_{j \in \tilde{J}} \sum_{t \in T} \tilde{v}_j \eta_j^t \quad (5)$$

$$\sum_{j \in \tilde{J}} d_j \sum_{t \in T} t \eta_j^t \leq f \quad (6)$$

$$d_j \sum_{t \in T} t \eta_j^t \leq f_j \quad j \in \tilde{J} \quad (7)$$

$$\sum_{t \in T \setminus \{0\}} \eta_j^t \leq \phi_k \quad j = (i, k) \in \tilde{J} \quad (8)$$

$$\eta_j^t \geq 0 \quad j \in \tilde{J}, t \in T \quad (9)$$

By (6), D_2 can distribute no more than f defects among the plates. By (7), no more than f_j defects can be distributed among the plates used for the production lot j (the larger the α in (2), the less the distribution per lot is forced to be proportional to the volume of the lot). Equation (8) ensures that only an appropriate fraction (given by (4)) of the plates used in production j can be affected by defects.

Normalizing η_j^t to ϕ_k , problem (5)-(9) can be seen as the continuous relaxation of $|\tilde{J}|$ multiple-choice knapsack

problems (Kellerer et al. (2004)), each with knapsack capacity f_j and a single class of $|T|$ items.

3. A NUMERICAL COMPARISON

In this section we compare the bilevel model $[D_1]$ - $[D_2]$ of Section 2, given by (1) and (5)-(9), to that presented in Arbib et al. (2022a), where:

- The leader's optimal decision is made using the same inequality set as (1).
- The follower has the objective of maximizing the leader's loss, expressed as the value of the items not produced: to this aim, it can arbitrarily distribute no more than f defects in the large plates totally produced by D_1 , without further conditions (such as inequalities (7)).

Unlike the more general assumptions of the present paper, in Arbib et al. (2022a) the follower cannot place more than one defect per plate. One consequence is that the probabilities π_{jq} introduced in Section 2.2 are reduced to $\pi_j = \pi_{j1}$, $j \in J$. That is, π_j indicates the probability that a unique defect of a large plate in lot j cannot be recovered, so one small item is lost. This follower, say $[\hat{D}_2]$ must then decide, for each lot $j \in J$, the number z_j of defects placed. Its decision is made, for a given BFS $\tilde{x}$ of (1), by solving the bounded integer knapsack problem

$$[\hat{D}_2] \quad \max \sum_{i \in S} v^i a^i \sum_{j \in J} \pi_j z_j \quad (10)$$

$$\sum_{j \in J} z_j \leq f$$

$$0 \leq z_j \leq d_j \tilde{x}_j \text{ and integer} \quad j \in J$$

Adapting the bilevel model (1) + (5)-(9) to ≤ 1 defects per plate, we rewrite (3)

$$v_j = v^i a^i d_j \pi_j$$

and reformulate (5)-(9)

$$[D_2] \quad \max_{\eta} \sum_{j \in \tilde{J}} v_j \eta_j \quad (11)$$

$$\sum_{j \in \tilde{J}} d_j \eta_j \leq f$$

$$d_j \eta_j \leq f_j \quad j \in \tilde{J}$$

$$\eta_j \leq \phi_k \quad j = (i, k) \in \tilde{J}$$

$$\eta_j \geq 0 \quad j \in \tilde{J}$$

where, for $j = (i, k)$, $\eta_j = \eta_j^i$ is the probability that a plate of size k used to produce items of size i is affected by a defect.

Computational experiments were carried out on an Intel® Core™ i7-7500U (2 cores) 2.90 GHz with 16 Gb RAM. Mathematical formulations were compared in 200 realistic instances of limited size, divided into homogeneous groups $I_1, \dots, I_8$. Each group counts 25 instances, has $|S| \in \{5, 6, 7, 8\}$, $p \in \{|S| - 1, |S| - 2\}$ and $v^i = 1$ for each $i \in S$. The probabilities π_j were computed according

to a policy named *constrained pattern reconfiguration*, see Arbib et al. (2022b). The expected number of defects, proportional to the total glass volume r , assumed a per-square-meter defect rate $\epsilon = 1.25\%$. A detailed presentation of the testbed is available in Arbib et al. (2022a). For comparison purposes, to isolate the impact of constraints (7) in the defect distribution chosen by the follower, we further assumed that $\phi_k = 1$ for every $k \in L$. The experiments were carried out for $\alpha \in \{0, 0.05, 0.10, 0.20, 0.50\}$. The results were achieved by Gurobi 12.0.0 with default settings and a time limit of 600 seconds.

3.1 Example

Let us use a small instance of group $|I_1|$ to comment on the solutions obtained with followers D_2 and $\hat{D}_2$, that is by the bilevel models (1) + (11), with $\alpha = 0$, and (1) + (10), respectively. In this instance, we have $|S| = 5$ small items and $|L| = 11$ plates, $p = 3$ of which must be selected. The total demand is roughly of 5×10^4 items, and we assume that the follower can place up to $f = 1,500$ defects. This instance, along with other few, exhibits the same leader's production policy $\tilde{J}$ for both models, that is (1, 2), (2, 10), (3, 4), (4, 4) and (5, 10). Notice that the same holds for all $\alpha \geq 0$.

Follower $\hat{D}_2$ assigns 974 defects to lot (5, 10), one per plate, and 526 to lot (1, 2), leaving 3,713 of these plates untouched, as well as all the other production lots.

The leader's total loss, of area 1,513.35, is achieved by a less credible distribution than that implemented by follower D_2 , which distributes defects more evenly among the lots: (5, 10), (1, 2) are in fact protected from the excessive penalization imposed by $\hat{D}_2$. Moreover, all the lots are hit by defects: 727 are assigned to (1, 2), 110 to (2, 10), 174 to (3, 4), 287 to (4, 4) and 202 to (5, 10). The loss incurred with D_2 is then 1,295.69, which means a reduction of 14,39% compared to $\hat{D}_2$.

Finally, and consistently with the definitions given, both defect distribution and total loss tend to those of $\hat{D}_2$ as α increases.

3.2 Numerical results

Table 1 shows the results averaged by instance group. The total expected loss (column ℓ) and the CPU time spent (column *time*) are reported for (1)+(10) and (1)+(11), the latter for α varying in the set defined before. The average number of distinct production lots hit by the follower is indicated in columns $\mathbf{z}^+$ and $\boldsymbol{\eta}^+$, respectively.

Follower $\hat{D}_2$ concentrates the defects in few different lots, only 259 in the entire numerical campaign (1.3 on average per instance), describing over-conservative worst-case scenarios in terms of defect realization. These scenarios imply an average expected loss of 3,545.60.

Model (1)+(11) with $\alpha = 0.50$ (the largest deviation from the average defect distribution on the lots) diversifies the lots hit and impairs 678 lots in total (3.4 on average per instance), that is, around 2.6 times the outcome of $\hat{D}_2$. The estimated loss is reduced by 12.7%. As one expects, tightening constraints (7) by reducing α leads to solutions

Group	ℓ	z^+	time	$\alpha = 0.00$			$\alpha = 0.05$			$\alpha = 0.10$			$\alpha = 0.20$			$\alpha = 0.50$		
				ℓ	η^+	time	ℓ	η^+	time	ℓ	η^+	time	ℓ	η^+	time	ℓ	η^+	time
I_1	3095.6	1.1	6.5	2096.9	4.2	11.4	2150.4	4.0	15.3	2201.6	3.8	16.4	2289.0	3.4	13.9	2482.1	2.7	15.2
I_2	2796.9	1.2	11.0	2016.8	4.3	11.2	2073.3	4.1	14.5	2125.7	3.8	15.4	2222.8	3.4	17.9	2423.9	2.4	20.6
I_3	3526.7	1.3	20.6	2608.9	5.2	32.9	2660.2	5.0	53.2	2709.0	4.8	50.1	2799.1	4.3	65.4	3019.4	3.3	38.0
I_4	2705.7	1.4	30.0	2139.4	5.2	46.3	2176.7	4.8	60.8	2210.9	4.6	50.0	2268.1	4.1	56.1	2399.6	3.2	57.2
I_5	3779.9	1.3	56.2	2836.5	5.9	105.5	2902.2	5.7	155.3	2961.9	5.6	121.0	3069.7	5.2	132.1	3280.7	3.8	124.3
I_6	4145.1	1.4	99.1	3257.1	6.4	138.0	3326.9	5.9	139.4	3387.5	5.4	151.2	3494.2	5.0	128.3	3725.5	3.8	129.2
I_7	4177.2	1.3	284.1	3370.5	6.7	347.8	3442.2	6.1	395.8	3503.3	5.6	401.9	3600.6	5.0	381.2	3788.2	3.9	384.9
I_8	4138.0	1.5	215.8	3142.2	7.0	334.2	3212.7	6.4	319.7	3276.5	6.0	278.3	3386.7	5.3	301.5	3641.7	4.0	292.8
Avg.	3545.6	1.3	90.4	2683.5	5.6	128.4	2743.1	5.2	144.3	2797.1	4.9	135.5	2891.3	4.5	137.0	3095.1	3.4	132.8

Table 1. Outcome of bilevel model (1)+(10) vs. (1)+(11) for different values of α .

with fairer defect distributions, up to 1,120 different lots hit in total (5.6 on average per instance) and an expected loss of 2,683.50, that is a reduction of 24.3% compared to that incurred with $\hat{D}_2$.

On the computational side, model (1)+(10) required 90.4 seconds on average and reached the time limit 8 times. As already observed in Arbib et al. (2022a), this is due to the weakness of the formulation's dual bound. We observe a huge optimality gap (above 90%) in half the cases; in the remaining cases, it is maintained below 10%.

On average, solving the new bilevel model (1)+(11) for different values of α required between 128.4 seconds ($\alpha = 0.00$) and 144.3 seconds ($\alpha = 0.05$). The time limit was reached between 13 and 17 times, 73 altogether considering all α values. In 23.3% of the cases the optimality gap was above 90%, and in 71.2% of the instances below 10%.

4. CONCLUSION

In this paper we considered the problem of simultaneously finding a limited assortment of large stock items, and cutting them with minimum trim loss to fulfill a known demand of different small parts. We presented a bilevel model in which an adversary can place a given number of defects and, within this limit, can arbitrarily choose the number of those placed in each plate. In this model, we also envisage an upper bound to the number of defects assigned to each production lot. These updates bypass some important limitations of an alternative model taken from literature. In fact, a numerical comparison of the two models highlights that, at a reasonable computational cost, the new model mitigates an overly conservative distribution of defects among the production lots, and so describes a much more credible scenario.

On the operational side, the proposed model can accommodate various pattern reconfiguration policies to react to defect detection, ranging from the basic *no reconfiguration* (presently adopted in the plant) to the more flexible *unconstrained reconfiguration* policy described in Arbib et al. (2022b). The latter reduces the probability π_{jq}^t that q items are discarded when lot j is hit by $t \geq q$ defects, at a cost of more articulated operation and set-ups. From a managerial perspective, this cost is to be compared to the improved efficiency of material utilization that one obtains by reducing product defectiveness.

An interesting research goal is to extend the bilevel formulation to alternative problem settings, like the one discussed in Arbib et al. (2022b) which focuses on meeting the demand for faultless items by extra production, while minimizing material usage.

ACKNOWLEDGEMENTS

Work supported by the Italian Ministry of Research via the PNRR Project *eAdaptive*, D.M. 12-31 2021 and D.D. 03-18 2022, contract F/310361/01-02/X56. The authors gratefully acknowledge the Associate Editor and the anonymous Referees who helped improve the quality of this contribution.

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