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A probabilistic study on impedances and kinematic response factors of square pile groups in homogeneous soils

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Abstract

The complexity of the Soil-Structure Interaction (SSI) problem is mainly due to difficulties related to the modelling of the soil-foundation behaviour, which is not only frequency-dependent, but is also largely influenced by uncertainties related to soil properties and stratigraphic conditions. According to a sub-structuring scheme for the analysis of SSI problems, a deterministic approach is usually adopted for the analysis of the soil-foundation system assuming properties and geotechnical models based on the expert judgment, despite the fact that uncertainties related to the intrinsic variability of soil parameters are widely recognised and confirmed by experimental campaigns and laboratory tests.

This paper presents a probabilistic study on the dynamic behaviour of square pile group foundations in homogeneous soils, focusing on the effects of the uncertainties related to (i) the frequency-dependent impedance functions and (ii) the kinematic response factors. Above quantities are required to define compliant restraints for the modelling of the soil-foundation system in performing inertial soil-structure interaction analysis, and for the definition of the foundation input motion starting from the free-field motion. Square pile groups are considered, and uncertainties are described through probabilistic distributions of parameters governing the soil-foundation dynamic response. The probabilistic analyses are performed through a numerical model developed by the Authors and some results are presented to show and discuss the variability of the results. In addition, a sensitivity analysis is performed to assess the influence of each variable uncertainty on the system response. Overall, response quantities are found to be very sensitive to shear wave velocity, although soil density and pile elastic modulus may often play a significant role.

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1. Introduction

It is nowadays well known from the literature that Soil-Structure Interaction (SSI) effects may play an important role on the dynamics of superstructures (Mylonakis and Gazetas 2000, Carbonari et al. 2017). One of the most common approaches used to properly account for SSI effects in seismic analyses is the substructure method (Wolf 1985); the approach allows to first evaluate the behaviour of the soil-foundation system in terms of frequency-dependent impedance matrix and kinematic response factors, and then the superstructure response subjected to the Foundation Input Motion (FIM). The latter represents the displacements actually experienced by the foundation, which are obtained on the basis of the kinematic response factors, representing the foundation displacement due to the propagation of unit amplitude harmonic waves in the soil, while the impedances define the force-displacement relationships that characterise restraints of the compliant base structures.

Classically, the SSI analysis follows a deterministic path, in which the geotechnical and the foundation parameters are chosen as average values representative of the embedded system. However, it is well known that both building materials and, above all, geotechnical parameters are affected by a certain variability. First attempts to consider the uncertainties of the input variables of the SSI problem and to capture the scattering of the response have been made by Lutes et al. (2000), Cottureau et al. (2007), Moghaddasi et al. (2011), but detailed studies on the probabilistic dynamic behaviour of pile foundations have not yet been made in a systematic way.

This paper presents a probabilistic approach for analysing the dynamic response of pile foundations in homogeneous soils. Uncertainties affecting both soil and piles are considered through probabilistic distributions of the main parameters governing the soil-foundation dynamic behaviour. Samples of the selected random variables are obtained through the Quasi-Random Sampling (QRS) technique by Sobol' (1992). Different pile groups configurations are assumed to derive the frequency-dependent impedance functions and the kinematic response factors. Probabilistic analyses are performed using the numerical model of Dezi et al. (2009) and some results of the applications are commented in terms of the variability of the output quantities, and sensitivity indexes of the random variables.

2. Modelling of uncertainties in the soil-foundation system

The use of statistical models and probabilistic laws to reproduce the aleatoric uncertainties affecting the soil-foundation system derives from the inherent variability of the mechanical parameters of both piles and soil. Concerning piles, the variability of the parameters can be closely associated with the uncertainties on construction material properties, mainly regarding the concrete. As for the soil, the literature offers a wide scenario of studies dealing with the probabilistic trends of geotechnical parameters such as the soil density, the cohesion, and the friction angle, based on results of field tests. In this study, the soil density ρ_s , the shear wave velocity V_s and the concrete elastic modulus E_p are assumed as independent random variables. The probabilistic models assumed for above variables are deduced from the available technical literature. In particular:

- a normal distribution is considered for the soil density (Jones et al. 2002);
- a lognormal distribution is considered for the shear wave velocity (Griffiths et al. 2016);
- a lognormal distribution is considered for the cylindrical compressive concrete strength f_c (CNR 2014), from which the elastic Young's modulus of the material is derived (EN1992-1-1):

$$E_p = 22000 \left(\frac{f_c + 8}{10} \right)^{0.3} \quad (1)$$

The remaining mechanical parameters, namely soil and concrete Poisson's ratios (ν_s, ν_p), soil and concrete damping ratios (ξ_s, ξ_p) and concrete density ρ_p , are assumed to be deterministic since their variability on the overall dynamic response of the foundation is limited under the assumption of linear soil behaviour (Cottureau et al. 2007).

The influence of the variability of the selected random parameters is studied evaluating the dynamic response of the soil-foundation system in the frequency domain, where the dynamic equilibrium for a group of vertical piles connected at the head by a rigid cap can be expressed by the following complex valued system of linear algebraic equations:

$$\left[\mathbf{A}^T \left(\mathbf{K}_p - \omega^2 \mathbf{M}_p + \mathbf{K}_s(\omega) \right) \mathbf{A} \right] \mathbf{d}(\omega) = \mathbf{A}^T \mathbf{f}(\omega) \quad (2)$$

where \mathbf{K}_p and \mathbf{M}_p are the global stiffness and mass matrices of piles, respectively, $\mathbf{K}_s(\omega)$ is the impedance matrix of the soil, $\mathbf{d}(\omega)$ is the vector collecting the six displacement components of the pile cap and the displacement components of the remaining non-constrained pile nodes, $\mathbf{f}(\omega)$ is the vector of external loads due to the free-field motion and the pile-soil-pile interaction forces, and \mathbf{A} is a geometric matrix imposing a constraint at the pile heads to simulate the presence of the cap. Both the impedance matrix of the soil and the external loads depend on the soil-pile and pile-soil-pile interactions that are described through elastodynamic Green's functions available in the literature (Dezi et al. 2009); the latter strongly depends on the selected random variables.

The impedance matrix of the soil-foundation system can be obtained through a condensation of the dynamic stiffness matrix appearing within square brackets in Eq. (2) on the pile cap degrees of freedom; for double-symmetric pile configurations, it can be expressed in a non-dimensional form according to the following expression:

$$\Pi(\rho_s, V_s, E_p; a_0) = \begin{bmatrix} \Pi_1 & 0 & 0 & 0 & \Pi_3 & 0 \\ & \Pi_1 & 0 & -\Pi_3 & 0 & 0 \\ & & \Pi_4 & 0 & 0 & 0 \\ & & & \Pi_2 & 0 & 0 \\ sym & & & & \Pi_2 & 0 \\ & & & & & \Pi_5 \end{bmatrix} \quad (3)$$

where $a_0 = \frac{\omega d}{\mu V_s}$ is the non-dimensional frequency and

$$\Pi_1(\rho_s, V_s, E_p; a_0) = \frac{\mathfrak{S}_i}{\rho_s \mu V_s^2 d} \quad \Pi_2(\rho_s, V_s, E_p; a_0) = \frac{\mathfrak{S}_{ri}}{\rho_s \mu V_s^2 d^3} \quad \text{for } i = x, y \quad (4a,b)$$

$$\Pi_3(\rho_s, V_s, E_p; a_0) = \frac{\mathfrak{S}_{i-rj}}{\rho_s \mu V_s^2 d^2} \quad \text{for } i = x, y \text{ and } j = y, x \quad (4c)$$

$$\Pi_4(\rho_s, V_s, E_p; a_0) = \frac{\mathfrak{S}_z}{\rho_s \mu V_s^2 d} \quad \Pi_5(\rho_s, V_s, E_p; a_0) = \frac{\mathfrak{S}_{rz}}{\rho_s \mu V_s^2 d^3} \quad (4d,e)$$

in which $\mathfrak{S}_x, \mathfrak{S}_y, \mathfrak{S}_z$ are the translational frequency dependent impedances along x, y and z , respectively, whereas $\mathfrak{S}_{rx}, \mathfrak{S}_{ry}, \mathfrak{S}_{rz}$ are the rotational impedance components and $\mathfrak{S}_{x-ry}, \mathfrak{S}_{y-rx}$ are the coupled roto-translational terms. Finally, μV_s is the mean value of the shear wave velocity probabilistic distribution.

The kinematic response factors I_U and I_Φ are obtained by solving system (2) for steady shear waves propagating vertically in the soil, obtaining the pile cap displacements. Kinematic factors, describe the foundation kinematic response and are expressed by the following equations:

$$I_U(\rho_s, V_s, E_p; a_0) = \frac{U_i}{U_{ff,i}} \quad I_\Phi(\rho_s, V_s, E_p; a_0) = \frac{\Phi_i d}{U_{ff,i}} \quad \text{for } i = x, y \quad (5a,b)$$

where $U_{ff,x}$ and $U_{ff,y}$ are the free field displacements at the soil outcrop, and U_x, U_y, Φ_y and Φ_x are the translational and rotational non-null displacement components of the foundation cap, respectively. Further details can be found in Minnucci et al. (2022).

3. Case studies and generation of samples

Square 2x2 and 3x3 groups of floating vertical piles embedded in a homogeneous soil deposit are considered with geometries chosen in a deterministic range. Two pile slenderness ratios L/d and three pile spacing-diameter ratios s/d are considered for a total of 12 geometric models (Fig. 1).

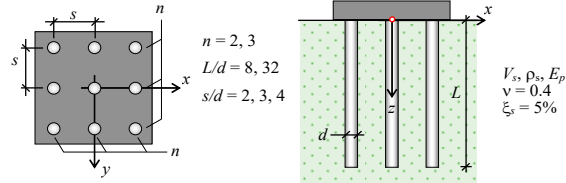


Fig. 1. Scheme of investigated soil-foundation systems.

As discussed in the previous section, soil density, shear wave velocity and concrete modulus of elasticity are considered to be aleatoric while the other mechanical parameters, necessary to define the model, are considered to be deterministic. Tab. 1 reports parameters (mean values and standard deviations) of the probability distributions adopted for the aleatoric parameters, while Tab. 2 lists the deterministic values adopted for the other parameters.

Table 1: Statistical distributions for ρ_s , V_s , E_c .

Uncertainties	Probabilistic Distribution	μ	σ
Soil density ρ_s [t/m^3]	Normal	1.75	$\sigma = 0.175$
Shear wave velocity V_s [m/s]	Lognormal	100, 200, 300	$\sigma_m = 0.10$
Concrete cylindrical strength [MPa]	Lognormal	20.12	$\sigma_m = 0.20$

Table 2: Deterministic parameters.

Mechanical parameters	value
Soil Poisson's ratio ν_s [-]	0.40
Soil damping ξ_s [-]	0.05
Pile material Poisson's ratio (concrete) ν_p [-]	0.20
Pile material density (concrete) ρ_p [t/m^3]	2.50

The Quasi-Random Sampling (QRS) technique, based on Sobol' Low Discrepancy Sequences (LDSs) (Saltelli et al. 2008), is adopted to generate samples. Compared to classic sampling techniques (Rubinstein et al. 2016; Scozzese et al. 2020), this approach allows to place sample points as uniformly as possible in the variability domain, reducing the generation of clusters (i.e. overlapping of samples) or gaps (i.e. empty spaces among samples). More details about the adopted sampling technique can be found in Minnucci et al. (2022). An optimal number 10.000 samples are generated for each foundation layout.

4. Results of the probabilistic SSI analyses

In this section, results of the probabilistic soil-foundation dynamic analyses are presented. Impedance functions and kinematic response factors are evaluated in the non-dimensional frequency range 0-1, covering the frequency range of practical interest (0-10 Hz) in the field of seismic engineering. For each case study, and for each response quantity appearing in Eq. (4) and Eq. (5), the variability of the results is elaborated in terms of probability density functions of realizations, depending on the selected value of the non-dimensional frequency. In the sequel, a limited set of results is presented to comment about the effects of uncertainties on the dynamic response of square pile groups. The latter are sufficient to support the conclusions; however, a more complete overview of the results can be found in Minnucci et al. (2022). In addition, sensitivity indexes of the output quantities are also shown for the 2X2 foundations; these are used to comment on how uncertainties of the generic variable affect uncertainties of the model. The sensitivity indexes are computed according to the following expression:

$$S_k(a_0) = \frac{v_{X_k}(E_{X \sim k}(Y|X_k))}{v(Y)} \quad (6)$$

where $Y = g(X_1, X_2, X_3; a_0)$ is the generic function (impedance or kinematic transfer function), X_k ($k = 1, \dots, 3$) the aleatoric variables (ρ_s , V_s and E_p), $V_{X_k} (E_{X \sim k}(Y|X_k))$ the conditional variance of Y and $V(Y)$ the total variance. It is worth noticing that the adopted sensitivity indexes vary within the nondimensional frequency (a_0) range. In an attempt to measure the overall importance of the variable parameter over the frequency range $0 - A_0$ considered, the following mean value will be considered for the analysis of the results:

$$\bar{S}_k = \frac{1}{A_0} \int_0^{A_0} S_k(a_0) da_0 \tag{7}$$

4.1. Statistical analysis of impedances and FIM

Fig. 2 shows the variability of the real and imaginary parts of the non-dimensional translation impedance Π_1 of the 2x2 foundation for $s/d = 4$, $L/d = 32$ and $V_s = 300$ m/s. In detail, Fig. 2a shows a planar view of the frequency-dependent impedance function realizations. Plots represent the probability density by means of a color scale (bright colors correspond to higher density values). In the same graph three significant quantities are highlighted to characterize the distribution in a probabilistic perspective: the red solid line fits the mean value of impedances, the red dashed curve is the median value and the dotted curve is the mode; furthermore 25th and 75th percentiles are reported. Fig. 2b shows curves representing the density distribution of impedances for selected a_0 values (0, 0.25, 0.50, 0.75 and 1.00). Regarding real parts, the data scattering, which is overall important, varies sensibly with frequency and presents a remarkable reduction in the 0.6-0.8 a_0 range. The mean and median curves are almost coincident within the investigated frequency range, while the mode curve differs sensibly from these at the higher frequencies. As for imaginary parts, a trend can be recognized for the data dispersions: an overall increase of the scattering is observed by increasing the frequency.

Similarly, Fig. 3 refers to real and imaginary parts of rotational non-dimensional impedance Π_2 for the 2x2 foundation with $s/d = 2$, $L/d = 8$ and $V_s = 100$ m/s.

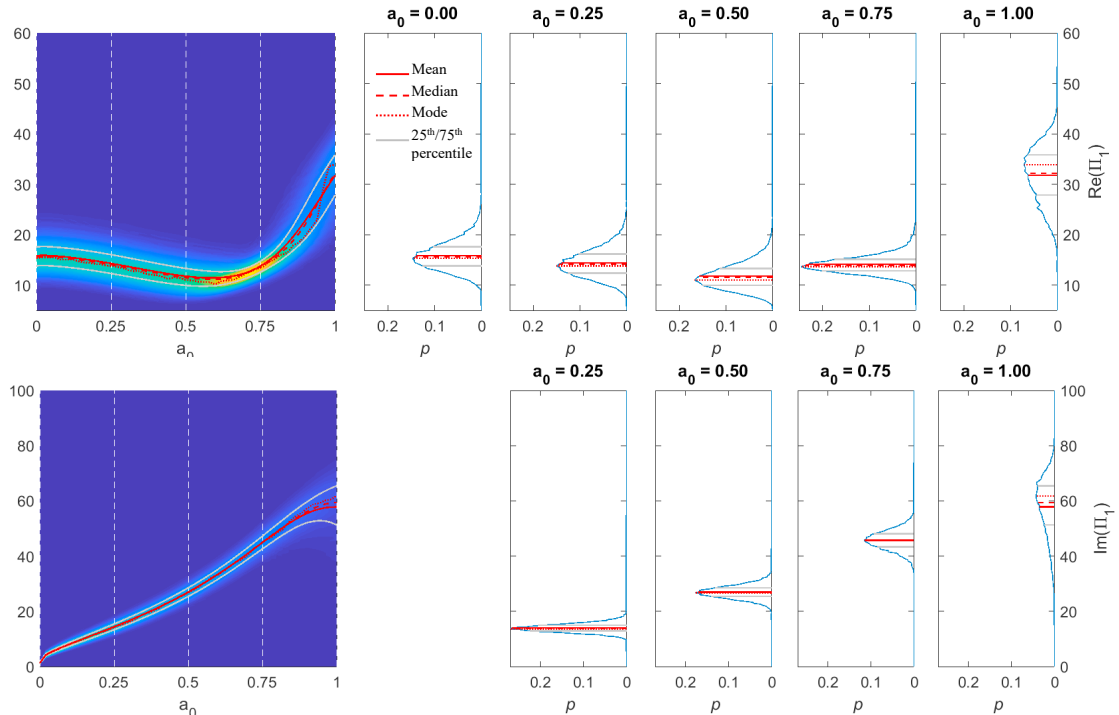


Fig. 2. (a) Variability of Π_1 (real and imaginary parts) and (b) distributions of values at selected frequencies
Case 2X2, $s/d = 4$, $L/d = 32$, $V_s = 300$ m/s.

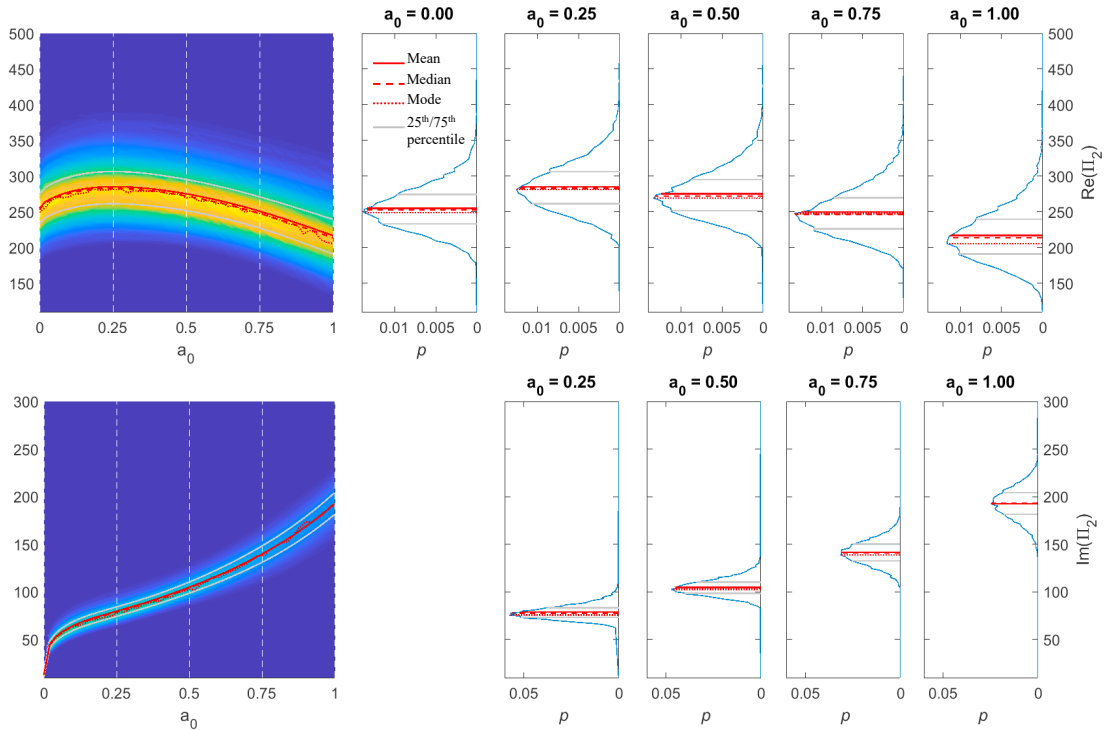


Fig. 3. (a) Variability of Π_2 (real and imaginary parts) and (b) distributions of values at selected frequencies
Case 2x2, $s/d = 2$, $L/d = 8$, $V_s = 100$ m/s.

Concerning trends of data scattering, the variability of real part turns out to be almost constant, holding a lognormal-like trend on the transverse probability distributions, while scattering of imaginary part increases progressively with frequency. A comprehensive exam of the variability results for all case studies considered and further comments are available in Minnucci et al. (2022); from an overall point of view, results lead to a general observation: scattering of impedances is very important in the frequency range where impedances are characterized by higher gradients. This is due to the fact that a variation of the aleatoric parameter produces a frequency shift of the impedance functions peaks.

Fig. 4a shows the variability of the translational (I_U) and rotational (I_Φ) kinematic response factors for a 3x3 foundation; a generalization of comments concerning the main statistical parameters and the results scattering of the other case studies. Differently from impedances, a small range at low non-dimensional frequency (0-0.25) can be identified in which the results dispersion of the translational factor is almost negligible, and the parameter can be described by its mean value; this is because at low frequencies the translational parameter is almost unitary. Overall, for $a_0 > 0.25$, the variability of I_U increases with frequency, consistently with the parameter decrement. This phenomenon confirms that the greater the gradient of the curves with frequency, the higher the dispersion of the parameter. The rotational kinematic response factor I_Φ also exhibits dispersions that follow the above rule. The density distributions at specific non-dimensional frequencies (0, 0.25, 0.50, 0.75 and 1.00), shown in Fig. 4b, confirm the dependence of the results scattering with the gradient of the function of the kinematic parameter. Above consideration reflects in the mode of both the translational and rotational factors that presents jumps due to the strong variation of data scattering with frequency. Jumps highlight that for certain frequencies the distributions can be bimodal with the most probable responses far from the mean one. Finally, the density distributions in Fig. 4b demonstrate the absence of a clear trend and the practical difficulty of defining a probabilistic model for the parameter.

4.2. Sensitivity indexes

Bar charts in Fig. 5a depict the sensitivity indexes of 3x3 foundations. Comments can be extended to 2x2 pile groups since they present similarities. Concerning real parts of impedances, very high values of the V_s sensitivity

indexes can be found for Π_1 and Π_5 for all the case studies, while Π_2 is slightly more sensitive to the effect of the other parameters variability, especially the soil density. The pile elastic modulus has a limited impact, mainly on the rotational (Π_2), component of the impedance matrix. Further comments can be found in Minnucci et al. (2022).

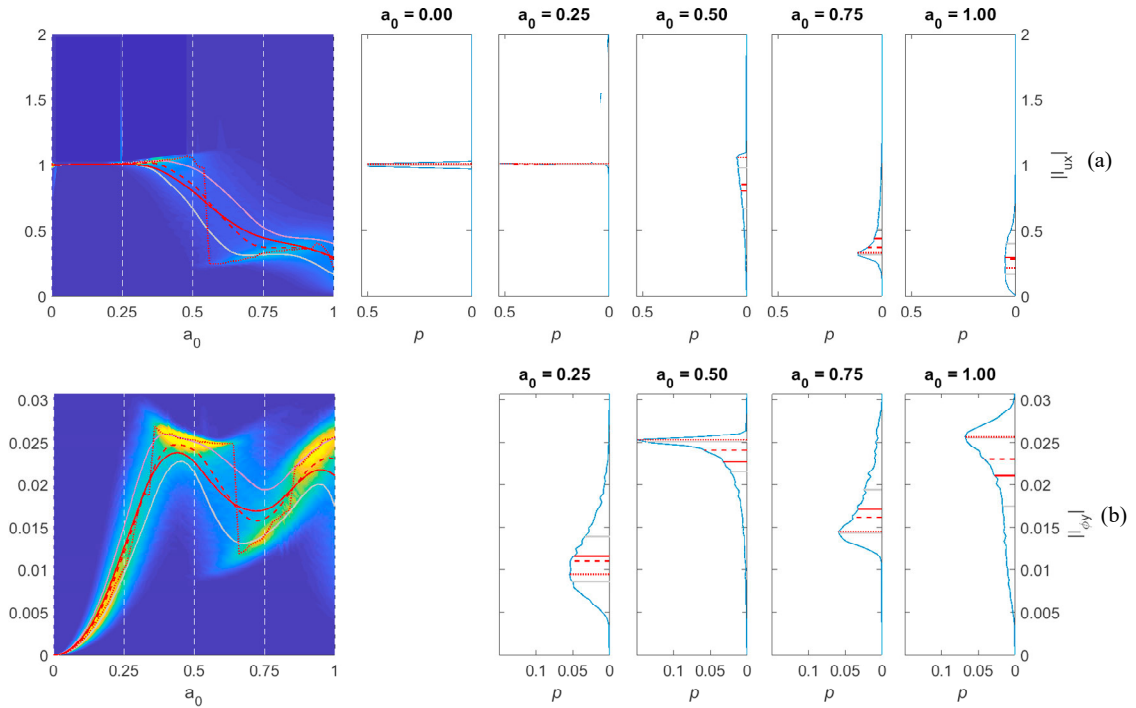


Fig. 4. Variability of (a) I_U and (b) I_Φ for 3x3 foundation, $s/d = 4$, $L/d = 32$, $V_s = 100$ m/s.

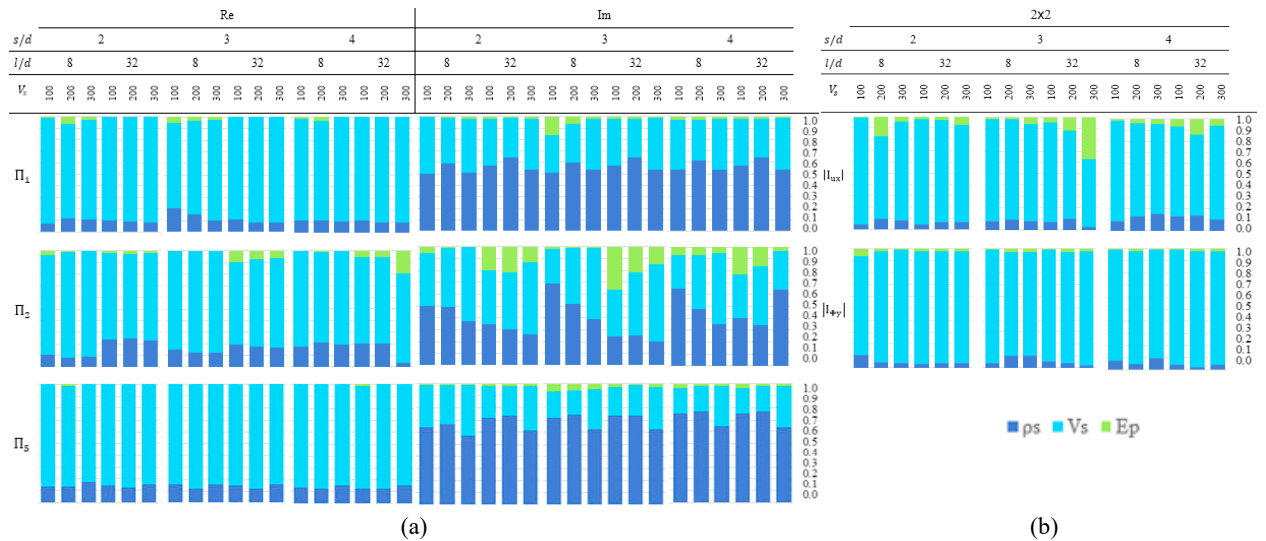


Fig. 5. (a) Sensitivity indexes for 3x3 foundations' impedances and (b) sensitivity indexes for 2x2 foundations' kinematic response factors

As for imaginary parts, an overall higher influence of the soil density for all the impedance components is observed; in particular, the sensitivity indexes of the soil density became comparable or even higher than those of the shear wave velocity, especially for the torsional (Π_5) terms. Conversely, the variability of E_p scarcely contributes to the variability of the translation (Π_1) and torsional impedances (Π_5); for the rotational (Π_2) impedance, E_p sensitivity indexes up to 0.3 are observed in some cases. The bar charts in Fig. 5b refer to all the investigated pile groups foundations' kinematic response factors for the 2x2 foundations. The sensitivity indexes of V_s are very high, especially for the rotational kinematic response factor. However, an effect of the soil density and of the pile elastic modulus uncertainties is evident in the variability of the translational kinematic response factor.

5. Conclusions

In this paper a probabilistic investigation on the dynamic behaviour of pile foundations has been presented, with the purpose of addressing the effects of intrinsic uncertainties of the main parameters governing the soil-piles interaction problem on the soil-foundation dynamic impedance and kinematic response factors of square floating pile groups. The soil density, the shear wave velocity and the concrete elastic modulus of piles are assumed as independent random variables and probabilistic distributions available in the literature are adopted to reproduce their uncertainties. A Quasi-Random simulation technique is exploited to generate samples. Floating piles in homogeneous deposits are analysed considering different layouts, and the results are presented and discussed in terms of significant statistical quantities and of sensitivity indexes, to highlight the influence of each independent variable on the output values. As a result, it is worth highlighting that scattering of results are higher for frequency ranges where impedances are characterised by high gradients. Moreover, sensitivity indexes reveal that the shear wave velocity is often mainly responsible for the global variability of the results; however, uncertainties in the pile elastic modulus and the soil density may also affect the dispersions of results: as an example, the latter sensibly affects the imaginary parts of impedances of pile groups. In the case of kinematic response factors, sensitivity indexes reveal that the shear wave velocity is always the main variable influencing the overall variability of the results.

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