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METHODS

# Assessment of Input-Output Dead Times Impact on Tuning Procedures and Grouping Policies for Output Soft Constraints in MPC

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**ABSTRACT** Model Predictive Control (MPC) is an Advanced Process Control technique able to handle constrained multivariable processes characterized by dead times. In these processes, dead times information covers a key role for control systems design and for the associated tuning procedures. This paper aims to assess the impact of input-output dead times on tuning procedures and grouping policies associated to output soft constraints in MPC based on linear models. An MPC problem characterized by hard constraints on the inputs magnitude and on the inputs slew rate and by soft constraints on the outputs magnitude is considered for the description of the proposed approach. Window parameters, tuning parameters associated to slack variables and grouping policies for constraints relaxation represent the key points investigated in the work. In addition, a characterization of the proposed methods based on different input-output configurations and on the control matrix to be used at each control instant is presented. Simulation results based on first-order plus deadtime models show how the proposed methods can improve the effectiveness and the efficiency of MPC systems design.

**INDEX TERMS** Model predictive control, controller tuning, linear model, dead time, window parameter, soft constraint, slack variable.

## I. INTRODUCTION

Model Predictive Control (MPC) represents one of the most predominant Advanced Process Control (APC) strategies, mainly due to its strong capability to deal with constrained multivariable processes characterized by very simple or complex dynamics. MPC refers to a set of control techniques that make explicit use of a model of the considered process to formulate an optimization problem, characterized by the minimization of an objective function (possibly) subject to constraints [1]. Among the major pros of MPC strategies, there are the options to directly include constraints in the optimization problem to be solved for the computation of the control law and to control processes characterized by dead times.

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MPC formulation is typically based on a constrained optimization problem. Usually, in industrial and non-industrial processes, there are constraints on the process variables (inputs and/or outputs) that have to be met. Frequently, the constraints are divided into two categories: *hard* constraints and *soft* constraints. *Hard* constraints cannot be violated; *soft* constraints can be violated only when needed. Usually, constraints on the inputs are defined as *hard* constraints, because for example actuators have limited ranges of action and limited slew rates; on the other hand, often the constraints on the outputs are defined as *soft* constraints. In order to define *soft* constraints, usually slack variables are introduced in the formulation of the MPC optimization problem: these variables are computed as nonzero values only if at least one of the associated constraints is violated [1]. The introduction of the slack variables in the MPC optimization problem requires including suitable coefficients and weights for the slack variables in the constraints and in the objective function that represent a subset of the tuning parameters [2], [3]; in addition, often constraints grouping policies must be defined so to limit the number of decision variables. For example, in [4], the slack variables are introduced in the linear inequality constraints through nonnegative Equal Concern for Relaxation (ECR) coefficients and in the cost function with a quadratic penalty through positive weights.

Dead times (also denoted as delays) on the input-output channels represent a typical feature of industrial and non-industrial processes. They are mainly due to the time needed to transport mass, energy, or information, but they can also be due to cascaded dynamic systems. The presence of dead times adds challenges for the design and implementation of control systems [5], [6]. Many dead-time compensators (DTCs) were developed, e.g., the Smith Predictor [7]. Together with DTCs, MPC, in its different formulations, e.g., Generalized Predictive Control (GPC) and Dynamic Matrix Control (DMC), showed its potential in the dead times handling within the different levels of the process plant automation (e.g., level 2).

With regard to dead times handling in MPC, different contributions on theory, tuning and applications were published by several research groups in the past two decades. With regard to theory on dead times handling in MPC, significant results are reported in [5], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], and [21]. With regard to tuning and applications of MPC on processes characterized by dead times, significant results are reported in [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32], [33], [34], [35], [36], [37], [38], [39], [40], [41], [42], [43], [44], [45], and [46].

In [5], dead times handling within GPC and DMC frameworks is discussed; SISO (single-input single-output) and MIMO (multi-input multi-output) cases are presented. In the MIMO case, the single dead time case and the multiple dead time case are formulated. The single dead time case refers to situations where the same time delay characterizes all input-output channels; in the multiple dead time case, different time delays characterize the different input-output channels. Furthermore, a distinction between the management of the dead times associated to manipulated inputs and to disturbance inputs is provided. Finally, the robustness problem is tackled. Robust stability and constraints satisfaction for discrete-time linear systems characterized by dead times are analyzed in [8] where an explicit dead time compensation method is proposed. In addition, the differences between implicit and explicit dead time compensation are presented highlighting the drawbacks tied to the use of an augmented state space representation for implicit dead time compensation. An explicit dead time compensation strategy is proposed also in [9], where robust tube based MPC for constrained linear systems with dead time is tackled, adopting a delay-free state space representation. A filtered Smith Predictor structure within MPC is proposed in [10], where the controller is tuned using a low pass filter to obtain a trade-off between robustness and performance; in particular, errors in the dead time estimation are considered. A filtered Smith Predictor structure is exploited also in [11], where a robust MPC approach to control multiple dead-time multivariable processes is presented that exploits terminal cost weighting. A robustness improvement strategy through explicit input-delay compensation is proposed in [12], where a filtered Smith Predictor delay compensation method is integrated into MPC. An explicit dead time compensation approach is proposed for Linear Parameter Varying MPC in [13], proposing a scheme that explicitly accounts for the dead time phenomena through a compensation loop. Delay estimation and model mismatch uncertainties are analyzed. Some results obtained for linear systems are extended to a class of nonlinear systems in [14], where the predictor structure is separated from the optimization stage.

In [15], a robust MPC design is proposed for time delayed systems where the control specifications require a zone control for the output variables while optimizing targets. In [16], integrating outputs with time delays in their relationships with the inputs are considered and an MPC scheme is proposed. Zone control is assumed as control specification for some outputs while the remaining outputs and/or the inputs are subjected to target tracking objectives. In [17], a robustly stabilizing MPC algorithm for multi-model integrating time delay processes is presented that takes also into account optimizing targets for some inputs/outputs and zone control for defined outputs.

In [18], invariance properties are defined for predictive control of linear systems with delayed inputs subject to input and state and/or output constraints. Uncertainties caused by the discretization procedure are taken into account. The variable time delay case is tackled in [19], where explicit formulations of the control law are obtained through multiparametric programming. In [20], variable feedback delay challenges are tackled in the predictive control of linear time invariant systems with constraints. In [21], distributed MPC for interconnected time delay systems is proposed, exploiting dual gradient method for optimization.

A review of tuning methods for MPC from a theoretical and practical point of view is proposed in [22]. SISO and MIMO cases are cited, and dead times are considered in the tuning procedures of the prediction horizon, of the control horizon and of window parameters. In addition, some remarks on processes that can be modellized through FOPDT (first-order plus deadtime) models are provided. In [23], an update of the insights given in [22] is provided. In particular, procedures based on multiobjective optimization and pole placement are proposed together with some autotuning methodologies. In [24], an extension of the FOPDT modified GPC algorithm is provided for SOPDT (second-order plus deadtime) models together with ad hoc tuning methods based on iterative procedures, focusing on nominal stability and desired performance. Simulations on a nonlinear non-isothermal CSTR (continuous stirred tank reactor) plant are provided. In [25], an analytical approach for tuning unconstrained MPC based on FOPDT models is proposed in a pole placement framework and the results are shown through simulations on FOPDT and high order plants. A general structure to control long time-delay plants is proposed in [26] together with a tailored procedure for parameters tuning. Advancements in the trade-off between performance and robustness are provided. To illustrate the proposed methods, simulations on processes characterized by integrators and dead times are reported. In [27], an analytical tuning approach for multivariable model predictive controllers is presented. Square plants modellized through FOPDT transfer functions are considered, e.g., multivariable pH process and Wood and Berry plant. This approach is extended to first-order plus fractional dead time models in [28]; simulation results on different FOPDT models and on the pH neutralization process are provided. Multi-objective optimization is exploited for MPC tuning in [29]. Two methods are proposed that consider the minimization of the output tracking error, considering square and non-square systems. Simulation results on a Shell heavy oil fractionator benchmark system are provided. In [30], PFC (Predictive Functional Controller) robust tuning procedures for processes modelled through FOPDT and SOPDT models are presented and applied to a laboratory pilot plant. Adaptive analytical tuning procedures for adaptive MPC based on FOPDT models are proposed in [31], proving the stability and the convergence of the proposed strategy. Simulation and experimental results are proposed considering a simulated pH neutralization process, a lab-scaled level control system and a lab-scaled pressure control system. In [32], a tuning strategy is introduced for nonsquare systems with more outputs than inputs. Set points or ranges for controlled variables are considered and the whole procedure is based on the creation of multiscenarios. Some simulation results for the zone region tracking on the Shell heavy oil fractionator benchmark are proposed. In [33], the approach presented in [32] is extended in order to ensure satisfactory performance in a wide operating region. A multi-objective optimization approach is proposed in [34]; the method provides the control and prediction horizons together with the weights of the objective function. Simulation results on a subsystem of the Shell heavy oil fractionator benchmark are presented. An autotuning method is presented in [35] for MPC subject to input constraints and for MPC with a terminal constraint. The associated tuning parameters are computed through a closed-loop simulation that takes into account model mismatch. A solar desalination plant is considered as case study in [36], where an optimization problem similar to that associated to a linear MPC is formulated. In addition, a robust dead-time compensation scheme is used. In [37], a crude distillation unit is analyzed and a tuning procedure for MPC with output zone control and input targets is proposed. The considered tuning parameters to be optimized are the weights used in the objective function. In [38], a two-layer MPC strategy oriented to control and optimize the clinker production phase of a cement industry is proposed; simulation and practical results associated to this process with significant time delays are reported. An MPC approach equipped with dead-time compensation procedures is proposed for a gas compression system in [39]. A linear model characterized by transfer functions with dead times is used for the predictions' computation; a filtered Smith predictor extension has been included in the MPC architecture. A single slack variable is exploited in the proposed formulation. In [40], a distributed MPC approach is proposed for linear interconnected time-delay systems; a series of chemical reactors is considered as case study. In [41], a multiobjective tuning technique is proposed for MPC in grinding circuits, i.e. processes characterized by long dead times, slow dynamics and variable coupling. DMC formulation is adopted, and the tuning goals are tied to the desired closed-loop performance associated to the process variables. The regulation of the pH value of the sewage treatment system is tackled in [42], where MPC is combined with machine learning. Neural networks are exploited for the formalization of a set of labels associated to different robust performance levels. In addition, an automated MPC tuning algorithm is proposed. In [43], the control of wet limestone flue gas desulfurization process is considered, and an economic MPC is utilized. A multi-objective approach is proposed, considering a two-layer architecture. A combination of utopia strategy and NSGA-II provides the Pareto front of the multi-objective optimization. In [44], a fluidized-bed gold ore roaster is considered as case study for practical implementation of an MPC system based on a simultaneous multistep transformer architecture. A comparison between transformer and Long-Short Term Memory (LSTM) models is proposed, focusing on MPC applications. The comparison shows that the presented multistep transformer architecture-based MPC is able to outperform LSTM-based MPC. In [45], a multi-scale nonlinear MPC system is proposed to control the temperature of a sintering furnace. A hybrid self-learning model is exploited considering time delay. In [46], an extracting dividing wall column represents the case study, and a LSTM-based MPC is designed. Simulation results show that the proposed approach outperforms temperature inferential control schemes.

According to the authors' knowledge, an overall assessment of the procedures based on input-output dead times information that can be used to make more effective and more efficient the definition of some tuning parameters and policies associated to output *soft* constraints in MPC based on linear models is not present in the literature. In addition, a characterization of these procedures based on different input-output configurations and on the control matrix to be used at each control instant represents another research gap. The research work presented in this paper aims to investigate these topics in order to give precise insights on the usefulness of input-output dead times information for the definition of tuning parameters and of policies associated to output *soft* constraints. The main contributions of the paper are:

- A method for the computation of a lower bound for an effective tuning of the window parameter associated to the output *soft* constraints.

- Two strategies oriented to provide an effective and efficient definition of the tuning parameters and of the grouping policies for the slack variables included in the MPC optimization problem formulation.
- The characterization of the solutions mentioned in the previous items based on different input-output configurations and on the control matrix to be used at each control instant.

The paper is organized as reported in the following: Section II reports some preliminaries and the problem statement; Section III covers the theoretical core of the paper while simulation results and discussion are reported in Section IV. Section V reports the conclusion and some ideas for future work. Table 1 reports a list of abbreviations used in the paper.

**TABLE 1.** List of abbreviations.

| Abbreviation | Description |
|---|---|
| APC | Advanced Process Control |
| CSTR | Continuous Stirred Tank Reactor |
| CV | Controlled Variable |
| DMC | Dynamic Matrix Control |
| DTC | Dead-Time Compensator |
| DV | Disturbance Variable |
| ECR | Equal Concern for Relaxation |
| FCC | Fluid Catalytic Cracking |
| FOPDT | First-Order Plus DeadTime |
| GPC | Generalized Predictive Control |
| IAE | Integral of Absolute Error |
| LSTM | Long-Short Term Memory |
| MIMO | Multi-Input Multi-Output |
| MISO | Multi-Input Single-Output |
| MPC | Model Predictive Control |
| MV | Manipulated Variable |
| PFC | Predictive Functional Controller |
| QP | Quadratic Programming |
| RTO | Real Time Optimization |
| SISO | Single-Input Single-Output |
| SOPDT | Second-Order Plus DeadTime |

## II. PRELIMINARIES AND PROBLEM STATEMENT

In this section, the MPC formulation that is considered in the present paper is detailed, together with some preliminaries. Finally, the statement of the problem that the present research work aims to solve is reported. In particular, after a description of the context where the present research work is located, process model assumptions are reported in Section II-A, while the considered MPC formulation is depicted in Section II-B. Finally, Section II-C focuses on the parameterization of outputs predictions and Section II-D reports the problem statement.

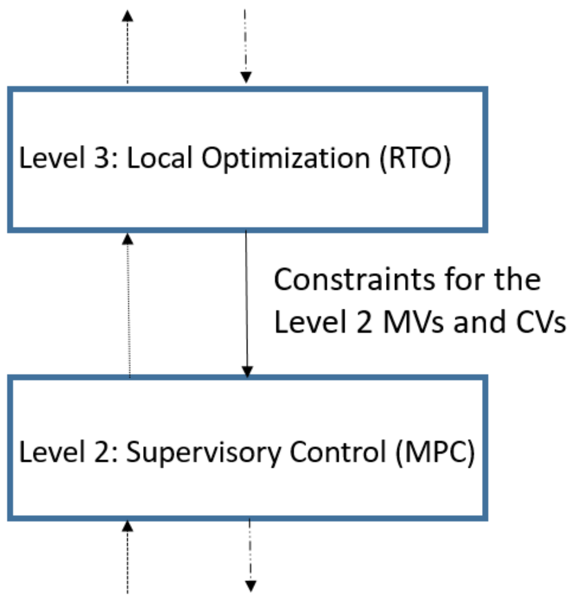


**FIGURE 1.** Hierarchical (vertical) decomposition of some control levels in a typical process plant with focus on levels 2-3.

Taking into account the typical hierarchical control system architecture (see [47], [48]), the considered MPC-based controller is positioned at level 2. In a typical process plant, the different control systems implemented are differentiated according to their execution frequency (temporal interval) and operating level (e.g., Device level, Loop level, Unit level, Plant/Asset level, Enterprise level); Fig. 1 focuses on level 3 and level 2 and their interactions. Usually, sampling times in the order of hours characterize level 3 optimizers while level 2 controllers are based on sampling times in the order of minutes. In many applications, Real Time Optimization (RTO) algorithms are located on level 3, while MPC algorithms characterize level 2.

Among the data exchanged between level 2 and level 3 (see Fig. 1), level 3 can supply to level 2 a set of constraints on Manipulated Variables (MVs) and Controlled Variables (CVs). This set of constraints can change online and identifies an optimal region where the process should be operated.

In addition to constraints, also targets on the process variables can be forwarded from level 3 to level 2 but they are not considered in the present paper because they do not represent crucial elements for the proposed methods.

Level 2 CVs are the output variables associated to level 2 subprocesses and they can be (eventually) measured. Level 2 MVs are measured inputs associated to level 2 subprocesses that the level 2 controllers can exploit (in the allowed region) for the achievement of the specifications on the satisfaction of the CVs constraints. Additional input variables can be included in the control problems' formulation, e.g., measured input Disturbance Variables (DVs). DVs are measured process inputs that affect the CVs behavior but, according to process requirements, they cannot be manipulated by the level 2 algorithms under consideration. Based on the available information, DVs future values can be known in advance and, if allowed by the selected control strategy, they can be exploited in the computation of the optimal control law (previewing or look-ahead feature, see [1], [4]).

Level 2 controllers, when adopting a zone control strategy (see [1]), have to compute the optimal MVs values within the assigned allowed region in order to maintain the CVs values within the defined constraints; in this context, an additional requirement could be the minimization of MVs moves magnitude (when possible). Furthermore, a typical requirement in industrial and non-industrial processes is the option to online define the variables involved in the control matrix, i.e. to online define the inputs and outputs that have to be considered in the MPC problem formulation at each control instant. This feature can be assessed through the definition of suitable status values for MVs and CVs [49]. For example, for each CV, two possible status values can be assumed: "1" and "0". A status value equal to "1" indicates that the level 2 scheme must have in control that variable, i.e. the CV is *active*. Conversely, a CV status value equal to "0" indicates that the CV is *inactive*: at the current control instant the level 2 scheme has not in control that CV, i.e. MVs must not act to satisfy its specifications. Typically, a CV is assigned an *inactive* status as a consequence of specific process conditions/requirements or when anomalies are detected, e.g., bad conditions on its measurement are observed [49]. For each MV, two possible status values can be assumed: "1" and "0". A status value equal to "1" indicates that at the current control instant that MV is *active*, i.e. it can be used (i.e. it can be manipulated or moved) by the controller. A status value equal to "0" indicates that the considered MV cannot be manipulated by the controller at the current control instant. In this case, the MV value considered in the predictions computation can be kept constant to the last MV value forwarded to the plant or it can be forced to predefined trajectories (if they are known in advance). Typically, an MV is considered *inactive* when another control system (e.g., manual or automatic) overrides the MPC-based controller and manipulates it [49].

### A. PROCESS MODEL ASSUMPTIONS

For each MV-CV and DV-CV pair, an asymptotically stable strictly proper linear model is assumed around suitable operating points. For example, an asymptotically stable FOPDT model can be considered for each MV-CV and DV-CV pair [50]. The assumption to consider a linear approximation (FOPDT models in the present paper) near suitable operating points for the design of a level 2 control system is well posed for practical industrial applications (see the literature review reported in Section I). For example, processes that can be represented through FOPDT models are used in [22] and [23] for different MPC paradigms which are widely used in practice, e.g., DMC and GPC. In addition, benchmark industrial processes, e.g., distillation columns and evaporators, on which different MPC applications are documented, can be modellized through FOPDT transfer functions [1]. Other types of linear models (e.g., SOPDT) can be exploited to design MPC strategies when higher order dynamics are needed to sensibly approximate the process nonlinearities, e.g., for CSTR plants [24] and for fluid catalytic cracking (FCC) units [15]. Furthermore, different level 2 control systems which exploit linear models for MPC practical industrial and non-industrial applications are present in literature, e.g., for cement industry and water distribution networks [51], [52], [53].

In (1), the parametric expression of a FOPDT model in Laplace domain is reported:

$$M(s) = \frac{K}{Ts+1}e^{-T_d s} \tag{1}$$

where $T$ is the time constant, $T_d$ is the dead time (or delay), and $K$ is the steady state gain of the considered input-output channel. Considering the different FOPDT models for each MV-CV and DV-CV pair, a MIMO transfer function matrix can be obtained. From this input-output representation, a continuous-time state-space realization [54] can be computed when a state-space description is adopted. Successively a discretization procedure can be performed, using a zero-order hold and a suitable sample time $T_s$. With regard to the input-output delays, they can be included into systems dynamics [55] or not [56], obtaining an implicit or explicit dead time compensation when the model is used for control purposes in model-based control techniques, e.g., MPC. In any case, assuming $l$ MVs, $l_d$ DVs and $m_z$ CVs, based on the considered discretized linear model, the steady state gain matrices $G_{zu} \in \mathbb{R}^{m_z \times l}$ (CVs-MVs) and $G_{zd} \in \mathbb{R}^{m_z \times l_d}$ (CVs-DVs) can be obtained, together with the dead time (or delay) matrices $D_{zu} \in \mathbb{N}^{m_z \times l}$ (CVs-MVs) and $D_{zd} \in \mathbb{N}^{m_z \times l_d}$ (CVs-DVs). In case of absence of dead time (or delay) on a CV-MV (CV-DV) channel, the associated element in $D_{zu}$ ($D_{zd}$) matrix is equal to zero.

The obtained linear model can be augmented and equipped with a state estimator in order to handle uncertainties and to guarantee an optimized feedback exploitation [57], [58].

### B. MPC FORMULATION

The level 2 MPC-based controller that is considered in the present paper performs a zone control. Its formulation is reported in the following:

$$\min_{\Delta U(k),\varepsilon(k)} V(k) = \min_{\Delta U(k),\varepsilon(k)} \left( \sum_{i=1}^{H_u} \| \Delta\hat{u}(k+M_i|k) \|^2_{R(i)} + \| \varepsilon(k) \|^2_{\rho} \right) \quad (2)$$

subject to

$$\begin{aligned}
&\text{a. } lb_{du}(i) \le \Delta\hat{u}(k+M_i|k) \le ub_{du}(i), \quad i = 1 \ldots H_u \\
&\text{b. } lb_u(i) \le \hat{u}(k+M_i|k) \le ub_u(i), \quad i = 1 \ldots H_u \\
&\text{c. } lb_{z_j}(i) - \gamma_{lbz_j}(i) \cdot \varepsilon(k) \le \hat{z}_j(k+i|k) \\
&\qquad \le ub_{z_j}(i) + \gamma_{ubz_j}(i) \cdot \varepsilon(k), \\
&\quad j = 1 \ldots m_z, \quad i = H_{w_j} \ldots H_p \\
&\text{d. } \varepsilon(k) \ge 0_{n_\varepsilon \times 1}
\end{aligned} \quad (3)$$

where $\|\cdot\|$ is the Euclidean norm, $0_{r\times 1}$ represents a $(r \times 1)$ vector with elements all equal to zero, $k$ is the current discrete time control instant, $\hat{u} \in \mathbb{R}^{l\times 1}$ is the MVs vector, and $\hat{z} \in \mathbb{R}^{m_z\times 1}$ is the CVs vector. $H_p$ is the prediction horizon while $H_u$ is the control horizon ($1 \le H_u \le H_p$). The future MVs moves, denoted by $\Delta\hat{u}(k+M_i|k) \in \mathbb{R}^{l\times 1}$ vectors ($i = 1 \ldots H_u$), are included in a $\Delta U(k) \in \mathbb{R}^{l\cdot H_u\times 1}$ vector. $\Delta U(k)$ vector represents the control moves and is part of the decision variables vector in the considered MPC problem (2)-(3). $M_i$ ($i = 1 \ldots H_u$) represent the future instants, within the prediction instants, where the future MVs moves to be computed are possibly nonzero; with regard to the $M_i$ definition, $M_1 = 0$ and $1 \le M_h \le H_p - 1$ ($M_{h-1} < M_h$, $h = 2 \ldots H_u$). $M_i$ may not be all consecutive instants; if this is the case, the MVs are kept constant over blocks of sampling intervals (*blocking* feature, see [59]). Practically, the definition of a control horizon and of the related MVs movement instants corresponds to the enforcement of implicit equality constraints ($\Delta\hat{u}(k+q|k) = 0_{l\times 1}$, $q \ne M_i$) within the MPC problem formulation.

MVs magnitude inequality constraints over the control horizon are reported in (3.a), while MVs moves inequality constraints over the control horizon are reported in (3.b); suitable $lb_{du}$, $ub_{du}$, $lb_u$, $ub_u$ ($\in \mathbb{R}^{l\times 1}$) vectors define the allowed region for the associated terms at each instant of the control horizon. As typical in the process control field (see [1] for further theoretical remarks), all MVs constraints have been assumed as *hard* constraints due to physical motivations (e.g., due to actuators' limitations): they can never be violated and, in case of online modification of these terms from a control instant to another, the feasibility must be suitably imposed. This design choice proved to be effective in many applications on real case studies, e.g., associated to cement industry and water distribution networks. In cement industry, examples of MVs magnitude and moves inequality constraints are constraints on fan speed and fuel charge rate set points, while in water distribution networks they can be associated to pressure set points [51], [52], [53]. The MVs moves, present also in the objective function reported in (2), are here weighted through suitable positive definite diagonal matrices $R(i) \in \mathbb{R}^{l\times l}$($i = 1 \ldots H_u$). Without loss of generality, only the $H_u$ prediction instants related to possibly nonzero MVs future moves (associated to the $M_i$ instants) are considered in (2), (3.a), and (3.b).

CVs magnitude inequality constraints over the prediction horizon $H_p$ are reported in (3.c); $H_w \in \mathbb{N}^{1\times m_z}$ represents a vector that contains a window parameter (in the range $1-H_p$) for each $j$th CV ($j = 1 \ldots m_z$) [1]. This parameter is crucial in the MPC optimization problem (2)-(3) because for example a too large value could cause a more sluggish and less aggressive control action; on the other hand, a too small value can cause "induced" constraints relaxations (this concept will be further stated in Section II-D, while it will be detailed and proved in Sections III-IV). $lb_z$, $ub_z$ ($\in \mathbb{R}^{m_z\times 1}$) vectors define the allowed region for the associated CVs predictions at each instant of the prediction horizon $H_p$. In (3.c), the index $j$ ($j = 1 \ldots m_z$) specifies the CV considered in each constraint. The terms in $lb_z$, $ub_z$ vectors can be changed online, e.g., based on process control specifications. In addition, they can change along the prediction horizon $H_p$, e.g., if future variations are known in advance, this information can be included in the MPC formulation. Based on the status value of each CV at the current control instant, suitable constraints cut off policies can be applied in order to consider only the *active* CVs in the control problem [49].

As typical in process control field, all CVs constraints have been assumed as *soft* constraints (see [1] for further theoretical remarks). This design choice proved to be effective in many applications on real case studies, e.g., associated to cement industry and water distribution networks. In cement industry, examples of CVs constraints are constraints on oxygen, temperature and nitrogen oxides, while in water distribution networks they can be associated to pressures at critical nodes [51], [52], [53]. In order to allow constraints' relaxation, $n_\varepsilon$ nonnegative slack variables have been included in the MPC formulation (vector $\varepsilon(k) \in \mathbb{R}_0^{+n_\varepsilon\times 1}$ in (2), (3.c), and (3.d)). Together with $\Delta U(k)$, the slack variables represent the decision variables vector in (2)-(3). The slack variables have been introduced in (3.c) through suitable $\gamma_{lbz}$, $\gamma_{ubz}$ ($\in \mathbb{R}_0^{+m_z\times n_\varepsilon}$) matrices composed by nonnegative elements; in (3.c), the index $j$ ($j = 1 \ldots m_z$) refers to the row vectors ($\in \mathbb{R}_0^{+1\times n_\varepsilon}$) of these matrices associated to the $j$th CV at each considered prediction instant; each row vector is characterized by a single positive element related to the slack variable that acts on the selected constraint.

The violation of the CVs constraints has to be discouraged; for this purpose, slack variables are introduced in the objective function reported in (2) through a suitable positive definite diagonal matrix $\rho \in \mathbb{R}^{n_\varepsilon\times n_\varepsilon}$.

In the MPC problem (2)-(3), the tuning parameters are represented by $H_p$, $H_u$, $M_i$ ($i = 1 \ldots H_u$), $H_w$, $n_\varepsilon$, $\gamma_{lbz}$, $\gamma_{ubz}$, $\rho$, and $R(i)$ ($i = 1 \ldots H_u$). Note that, with regard to $\gamma_{lbz}$ and $\gamma_{ubz}$, they can be potentially characterized by different values at each step of the prediction horizon $H_p$. In addition, in some formulations (e.g., the formulation reported in [4]), suitable scaling factors can be introduced in order to simplify the tuning procedures.

Having assumed a linear model for the considered process (see Section II-A), the optimization problem (2)-(3) can be recast as a Quadratic Programming (QP) problem. For example, formulating all terms as a function of the $l \cdot H_u + n_\varepsilon$ decision variables and of known values, the objective function reported in (2) is quadratic and the constraints reported in (3) are linear; with a suitable definition of the matrices $R(i) \in \mathbb{R}^{l\times l}(i = 1 \dots H_u)$ and $\rho \in \mathbb{R}^{n_\varepsilon \times n_\varepsilon}$, a positive definite Hessian of the QP problem can be obtained. Due to the presence of constraints, an iterative optimization method has to be used to solve the QP problem, e.g., active-set or interior point methods [1], [4], [56]. Exploiting the *receding horizon* strategy, only the first MVs move $\Delta\hat{u}(k + M_1|k) = \Delta\hat{u}(k|k) \in \mathbb{R}^{l\times 1}$ is taken into account for the control signals ($u(k) = \hat{u}(k|k) = u(k-1) + \Delta\hat{u}(k|k) \in \mathbb{R}^{l\times 1}$) to be forwarded to the plant at the current control instant $k$; the whole procedure is repeated at the next control instant $k + 1$ [1].

### C. CVS PREDICTIONS PARAMETERIZATION

As described in Section II-B, the MPC optimization problem (2)-(3) is characterized by a set of decision variables consisting of $\Delta U(k) \in \mathbb{R}^{l\cdot H_u \times 1}$ and $\varepsilon(k) \in \mathbb{R}_0^{+n_\varepsilon \times 1}$. Adopting this MPC problem formulation, all terms present in (2) and (3) must be parameterized as a function of the decision variables. For example, the CVs predictions $\hat{z} \in \mathbb{R}^{m_z \times 1}$ over the prediction horizon $H_p$ can be parameterized as a function of $\Delta U(k) \in \mathbb{R}^{l\cdot H_u \times 1}$ through the following equations [1]:

$$Z(k) = \begin{bmatrix} \hat{z}(k+1|k) \\ \hat{z}(k+2|k) \\ \vdots \\ \hat{z}\left(k+H_p|k\right) \end{bmatrix} = Z(k)_{free} + \theta \cdot \Delta U(k)$$
$$Z(k)_{free} = Z(k)\,|_{\Delta U(k) = 0_{l\cdot H_u \times 1}} \tag{4}$$

where $Z(k)_{free}$ represents the CVs *free response*, namely the response that would be obtained if the MVs future values are kept constant to the last value $u(k-1)$ forwarded to the plant [1]. For the computation of the CVs *free response* over the prediction horizon $H_p$, all information up to $k$ instant is taken into account, included the information on DVs effect. In (4), $\theta \in \mathbb{R}^{m_z \cdot H_p \times l\cdot H_u}$ is a matrix, suitably obtained considering the discretized linear model (see Section II-A), which models the effect of the MVs moves on the CVs predictions over the prediction horizon $H_p$. From (4), the following equations can be obtained for the CVs prediction at the $i$th prediction instant ($i = 1 \dots H_p$):

$$\hat{z}(k+i|k) = \hat{z}(k+i|k)_{free} + \Delta\hat{z}(k+i|k)$$
$$\Delta\hat{z}(k+i|k) = \theta_{((i-1)\cdot m_z+1:i\cdot m_z,:)} \cdot \Delta U(k) \tag{5}$$

where $\theta_{(r:s,:)} \in \mathbb{R}^{m_z \times l\cdot H_u}$ is composed by $m_z$ rows of $\theta$, from the $r$th to the $s$th. The described $\theta \in \mathbb{R}^{m_z\cdot H_p \times l\cdot H_u}$ matrix will be exploited in Section III.

### D. PROBLEM STATEMENT

In the MPC optimization problem (2)-(3), the *active* CVs, that are characterized by constraints over the prediction horizon $H_p$, require a joint tuning of $H_w$, $n_\varepsilon$, $\gamma_{lbz}$, $\gamma_{ubz}$ and $\rho$.

To account for the complexity of the MPC optimization problem, the number $n_\varepsilon$ of nonnegative slack variables has to be properly chosen. In some cases, a single slack variable is chosen for all the CVs *soft* constraints, but this choice can result in an excessive limitation. Another solution is the definition of suitable constraints grouping policies with regard to the slack variables.

The tuning procedures of the cited terms and the definition of the mentioned grouping policies are critical to obtain an effective control action; in addition, they make it possible to assign a priority order between CVs constraints by suitably ranking the importance of the minimization of the slack variables associated to constraints relaxations. In general, a slack variable is computed as nonzero value only if at least one of the associated constraints must be violated in order to find a feasible solution; for this purpose, high priority has to be given to the minimization of slack variables magnitude over the $H_u$ MVs moves magnitude in (2). With regard to the grouping policies for the constraints associated to *active* CVs, the basic case is $n_\varepsilon = 1$: in this case, all *soft* constraints share the same single slack variable and a priority order between constraints relaxations can be only defined through $\gamma_{lbz}$ and $\gamma_{ubz}$. Furthermore, in this case, $\rho$ is a positive scalar and it is only exploited to weight the magnitude of the single slack variable in the optimization problem (2)-(3). The inclusion of a single slack variable is proposed in different works, e.g., [39] and [46], but no mentions can be found in literature regarding the correlation with the status values (see the works reported in Section I). On the other hand, when each *soft* constraint is equipped with its own slack variable, there is a structural independence between constraints. In this case, $\rho$ is a matrix and a joint tuning of $\gamma_{lbz}$, $\gamma_{ubz}$ and $\rho$ can assign the desired importance to the associated slack variables and the desired priority order between constraints relaxations. Finally, if there are some groups composed by two or more constraints that share the same slack variable, a joint tuning of $\gamma_{lbz}$, $\gamma_{ubz}$ and $\rho$ allows to assign the desired importance to the associated slack variables and the desired priority order between/within the different groups. This solution is adopted in different works, e.g., [16], [17], [32], and [35], where a single slack variable is considered for each CV. When two or more constraints share the same slack variable, the shared slack variable is computed by the optimizer taking into account the worst-case constraint violation.

Clearly, as stated before, $n_\varepsilon$ affects the number of the decision variables of the MPC optimization problem (2)-(3); for this reason, an optimal trade-off between computational load retaining and the achievement of the desired performance has to be guaranteed.

In general, when two or more constraints are grouped under the same slack variable, i.e. they share the same slack variable, "induced" constraints relaxations may happen: if at least one constraint of a generic group requires to be softened, the relaxation is propagated to all the constraints of the same group, weighted by the associated coefficient of $\gamma_{lbz}$ or $\gamma_{ubz}$. The described "induced" constraints relaxations may affect the controller performance causing, for example, a less prompt response of the control action and/or an improper control action [52].

Based on the assumptions and preliminaries reported in the previous, the main scope associated to the MVs in the solution of the considered MPC zone control optimization problem (2)-(3) is the following: the *active* MVs have to be moved for *active* CVs control requirements only to prevent the CVs constraints violation (as stated in the previous, all CVs constraints are assumed as *soft* constraints). To better clarify, no tracking is required so the *active* MVs must just perform the smallest action able to guarantee the meeting of the constraints associated to the *active* CVs.

Within this context, the research work presented in this paper aims to investigate how the information on the dead times associated to each CV-MV channel can be exploited to make more effective and more efficient the tuning procedures of some parameters involved in the MPC formulation (2)-(3). Furthermore, this research work proposes some policies related to these parameters. The considered parameters of the proposed methods are those associated to the CVs constraints in the MPC optimization problem (2)-(3), i.e. $H_w$, $n_\varepsilon$, $\gamma_{lbz}$, and $\gamma_{ubz}$; the considered policies are the constraints grouping policies that have to be defined with regard to the slack variables.

## III. DEAD TIMES IMPACT ON TUNING PROCEDURES AND GROUPING POLICIES ASSOCIATED TO CVS SOFT CONSTRAINTS

In this section, an analysis of dead times impact on CVs *soft* constraints tuning parameters and grouping policies with regard to the MPC optimization problem (2)-(3) is performed. The SISO control problem as well as the multi-input single-output (MISO) and the MIMO cases are tackled; different configurations associated to CVs-MVs relationships, and the status value associated to CVs and MVs are taken into account. Section III-A describes the SISO case, while the MISO case is reported in Section III-B. Section III-C reports the MIMO case.

### A. SISO CASE

In the SISO case, a single CV and a single MV are present ($m_z = 1$, $l = 1$, $l_d = 0$) and it is assumed that they are both *active* (see Section II-A); $\theta$ matrix reported in (4)-(5) is characterized by $H_p$ rows and $H_u$ columns and the CV-MV dead time matrix $D_{zu}$ (see Section II-A) is a scalar parameter. In the SISO case, $H_w$ is a scalar parameter ($1 \leq H_w \leq H_p$). The proposed method suggests a lower bound $lb_{H_w}$ for $H_w$ parameter tuning ($H_w \geq lb_{H_w}$).

*Definition 1: In the SISO case, the window parameter lower bound $lb_{H_w}$ indicates the first CV prediction instant on which the first MV move (contained in $\Delta\hat{u}(k|k) \in \mathbb{R}^{1\times 1}$ scalar) will show its effect, i.e. $lb_{H_w}$ represents the first controllable CV prediction instant.*

Based on *Definition 1*, $lb_{H_w}$ can be computed as in the following:

$$lb_{H_w} = lb_{H_{w_1}} = D_{zu} + 1 \tag{6}$$

In the SISO case, because a single CV is present, $lb_{H_w}$ is a scalar. For notation consistency with respect to the cases with more than a single CV (see Section III-C), $lb_{H_{w_1}}$ has been specified in (6). $lb_{H_w}$ can also be determined through $\theta$ matrix: taking into account only the first column of $\theta$ matrix (associated to the first MV move, i.e. $\Delta\hat{u}(k|k) \in \mathbb{R}^{1\times 1}$), $lb_{H_w}$ is equal to the index of the first row where there is a nonzero entry. In the SISO case, $lb_{H_w}$ can be considered as a fixed parameter. According to the authors' knowledge, the proposed definition of the parameter $lb_{H_w}$ and its interpretation in term of the $\theta$ matrix in the SISO case is innovative. In fact, a window parameter lower bound definition and interpretation in term of $\theta$ matrix was never proposed in the literature, even in cases where guidelines on the window parameter setup have been given, e.g., in [22] and [23].

In the SISO case, the information on the dead time associated to the CV-MV channel must be exploited for the computation of the $H_w$ parameter; this parameter is crucial in the MPC optimization problem (2)-(3). A $H_w$ setup that does not take into account the dead time ($H_w < lb_{H_w}$) could cause "induced" constraints relaxations if the same slack variable is shared by at least one constraint associated to the not controllable prediction instants and at least one constraint associated to the controllable prediction instants. Based on the previous statement, in the SISO case, the information on the dead times can be exploited, through the window parameter lower bound $lb_{H_w}$, to make more effective the tuning procedure associated to the $H_w$ parameter.

### B. MISO CASE

The considered MISO case assumes that all inputs are tied to the single CV. In addition, it is assumed that the CV is *active* (see Section II-A).

In the MISO case, two main subcases must be distinguished: the presence of a single MV and the presence of more than one MV. In the first subcase, a single MV is present together with one or more DVs. In this subcase, $m_z = 1$, $l = 1$, $l_d \geq 1$. As reported in Section II-C, the DVs contribution is included in the CV *free response*. For the MISO case with a single MV (that is assumed to be *active*), the same rationales and conclusions associated to the SISO case (see Section III-A) hold. In the second subcase, two or more MVs are present together with zero or more DVs. In this subcase, $m_z = 1$, $l \geq 2$, $l_d \geq 0$. As in the previous subcase, if DVs are present, their contribution is included in the CV *free response*. In the MISO case with two or more MVs, only the *active* MV/MVs at the current control instant must be considered in the control problem formulation. $\theta$ matrix reported in (4)-(5) is characterized by $H_p$ rows and $l \cdot H_u$ columns. The CV-MVs dead time matrix $D_{zu} \in \mathbb{N}^{1\times l}$ (see Section II-A) is a row vector. In the MISO case, $H_w$ is a scalar parameter ($1 \leq H_w \leq H_p$). The proposed method suggests a lower bound $lb_{H_w}$ for $H_w$ parameter tuning ($H_w \geq lb_{H_w}$).

*Definition 2: In the MISO case, the window parameter lower bound $lb_{H_w}$ indicates the first CV prediction instant on which the first MV move (contained in $\Delta\hat{u}(k|k) \in \mathbb{R}^{l\times 1}$ vector) related to at least one active MV will show its effect, i.e. $lb_{H_w}$ represents the first controllable CV prediction instant.*

*Definition 3: In the MISO case, the activation mask $\in \mathbb{N}^{1\times l}$ vector is a row binary vector composed by $l$ elements that allows selecting the $D_{zu}$ columns associated to the active MV/MVs.*

Based on *Definition 2* and on *Definition 3*, $lb_{H_w}$ can be computed as in the following:

$$lb_{H_w} = lb_{H_{w_1}} = \min(D_{zu_{(1,mask)}}) + 1 \tag{7}$$

where min indicates the minimum operation on vectors. Therefore, in (7), the minimum operation is performed on the $D_{zu}$ columns defined by *mask* vector. As in the SISO case, $lb_{H_w}$ can also be determined through $\theta$ matrix. In fact, considering all the $H_p$ rows of $\theta$ matrix (associated to the single CV) and only the first $l$ columns of $\theta$ matrix related to the *active* MV/MVs (associated to the first MVs move, i.e. $\Delta\hat{u}(k|k) \in \mathbb{R}^{l\times 1}$), $lb_{H_w}$ is equal to the index of the first row where there is at least a nonzero entry. Given these considerations, in the MISO case with two or more MVs, if there are at least two different dead times (zero included) in the dead time matrix $D_{zu} \in \mathbb{N}^{1\times l}$, $lb_{H_w}$ may change at different control instants as a consequence of the online variation of the status value associated to the MVs (see Section II-A). On the other hand, if all CV-MV channels are characterized by the same dead time (zero included), $lb_{H_w}$ can be considered as a fixed parameter. According to the authors' knowledge, the proposed definition of the parameter $lb_{H_w}$ and its interpretation in term of the $\theta$ matrix in the MISO case is innovative. In fact, as for the SISO case (see Section III-A), a window parameter lower bound definition and interpretation in term of $\theta$ matrix was never proposed in the literature. Furthermore, in the MISO case, an additional novelty of the proposed definition of $lb_{H_w}$ is that it takes into account the status value associated to the MVs.

The same considerations reported in Section III-A about possible "induced" relaxations caused by an incorrect tuning of the $H_w$ parameter hold. Based on the previous statements, in the MISO case, the information on the dead times can be exploited, through the window parameter lower bound $lb_{H_w}$, to make more effective the tuning procedure associated to the $H_w$ parameter.

In the MISO case with two or more MVs, when at least two different dead times (zero included) are present in the CV-MVs relationships which involve *active* MVs, in addition to the just mentioned $H_w$ parameter tuning procedures, some insights can be inferred from dead times information for enhancing the effectiveness and the efficiency of the tuning procedures and of the associated policies. As it will be shown in the following, the parameters involved in the proposed method are $n_\varepsilon$, $\gamma_{lbz}$, $\gamma_{ubz}$, together with the constraints grouping policies that have to be defined with regard to the slack variables.

In general, considering all single CV-MV pairs, there could be up to $l$ different dead times (zero included). At each control instant $k$, an $f \in \mathbb{N}^{1\times h}$ row vector is defined, named *sorting d-vector*.

*Definition 4: In the MISO case, the sorting d-vector $f \in \mathbb{N}^{1\times h}$ is a row vector that contains the elements of $D_{zu} \in \mathbb{N}^{1\times l}$ associated to the active MVs (see mask in (7)), increased by one, sorted in ascending order, and without duplicates.*

Based on *Definition 4*, $f$ can be expressed as:

$$f = \begin{bmatrix} f_1 & \cdots & f_h \end{bmatrix} \tag{8}$$

In (8), $h$ indicates the number of different dead times that are present in the relationships between the CV and the *active* MVs. $f$ components represent all process delays increased by one. Based on (8) and (7), the following equation holds:

$$lb_{H_w} = f_1 \tag{9}$$

In general, $1 \leq h \leq l$ holds, but let us assume $2 \leq h \leq l$ as stated in the previous. In addition, without loss of generality, assume to choose $H_w = lb_{H_w}$. As described in Section II-D, in the MPC optimization problem (2)-(3), some issues related to constrained control may occur, caused by "induced" constraints relaxations. These relaxations can affect the promptness and the correctness of the overall control action. These issues are connected to some of the CVs constraints parameters in the MPC optimization problem (2)-(3), i.e. $n_\varepsilon$, $\gamma_{lbz}$, $\gamma_{ubz}$, and the constraints grouping policies that have to be defined with regard to the slack variables.

The violation of the CVs constraints has to be discouraged, as reported in Section II-B and in Section II-D, thus the needed priority is given to the minimization of $n_\varepsilon$ slack variables magnitude with respect to the $H_u$ MVs moves magnitude in (2). In this way, values different from zero of the slack variables will be strongly discouraged and consequently the related constraints violation will be discouraged as well. Two different strategies are proposed to guarantee the effectiveness of the control action. The first strategy assumes $n_\varepsilon = 1$, i.e. a single slack variable is considered in the MPC optimization problem formulation (2)-(3). The second strategy assumes $n_\varepsilon > 1$ and the effectiveness of the control action is ensured independently on the value of the positive elements in $\gamma_{lbz}$ and $\gamma_{ubz}$ parameters (under the previous assumption on the ranking between the $n_\varepsilon$ slack variables and the $H_u$ MVs moves in (2)).

The first strategy ($n_\varepsilon = 1$), hereafter denoted as S1, consists of an innovative method for the definition of the minimum number of different parameters that have to be associated to the CV in the tuning of $\gamma_{lbz}$ and $\gamma_{ubz}$. This number is defined equal to $h$, i.e. to the number of different dead times that are present in the relationships between the CV and the *active* MVs. In particular, the following tuning is proposed:

$$\begin{aligned}
&\gamma_{lbz_1}(i) = \alpha_{lbz_{1_1}}, \quad \gamma_{ubz_1}(i) = \alpha_{ubz_{1_1}} \quad i = f_1 \ldots f_2 - 1 \\
&\gamma_{lbz_1}(i) = \alpha_{lbz_{1_2}}, \quad \gamma_{ubz_1}(i) = \alpha_{ubz_{1_2}} \quad i = f_2 \ldots f_3 - 1 \\
&\quad \ldots \\
&\gamma_{lbz_1}(i) = \alpha_{lbz_{1_{h-1}}}, \quad \gamma_{ubz_1}(i) = \alpha_{ubz_{1_{h-1}}} \quad i = f_{h-1} \ldots f_h - 1 \\
&\gamma_{lbz_1}(i) = \alpha_{lbz_{1_h}}, \quad \gamma_{ubz_1}(i) = \alpha_{ubz_{1_h}} \quad i = f_h \ldots H_p
\end{aligned} \tag{10}$$

where

$$\begin{aligned}
&\alpha_{lbz_{1_1}} > \alpha_{lbz_{1_2}} > \ldots > \alpha_{lbz_{1_{h-1}}} > \alpha_{lbz_{1_h}} \\
&\alpha_{ubz_{1_1}} > \alpha_{ubz_{1_2}} > \ldots > \alpha_{ubz_{1_{h-1}}} > \alpha_{ubz_{1_h}}
\end{aligned} \tag{11}$$

Practically, $h$ different parameters for the CV have to be set in the tuning of $\gamma_{lbz}$ and $\gamma_{ubz}$; no associations are inferred between the positive elements of $\gamma_{lbz}$ and the positive elements of $\gamma_{ubz}$: none, some or all of them can be characterized by the same value. CV predictions on which there is the action of the same set of MVs are characterized by the same tuning parameters of $\gamma_{lbz}$ and $\gamma_{ubz}$. The relationships between the different parameters have been reported in (11): the priority of the CV constraints is increased over the prediction horizon $H_p$. Note that the proposed approach does not lack generalization. In fact, it can also be applied to the case with a single dead time (zero included) in the CV-MVs relationships ($h = 1$) and to the case where $H_w > lb_{H_w}$ is assumed.

The just described S1 strategy ($n_\varepsilon = 1$) defines a method that takes into account dead times information and that is adapted online between different control instants based on the *active* MVs, in accordance with *Definitions 3-4* and with (8)-(11). The online adaptation procedure is referred to $lb_{H_w}$ (and, as a consequence, $H_w$), $\gamma_{lbz}$ and $\gamma_{ubz}$ parameters. The proposed procedures can be efficiently implemented in order to save memory and computation time. For example, $\gamma_{lbz}$ and $\gamma_{ubz}$ parameters can be initialized assuming that all MVs are *active* through (10)-(11), thus defining a basic tuning associated to the most comprehensive configuration. At each control instant, $f$ vector (see (8)) is computed. $lb_{H_w}$ parameter is adapted through (9). $\gamma_{lbz}$ and $\gamma_{ubz}$ parameters are adapted with respect to their initial structure excluding $\alpha_{lbz}$ and $\alpha_{ubz}$ terms (see (10)-(11)) conceptually associated to the dead times (see *Definition 4* and (8)) that are no longer present due to *inactive* MV/MVs. Thanks to the fulfillment of the relationships reported in (11), the proposed adaptation law ensures that the priority of the CV constraints is always increased over the prediction horizon $H_p$ thus guaranteeing the "best" control action in all possible MVs status value configurations. A draft sensitivity analysis of the first strategy proposed with respect to $\gamma_{lbz}$ and $\gamma_{ubz}$ parameters will be shown in Section IV-C. According to the authors' knowledge, this first strategy is innovative. In fact, with respect to the strategies ($n_\varepsilon = 1$) associated to the inclusion of CV *soft* constraints reported in different works, e.g., [16], [17], [32], [35], [39], and [46], it can be noticed that the S1 strategy proposed is able to exploit dead times information for the tuning of $\gamma_{lbz}$ and $\gamma_{ubz}$, and, in addition, it takes into account the MVs status value.

In the second strategy ($n_\varepsilon > 1$), hereafter denoted as S2, in order to guarantee the effectiveness of the control action regardless from the value of the positive elements in $\gamma_{lbz}$ and $\gamma_{ubz}$ parameters (under the previous assumption on the ranking between the $n_\varepsilon$ slack variables and the $H_u$ MVs moves in (2)), an innovative structure is proposed for the slack variables vector $\varepsilon(k)$. The proposed structure is reported in the following:

$$\varepsilon(k) = \begin{bmatrix} \varepsilon_1(k) \\ \vdots \\ \varepsilon_h(k) \end{bmatrix} \tag{12}$$

In (12), $h$ different slack variables groups for the CV constraints have been assumed, i.e. the number of slack variables groups matches the number of different dead times that are present in the relationships between the CV and the *active* MVs. In this sense, structural independence is enforced based on dead times information. In each group, there could be one or more slack variables (up to the number of constraints associated to the same group), i.e. each of the terms $\varepsilon_i(k)(i = 1 \ldots h)$ can be a scalar or a vector. In any case, in the proposed structure, a minimum number of $h$ slack variables is imposed in the MPC formulation ($n_\varepsilon \geq h$). When $n_\varepsilon = h$, a number of slack variables equal to the number of different dead times that are present in the relationships between the CV and the *active* MVs is imposed in the MPC optimization problem (2)-(3); in addition, in this configuration, lower and upper CV constraints associated to the same prediction instant $i$ share the same slack variable. In addition, when $n_\varepsilon = h$, $\gamma_{lbz}(i)$ and $\gamma_{ubz}(i)$ row vectors, characterized by a single positive element (the remaining elements are equal to zero), have the following structure:

$$\begin{bmatrix} 0_{1\times(c_i-1)} & \mathrm{P} & 0_{1\times(h-c_i)} \end{bmatrix} \tag{13}$$

where

$$c_i = \max(t : t \in (1 \ldots h) \wedge i \geq f_t) \tag{14}$$

In (13), the P symbol indicates the column of the nonzero value (positive value) in $\gamma_{lbz}(i)$ and $\gamma_{ubz}(i)$ row vectors. Note that the proposed approach does not lack generalization. In fact, on the one hand, it can be applied to the case with a single dead time (zero included) in the CV-MVs relationships ($h = 1$). On the other hand, it covers the case where, for each CV constraint present in the MPC formulation (2)-(3) at the current control instant, its individual slack variable is assigned. In addition, the proposed approach can be applied to the case where $H_w > lb_{H_w}$ is assumed.

Within the S2 strategy, thanks to the proposed structure of the slack variables vector (see (12)), no specific restrictions are imposed for the tuning of $\gamma_{lbz}$. $\gamma_{lbz}$ can be characterized by the same values along the whole prediction horizon $H_p$, i.e. a tuning similar to that proposed in (10)-(11) is not required. The same rationale holds for $\gamma_{ubz}$.

The proposed structure of the slack variables vector associated to the second strategy, that takes into account dead times information and that can change online between different control instants based on the *active* MVs, ensures that the "best" control action is always computed. According to the authors' knowledge, the proposed structure of the slack variables vector is innovative. In fact, the S2 strategy proposed adapts the structure of the slack variables vector based on dead times information and on MVs status value. These features are not present in the cited works which detail the exploited strategy associated to the inclusion of CV *soft* constraints, e.g., [16], [17], [32], [35], [39], and [46].

### C. MIMO CASE

The considered MIMO case assumes that all inputs are not necessarily tied to each single CV (this information can be inferred through the steady state gain matrices reported in Section II-A). In addition, each MV and each CV can be *active* or *inactive* (see Section II-A).

In the following, all concepts and insights reported for the MISO case (see Section III-B) are extended to the MIMO case. As in the previous cases, if DVs are present, their contribution is included in the CV *free response* (see Section II-C).

As a first step, at each control instant, based on the status values of MVs and CVs (see Section II-A), a conditioning between MVs status values and CVs status values must be established. In fact, a MV cannot be *active* if there is not at least an *active* CV tied to that MV. On the other hand, a CV should not be *active* if there is not at least an *active* MV tied to that CV (it would be of no benefit to keep that CV *active*). Exploiting these statements, a suitable status value can be obtained for each MV and CV, also taking into account the impact of CVs status values on MVs status values and vice versa.

In the MIMO case, $H_w$ is a vector ($H_w \in \mathbb{N}^{1 \times m_z}$). The proposed method suggests a lower bound $lb_{H_{w_o}}$ for the tuning of the window parameter $H_{w_o}$ ($H_{w_o} \geq lb_{H_{w_o}}$) associated to each single $o$th *active* CV.

*Definition 5: In the MIMO case, the window parameter lower bound $lb_{H_{w_o}}$ indicates the first CV prediction instant on which the first MVs move (contained in $\Delta\hat{u}(k|k) \in \mathbb{R}^{l \times 1}$ vector) related to at least one active MV tied to the oth active CV will show its effect, i.e. it represents the first controllable prediction instant for the considered CV.*

In addition, for each single CV, an activation *mask* is defined.

*Definition 6: In the MIMO case, the activation $mask_o \in \mathbb{N}^{1 \times l}$ vector is a row binary vector composed by $l$ elements that allows selecting the $D_{zu}$ columns associated to the active MV/MVs that is/are tied to the oth active CV.*

Based on *Definition 5* and on *Definition 6*, $lb_{H_{w_o}}$ can be computed as in the following:

$$lb_{H_{w_o}} = \min(D_{zu_{(o,mask_o)}}) + 1 \quad (15)$$

where min indicates the minimum operation on vectors. Therefore, in (15), the minimum operation is performed on the $D_{zu}$ columns defined by $mask_o$ vector. As in the SISO and MISO cases, $lb_{H_{w_o}}$ can also be determined through $\theta \in \mathbb{R}^{m_z \cdot H_p \times l \cdot H_u}$ matrix. Considering the $H_p$ rows of $\theta$ matrix associated to the $o$th *active* CV and taking into account only the columns associated to the *active* MVs (that are tied to that CV) within the first $l$ columns of $\theta$ matrix (associated to the first MVs move, i.e. $\Delta\hat{u}(k|k) \in \mathbb{R}^{l \times 1}$), $lb_{H_{w_o}}$ is equal to the index of the first row where there is at least a nonzero entry. For each $r$th *inactive* CV, $lb_{H_{w_r}}$ is irrelevant (see the cut off policies cited in Section II-B). Given these considerations, in the MIMO case with a single MV, the lower bound of the window parameter associated to the $o$th *active* CV is a fixed parameter. On the other hand, in the MIMO case with two or more MVs, if there are at least two different dead times (zero included) in the dead time matrix $D_{zu} \in \mathbb{N}^{m_z \times l}$, $lb_{H_{w_o}}$ may change according to the status value of the MVs that can change online (see Section II-A). On the other hand, if all CV-MV channels are characterized by the same dead time (zero included), $lb_{H_{w_o}}$ can be considered as a fixed parameter for the considered CV. According to the authors' knowledge, the proposed definition of the parameter $lb_{H_{w_o}}$ and its interpretation in term of the $\theta$ matrix in the MIMO case is innovative. In fact, as for the SISO and MISO case (see Section III-A and Section III-B), a window parameter lower bound definition and interpretation in term of $\theta$ matrix was never proposed in the literature. Furthermore, in the MIMO case, an additional novelty provided by the proposed definition is that the $lb_{H_{w_o}}$ computation takes into account the status value associated to each CV and MV.

With regard to the MIMO case, the same considerations reported in Section III-A and in Section III-B about possible "induced" relaxations caused by an incorrect tuning of the window parameters hold. Based on the previous statement, it is also possible in the MIMO case to exploit the dead time information through the window parameter lower bound $lb_{H_{w_o}}$, to make the tuning procedure associated to the window parameters more effective. Furthermore, the generalization insights mentioned in Section III-B can also be extended to the MIMO case.

As in the MISO case (see Section III-B), when a single MV is present, no further insights on the tuning procedures can be obtained from the dead times information. Conversely, if two or more MVs are present, from the information on dead times, when at least two different dead times (zero included) are present in the relationships between (at least) an *active* CV and the *active* MVs tied to that CV, some insights can be inferred to improve the effectiveness and efficiency of the tuning procedures and associated policies. As it will be shown in the following, the parameters involved in the proposed method are $n_\varepsilon$, $\gamma_{lbz}$, $\gamma_{ubz}$, together with the constraints grouping policies that have to be defined with regard to the slack variables.

In general, considering all single CV-MV pairs associated to the generic $o$th CV, there could be up to $l$ different dead times (zero included). At each control instant $k$, a $f_o \in \mathbb{N}^{1\times h_o}$ row vector can be obtained for each *active* CV, named *sorting d-vector*.

*Definition 7: In the MIMO case, the sorting d-vector* $f_o \in \mathbb{N}^{1\times h_o}$ *is a row vector that contains the elements of* $D_{zu}(o,:) \in \mathbb{N}^{1\times l}$ *(i.e. the row of* $D_{zu} \in \mathbb{N}^{m_z\times l}$ *associated to the considered oth active CV) associated to the active MV/MVs that is/are tied to the considered CV (see* $mask_o$ *in (15)), increased by one, sorted in ascending order, and without duplicates.*

Based on *Definition 7*, $f_o$ can be expressed as:

$$f_o = \begin{bmatrix} f_{o_1} & \dots & f_{o_{h_o}} \end{bmatrix} \tag{16}$$

Because each *active* CV is characterized by its own dead times in its relationships with the *active* MVs that are tied to that CV, the dimension of the vector reported in (16) could not be the same for all *active* CVs. Based on (16), equation (9) can be straightly extended to all *active* CVs.

As in the MISO case, two different strategies are proposed to guarantee the effectiveness of the control action. Without loss of generality, assume to choose $H_{w_o} = lb_{H_{w_o}}$ for each $o$th *active* CV. The first strategy assumes $n_\varepsilon = 1$, i.e. a single slack variable is considered in the MPC optimization problem formulation (2)-(3). The second strategy assumes $n_\varepsilon > 1$.

The first strategy ($n_\varepsilon = 1$) straightly extends to the MIMO case the first strategy described for the MISO case (see Section III-B) and it is not detailed for brevity.

The second strategy ($n_\varepsilon > 1$) exploits the insights provided by the authors in [52], where a structural independence is imposed between the CVs with regard to constraints relaxations, i.e. each CV is equipped with an own set of slack variables. In order to guarantee the effectiveness of the control action, an innovative structure is proposed for the slack variables vector $\varepsilon(k)$. The proposed structure is reported in the following (assuming that all CVs are *active*):

$$\varepsilon(k) = \begin{bmatrix} \varepsilon_1(k) \\ \vdots \\ \varepsilon_{m_z}(k) \end{bmatrix}$$
$$\varepsilon_o(k) = \begin{bmatrix} \varepsilon_{o_1}(k) \\ \vdots \\ \varepsilon_{o_{h_o}}(k) \end{bmatrix} \quad o = 1 \dots m_z \tag{17}$$

In (17), each *active* CV is structurally independent from other CVs with regard to constraints relaxations, i.e. each CV is equipped by an own set of slack variables. In addition, $h_o$ slack variables groups for each *active* CV have been assumed, i.e. the number of slack variables groups matches the number of different dead times that are present in the relationships between the considered CV and the *active* MVs tied to that CV. In this sense, structural independence is enforced based on dead times information.

In (17), $\varepsilon_o(k)$ represents the slack variables subset for the *active* CV. In any case, in the proposed structure, a minimum number of $h_o$ slack variables for each *active* CV is imposed in the MPC formulation; for this reason, when all CVs are *active*, $n_\varepsilon \geq \sum_{o=1}^{m_z} h_o$ holds. With regard to the *active* CVs, the introduction of the slack variables in the constraints (3.c) at the $i$th prediction instant is performed through $\gamma_{lbz}(i)$ and $\gamma_{ubz}(i)$ matrices. If $n_\varepsilon = \sum_{o=1}^{m_z} h_o$, each CV is characterized by a number of slack variables equal to the number of different dead times of its relationships with the *active* MVs tied to it; in addition, in this configuration, lower and upper CV constraints associated to the same prediction instant share the same slack variable. In this case, considering a generic $o$th row vector of $\gamma_{lbz}(i)$ and $\gamma_{ubz}(i)$ matrices, characterized by only one positive element (the remaining elements are equal to zero), it has the following structure:

$$\begin{bmatrix} 0_{1\times(\sum_{n=1}^{o-1} h_n + c_{o_i} - 1)} & \mathrm{P} & 0_{1\times(\sum_{n=o}^{m_z} h_n - c_{o_i})} \end{bmatrix} \tag{18}$$

where

$$c_{o_i} = \max(t : t \in (1 \dots h_o) \wedge i \geq f_{o_t}) \tag{19}$$

In (18), the P symbol indicates the column of the nonzero value (positive value) in the $o$th row vector of the $\gamma_{lbz}(i)$ and $\gamma_{ubz}(i)$ matrices. $0_{1\times 0}$ represents an empty vector. As described in Section II-B, in (2) the introduction of the slack variables vector takes place through the $\rho$ positive definite diagonal matrix. Based on the previous considerations, this matrix is characterized by the following structure:

$$\begin{aligned} &\rho = blkdiag(\rho_1, \rho_2, \dots, \rho_{m_z}) \in \mathbb{R}^{\sum_{o=1}^{m_z} h_o \times \sum_{o=1}^{m_z} h_o} \\ &\rho_o = blkdiag(\rho_{o_1}, \rho_{o_2}, \dots, \rho_{o_{h_o}}) \in \mathbb{R}^{h_o\times h_o} \\ &o = 1 \dots m_z \end{aligned} \tag{20}$$

where $blkdiag(a, b)$ represents the diagonal composition of the matrices $a$ and $b$. In (20), $\rho_o$ is a positive definite diagonal matrix composed by positive scalars (remember that, as an example, we are considering $n_\varepsilon = \sum_{o=1}^{m_z} h_o$).

As in the MISO case (see Section III-B), within the second strategy, thanks to the proposed structure of the slack variables vector, both $\gamma_{lbz}$ and $\gamma_{ubz}$ can be characterized by the same values along the prediction horizon $H_p$. Furthermore, the generalization insights mentioned in Section III-B can be extended also to the MIMO case.

Based on the *active* CVs and on the *active* MVs at the considered control instant, the dimension of the slack variables vector can change dynamically (see (17)). The possible dynamic adaptation of the dimension of the slack variables vector determines an adaptation of $\gamma_{lbz}(i)$, $\gamma_{ubz}(i)$ and $\rho$ matrices in accordance with (18)-(20). The proposed structure of the slack variables vector associated to the second strategy, that takes into account dead times information, ensures that the "best" (in terms of constraints relaxation) control action is always computed. According to the authors' knowledge, the proposed structure of the slack variables vector is innovative. In fact, as for the MISO case (see Section III-B), a strategy that takes into account dead times information and the status value of each MV for the definition of the structure of the slack variables vector was never proposed in the literature. Furthermore, in the MIMO case, an additional novelty is provided, i.e. the proposed structure of the slack variables vector takes into account the status value associated to each CV.

## IV. RESULTS AND DISCUSSION

In this section, simulation results and discussion are reported. Different scenarios are considered and a comparison between different strategies associated to the tackled problem is proposed. Model mismatch and previewing (or look-ahead) features are not considered in the simulations because they do not represent crucial elements for the scope of the paper. After a description of the considered process, of the constraints and of tuning procedures, a first scenario is simulated in Section IV-A, considering CV constraints change and focusing on $H_w$ parameter. A second scenario is simulated in Section IV-B, where a CV constraints change is proposed focusing on tuning procedures and grouping policies associated to CV *soft* constraints. Section IV-C reports a third scenario characterized by DV changes and the focus is on tuning procedures and grouping policies.

A MISO process modellized through FOPDT models is considered for the MPC optimization problem (2)-(3). The process is characterized by $m_z = 1$, $l = 2$, $l_d = 1$: there are one CV (denoted by CV1), two MVs (denoted by MV1 and MV2) and one DV (denoted by DV1). The involved variables are assumed to be characterized by similar spans thus scaling is not performed; in addition, CV, MVs and DV measurement units are not specified. In the following, the transfer functions in Laplace domain are reported:

$$\begin{aligned} M_{CVMV1}(s) &= \frac{1}{300s+1}e^{-600s} \\ M_{CVMV2}(s) &= \frac{0.75}{600s+1}e^{-240s} \\ M_{CVDV1}(s) &= \frac{0.25}{720s+1} \end{aligned} \tag{21}$$

In (21), the measurement unit of each time constant and of each dead time is seconds; in addition, a sample time $T_s$ equal to 60 seconds (1 minute) is assumed for the discretization procedure (see Section II-A). This sample time is considered also for the simulation of the different scenarios. Note that each time constant and each dead time is a multiple of the sample time.

As mentioned in the previous, different scenarios are considered. Table 2 reports the MVs *hard* constraints associated to the MVs magnitude and to the MVs slew rate. These constraints hold in all scenarios.

Different strategies associated with the tackled problem are compared. In particular, the S1 strategy and the S2 strategy proposed in Section III-B will be considered together with three alternative strategies (denoted by AS1, AS2, AS3). Some tuning parameters are assumed identical for all strategies (see Table 3), while other parameters (see Table 4 and Table 5) can change according to the considered strategy.

In Table 3, $H_p$, $H_u$, $M_i$ ($i = 1 \ldots H_u$), $\rho$ and $R(i)$ ($i = 1 \ldots H_u$) parameters are reported. With regard to $H_p$, $H_u$ and $M_i$ ($i = 1 \ldots H_u$), a tuning method proposed in [22] for processes modellized through FOPDT models is exploited; in particular, $M_i = i - 1$ ($i = 1 \ldots H_u$) while $H_p$ and $H_u$ are given by:

$$\begin{aligned} H_p &= \max\left(5\frac{T_{c,v}}{T_s} + \frac{T_{d_{c,v}}}{T_s} + 1\right) \quad c = 1 \ldots m_z, \\ &\quad v = 1 \ldots (l + l_d) \\ H_u &= \max\left(\frac{T_{c,v}}{T_s} + \frac{T_{d_{c,v}}}{T_s} + 1\right) \quad c = 1 \ldots m_z, \\ &\quad v = 1 \ldots (l + l_d) \end{aligned} \tag{22}$$

where, for each input-output channel (specified through $c$ and $v$ indexes), $T_{c,v}$ is the time constant and $T_{d_{c,v}}$ is the dead time (or delay). With regard to $\rho$ and $R(i)$ ($i = 1 \ldots H_u$) parameters (see Table 3), they have been jointly tuned in order to ensure that greater importance is given to the minimization of the $n_\varepsilon$ slack variables with respect to the $H_u$ MVs moves [1], [4].

$H_w$, $n_\varepsilon$, $\gamma_{lbz}$ and $\gamma_{ubz}$ parameters are reported in Table 4 and Table 5.

With regard to S2 strategy, it is worth remembering that CV1 upper and lower constraints referred to the same prediction instant share the same slack variable. In addition, a number of slack variables equal to $h$ (number of different dead times that are present in the relationships between the CV and the *active* MVs) is imposed in the MPC optimization problem (2)-(3). For this reason, based on the *active* MVs, $1 \leq n_\varepsilon \leq 2$ holds. In both S1 and S2 strategies (see Table 4), assuming that MV1 and MV2 are both *active*, the *sorting d-vector f* is:

$$f = [f_1 f_2] = [5 11] \tag{23}$$

From (23), considering (7) and (9), it follows that $lb_{H_w} = f_1 = 5$.

In case just one MV is *active*, the *sorting d-vector f* has the only component $f_1$; if MV1 is the *active* MV, its value (and $lb_{H_w}$ value) is equal to 11, while, if MV2 is the *active* MV, its value is equal to 5.

The tuning when adopting the S2 strategy assumes that the positive elements of $\gamma_{lbz}$ and $\gamma_{ubz}$ are identical (equal to 1 in the proposed simulations), while S1 strategy prescribes specific procedures as suggested in Section III-B. In particular, 1.25 and 1 values are initially set in the proposed simulations (see Table 4); successively, within S1 strategy, 1.25 value will be replaced by other values in order to provide further comparisons.

Taking into account the considerations and comparisons proposed in Section II-D and in Section III with the current state of the art, in addition to S1 and S2, three alternative strategies (AS1, AS2, AS3) are considered so to compare the performances of the proposed control solutions with existing ones.

All the three proposed alternative strategies (AS1, AS2, AS3) assume a single slack variable in the MPC optimization

**TABLE 2.** MVs *hard* constraints.

| Parameter | Value |
|---|---|
| $lb_{du_j}(i)$ $(j = 1 \dots l,\ i = 1 \dots H_u)$ | −2 |
| $ub_{du_j}(i)$ $(j = 1 \dots l,\ i = 1 \dots H_u)$ | +2 |
| $lb_{u_j}(i)$ $(j = 1 \dots l,\ i = 1 \dots H_u)$ | 0 |
| $ub_{u_j}(i)$ $(j = 1 \dots l,\ i = 1 \dots H_u)$ | 20 |

**TABLE 3.** Tuning parameters identical for all strategies.

| Parameter | Value |
|---|---|
| $H_p$ | 61 |
| $H_u$ | 16 |
| $M_i$ $(i = 1 \dots H_u)$ | $i - 1$ |
| diagonal elements of $\rho$ | 100 |
| diagonal elements of $R(i)$ $(i = 1 \dots H_u)$ | 1 |

**TABLE 4.** Tuning parameters based on the selected strategy (I).

| Parameter | S1 | S2 | AS1 |
|---|---|---|---|
| $H_w$ | $lb_{H_w}$* | $lb_{H_w}$* | $lb_{H_w}$* |
| $n_\varepsilon$ | 1 | * | 1 |
| positive elements of $\gamma_{lbz}, \gamma_{ubz}$ | $\alpha_{lbz_{1_1}} = \alpha_{ubz_{1_1}} = 1.25$<br>$\alpha_{lbz_{1_2}} = \alpha_{ubz_{1_2}} = 1$<br>** | 1 | 1 |

*$lb_{H_w}$ computation is based on *active* MV/MVs (see Section III.B).
**These values are used if MV1 and MV2 are *active*; otherwise, the positive elements of $\gamma_{lbz}, \gamma_{ubz}$ are equal to 1 (see (10)-(11)).

**TABLE 5.** Tuning parameters based on the selected strategy (II).

| Parameter | AS2 | AS3 |
|---|---|---|
| $H_w$ | 5 | 5 |
| $n_\varepsilon$ | 1 | 1 |
| positive elements of $\gamma_{lbz}, \gamma_{ubz}$ | $\alpha_{lbz_{1_1}} = \alpha_{ubz_{1_1}} = 1.25$<br>$\alpha_{lbz_{1_2}} = \alpha_{ubz_{1_2}} = 1$ | 1 |

problem formulation (see Table 4 and Table 5). In addition, AS1 and AS3 assume the same value for the positive elements of $\gamma_{lbz}$ and $\gamma_{ubz}$ while AS2 assumes the same tuning rule associated to S1 when MV1 and MV2 are *active*. With regard to $H_w$, AS2 and AS3 assume that $H_w$ is a fixed parameter (equal to 5, see (23)), while AS1 assumes the same $H_w$ tuning rule associated to S1 and S2.

AS3 represents an example of strategy ($n_\varepsilon = 1$), associated to the inclusion of CV *soft* constraints, as reported in different works, e.g., [16], [17], [32], [35], [39], and [46]. In order to achieve a thorough comparison, also enhanced versions of AS3 (i.e. AS2 and AS1) are considered which include some of the innovative features proposed in the present paper, i.e. the tuning rule associated to the positive elements of $\gamma_{lbz}$ and $\gamma_{ubz}$ (for AS2) or to $H_w$ (for AS1).

In order to quantify the goodness of the performance associated to each strategy, qualitative and quantitative comments will be provided. From a quantitative point of view, a tailored performance indicator $t_{restore}$ is proposed.

*Definition 8: Let assume that an event (e.g., a constraint change or a DV1 change) causes CV1 to fail to meet its lower (upper) constraint at the simulation instant $n_{out}$. Let assume that CV1 is restored within the violated lower (upper) constraint at the simulation instant $n_{restore}$. A performance indicator $t_{restore}$ is defined as:*

$$t_{restore} = \begin{cases} n_{restore} - n_{out}, & \text{if } \exists n_{restore} \\ \infty, & \text{otherwise} \end{cases} \tag{24}$$

The proposed performance indicator $t_{restore}$ quantifies the promptness of the considered controller to restore CV1 within the previously violated constraint. A tailored performance indicator is proposed because other types of indicators, e.g., Integral of Absolute Error (IAE), are not suitable for the considered zone control problem since they are referred to tracking control problems.

### A. FIRST SCENARIO: CV CONSTRAINTS CHANGE AND FOCUS ON $H_W$ PARAMETER

The first proposed scenario aims to prove the importance of MVs status value for an effective computation of the $H_w$ parameter.

In the initial configuration, CV1, MV1 and MV2 are assumed *active*; online modifications of CV1 constraints are proposed in the current scenario. In addition, DV1 remains constant at its initial value thus not influencing the simulation. MV1 and MV2 constraints are reported in Table 2 (see in the previous).

Fig. 2 shows the behavior of CV1 when the different strategies are applied, while Fig. 3-4 are associated to MV1 and MV2, respectively. The constraints are depicted through red lines, while the process variables (CV1, MV1, MV2) are depicted through blue (S1), green (S2), black (AS1), cyan (AS2) and magenta (AS3) lines.

In the first 29 simulation instants, CV1 fulfills the assigned lower-upper constraints (0–6, see Fig. 2) thus no moves are computed for MV1 (see Fig. 3) and MV2 (see Fig. 4): all the considered strategies provide the same results with regard to the process variables, i.e. blue, green, black, cyan and magenta lines overlap in Fig. 2-4.

At simulation instant 30, CV1 lower and upper constraints are changed to 2 and 8, respectively (see red lines in Fig. 2). The constraints change considered (not a priori known by

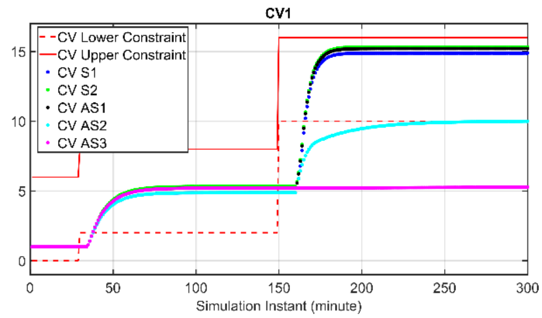


**FIGURE 2.** First scenario: CV1.

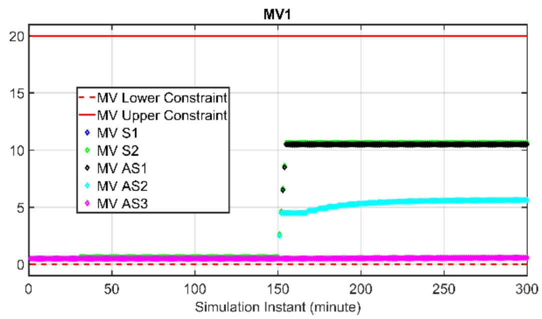


**FIGURE 3.** First scenario: MV1.

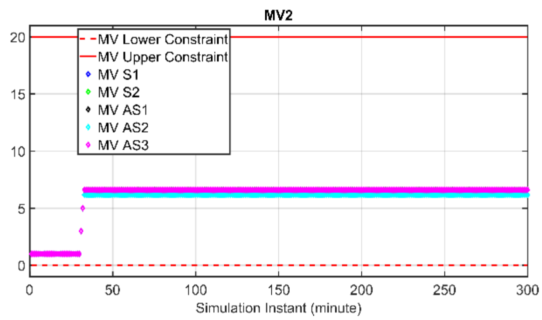


**FIGURE 4.** First scenario: MV2.

the control system) is feasible at steady state (see (21) and Fig. 2-4). Being both MV1 and MV2 *active*, $H_w = 5$ for all strategies. For this reason, all strategies compute to move MV2 (see Fig. 4); in addition, all strategies except S2 decide to not move MV1 (see Fig. 3). S2 strategy decides to move both MV1 and MV2 due to the higher flexibility given by $n_\varepsilon = 2$ and to the proposed constraints grouping policies: the first slack variable ($\varepsilon_1$, see in the previous) is associated to CV1 upper and lower constraints related to prediction instants 5–10 while the second slack variable ($\varepsilon_2$, see in the previous) is associated to CV1 upper and lower constraints related to the remaining prediction instants 11–61. In this part of the simulation, blue (S1) and cyan (AS2) lines overlap in Fig. 2 and Fig. 4; in addition, black (AS1) and magenta (AS3) lines overlap in Fig. 2 and Fig. 4. The identical behavior of different strategies is motivated by the tuning parameters definition reported in Table 3-5. Practically, since $H_w = 5$, S1 and AS2 strategies are equivalent in this part of the simulation; furthermore, for the same motivation, AS1 and AS3 strategies are equivalent in this part of the simulation.

At simulation instant 150, another modification of CV1 lower and upper constraints is performed: they are changed to 10 and 16, respectively (see red lines in Fig. 2); in addition, MV2 status is changed from *active* to *inactive*; MV2 is kept constant at its last from here onward. The considered constraints change (not a priori known by the control system) is feasible at steady state (see (21) and Fig. 2-3). Due to the fact that only MV1 is *active* from here onward, S1, S2 and AS1 strategies (see Table 4) compute $H_w$ value equal to 11. In addition, in this part of the simulation, $n_\varepsilon$ is mandatorily equal to 1 for all strategies. Furthermore, $H_w = 5$ for AS2 and AS3 strategies (see Table 5).

As can be observed in Fig. 2 (see magenta line), considering the typical constraints handling performed by AS3 strategy, common of many research works, e.g., [16], [17], [32], [35], [39], and [46], a satisfactory control action is not ensured. AS3 strategy does not perform any control action (no moves on MV1, see magenta line in Fig. 3) because there is an "induced" constraints relaxation that takes place at the not controllable prediction instants (from 5th to 10th prediction instant). This behavior can be explained by the lack of adaptation with regard to $H_w$ based on the MVs status value. This "induced" constraints relaxation occurs also for AS2 strategy (see cyan lines in Fig. 2 and Fig. 3) but it is mitigated by the fact that the constraints at the not controllable prediction instants (from 5th to 10th prediction instant) are considered with a lower priority with respect to the constraints that take place at the subsequent prediction instants (see Table 5). This behavior is due to the different tuning of $\gamma_{lbz}$ and $\gamma_{ubz}$ parameters with respect to AS3 (see Table 5). In this case, moves are performed also on MV1 (see cyan line in Fig. 3) but they are not enough to ensure a satisfactory CV1 behavior (see cyan line in Fig. 2).

The effectiveness of the proposed method for the adaptation of $H_w$ parameter based on the MVs status value can be observed for S1, S2 and AS1 strategies, which all ensure a satisfactory control action. In particular, the same behavior is computed for MV1 moves (blue and black lines overlap in Fig. 3) thus ensuring that CV1 satisfies the imposed constraints (see blue, green and black lines in Fig. 2). Note that S1, S2 and AS1 strategies are characterized by the same tuning parameters from simulation instant 150 onward because $H_w = 11$ and, in addition, also for S2 strategy $n_\varepsilon = 1$ holds since only MV1 is *active* (see Table 4).

**TABLE 6.** Performance indicator for each strategy (first scenario).

| Performance Indicator (CV1 lower constraint) | S1 | S2 | AS1 | AS2 | AS3 |
|---|---|---|---|---|---|
| $t_{restore}$ (first change) | 8 | 8 | 8 | 8 | 8 |
| $t_{restore}$ (second change) | 16 | 16 | 16 | 100 | $\infty$ |

The previous considerations are proven also from a quantitative point of view through the performance indicator $t_{restore}$ (see (24) and Table 6). With regard to the first violation of the CV1 lower constraint (simulation instant 30, see Fig. 2), all strategies are able to restore CV1 within the updated lower constraint in 8 simulation instants ($t_{restore} = 8$, see Table 6). With regard to the second violation of the CV1 lower constraint (simulation instant 150, see Fig. 2), S1, S2 and AS1 strategies are able to restore CV1 within the updated lower constraint in 16 simulation instants ($t_{restore} = 16$, see Table 6), while AS2 strategy requires a higher time to restore CV1 ($t_{restore} = 100$). Finally, AS3 strategy ($t_{restore} = \infty$) is not successfully in the recovery of CV1. In the considered scenario, S1, S2 and AS1 strategies ensure a 84% decrease of the indicator with respect to AS2 strategy.

Due to the issues proven for AS2 and AS3 in this context, only S1, S2 and AS1 strategies will be considered for the next scenarios.

### B. SECOND SCENARIO: CV CONSTRAINTS CHANGE AND FOCUS ON TUNING PROCEDURES AND GROUPING POLICIES ASSOCIATED TO CV SOFT CONSTRAINTS

The second scenario that is simulated proposes an online modification of CV1 constraints, highlighting the differences between S1, S2 and AS1 strategies when an MV is saturated on a constraint.

In the present scenario, CV1, MV1 and MV2 are assumed *active*; being all MVs *active*, $H_w = 5$ for all strategies. As in the previous scenario, DV1 remains constant at its initial value thus not influencing the simulation. MV1 and MV2 constraints are reported in Table 2 (see in the previous).

Fig. 5 shows the behavior of CV1 when the different strategies are applied, while Fig. 6-7 are associated to MV1 and MV2, respectively. The constraints are depicted through red lines, while the process variables (CV1, MV1, MV2) are depicted through blue (S1), green (S2) and black (AS1) lines.

In the first 49 simulation instants, CV1 fulfills the assigned lower and upper constraints (0 and 6 respectively, see Fig. 5) thus no control actions are performed (no moves for MV1 and MV2, see Fig. 6 and Fig. 7): all the considered strategies provide the same results with regard to the process variables, i.e. blue, green and black lines overlap in Fig. 5-7.

At simulation instant 50, CV1 lower and upper constraints are changed to 4 and 10 respectively (see red lines in Fig. 5). The constraints change considered (not a priori known by the control system) is feasible at steady state (see (21) and Fig. 5-7). In general, at each control instant $k$, given the presence of time delays on CV1-MVs channels, the control system can ensure a response of CV1 from the 5th prediction instant onward ($H_w = 5$ for all strategies). In the proposed plant condition, in order to satisfy the request, the controller can only increase MV1, because an increase of MV2 is not admitted (MV2 is saturated at its upper constraint; see Fig. 7 where blue, green and black lines overlap). For this reason, the control system can ensure a response of CV1 from 11th prediction instant onward (remember that the delay sample instants on CV1-MV1 channel are 10, as previously stated).

In the AS1 strategy (see black lines in Fig. 5-7), a single slack variable is included in the MPC optimization problem formulation (see Table 4). For this reason, this slack variable is shared by all CV1 constraints on the time window $(k + 5) - (k + 61)$. A softening of the updated CV1 lower constraint is observed and, consequently, the controller does not perform control actions on MV1 (see Fig. 6, black line) to restore CV1 within the required constraint and a failure of the overall control action is observed (see Fig. 5, black line). This softening is induced on the constraints of 11th-61th prediction instants by the constraints of 5th-10th prediction instants that are not controllable because MV2 lies on its upper constraint (see Fig. 7).

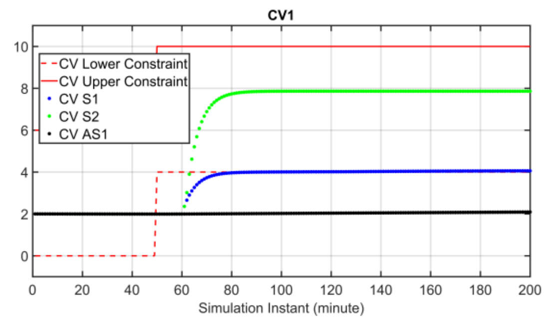


**FIGURE 5. Second scenario: CV1.**

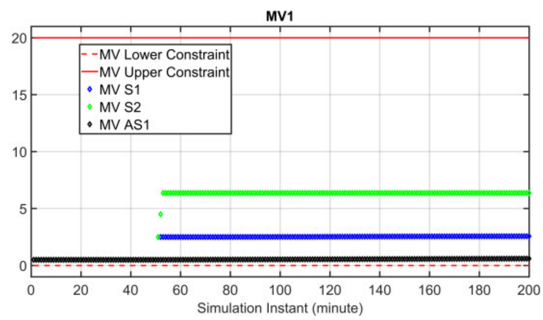


**FIGURE 6. Second scenario: MV1.**

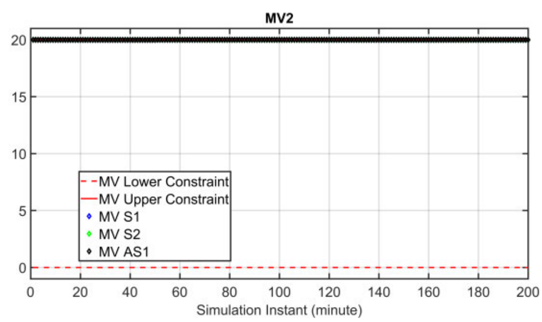


**FIGURE 7. Second scenario: MV2.**

S1 strategy, thanks to the proposed tuning procedure for $\gamma_{lbz}$ and $\gamma_{ubz}$ parameters (see Table 4) which takes into account dead times information, guarantees that CV1 is restored within the updated constraints (see Fig. 5, blue line). The restore of CV1 within the updated constraints is obtained through MV1 moves (see Fig. 6, blue line). The proposed S1 strategy is characterized by the possibility to set a more effective tuning of $\gamma_{lbz}$ and $\gamma_{ubz}$ parameters with respect to AS1 strategy. In fact, because a lower priority can be assigned to the constraints associated to 5th-10th prediction instants (see Table 4), the impact of their softening on the remaining constraints is mitigated.

S2 strategy involves two slack variables in the MPC optimization problem formulation because both MV1 and MV2 are *active*. These two slack variables are responsible for all CV1 constraints on the time window $(k + 5) - (k + 61)$. In particular, the first slack variable is responsible for constraints at 5th-10th prediction instants, while the second slack variable is responsible for constraints at 11th-61th prediction instants. Therefore, a structural independence between constraints related to 5th-10th prediction instants and the ones of 11th-61th prediction instants is imposed. Given the possibility to impose this structural independence when adopting the S2 strategy, the softening induction between the two constraints sets when CV1 constraints are changed does not take place. On the contrary, at the control instant related to the constraints change, the same relaxation of the single slack variable related to the AS1 strategy is performed by $\varepsilon_1$ slack variable on the constraints of 5th-10th prediction instants. When adopting S2 strategy, thanks to the action performed by $\varepsilon_2$ slack variable on the constraints related to 11th-61th prediction instants, the control system acts on MV1 (Fig. 6, green line) thus performing a gradual zeroing of the relaxation on the updated CV1 constraints and meeting the newly set constraints (Fig. 5, green line).

The previous considerations are proven also from a quantitative point of view through the performance indicator $t_{restore}$ (see (24) and Table 7). As shown in Fig. 5, the more effective strategy is S2 while AS1 strategy is not able to ensure a satisfactory control action (see also Table 7). The different behavior of S1 with respect to S2 is due to the possibility of tuning of the positive elements of $\gamma_{lbz}$ and $\gamma_{ubz}$ parameters in S1 strategy (see Table 4). In this context, as previously explained, the priority assigned to the CV1 constraints associated to 5th-10th prediction instants covers a key role; in this case, $\alpha_{lbz_{1_1}} = \alpha_{ubz_{1_1}} = 1.25$ (see Table 4). At this regard, a draft sensitivity analysis of S1 strategy with respect to $\gamma_{lbz}$ and $\gamma_{ubz}$ parameters will be shown in Section IV-C.

**TABLE 7. Performance indicator for each strategy (second scenario).**

| Performance Indicator (CV1 lower constraint) | S1 | S2 | AS1 |
|---|---|---|---|
| $t_{restore}$ | 43 | 14 | $\infty$ |

### C. THIRD SCENARIO: DV REJECTION AND FOCUS ON TUNING PROCEDURES AND GROUPING POLICIES

In the third simulated scenario, the occurrence of an online change on DV1 is considered. The scope of this simulation is to highlight the differences between S1, S2 and AS1 strategies when an MV lies on a constraint. In addition, different tuning of $\gamma_{lbz}$ and $\gamma_{ubz}$ parameters for S1 strategy are compared.

In this scenario, CV1, MV1 and MV2 are assumed *active*; thus, being all MVs *active*, $H_w = 5$ for all strategies. MV1 and MV2 constraints are reported in Table 2 (see in the previous), while CV1 lower and upper constraints are 15 and 20, respectively.

Fig. 7 depicts DV1, Fig. 9 and Fig. 12 are associated to CV1, while Fig. 10-11 and Fig. 13-14 are associated to MV1 and MV2.

In the first 49 simulation instants, CV1 fulfills the assigned lower-upper constraints (15–20, see Fig. 9 and Fig. 12) thus no moves are computed for MV1 (see Fig. 10 and Fig. 13) and MV2 (see Fig. 11 and Fig. 14): all the considered strategies provide the same results with regard to the process variables.

At simulation instant 50 (see Fig. 7), the occurrence of an abrupt DV1 increase (not a priori known by the control system) is simulated: DV1 is increased from 0 to 15. This variation on DV1 causes CV1 to increase (see (21)), and, starting at a certain instant, CV1's failure to meet its upper constraint (see Fig. 9 and Fig. 12).

The MV1 change required to counteract the effects of the occurred DV1 variation is feasible at steady state (see (21), Fig. 9-10 and Fig. 12-13): with suitable MV1 moves, CV1 can be restored within its constraints. In general, at each control instant $k$, given the presence of time delays on CV1-MVs channels, the control system can affect the response of CV1 from the 5th prediction instant onward. In the proposed plant condition, in order to satisfy the request, the controller can only act on MV1, decreasing it, because a decrease of MV2 is not admitted (MV2 lies on its lower constraint; see Fig. 11 and Fig. 14). For this reason, the control system can affect the response of CV1 only from 11th prediction instant onward (remember that the delay sample instants on CV1-MV1 channel are 10). A more prompt response would have been observed if MV2 could be exploited given the lower dead time associated to it.

In the AS1 strategy (see Fig. 9-10, black lines), where a single slack variable is responsible for all CV1 constraints on the time window $(k + 5) - (k + 61)$, a softening of the CV1 upper constraint is performed (see Fig. 9, black line). This softening is caused by the effect on CV1 of the occurred DV1 increase. Recalling that $H_w = 5$ and that MV2 could not be lowered, being already on its lower constraint (see Fig. 11), this softening is induced on the constraints at 11th-61th prediction instants by the constraints at 5th-10th prediction instants since no change can be forced on CV1 on that interval. In this case, the side effect of the "induced" constraints relaxation is that the expected moves on MV1 are no longer performed (see Fig. 10, black line) because they are no longer required given the relaxed constraints. Consequently, DV1 variation cannot be compensated and CV1 is not restored within its constraints: a failure of the overall control action (see Fig. 9, black line) takes place.

In the S2 strategy (see Fig. 9-10, green lines), two slack variables are involved in the MPC optimization problem formulation because both MV1 and MV2 are *active*. These two slack variables are disjointly responsible for all CV1 constraints on the time window $(k+5)-(k+61)$. In particular, the first slack variable is responsible for constraints at 5th-10th prediction instants, while the second slack variable is responsible for constraints at 11th-61th prediction instants. Therefore, a structural independence between constraints at 5th-10th prediction instants and the ones at 11th-61th prediction instants is imposed. Given this structural independence, when adopting the S2 strategy, no softening "induction" between the two constraints sets takes places when CV1 is dragged out from its constraints as a consequence of DV1 change. At the control instant related to the DV1 variation, the same relaxation of the single slack variable related to the AS1 strategy is performed by $\varepsilon_1$ slack variable on the upper constraints of 5th-10th prediction instants. However, adopting the S2 strategy, the control system, acting on MV1, is able to gradually zeroing the relaxation on the CV1 upper constraint and restoring CV1 within its defined constraints (Fig. 10, green line). In fact, the action performed by $\varepsilon_2$ slack variable on the constraints related to 11th-61th prediction instants is disjoint from that of $\varepsilon_1$ slack variable on the previous prediction instants.

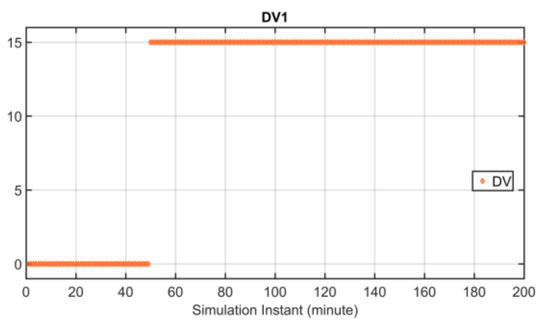


**FIGURE 8.** Third scenario: DV1.

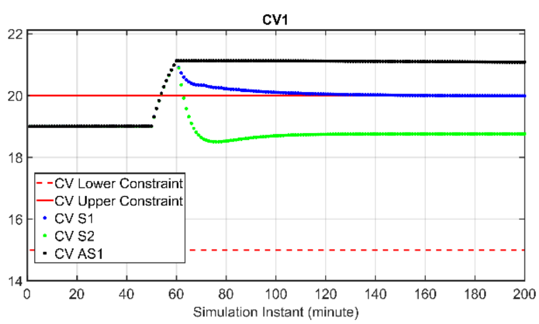


**FIGURE 9.** Third scenario: CV1.

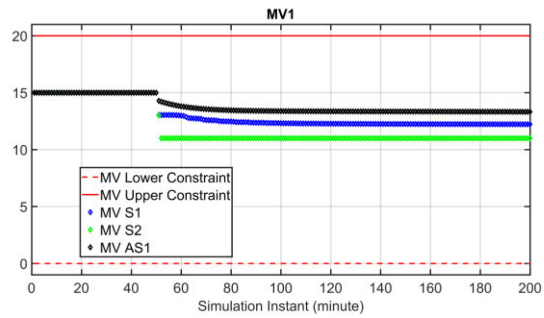


**FIGURE 10.** Third scenario: MV1.

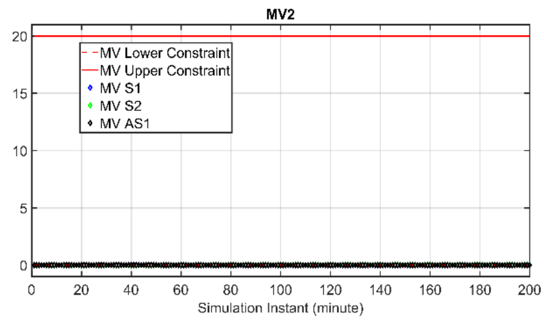


**FIGURE 11.** Third scenario: MV2.

**TABLE 8.** Performance indicator for each strategy (third scenario).

| Performance Indicator (CV1 upper constraint) | S1 | S2 | AS1 |
|---|---|---|---|
| $t_{restore}$ | 165 | 9 | $\infty$ |

From Fig. 9 (blue line), it can be observed that the application of the S1 strategy also succeeds in restoring CV1 within the defined constraints. This is thanks to the proposed tuning procedure for $\gamma_{lbz}$ and $\gamma_{ubz}$ parameters (see Table 4) which takes into account dead times information, as already discussed in the previous. The restore of CV1 within the defined constraints is obtained through MV1 moves (see Fig. 10, blue line). The proposed S1 strategy is characterized by a more effective tuning of $\gamma_{lbz}$ and $\gamma_{ubz}$ parameters with respect to AS1 strategy. In fact, because a lower priority is assigned to the constraints associated to 5th-10th prediction instants (see Table 4), the impact of their softening on the remaining constraints is mitigated. Given the limitation of MV2, the restore of CV1 within the defined constraints is obtained, as in S2 strategy, through MV1 moves (see Fig. 10, blue line).

The previous considerations are also proven from a quantitative point of view through the performance indicator $t_{restore}$ (see (24) and Table 8). As shown in Fig. 9, AS1 strategy is not able to ensure a satisfactory control action ($t_{restore} = \infty$). On the other hand, both S1 and S2 strategies are able to restore CV1 within its constraints; from the values of the indicator $t_{restore}$ (see Table 8), it can be concluded that the

S2 strategy performs better. With regard to the S1 strategy the priority assigned to the CV1 constraints associated to 5th-10th prediction instants covers a key role; in this case, $\alpha_{lbz_{1_1}} = \alpha_{ubz_{1_1}} = 1.25$ (see Table 4). At this regard, a draft sensitivity analysis of S1 strategy with respect to $\gamma_{lbz}$ and $\gamma_{ubz}$ parameters is performed in the following.

Two different settings have been analyzed, i.e. $\alpha_{lbz_{1_1}} = \alpha_{ubz_{1_1}} = 1.5$ and $\alpha_{lbz_{1_1}} = \alpha_{ubz_{1_1}} = 1.75$, thus lowering further the priority assigned to the constraints associated to 5th-10th prediction instants with respect to the original setting. The comparison of the system performance when adopting the S1 strategy with the three different settings is shown in Fig. 12-14. In Fig. 12-14, the blue line refers to the original setting ($\alpha_{lbz_{1_1}} = \alpha_{ubz_{1_1}} = 1.25$) while ochre and purple lines refer to the setting of $\alpha_{lbz_{1_1}} = \alpha_{ubz_{1_1}} = 1.5$ and $\alpha_{lbz_{1_1}} = \alpha_{ubz_{1_1}} = 1.75$, respectively. As can be observed in Fig. 12-13, a decrease of the priority assigned to the constraints at 5th-10th prediction instants results in a more prompt control action and increased control efforts (see Fig. 13).

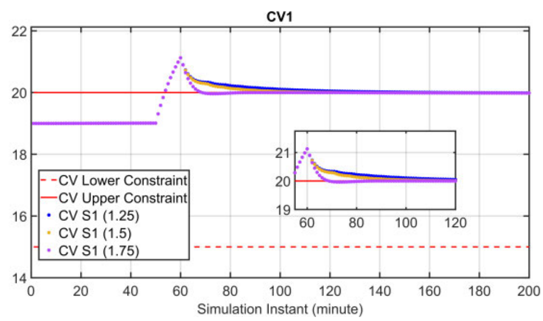


**FIGURE 12. Third scenario (S1 strategy addendum): CV1.**

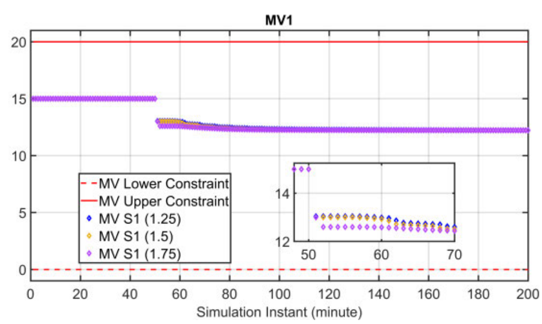


**FIGURE 13. Third scenario (S1 strategy addendum): MV1.**

**TABLE 9. Performance indicator for each strategy (third scenario, S1 strategy addendum).**

| Performance Indicator (CV1 upper constraint) | S1 (1.25) | S1 (1.5) | S1 (1.75) |
|---|---|---|---|
| $t_{restore}$ | 165 | 129 | 15 |

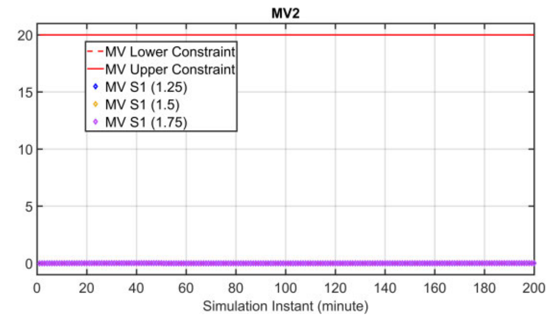


**FIGURE 14. Third scenario (S1 strategy addendum): MV2.**

The increasing promptness related to increasing values of $\alpha_{lbz_{1_1}} (= \alpha_{ubz_{1_1}})$, i.e. to the decrease of the priority assigned to the constraints at 5th-10th prediction instants, can be verified also through the performance indicator $t_{restore}$ (see (24) and Table 9). In addition, it can be observed that the indicator computed for S1 strategy with $\alpha_{lbz_{1_1}} = \alpha_{ubz_{1_1}} = 1.75$ ($t_{restore} = 15$) is comparable to the indicator computed for S2 strategy ($t_{restore} = 9$) in the same scenario (see Table 8).

### D. FURTHER DISCUSSION

The depicted scenarios have proven the effectiveness of the proposed S1 and S2 strategies and their performances have been compared from a qualitative and from a quantitative point of view to MPC strategies reported in different works, e.g., [16], [17], [32], [35], [39], and [46]. Similar results are obtained also in SISO and MIMO cases; an additional aspect that must be considered in the MIMO case is that the tuning parameters can take into account also the ranking between different CVs ($m_z > 1$).

The effectiveness of the proposed strategies with respect to strategies that do not fully exploit dead times information is more evident with the increase of the different dead times associated to the CV-MVs relationships and with the increase of the difference between subsequent different dead times (see (8) and (23)).

An effective $H_w$ parameter tuning procedure characterizes both the proposed S1 and S2 strategies; this procedure takes into account dead times information and the status value associated to the MVs tied to the considered CV. S1 and S2 strategies exploit dead times information in different ways in order to tune $n_\varepsilon$, $\gamma_{lbz}$, $\gamma_{ubz}$, and the constraints grouping policies that have to be defined with regard to the slack variables.

S1 strategy considers a single slack variable ($n_\varepsilon = 1$) and it is characterized by a tailored effective tuning of $\gamma_{lbz}$ and $\gamma_{ubz}$ parameters. In particular, if all CVs are *active*, at least $\sum_{o=1}^{m_z} h_o$ values must be defined for the positive elements of $\gamma_{lbz}$ and $\gamma_{ubz}$ parameters. As described in Section III-C, $h_o$ is the number of different dead times that are present in the relationships between the considered $o$th CV and the *active* MVs tied to that CV. The definition of $\sum_{o=1}^{m_z} h_o$ values for the positive elements of $\gamma_{lbz}$ and $\gamma_{ubz}$ parameters must take into account both the ranking between constraints associated to the same CV (as shown in the proposed scenarios on MISO cases) and the ranking between different CVs.

S2 strategy considers $n_\varepsilon \geq \sum_{o=1}^{m_z} h_o$ (if all CVs are *active* and the $H_w$ window parameter is set to the proposed lower bound) but it does not require an ad hoc tuning of $\gamma_{lbz}$ and $\gamma_{ubz}$ parameters of a selected *active* CV for the management of the priority of the constraints related to different prediction instants. If $n_\varepsilon = \sum_{o=1}^{m_z} h_o$, each $o$th CV is characterized by a number of slack variables equal to the number of different dead times of its relationships with the *active* MVs (tied to that CV); in addition, in this configuration, lower and upper CV constraints associated to the same prediction instant share the same slack variable.

Comparing S1 and S2 strategy, the latter is characterized by a higher computational load because $n_\varepsilon \geq \sum_{o=1}^{m_z} h_o$. On the other hand, S2 strategy provides a more efficient tuning procedure of the positive elements of $\gamma_{lbz}$ and $\gamma_{ubz}$ parameters, because only the ranking between different CVs must be handled.

The time required to tune the MPC system is optimized more in S2 strategy with respect to S1 strategy, because S1 strategy requires also taking into account the ranking between constraints associated to the same CV. On the other hand, S1 strategy optimizes the required computational load because it includes a single slack variable for each CV.

In many real case studies (e.g., industrial or non-industrial processes) the presence of different dead times associated to the CV-MVs relationships is a widespread feature, together with the assignment of zone control specifications. For example, in a distillation column (heavy oil fractionator), some CVs, e.g., the composition of the top end point product, the composition of the side end point product, and the bottoms reflux temperature, are often characterized by at least two different dead times in their relationships with the MVs, e.g., draw flow rate, side draw flow rate, and bottoms reflux duty [1], [32]. Additional significant examples are represented by the industrial ethylene oxide production system, the ethylene oxide reactor system, and the gas compression system, where several CVs are often characterized by at least two different dead times in their relationships with the MVs [16], [17]. In these contexts, potential challenges in implementation can be referred to guarantee a trade-off between the optimization of performance indicators (e.g., the proposed $t_{restore}$) in terms of CVs constraints fulfillment, the minimization of the time required to tune the MPC system and the minimization of the required computational load. In these cases, the proposed S1 and S2 strategies provide all the elements to find the desired trade-off, because both the time required to tune the MPC system and the required computational load can be taken into account, while trying to guarantee satisfactory performance in terms of zone control in response to different events.

## V. CONCLUSION

The present research work proposes a method to assess the input-output dead times impact on tuning procedures and grouping policies associated to output *soft* constraints in Model Predictive Control (MPC) based on linear models. An MPC problem characterized by *hard* constraints on the Manipulated Variables (MVs) magnitude and on the MVs slew rate and by *soft* constraints on the Controlled Variables (CVs) magnitude has been considered. In addition, the option to online define the control matrix to be considered in the control problem formulation has been included through suitable MVs and CVs status values.

Window parameters, tuning parameters associated to slack variables and grouping policies for constraints relaxation have been considered. The present paper established some effective and efficient rules for the definition of these parameters and policies that take into account input-output dead time information in different manners. Two effective strategies have been proposed in this context. The first strategy includes a single slack variable in the MPC optimization problem formulation and has to be preferred when a low computational load is sought. On the other hand, it requires the tuning of a number of parameters that is tied to the number of different dead times that are present in the relationships between CVs and MVs included in the control problem. A lower number of parameters to be tuned characterizes the second proposed strategy. On the other hand, it needs a greater computational load with respect to the first proposed strategy because it requires a number of slack variables tied to the number of different dead times that are present in the relationships between CVs and MVs included in the control problem.

Simulation results based on first-order plus deadtime models showed how the given insights can improve the effectiveness and the efficiency of MPC systems design.

Future work will extend the proposed procedures to processes that can be modellized through other types of models; in particular, supplementary types of linear models will be considered, e.g., second-order plus deadtime models and models characterized by higher order dynamics. In addition, the proposed methods will be customized for processes that necessarily require the exploitation of nonlinear models, considering also combinations with machine learning and deep learning techniques. Furthermore, alternative objective functions will be considered, e.g., including tracking objectives for the CVs and/or the MVs.

## REFERENCES

[1] J. M. Maciejowski, *Predictive Control: With Constraints*. London, U.K.: Prentice-Hall, 2002.
[2] P. O. M. Scokaert and J. B. Rawlings, "Feasibility issues in linear model predictive control," *AIChE J.*, vol. 45, no. 8, pp. 1649–1659, Aug. 1999, doi: 10.1002/aic.690450805.
[3] E. Kerrigan and J. Maciejowski, "Soft constraints and exact penalty functions in model predictive control," in *Proc. UKACC Int. Conf. Control*, Cambridge, U.K., Sep. 2000, pp. 1–6.
[4] A. Bemporad, N. L. Ricker, and M. Morari, "Model predictive control toolbox user's guide," Mathworks Inc., Natick, MA, USA, Tech. Rep., 2023.
[5] J. E. Normey-Rico and E. F. Camacho, *Control of Dead-time Processes*. London, U.K.: Springer, 2007, doi: 10.1007/978-1-84628-829-6.

[6] L. R. Da Silva, R. C. C. Flesch, and J. E. Normey-Rico, "Controlling industrial dead-time systems: When to use a PID or an advanced controller," *ISA Trans.*, vol. 99, pp. 339–350, Apr. 2020, doi: 10.1016/j.isatra.2019.09.008.

[7] C. G. Sigurd Skogestad, "Should we forget the Smith predictor?" *IFAC-PapersOnLine*, vol. 51, no. 4, pp. 769–774, 2018, doi: 10.1016/j.ifacol.2018.06.203.

[8] T. L. M. Santos, D. Limon, J. E. Normey-Rico, and T. Alamo, "On the explicit dead-time compensation for robust model predictive control," *J. Process Control*, vol. 22, no. 1, pp. 236–246, Jan. 2012, doi: 10.1016/j.jprocont.2011.08.003.

[9] T. L. M. Santos, D. Limon, T. Alamo, and J. E. Normey-Rico, "Robust tube based model predictive control for constrained systems with dead-time," in *Proc. UKACC Int. Conf. Control*, Sep. 2010, pp. 1–6, doi: 10.1049/ic.2010.0404.

[10] B. C. Torrico and J. E. Normey-Rico, "Robust constrained horizon predictive controller for dead time systems," *IFAC Proc. Volumes*, vol. 38, no. 1, pp. 290–295, 2005, doi: 10.3182/20050703-6-cz-1902.00619.

[11] T. L. M. Santos, J. E. Normey-Rico, and D. L. Marruedo, "Robust model predictive controller with terminal weighting for multivariable dead-time processes," *IFAC Proc. Volumes*, vol. 42, no. 14, pp. 248–253, 2009, doi: 10.3182/20090901-3-ro-4009.00040.

[12] T. L. M. Santos, J. E. Normey-Rico, and D. Limón, "Explicit input-delay compensation for robustness improvement in MPC," *IFAC Proc. Volumes*, vol. 43, no. 2, pp. 384–389, 2010, doi: 10.3182/20100607-3-cz-4010.00068.

[13] M. M. Morato, T. L. M. Santos, and J. E. Normey-Rico, "Explicit dead-time compensation in linear parameter varying model predictive control," *IFAC-PapersOnLine*, vol. 55, no. 16, pp. 278–283, 2022, doi: 10.1016/j.ifacol.2022.09.037.

[14] J. E. Normey-Rico, D. M. Lima, and T. L. M. Santos, "Robustness of nonlinear MPC for dead-time processes," *IFAC-PapersOnLine*, vol. 48, no. 23, pp. 332–341, 2015, doi: 10.1016/j.ifacol.2015.11.303.

[15] A. H. González and D. Odloak, "Robust model predictive control for time delayed systems with optimizing targets and zone control," in *Robust Control, Theory and Applications*, A. Bartoszewicz, Ed., Rijeka, Croatia: InTech, Apr. 2011.

[16] B. F. Santoro and D. Odloak, "Closed-loop stable model predictive control of integrating systems with dead time," *J. Process Control*, vol. 22, no. 7, pp. 1209–1218, Aug. 2012, doi: 10.1016/j.jprocont.2012.05.005.

[17] M. A. F. Martins, A. S. Yamashita, B. F. Santoro, and D. Odloak, "Robust model predictive control of integrating time delay processes," *J. Process Control*, vol. 23, no. 7, pp. 917–932, Aug. 2013, doi: 10.1016/j.jprocont.2013.05.002.

[18] S. Olaru and S.-I. Niculescu, "Predictive control for linear systems with delayed input subject to constraints," *IFAC Proc. Volumes*, vol. 41, no. 2, pp. 11208–11213, 2008, doi: 10.3182/20080706-5-kr-1001.01899.

[19] S. Olaru, H. Benlaoukl, and S. Niculescu, "MPC for systems with variable time-delay robust positive invariant set approximations," in *Proc. 5th Int. Conf. Informat. Control, Autom. Robot. (ICINCO)*, Madeira, Portugal, May 2008, pp. 177–182.

[20] W. Lombardi, S. Olaru, S.-I. Niculescu, and L. Hetel, "A predictive control scheme for systems with variable time-delay," *Int. J. Control*, vol. 85, no. 7, pp. 915–932, Jul. 2012, doi: 10.1080/00207179.2012.669847.

[21] A. Grancharova and S. Olaru, "Distributed model predictive control of time-delay systems," *IFAC-PapersOnLine*, vol. 55, no. 40, pp. 85–90, 2022, doi: 10.1016/j.ifacol.2023.01.053.

[22] J. L. Garriga and M. Soroush, "Model predictive control tuning methods: A review," *Ind. Eng. Chem. Res.*, vol. 49, no. 8, pp. 3505–3515, Mar. 2010, doi: 10.1021/ie900323c.

[23] M. Alhajeri and M. Soroush, "Tuning guidelines for model-predictive control," *Ind. Eng. Chem. Res.*, vol. 59, no. 10, pp. 4177–4191, Feb. 2020, doi: 10.1021/acs.iecr.9b05931.

[24] A. R. Neshasteriz, A. Khaki Sedigh, and H. Sadjadian, "Generalized predictive control and tuning of industrial processes with second order plus dead time models," *J. Process Control*, vol. 20, no. 1, pp. 63–72, Jan. 2010, doi: 10.1016/j.jprocont.2009.10.003.

[25] P. Bagheri and A. K. Sedigh, "Analytical approach to tuning of model predictive control for first-order plus dead time models," *IET Control Theory Appl.*, vol. 7, no. 14, pp. 1806–1817, Sep. 2013, doi: 10.1049/iet-cta.2012.0934.

[26] P. García and P. Albertos, "Robust tuning of a generalized predictor-based controller for integrating and unstable systems with long time-delay," *J. Process Control*, vol. 23, no. 8, pp. 1205–1216, Sep. 2013, doi: 10.1016/j.jprocont.2013.07.008.

[27] P. Bagheri and A. Khaki-Sedigh, "An analytical tuning approach to multivariable model predictive controllers," *J. Process Control*, vol. 24, no. 12, pp. 41–54, Dec. 2014, doi: 10.1016/j.jprocont.2014.09.002.

[28] P. Bagheri and A. Khaki-Sedigh, "Closed form tuning equations for model predictive control of first-order plus fractional dead time models," *Int. J. Control, Autom. Syst.*, vol. 13, no. 1, pp. 73–80, Feb. 2015, doi: 10.1007/s12555-014-0007-6.

[29] A. S. Yamashita, A. C. Zanin, and D. Odloak, "Tuning of model predictive control with multi-objective optimization," *Brazilian J. Chem. Eng.*, vol. 33, no. 2, pp. 333–346, Jun. 2016, doi: 10.1590/0104-6632.20160332s20140212.

[30] K. Zabet and R. Haber, "Robust tuning of PFC (predictive functional control) based on first- and aperiodic second-order plus time delay models," *J. Process Control*, vol. 54, pp. 25–37, Jun. 2017, doi: 10.1016/j.jprocont.2017.01.008.

[31] T. Gholaminejad, A. Khaki-Sedigh, and P. Bagheri, "Direct adaptive model predictive control tuning based on the first-order plus dead time models," *IET Control Theory Appl.*, vol. 11, no. 16, pp. 2858–2869, Nov. 2017, doi: 10.1049/iet-cta.2016.1174.

[32] J. E. W. Santos, J. O. Trierweiler, and M. Farenzena, "Model predictive control tuning strategy for non-square systems and range controlled variables based on multi-scenarios approach," *Ind. Eng. Chem. Res.*, vol. 56, no. 40, pp. 11496–11506, Oct. 2017, doi: 10.1021/acs.iecr.7b01531.

[33] J. E. W. Santos, J. O. Trierweiler, and M. Farenzena, "Robust tuning for classical MPC through the multi-scenarios approach," *Ind. Eng. Chem. Res.*, vol. 58, no. 8, pp. 3146–3158, Jan. 2019, doi: 10.1021/acs.iecr.8b05485.

[34] S. A. C. Giraldo, P. A. Melo, and A. R. Secchi, "Tuning of model predictive control based on hybrid optimization," *IFAC-PapersOnLine*, vol. 52, no. 1, pp. 136–141, 2019, doi: 10.1016/j.ifacol.2019.06.050.

[35] R. M. Fontes, M. A. F. Martins, and D. Odloak, "An automatic tuning method for model predictive control strategies," *Ind. Eng. Chem. Res.*, vol. 58, no. 47, pp. 21602–21613, Oct. 2019, doi: 10.1021/acs.iecr.9b03502.

[36] T. L. M. Santos, L. Roca, J. L. Guzman, J. E. Normey-Rico, and M. Berenguel, "Practical MPC with robust dead-time compensation applied to a solar desalination plant," *IFAC Proc. Volumes*, vol. 44, no. 1, pp. 4909–4914, Jan. 2011, doi: 10.3182/20110828-6-it-1002.01212.

[37] A. S. Yamashita, A. C. Zanin, and D. Odloak, "Tuning the model predictive control of a crude distillation unit," *ISA Trans.*, vol. 60, pp. 178–190, Jan. 2016, doi: 10.1016/j.isatra.2015.10.017.

[38] S. M. Zanoli and C. Pepe, "A constraints softening decoupling strategy oriented to time delays handling with model predictive control," in *Proc. Amer. Control Conf. (ACC)*, Boston, MA, USA, Jul. 2016, pp. 2687–2692, doi: 10.1109/ACC.2016.7525324.

[39] S. A. C. Giraldo, R. C. Supelano, T. C. D'Avila, B. D. O. Capron, L. D. Ribeiro, M. M. Campos, and A. R. Secchi, "Model predictive control with dead-time compensation applied to a gas compression system," *J. Petroleum Sci. Eng.*, vol. 203, Aug. 2021, Art. no. 108580, doi: 10.1016/j.petrol.2021.108580.

[40] A. Grancharova, S. Olaru, and I. Valkova, "Distributed model predictive control of time-delay systems: Application to a series of chemical reactors," in *Proc. Int. Conf. Automatics Informat. (ICAI)*, Varna, Bulgaria, Oct. 2022, pp. 199–203, doi: 10.1109/ICAI55857.2022.9960017.

[41] A. S. Yamashita, W. T. Martins, T. V. B. Pinto, G. V. Raffo, and T. A. M. Euzébio, "Multiobjective tuning technique for MPC in grinding circuits," *IEEE Access*, vol. 11, pp. 43041–43054, 2023, doi: 10.1109/ACCESS.2023.3269559.

[42] N. He, M. Zhang, and R. Li, "An improved approach for robust MPC tuning based on machine learning," *Math. Problems Eng.*, vol. 2021, pp. 1–16, May 2021, doi: 10.1155/2021/5518950.

[43] P. Liu, L. Yang, and L. Sun, "Multi-objective economic model predictive control of wet limestone flue gas desulfurisation system," *Process Saf. Environ. Protection*, vol. 150, pp. 269–280, Jun. 2021, doi: 10.1016/j.psep.2021.04.012.

[44] J. Park, M. R. Babaei, S. A. Munoz, A. N. Venkat, and J. D. Hedengren, "Simultaneous multistep transformer architecture for model predictive control," *Comput. Chem. Eng.*, vol. 178, Oct. 2023, Art. no. 108396, doi: 10.1016/j.compchemeng.2023.108396.

[45] Y. Dai, N. Chen, and Z. Shao, "Hybrid self-learning model for the prediction and control of sintering furnace temperature," *Control Eng. Pract.*, vol. 154, Jan. 2025, Art. no. 106159, doi: 10.1016/j.conengprac.2024.106159.
[46] H. Zhang, Z. Wu, Q. Yuan, L. Guo, X. Li, C. Hua, and P. Lu, "Control of extractive dividing wall column using model predictive control based on long short-term memory networks," *Separat. Purification Technol.*, vol. 361, Jul. 2025, Art. no. 131351, doi: 10.1016/j.seppur.2024.131351.
[47] S. Skogestad, "Advanced control using decomposition and simple elements," *Annu. Rev. Control*, vol. 56, May 2023, Art. no. 100903, doi: 10.1016/j.arcontrol.2023.100903.
[48] J. Richalet, A. Rault, J. L. Testud, and J. Papon, "Model predictive heuristic control," *Automatica*, vol. 14, no. 5, pp. 413–428, Sep. 1978, doi: 10.1016/0005-1098(78)90001-8.
[49] C. Pepe and S. M. Zanoli, "A two-layer model predictive control system with adaptation to variables status values," in *Proc. 17th Int. Carpathian Control Conf. (ICCC)*, High Tatras, Slovakia, May 2016, pp. 573–578, doi: 10.1109/CarpathianCC.2016.7501162.
[50] L. Ljung, *System Identification: Theory for the User*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1999.
[51] S. M. Zanoli, L. Orlietti, F. Cocchioni, G. Astolfi, and C. Pepe, "Optimization of the clinker production phase in a cement plant," in *Proc. 14th APCA Int. Conf. Autom. Control Soft Comput.*, in Lecture Notes in Electrical Engineering, vol. 695, J. A. Gonçalves, M. Braz-César, and J. P. Coelho, Eds., Cham, Switzerland: Springer, doi: 10.1007/978-3-030-58653-9_25.
[52] S. M. Zanoli, C. Pepe, and M. Rocchi, "Cement rotary kiln: Constraints handling and optimization via model predictive control techniques," in *Proc. 5th Austral. Control Conf. (AUCC)*, Nov. 2015, pp. 288–293.
[53] S. M. Zanoli, G. Astolfi, L. Orlietti, M. Frisinghelli, and C. Pepe, "Water distribution networks optimization: A real case study," *IFAC-PapersOnLine*, vol. 53, no. 2, pp. 16644–16650, 2020, doi: 10.1016/j.ifacol.2020.12.794.
[54] B. D. Schutter, "Minimal state-space realization in linear system theory: An overview," *J. Comput. Appl. Math.*, vol. 121, nos. 1–2, pp. 331–354, Sep. 2000, doi: 10.1016/s0377-0427(00)00341-1.
[55] M. V. Kothare, V. Balakrishnan, and M. Morari, "Robust constrained model predictive control using linear matrix inequalities," *Automatica*, vol. 32, no. 10, pp. 1361–1379, Oct. 1996, doi: 10.1016/0005-1098(96)00063-5.
[56] C. V. Rao, S. J. Wright, and J. B. Rawlings, "Application of interior-point methods to model predictive control," *J. Optim. Theory Appl.*, vol. 99, no. 3, pp. 723–757, Dec. 1998, doi: 10.1023/a:1021711402723.
[57] K. R. Muske and T. A. Badgwell, "Disturbance modeling for offset-free linear model predictive control," *J. Process Control*, vol. 12, no. 5, pp. 617–632, Aug. 2002, doi: 10.1016/S0959-1524(01)00051-8.
[58] G. Pannocchia and J. B. Rawlings, "Disturbance models for offset-free model-predictive control," *AIChE J.*, vol. 49, no. 2, pp. 426–437, Feb. 2003, doi: 10.1002/aic.690490213.
[59] R. Cagienard, P. Grieder, E. C. Kerrigan, and M. Morari, "Move blocking strategies in receding horizon control," *J. Process Control*, vol. 17, no. 6, pp. 563–570, Jul. 2007, doi: 10.1016/j.jprocont.2007.01.001.

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