Influence of Dynamic Properties Variability of Elastomeric Bearings on the Seismic Response of Base-Isolated Structures

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Abstract. The response of bae-isolated structures is affected by the variability of dynamic properties of isolation bearings due to environmental effects, such as the temperature and ageing, and/or behavioral effects. For elastomeric bearings made by High Damping Rubber (HDR), the main behavioral effects are the strain amplitude and the strain rate dependence of the shear response and the load history dependence. They can be all included in advanced numerical models, alternatively current seismic codes allow to adopt less accurate models in combination with property modification factors (also called λ-factors) defined for both behavioral and environmental effects.
These factors should be experimentally assessed during the qualification process of bearings but, in some cases such as in the European context, the product standard on anti-seismic devices (EN 15129) allows the manufacturers to deduce these variabilities from tests performed on material specimens. In this paper a statistical analysis is carried out on a large set of material tests carried out according to EN 15129 on low and high damping rubber compounds provided from different suppliers. Based on the results, new λ-factors for stiffness and damping are proposed and compared with those deduced from the "λ-factors" suggested in the informative annexes of EN 15129 and EC8-part 2. In the last part of the paper some preliminary considerations are made on the influence of such variability on the vulnerability assessment of base-isolation structures.

Keywords: seismic isolation, elastomeric bearings, upper lower bound analysis.

1 Introduction

Base isolation is a very efficient and widespread technique for seismic protection of buildings and bridges. Nevertheless, the variability of dynamic properties of isolation bearings may notably affect the response of such structures and should be considered in the analysis and design of base isolated structures. Different variability sources are suggested by the most advanced seismic codes [1-6], distinguished between behavioral and environmental effects. For Low Damping Rubber (LDR) and more significantly for High Damping Rubber (HDR) bearings, the main behavioral effects are the strain amplitude and the strain rate dependence of the shear response, the load history dependence related to the so-called Mullins or scragging effect [7-11], and the influence of the vertical load [12-14]. The main environmental effects are the temperature variation and long-term environmental action (ageing).
Currently, most of the behavioral effects [10], [11], [15], [16] as well as the effects of temperature [17-19] and ageing [18] are included in advanced numerical models. However, current seismic codes also allow to adopt a simplified approach based on simplified models in combination with property modification factors (called λ-factors in the European context) that are defined for both behavioral and environmental effects. By combining such factors and by applying the results to the nominal properties of the isolation bearings two sets of design properties are obtained: the Upper Bound Design Properties (UBDPs) and the Lower Bound Design Properties (LBDPs). This way, two structural analyses should be separately carried out, that are UB and LB analyses. The former usually leads to the maximum base shear force and to the maxima superstructure accelerations, while the latter to the maximum displacement of the isolation system. All current seismic standards suggest deriving these factors from experimental data obtained from qualification tests on bearings. However, in some cases such as in the European context, the product standard EN 15129 [3], allows the manufacturers to deduce the variability of bearings properties also from tests performed on material specimens.

For this reason, a statistical analysis is carried out in this paper on a large set of material tests carried out according to [3] on low and high damping rubber compounds from different European suppliers currently used for elastomeric bearings, with shear modulus ranging from 0.4 MPa to 1.3 MPa. For each behavioral effect and environmental condition (i.e., shear strain amplitude and strain rate, cyclic degradation, ageing, and temperature) the response variability has been analyzed in terms of equivalent linear properties, i.e., the dynamic shear modulus (G) and the damping ratio (ξ). Results are reported in terms of statistical distribution of the relative variation with respect to values observed at the reference condition and are shown separately for HDRs and LDRs. The feasibility of deriving full-scale device properties from material tests are also investigated utilizing some available experimental data on full-scale bearings. Based on these results, updated λ-factors for equivalent stiffness and damping are proposed and compared with those deduced from the informative annexes of EN 15129 [3] and Eurocode 8-part 2 [4]. At this regard, it should be noted that the modification factors given in these informative annexes and called λ-factors, namely the post-yield stiffness and strength at zero displacement, refer to a bi-linear response model, consequently they cannot be directly implemented in linear models usually adopted for the preliminary analysis of base-isolated structures. Moreover, they are taken from outdated documents [19-21] based on limited data not referring to the current European production of elastomeric bearings [22]. For this reason, the main differences are highlighted and commented. Finally, the impact on the seismic response of base-isolated structures of the obtained λ-factors has been evaluated, using a simplified S-DOF linear model (i.e., rigid superstructure). These preliminary results show that these proposed values could be a new generally accepted set of λ-factors agreed between suppliers and structural engineers, in compliance with the new procedure suggested by the ISO 22762 [23].

2 Experimental data on elastomeric compounds

Dataset used in this study collects results of qualification tests carried out by the Materials Testing Laboratory of Politecnico di Milano, which is a Notified Body for Testing, Inspection and Certification of anti-seismic devices according to EN 15129, since 2011 [24]. The dataset is composed by 18 rubber compounds (7 LDR and 11 HDR) belonging to 5 different European suppliers, with a shear modulus (*G*) in the range 0.4-1.3 MPa (Tab. 1). Data refer to the results of dynamic tests carried out to evaluate the influence on the dynamic properties (dynamic shear modulus, *G*, and damping, *ξ*, for HDR and only the dynamic shear modulus, *G*, for LDR) of different effects: strain amplitude, frequency, temperature, ageing and repeated cycles. These tests are all part of Type Tests carried out according to the EN 15129 - §8.2.2 [3] to verify that the compounds meet code requirements for their use in isolation bearings. The other tests required by the qualification procedure (shear bond test, resistance to low temperature crystallization, resistance to slow crack growth) are not object of this study. Table 1 reports the list of rubber compounds with the indication of the dynamic properties (including for completeness the damping for LDR even though not object of qualification) related to specimens used for the strain amplitude effect evaluation. It can be observed that values of *G* are all close to nominal values reported in the compound identifier and that values of *ξ* are all lower than 6% for LDR and lower than 17% for HDR. According to the EN 15129, tests to assess the property variation are made on a set of 3 samples for each effect, and the reported values are the averaged results. In detail, dynamic parameters considered in this paper are defined according to §G.5 of EN 15129 and computed by the following expressions:

$$G=(\tau^{+}-\tau^{-})/(\gamma^{+}-\gamma^{-}) \qquad (1)$$

$$\xi=2H/(\pi G(\gamma^{+}-\gamma^{-})) \qquad (2)$$

where γ^{+} and γ^{-} are the maximum and minimum values of shear strain of the cycle, τ^{+} and τ^{-} are the shear stress at those strains and *H* is the area of the hysteresis loop. For each test sequence, reference values of the dynamic parameters (G_0 and ξ_0), i.e., values measured at the reference condition (the 3rd cycle at shear strain $\gamma = 1$, temperature $T = 23$°C and frequency $f = 0.5$ Hz), are identified.

Table 1. List of rubber compounds.

Compound	Rubber type	Supplier	G_0 [MPa]	ξ_0 [-]
1 A (0.7)	*LDR*	supplier 1	0.74	0.058
1 B (0.7)	*LDR*	supplier 1	0.75	0.042
1 C (0.6)	*LDR*	supplier 1	0.58	0.034
1 D (0.9)	*LDR*	supplier 1	0.91	0.062
1 E (1.2)	*LDR*	supplier 1	1.17	0.049
2 F (0.6)	*LDR*	supplier 2	0.61	0.050
3 G (0.4)	*LDR*	supplier 3	0.45	0.059
3 H (0.4)	*HDR*	supplier 3	0.45	0.117
3 I (0.8)	*HDR*	supplier 3	0.81	0.155
3 L (1.2)	*HDR*	supplier 3	1.18	0.151
3 M (0.7)	*HDR*	supplier 3	0.72	0.083
4 O (0.7)	*HDR*	supplier 4	0.74	0.092
5 P (0.4)	*HDR*	supplier 5	0.39	0.132
5 Q (0.5)	*HDR*	supplier 5	0.51	0.086
5 R (0.7)	*HDR*	supplier 5	0.66	0.113
5 S (0.9)	*HDR*	supplier 5	0.86	0.126
2 T (1.1)	*HDR*	supplier 2	1.09	0.077
2 U (1.3)	*HDR*	supplier 2	1.30	0.167

In the following, to focus on the effect of each behavioral and environmental factors, results are analyzed separately in terms of ratio between the values measured at a specific condition (G and ξ), and the values measured in the same test at the reference condition (G_0 and ξ_0). This way the production variability is removed and discussed later. Moreover, results are separately presented for LDR and HDR. The effects on ξ of each phenomenon on LDR is reported only for comparison purpose since it is usually neglected in the design and not object of the qualification procedure. More in detail results about the shear strain, cyclic degradation and temperature are reported in Fig.1, 2 and 3 respectively. For each set of data (HDR in blue and LDR in orange) the solid line represents median values, while the area in the background represents the range between the 25th and 75th percentile [25]. The effect on ξ for LDR is reported for comparison purpose only (dashed line), as damping of this kind of rubber is not considered in common design practice.

As concerns the strain amplitude (γ) the effect, which is commonly referred to as Payne effect in the literature [8], results in an increment of the dynamic modulus G for small shear strain amplitudes, whereas for the damping ratio ξ the trend follows an almost linearly decreasing pattern with γ.

For what concern the repeated cycles effect, it is known that all rubber compounds during cyclic tests show a progressive reduction of G and ξ up to a stable or "fully scragged" behavior [10]. This phenomenon is commonly called Mullins effect [7] or "scragging" [9]. According to [3], its evaluation may be done by performing material tests at the reference strain of $\gamma = 1$ and by verifying that the ratios between the values of G and ξ measured at the 10th cycle and the respective values at the 2nd cycle are not lower than 0.7, and that the ratio between G values measured at the 10th and at the 1st cycle is not lower than 0.6 (whereas there is no limit for *ξ* between the 1st and the 10th cycle). For this reason, Fig. 2 (*a* and *b*) shows the ratios between G and ξ values measured at each cycle, with respect to the values recorded at the 2nd cycle (G_2 and ξ_2), whereas Fig. 2 (*c* and *d*) show the ratios G/G_0 and ξ/ξ_0 with reference to 3rd cycle data. It can be observed that all the rubber compounds fulfill the code limits and that in general, differently form the past, current rubber compounds are less sensitive to the effect of repeated cycles.

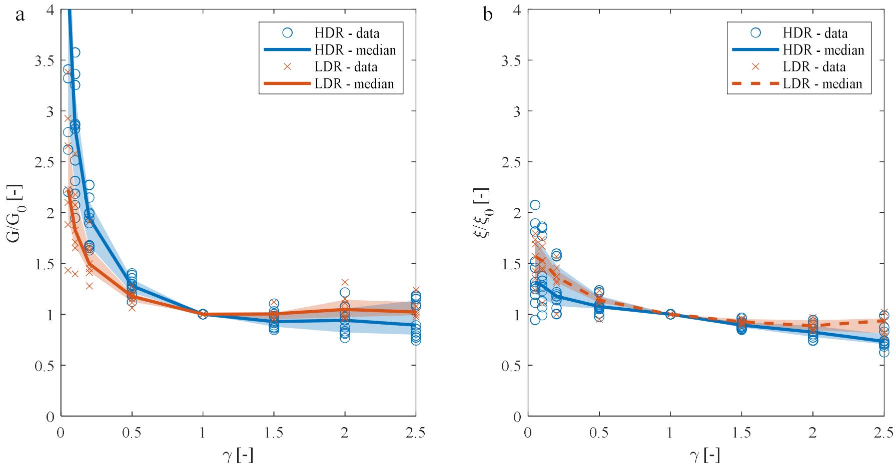
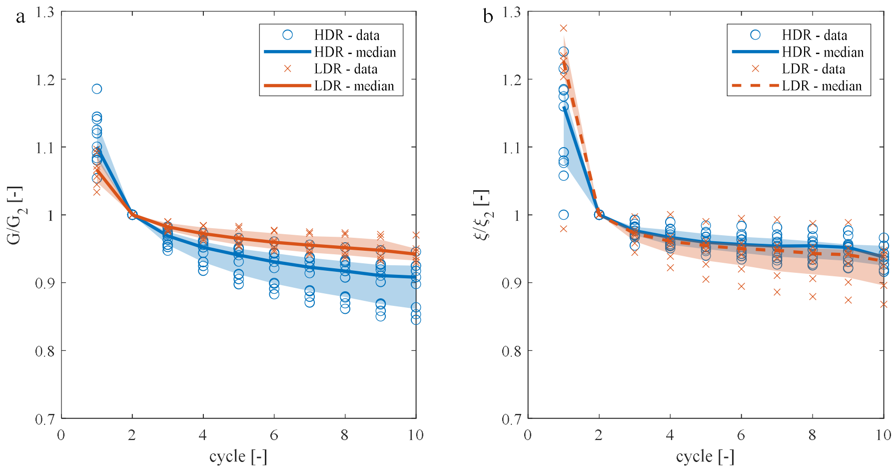


Fig. 1. Strain-amplitude effect for shear modulus (a) and damping ratio (b)



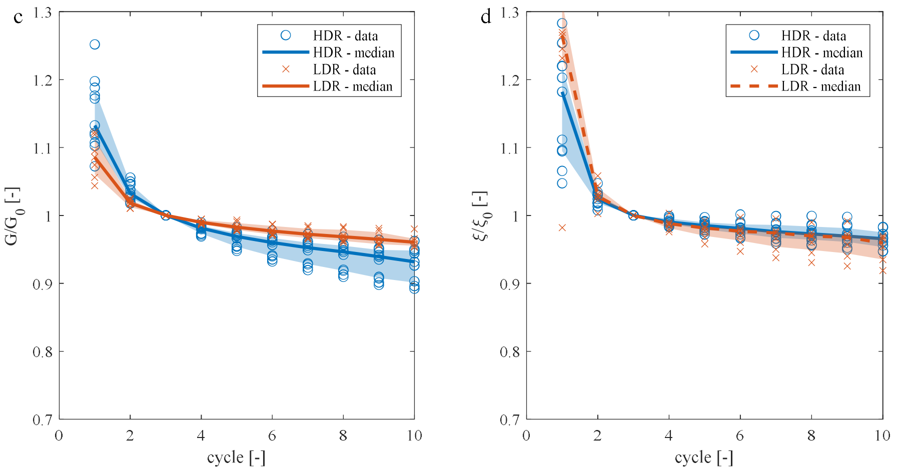


Fig. 2. Cyclic effect for shear modulus and damping ratio with respect to the second cycle (a and b) and the third cycle (c and d).

Regarding the temperature effect, according to [3], the evaluation may be done by performing material tests at different temperatures, usually ranging from -20°C to 40°C Fig. 3 shows single data and median curves for the temperature effect analysed in the range of -40°C +50°C. As the numerosity of data at each temperature is quite different in the dataset analysed in this paper, the 25th - 75th percentiles range is not represented. In the same figure code limits are also reported (dashed lines). It can be observed that all LDR compounds comply with the code prescriptions while for HDR compounds many data are out of the boundaries, especially for the G/G_0 ratio at low temperatures. As expected, HDR has higher variability with temperature with respect to LDR for G/G_0.

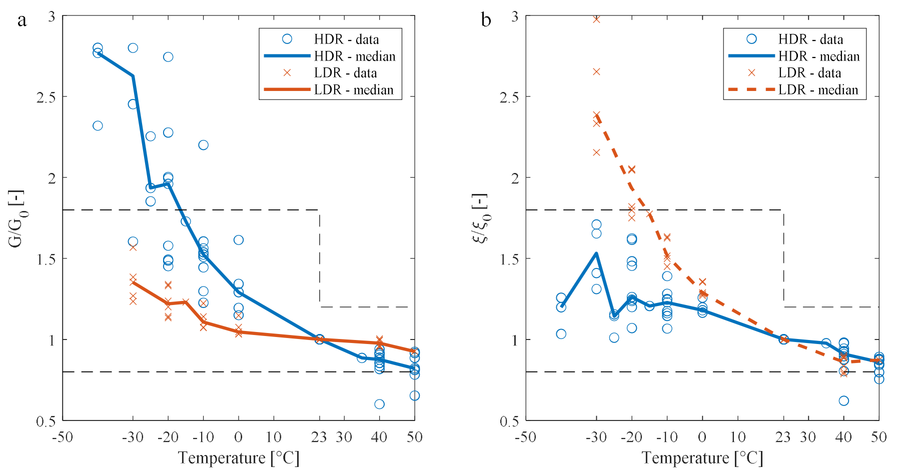


Fig. 3. Temperature effect for shear modulus (a) and damping ratio (b).

Figs. 4, 5 and 6 shows results about the aging effect, the strain rate o frequency effect and the production variability.

Differently from the previous factors, for the ageing effect only one ratio can be computed (aged divided unaged values), thus results are reported as boxplots always in terms of the ratio G/G_0 and ξ/ξ_0 (where G_0 and ξ_0 are unaged values). Each colored box represents the interquartile range (25th and 75th percentiles of the samples), while the line inside the box is the sample median. Moreover, values are considered outliers if they are out the range of the box limits plus-minus 1.5 times the interquartile range. The results show, as expected, an increase of stiffness, which is similar for HDR and LDR, (with a median value around 1.11) but with a higher dispersion for HDR. The damping, instead, decreases for both the compounds.

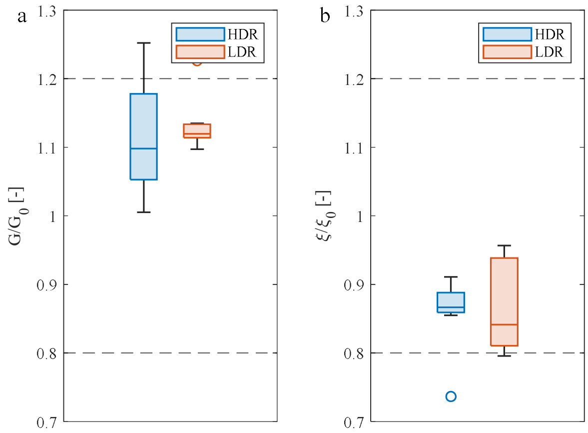


Fig. 4. Aging effect for shear modulus and damping ratio.

According to EN 15129, the effect of strain rate on G and ξ can be evaluated by performing cyclic tests on material specimens at frequencies in the range between 0.1 Hz and 2 Hz, which is quite larger with respect to the dynamic response of isolated structures. As previously described for the ageing effect, Figure 5 shows the 25th-75th percentile range, median values, and the minimum-maximum range of ratios G/G_0 and ξ/ξ_0 (where G_0 and ξ_0 are values computed at 0.5 Hz) for the two boundary frequencies 0.1 Hz and 2 Hz. Code limits are also reported (dashed lines). It can be observed that in the range of frequencies of interest for seismic applications (from 0.5 Hz to 2 Hz), the dependence of the response due to viscous effects is not very high and within the code limits.

Finally, the resulting statistical distributions of the ratio G_0/G_{nom} and ξ_0/ξ_{nom} are reported in Fig.6. In general, median values are very close to one and all the values are within the range prescribed by the code (±20%). Only some values of the damping ratio of LDR compounds are out of bounds, because damping for LDR compounds is not relevant (lower than 0.06) and disregarded in the design of seismic isolation, thus not controlled by the manufacturers.

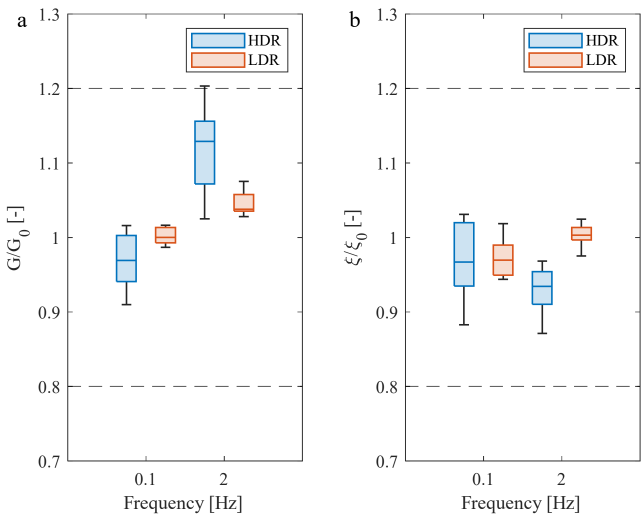


Fig. 5. Frequency effect for shear modulus and damping ratio.

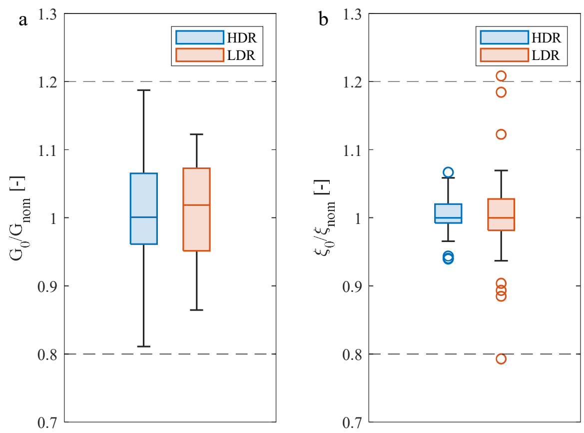


Fig. 6. Production variability for shear modulus and damping ratio

Finally, to compare the behaviors at the material scale and at the whole bearing scale a comparison is made between material tests of a compound belonging to the analyzed dataset (compound 5 P (0.4) of Table 1), and type tests (TT) and factory production control tests (FPCT) carried out on full-scale bearings made of that compound. All the tests were carried out according to EN 15129 [3], for a total of 8 bearings.. Bearings have a diameter of 800 mm and a total rubber thickness of 184 mm and they are tested up to $\gamma = 2$ for dynamic cyclic test (EN 15129 §8.2.1.2.2) and up to $\gamma = 2.18$ for lateral displacement capacity (EN 15129 §8.2.1.2.7). Fig.7 shows a direct comparison between G and ξ for the material specimen and the bearing results. There is a quite good agreement between mean values of G for the compound and the bearings with a larger dispersion of the bearing values, as expected, due to the manufacturing process. On the contrary, mean values of ξ are always higher for the bearings with respect to the compound, due to the effect of vertical pressure [26]. The same results are plotted by dividing G and ξ values by the recorded ones at the nominal conditions (shear strain $\gamma = 1$, 3rd cycle) to estimate the repeated cycles effect (Fig. 8). It can be observed that the shear modulus reduction for the compound and the bearings is similar and a good agreement, even though not as close as for the ratio G/G_0, is obtained also for the ratio ξ/ξ_0.

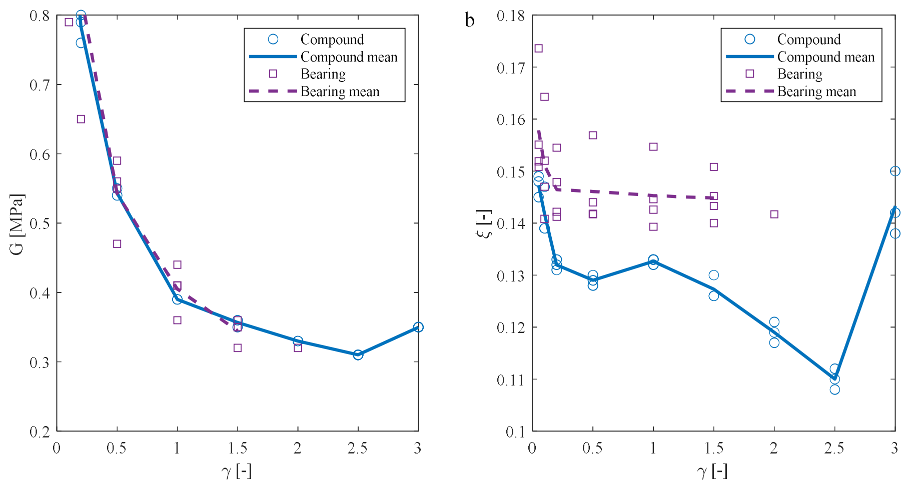


Fig. 7. Compound-bearing comparison of G and ξ for the deformation-amplitude effect.

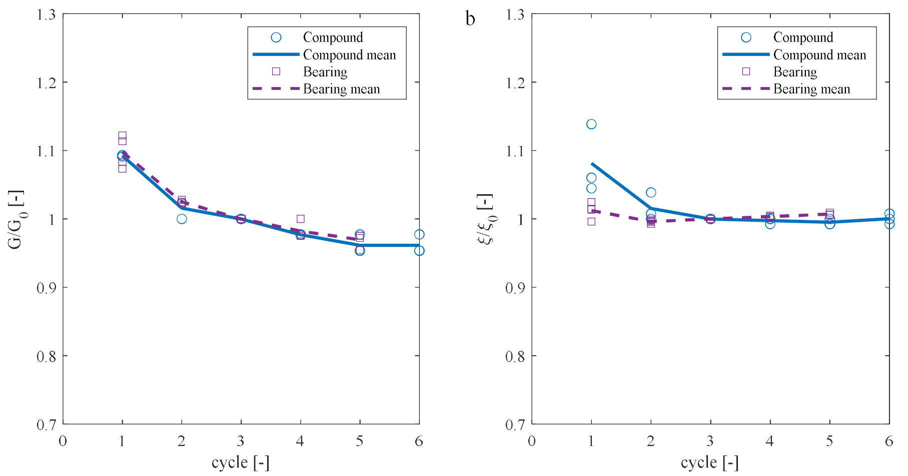


Fig. 8. Compound-bearing comparison of G/G_0 and ξ/ξ_0 ratios for the cyclic effect.

3 Proposal for updated λ-factors and potential impact on structural seismic response

Based on the previous results a set of new λ-factors are proposed in terms of G and ξ for each environmental and behavioral effect, namely strain rate, repeated cycles, ageing, temperature, and production variability. In this paper only HDRs are considered (Table 2), results included LDRs may be found in [27]. More in detail, two values are provided for both UB and LB: one is related to the experimental data (for G the 75th and the 25th percentiles have been considered for UB and LB values respectively, whereas for ξ, instead, the 25th percentile has been chosen for both UB and LB values as lower damping is always a conservative value) while the other (bracketed) is the limit imposed by the EN15129 code during the elastomer type test (see §8.2.2.1 [3]). For the experimentally derived values, an exception is made for the temperature for which the median values have been chosen since the sample size at each temperature is not large enough to derive a reliable 25th-75th percentile range. As also shown in the previous sections, most of the experimental data are largely within the limits prescribed by the code, since most of the compound suppliers (in the European context) have de facto updated their products to agree with code prescriptions. For this reason, code-derived values in bracket can be considered as extreme limits for the variability of dynamic properties of bearings.

For comparison purpose, values suggested by the informative annex J of EN 15129 [3] (and informative annex JJ of EN 1998 – part 2 [4]) are also shown in Table 2 and for ageing and temperature. However, since values suggested in the annexes are given in terms of K_p (post-elastic stiffness) and F_0 (strength at zero displacement) of the bi-linear model, equivalent values in terms of G and ξ are computed by using formulas reported in ISO 22762 [28]. It should be observed that last values come from outdated experimental campaigns performed on rubber compounds produced by not European suppliers [22], thus they are not fully coherent to the qualification process currently required by the code itself. It is also worth to highlight that the values relevant to the temperature effect provided by [3] and [4] are inverted with respect to what is reported in the original table of AASHTO [19] and consequently they have been corrected for this elaboration [29]. Moreover, in the EN 15129 [3] HDR compounds are divided in two categories, based on the nominal value of ξ. In **Errore. L'origine riferimento non è stata trovata.** only factors related to $\xi_0 \leq 15\%$ are reported, as almost all the compounds analyzed are within this limit.

For frequency and repeated cycles, the annex J does not suggest any value, so the proposal is completely new. In particular, for the effect of repeated cycles, the proposal has been formulated for the 1st and the 10th cycle with respect to the reference condition (3rd cycle) in compliance with limits for the material tests (EN 15129 §8.2.2.1) and calibrated on the experimental data. The result is an UB value of 1.35 and a LB value of 0.8 for G, consistent with the limit of 0.6 for the variation between the 10th and the 1st cycle (0.8/1.35 = 0.6) but also in reasonable agreement with the 0.7 limit for the variation between the 10th and the 2nd cycle. For ξ, value of 0.8 and 1.0 are suggested, respectively, for LB and for UB. For the production variability, the experimental dataset shows a lower variation than code indications of 1.2 and 0.8.

However, it is worth to highlight that variations at the material scale are lower than full scale bearing due to more complex manufacturing process.

Table 2. λ-factors from experimental data, material test limits of EN 15129 §8.2.2.1 (bracketed), and Annex J for HDR.

		Experimental data (EN 15129 §8.2.2.1)				EN 15129 ANNEX J	
		G		*ξ*		*G*	*ξ*
		LB	UB	LB	UB		
Temperature	50°C	0.82 (0.80)	-	0.87 (0.80)	-	-	-
	40°C	0.88 (0.80)	-	0.91 (0.80)	-	-	-
	0°C	-	1.30	-	1.18	1.15	1.10
	-10°C	-	1.52 (1.80)	-	1.23 (1.8)	1.25	1.09
	-20°C	-	- (1.80)	-	1.26 (1.8)	-	-
	-30°C	-	- (1.80)	-	1.53 (1.80)	1.56	1.20
Ageing		1.00 (1.00)	1.18 (1.20)	1.00 (1.00)	0.86 (0.80)	1.20	1.00
Frequency		0.94 (0.80)	1.16 (1.20)	0.93 (0.80)	0.91 (0.80)	-	-
Rep. cycles		0.90 (0.80)	1.18 (1.35)	0.95 (0.80)	1.09 (1.00)	-	-
Production Variability		0.95 (0.80)	1.07 (1.20)	1.00 (0.80)	1.00 (0.80)	-	-

To better understand the impact of the new proposed λ-factors on the seismic response of base-isolated structures, a simplified application is presented analyzing a single degree of freedom model through response spectra in terms of pseudo-acceleration and displacement. More in detail, such application compares the variability of the response between the proposed λ-factors and those currently suggested by EN 15129 (Annex J) for HDRBs. The variation of both the isolation period (T_{is}) and the isolation damping are evaluated by using the λ-factors of G and ξ previously summarized in Table 2 and summarized in Table 3. In both the cases the production variability is assumed equal to the allowed by EN 15129 (0.8-1.2). In the last row of Table 3 the combined λ-factors for G and ξ for UB and LB analyses are also provided. They are computed according to the combination rule proposed by the annex J of EC8-part2 [4]. A combination factor ψ of 0.7 is used to account for the low probability of the simultaneous occurrence of all the maximum adverse effects, except for ψ of ageing assumed equal to 1, according to EN15129 [3]. The combined values are similar for the LB conditions. On the contrary, for UB the proposed values lead to an higher stiffness (due to a higher temperature effect: 1.52 with respect to 1.25) and a lower damping mainly related to the ageing effect (0.86 with respect to 1.2).

Table 3. Proposed experimental λ-factors, λ-factors from EN 15129 annex J and related coefficient of combinations ψ for the UB/LB analyses.

	Proposed				**EN 15129**				**ψ**
	G		***ξ***		***G***		***ξ***		
	LB	**UB**	**LB**	**UB**	**LB**	**UB**	**LB**	**UB**	**LB/UB**
Temperature (40°/-10°)	0.88	1.52	0.91	1.23	1.00	1.25	1.00	1.09	0.70
Ageing	1.00	1.18	1.00	0.86	1.00	1.20	1.00	1.20	1.00
Frequency	0.94	1.16	0.93	0.91	-	-	-	-	0.70
Rep. cycles	0.90	1.18	0.95	1.09	-	-	-	-	0.70
Production	0.95	1.07	1.00	1.00	0.80	1.20	0.80	1.20	0.70
Combined	**0.79**	**2.11**	**0.86**	**0.86**	**0.80**	**1.61**	**0.80**	**1.09**	

Fig. 9 reports response spectra in terms of pseudo-acceleration (S_a) and displacement (S_d) for two sites in Italy, L'Aquila and Naples, representative of high- and medium-hazard seismic areas respectively. Response spectra have been evaluated according to NTC 2018 [30] with a return period equal to T_R = 950 years (corresponding to a 5% probability of exceedance in 50 years).

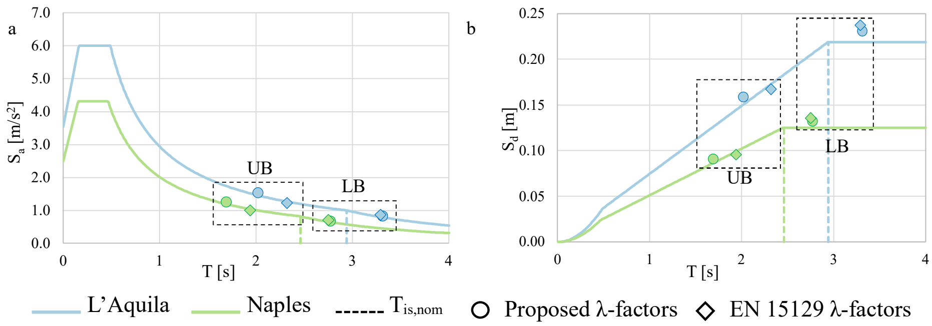


Fig. 9. Pseudo-acceleration (a) and displacement (b) response spectra at the design and U/L bound condition for high- (cyan) and medium-hazard (green) seismic areas.

The design (nominal) isolation periods in the two sites are assumed equal to the start period of the constant branch of the displacement spectrum, that are $T_{is,nom}$ = 2.45s for Naples and $T_{is,nom}$ = 2.95s for L'Aquila. The nominal isolation damping is $\xi_{is,nom} = 0.15$ for both the cases. The effect of the variability of G can be easily converted in terms of modified isolation period T_{is}, as it is proportional to the inverse of the square root of G (disregarding the effect of damping variability on the isolation period). As expected, UBDPs lead to the most demanding condition in terms of base shear and floor absolute accelerations for the superstructure, while the LBDPs provide the highest displacement for the isolation devices. However, an isolation period in the range of constant displacement branch of the spectrum is a robust solution since the increment of the period does not lead to an increment of the isolation system displacement. There is only a little increment of the displacement due to the damping ratio decrement. Moreover, it should be observed that the periods of vibration derived from the LB values are similar, thus the difference between displacements obtained with proposed and EN 15129 λ-factors for LB is reduced because it is only related to ξ. Differently, periods derived from UB values of the proposed approach are lower than ones computed according to EN 15129. Consequently, for what concerns the spectral acceleration, the UB analysis carried out with the proposed λ-factors gives pseudo-accelerations with a +25% respect to λ-factors provided by EN 15129, due mainly to the temperature effects. However, it is worth to recall that the temperature effect shall be disregarded if the rubber bearings are protected.

4 Conclusions

In conclusion, by using the proposed λ-factors all the effects, including the frequency and the repeated cycles, are correctly considered in the design, giving a more robust designed solution. The application shows the proposed values derived from the experimental campaign provide no significant differences in terms of isolation system the displacement and moderate increments of the spectral acceleration. This is a very important point since in the past HDR bearings with low shear modulus and/or high damping ratios were considered affected by a large "scragging" and this have significantly limited their use, especially for strategic structures in the U.S. Finally, as no major differences in the design procedure of the isolation system are recognized and most of the rubber suppliers can be in compliance with these variabilities, these preliminary results also highlight that the proposed values could be used as λ-factors generally agreed by both the manufacturers and the engineers, according to the procedure recently suggested by the ISO 22762.

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