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Floating particles transport through the free surface vortex technique: A novel numerical study to assess the interaction among different scales of vortex structures

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Floating particles transport through the free surface vortex technique: A novel numerical study to assess the interaction among different scales of vortex structures / Li, Zhixiang; Chen, Huixiang; Xu, Hui; Feng, Jiangang; Rossi, Mose; Qian, Shangtuo; Yang, Zixuan; Kan, Kan. - In: PHYSICS OF FLUIDS. - ISSN 1070-6631. - 36:10(2024). [10.1063/5.0232526]

Availability:

This version is available at: 11566/337214 since: 2024-11-14T13:24:36Z

Publisher:

Published

DOI:10.1063/5.0232526

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(Article begins on next page)

30 **1 Introduction**

31 Sewage pumping stations are a major component of sewage services, collecting sewage from urban pipelines
32 via sumps and lifting sewage to higher water levels to assist normal gravity flow in reaching its final destination,
33 a sewage treatment plant. Sewage pumping stations play a crucial role in ensuring urban sanitation and
34 environmental protection. In urban areas across most countries, these stations often encounter challenges
35 associated with the accumulation of floating fat and other debris, collectively referred to as "floating debris" in
36 their sumps. While large floating debris typically gets stacked and filtered out by trash racks before entering the
37 sump, smaller and more pliable floating debris like fat, oil, and grease (FOG) particles might accumulate above
38 the pump intake. Over time, these FOG floating particles undergo a series of chemical and physical transformations
39 until it becomes a rigid layer that covers the entire sump surface.¹ The presence of such rigid layers within urban
40 drainage systems blocks drainage and leads to rainwater and sewage overflow, which can damage sewage system
41 and pose environmental and health risks.^{2,3} As a result, the timely removal of this rigid layer from the sump is
42 crucial for both their correct operation and reliability as well. Nevertheless, this kind of intervention is not only
43 labor-intensive, hazardous, and unhygienic but also cost-consuming. Duinmeijer⁴ proposed the creation of FSV
44 within the sump to overcome the criticalities previously mentioned, which are a means of transport for floating
45 particles from the water surface down to the suction pipe. Since vortices have a strong suction capacity for
46 particles.⁵

47 To evaluate the transport capacity of FSV, it is necessary to understand the three-dimensional flow field
48 information of the vortices. Based on Navier-Stokes governing equations, different vortex theoretical models have
49 been derived to characterize and describe the tangential velocity, the water surface profile, and other parameters
50 such as the Rankine and Burgers vortex models.^{6,7} Starting from these models, the simplified vortex theoretical
51 models have been further improved and assessed through experimental data and then analyzed deeply with
52 computational fluid dynamics.⁸ It is all known that the presence of multiple phases and the associated mass
53 transport, including the particle movement and air entrainment, heighten the operational risks faced by pumping
54 stations. Scientists and technicians have focused their studies on establishing relationships between boundary
55 conditions, operational parameters, and the occurrence of FSV to provide a better prediction of such a physical
56 phenomenon. They also investigated the mechanisms underlying the formation of FSV, the rate of air entrainment,
57 and their impact on pump devices. In the investigation of the entrainment of floating particles by FSV, Naderi *et*
58 *al.*⁹ and Duinmeijer *et al.*¹⁰ used high-speed digital cameras to visually track and calculate the trajectories, as well
59 as the three-dimensional velocity components, of particles within FSV. Notably, Duinmeijer and Clemens²
60 observed that the motion trajectories of particles driven by FSV exhibit conspicuous chaotic behavior and are
61 highly sensitive to the initial conditions of these particles.

62 While most of the aforementioned studies have analyzed the motion of individual or multiple particles
63 propelled by FSV, there is a paucity of research delving into the motion of particle swarm and exploring the
64 potential coupling effects among coherent structures induced by these FSVs. At present, the discussions of FSV
65 flow predominantly rely on the Eulerian perspective, which is effective but cannot provide the transport and mixing
66 process of the coherent structures.¹¹⁻¹³ The Lagrangian system of flow structure identification method has gradually
67 developed. It views the flow field as a dynamic system comprising discrete particles and leverages disparities in
68 particle trajectories to identify Lagrangian coherent structures (LCS) within the flow field. Due to its inherently
69 swarm-based nature, which transcends the characteristics of individual particle trajectories, LCS offer precise
70 material transport evaluation and provides powerful opportunities to predict, and even influence, large-scale flow
71 characteristics and mixing events.^{14, 15} This paper aims to provide insights into the transport mechanism of FOG
72 particles and the dynamic characteristics of the flow structure, which will aid in making informed decisions
73 regarding operations and management, such as employing FSV for floating particles transport or undertaking
74 manual salvage operations.

75 While LCS have demonstrated their utility in various geophysical and biomedical applications, their
76 applicability within the context of FSV flows remains unexplored. In this paper, we employ the coupled level-set
77 and volume-of-fluid (CLSVOF) method in conjunction with large-eddy simulation (LES) methods to reproduce
78 the two-phase vortex flow phenomena in the sewage sump. Through proper orthogonal decomposition (POD), the
79 intricate interaction among different-scale vortex structures is unveiled. The entrainment process of particles under
80 the force of FSV and the LCS transport mechanism are further analyzed, which are the key to understanding the
81 hydrodynamic mixing characteristics in the FSV system. The main content of this paper has been structured as
82 follows: Section 2 presents the geometric parameters and the numerical methods; in Section 3, two analysis
83 methods, namely POD and the finite-time Lyapunov exponent (FTLE), are introduced; in Section 4, the analysis
84 of the spiral vortex band, vortex structures, and particle trajectories constitutes the primary research content; lastly,
85 Section 5 gives the main conclusions derived from the research.

86 **2 Numerical model**

87 ***2.1 Governing equations***

88 Urban sewage pumping stations are composed of submerged-type pumps so that a free surface, which is the
89 interface between water and air, is present above them. When dealing with more than two fluids, numerical
90 simulations of water flow in pumps become a challenging task to deal with, especially when floating particles are
91 grouped on the sump free surface.¹⁶ Therefore, LES combined with CLSVOF to accurately capture the water-air
92 interface, particularly in regions where high curvature deformation occurs, and simulate the coherent structures
93 within the sump. The continuity and momentum equations of LES combined with CLSVOF are the following:

94
$$\frac{\partial \tilde{u}_i}{\partial x_i} = 0, \quad (1)$$

95
$$\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \frac{\partial \tilde{u}_i}{\partial x_j} = -\frac{\partial \tilde{p}}{\rho \partial x_i} + \nu \frac{\partial^2 \tilde{u}_i}{\partial x_j \partial x_j} + \frac{\partial \tau_{ij}}{\partial x_j} + \frac{\sigma \kappa \delta_s n_i}{\rho} - g \delta_{i3}. \quad (2)$$

96 Here, p is the pressure; u_i stands for the fluids velocity vector; the tilde (\sim) denotes the filtered quantities in LES
 97 governing equations; σ is the surface tension coefficient; δ_{ij} is the Kronecker delta symbol (If $i=j$, $\delta_{ij}=1$, otherwise
 98 $\delta_{ij}=0$); κ is local mean curvature of the immiscible fluids interface; ρ and ν are the fluids mixture density and
 99 dynamic viscosity, respectively; g represents the effect of gravity and is the acceleration of gravity; The subgrid-
 100 scale stress τ_{ij} resulting from filtering is calculated by the Boussinesq hypothesis:¹⁷

101
$$\tau_{ij} = 2\nu_{SGS} \tilde{S}_{ij} - \frac{1}{3} \tau_{kk} \delta_{ij}, \quad (3)$$

102
$$\tilde{S}_{ij} = \frac{1}{2} \left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right), \quad (4)$$

103 where τ_{kk} is the isotropic part of τ_{ij} and ν_{SGS} is the sub-grid eddy viscosity. The wall-modeled LES (WMLES)
 104 method is adopted in the simulation. As opposed to the traditional LES method, WMLES does not need a grid
 105 refinement within the boundary layer, thus saving computational efforts.^{18, 19} Currently, this method is being used
 106 in numerical simulations of two-phase flow²⁰. In the WMLES method, ν_{SGS} is calculated by:

107
$$\nu_{SGS} = \min \left[(0.41d_w)^2, (0.2\Delta)^2 \right] \cdot \sqrt{2\tilde{S}_{ij}\tilde{S}_{ij}} \cdot \{1 - \exp[-(y^+ / 25)^3]\}, \quad (5)$$

108 where d_w is the normal distance from the grid node to the wall and y^+ is the dimensionless wall distance for a wall-
 109 bounded flow. When calculating the filter size, a wall damping function is introduced in the WMLES method,
 110 written as equation (6). Thus, this method is well suited for anisotropic grids reducing the requirement for grids
 111 near the wall and improving calculation efficiency:

112
$$\Delta = \min(\max(0.15d_w, 0.15h_{\max}, h_{wn}), h_{\max}). \quad (6)$$

113 Here, h_{\max} is the longest edge of a hexahedral cell (for other cell types, this concept is extended accordingly). h_{wn}
 114 is the grid spacing in the wall-normal direction.

115 The CLSVOF method has been well applied in simulating vortex phenomena in the sump or pipe.^{21, 22} This
 116 approach offers an accurate representation of the air-water interface while maintaining mass conservation.²³ The
 117 CLSVOF method tracks the interface of immiscible fluids by establishing a convection equation, from which it
 118 derives a volume fraction function F and a signed distance ϕ . The signed distance ϕ determines ρ and ν of the
 119 immiscible fluids in equation (2):

120
$$\phi(x, t) = \begin{cases} +|d| & x \in \text{water} \\ 0 & x \in \text{interface} \\ -|d| & x \in \text{air} \end{cases}, \quad (7)$$

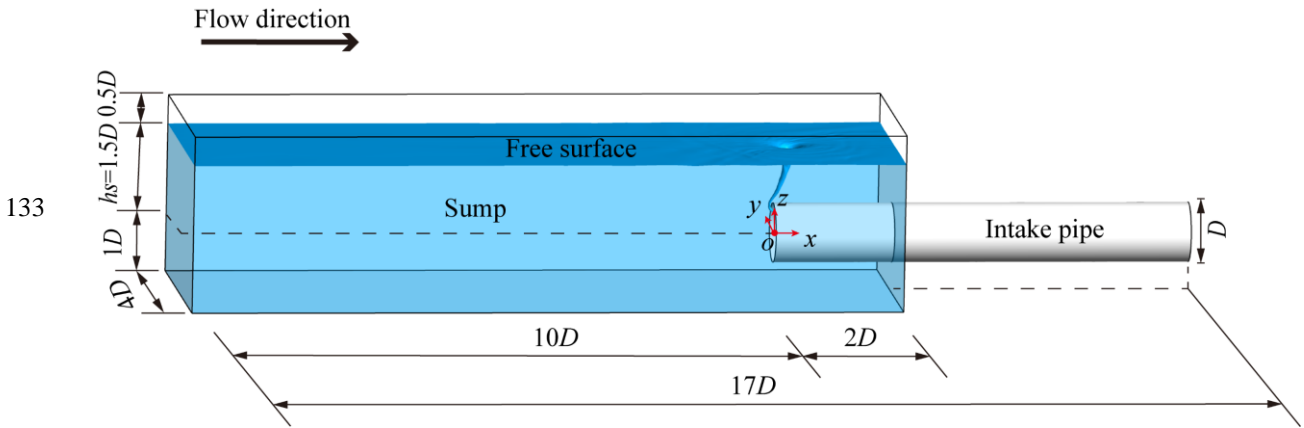
121
$$\rho = \rho_w H(\phi) + \rho_a [1 - H(\phi)], \quad (8)$$

122
$$v = v_w H(\phi) + v_a [1 - H(\phi)], \quad (9)$$

123 where d is the distance from the interface. Positive and negative values of ϕ represent water and air phases
 124 respectively. $H(\phi)$ stands for the smoothed Heaviside function.^{24, 25} All velocities are filtered in the subsequent
 125 section, while the tilde (\sim) is omitted for simplicity.

126 **2.2 Setup of the simulation**

127 Large-scale water supply and drainage pumping stations commonly employ a horizontal inflow configuration.
 128 ²⁶ As shown in Fig. 1, this layout simplified an inflow system with a horizontal pipe as a water inlet,^{27, 28} where D
 129 $= 0.4$ m is the pipe mouth diameter. The submergence hs and the suspended height of the pipe mouth are $1.5D$ and
 130 $1.0D$, respectively. To minimize the interference of domain inlet boundary on the flow near the pipe mouth, the
 131 maximum edge length for the sump is set to $12.0D$. The coordinates origin is established at the pipe mouth center.
 132 The main flow direction coincides with the positive direction of the x -axis.



134 Fig.1. Sketch of the sump model in a drainage pumping station.

135 The calculation conditions settings are shown in Table 1, where Q is the discharge. $Fr = u_D / \sqrt{gD}$,
 136 $Ds = Q / g^{0.5} hs^{2.5}$, $Re = u_D D / \nu$, and $We = \rho u_D D / \sigma$. Ds is the discharge-to-submergence ratio, quantifying the
 137 relationship between Q and hs .²⁰ Taking Case M3 as an illustrative example, the corresponding boundary
 138 conditions have been defined as follows: the upper section of the sump is defined as an opening surface with an
 139 atmospheric pressure. The walls of the sump and intake pipe have been configured as non-slip walls. The pipeline
 140 outlet employs a pressure outlet that adjusts in real-time with variations in the flow rate. When the discharge
 141 overcomes the expected value, the outlet pressure increases, and conversely, it decreases to maintain a steady
 142 discharge of $0.785u_D D^2$ throughout the computation process. The inlet of the sump adopts the air-water two-phase
 143 flow inlet. Pure water inlet ($\phi > 0$) is below $z = 1.5D$ with a pressure distribution conforming to the hydrostatic
 144 pressure. Above $z = 1.5D$, identical settings equal to the upper section have been applied.^{29, 30} The LES simulation

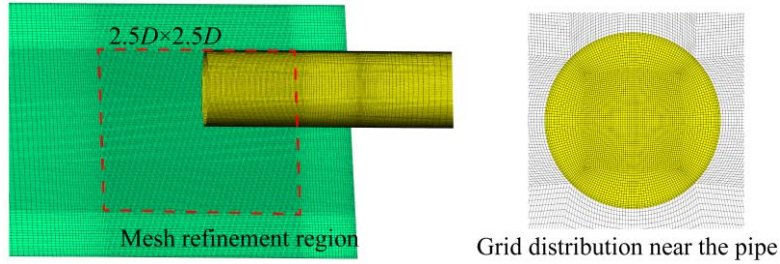
145 employed a time step of $6.41 \times 10^{-3} D/u_D$, with up to 20 iterations per step. The governing equation discretization is
 146 detailed in our previous studies.^{20, 31} The simulations of FSV were executed utilizing FLUENT 18.0 software,
 147 requiring an extensive computational time of approximately 210,000 CPU hours to complete.

148 Table 1 Parameters for FSV flow simulation cases.

Case Name	$Q/u_D D^2$	hs/D	Ds	Fr	Re	We
M1	0.392	1.50	0.21	0.75	5.88×10^5	1.82×10^4
M2	0.785	1.75	0.29	1.49	1.18×10^6	4.86×10^4
M3	0.785	1.50	0.43	1.49	1.18×10^6	4.86×10^4
M4	0.785	1.25	0.67	1.49	1.18×10^6	4.86×10^4
M5	1.375	1.50	0.75	2.62	2.06×10^6	1.49×10^5

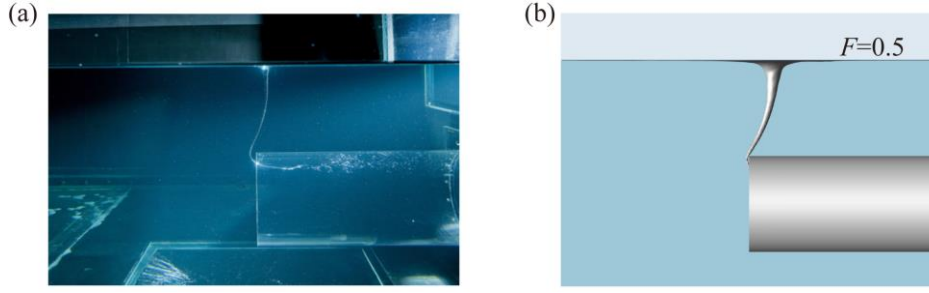
149 2.3 Validation of numerical results

150 ICEM has been used to mesh the entire computational domain, where the vortex generation region is refined.
 151 Grid independence analysis has been performed using the widely accepted GCI criterion.^{32, 33} In FSV flow, vertical
 152 vorticity ω_z is the important characteristic parameter and is chosen as the evaluation parameter for the grid
 153 convergence analysis. Three sets of grid schemes, ranging from fine to coarse ($G1 = 17,456,622$; $G2 = 8,645,974$;
 154 $G3 = 4,224,656$) are designed for grid error analyses. The convergence index GCI_{fine}^{21} of ω_z has fallen below 3%,
 155 signifying that the refinement of Grids G1 and G2 adequately meets the required convergence criterion.^{31, 34} Grid
 156 G2 has subsequently been selected for the simulations due to its computational efficiency, as depicted in Fig. 2.



158 Fig. 2. Mesh generation of Grid G2.

159 Fig. 3 presents the comparison of the air-water interface between the experimental result from Möller *et al.*³⁵
 160 and the simulation result. The representation of the free surface is achieved using $F = 0.5$. In general agreement
 161 with the results of experiment, numerical calculation effectively captures the overall air-water interface shape as
 162 it develops from the water surface to the pipe mouth. The upper part of FSV forms a funnel shape structure, with
 163 its central part resembling an elongated air tube, while the tail transitions into a pronounced hook-shaped pattern.
 164 The simulation employed a higher flow rate compared to the experimental setup, resulting in the formation of a
 165 more pronounced vortex and an increased diameter of the air tube.



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167

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Fig. 3. Comparison of free surfaces: (a) experimental picture from Möller *et al.*³⁵ and (b) $F = 0.5$ iso-surface of Case M3.

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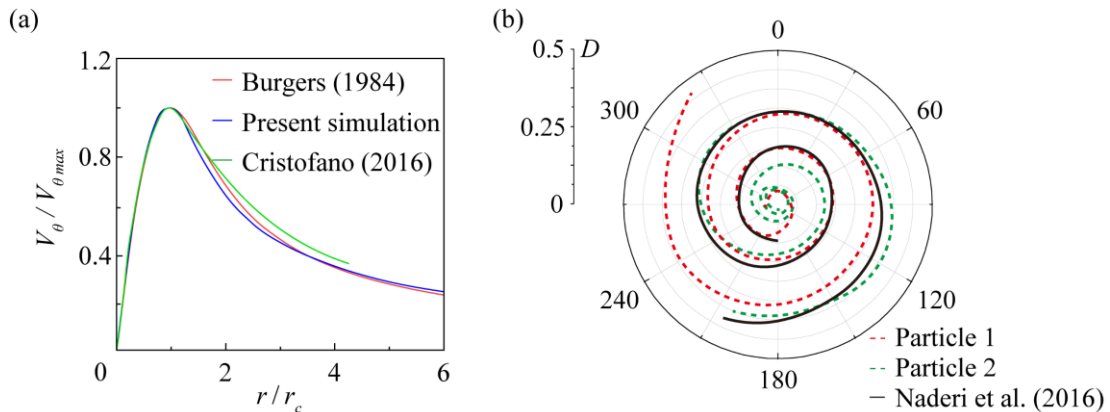
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Fig. 4(a) presents the comparison of tangential velocities distribution of FSV between the simulation, experiment, and the theoretical results. Burgers vortex model has been selected as the theoretical model, given its well-verified accuracy in representing tangential velocity according to previous experiments.³⁶ The graph demonstrates that the calculation result is consistent with Burgers theoretical model. Tangential velocity V_θ increases approximately linearly for radius as $r/r_c < 1$, and then slowly decreases. r_c denotes the radius of the vortex core with maximum V_θ . Considering the primary focus on particle transport and mixing mechanisms, it is imperative to compare the differences in particle trajectories between the experiment and simulation results, as plotted in Fig. 4(b). It illustrates that the particle motion trajectories, which are obtained from the simulation results closely correspond to the measured trajectories from the experiment, while some deviation exists due to the different arrangement of the intake pipe in the experiment that is situated at the bottom, unlike in the simulation case.



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Fig. 4. (a) radial profiles of the tangential velocity and (b) particle trajectories from experiment and simulation results.

183 3 Coherent structure analysis

184 3.1 POD analysis

185

186

187

Proper orthogonal decomposition (POD) is a data processing technique commonly employed for analyzing large volumes of data obtained from experiments or numerical simulations of unsteady flow fields. POD decomposes these physical quantities into a series of orthogonal basis functions, referred to as "POD modes", that

188 are optimal in a least-squares sense. Each mode is associated with eigenvalues that represent its energy content.
 189 Due to the ability to establish correlations between coherent structures and the energy they encompass, POD has
 190 been used to identify structures at different energy levels within turbulent flows. With respect to the motion of
 191 particles within the flow field, various flow structures (coherence structures) serve as carriers to determine particle
 192 trajectories.

193 Currently, the most widely used POD method, based on the snapshot method introduced by Sirovich³⁷, is a
 194 mathematical improvement over direct POD that addresses the challenge posed by large spatial point datasets
 195 where the spatial correlation matrix cannot be efficiently computed due to the sheer number of spatial points. The
 196 snapshot POD method is particularly well-suited for Particle Image Velocimetry (PIV) experiments and numerical
 197 simulations, where the count of spatial points m substantially exceeds that of sampling points n .

198 The central idea of the snapshot POD method is to express the pulsation of a certain physical quantity as the
 199 product of a few POD bases φ_j and time modal coefficients $a_j(t)$ through the orthogonal decomposition:

$$200 \quad \mathbf{X}' = \sum_j a_j(t) \varphi_j, \quad (10)$$

$$201 \quad \mathbf{X}' = \mathbf{X} - \bar{\mathbf{X}}. \quad (11)$$

202 Here, $\mathbf{X} = [\mathbf{x}(t_1) \ \mathbf{x}(t_2) \ \dots \ \mathbf{x}(t_m)] \in \mathbb{R}^{n \times m}$ donates instantaneous physical field at m moments and $\bar{\mathbf{X}}$ is
 203 it's the average part. The eigenvector $\boldsymbol{\psi}$ and its corresponding eigenvalue λ of the constructed covariance matrix
 204 $\mathbf{X}^T \mathbf{X}$ can be solved:

$$205 \quad \mathbf{X}^{T'} \mathbf{X}' \boldsymbol{\psi}_j = \lambda_j \boldsymbol{\psi}_j, \quad (12)$$

206 where the symbol T indicates matrix transposition. The mode of the snapshot POD method can be solved as:

$$207 \quad \varphi_j = \frac{1}{\sqrt{\lambda_j}} \mathbf{X}' \boldsymbol{\psi}_j, \quad (13)$$

208 and time modal coefficient can be written as:

$$209 \quad a_j(t) = \langle \mathbf{X}'(t), \varphi_j \rangle. \quad (14)$$

210 POD modes are organized in descending order based on their associated eigenvalues. The energy fraction of
 211 a specific POD mode, relative to the total resolved energy, is proportional to its corresponding eigenvalue.³⁸
 212 Typically, the first k -order modes, whose energy is close to the overall kinetic energy of the flow field, are selected
 213 for analysis as follows:

$$214 \quad \sum_{j=1}^k \lambda_j / \sum_{j=1}^m \lambda_j \approx 1. \quad (15)$$

215 **3.2 Lagrangian approach**

216 This paper focuses on the dynamic process of FOG floating particles entrainment by vortices, a phenomenon
 217 predominantly governed by vortex structures. Most of the vortex feature extraction approaches are based on the
 218 Euler framework for analyzing vortices in unsteady flow fields, but these methods often lack a strict physical
 219 interpretation.¹⁴ In contrast, Lagrangian approaches exploit properties of fluid particle trajectories to identify
 220 coherent structures.^{14, 39} On the contrary, Lagrangian approaches offer objectivity, incorporate flow history
 221 information, and provide a clear physical interpretation. The finite-time Lyapunov exponent (FTLE) serves as a
 222 widely used method for identifying Lagrangian coherent structures (LCS) within velocity fields that are spatially
 223 and temporally discretized across domains with finite spatial and temporal dimensions. The "ridge" structure
 224 formed by the maxima in the FTLE field effectively reveals hidden coherent structures. Importantly, LCS does not
 225 necessitate artificial threshold value settings, unlike vortex criteria based on the Euler framework that heavily
 226 relies on threshold values. f-FTLE and b-FTLE can be obtained by integrating in the forward and backward time
 227 directions, respectively. Between them, the ridge of b-FTLE reveals the flow structures.^{40, 41} Ridges in the b-FTLE
 228 field can visually attract materials or unstable manifolds. In other words, along these ridges, fluid particles
 229 experience elongation when their trajectories are traced backward in time. To illustrate the entrainment within the
 230 vortex, the FTLE related to attracting material lines has been computed as depicted in Fig. 7.

231 A two-dimensional flow field represented by a transient velocity field $v(x, t)$, is known over a finite time
 232 interval $[t_0, t_1]$. The movement of fluid particles can be described by the following differential equation:¹⁵

$$\dot{x} = v(x, t), \quad (16)$$

234 where trajectories of fluid particles are represented by $x(t; t_0, x_0)$, t_0 stands for initial time and x_0 referring to initial
 235 positions. The displacement field $x(t; t_0, x_0)$ maps the movement from the initial positions x_0 to the current positions
 236 x over the time interval $[t_0, t]$. At position x_0 , the right Cauchy–Green strain tensor $C_{t_0}^t$ is adopted to capture the
 237 stretch induced by the velocity gradient, which is written as:

$$C_{t_0}^t(x_0) = \left(\frac{\partial x(t; t_0, x_0)}{\partial x_0} \right)^T \left(\frac{\partial x(t; t_0, x_0)}{\partial x_0} \right). \quad (17)$$

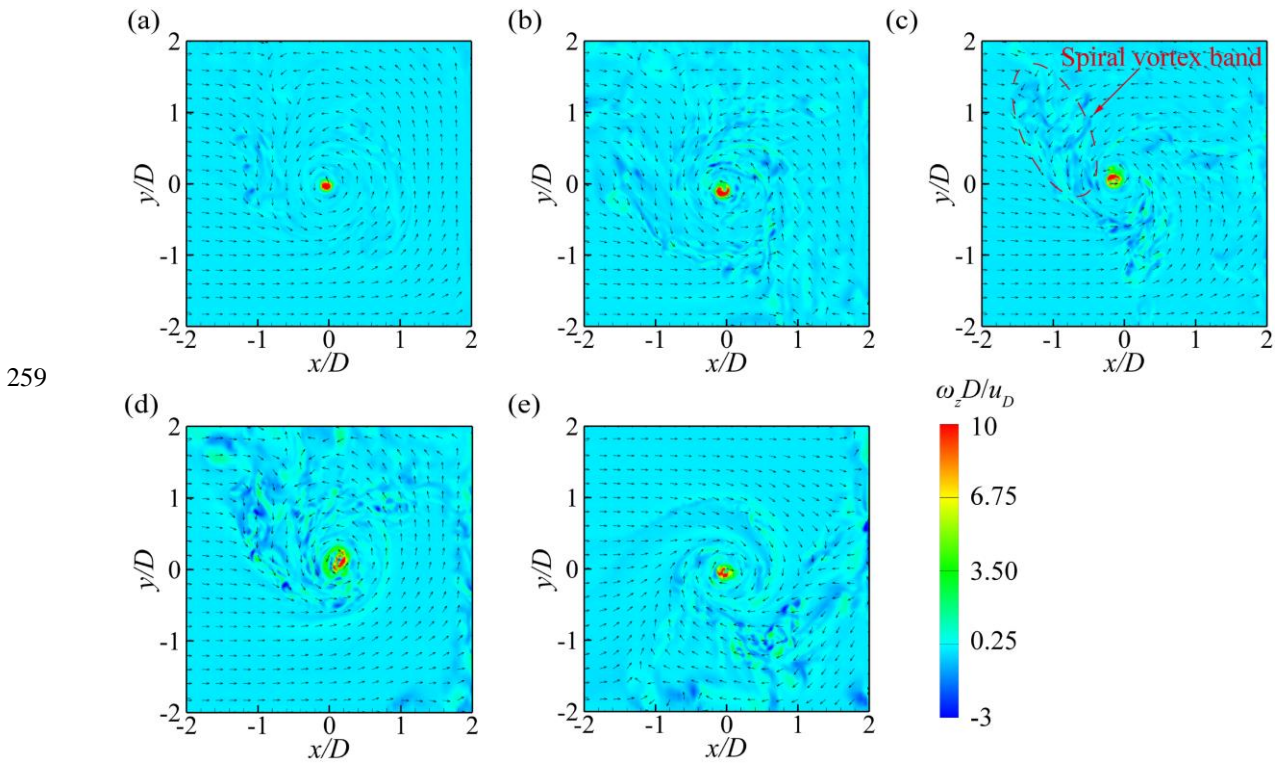
239 $C_{t_0}^t$ has two positive eigenvalues. The larger of the two eigenvalues λ_2 represents the maximum extension of the
 240 particle, and its corresponding eigenvector indicates the direction of the deformation. Following Haller³⁹, the FTLE
 241 for a period of time $[t_0, t_1]$ is defined as:

$$\text{FTLE}_{t_0}^{t_1}(x_0) = \frac{1}{|t_1 - t_0|} \log \sqrt{\lambda_2 [C_{t_0}^{t_1}(x_0)]} \quad (18)$$

243 **4 Results and analyses**

244 **4.1 Spiral vortex band**

245 Fig. 5 shows the vertical vorticity distribution on the horizontal plane $z = 0.9hs$ under various operating
 246 conditions at different discharges and submergences in the sump. Plane $z = 0.9hs$ is selected as the characteristic
 247 plane because the vortex behavior on this particular plane exhibits a relatively stable pattern and can also well
 248 reflect the flow structure beneath the surface.^{24, 35} Notably, the velocity vectors in Figs. 5(a)-(d) depict the
 249 counterclockwise rotation of all the vortices, while in Fig. 5(e) the vortex rotates clockwise. Provided that the
 250 inlet/outlet boundaries, as well as the sump geometric model, are entirely symmetrical, the FSV rotation direction
 251 is random. For ease of discussion and comparison, the vertical vorticity values for Case M5 in Fig. 5(e) have been
 252 inverted. In all subplots, the maximum value of the vertical vorticity is focused at the vortex center, while in a
 253 specific region outside the center, the vorticity approaches zero. The velocity vector diagrams reveal the presence
 254 of a spiral vortex band surrounding the FSV in regions where the water flow convergence or significant flow
 255 velocity gradient changes occur. These bands originate either at the side wall or at the confluence points of water
 256 flows, spiral in alignment with the rotational direction of FSV, and ultimately converge into it. There are counter-
 257 rotating vortices in the vortex band opposite to the vorticity of FSV. As Ds increases, both the extent and intensity
 258 of the spiral vortex bands increase as well.



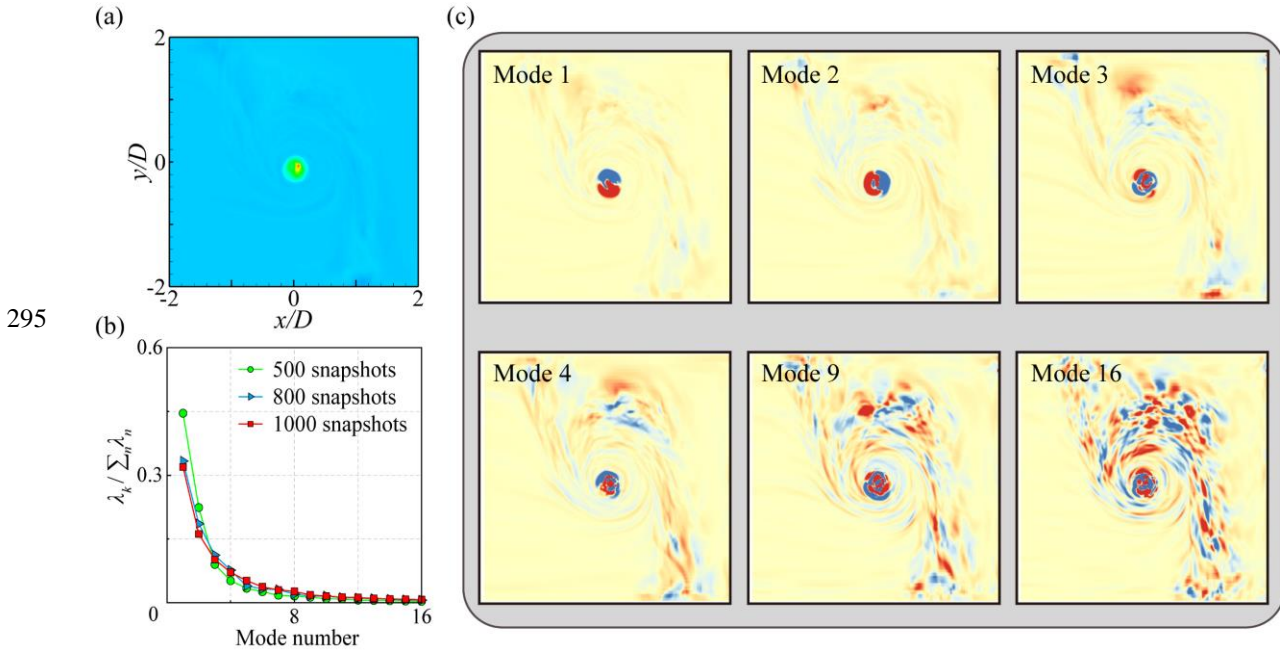
260 Fig. 5. Distribution of vertical vorticity around FSV: (a) M1, $Ds = 0.21$; (b) M2, $Ds = 0.29$; (c) M3, $Ds = 0.43$; (d)
 261 M4, $Ds = 0.67$ and (e) M5, $Ds = 0.75$.

262 4.2 POD analyses of FSV *flied*

263 Following the analysis of the vorticity distribution on the characteristic planes under various flow parameter
264 conditions reported in Section 4.1, it has been determined that there are consistent flow phenomena among them.
265 As a result, Case M3 has been selected as a representative case. To analyze various coherent structures and their
266 transport regions in the suction vortex flow, the transient vertical vorticity of the characteristic plane is selected as
267 the physical quantity for the POD method, and the decomposition results are shown in Fig. 6. Precisely, Fig. 6(a)
268 presents the time-averaged vertical vorticity with a legend that is consistent with the one of Fig. 5. The POD modal
269 eigenvalues calculated with different numbers of snapshots are plotted in Fig. 6(b). POD results are sensitive to
270 the number of snapshots. Generally, eigenvalues exhibit rapid decay as the modal order increases, and then the
271 decay becomes more gradual when the mode order exceeds 12. Comparing 800 and 1,000 snapshots, eigenvalues
272 from both sets are consistent, indicating that the increase in the number of snapshots does not significantly alter
273 the decomposition results. Therefore, POD results obtained from 1,000 snapshots have been chosen for further
274 analysis. The first two modes have the largest eigenvalues and carry the most energy accounting for 32.9% and
275 17.2% of the total energy, respectively. As the mode order increases, the energy carried by the third and fourth
276 modes decays by 10.1% and 7.1%, respectively. The energy carried by higher-order modes, such as the 9th and
277 16th modes, is even lower and essentially negligible.

278 To further explore the relationship between POD modes and vortex structures of various spatial scales within
279 the flow field, the first four high-energy modes were plotted, as well as the 9th and 16th modes in Fig. 6(c). In the
280 first mode, it can be noticed that there is a pair of counter-rotating vortices at the position of FSV, which is
281 approximately distributed in a symmetrical manner along the spanwise direction. In the second mode, the vortex
282 pair rotates almost 90°, parallel to the main flow direction. These findings are consistent with those from POD
283 analyses performed on trailing vortices from wings and inlet vortices from engines.^{38, 42, 43} They argued that the
284 first two modes are strongly correlated with the displacement of the vortex; however, when pioneers explained the
285 physical meaning of higher-order modes, they tended to downplay the effect of these modes due to their lower
286 energy content, as depicted in Fig. 6(b). Nevertheless, even though the energies associated with higher-order modes
287 are lower compared to the previous modes, they represent the patterns of small-scale coherent structures near the
288 FSV in the sump. Moving from the first mode to the fourth mode, it is clear that the absolute value of ω_z at the
289 location where the spiral vortex band is situated in Fig. 5 gradually increases with the increasing order of modes.
290 This trend is especially pronounced in the high-order mode (16th mode) where the vortex distribution within the
291 spiral vortex bands closely resembles the results of POD decomposition observed in a von Kármán vortex street.
292 In these regions, small-scale vortices form spiral vortex bands and gradually converge toward the center under the
293 rotational influence of FSV. Consequently, each order mode in the POD results effectively represents different-

294 scale coherent structures that act as carriers for floating particles in the flow field.



296 Fig. 6. POD analysis of FSV flow: (a) time-average vortical vorticity flowfield; (b) relative energy fluctuations
 297 captured by the POD modes, and (c) first four dominant POD modes, 9th and 16th POD modes.

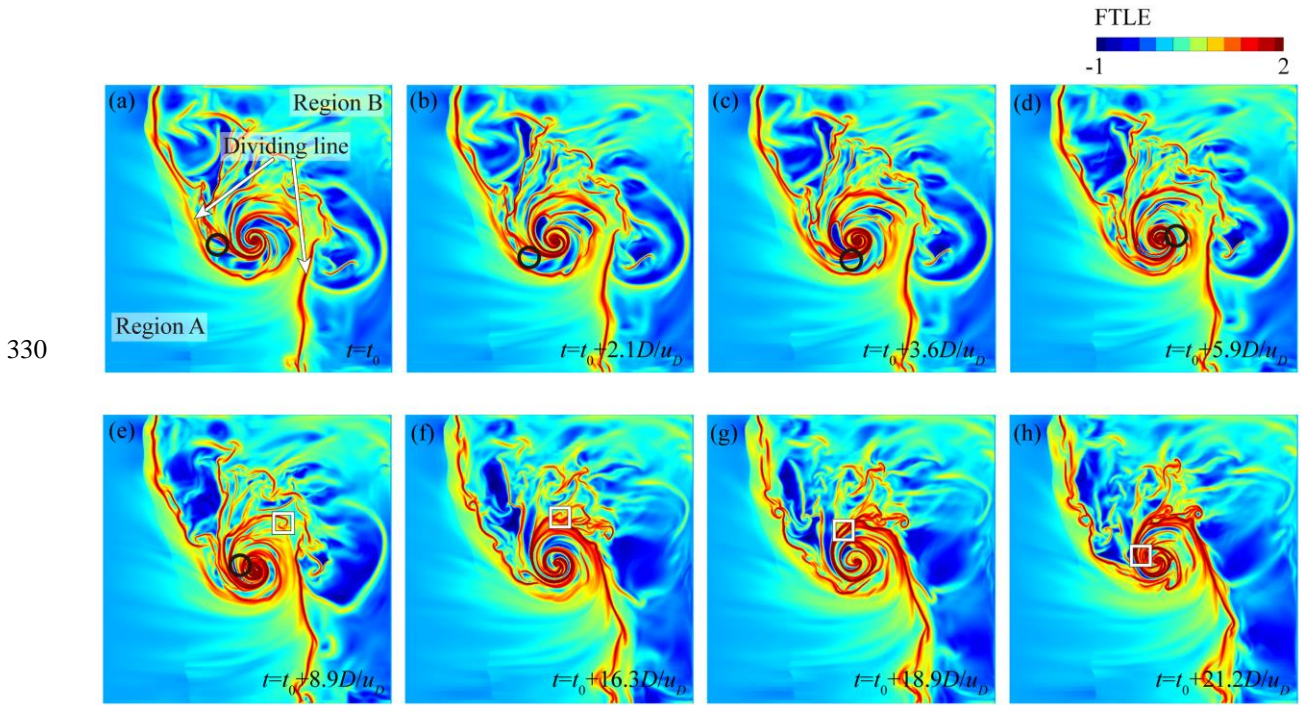
298 4.3 Lagrangian analysis

299 4.3.1 FTLE fields

300 While vortex bands are discernible in Fig. 5, it is crucial to notice that the selection of the threshold in the
 301 Euler method, which employs vorticity for vortex identification, plays a pivotal role in displaying the vortex
 302 morphology. This choice heavily relies on the sensitivity and experience of the researcher regarding vortex-related
 303 issues. Additionally, as it is clear in Figure 5, despite the author's efforts to optimize the vorticity display range,
 304 both the vortex motion patterns and boundaries of the vortex bands appear somewhat blurred. Although the POD
 305 decomposition can effectively decompose vortices of different scales (as shown in Fig. 6), the calculation process
 306 is relatively complex and depicts their instantaneous evolution. To this end, it is imperative to employ the FTLE
 307 method to capture the LCS within the FSV flow field. This approach allows the examination of the overall
 308 geometric structure of the vortex flow and provides a precise quantification of the flow characteristics of vortices
 309 at various scales, as well as a thorough analysis of the nearby particle transport and mixing processes.

310 Fig. 7 shows instantaneous FTLE fields on the characteristic plane. The dividing line formed by the upper
 311 and lower ridges surrounding the vortex center divides the flow field into two parts: the upstream main flow
 312 (Region A) and the reflected flow from the back wall (Region B). The material flux is very small at the clear
 313 dividing line formed by the flows on both sides, where the material and energy located in Region A will not cross
 314 the dividing line into Region B. This finding aligns with the research conducted by Shadden *et al.*⁴⁴ that analyzed
 315 observational data of coastal flows through LCS and identified separation areas in ocean currents, namely inner

316 and outer circulation. It is worth highlighting that particles within the water tend to converge towards nearby FTLE
317 ridges and are often captured by them. Comparing the results of the POD analysis in Fig. 6, the ridges of the FTLE
318 field clearly delineate the boundaries of energetic regions, which shows that the FTLE ridges can describe the
319 energy transport pattern carried by the coherent structure in the FSV field. Figs. 7(a-e) show the vortex structure
320 rolled up in the main flow direction. With the entrainment of FSV, it gradually enters the center. When the elliptical
321 LCS (marked with a black circle in Fig. 7(a)) approaches the vortex, the flow velocity increases and the structure
322 is stretched to become flatter, as depicted in Fig. 7(c). As the LCS progress towards the FSV center, it undergoes
323 further stretching, eventually forming a long and narrow spiral line that merges into the FSV center, as seen in Fig.
324 7(e). In Figs. 7(a-c), the coherent structure remains relatively stable, whereas in Figs. 7(d-e) its shape is influenced
325 by the FSV, thus leading to the gradual disappearance of the LCS structure. Similarly, Figs. 7(e-h) show the
326 evolution of LCS in the backflow from the back wall, which is consistent with the LCS evolution in Region A. In
327 summary, FTLE clearly shows the characteristics of the formation and evolution of the LCS due to the roll-up of
328 FSV existing in the pumping station sump, which can provide insights into the transport mechanisms of coherent
329 structures.



331 Fig. 7. Instantaneous FTLE flow fields.

332 4.3.2 Transport process based on trajectories

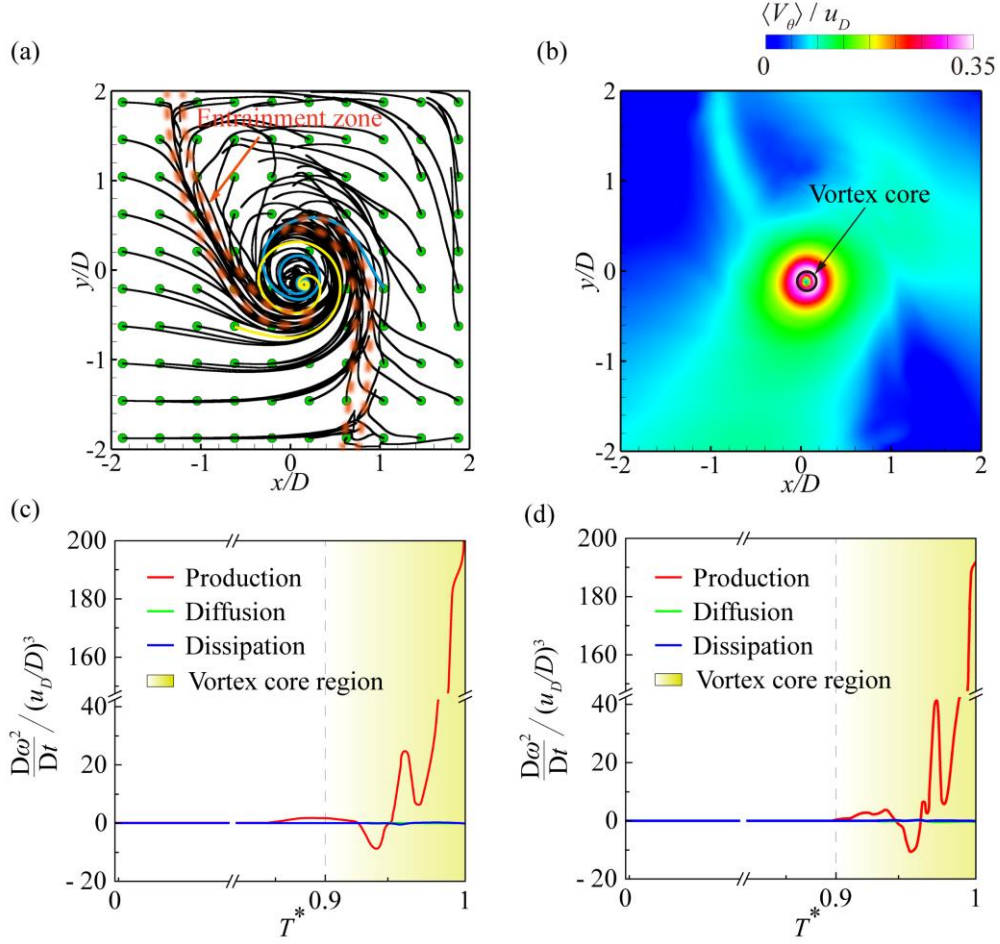
333 From the evolution of the aforementioned LCS, the coherent structures undergo significant stretching,
334 resulting in significant alterations in the shape and intensity of the vortex. Enstrophy $\omega^2/2$ can represent the vortex
335 intensity. To examine the dynamics of the vortex, the enstrophy transport equation is utilized, expressed as:⁴⁵

336
$$\frac{D\omega^2/2}{Dt} = \omega_i\omega_j \frac{\partial u_i}{\partial x_j} + \nu \nabla^2 \omega^2/2 - \nu \frac{\partial \omega_i}{\partial x_j} \frac{\partial \omega_i}{\partial x_j}. \quad (19)$$

337 On the right hand of the equation, the first term represents the effects of the production term, due to the vortex
 338 stretching and tilting. The second term accounts for the effects of viscous diffusion, and the third term represents
 339 the viscous dissipation of the fluids.

340 From the Lagrangian perspective, the motion trajectories of particles at different positions in the system are
 341 captured and plotted in Fig. 8(a). Green points donate the initial locations of particles. The trajectories of particles
 342 illustrate how floating particles, represented by particles with good water flow followability, are entrained by FSV
 343 within the sump. Particles from the inflow and those from the backflow mix together in the entrainment zone.
 344 Comparing the FTLE field in Fig. 7, it is evident that the entrainment zone is actually a strip region with the
 345 dividing line as the bone line. Sultana *et al.*⁴⁶ showed that more than 85% of the FOG particles were between 373-
 346 2280 μm in size. Consequently, in this study, the behavior of FOG particles in the sump has been represented with
 347 fluid particles. While there may be deviations, this approach effectively models the dynamic process of the vortex
 348 entrainment of FOG particles, as tested in previous researches.^{4, 47, 48} When FSV entrains FOG particles within the
 349 sump, particles from different areas initially converge along the dividing line formed by the FTLE ridge, mixing
 350 within the entrainment zone. Subsequently, these particles follow the FTLE ridges, spiraling into the vortex at the
 351 center of the sump and ultimately being drawn into the suction pipe along with the movement of FSV.

352 In Fig. 8(a), two trajectories, each marked with a different color, are selected to compare the variations in
 353 each term of the equation (18) along distinct pathways. The horizontal axes T^* in Figs. 8(c) and (d) represent the
 354 non-dimensional time, which has been normalized by the time required for each particle to move to the center of
 355 the vortex. In Fig. 8(c), it is observable that, throughout much of the particle entrainment process, each term of the
 356 enstrophy transport equation remains relatively constant. Once the particle enters the vortex core region, as
 357 depicted in Fig. 8(b), the production term begins to rise rapidly, while the other two do not exhibit significant
 358 changes. This distinction indicates that only the vortex core region, representing the rotational motion of a rigid
 359 body, contributes to the propagation of the vortex. The stretching and tilting effects of the vortex represented by
 360 the production term are the primary factors contributing to the increased intensity of the vortex. The budgets and
 361 the mechanisms described next are found to be the same along the two trajectories, so only the yellow one is further
 362 analyzed.



363

364 Fig. 8. Trajectories analysis based on Lagrangian perspective: (a) trajectories of the particles entrained into FSV;
 365 (b) the tangential velocity contour; (c) enstrophy budget of the trajectory of Particle A (yellow) and (d) enstrophy
 366 budget of the trajectory of Particle B (blue).

367 From the Lagrangian perspective, the variation in the three terms of the enstrophy transport equation is
 368 analyzed, concluding that the dominant term is the production one. Given the rotational symmetry of the vortex
 369 flow, it becomes challenging to concisely analyze each term of the equation within the Cartesian coordinate system.
 370 Consequently, FSV from the Cartesian system is translated into the cylindrical coordinate system. This conversion
 371 facilitates a more effective analysis of both primary and secondary components of each term. The expanded form
 372 of the production term in the cylindrical coordinate system includes:

$$P_{\omega} = \begin{bmatrix} P_{\omega 1} \\ P_{\omega 2} \\ P_{\omega 3} \end{bmatrix} = \begin{bmatrix} \omega_r \omega_r \frac{\partial V_r}{\partial r} & \omega_r \omega_{\theta} \left(\frac{\partial V_r}{r \partial \theta} - \frac{V_{\theta}}{r} \right) & \omega_r \omega_z \frac{\partial V_r}{\partial z} \\ \omega_{\theta} \omega_r \frac{\partial V_{\theta}}{\partial r} & \omega_{\theta} \omega_{\theta} \left(\frac{\partial V_{\theta}}{r \partial \theta} + \frac{V_r}{r} \right) & \omega_{\theta} \omega_z \frac{\partial V_{\theta}}{\partial z} \\ \omega_z \omega_r \frac{\partial V_z}{\partial r} & \omega_z \omega_{\theta} \frac{\partial V_z}{r \partial \theta} & \omega_z \omega_z \frac{\partial V_z}{\partial z} \end{bmatrix}, \quad (20)$$

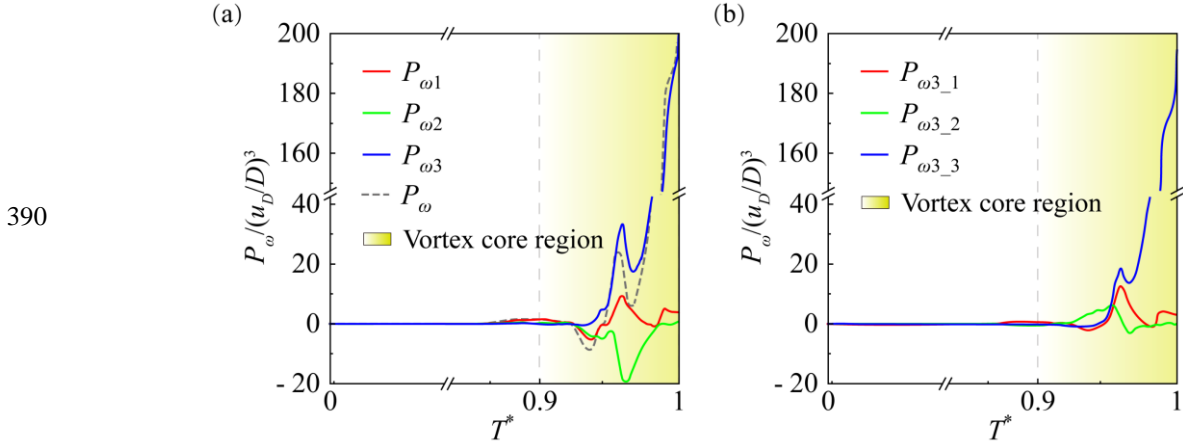
374 where the subscripts 1-3 of P_{ω} represent the radial r , tangential θ , and axial z direction components in the
 375 cylindrical coordinate system, respectively. The transformations between the variables of cylindrical and Cartesian

376 coordinate systems read as:

$$377 \begin{bmatrix} V_r \\ V_\theta \\ V_z \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}, \quad (21)$$

$$378 \theta = \tan^{-1} \left(\frac{y - y_0}{x - x_0} \right). \quad (22)$$

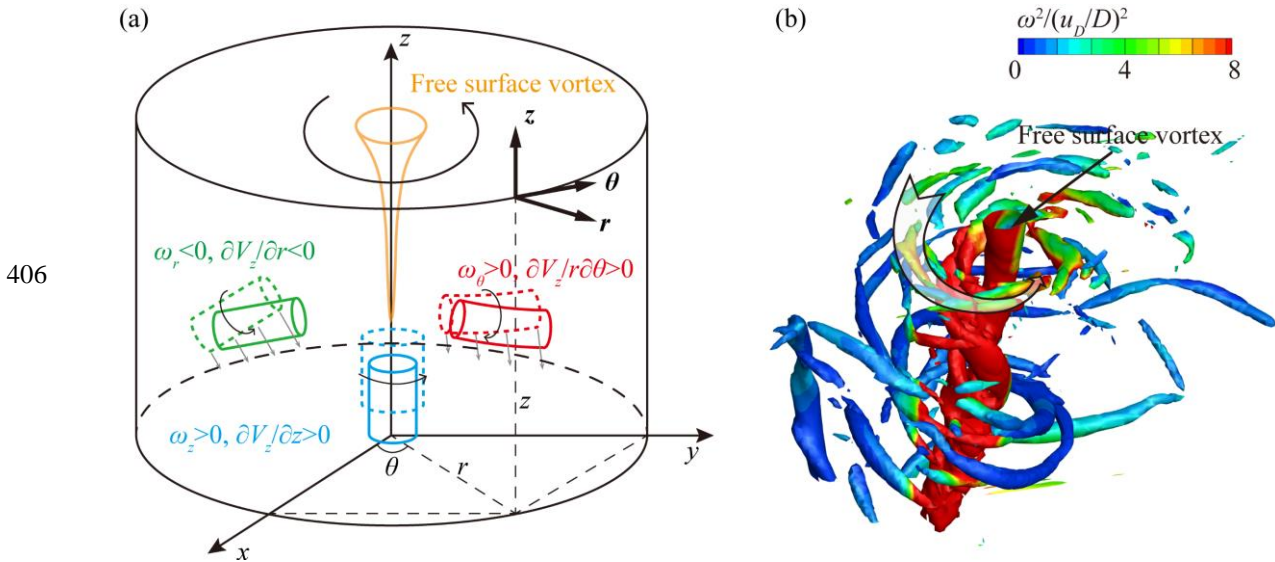
379 Here, (x_0, y_0) is the FSV coordinate. Fig. 9(a) illustrates the variation in the production term across the three
 380 directional components along the particle trajectory. The axial component of the production term (P_{ω_3}) is
 381 particularly significant and exhibits a high degree of similarity with the overall production term P_ω . The budget of
 382 the other two direction components is opposite and tends to cancel each other. This phenomenon can be also found
 383 in the stable laminar rotating thermal plumes, where the vortex transport equation is employed in the cylindrical
 384 coordinate system.⁴⁹ Moreover, the budgets of the tilting ($\omega_z \omega_r \frac{\partial V_z}{\partial r}$ and $\omega_z \omega_\theta \frac{\partial V_z}{r \partial \theta}$, i.e., $P_{\omega_3_1}$ and $P_{\omega_3_2}$) and
 385 stretching terms ($\omega_z \omega_z \frac{\partial V_z}{\partial z}$, i.e., $P_{\omega_3_3}$) of P_{ω_3} are plotted in Fig. 10(b), revealing that the stretching term plays a
 386 dominant role in maintaining the high-intensity entrainment of the vortex. Although the other two terms (tilting
 387 terms) fluctuated during the transportation, they eventually converged to zero. These two terms are tilted by radial
 388 and tangential velocity gradients ($\frac{\partial V_z}{\partial r}$ and $\frac{\partial V_z}{r \partial \theta}$) during the transportation process and shifted from ω_r and ω_θ
 389 to ω_z .^{20, 49}



391 Fig. 9. Budgets of production term: (a) the radial, tangential, and axial direction components of P_ω and (b) the
 392 tilting and stretching components of P_{ω_3} .

393 Removing the communal terms ω_z in P_{ω_3} , the physical meaning of three items ($\omega_z \omega_r \frac{\partial V_z}{\partial r}$, $\omega_z \omega_\theta \frac{\partial V_z}{r \partial \theta}$ and
 394 $\omega_z \omega_z \frac{\partial V_z}{\partial z}$) can be clarified: $\omega_r \frac{\partial V_z}{\partial r}$ and $\omega_\theta \frac{\partial V_z}{r \partial \theta}$ represent the tilting effect of radial and tangential velocity
 395 gradients on radial vorticity and tangential vorticity respectively. $\omega_z \frac{\partial V_z}{\partial z}$ represents the stretching effect of the

396 axial velocity gradient on the axial vorticity. Fig. 10(a) shows the process in which the radial and tangential vortices
 397 are distorted by the velocity gradients ($\frac{\partial V_z}{\partial r}$ and $\frac{\partial V_z}{r\partial\theta}$) and tilt toward the axial vortex. Red, green, and blue
 398 columns represent radial, tangential, and axial vortices, respectively. The dashed and solid columns represent the
 399 early and late stages of the vortices, respectively. The vortex evolution in Fig. 10(a) shows that, under the action
 400 of the velocity gradient, ω_r and ω_θ will eventually transform into ω_z . Fig. 10(b) shows vortex structures from the
 401 simulation results. The flow structure near the FSV is visualized by using the Q -criterion and dyed by ω^2 , which
 402 demonstrates how radial and tangential vortices spiral and merge into the FSV center due to the influence of the
 403 velocity gradient tilting terms. These results do not only provide strong support for the previous discussion
 404 regarding the transformation of vortices with different directions but also exhibit a consistent evolution with the
 405 LCS observed in Fig. 7.



407 Fig. 10. (a) schematics of distortion effect from the velocity gradients and (b) vortex
 408 structures (iso-surface of $QD^2/u_b^2 = 5.5$).

409 5 Conclusions

410 The LES turbulence model with the CLSVOF method has been used to simulate the FSV phenomenon in the
 411 sewage pumping station. Based on these comprehensive flow fields, the transport and mixing characteristics of
 412 floating particles are studied when there is a FSV at the sump. The main results are the following:

413 (1) The spiral vortex band induced by FSV is studied for the first time and decomposed by the POD method.
 414 Two coherent structures have been identified: FSV represented by the first two high-energy orders, exceeding half
 415 of the total resolved turbulent energy, and the spiral vortex band represented by the low-energy higher orders,
 416 which consists of numerous small-scale vortices with different rotation directions.

417 (2) The LCS approach together with particle tracking is applied to reveal the evolution of the LCS and particle
 418 transport in the sump. LCS can be well captured by the FTLE method. As the LCS flows towards the FSV center,

419 it undergoes significant elongation by forming an elongated spiral shape before ultimately converging at the vortex
420 center. Particles at different positions in the sump are first caught by the dividing line formed by FTLE ridges,
421 mixed in the entrainment zone, then move along the FTLE ridge pathways and spiral into the vortex center of the
422 sump. Eventually, they are drawn into the intake pipe and discharged in conjunction with the movement of the
423 vortex.

424 (3) The dynamic spiral motion of particles is analyzed using the enstrophy transport equation in cylindrical
425 coordinates to fully understand the entrainment mechanism of FSV on particles. It can be found that the dominant
426 term responsible for the vortex entrainment of particles is the production term, which increases as a result of the
427 stretching of the axially moving flow. The vorticity in the other two directions is converted into axial vorticity due
428 to the inclination of the velocity gradient, further enhancing the entrainment effect of FSV on the particles in the
429 vortex core region and accelerating the particle motion towards the pipe mouth.

430 The analytical methods used in this study have proven to be effective tools for predicting transport capacity
431 and understanding the mechanisms of the FSV. The research results can provide a reference for the renovation
432 design of existing sewage pumping stations and the operation of various urban drainage pumping stations.
433 Furthermore, the design specifications for pumping stations can be supplemented and improved regarding the
434 accumulation of floating debris in the sump. When utilizing FSV for FOG removal, the velocity gradients in the
435 sump should be enhanced, for example by adjusting valves to lower the submergence or by activating additional
436 units to increase the flow rate. The properties of FOG particles (e.g., composition and shape) influence their
437 transport and mixing processes induced by FSV, a topic that warrants further exploration in future studies.

438 **Acknowledgments**

439 This work was supported by the National Natural Science Foundation of China (52379086; 52009033), the
440 Jiangsu Innovation Support Programme for International Science and Technology Cooperation (BZ2023047), the
441 Postdoctoral Research Foundation of China (2022T150185; 2022M711021). The computational work was
442 supported by the High-Performance Computing Platform at Hohai University. The support of Hohai University is
443 also gratefully acknowledged.

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