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ABSTRACT

The aim of this paper is to model trading decisions of financial investors based on a sentiment index. For this purpose, we analyze a dynamical model, which includes the sentiment index in the agents' trading behavior. We consider the setup of a discrete dynamical system, assuming that in financial markets, transactions take place between two groups of fundamentalists that differ in their perception of fundamental value. This assumption is motivated by a degree of uncertainty about the true fundamental value. The proportion of fundamentalists in the two groups is assumed to depend on the sentiment index. The sentiment index used is related to the risk asymmetry index, enabling us to consider both the variance and the asymmetry of the prediction error between the two groups of fundamentalists. We identify the equilibria of the model and conduct a numerical analysis in order to capture stylized facts documented empirically in the financial literature.

Financial markets play a central role in the economy. An understanding of financial mechanisms, such as stock price formation, is fundamental for policy-makers, regulators, and investors. Moreover, empirical studies have shown that returns in financial markets show some regularities, in particular, asymmetry, excess of kurtosis, and volatility clustering. Heterogeneous agent models (HAMs) are able to replicate these stylized facts. A number of empirical studies have shown that investor sentiment plays a significant role in asset pricing (see, e.g., Jawadi *et al.*, 2018). In order to investigate the role of investor sentiment in asset pricing, we combine two strands of literature: the theoretical HAM models and the empirical literature on investor sentiment and financial returns. To this end, we examine a financial market with two groups of traders that are homogeneous in their trading strategy, but heterogeneous in their beliefs about the fundamental value of the asset. This assumption is implicitly motivated by a degree of uncertainty about the true fundamental value as considered in He and Zheng (2016). In particular, by allowing traders to switch, this means that all fundamentalists not only use the same sentiment index for switching but also realize the two different fundamental values explicitly. However, unlike previous studies (e.g., Naimzada and Pireddu, 2015), we assume that all agents take into consideration the same sentiment index in forming their expectations. As a proxy for the sentiment index, we adopt the Risk Asymmetry Index (RAX) introduced by Elyasiani *et al.* (2018). The RAX index aims to capture risk asymmetry in the market, i.e., the higher volatility of negative returns, compared to the volatility of positive returns. We examine the joint role of heterogeneity and non-linearity (introduced by the sentiment index) in financial markets.

Our contribution to the literature is twofold. First, we introduce for the first time a new sentiment index relying on the work of Elyasiani *et al.* (2018), taking into consideration the difference between the market price and the two fundamental prices, representing the sentiment of the market. Thanks to the sentiment index, traders modify their expectations about fundamental value and consequently they switch to the strategy that they believe to be performing better. Second, by using the sentiment index, we model theoretically the empirical evidence on the fact that periods of declining prices (fear scenario) have a greater impact on volatility than periods of rising prices (greed scenario).

Our analysis confirms the findings of Elyasiani *et al.* (2018), that is, that the RAX index has the advantage of signaling the prevailing market sentiment. In particular, it is a sentiment index of fear in the sense that it is able to signal downturn moves to investors that can modify their strategies in order to avoid huge losses. Moreover, we find that a period of stability in the market is possible only when the proportion of one type of trader is very small.

I. INTRODUCTION

Many theoretical and empirical studies have analyzed asset price dynamics in financial markets in order to explain stylized facts of stock returns, such as asymmetry, excess of kurtosis, and volatility clustering. An important role in this sense is played by models with heterogeneous agents (see Chiarella *et al.*, 2002, 2009; Gaunersdorfer, 2000, among others). Models with heterogeneous agents exhibit good performance in capturing market behavior and in the replication of econometric properties and stylized facts of financial time series (see, for example, Franke and Westerhoff, 2016; He and Le, 2007). Moreover, it is well known that models involving heterogeneous agents allow for considerable flexibility in investor behavior because they have the opportunity to switch between different trading rules according to certain fitness measures Brock and Hommes (1998). Our work is closely related to the nonlinear dynamic approach that views a financial market as a result of the nonlinear interaction of heterogeneous investors with different expectations that are characterized by the expectation feedback mechanism. Heterogeneous agent models are characterized by substantial simplifications at the modeling level (few belief-types, simplified interaction structures, and reduced number of parameters) that make them amenable to analysis mostly within the theoretical framework of nonlinear dynamical systems (see Dieci and He, 2018; He *et al.*, 2019 for a survey). The majority of models with heterogeneous agents consider two types of traders: fundamentalists and chartists. Fundamentalists are aware of market fundamentals and they believe in reversion to the mean, while chartists are considered as a source of instability in the model because, with their speculative behavior, they destabilize the market leading to intricate scenarios. As a result, these studies consider heterogeneity both in the expectations of agents and their trading decision rules.

This work contributes to the literature on heterogeneous agent models in finance. Unlike most heterogeneous financial market models, we consider two groups of fundamentalists, instead of fundamentalists and chartists. A key assumption is that both groups of fundamentalists have different beliefs on the fundamental value of the risky asset. This assumption is motivated by a degree of uncertainty about the true fundamental value as considered in He and Zheng (2016). In particular, by allowing traders to switch, this means that all the fundamentalists not only use the same sentiment index for switching but also realize the two different fundamental values explicitly.

In the financial literature, investors' preferences are assumed to be increasing in odd moments and decreasing in even moments (see, e.g., Kraus and Litzenberger, 1976). The normality assumption about returns, which focus only on the first two moments, mean and variance, and overlooks the empirical evidence about the distribution of stock returns which are found to be negatively skewed. In other words, extreme and negative events are more probable than positive ones, in contrast with the assumption of a normal distribution that assigns the same probability to positive and negative returns (see, e.g., Elyasiani *et al.*, 2020; Sasaki, 2016). In fact, asset volatility can be broken down into good and bad volatility, accounting for the fact that investors like positive spikes in returns while they dislike negative ones (Elyasiani *et al.*, 2017). Based on this line of reasoning, some studies have analyzed the role of investor sentiment in stock price formation, suggesting that investor sentiment is one of the main determinants of asymmetry in stock returns (Jawadi *et al.*, 2018; Verma and Soydemir, 2009).

The aim of our paper is to highlight the role of a sentiment index in the market with heterogeneous agents. As a proxy for the sentiment index, we adopt the Risk Asymmetry Index (RAX) introduced by Elyasiani *et al.* (2018), The RAX index aims to capture the risk asymmetry in the market, i.e., the higher volatility of negative returns, compared to the volatility of positive returns. We examine the joint role of heterogeneity and non-linearity (introduced by the sentiment index) in financial markets. We model an endogenous switching mechanism between traders relying on the sentiment index, which is considered as a benchmark index for all agents. We consider two groups of fundamentalists that adopt the same trading rule but heterogeneous beliefs about fundamental value. In this way, we implicitly introduce a degree of uncertainty about the true fundamental value [which in He and Zheng (2016) is modeled directly in the utility function].

There are a few studies that consider only fundamentalists in asset pricing models. In particular, Naimzada and Ricchiuti (2008) analyze a financial market with two fundamentalists acting as gurus. Traders followed one of the two fundamentalists, by relying on a fitness measure that involves the distance between the fundamental value and the current price. They found that complex dynamics arise when market maker and agents overreact to price misalignment. However, this model does not consider market sentiment: fundamentalists take trading decisions by looking at the distance between the fundamental price and the current price. On the other hand, we assume that all the agents in the market take trading decisions by looking at the same sentiment index. Second, unlike Naimzada and Ricchiuti (2008), we do not consider any kind of imitation, in the sense that in the market there are only two types of agents and no other trader is allowed to enter in the market. The introduction of a sentiment index is the innovative aspect of our model, and it is the main reason that leads us to consider only one group of traders. In this way, we can analyze the effect of the sentiment index on the complex scenarios without the destabilization in the market introduced by chartists. Another study considering only fundamentalists is that of Kaltwasser (2010). The author analyzes a heterogeneous agent model of the FOREX market, building on the model of Alfarano *et al.* (2008), considering only fundamentalists. Unlike our study, the author considers the FOREX market and relies on a probabilistic approach, focusing mainly on numerical results. On the other hand, we adopt a qualitative analysis, both analytically and numerically. Moreover, we extend our model by including stochastic shocks to the demand of both fundamentalists. In this way, we show that our model is capable of matching the stylized facts observed in financial markets.

Our contribution to the literature is twofold. First, for the first time, we introduce a new sentiment index relying on the work of Elyasiani *et al.* (2018), which takes into consideration the difference between the market price and the two fundamental prices, representing market sentiment. Thanks to the sentiment index, traders modify their expectations about fundamental value and consequently they switch to the strategy that they believe to be performing better. Second, by using the sentiment index, we model theoretically the empirical evidence on the fact that periods of declining prices (fear scenario) have a greater impact on volatility than periods of rising prices (greed scenario). Instead, following literature on nonlinear dynamics to financial markets, we focus on the role of our model to replicate these stylized facts from a mathematical and a numerical point of view (see Dieci and He, 2018; He *et al.*, 2019).

Our analysis confirms the findings of Elyasiani *et al.* (2018), that is, the RAX index has the advantage of signaling prevailing market sentiment. In particular, it is a sentiment index of fear in the sense that it is able to signal downturn moves to investors that can modify their strategies in order to avoid huge losses. Moreover, we find that a period of stability in the market is possible only when the fraction of one type of trader is very low.

The paper proceeds as follows. In Sec. II, we describe the model setup. Section III focuses on the study of fixed points and their stability analysis. Section IV analyzes the economic implications of our model with a bifurcation analysis and a global analysis. In Sec. V, we extend the deterministic model introducing stochastic shock to the fundamental demand of both types of trader and we carry out an in-depth statistical analysis comparing our results with those of the *S&P*500 index. Section VI concludes our work.

II. THE MODEL

We outline a financial market model, which describes price dynamics in the presence of traders with different beliefs about fundamental value. The model includes a *market maker* who adjusts the price based on *order imbalances* and two fundamentalists who believe in reversion to the mean (i.e., they expect the price to return to the fundamental value F). As a result, fundamentalists place orders to buy when the price is below F because in this case, they believe that the market is undervalued; they place orders to sell when the price is above F because they believe that the market is overvalued.

Our model incorporates two types of fundamentalists: in particular, we assume that type-2 fundamentalists (f_2) underestimate the fundamental value with respect to type-1 fundamentalists (f_1), i.e., $F_2 < F_1$. This implies that when the price (P_t) is lower than F_2, type-2 fundamentalists overestimate the price less than type-1 fundamentalists. On the other hand, when price P_t is higher than F_1, type-2 fundamentalists underestimate the price P_t more than type-1 fundamentalists. We assume that both types of fundamentalist have the same excess demand function,

$$D_t^{f_1} = \lambda(F_1 - P_t), \tag{1}$$

$$D_t^{f_2} = \lambda(F_2 - P_t), \tag{2}$$

where $D_t^{f_i}$ is the excess demand function for type-i fundamentalists; $i = 1, 2$, λ is a positive parameter and indicates how aggressively the fundamentalist reacts to the distance of the price to the corresponding fundamental value (F_1, F_2).

Within this framework, when the price is below F_2 type-2 fundamentalists buy less than type-1 fundamentalists. When the price is above F_1, type-2 fundamentalists sell more than type-1 fundamentalists.

Depending on the price, we can distinguish the following fear or greed predominance regions (see Fig. 1):

(a) $P_t > F_1 > F_2$, in this case, both fundamentalists sell, and type-2 fundamentalists sell more than type-1 (fear predominance region).
(b) $F_2 < P_t < F_1$, type-2 fundamentalists sell, whereas type-1 fundamentalists buy. This is similar to the bull and bear regime described in Day and Huang (1990) or Tramontana *et al.* (2009) (fear and greed mixed predominance region).
(c) $P_t < F_2 < F_1$, both types of fundamentalists buy, but type-2 fundamentalists buy less than type-1 fundamentalists (greed predominance region).

The stock market is characterized by the presence of a market maker that sets the stock price P_{t+1} according to total excess demand,

$$P_{t+1} = P_t + (w_1 D_t^{f_1} + w_2 D_t^{f_2}), \tag{3}$$

where w_i is the proportion of fundamentalists of type i, $i = 1, 2$ and $w_1 + w_2 = 1$.

We consider a fixed number (we assume that a trader can only trade one unit at a time) of traders equal to $2N$, n_i is the number of fundamentalists of type $i = 1, 2$,

$$n_1 + n_2 = 2N. \tag{4}$$

Therefore,

$$w_1 = \frac{n_1}{2N} \quad \text{and} \quad w_2 = \frac{n_2}{2N}. \tag{5}$$

We assume that the number of type-1 and type-2 fundamentalists varies according to the following market sentiment index:

$$\eta_t = \frac{(F_1 - P_t)^2 - (F_2 - P_t)^2}{(F_1 - P_t)^2 + (F_2 - P_t)^2}. \tag{6}$$

The sentiment index measures in relative terms the distance between the price P_t and the two fundamental prices, F_1 and F_2.

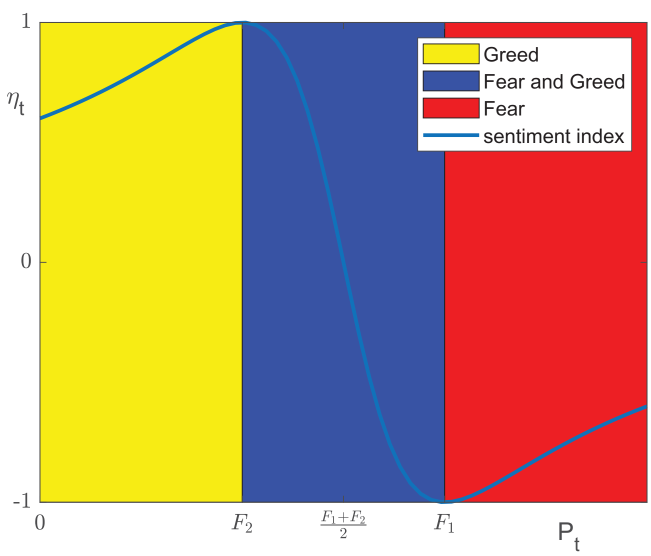

FIG. 1. Fear and greed predominance regions.

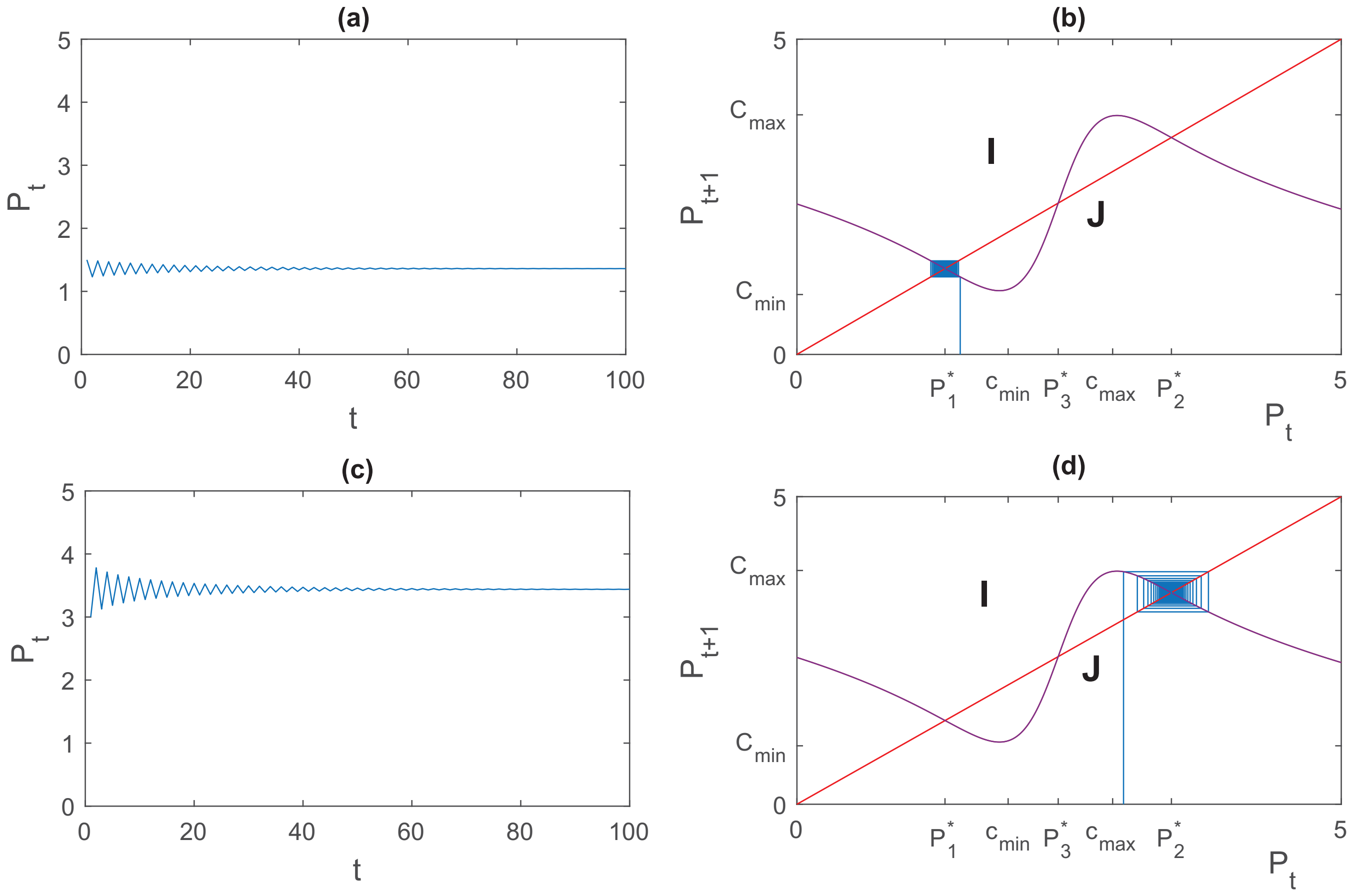

FIG. 2. Coexistence of attractors for parameters $n_2 = 0.5$, $F_1 = 3.44$, $F_2 = 1.36$, $\lambda = 1.3$, and $N = 0.5$. In (a) and (b) (greed scenario), an i.c. $P_0 = 1.5$ generates a trajectory converging to the fixed point P_1^*. In (c) and (d) (fear scenario), for $P_0 = 3$, the trajectory converges to the fixed point P_2^*.

In particular, the numerator measures how close the price is to the fundamental value F_1 and subtracts how close the price is to the fundamental value F_2. This difference is normalized by the sum of the two distances (the denominator). In this way, the sentiment index is bounded in the interval $[-1, 1]$ (see Fig. 1). In particular, if the price is close to the fundamental value F_1, then the index is negative since for prices higher than F_1 both types of investors sell and for prices $P_t > \frac{F_1+F_2}{2}$ and lower than F_1 type-1 investors buy with a lower intensity than that which characterizes the selling behavior of type-2 investors, yielding an overall sell signal. In fact, when $P_t > \frac{F_1+F_2}{2}$, then the sentiment index is negative. When $P_t = F_1$, then the sentiment index reaches its minimum value equal to -1. The sentiment index is such that if the price is higher than F_1 it gradually returns to zero, since the selling behavior of both types of fundamentalists determines an oversold market. On the other hand, if the price is close to the fundamental value F_2, then the index is positive since for prices lower than F_2, both types of investors buy and for prices $P_t < \frac{F_1+F_2}{2}$ and higher than F_2, type-1 investors buy with a higher intensity than that which characterizes the selling behavior of type-2 investors, yielding an overall buy signal. In fact, when $P_t < \frac{F_1+F_2}{2}$, then the sentiment index is positive. When $P_t = F_2$, then the sentiment index reaches its maximum value equal to 1. The sentiment index is such that if the price is lower than F_2, it gradually returns to zero, since the buying behavior of both types of fundamentalists determines an overbought market. Last, if $P_t = \frac{F_1+F_2}{2}$, the sentiment index is equal to zero, indicating no buy or sell signal.

The number of type-1 and type-2 fundamentalists varies according to the sentiment index in the following way (see (Lux, 1995)):

$$n_1 = n_2 - 2N\eta. \tag{7}$$

As a result, the number of type-1 fundamentalists is equal to the number of type-2 fundamentalists when $\eta = 0$. The number of type-1 fundamentalists is greater than the number of type-2 fundamentalists when $\eta < 0$. The number of type-1 fundamentalists is smaller than the number of type-2 fundamentalists when $\eta > 0$. The

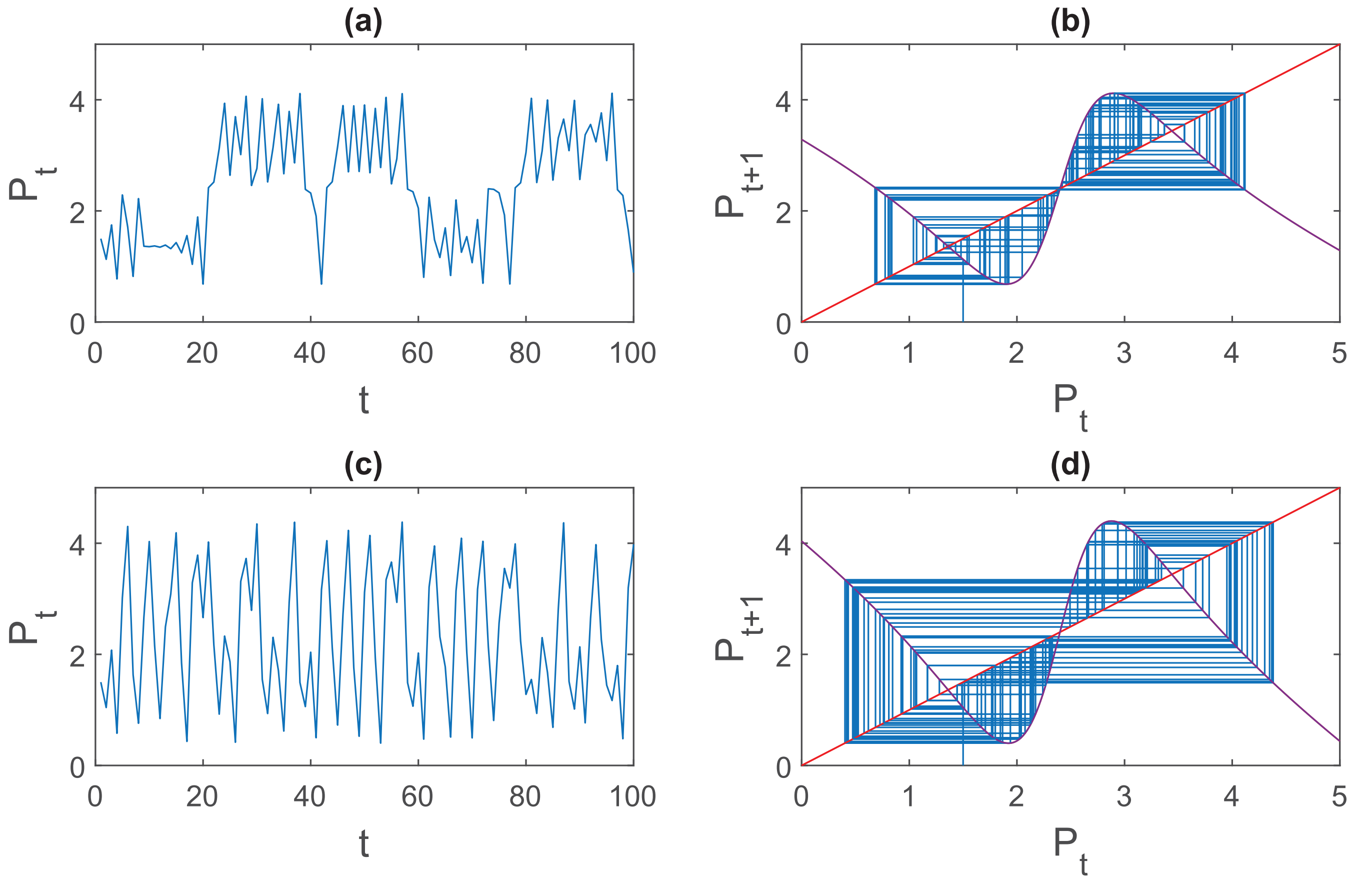

FIG. 3. Homoclinic bifurcations scenario. In (a) and (b), two complex attractors merge for $n_2 = 0.5$, $F_1 = 3.44$, $F_2 = 1.36$, $\lambda = 1.79$, $N = 0.5$, and i.c. $P_0 = 1.5$. In (c) and (d), there is the occurrence of an homoclinic bifurcation for $n_2 = 0.5$, $F_1 = 3.44$, $F_2 = 1.36$, $\lambda = 2.2$, $N = 0.5$, and i.c. $P_0 = 1.5$.

closer the price to the fundamental value F_1, the more the proportion of type-1 fundamentalists increases since they performed better than the other group in forecasting the equilibrium price (and in the market, we have an overall fear predominance since investors expect the price to decrease). On the other hand, the closer the price to the fundamental value F_2, the more the proportion of type-2 fundamentalists increases since they performed better than the other group in forecasting the equilibrium price (and in the market, we have an overall greed predominance since investors expect prices to increase).

The final map: Based on the above considerations, we obtain the first order nonlinear discrete dynamical equation, which describes the price evolution over time. It takes the following form:

$$P_{t+1} = P_t + \left\{ \left[\frac{n_2}{2N} - \frac{(F_1 - P_t)^2 - (F_2 - P_t)^2}{(F_1 - P_t)^2 + (F_2 - P_t)^2} \right] \lambda (F_1 - P_t) + \left[\frac{n_1}{2N} + \frac{(F_1 - P_t)^2 - (F_2 - P_t)^2}{(F_1 - P_t)^2 + (F_2 - P_t)^2} \right] \lambda (F_2 - P_t) \right\}. \quad (8)$$

III. STABILITY ANALYSIS OF EQUILIBRIUM POINTS

In this section, we explore the qualitative properties of Map (8). We first consider, for the sake of completeness, the case of homogeneous fundamentalists, then the more interesting case of heterogeneous fundamentalists.

Homogeneous fundamentalists: If both fundamentalists are of the same type, they have equal beliefs in the fundamental price, that is, $F_1 = F_2$, then the final Map (8) becomes

$$P_{t+1} = P_t + \left(1 - \frac{n_1}{2N}\right) \lambda (F - P_t) + \left(\frac{n_1}{2N}\right) \lambda (F - P_t),$$

and the fixed point is the fundamental price $P^* = F$. In this case, it turns out that

$$\frac{dP_{t+1}}{dP_t} = 1 - \lambda,$$

and we have a situation in which the market reaches a stable equilibrium if $0 < \lambda < 2$.

Heterogeneous fundamentalists: We first show the existence of the fixed points analytically and then we study their local stability

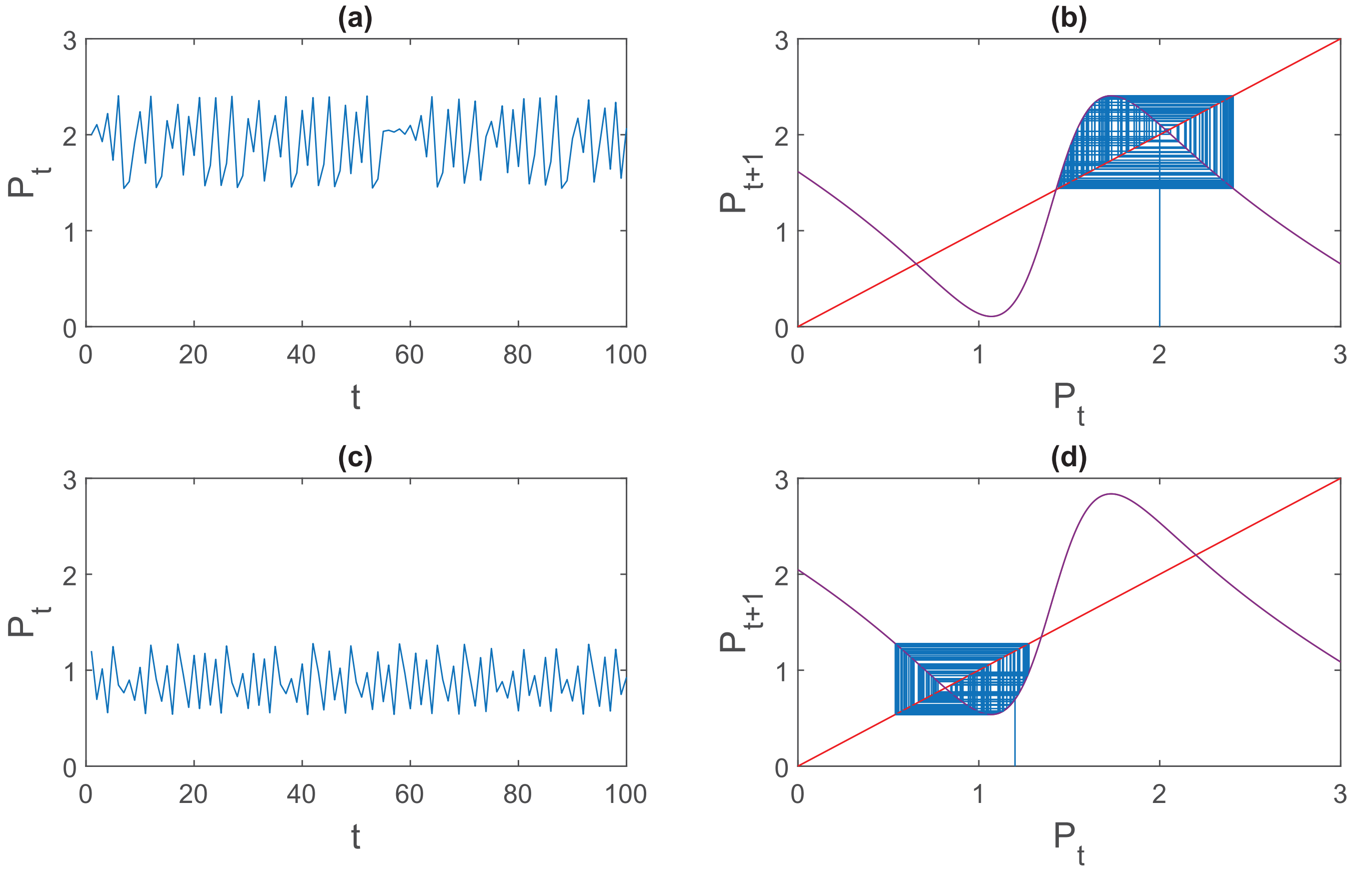

FIG. 4. Asymmetric mid fixed point. In (a) and (b), the trajectory converges to the fear attractor for $n_2 = 0.7$, $F_1 = 2.04$, $F_2 = 0.65$, $\lambda = 1.8$, $N = 0.5$, and i.c. $P_0 = 2$. In (c) and (d), the trajectory leads to the greed attractor for $n_2 = 0.4$, $F_1 = 2.2$, $F_2 = 0.81$, $\lambda = 1.8$, $N = 0.5$, and i.c. $P_0 = 1.2$.

by means of graphical analysis. The condition for the existence of the steady state of the Map (8) is $P_{t+1} = P_t = P^*$. Therefore, we solve the following equation in the variable P^*:

$$T(P^*) = \left[\frac{n_2}{2N} - \frac{(F_1 - P^*)^2 - (F_2 - P^*)^2}{(F_1 - P^*)^2 + (F_2 - P^*)^2}\right]\lambda(F_1 - P^*) + \left[\left(1 - \frac{n_2}{2N}\right) + \frac{(F_1 - P^*)^2 - (F_2 - P^*)^2}{(F_1 - P^*)^2 + (F_2 - P^*)^2}\right]\lambda(F_2 - P^*) = 0. \tag{9}$$

The following proposition holds.

Proposition 1. *Assume $P_t > 0\,\forall t$ and $F_1 > F_2$. Then, Map (8) admits three real fixed points P_i^* with $i = 1, 2, 3$ given by*

$$P_1^* = F_2, \tag{10}$$

$$P_2^* = F_1, \tag{11}$$

$$P_3^* \in (F_2, F_1). \tag{12}$$

The fixed points belong to the interval $[F_2, F_1]$.

Proof. In order to compute the fixed points, we have to solve Eq. (9). It may be seen that the two fundamental prices F_1 and F_2 are two of the fixed points of the model. Indeed, if we substitute $P^* = F_1$ we have

$$-\frac{n_2}{2N}\lambda(F_2 - F_1) = 0.$$

Note that, assuming $P^* = F_1$, we have a situation in which type-1 fundamentalists have performed better than type-2 fundamentalists, and in this case, all fundamentalists become type 1, which implies $\frac{n_2}{2N} = 0$. Therefore, F_1 solves Eq. (9) and it is a solution.

A similar argument holds with regard to the second fixed point, F_2. Assuming $P^* = F_2$, we have that type-2 fundamentalists have performed better than type-1 fundamentalists, and in this case, all fundamentalists become type 2, which implies $\frac{n_1}{2N} = 0$. Therefore, F_2 solves Eq. (9) and it is a solution.

For the existence of the third fixed point in the interval (F_2, F_1), we make use of the intermediate value theorem. In particular, consider the midpoint of the segment $\bar{F_2 F_1}$, i.e., $\frac{F_1+F_2}{2}$. Now, we take the midpoints of the two intervals $(F_2, \frac{F_1+F_2}{2})$ and $(\frac{F_1+F_2}{2}, F_1)$, namely, points I_1 and I_2 and evaluate Eq. (9) in each of the points. Given that

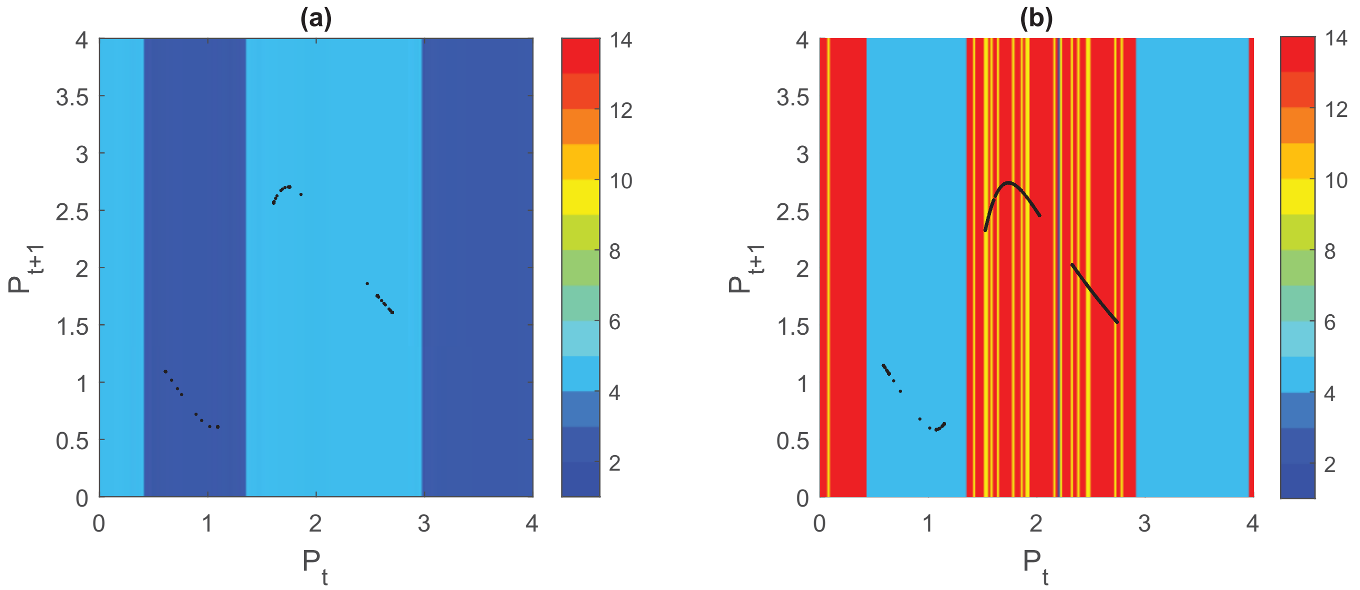

FIG. 5. The role of parameter λ. Two pictures showing the basins of attraction of Map (8). In (a), $n_2 = 0.7$, $F_1 = 1.8$, $F_2 = 1$, $\lambda = 1.58$, and $N = 0.5$. In (b), we use the set of parameters $n_2 = 0.7$, $F_1 = 1.8$, $F_2 = 1$, $\lambda = 1.64$, and $N = 0.5$.

$T(P(I_1)) < 0$ and $T(P(I_2)) > 0$ and that the function $T(P)$ is continuous in the interval (F_2, F_1), it may be said that in this interval, there is a solution for Eq. (9) and it is unique. This concludes the proof. □

Note that in the steady state $P_2^* = F_1$ ($P_1^* = F_2$), the share of type-2 fundamentalists (1) is zero. The presence of both types of fundamentalists is possible only in the third steady state P_3^*. Thanks to graphical analysis, it will be demonstrated that the steady state P_3^* is

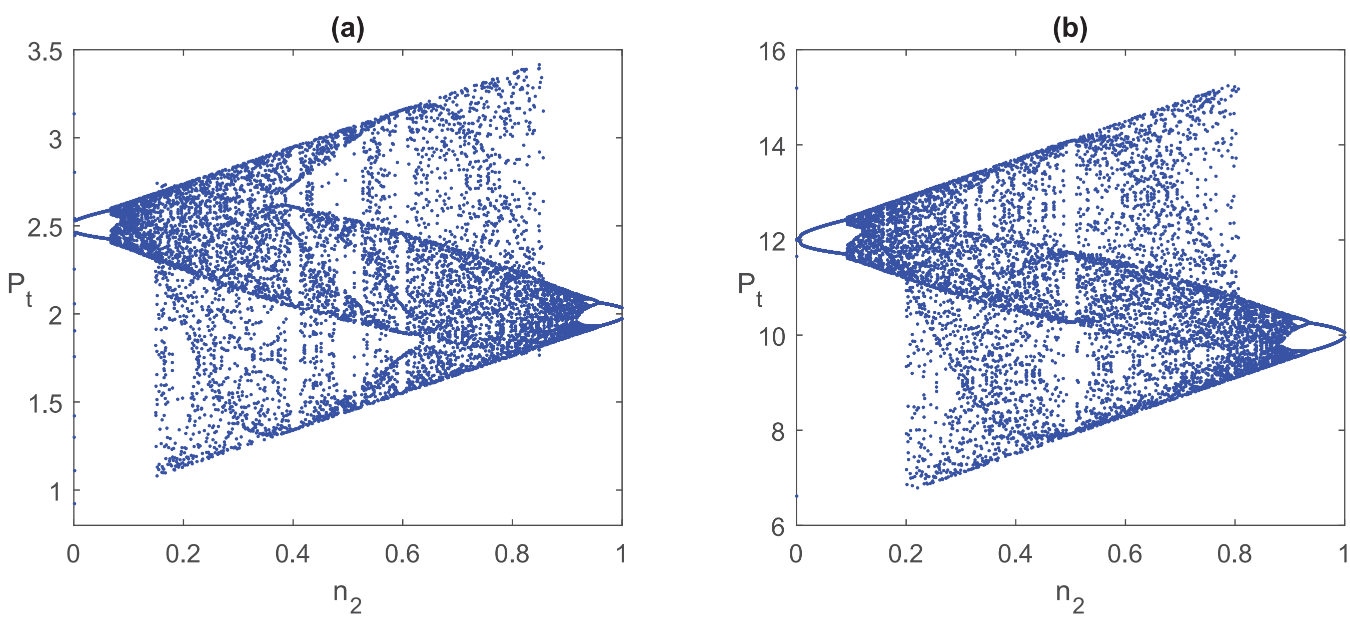

FIG. 6. The role of the difference between F_1 and F_2. Two pictures showing the bifurcation diagram with respect to n_2 in the fear and greed scenario. In (a), parameters $F_1 = 2.5$, $F_2 = 2$, $\lambda = 2$, $N = 0.5$, and i.c. $P_0 = 1.5$. In (b), parameters $F_1 = 12$, $F_2 = 10$, $\lambda = 2$, $N = 0.5$, and i.c. $P_0 = 1.5$.

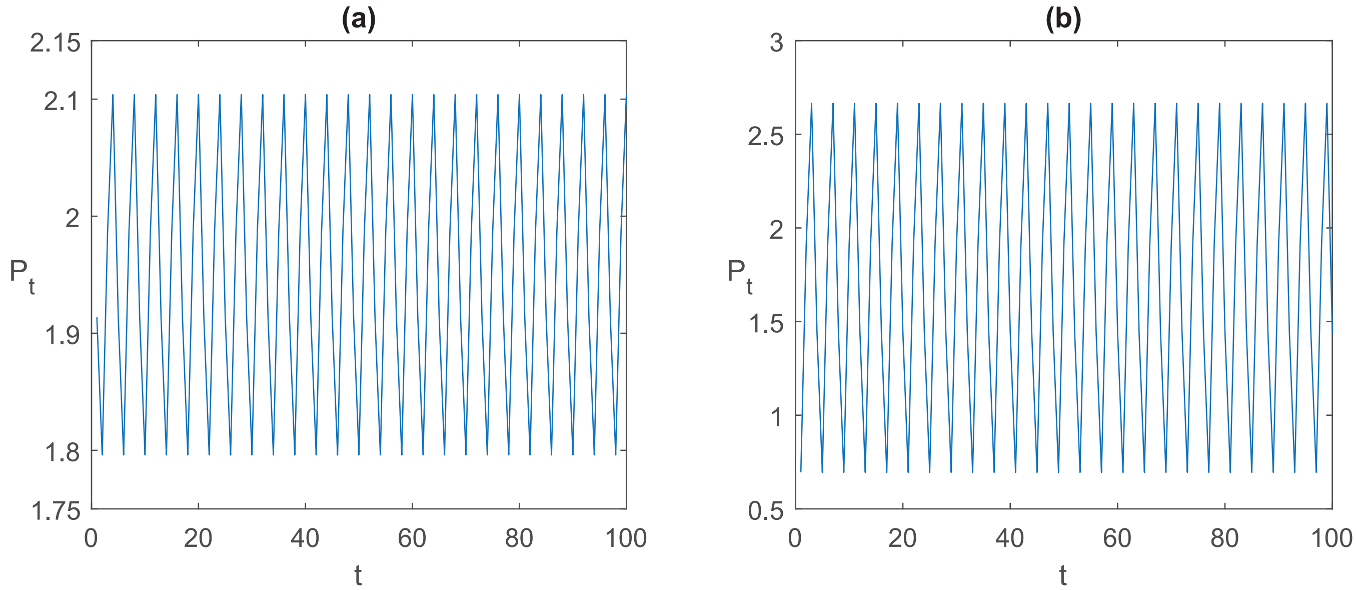

FIG. 7. The role of the difference between F_1 and F_2. A stable two-cycle in (a) greed scenario for $n_2 = 0.5$, $F_1 = 2$, $F_2 = 1.9$, $\lambda = 2$, $N = 0.5$, and i.c. $P_0 = 1.5$; and (b) fear scenario for $n_2 = 0.5$, $F_1 = 2$, $F_2 = 1.36$, $\lambda = 2$, $N = 0.5$, and i.c. $P_0 = 1.5$.

always unstable since it delimits the basins of attraction of the other two steady states. For appropriate values of parameters, we find that it does not coincide with $(\frac{F_1+F_2}{2})$.

The possible scenarios arising in our model are described in the following Proposition 2.

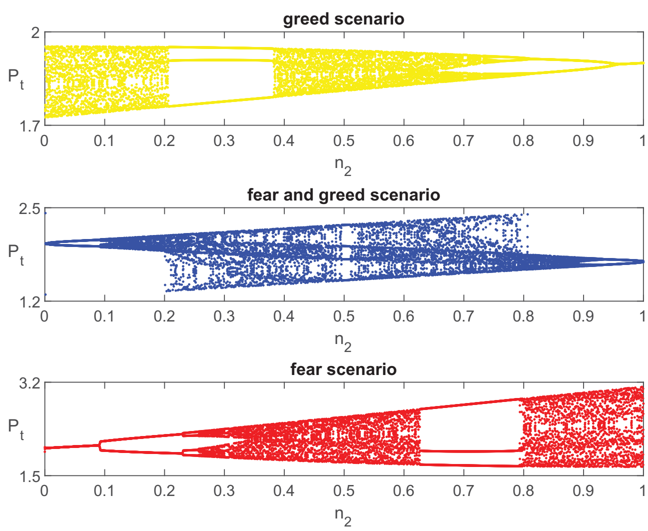

FIG. 8. The three scenarios. Greed scenario (yellow diagram) with parameters $F_1 = 2$, $F_2 = 1.9$, $\lambda = 1.72$, $N = 0.5$, and i.c. $P_0 = 1.8$. Fear and greed scenario (blue diagram) with parameters $F_1 = 2$, $F_2 = 1.75$, $\lambda = 2$, $N = 0.5$, and i.c. $P_0 = 1.8$. Fear scenario (red diagram) with parameters $F_1 = 2$, $F_2 = 1.36$, $\lambda = 1.72$, $N = 0.5$, and i.c. $P_0 = 1.8$.

Proposition 2. *Assume $F_1 \neq F_2$. Let $c_{\min}$, $c_{\max}$ be the local minimum and the local maximum of the Map (8); $C_{\min}$ and $C_{\max}$ are their iterates, respectively, that is, $C_{\min} = P_{t+1}(c_{\min})$ and $C_{\max} = P_{t+1}(c_{\max})$, then there exist two disjoint invariant [A set $I \subseteq \mathbb{R}_+$ is positively (negatively) invariant if $T^i(I) \subseteq I$ ($T^i(I) \supseteq I$) $\forall i \in \mathbb{Z}_+$. Moreover, I is invariant when it is both positively and negatively invariant. Finally, a closed and positively invariant region is called trapping.] intervals $I = [P_{t+1}(c_{\min}), P_{t+1}(C_{\min})]$ and $J = [P_{t+1}(c_{\max}), P_{t+1}(C_{\max})]$ such that:*

1. ***Fear or Greed scenario:*** *if $P_{t+1}(C_{\min}) < P_3^*$ and $P_{t+1}(C_{\max}) > P_3^*$, then Map (8) has two coexistent attractors, P_1^* and P_2^*.*
2. ***Fear and Greed scenario:*** *for $P_{t+1}(C_{\min}) = P_{t+1}(C_{\max}) = P_3^*$ the two attractors merge and a contact bifurcation occurs.*

In scenario (1) depicted in Fig. 2, we observe the trajectory of the price converging to one of the two coexisting attractors, P_1^* or P_2^*, depending on the initial condition P_0. Indeed, if we take an initial condition P_0 close to P_1^*, then the price converges to P_1^* [Figs. 2(a) and 2(b), greed scenario]. On the other hand, taking an initial condition P_0 close to P_2^*, then the price converges to P_2^* [Fig. 2(c) and 2(d), fear scenario]. It may be noted that the attractor may be a fixed point, an n-period cycle, a strange attractor, or the union of two coexisting attractors. Therefore, in scenario (1), we have a greed or a fear scenario depending on the initial condition.

In scenario (2) shown in Fig. 3, we have the occurrence of a homoclinic bifurcation. In particular, in Figs. 3(a) and 3(b) the two absorbing intervals are merged. In this case, the two chaotic attractors are transformed into a one-piece chaotic attractor. Figure 3(c) and 3(d) shows a more chaotic dynamic, with respect to the former, due to the merging of the two attractors, which now form a q-piece chaotic attractor where complex price dynamics arise. Therefore, in this scenario, we have a recursive flip of the price between fear and greed regions.

Another important feature of our model is that the fixed point P_3^* could be asymmetrically distributed in the interval (F_2, F_1). In Fig. 4, we find two trajectories leading to the two coexisting attractors. This case arises for a different value of the parameter n_2 (the fraction of fundamentalists of type 2). Indeed, when we decrease the value of n_2, the basin of attraction of the fixed point F_2 enlarges with respect to that of the other fixed point F_1.

Both the scenarios outlined in Figs. 2 and 4 are related to the co-existence of multiple equilibria. As stressed by He *et al.* (2018), the coexistence of multiple equilibria is the source of interesting dynamics of the price. In fact, in line with these authors, we identify the coexistence of two local attractors (the fixed points P_1^* and P_2^*). Depending on the initial condition, and on the model parameters, the model may converge to one of the two attractors. However, unlike He *et al.* (2018), our model does not exhibit bistable dynamics, given that we have a multiple equilibria mechanism. As a result, the (He *et al.*, 2018) model is able to characterize seemingly unrelated or even opposite market phenomena, while our model is able to characterize one single market phenomenon: volatility in the fear scenario is greater than volatility in the greed scenario.

Finally, in Fig. 5, we investigate the role of the reactivity parameter λ in determining the structure of the basins of attraction. In particular, in Figs. 5(a) and 5(b), we have two disconnected basins of attraction, i.e., the basins of the two fixed points are located also in regions that do not contain the relative fixed points (see Abraham *et al.*, 1997). The greater the value of λ, the more complex the structure of the basins of attraction.

In Sec. IV, we describe in detail the scenarios analyzed in the graphical analysis from an economic perspective in order to match our findings with the stylized facts.

IV. NUMERICAL ANALYSIS

In this section, we shed light on the main insights of our work. From the stability analysis, we know that the three equilibria always exist. From an economic point of view, instead, our model is able to capture interesting features of financial markets. In particular, our goal is the introduction of the RAX sentiment index introduced by Elyasiani *et al.* (2018) observing how it

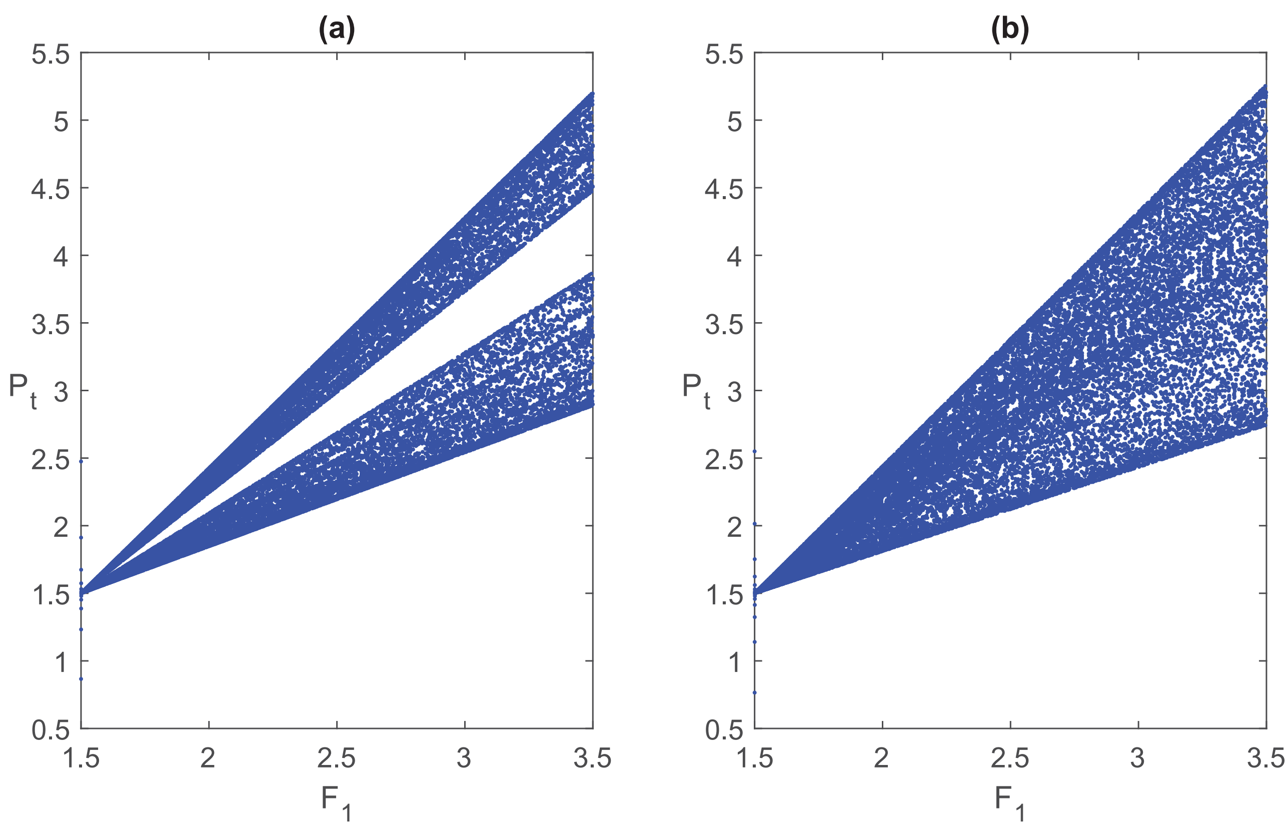

FIG. 9. Bifurcation diagrams with respect to F_1 showing the transition from a two-piece chaotic attractor to a one-piece chaotic attractor for given values of parameters. In (a), $n_2 = 0.5$, $F_2 = 1.5$, $\lambda = 1.65$, $N = 0.5$, and i.c. $P_0 = 1.5$. In (b), $n_2 = 0.5$, $F_2 = 1.5$, $\lambda = 1.7$, $N = 0.5$, and i.c. $P_0 = 1.5$.

TABLE I. Summary statistics of returns. The table reports the summary statistics including mean, standard deviation (SD), skewness, kurtosis, minimum and maximum value, Jarque–Bera test, and statistic of *S&P*500, *SM*.

	Mean	SD	Min	Max	Skewness	Kurtosis	J–B	J–B statistic
*S&P*500	0.0000	0.0093	−0.0001	0.0000	−0.4976	7.5827	1	2324.90
SM	0.0000	0.1175	−0.0006	0.0005	−0.1493	4.9563	1	1240.10

works as a reference index of market sentiment. Instead of introducing heterogeneity into the behavioral rules of each group of agents, we highlight how agents with the same trading behavior (fundamentalists) influence market sentiment. In this sense, the RAX index is a sentiment index because it records the prevalent trend of the market (which we have referred to as fear and greed predominance).

Another important element of our analysis is the role of the fundamental values perceived by the two groups of traders. In particular, the difference in the two fundamental prices matters not only in a mixed fear and greed scenario Fig. 6 but also in a fear or greed scenario Fig. 7 thanks to the signal contained in the sentiment index. Indeed, in Fig. 6(a), it is apparent how the interaction of both types of fundamentalist leads to complicated dynamics and a period of volatility of the price. On the other hand, if we look at Fig. 6(b) for the same level of reaction of traders, we see an amplification of price volatility when the distance between the two fundamental prices increases. Moreover, in both cases, when the fraction of fundamentalists of type i, with $i = 1, 2$, is sufficiently lower than the proportion of fundamentalists of type j, with $j = 1, 2$ and $j \neq i$, then the market reaches a stable equilibrium. Considering Fig. 6(a), for $n_2 \approx 0.1$, the stable two-cycle scenario disappears and a cascade of flip bifurcation becomes apparent. We note that two different attractors coexist, implying that the asymptotic dynamic of price can lead to one of the two attractors depending on the initial condition. At $n_2 \approx 0.2$, the two attractors merge into a single attractor generating a homoclinic bifurcation. In line with (He and Zheng, 2016), we can attribute the difference between the two fundamental prices to the uncertainty about the true value of the fundamental. Traders try to guess the fundamental price, taking into account private information in addition to the public information given by the sentiment index. This gives rise to endogenous heterogeneity and switching behavior on the part of agents.

Based on our analysis, a period of stability is possible when the proportion of one type of trader is very low (e.g., n_2, see Fig. 8), implying that when the market is mainly populated by one type of fundamentalist, the level of uncertainty diminishes significantly. However, also in a period of stability, the difference between the two fundamental prices determines a higher price volatility in a fear scenario [Fig. 7(b)] compared to a greed scenario [Fig. 7(a)]. [In the empirical literature (see, e.g., Caloia *et al.*, 2018), the volatility of negative returns is higher than that of positive returns. This difference is exacerbated in high volatility periods.]

Our framework is also able to capture the asymmetry in the return distribution of prices. In this connection, the introduction of the RAX index has the property of signaling the sentiment prevailing in the market, in order to allow traders to take their final decision on whether to buy or sell. It is self-evident that investors prefer positive returns to negative ones, and the literature provides evidence that the return distribution is negatively asymmetric which implies in turn many low positive returns and a small number of large losses. This is captured in our model. In the sentiment index positive and negative returns are weighted asymmetrically, as reflected in the behavior of agents that buy and sell on the basis of

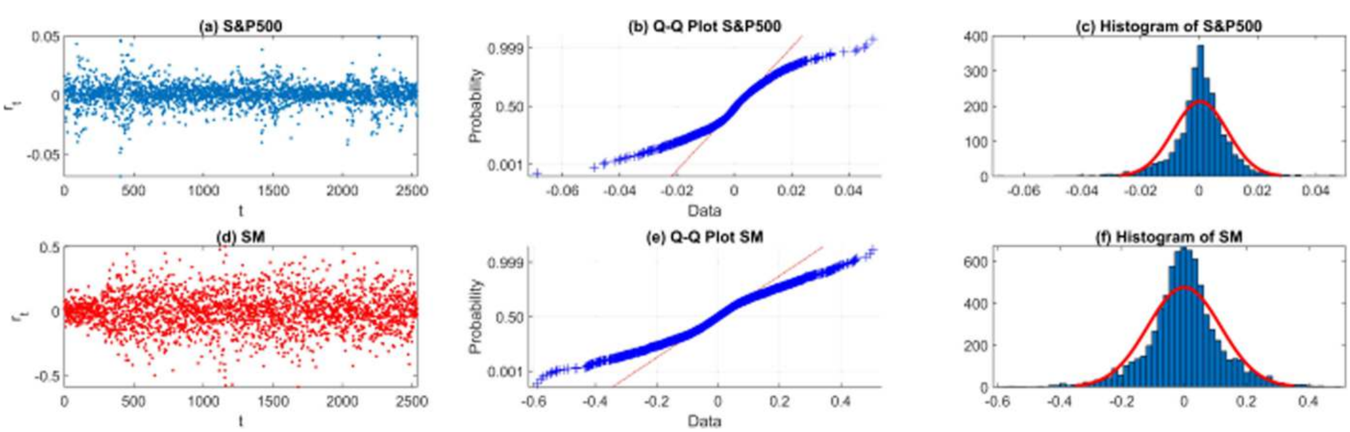

FIG. 10. Returns series [panel (a)], Q–Q plot [panel (b)], and the histogram [panel (c)] for *S&P*500 index for time periods $t \in [2010, 2020]$. The same diagrams for *SM* are figured in panels (c)–(e), respectively.

TABLE II. Power law of returns. For each return series in $S\&P500$ and SM, we estimate $\ln P(\left|\frac{r_t-\bar{r}}{SD}\right| > X) = -\zeta \ln X + b$ with ordinary least squares and report ζ, the number of observation N, and R^2.

	$S\&P500 \ln P(\left\|\frac{r_t-\bar{r}}{SD}\right\| > X)$	$SM \ln P(\left\|\frac{r_t-\bar{r}}{SD}\right\| > X)$
ζ	2.9074[a]	2.7087[a]
	(0.2564)	(0.1781)
N	46	46
R^2	0.8533	0.9117

[a]Significant at 1%.

the sentiment index and generate skewness in returns. As a result, the volatility of prices when the market faces a fear scenario is greater than the volatility occurring in the greed scenario or in the mixed fear-and-greed scenario. In Fig. 8, we show the three possible scenarios of our model. It is evident that, in the fear scenario, the volatility of the price is much larger than in the other two cases. These findings confirm that the RAX index is a sentiment index of fear in the sense that it is able to signal downturn periods to investors who can modify their strategies in order to avoid huge losses.

Finally, we wish to stress the role of the reactivity parameter λ and heterogeneity $(F_2 - F_1)$ in determining chaotic dynamics. This situation is described in Fig. 9 where we note the transition to more complicated dynamics when both λ and F_1 increase. Figure 9(a) shows the origin of a two-piece chaotic attractor, while in Fig. 9(b), we see that the two attractors give rise to a homoclinic bifurcation.

V. STOCHASTIC MODEL

In this section, we explore the statistical properties generated by our model, showing that it is able to match a rich set of empirically observed stylized facts. The main empirical evidence that we will deal with in this section will concern non-normality of the returns, heavy tails, and volatility clustering (see Cont, 2002; He and Le, 2007; He and Zheng, 2016; Lux and Alfarano, 2016; Lux and Marchesi, 2006, for example). In particular, we add noise to both the fundamental demand processes and examine the dynamics of the model using numerical simulations. Combining the deterministic and stochastic elements, the demand of type-1 fundamentalists is

$$D_t^{f_1} = \lambda(F_1 - P_t) + \epsilon_t^{f_1}, \quad \epsilon_t^{f_1} \sim N(0, (\sigma)^2), \tag{13}$$

and, similarly, the demand of type-2 fundamentalists is given by

$$D_t^{f_2} = \lambda(F_2 - P_t) + \epsilon_t^{f_2}, \quad \epsilon_t^{f_2} \sim N(0, (\sigma)^2), \tag{14}$$

where σ is a positive parameter representing the standard deviations of the normal random variables that we assume equal for both types of fundamentalists. In our analysis, we consider the daily log-returns (In the simulations, we analyze the most interesting case of $F_2 < P_t < F_1$. Given the initial value of the price P_t, we do not face non-positive prices.), defined as

$$r_t = \ln(P_t) - \ln(P_{t-1}), \tag{15}$$

focusing on the from normality of their distribution. As a benchmark, we consider the daily log-returns of the $S\&P500$ from January 02, 2010 to January 28, 2020. To differentiate the time series generated from the $S\&P500$ index ($S\&P500$) and the simulated stochastic model (SM) we add $S\&P500$ and SM in front of the name of each time series. In all the simulations performed, we consider the following parameter set: $N = 0.5$; $n_2 = 0.5$; $F_1 = 2000$; $F_2 = 1200$; $\lambda = 0.1$, $P_0 = 1500$,and $\sigma = 0.09$. In Table I, we provide some useful summary statistics for the returns of the $S\&P500$ and the SM and the diagrams shown in Fig. 10 will help us to interpret these statistics.

Figures 10(a) and 10(d) show that the time series exhibit volatility clustering, which is characterized by intermittent and large fluctuations. Actually, the heterogeneity regarding the beliefs about the fundamental price is responsible for producing such patterns. In Figs. 10(c) and 10(f), we compare the shape of the distribution with a normal distribution with variance identical to the sample variance (depicted by the solid line). We note a stronger concentration around the mean, with a greater probability mass in the tails of the distribution and thinner shoulders. All these aspects are a typical deviation from the normal distribution and are to be found in the empirical literature (see Lux, 1998; Lux and Marchesi, 2006).

In Figs. 10(b) and 10(e), we report the Q–Q plot of the returns for $S\&P500$ and SM. They display the quantile of the sample data (returns) vs the theoretical quantiles of the normal distribution. If the distribution of returns is normal, then the plot appears to be linear, which is not the case in this instance. Indeed, the tails lay below and above the 45° line implying that their distributions are fat-tailed, as well [see Figs. 10(b) and 10(e)]. We also analyze the non-normality of the returns conducting the Jarque–Bera test (see Table I). In particular, we test the null hypothesis that returns follow a normal distribution at the 1% significant level. The Jarque–Bera test rejects the null hypothesis at the 1% significance level (as indicated by the value 1 in the J–B column); in fact, the test statistic, J–B statistic, is greater than the critical value, which is 5.8461.

Some of the empirical quantitative properties related to returns include a power-law behavior, long memory, and correlation to volatility. We intend to demonstrate all these stylized facts with our model. Lux and Alfarano (2016) review a number of universal power laws characterizing financial markets. We take into account two of these in our study. The first concerns the distribution of the returns, which is characterized by the presence of fat tails. It is found that

TABLE III. Persistence of ACFs of absolute returns. For each return series in $S\&P500$ and SM, we estimate $corr(|r_{t+q}|, |r_t|) \simeq \zeta/q^d$ with nonlinear least squares and report ζ and d.

	$S\&P500$ $corr(\|r_{t+q}\|, \|r_t\|)$	SM $corr(\|r_{t+q}\|, \|r_t\|)$
d	0.3692[a]	0.2218[a]
	(0.0615)	(0.0163)
ζ	0.3360[a]	0.3219[a]
N	150	150
R^2	0.7359	0.8275

[a]Significant at 1%.

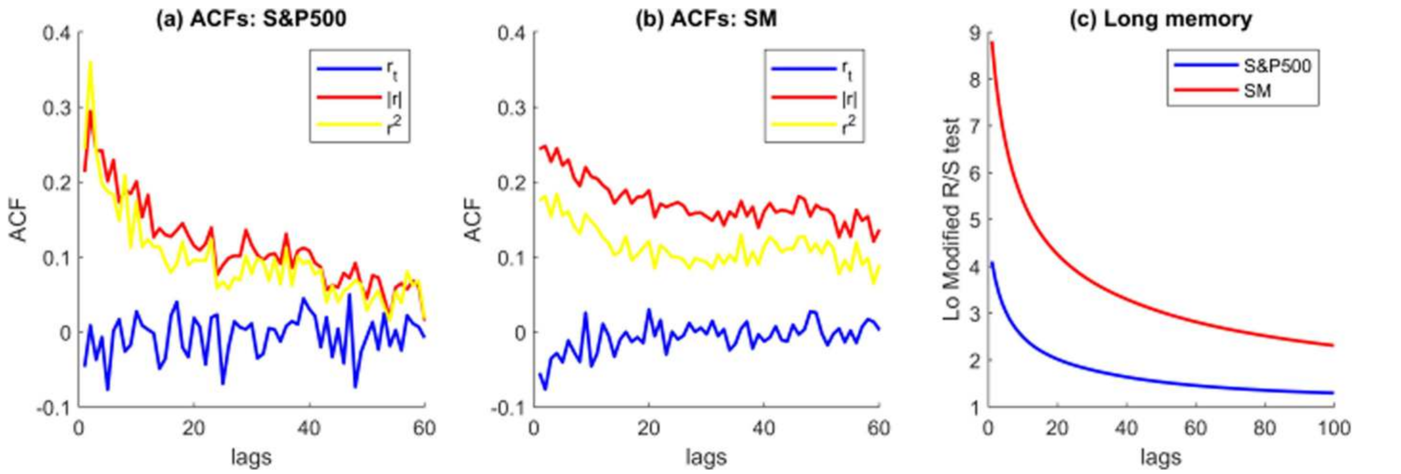

FIG. 11. Volatility clustering and long-range dependence. Panels (a) and (b) plot the ACFs (autocorrelation functions) of returns (blue line), the absolute returns (red line), and the squared returns (yellow line) for *S&P*500 and *SM*, respectively. Panel (c) plots the Lo-modified R/S statistic of the absolute returns.

fat-tailed distributed returns approximately follow an inverse cubic power law described by

$$P\left(\left|\frac{r_t - \bar{r}}{SD}\right| > X\right) \sim X^{-\zeta}, \tag{16}$$

where $\zeta \simeq 3$ is the Pareto exponent (also called the characteristic exponent), $\bar{r}$ and SD are the mean and the standard deviation of the returns r_t, and $\frac{r_t-\bar{r}}{SD}$ is the normalized return. The pertinent literature converged on the insight of an exponent close to 3.

Following He and Zheng (2016) and Gabaix *et al.* (2006), we estimate the characteristic exponent with ordinary least squares (OLSs) estimating the logarithm of Eq. (16), that is,

$$\ln P(\left|\frac{r_t - \bar{r}}{SD}\right| > X) = -\zeta \ln X + b. \tag{17}$$

As shown in Table II, for both *S&P*500 and *SM*, the value of the characteristic exponent, ζ, is close to 3, confirming the findings of the empirical literature.

The second power law relates to the concept of *volatility clustering*. We can see how persistent the volatility is by estimating the following power component in the ACFs of absolute returns (see Cont, 2002; He and Zheng, 2016)

$$corr(|r_t|, |r_{t+q}|) \simeq \frac{\zeta}{q^d}, \tag{18}$$

where q is the number of lags, ζ is a parameter capturing the ACF of absolute returns with lag one, and d is the power exponent capturing the decay of the ACFs. We estimated Eq. (18) with non-linear least squares and the results of each model are shown in Table III. As we can see, our estimates are in line with the empirical evidence, which postulates a value of d in the interval [0.2 0.4]. Figures 11(a) and 11(b) plot the series of sample autocorrelation of the log-returns, absolute returns, and squared returns of the simulated data and of the *S&P*500. Both the stochastic and empirical series of absolute and squared returns are characterized by autocorrelation persistent up to more than 60 lags, in line with empirical findings observed in financial time series.

Finally, we complete the current analysis by testing the hypothesis of long-range dependence in the volatility measured by the time series of $|r_t|$. To this end, we compute the Lo-modified range over standard deviation or R/S statistic (also called re-scaled range) (see Lo, 1991). In Fig. 11(c), we show the R/S statistic for lags ranging from 1 to 100, and it is possible to see that we reject the null hypothesis of no long-range dependence when the number is not particularly large, i.e., $q \leq 30$. Indeed, for these values of the lags, the Lo-modified R/S statistic for *S&P*500 and *SM* all fall out of the 95% critical interval [0.809, 1.862] suggesting the presence of long memory in absolute return series.

VI. CONCLUSIONS

The aim of the paper is to model trading decisions of financial investors based on a sentiment index introduced by Elyasiani *et al.* (2018). We consider two groups of fundamentalists with heterogeneous beliefs about the fundamental value and we model an endogenous switching mechanism between the two groups of traders, relying on the sentiment index. The sentiment index is considered as a benchmark index for all agents. Depending on the value of the price, one group of agents is more aggressive than the other in buying or selling. The main finding of our work is that the introduction of the sentiment index allows the model to generate complex price dynamics. The model is able to distinguish a fear scenario, a greed scenario, or a mixed fear and greed one. The difference in the two fundamental prices perceived, thanks to the sentiment index, matters in all three scenarios.

In particular, when the proportion of the two groups of fundamentalists is similar, we observe sudden changes from fear to greed scenarios and vice versa. Based on our analysis, a stability period is possible when the proportion of one type of trader is very low. However, also in a stable period, the difference in the two fundamental prices determines greater uncertainty in the fear compared to the greed scenario.

Moreover, we observe a higher uncertainty, proxied by the range of prices attained, in a fear than in a greed scenario. As a result, our paper casts light on investor sentiment as one of the main drivers of asymmetry in stock returns (Jawadi *et al.*, 2018; Verma and Soydemir, 2009).

Finally, we note the transition to more complicated dynamics when the reactivity to price changes of each group of traders increases.

The model is able to match many empirically observed stylized facts such as non-normality of the returns, heavy tails, and volatility clustering.

It is possible to extend our work in several ways. First, we could introduce chartists or other types of noisy traders in order to model speculative agent behavior. Second, we could increase the heterogeneity in trading decisions by assuming that each group of agents behaves according to a different sentiment index.

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APPENDIX: TWO ALTERNATIVE FORMULATIONS OF THE PRICE PROCESS

In this appendix, we show two equivalent representations of the price process (8), Model 1 and Model 2.

Model 1

Based on Eqs. (4), (5), and (7), we have

$$\begin{cases} w_1 + w_2 = 1, \\ w_1 - w_2 = -\eta, \end{cases} \tag{A1}$$

leading to

$$w_1 = \frac{1-\eta}{2}, \quad w_2 = \frac{1+\eta}{2}. \tag{A2}$$

Consequently, Eq. (8) results

$$P_{t+1} = P_t + \frac{1-\eta}{2}\lambda(F_1 - P_t) + \frac{1+\eta}{2}\lambda(F_2 - P_t). \tag{A3}$$

With this formulation the model rests on the three parameters, λ, F_1, and F_2.

Model 2

Note that Eq. (A3) can be expressed in terms of n_1 and n_2, which is the representation we have used in the paper. In this case, we have

$$n_1 + n_2 = 2N. \tag{A4}$$

Moreover, it is

$$\eta = \frac{n_2 - n_1}{2N}. \tag{A5}$$

Combining (A4) and (A5), we obtain

$$n_1 = n_2 - 2N\eta, \tag{A6}$$

which is equivalent to (7).

From (A4), we have $n_2 = 2N - n_1$; substituting in (A6), we have

$$n_1 = N(1-\eta), \quad n_2 = N(1+\eta). \tag{A7}$$

In order to normalize the total population to 1, we divide by $2N$ obtaining

$$w_1 = \frac{n_1}{2N} = \frac{1-\eta}{2}, \quad w_2 = \frac{n_2}{2N} = \frac{1+\eta}{2}, \tag{A8}$$

and this shows the equivalence between (A2) and (5).

However, we wish to stress the role of the switching mechanism between fractions n_1 and n_2. For this purpose, we consider an alternative formulation of the fractions. By using Eq. (A8), system (A1) is the following:

$$\begin{cases} \frac{n_1}{2N} + \frac{n_2}{2N} = 1, \\ n_1 - n_2 = -2N\eta, \end{cases} \tag{A9}$$

which leads to

$$\begin{cases} w_1 = \frac{n_2}{2N} - \eta, \\ w_2 = \frac{n_1}{2N} + \eta, \end{cases} \tag{A10}$$

using the fractions expressed in (A10) and the expression for the sentiment index η in Eq. (6), we obtain the price process (8),

$$\begin{aligned} P_{t+1} = P_t + \Bigg\{ &\left[\frac{n_2}{2N} - \frac{(F_1-P_t)^2 - (F_2-P_t)^2}{(F_1-P_t)^2 + (F_2-P_t)^2}\right]\lambda(F_1 - P_t) \\ &+ \left[\frac{n_1}{2N} + \frac{(F_1-P_t)^2 - (F_2-P_t)^2}{(F_1-P_t)^2 + (F_2-P_t)^2}\right]\lambda(F_2 - P_t)\Bigg\}, \end{aligned} \tag{A11}$$

which is equivalent to (A3).

By recalling Eq. (A4), the representation (A11) relies on five parameters, i.e., λ, F_1, F_2, n_1, and n_2. In this paper, we used Model 2 since it allows us to explicitly tune the amounts of the two fundamentalists n_1 and n_2. Model 1, which relies on only three parameters, is more suitable in financial models that have to be calibrated.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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