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Stability in Parametric Resonance of a Controlled Stay Cable with Time Delay

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Stability in Parametric Resonance of a Controlled Stay Cable with Time Delay

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The stability of the parametric resonance of a controlled stay cable with time delay is investigated. The in-plane nonlinear equations of motion are initially determined via the Hamilton principle. Then, utilizing the method of multiple scales, the modulation equations that govern the nonlinear dynamics are obtained. These equations are then utilized to investigate the effect of time delays on the amplitude and frequency-response behavior and, subsequently, on the stability of the parametric resonance of the controlled cable, that it is shown to depend on the excitation amplitude and the commensurability of the delayed-response frequency to the excitation frequency. The stability region of the parametric resonance is shifted, and the effects of control on the cable become worse by increasing time delay. The work plays a guiding role in the parametric design of the control system for stay cables.

Keywords: stay cable; parametric resonance; vibration control; time delay; stability.

1. Introduction

As the primary load-bearing component in long-span structures, stay cables are characterized by their light weight and low damping, making them susceptible to vibrations induced by external factors such as wind, rain, traffic, or earthquakes^{1,2,3}. In recent years, large-scale cable vibrations on bridges have been observed even at low wind speeds, often attributed to a parametric resonance phenomenon^{4,5}. Therefore, it is crucial to investigate the vibration mechanisms of these cables.

With the combined contributions of multiple researchers, the study of cable vibration mechanisms has been substantially advanced. Hikami et al.⁶ and Shiraiishi et al.⁷ investigated the mechanism of rain-wind induced vibration of cable of cable-stayed bridges and proposed aerodynamic countermeasures to suppress the

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vibration. Jafari et al.⁸ conducted a meticulous review of past studies, exploring various forms of cable vibrations caused by wind. This extensive survey of the existing literature contributes to our understanding of the diverse manifestations of wind-induced cable vibrations. Furthermore, Wang et al.⁹ addressed the large amplitude motion mechanism and the non-planar vibration characteristics of stay cables subject to the support motions. Gattulli et al.¹⁰ analyzed the modal interactions in the nonlinear dynamics of a beam-cable-beam. Guo et al.^{11,12} studied the modal dynamics of boundary-interior coupled structures. Zhao et al.^{13,14} discussed the analytical solutions for resonant response of suspended cables subjected to external excitation. It's worth noting that among these studies, parametric vibrations of the stay cable have been a significant focus. Ying et al.¹⁵ investigated the parametrically excited instability of a cable under two support motions. Lu et al.¹⁶ studied nonlinear parametric vibration with different orders of small parameters for stayed cables, further enhancing our understanding of this intricate domain.

In parallel with the studies focusing on understanding complex nonlinear phenomena, other research has concentrated on vibration control in stay cables. Passive control was initially employed, but its effectiveness was limited by external excitations^{17,18,19}. With the continuous development of modern control theory, active control was applied in cable vibration control^{20,21,22,23}. However, after adopting active control techniques, unavoidable time delays were introduced in various components, including sensors, filters, controllers, and actuators, which in turn affected the stability and control performance of the entire controlled system^{24,25,26,27}. In response to this challenge, researchers have conducted a series of in-depth studies. Cha et al.²⁸ studied time delay effects on large-scale MR damper based semi-active control strategies. Udwadia et al.²⁹ presented principles and applications of time-delayed control design for active control of structures. Yan et al.³⁰ considered energy determining multiple stability in time-delayed systems. Furthermore, Wang et al.^{31,32} studied effect of delay combinations on stability and Hopf bifurcation of an oscillator with acceleration-derivative feedback. Sun et al.³³ studied parameter design of a multi-delayed isolator with asymmetrical nonlinearity. Their results showed that time delay can affect the damping performance of the control system, and, on the other hand, making good use of it can provide another control idea and improve control performance. In this context, the time-delay dynamics of the damper-cable system under uniformly distributed harmonic loads and in condition of 1:1 internal resonance have been investigated by the author, that found that time-delay is an unavoidable factor to be considered³⁴.

As a matter of fact, to the best of the authors' knowledge, no studies have addressed the time delay effects in nonlinear parametric resonance of controlled cables, that occurs when one end of the cable is subjected to an harmonic motion, and filling this gap is the main goal of this work. It leads to interesting and partially unexpected results in terms of performance (or better, loss of performance) of the considered control. The mechanical model of controlled cable under axial excitation

is considered. The method of multiple scales is used to analyze the parametric vibration under the influence of time delay. The stability of parametric resonance of the controlled stay cable is discussed, and the time delay effect of the parametric vibration system is discussed by numerical examples.

2. Controlled cable model and equations of motion

As shown in Figure 1, a stayed cable subject to a vertical sinusoidal support motion $Z \sin \omega t$ (where Z and ω denote the amplitude and frequency, respectively), is considered. A Cartesian coordinate system $O - xy$ is chosen, with the origin O placed at the left fixed support A of the cable. The displacements of the points are denoted by $u(x, t)$ and $v(x, t)$ along the x and y directions, respectively. a is the distance between the right oscillating boundary B and the MR damper.

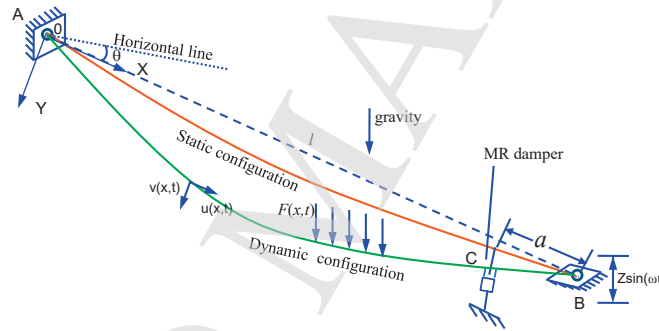


Fig. 1. The configuration of the controlled cable model.

The axial Lagrangian strain of the inclined cable can be written as

$$\varepsilon(x, t) = u' + y'v' + \frac{v'^2}{2}, \quad (2.1)$$

where prime indicates differentiation with respect to the spatial coordinate x and $y(x)$ is the static configuration of the cable, that can be approximately written as $y(x) = \frac{mg l \cos \theta}{2H} x(1-x)$. The equations of motion can be obtained by means of the Hamilton principle¹¹

$$m\ddot{u} + c_u \dot{u} - \left\{ EA \left[u' + y'v' + \frac{v'^2}{2} \right] \right\}' = 0, \quad (2.2)$$

$$m\ddot{v} + c_v \dot{v} - \left\{ H v' + EA(y' + v') \left[u' + y'v' + \frac{v'^2}{2} \right] \right\}' = 0, \quad (2.3)$$

where dot indicates differentiation with respect to time t , m is the mass per unit length; E is the Young modulus, A is the area of the cross-section, c_u and c_v are the

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viscous damping coefficients per unit length, H is the axial component of the initial tension ($H \ll EA$) and g is the gravity acceleration. The boundary conditions can be written as

$$\begin{aligned} u(0, t) &= v(0, t) = 0, \\ u(l, t) &= Z \sin \theta \sin(\omega t), \\ v(l, t) &= Z \cos \theta \sin(\omega t) \end{aligned} \quad (2.4)$$

where l is the cable span and θ is the angle of inclination of the cable (see Figure 1). It is worth noting that the boundary conditions are nonhomogeneous both in the axial displacement component $u(x, t)$ and in-plane transverse displacement component $v(x, t)$.

Under the quasi-static assumption in the axial direction, i.e., neglecting the acceleration and velocity term in equation (2.2), and taking into account the boundary conditions, the displacement $u(x, t)$ can be expressed by

$$\begin{aligned} u(x, t) &= Z \sin \theta \sin(\omega t) \frac{x}{l} + \frac{x}{l} \int_0^l \left(y' v' + \frac{v'^2}{2} \right) dx \\ &\quad - \int_0^x \left(y' v' + \frac{v'^2}{2} \right) dx. \end{aligned} \quad (2.5)$$

Inserting equation (2.5) in equation (2.3), it is possible to obtain an equation in the primary unknown $v(x, t)$. Then, considering the concentrated force at $x = l - a$ due to the damper (introduced to reduce the cable oscillations), and proceeding in a manner similar to references²⁶, the non-dimensional equations of motion can be written as

$$\begin{aligned} &\ddot{v} + c_v \dot{v} - v'' - \alpha(y'' + v'') \\ &\left\{ z_0 \sin \theta \sin(\Omega t) + \int_0^l \left(y' v' + \frac{v'^2}{2} \right) dx \right\} \\ &= F_d \delta(x - (l - a)). \end{aligned} \quad (2.6)$$

where $F_d = -C_{eq} \dot{v}(t - \tau)$ is the control force of the damper, τ is the time delay of the control system and δ is the Dirac delta function. The non-dimensional variables are $x^* = x/l$, $a^* = a/l$, $y^* = y/l$, $z_0 = Z/l$ (the excitation amplitude), $v^* = v/l$, $\alpha = EA/H$, $t^* = t/l\sqrt{H/m}$, $\Omega = \omega l/\sqrt{m/H}$ (the excitation frequency), $C_{eq}^* = (C_{eq}l/m)\sqrt{m/H}$. Asterisks in Eq. (2.6) are dropped for simplicity.

For the nonhomogeneous boundary value problem, it is convenient to introduce a suitable chosen particular solution that satisfies the nonhomogeneous boundary conditions, to transform the nonhomogeneous problem to a homogeneous one. Then, the solution of the homogeneous problem can be approximated by a time-varying linear combination of known (and fixed) spatial functions, assumed to be the eigenfunctions of the homogeneous problem. In this study, according to the boundary condition of the inclined cable, the non-dimensional displacements $v(x, t)$ is sought

after in the form

$$v(x, t) = \sum_{i=1}^N \phi_i(x) q_i(t) + x z_0 \cos \theta \sin(\Omega t), \quad (2.7)$$

where $q_i(t)$ are the generalized displacements, and $\phi_i(x) = \sqrt{2} \sin(i\pi x)$ are the i th in-plane mode shapes. Substituting equation (2.7) into equation (2.6) and applying the Galerkin method yield a set of nonlinear ordinary differential equations

$$\begin{aligned} & \ddot{q}_i + 2\omega_i \xi_i \dot{q}_i + \omega_i^2 q_i + \Gamma_{1i} \sin(\Omega t) q_i + \Gamma_{2i} \sin^2(\Omega t) q_i \\ & + \sum_{j=1}^N (\Gamma_{3ij} q_j^2 + \Gamma_{4ij} q_j \dot{q}_i + \Gamma_{5ij} \dot{q}_j^2 q_i) = -k_i \dot{q}_i(t - \tau) \\ & + \Gamma_{6i} \sin(\Omega t) + \Gamma_{7i} \cos(\Omega t) + \Gamma_{8i} \sin^2(\Omega t), \\ & i = 1, 2, \dots, n, \end{aligned} \quad (2.8)$$

where ξ_i are the viscous damping ratios, ω_i are the i th in-plane natural frequencies, and the other coefficients are given by

$$\begin{aligned} k_i &= C_{eq} \phi_i^2(l - a), \Gamma_{1i} = i^2 \alpha z_0 \sin \theta, \Gamma_{2i} = \frac{1}{2} i^2 \alpha z_0^2 \cos^2 \theta, \\ \Gamma_{3ij} &= \frac{\alpha}{2} j^2 \int_0^1 y' \phi_i'(x) dx, \Gamma_{4ij} = \alpha i^2 \int_0^1 y' \phi_j'(x) dx, \Gamma_{5ij} = \frac{\alpha}{2} i^2 j^2, \\ \Gamma_{6i} &= z_0 \Omega^2 \cos \theta \int_0^1 x \phi_i(x) dx - \alpha z_0 \sin \theta \int_0^1 y' \phi_i'(x) dx, \\ \Gamma_{7i} &= -2\xi_i i z_0 \Omega \cos \theta \int_0^1 x \phi_i(x) dx, \Gamma_{8i} = \frac{\alpha}{2} z_0^2 \cos^2 \theta \int_0^1 y'' \phi_i(x) dx, \end{aligned} \quad (2.9)$$

3. Linear stability with delay

In this section, the linear stability analysis of the single degree of freedom vibration mode is investigated. Considering only one equation $i = n$ in equation (2.8), neglecting the nonlinear terms and the external excitation ($z_0 = 0$), the following equation is obtained ($\mu_n = 2\omega_n \xi_n$)

$$\ddot{q}_n(t) + \mu_n \dot{q}_n(t) + \omega_n^2 q_n(t) = -k_n \dot{q}_n(t - \tau). \quad (3.1)$$

The solution of equation (3.1) is given by

$$q_n = e^{(\zeta_n + i\lambda_n)t} \quad (3.2)$$

where ζ_n and λ_n are real numbers. Substituting equation (3.2) into equation (3.1), and separating real and imaginary parts, gives

$$\begin{aligned} & \lambda_n (2\zeta_n + \mu_n) e^{\zeta_n \tau} \\ & + k_n [\lambda_n \cos(\lambda_n \tau) - \zeta_n \sin(\lambda_n \tau)] = 0 \end{aligned} \quad (3.3)$$

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$$\begin{aligned} & (\lambda_n^2 - \zeta_n^2 - \mu_n \zeta_n - \omega_n^2) e^{\zeta_n \tau} \\ & - k_n [\zeta_n \cos(\lambda_n \tau) + \lambda_n \sin(\lambda_n \tau)] = 0. \end{aligned} \quad (3.4)$$

When $\zeta_n < 0$ the solution (3.2) converges to 0 for $t \rightarrow \infty$ and thus is stable, while for $\zeta_n > 0$ the solution diverges to infinity and thus is unstable. The stability limit is then given by $\zeta_n = 0$. Substituting this value in equation (3.3) and equation (3.4) we obtain

$$\cos(\lambda_n \tau) = -\frac{\mu_n}{k_n}, \quad \sin(\lambda_n \tau) = \frac{\lambda_n^2 - \omega_n^2}{k_n \lambda_n}, \quad (3.5)$$

and thus the boundary of linear stability are

$$\begin{aligned} \tau &= \frac{1}{\lambda_n} \left[\tan^{-1} \left(-\frac{\lambda_n^2 - \omega_n^2}{\lambda_n \mu_n} \right) + j\pi \right], \\ j &= 0, 1, \dots, \\ k_n &= \pm \frac{\sqrt{\lambda_n^2 \mu_n^2 + (\lambda_n^2 - \omega_n^2)^2}}{\lambda_n}, \end{aligned} \quad (3.6)$$

which is a parametric representation of the stability curve $k_n = k_n(\tau)$ (λ_n is the freely varying parameter).

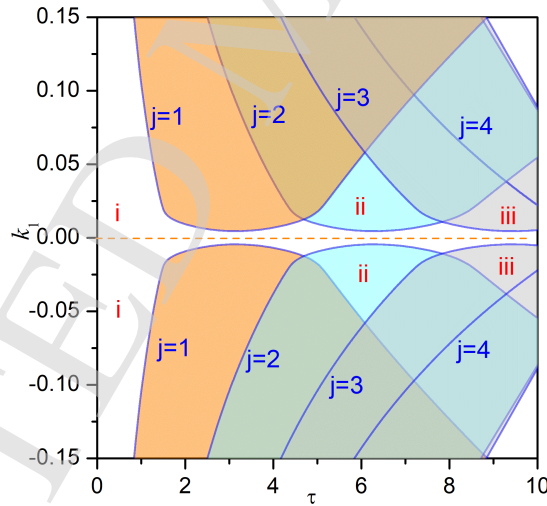


Fig. 2. Stability region (in white) of the single mode response of the controlled cable system.

The stability regions described by equation(3.6) are shown in Figure 2, where regions i, ii and iii correspond to a small, medium and large values of time delay, respectively. The figure clearly shows that for small values of the delay the system is stable, and thus the control effective, even for very large values of the gain k_n . For medium and large values of τ , on the other hand, the stability region

is a narrow strip around $k_n = 0$, namely the system is stable only for very low values of k_n , giving not good performance because with small values of the gain the damping is low and the vibration reduction is ineffective. For quite large values of k_n , the system remains stable for low values of the delay, but loses stability as τ increases. This could be very dangerous from a practical point of view, because unplanned increasing delay of the control, due for example to the ageing of the structure, can destabilize the system, with unwanted phenomena up to collapse. We are considering the use of high-bandwidth controllers to reduce the impact of time delay, as they can respond more quickly to changes in the system. Furthermore, we can also employ feedback control to monitor the actual system response and adjust control inputs accordingly to mitigate the effects of time delay. Continuous adjustment of control inputs allows us to better counteract the instability caused by time delay.

4. Nonlinear analysis with parametric excitation

In this section, given that in practical engineering applications, the first-order mode of the cable is more susceptible to excitation and easier to test, for the sake of simplicity, this section exclusively considers a single-mode model³⁵. we continue to consider the single degree of freedom vibration mode, but extend the analysis to the forced and nonlinear regime, utilizing the method of multiple scales³⁶.

It is convenient to introduce a small bookkeeping parameter ε to obtain the solution. The equation of the motion can be written as

$$\begin{aligned} \ddot{q}_n + \omega_n^2 q_n + \varepsilon \mu_n \dot{q}_n + \varepsilon \Gamma_{1n} q_n \sin(\Omega t) + \varepsilon \Gamma_{2n} q_n \sin^2(\Omega t) \\ + \varepsilon (\Gamma_{3nn} + \Gamma_{4nn}) q_n^2 + \varepsilon \Gamma_{5nn} q_n^3 = -\varepsilon k_n \dot{q}_n(t - \tau) \\ + \varepsilon \Gamma_{6n} \sin(\Omega t) + \varepsilon \Gamma_{7n} \cos(\Omega t) + \varepsilon \Gamma_{8n} \sin^2(\Omega t), \end{aligned} \quad (4.1)$$

The solution of equation (4.1) is sought after in the form

$$q_n(t; \varepsilon) = q_{n0}(T_0, T_1, \dots) + \varepsilon q_{n1}(T_0, T_1, \dots) + \dots \quad (4.2)$$

where $T_n = \varepsilon^n t$, $n = 0, 1, 2$, it is further assumed that

$$\omega_n = \frac{\Omega}{2} + \varepsilon \omega_1 + \varepsilon^2 \omega_2 + \dots \quad (4.3)$$

Substituting equation (4.2) and equation (4.3) into equation(4.1), and equating the coefficients of ε^0 and ε^1 on both sides, we obtain

$$D_0^2 q_{n0} + \frac{\Omega^2}{4} q_{n0} = 0, \quad (4.4)$$

$$\begin{aligned} D_0^2 q_{n1} + \frac{\Omega^2}{4} q_{n1} = -2D_0 D_1 q_{n0} - [\Omega \omega_1 + \Gamma_{1n} \sin(\Omega t) \\ + \Gamma_{2n} \sin^2(\Omega t)] q_{n0} - \mu D_0 q_{n0} - (\Gamma_{3nn} + \Gamma_{4nn}) q_{n0}^2 \\ - \Gamma_{5nn} q_{n0}^3 - k_n \dot{q}_{n0}(t - \tau) + \Gamma_{6n} \sin(\Omega t) \\ + \Gamma_{7n} \cos(\Omega t) + \Gamma_{8n} \sin^2(\Omega t), \end{aligned} \quad (4.5)$$

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where D_n denotes the derivatives with respect to T_n .

The general solution of equation (4.4) can be written as

$$q_{n0} = A_n(T_1) \exp\left(\frac{i\Omega T_0}{2}\right) + \bar{A}_n(T_1) \exp\left(\frac{-i\Omega T_0}{2}\right). \quad (4.6)$$

Substituting equation (4.6) into equation (4.5) we obtain

$$\begin{aligned} D_0^2 q_{n1} + \frac{\Omega^2}{4} q_{n1} = & - \left[i\Omega A'_n + \Omega\omega_1 A_n + \frac{\Gamma_{1n}}{2} \bar{A}_n \right. \\ & \left. + \frac{1}{2} i\mu\Omega A_n + 3\Gamma_{5nn} A_n^2 \bar{A}_n + \frac{1}{2} k_n i\Omega A_n \exp\left(-\frac{i\Omega\tau}{2}\right) \right] \\ & \exp\left(\frac{i\Omega T_0}{2}\right) - \left(\frac{\Gamma_{1n}}{2} A_n - \frac{\Gamma_{2n}}{4} A_n + \Gamma_{5nn} A_n^3 \right) \\ & \exp\left(\frac{3i\Omega T_0}{2}\right) + \frac{\Gamma_{2n}}{4} A_n \exp\left(\frac{5i\Omega T_0}{2}\right) \\ & - (\Gamma_{3nn} + \Gamma_{4nn}) A_n^2 \exp(i\Omega T_0) - (\Gamma_{3nn} + \Gamma_{4nn}) A_n \bar{A}_n \\ & - \frac{i\Gamma_{6n}}{2} \exp(i\Omega T_0) + \frac{\Gamma_{7n}}{2} \exp(i\Omega T_0) + cc \\ & - \frac{\Gamma_{8n}}{4} [\exp(i\Omega T_0) + \exp(-i\Omega T_0) - 2], \end{aligned} \quad (4.7)$$

where cc denotes the complex conjugate of the preceding terms. To eliminate secular terms from q_{n1} we must put

$$\begin{aligned} i\Omega A'_n + \Omega\omega_1 A_n + \frac{\Gamma_{1n}}{2} \bar{A}_n + \frac{1}{2} i\mu\Omega A_n + 3\Gamma_{5nn} A_n^2 \bar{A}_n \\ + \frac{1}{2} k_n i\Omega A_n \exp\left(-\frac{i\Omega\tau}{2}\right) = 0. \end{aligned} \quad (4.8)$$

To solve equation (4.8), we write A_n in the polar form

$$A_n = \frac{1}{2} a_n \exp(i\beta_n), \quad (4.9)$$

where a_n and β_n are real functions of T_1 . Substituting equation (4.9) into equation (4.8) and separating real and imaginary parts, we have

$$a'_n = -\frac{1}{2} \mu_e a_n + \frac{\Gamma_{1n} a_n}{2\Omega} \sin 2\beta_n, \quad (4.10)$$

$$\beta'_n = \omega_1 + \frac{k_n}{2} \sin\left(\frac{\Omega\tau}{2}\right) + \frac{3\Gamma_{5nn}}{4\Omega} a_n^2 + \frac{2\Gamma_{1n}}{\Omega} \cos 2\beta_n, \quad (4.11)$$

where $\mu_e = \mu + k_n \cos\left(\frac{\Omega\tau}{2}\right)$.

When $a'_n = \beta'_n = 0$, the sought periodic solution is obtained. Considering the nontrivial solutions ($a_n \neq 0$), from equation (4.10) and equation (4.11) we can then obtain

$$\sin 2\beta_n = \frac{\Omega\mu_e}{\Gamma_{1n}} \quad (4.12)$$

Remembering that $\cos(2\beta_n) = \pm\sqrt{1 - \sin^2(2\beta_n)}$ and substituting equation (4.12) in equation (4.11), we obtain the amplitude of the steady solution

$$a_n^2 = -\frac{4\Omega}{3\Gamma_{5nn}} \left[\omega_1 + \frac{k_n}{2} \sin\left(\frac{\Omega\tau}{2}\right) \right] \pm \frac{8\Gamma_{1n}}{3\Gamma_{5nn}} \sqrt{1 - \frac{\Omega^2 \mu_e^2}{\Gamma_{1n}^2}}, \quad (4.13)$$

which is the frequency-response equation, since the excitation amplitude z_0 is within Γ_{1nn} .

It is worth to underline that Γ_{6n} , Γ_{7n} and Γ_{8n} do not appear in equation (4.13) because we are focusing on the parametric excitation (as reflected in the choice (4.3)). They would appear if one considers the external resonance, i.e. $\omega_n \approx \Omega$. This is left for future work.

Since a_n is a real function, from $a_n^2 > 0$ we obtain first order approximate region of existence of the periodic solution

$$\omega_1 < -\frac{k_n}{2} \sin\left(\frac{\Omega\tau}{2}\right) \pm \frac{2\Gamma_{1n}}{\Omega} \sqrt{1 - \frac{\Omega^2 \mu_e^2}{\Gamma_{1n}^2}}. \quad (4.14)$$

Inserting this expression in equation (4.3), and remembering that $\varepsilon k_n = \hat{k}_n$, $\varepsilon \mu_e = \hat{\mu}_e$ and $\varepsilon \Gamma_{1n} = \hat{\Gamma}_{1n}$, yields

$$\frac{2\omega}{\Omega} < 1 - \frac{\hat{k}_n}{\Omega} \sin\left(\frac{\Omega\tau}{2}\right) \pm \frac{4\hat{\Gamma}_{1n}}{\Omega^2} \sqrt{1 - \frac{\Omega^2 \hat{\mu}_e^2}{\hat{\Gamma}_{1n}^2}}. \quad (4.15)$$

In the frequency/amplitude parameter space (Ω, z_0) , the boundary of the existence region—coinciding with the stability region—is obtained by considering equality instead of inequality in equation (4.15). It forms the classical V-shape with the vertex at $\omega = \Omega/2$ (refer to forthcoming Figure 3, for example).

5. Numerical results and discussions

A stay cable of the Dongting Lake Bridge, in China, was chosen as an example to verify the spatial motions of the cable. The dimensional parameters and material properties of the sample stay cable are as follows⁹: span $l = 121.9\text{m}$; inclination angle $\theta = 35.2^\circ$; cross-sectional area $A = 6\,237 \times 10^{-6}\text{m}^2$; initial tension $H = 3\,150\text{kN}$; elastic modulus $E = 2.0 \times 10^5\text{MPa}$; mass per unit length $m = 51.8\text{kg/m}$.

Figures 3-5 show the stability regions given by equation (4.15) for the controlled cable under different values of the parameters.

The effect of the time delay on the stability of the parametric resonance of the controlled cable is illustrated in Figure 3. It is clear that with increasing delay τ the unstable region (above the stability boundary) increases in magnitude, confirming the findings of Sect. 2 that the delay has a destabilizing effect. Actually, τ has a significant effect on the minimum values of the curve, while mildly affects the

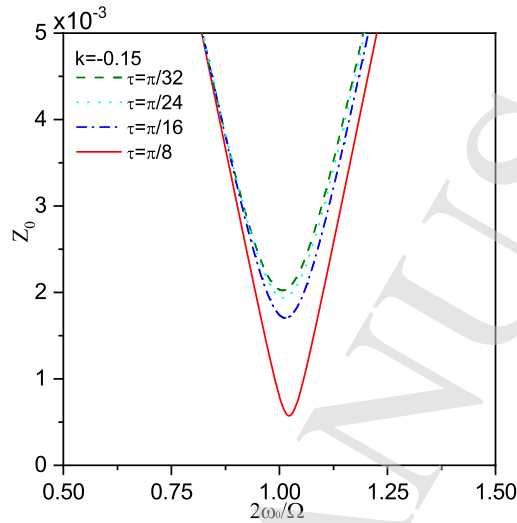


Fig. 3. The effect of the time delay on the stability of the controlled cable.

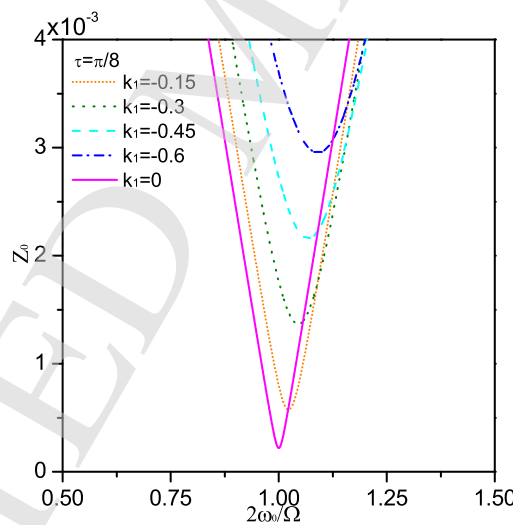


Fig. 4. The effect of the control gain on the stability of the controlled cable.

frequency where this minimum occurs (always in the neighborhood of the perfect parametric resonance $\omega = \Omega/2$).

Figure 4 analyzes the impact of the control gain on the stability of the controlled cable. As expected, increasing the absolute value of the feedback control gain k_n , enlarges the unstable region. The minimum value of the limit curve is almost proportional to k_n , demonstrating the effectiveness of control in mitigating

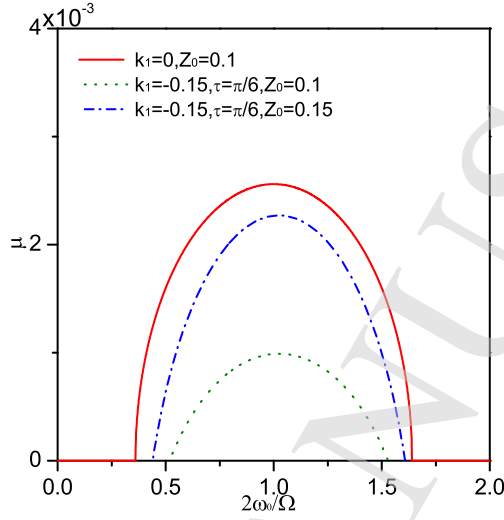


Fig. 5. The effect of the amplitude of the excitation on the stability of the controlled cable.

the instability of parametric resonance. The frequency where the minimum occurs slightly increases, even if this is not expected to be relevant in practical applications.

We now illustrate the effect of control gain (Figure 6) and control delay (Figure 7) on the frequency response curves, which are crucial for practical applications and for design ($z_0 = 0.005$). Figure 6 demonstrates that the frequency response curve shifts to the right when the control gain increases (in absolute value), confirming the beneficial effect of control in raising the instability threshold of the rest position. The stable curve, occurring for “large” displacements (experienced by the system after the loss of the stability of the rest position), is not significantly affected by k_n , except at the left Saddle-Node bifurcation where it originates. However, this curve is much more influenced by the delay τ , as shown in Figure 7, which also confirms that increasing the delay destabilizes the rest position.

In the frequency/damping parameter space (Ω, μ) , the effect of the excitation amplitude z_0 on the stability of the controlled cable is shown in Figure 5. As expected, the larger z_0 the larger is the instability region (now below the reported curves), indicating that a large gain is needed to control large excitation amplitudes. The case with no control ($k_1 = 0$) is also reported to appreciate the beneficial effect of control.

Finally, in Figure 8, the time history of the uncontrolled and controlled cable with different time delays is compared. The amplitude of the support motion used here is $z_o = EA/4mL^4, \Omega = 6.25$. Comparing Figure 8(a) with Figure 8(b) it can be observed that with a small delay the control is very effective in reducing the vibration amplitudes of the cable. However, with increasing delay, the destabilizing effect, already illustrated, can be seen also in the time history of Figure 8(c). Note that the maximum displacement in Figure 8(c) is quite similar to that of Figure 8(a),

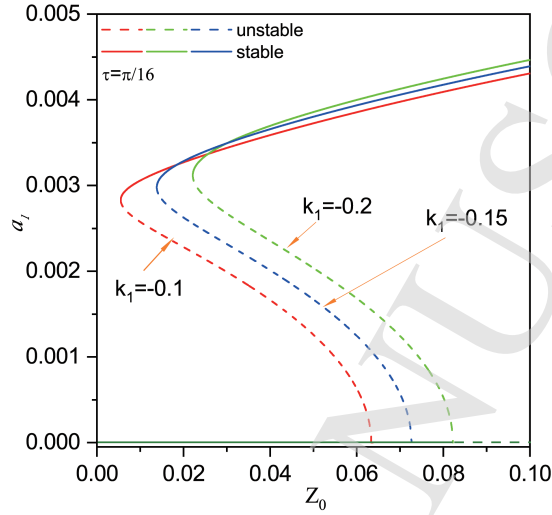


Fig. 6. The frequency response curve of the controlled cable with time delay $\tau = \pi/16$.

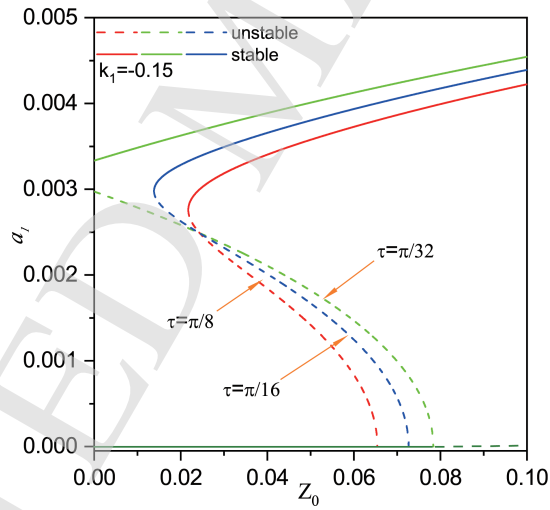


Fig. 7. The frequency response curve of the controlled cable with control gain $k_1 = -0.15$.

demonstrating how the large delay nullifies the effect of control.

6. Conclusions

The stability of the parametric resonance of the controlled cable under the influence of time delay has been investigated. The results show that the unstable region increases with the time delay and decreases with the increase of the absolute value

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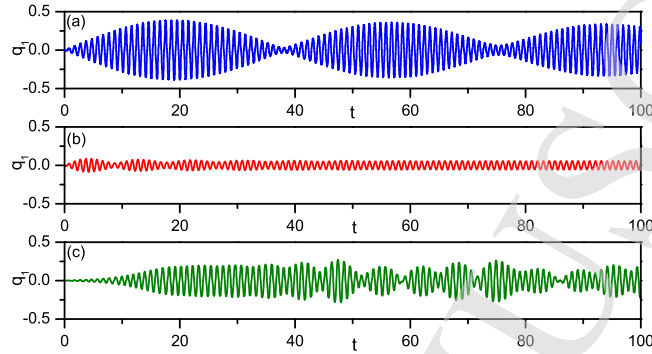


Fig. 8. Comparison of the time history of the controlled cable. (a) no control; (b) $k_1 = -1, \tau = \pi/2$; (c) $k_1 = -1, \tau = \pi$.

of the control gain. These findings have been obtained theoretically by analyzing the closed-form solutions, and have been confirmed through numerical simulations. Furthermore, with an increase in the absolute value of the control gain, the frequency response curve shifts to the right, confirming the beneficial effect of control on increasing the instability threshold of the rest position. The stable curve, occurring for “large” displacements (experienced by the system after the loss of the stability of the rest position), is not significantly affected by the control gain. Instead, this curve is much more influenced by the time delay, further confirming that increasing the delay destabilizes the rest position.

The overarching conclusion drawn from this study is that, in the realm of control design, particularly when contemplating active and semi-active control strategies, meticulous consideration of the impact of time delay is paramount.

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