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A max-plus algebra approach to model laboratory experiences in blended control engineering education

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Abstract: Internet-based control engineering education combines traditional methods with modern technology, creating a dynamic and accessible learning environment. Both online and traditional approaches have unique advantages and challenges, with the choice depending on factors like student preferences, geographic location, and available resources. A blended approach, integrating online and in-person activities, leverages the strengths of both methods to offer a comprehensive and flexible learning experience. In this context, managing laboratory resources and activities scheduling may be challenging. For this reason, this paper introduces a novel approach to modelling these activities and resources through the mathematical framework of max-plus algebra. This algebra can improve the use of finite resources, meeting educational goals without concurrency issues. Moreover, the resolution of a model matching problem shows how lessons can be structured to meet time constraints and maximize flexibility, accessibility, and resource efficiency, thus preparing students effectively for the field.

Keywords: Control education, internet-based education, control engineering, max-plus algebra

1. INTRODUCTION

The integration of laboratory activities with internet-based resources is a main challenge of engineering education nowadays. Traditionally, laboratory activities have been confined to dedicated modules, whereas modern pedagogies advocate for the seamless integration of physical and digital learning environments (Rossiter, 2020). Web-accessible resources, including virtual spaces, offer continuous access to students, overcoming the limitations of traditional labs. Yet, they may lack the hands-on experience, crucial for developing practical skills. An alternative, intermediate solution involves providing take-home laboratory kits, offering students the opportunity for independent experimentation outside of scheduled class time (Rossiter et al., 2019; Sotelo et al., 2022). Thus, the challenge lies in balancing the advantages of internet-based resources with the necessity for hands-on experience in engineering education.

University courses have been increasingly adopting internet-based control education to innovate and meet the evolving needs of learners in the digital age(Anthony et al., 2022; Rossiter, 2022; Abramovitch, 2023). Internet-based education allows access to learning materials from anywhere with an internet connection and can be realized in a variety of ways. In this context, Blended Learning (BL) combines online and face-to-face education, extending the learning process through additional resources and multimedia elements, available with flexible time and place. This offers a dynamic and interactive environment, fostering student engagement and personalized learning without compromising the depth of knowledge or quality of instruction. Moreover, since employers value candidates with experience in real-world applications, laboratory work is fundamental in control engineering education.

Integrating real-world laboratories into online learning experiences within a BL course requires the careful design of activities along several dimensions, like laboratory kits, experiment modules, collaboration, and online tools. In this paper, the modelling approach to a real laboratory experience integrated into a BL course is presented. The course at issue deals with system identification and it was formerly considered as a case study for the implementation of real-time assessment in (Screpanti et al., 2024). The course is designed to give students real-life skills in system assembling and data collection, by applying theoretical concepts to effectively gather data, prepare them for the identification process and identify the system's parameters. Students also have to deal with diverse laboratory equipment.

Laboratories usually have limited resources and their accessibility depends on the opening hours of the university and on the supervisors' availability. Moreover, students enrolled in undergraduate programs have limited hours to at-

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tend the course. So, time is very important when designing lab activities within traditional courses. In this context, a simulator can provide the lecturer with a support system to organize resources and time for each lab activity. To model this kind of situation, the max-plus algebra seems to be a suitable tool, since it is used to represent man-made systems where finite resources are shared among multiple "users" and the purpose is to achieve common objectives in the absence of concurrency.

The paper is organized as follows. Section 2 introduces the methodological background of max-plus algebra in general terms. Section 3 provides a qualitative description of a real laboratory experience in a BL course on system identification and shows how this description can be formalized in the framework of the max-plus algebra. Section 4 concerns the problem of imposing a predefined schedule and tackles it as a model matching problem (MMP) within the max-plus algebra framework. Section 4 reports then the final results of the simulation and Section 5 presents the conclusions and future ideas.

2. METHODOLOGICAL BACKGROUND

Max-plus algebra has always been used to model and analyze Discrete Event Systems (DES) when considering synchronization without concurrency. This mathematical framework has the operations *max* ($\oplus$) and *plus* ($\otimes$) of traditional algebra, defined as addition and multiplication. The max-plus semi-ring is $\mathbb{R}_{\max} = \mathbb{R} \cup \{-\infty\}$ and for any $x, y \in \mathbb{R}_{\max}$, addition and multiplication are defined as:

$$x \oplus y = \max(x, y) \qquad x \otimes y = x + y$$

The neutral element for $\oplus$ is $\epsilon = -\infty$ and the neutral element for $\otimes$ is $e = 0 \in \mathbb{R}$.

Generally, for a DES, two operations can be modelled: concatenation and synchronization. The former is used when two processes or events are in sequential order, one before the other, and the related operator is the o-times ($\otimes$) for the sum of the times of each task. On the other hand, for the synchronization, the o-plus ($\oplus$) is used, to calculate the maximum time among different tasks, e.g., when tasks can progress independently, but the whole activity is not finished until both tasks are completed.

Concerning the modelling of DES with this algebra, the first authors who worked on this topic were Cohen et al. (1985), who introduced linear systems over max-plus algebra as early as the eighties and then worked on applying this approach in systems and control theory (Cohen et al., 1999). The history of max-plus algebra for discrete event systems can be found in Komenda et al. (2018).

Throughout the years, further work has been carried out on this topic, especially at the theoretical level and by means of structural geometric methods. Several contributions have been focused on the study of the structural properties of max-plus linear systems (Di Loreto et al., 2010; Hardouin et al., 2011; Cárdenas et al., 2015). More recently, the model matching problem has been investigated and solved for max-plus linear systems (Animobono et al., 2023) as well as for switching max-plus linear systems Animobono et al. (2022) and for periodic max-plus linear systems (Animobono et al., 2024a). A more general formulation of the model matching problem for switching max-plus linear systems was given in terms of synchronization and subsynchronization problems in (Animobono et al., 2024b), where structural solvability conditions were derived.

As to applications, max-plus algebra is often exploited to deal with production planning in industry and manufacturing (Chen et al., 2020; Majdzik, 2020; Oliveira et al., 2020). However, some recent works have shown that the use of this algebra is appropriate and successful also in other relevant fields. In Bartolucci et al. (2024), a max-plus linear system is employed to effectively model the repetitive patrolling of an area by a shoal of fish robots whereas the problem of forcing the fish robots to follow a predefined, iterative schedule has been formalized and solved as a model matching problem in Bartolucci (2024).

3. THE CASE STUDY

3.1 Qualitative description of the activities

European universities require students to collect a finite amount of European Credit Transfer System (ECTS) credits to obtain a bachelor's or a master's degree (European Commission and Directorate-General for Education, Youth, Sport and Culture, 2015). Each credit quantifies the work needed to acquire skills and is equivalent to around 25 hours of commitment, divided into classroom activities and individual study. The classroom hours to obtain 1 ECTS may vary depending on the course and university policies, although a rough estimate is 8 hours. The course on "System Identification and Modelling" of the Università Politecnica delle Marche awards the students 9 CFU, i.e. 72 hours of lessons, of which 50 of frontal lessons and 22 of exercise (around 4 hours) and laboratory. In the academic year 2023/2024, the course was in a blended modality (Screpanti et al., 2024). Students were trained using the Moodle LMS which hosted online resources, like video lessons, and e-tivities, like interactive graded assignments thanks to the MATLAB Grader add-on (The MathWorks, 2023). At the end of the educational path, students worked on a final project in the lab to set up and carry out a real-world black box identification for around 18 hours. Students were divided into groups and only four groups at a time could access the lab to work on three tasks:

(1) realizing the RC circuit: the lab equipment entails 3 breadboards and a soldering station; each group must describe and realize its own RC circuit on the breadboard and then solder it; at the end, the group must complete the assigned Matlab Grader activity on the Moodle platform using the actual parameters chosen for their RC circuit;
(2) building a Bode diagram: groups can exploit either the PicoScope oscilloscope, the Virtual Bench or two pairs Oscilloscope/Function generator with no logging capability; at the end, they must complete an e-tivity on the MATLAB Grader to draw the bode diagram of their RC circuit.
(3) generating white noise, giving it as input of the system and measuring the output: this task can be carried out using the PicoScope or two sets of tools (an ESP32 microcontroller with DAC/ADC); then

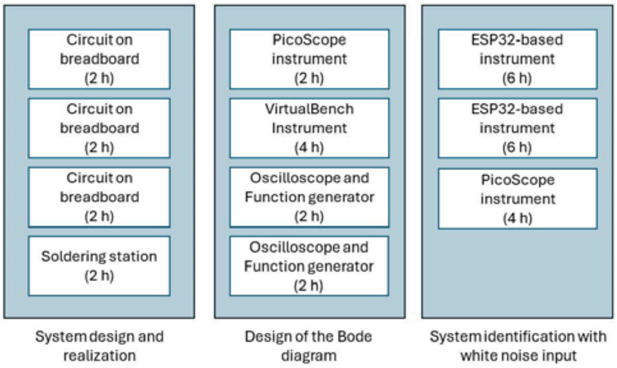

Fig. 1. Resources available in the lab for each task, with esteemed time to complete the assignment.

students must upload the report of their identification activity into the Moodle LMS.

Fig. 1 shows the equipment for the three tasks of the lab experience; the time esteemed to complete the task using one of the available tools is reported in brackets. In this context, the lecturer aims at respecting time, considering the allocation of both time and resources.

3.2 Formal description of the activities

Before formalizing the problem, the concept of a linear max-plus system is necessary. The latter is a dynamical system that has the following first law that guides its evolution:

$$\Sigma \equiv \begin{cases} x(k+1) = A \otimes x(k) \oplus B \otimes u(k+1) \\ y(k) = Cx(k) \\ x(0) = \epsilon \end{cases} \tag{1}$$

with $A \in \mathbb{R}_{\max}^{n\times n}$, $B \in \mathbb{R}_{\max}^{n\times m}$ and $C \in \mathbb{R}_{max}^{p\times n}$.

In this context, $k \in \mathbb{N}$ is the event instance index, $x(k) : \mathbb{N} \to \mathbb{R}^n_{max}$ is the internal events' dater or state vector, $u(k) : \mathbb{N} \to \mathbb{R}^m_{max}$ is the input events' dater or control input, and $y(k) : \mathbb{N} \to \mathbb{R}^p_{max}$ is the output dater, for integers $k > 0$. A dater is an n-dimensional function $d(.) : \mathbb{N} \to \mathbb{R}^n_{max}$, and its value at $k \in \mathbb{N}$ is a vector $d(k) = (d_1(k), ..., d_n(k))^\top$. The i-th component $d_i(k)$ contains the instant when an event of the i-th type happens for the k-th time. To have physical meaning, daters have to be non-decreasing: $d(k+1) \geq d(k)\ \forall k \in \mathbb{N}$. Moreover, as regards the solution of (1), it is uniquely achieved starting from u and the initial condition $x(0) \in \mathbb{R}^n_{max}$.

Following the previous section, the list of activities and resources divided among the 4 groups, called G1, G2, G3 and G4, was defined as in Fig. 2. Starting from the scheme, four phases can be depicted where some resources have to be used by more groups:

- In phase 1, which is the RC circuit realization, G1 will use one breadboard, G2 will use another breadboard, whereas the third breadboard has to be used by G3 and G4, one after the other.
- In phase 2, all groups will have to use the same soldering station, one after the other.
- In phase 3, G1 will use the picoscope, G2 the virtual bench, G3 and G4 one oscilloscope each.
- In phase 4, G2 and G3 will use, one after the other, the picoscope, G1 and G4 one ESP-32-based instrument each.

To model this situation, another concept related to the priority of some groups has to be introduced. Since there are some common resources, first it has to be decided which will be the first group in every common resource. In this case, the decision is that the lower the number of the group, the higher the priority, which means that G1 has priority over G2, that has priority over G3, that is prioritised over G4. Another important assumption is that, when all groups have finished, it is supposed that other 4 groups are ready to do the same activities.

This situation can be modelled through the max-plus algebra theoretical framework, as it is presented in Table 1, which contains the definition of all variables. In this case, the index k represents the round number: for example, the first round will be represented by the first 4 groups that carry out the activities, the second round will be composed of new 4 groups of people, etc. until all students have done the activities.

In the system, there will be 16 input events, triggered from outside of the system and defined as u_{ij}, where i represents the number of the group and j is the activity that has to be performed, also defined as "Phase" in Fig. 2 (e.g. the first activity is the RC circuit, then the welding etc.). This input represents the "time when group i can start the j-th activity" and this information can be said by the teacher or by Moodle when the previous activity has properly been uploaded on the platform. Then, we consider as internal events x_{ij}, that represent the "completion time by group i of the j-th activity": events of this type can here only occur after the group has finished the previous activity and, if the instrument is in common with a higher priority group, when the other groups have finished and when the group receive the start command. Finally, the "global end of activity" is an output event, triggered internally by the system and visible from outside: it represents the time when all four groups have finished their work.

Following this information, the model can be represented in the following way:

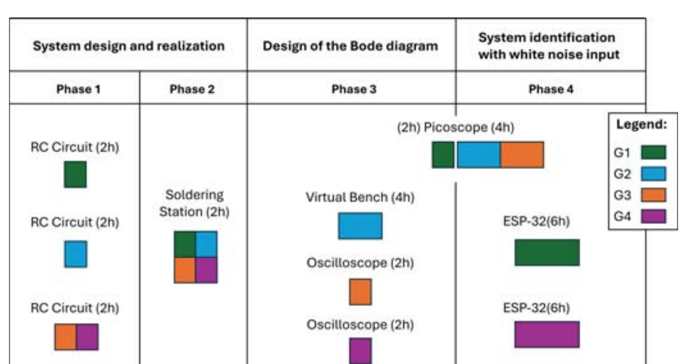

Fig. 2. Scheme of the activities divided into the four groups

Table 1. Involved Variables

Variable name	Meaning
$u_ij(k)$	Time when group i can start the j-th activity
$x_{ij}(k)$	Completion time by group i of the j-th activity
$y(k)$	Global end of activities

$$
\begin{aligned}
x_{11}(k+1) &= 2\otimes(u_{11}(k+1)\oplus x_{14}(k)\oplus x_{24}(k)\oplus x_{34}(k)\\
&\quad \oplus x_{44}(k))\\
x_{21}(k+1) &= 2\otimes(u_{21}(k+1)\oplus x_{14}(k)\oplus x_{24}(k)\oplus x_{34}(k)\\
&\quad \oplus x_{44}(k))\\
x_{31}(k+1) &= 2\otimes(u_{31}(k+1)\oplus x_{14}(k)\oplus x_{24}(k)\oplus x_{34}(k)\\
&\quad \oplus x_{44}(k))\\
x_{41}(k+1) &= 2\otimes(u_{41}(k+1)\oplus x_{14}(k)\oplus x_{24}(k)\oplus x_{34}(k)\\
&\quad \oplus x_{44}(k)\oplus x_{31}(k+1))\\
x_{12}(k+1) &= 2\otimes(u_{12}(k+1)\oplus x_{11}(k+1))\\
x_{22}(k+1) &= 2\otimes(u_{22}(k+1)\oplus x_{21}(k+1)\oplus x_{12}(k+1))\\
x_{32}(k+1) &= 2\otimes(u_{32}(k+1)\oplus x_{31}(k+1)\oplus x_{22}(k+1))\\
x_{42}(k+1) &= 2\otimes(u_{42}(k+1)\oplus x_{41}(k+1)\oplus x_{32}(k+1))\\
x_{13}(k+1) &= 2\otimes(u_{13}(k+1)\oplus x_{12}(k))\\
x_{23}(k+1) &= 4\otimes(u_{23}(k+1)\oplus x_{22}(k))\\
x_{33}(k+1) &= 2\otimes(u_{33}(k+1)\oplus x_{32}(k))\\
x_{43}(k+1) &= 2\otimes(u_{43}(k+1)\oplus x_{42}(k))\\
x_{14}(k+1) &= 6\otimes(u_{14}(k+1)\oplus x_{13}(k+1))\\
x_{24}(k+1) &= 4\otimes(u_{24}(k+1)\oplus x_{23}(k+1)\oplus x_{13}(k+1))\\
x_{34}(k+1) &= 4\otimes(u_{34}(k+1)\oplus x_{33}(k+1)\oplus x_{24}(k+1))\\
x_{44}(k+1) &= 6\otimes(u_{44}(k+1)\oplus x_{43}(k+1))\\
y(k) &= x_{14}(k)\oplus x_{24}(k)\oplus x_{34}(k)\oplus x_{44}(k)
\end{aligned}
$$

Through a matrix form, the equations are written as:

$$
\begin{cases}
x(k+1) = A_0 x(k+1)\oplus A_1 x(k)\oplus B' u(k+1)\\
\quad y(k) = Cx(k)
\end{cases}
$$

Matrices A_0, A_1 and B' are the max-plus equivalent of sparse matrices, where most entries are ϵ. For these matrices, the elements different from ϵ are as follows:

$A_0(4,3) = A_0(5,1) = A_0(6,2) = A_0(6,5) = A_0(7,3) = A_0(7,6) = A_0(8,4) = A_0(8,7) = A_0(9,5) = A_0(11,7) = A_0(12,8) = 2;$

$A_0(10,6) = A_0(14,9) = A_0(14,10) = A_0(15,11) = A_0(15,14) = 4;$

$A_0(13,9) = A_0(16,12) = 6;$

$A_1(1,13) = A_1(2,14) = A_1(3,15) = A_1(4,16) = 2;$

$B'(1,1) = B'(2,2) = B'(3,3) = B'(4,4) = B'(5,5) = B'(6,6) = B'(7,7) = B'(8,8) = B'(9,9) = B'(11,11) = B'(12,12) = 2;$

$B'(10,10) = B'(14,14) = B'(15,15) = 4;$

$B'(13,13) = B'(16,16) = 6;$

$C(1,13) = C(1,14) = C(1,15) = C(1,16) = e = 0.$

The presence of $A_0 x(k+1)$ makes the equation describing an implicit relation that has to be explicit as in (1), that is:

$$
\begin{cases}
x(k+1) = Ax(k)\oplus Bu(k+1)\\
\quad y(k) = Cx(k)\\
\quad x(0) = \epsilon
\end{cases}
$$

Now it has to be recalled that the least solution of $x = Ax \oplus b$ can be here obtained with the Kleene star operator $A^* = \bigoplus_{n\in\mathbb{N}} A^n$, as $x = A^*b$ and since A_0 is lower triangular, $A_0^i = \epsilon \;\forall i \geq \dim A_0$. So, $A_0^* = \bigoplus_{n\in\mathbb{N}} A_0^n$ with $A_0^i = \epsilon \;\forall i \geq 16$.

Once obtained A_0^*, the system can be represented as in (1) in the following way, with $A = A_P = A_0^* A_1$, $B = B_P = A_0^* B'$ and $C = C_P$:

$$
\begin{cases}
x(k+1) = A_0^* A_1 x(k) \oplus A_0^* B' u(k+1)\\
\quad y(k) = Cx(k)\\
\quad x(0) = \epsilon
\end{cases}
\tag{2}
$$

Once computed, A_P, B_P and C_P become as follows:

$$
A_P = \begin{pmatrix}
\epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & 2 & \epsilon & \epsilon & \epsilon\\
\epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & 2 & \epsilon & \epsilon\\
\epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & 2 & \epsilon\\
\epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & 4 & 2\\
\epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & 4 & \epsilon & \epsilon & \epsilon\\
\epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & 6 & 4 & \epsilon & \epsilon\\
\epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & 8 & 6 & 4 & \epsilon\\
\epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & 10 & 8 & 6 & 4\\
\epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & 6 & \epsilon & \epsilon & \epsilon\\
\epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & 10 & 8 & \epsilon & \epsilon\\
\epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & 10 & 8 & 6 & \epsilon\\
\epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & 12 & 10 & 8 & 6\\
\epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & 12 & \epsilon & \epsilon & \epsilon\\
\epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & 14 & 12 & \epsilon & \epsilon\\
\epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & 18 & 16 & 10 & \epsilon\\
\epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & 18 & 16 & 14 & 12
\end{pmatrix}
$$

$$
B_P = \begin{pmatrix}
2 & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon\\
\epsilon & 2 & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon\\
\epsilon & \epsilon & 2 & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon\\
\epsilon & \epsilon & 4 & 2 & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon\\
4 & \epsilon & \epsilon & \epsilon & 2 & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon\\
6 & 4 & \epsilon & \epsilon & 4 & 2 & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon\\
8 & 6 & 4 & \epsilon & 6 & 4 & 2 & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon\\
10 & 8 & 6 & 4 & 8 & 6 & 4 & 2 & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon\\
6 & \epsilon & \epsilon & \epsilon & 4 & \epsilon & \epsilon & \epsilon & 2 & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon\\
10 & 8 & \epsilon & \epsilon & 8 & 6 & \epsilon & \epsilon & \epsilon & 4 & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon\\
10 & 8 & 6 & \epsilon & 8 & 6 & 4 & \epsilon & \epsilon & \epsilon & 2 & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon\\
12 & 10 & 8 & 6 & 10 & 8 & 6 & 4 & \epsilon & \epsilon & \epsilon & 2 & \epsilon & \epsilon & \epsilon & \epsilon\\
12 & \epsilon & \epsilon & \epsilon & 10 & \epsilon & \epsilon & \epsilon & 8 & \epsilon & \epsilon & \epsilon & 6 & \epsilon & \epsilon & \epsilon\\
14 & 12 & \epsilon & \epsilon & 12 & 10 & \epsilon & \epsilon & 6 & 8 & \epsilon & \epsilon & \epsilon & 4 & \epsilon & \epsilon\\
18 & 16 & 10 & \epsilon & 16 & 14 & 8 & \epsilon & 10 & 12 & 6 & \epsilon & \epsilon & 8 & 4 & \epsilon\\
18 & 16 & 14 & 12 & 16 & 14 & 12 & 10 & \epsilon & \epsilon & \epsilon & 8 & \epsilon & \epsilon & \epsilon & 6
\end{pmatrix}
$$

$$
C_P = \begin{pmatrix} \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & \epsilon & e & e & e & e \end{pmatrix}
$$

Assuming that G1, G2, G3 and G4 are commanded to start their work respectively at instants 0, 1, 2 and 3, to then increment 1 hour at a time, from the simulation it emerges that the first round ends after 21 hours:

$$
\begin{aligned}
&u(1) = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \end{pmatrix}^T\\
&\rightarrow x(1) = Ax(0) \oplus Bu(1) =\\
&= \begin{pmatrix} 2 & 3 & 4 & 6 & 6 & 8 & 10 & 12 & 10 & 13 & 12 & 14 & 18 & 17 & 21 & 21 \end{pmatrix}^T
\end{aligned}
$$

$y(1) = Cx(1) = 21$

4. THE MODEL MATCHING PROBLEM

At this point, to check if the plant, i.e. the system modelled, can effectively respect a predefined schedule, a so-called MMP has to be presented and solved, following the steps detailed in Animobono et al. (2023) and here briefly reported.

Problem 1. (MMP, Animobono et al. (2023)). Given a linear max-plus system

$$\Sigma_P \equiv \begin{cases} x_P(k+1) = A_P x_P(k) \oplus B_P u_P(k+1) \\ y_P(k) = C_P x_P(k) \\ x_P(0) = \epsilon \end{cases} \tag{3}$$

called the plant, and a linear max-plus system

$$\Sigma_M \equiv \begin{cases} x_M(k+1) = A_M x_M(k) \oplus B_M u_M(k+1) \\ y_M(k) = C_M x_M(k) \\ x_M(0) = \epsilon \end{cases} \tag{4}$$

called the model, with $x_P : \mathbb{N} \to \mathbb{R}^{n_P}_{max}$, $x_M : \mathbb{N} \to \mathbb{R}^{n_M}_{max}$, $u_P : \mathbb{N} \to \mathbb{R}^{m_P}_{max}$, $u_M : \mathbb{N} \to \mathbb{R}^{m_M}_{max}$ and $y_P, y_M : \mathbb{N} \to \mathbb{R}^{p}_{max}$, the MMP consists in finding, for all possible non-decreasing input sequences of the model $\{u_M(k)\}_{k\in\mathbb{N}}$, an appropriate non-decreasing control input sequence for the plant $\{u_P(k)\}_{k\in\mathbb{N}}$, such that $y_P(k) = y_M(k)$ for all $k \in \mathbb{N}$.

In summary, the MMP involves determining a control law that ensures the plant behaves according to the model.

To solve this problem, the so-called joint internal event dater $x_E(.) = \begin{pmatrix} x_P(.) \\ x_M(.) \end{pmatrix} : \mathbb{N} \to \mathbb{R}^{(n_P+n_M)}_{max}$, is needed, as well as its joint dynamics:

$$x_E(k) = A_E x_E(k-1) \oplus B_1 u_P(k) \oplus B_2 u_M(k)$$

with $A_E = \begin{pmatrix} A_P & \epsilon \\ \epsilon & A_M \end{pmatrix}$, $B_1 = \begin{pmatrix} B_P \\ \epsilon \end{pmatrix}$, $B_2 = \begin{pmatrix} \epsilon \\ B_M \end{pmatrix}$, $x_E(0) = \epsilon$.

At this point, Problem 1 can be restated as the task of determining, for any $\{u_M(k)\}_{k\in\mathbb{N}}$, an $\{u_P(k)\}_{k\in\mathbb{N}}$ that keeps $\{x_E(k)\}_{k\in\mathbb{N}}$ in the *output equalizer* sub semi-module

$$\mathcal{K} = \left\{ x_E = \begin{pmatrix} x_P \\ x_M \end{pmatrix} \in \mathbb{R}^{(n_p+n_M)}_{max}, \text{ s. t. } C_P x_P = C_M x_M \right\}$$

with $\mathcal{K} \subseteq \mathbb{R}^{(n_p+n_M)}_{max}$

Theorem 1. (Animobono et al. (2023)). Given a non-anticipative plant Σ_P (3) and a non-anticipative model Σ_M (4), their MMP is solvable if and only if for all $x \in \operatorname{Im} B_2 = \operatorname{Im} \begin{pmatrix} \epsilon \\ B_M \end{pmatrix}$ there exists $z \in \operatorname{Im} B_1 = \operatorname{Im} \begin{pmatrix} B_P \\ \epsilon \end{pmatrix}$ such that $x \oplus z$ belongs to $\mathcal{V}^*(\mathcal{K})$, where $\mathcal{V}^*(\mathcal{K})$ is the maximum (A_E, B_1)-invariant semimodule contained in the output equalizer semimodule $\mathcal{K}$ defined by (4).

The final aim of the work is to verify if the current plant can synchronize on a predefined completion time. To do this, an MMP is solved, through the steps presented above. The model imposes that each round ends exactly 18 hours after the first start command or 18 hours after the previous round. This requirement aims to match the plant's output with the output of this max-plus stationary linear model:

$$\begin{cases} x_M(k+1) = 18\ x_M(k) \oplus 18\ u_M(k+1) \\ y_M(k) = x_M(k) \\ x_M(0) = \epsilon \end{cases} \tag{5}$$

with $x_M : \mathbb{N} \to \mathbb{R}_{max}$ as state vector, $u_M : \mathbb{N} \to \mathbb{R}_{max}$ as control input and $y_M : \mathbb{N} \to \mathbb{R}_{max}$ as output.

Once A_E, B_1 and B_2 were obtained, matrices $\mathcal{V}^*$, $\mathcal{K}$ and U were computed via ScicosLab, resulting in the MMP being solvable, with the following input sequence, that makes the plant's output equal to that of the model:

$$u_M(1) = (0\ 2\ 4\ 6\ 2\ 4\ 6\ 8\ 8\ 6\ 12\ 10\ 12\ 10\ 14\ 12)^T$$

This means that G1 is commanded to start its first activity at time instant 0 and then G2 will start 2 hours later, G3 4 hours later (with respect to G1), and G4 after 6 hours. Then, G1 will start the second activity after 2 hours, G2 after 4 hours, and so on.

5. CONCLUDING REMARKS AND FUTURE WORK

Integrating internet-based and traditional control engineering education presents a promising approach to modernizing and enhancing the learning experience. By combining the flexibility and accessibility of online education with the hands-on, interactive benefits of traditional methods, a blended approach effectively addresses the diverse needs of students, particularly in laboratory experiences, providing a comprehensive and flexible educational environment. However, managing time and resources remains a significant challenge. Employing modelling techniques such as max-plus algebra can improve resource allocation, ensuring efficient use of finite resources and adherence to educational standards. In fact, in this paper, this framework is applied in activity scheduling and laboratory resource management through a case study, leading to the modelling of classroom activities through a linear max-plus system. The resolution of a Model Matching Problem shows how such tasks can be structured to meet time constraints. This approach fosters a more adaptable, efficient, and high-quality educational environment, meeting the evolving needs of both students and lecturers.

Future work could focus on investigating the scalability of these systems and exploring and extending their applications to other educational contexts, to take full advantage of the benefits of this approach. For example, software could be devised to automate the methodology and generalize it to make it applicable to other courses. This would involve adding code and an interface that would facilitate structuring the activities by allowing users to input the type, number of tasks, and expected timing for each. In this way, this type of work would become easier and more accessible even for educators who are not familiar with max-plus algebra. Moreover, once the entire procedure has been implemented, it would be valuable to include a survey at the end of the course activities to assess the students' opinions and satisfaction with the management of laboratory resources and their time. This would allow lecturers to understand whether this approach is also perceived as effective by the students.

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REFERENCES

Abramovitch, D.Y. (2023). Thoughts on furthering the control education of practicing engineers [Focus on education]. *IEEE Control Systems*, 43(1), 64–88. doi: 10.1109/MCS.2022.3216745.

Animobono, D., Scaradozzi, D., Zattoni, E., Perdon, A.M., and Conte, G. (2022). The model matching problem for switching Max-Plus systems: A geometric approach. *IFAC-PapersOnLine*, 55(40), 7–12. doi: 10.1016/j.ifacol.2023.01.040.

Animobono, D., Scaradozzi, D., Zattoni, E., Perdon, A.M., and Conte, G. (2023). The model matching problem for Max-Plus linear systems: A geometric approach. *IEEE Transactions on Automatic Control*, 68(6), 3581–3587. doi:10.1109/TAC.2022.3191362.

Animobono, D., Scaradozzi, D., Zattoni, E., Perdon, A.M., and Conte, G. (2024a). The model matching problem for periodic max-plus systems. In *63rd IEEE Conference on Decision and Control, Milano, Italy, December 16–19, 2024*.

Animobono, D., Zattoni, E., Scaradozzi, D., Perdon, A.M., and Conte, G. (2024b). Synchronization and sub-synchronization problems for switching Max-Plus systems: Structural solvability conditions. *IEEE Transactions on Automatic Control*, 69(8), 5613–5619. doi: 10.1109/TAC.2024.3368298.

Anthony, B., Kamaludin, A., Romli, A., Raffei, A.F.M., Phon, D.N.A.L.E., Abdullah, A., and Ming, G.L. (2022). Blended learning adoption and implementation in higher education: A theoretical and systematic review. *Technology, Knowledge and Learning*, 27(2), 531–578. doi:10.1007/s10758-020-09477-z.

Bartolucci, V. (2024). *Modelling a Shoal of Marine Biomimetic Vehicles: A Max-Plus Algebra Approach.* Ph.D. thesis, Università Politecnica delle Marche, Ancona, Italy.

Bartolucci, V., Scaradozzi, D., Zattoni, E., Loiseau, J.J., Martinez, C., and Conte, G. (2024). A max-plus algebra-based approach for modelling shoals of fish robots during underwater exploration. In *34th International Ocean and Polar Engineering Conference, Rhodes, Greece, June 16–21, 2024*, 1966–1973.

Cárdenas, C., Loiseau, J.J., and Martinez, C. (2015). Controlled invariance and dynamic feedback for systems over semirings. In *SIAM Conference on Control and Its Applications, Paris, France, July 8–10, 2015*, 1–8. doi: 10.1137/1.9781611974072.1.

Chen, W., Liu, H., and Qi, E. (2020). Discrete event-driven model predictive control for real-time work-in-process optimization in serial production systems. *Journal of Manufacturing Systems*, 55, 132–142. doi: 10.1016/j.jmsy.2020.03.002.

Cohen, G., Dubois, D., Quadrat, J.P., and Viot, M. (1985). A linear-system-theoretic view of discrete-event processes and its use for performance evaluation in manufacturing. *IEEE Transactions on Automatic Control*, 30(3), 210–220. doi:10.1109/TAC.1985.1103925.

Cohen, G., Gaubert, S., and Quadrat, J.P. (1999). Max-plus algebra and system theory: Where we are and where to go now. *Annual Reviews in Control*, 23, 207–219. doi:10.1016/S1367-5788(99)00023-1.

Di Loreto, M., Gaubert, S., Katz, R.D., and Loiseau, J.J. (2010). Duality between invariant spaces for max-plus linear discrete event systems. *SIAM Journal on Control and Optimization*, 48(8), 5606–5628. doi: 10.1137/090747191.

European Commission and Directorate-General for Education, Youth, Sport and Culture (2015). *ECTS Users' Guide 2015*. Publications Office of the European Union, Luxembourg. doi:10.2766/87192.

Hardouin, L., Lhommeau, M., and Shang, Y. (2011). Towards geometric control of max-plus linear systems with applications to manufacturing systems. In *50th IEEE Conference on Decision and Control and European Control Conference, Orlando, FL, December 12–15, 2011*, 1149–1154. doi:10.1109/CDC.2011.6160489.

Komenda, J., Lahaye, S., Boimond, J.L., and van den Boom, T. (2018). Max-plus algebra in the history of discrete event systems. *Annual Reviews in Control*, 45, 240–249. doi:10.1016/j.arcontrol.2018.04.004.

Majdzik, P. (2020). Feasible schedule under faults in the assembly system. In *16th IEEE International Conference on Control, Automation, Robotics and Vision, Shenzhen, China, December 13–15, 2020*, 1049–1054. doi:10.1109/ICARCV50220.2020.9305377.

Oliveira, G.F., Ferreira Candido, R.M., Gonçalves, V.M., Maia, C.A., Cottenceau, B., and Hardouin, L. (2020). Discrete event system control in max-plus algebra: Application to manufacturing systems. *IFAC-PapersOnLine*, 53(4), 143–150. doi: 10.1016/j.ifacol.2021.04.014.

Rossiter, J.A. (2020). Blended learning in control engineering teaching; An example of good practice. *IFAC-PapersOnLine*, 53(2), 17252–17257. doi: 10.1016/j.ifacol.2020.12.1797.

Rossiter, J.A., Pope, S.A., Jones, B.L., and Hedengren, J.D. (2019). Evaluation and demonstration of take home laboratory kit. *IFAC-PapersOnLine*, 52(9), 56–61. doi: 10.1016/j.ifacol.2019.08.124.

Rossiter, J.A. (2022). Future trends for a first course in control engineering. *Frontiers in Control Engineering*, 3, art. n. 956665, 1–5. doi:10.3389/fcteg.2022.956665.

Screpanti, L., Morano, M., and Scaradozzi, D. (2024). Designing courses in modeling and identification of dynamic systems for real-time assessment. In *32nd Mediterranean Conference on Control and Automation, Chania - Crete, Greece, 11–14 June 2024*, 304–309. doi: 10.1109/MED61351.2024.10566180.

Sotelo, D., Sotelo, C., Ramirez-Mendoza, R.A., López-Guajardo, E.A., Navarro-Duran, D., Niño-Juárez, E., and Vargas-Martinez, A. (2022). Lab-Tec@Home: A cost-effective kit for online control engineering education. *Electronics*, 11(6), 907. doi: 10.3390/electronics11060907.

The MathWorks (2023). MATLAB Grader. https://grader.mathworks.com/. Accessed: 2024-08-02.