



UNIVERSITÀ POLITECNICA DELLE MARCHE  
Repository ISTITUZIONALE

Automatic generation of synthetic heritage point clouds: Analysis and segmentation based on shape grammar for historical vaults

This is the peer reviewed version of the following article:

*Original*

Automatic generation of synthetic heritage point clouds: Analysis and segmentation based on shape grammar for historical vaults / Battini, Carlo; Ferretti, Umberto; De Angelis, Giorgia; Pierdicca, Roberto; Paolanti, Marina; Quattrini, Ramona. - In: JOURNAL OF CULTURAL HERITAGE. - ISSN 1296-2074. - ELETTRONICO. - 66:(2024), pp. 37-47. [10.1016/j.culher.2023.10.003]

*Availability:*

This version is available at: 11566/325353 since: 2023-12-21T19:20:17Z

*Publisher:*

*Published*

DOI:10.1016/j.culher.2023.10.003

*Terms of use:*

The terms and conditions for the reuse of this version of the manuscript are specified in the publishing policy. The use of copyrighted works requires the consent of the rights' holder (author or publisher). Works made available under a Creative Commons license or a Publisher's custom-made license can be used according to the terms and conditions contained therein. See editor's website for further information and terms and conditions.

This item was downloaded from IRIS Università Politecnica delle Marche (<https://iris.univpm.it>). When citing, please refer to the published version.

(Article begins on next page)

# Automatic generation of synthetic heritage point clouds: analysis and segmentation based on shape grammar for historical vaults

Carlo Battini<sup>a</sup>, Umberto Ferretti<sup>b,d,\*</sup>, Giorgia De Angelis<sup>b</sup>, Roberto Pierdicca<sup>b</sup>, Marina Paolanti<sup>c</sup>, Ramona Quattrini<sup>b</sup>.

## Affiliation:

<sup>a</sup>Università degli Studi di Genova, Dipartimento di Ingegneria Civile, Chimica e Ambientale (DICCA), 16145 Genova, Italy. [carlo.battini@unige.it](mailto:carlo.battini@unige.it) (C. B.).

<sup>b</sup>Università Politecnica delle Marche, Dipartimento di Ingegneria Civile, Edile e dell'Architettura, 60131, Ancona, Italy, [giorgiadean@libero.it](mailto:giorgiadean@libero.it) (G. D.); [r.pierdicca@staff.univpm.it](mailto:r.pierdicca@staff.univpm.it) (R. P.); [r.quattrini@staff.univpm.it](mailto:r.quattrini@staff.univpm.it) (R. Q.).

<sup>c</sup>University of Macerata, Department of Political Sciences, Communication and International Relations, VRAI Vision Robotics and Artificial Intelligence Lab, 62100 Macerata, Italy, [marina.paolanti@unimc.it](mailto:marina.paolanti@unimc.it) (M. P.).

<sup>d</sup>Sapienza University of Rome, Department of Science of Antiquities, 00185, Roma, Italy [ferretti.2079882@studenti.uniroma1.it](mailto:ferretti.2079882@studenti.uniroma1.it) (U. F.)

\* Corresponding author:

Email address: [ferretti.2079882@studenti.uniroma1.it](mailto:ferretti.2079882@studenti.uniroma1.it) (Umberto Ferretti).

## Abstract

Historical heritage is demanding robust pipelines for preserving, enhancing, and disseminating its prominent value. Semantic segmentation of 3D Point Clouds has gained increasing attention over the years, since it might assist in automatically recognizing historical architectural elements, thus facilitating large dataset management. Nonetheless, semantic segmentation is particularly challenging in Cultural Heritage (CH) domain, due to the shapes complexity and the limited repeatability of elements across different architectures, which strengthens the difficulty to define common patterns within the same class of elements. Besides, as Deep Neural Networks demand an appreciable amount of labelled data to be trained, the lack of available annotated heritage point clouds prevent the research in this direction. To tackle these issues, in this paper it is proposed a Deep Learning system able to recognize historical building elements by leveraging synthetic point cloud. The generation of the 3D models, vaults, is based on a procedural modelling approach that follows the ideal shapes, according to the rules of descriptive geometry for the main types of vaults. The approach has been applied to a newly synthetic dataset which is publicly available. This dataset comprises 6 labelled points clouds, derived from a comprehensive ontological taxonomy in order to describe an univocal and robust architectural hierarchy: barrel vaults, groined vaults, mirror vaults, barrel vaults with cloister heads and lunettes, barrel vaults with lunettes, sail vaults. The experiments yield high accuracy, demonstrating the effectiveness and suitability of the proposed approach. Keywords: 3D point clouds, Deep Learning, Cultural Heritage, Synthetic Data, Architecture, Semantic segmentation, DGCNN

## 1. Introduction

**Motivation:** In the Digital Cultural Heritage (DCH) domain a significant number of technologies for 3D documentation and semantic intelligence are currently available in order to preserve, enhance and disseminate the Cultural Heritage (CH) [1]. The process of digitization involves archival and management, representation and reproduction [2] and enables a number of possibilities which suggest the merging between different branches of knowledge [3]. The generation of 3D Point Clouds (PC) is, currently, the most suitable way to manage CH assets. The representation of CH goods through 3D data is a well-established methodology to perform several tasks, e.g., map degradation, data enrichment or morphological analysis. These are only some examples of attainable modality to exploit such rich informative virtual representation [4]. Considering the Historical Building Information Modeling (HBIM) one of the most effective tools for managing CH assets, a crucial point is the segmentation of unstructured collection of 3D coordinate that transforms the points into structured elements [5].

In the last years, research focused on the automation of the Scan to HBIM process. A recent review revealed that the semantic segmentation of 3D heritage geometric models or 2D imagery is widely adopted, although such an application needs a large amount of training data and addresses simple scenes [6]. The manual segmentation and data enrichment is still a very reliable process, since it's implemented by domain experts, but it needs a significant investment of time. A very promising research field is the development of Deep Learning (DL) frameworks for PC such as Point-Net/Pointnet++ [7], [8], which has opened up more powerful and efficient ways to handle 3D data. These methods are designed to deal with PC classification and segmentation. The Point Cloud classification takes the whole Point Cloud as input and output the category of the input Point Cloud. The segmentation aims at classifying each point to a specific part of the Point Cloud. Albeit the literature for 3D instance segmentation is limited especially in CH, if compared to its 2D counterpart (mainly due to the high memory and computational cost required by Convolutional Neural Network (CNN) for scene understanding), these frameworks may facilitate the recognition of historical architectural elements, at an appropriate level of detail, and thus speeding up the process of reconstruction of geometries in the HBIM environment or in object-oriented software.

While well-established methods like RANSAC [4] are widely adopted for those regular elements like beams or planes [9], [10] researchers are more focused on automatic methods based on Deep Learning [11] given the sparse and uncertain nature of unstructured point cloud for complex elements [12]. Besides, AI-based approaches present remarkable advantages over traditional techniques such as RANSAC and region growing in DCH domain. The high degree of automation offered by AI streamlines the segmentation process, particularly for extensive heritage site analyses, significantly reducing the need for manual intervention and expediting the overall workflow. Furthermore, AI models possess the capacity for continuous learning, enabling them to adapt and improve over time, making them flexible and well-suited for future developments and challenges in the field. By leveraging the power of AI, we aim to overcome the limitations of conventional techniques and achieve an optimal and robust semantic segmentation solution tailored specifically to the unique demands of CH point clouds [13].

Despite these neural networks are not specifically designed for the CH domain, they proved to be suitable in domain shift application, although suffering of two main bottlenecks: the uniqueness of the objects, that hamper the learning phase from being straightforward, and the resolution, since the description of ornaments or complex shapes require a very detailed point cloud at the expenses of computational capabilities.

**Research aims:** Given the above-mentioned limitations, this study aims at developing a system which is able to feed a neural network with synthetic data in order to increase its performances. As it is well known, the lack of dataset results in poorly trained systems. Our method moves toward this direction but proposing a solution to increased labelled point cloud dataset. All these topics are implemented in a real case of study, that presents a remarkable architectural complexity especially referring to the vaults system, which includes a large taxonomy and variety.

**Contributions:** The main contributions of this paper are: (i) the release of a DCH dataset of vaults both in parametric/geometric shape both exported as synthetic point cloud, based on shape grammar discussion and recognized thesaurus; (ii) the development of a data augmentation method for generating training data automatically when real 3D objects are insufficient; (iii) the development of a DL system able to segments and distinguish types of architectural vaults according to standard classifications. The article is structured as follows: after the state of art paragraph presenting different papers that use DL in segmentation of 3D point clouds (Section 2), the methodology proposes a shape-grammar based taxonomy about vaulted roofs related to standard thesauri (Section 3.2.1) and a procedural method to generate synthetic data according to the presented ontology (Section 3.2.2). In Section 3 are also included the description of used Neural Networks (Subsection 3.3) and of the case study (Subsection 3.1). Section 4 presents 3 kind of experiments: two tests aimed to classify and segment the synthetic vaults and the last

attempt dealing with segmentation of scenes consisting in reality-based point clouds. Finally, in Section 5, conclusions and discussion about future directions for this field of research are drawn.

## 2. State of the Art

Several methods have been used in the literature for the segmentation and classification of historical buildings. These methods often leverage DL and computer vision techniques to process historical building data, including images, point clouds, and 3D models. It is noticeable that point clouds are an ongoing research field that can take inspiration and rely on the advancements deriving from other more developed disciplines such as autonomous navigation and in-door positioning [14]. Even if, in the literature, there are limited studies that use DL approaches to classify 3D point clouds in different objects belonging to CH scenes, according to [15], these methods had great potential to this regard. For this purpose, in this section we briefly review some relevant background works concerning AI techniques for the analysis of digital representation of CH.

CNNs and their variants have been widely employed for semantic segmentation of historical buildings in images and point clouds. Inspired by the great results obtained in [16], which introduced a module called EdgeConv, that constructs a local neighborhood graph and applies convolution-like operations and developed a new DL model named DGCNN (Dynamic Graph Convolutional Neural Network), dynamically updates the graph, changing the set of k-nearest 100 neighbors of a point from layer to layer of the network, the authors in [15] made an extension of the previous work and exploited the novelties offered by the DGCNN. Thus, they proposed a modified version of DGCNN by adding relevant features such as normal and HSV encoded color. This improved version aimed at facilitating the management of DCH assets that have complex geometries, extremely variable and defined with a high level of detail. Another research work that adopted DL strategies to classify CH objects is presented by Grilli et al. in [17]. The authors made experiments in order to compare the performances of ML and DL approaches in classifying two different CH datasets. Using features-based approaches such as Random Forest and One-versus-One the performances achieved are preeminent in almost all the chosen classes, but there is no correlation between the characteristics. The results of the DL counterpart with the adoption of 1D CNN, 2D CNN and Recurrent Neural Network (RNN) Bi-directional long short term memory (Bi-LSTM) by considering point clouds as sequence of points encountered several difficulties in the classes recognition. This was due to the implementation of models not properly designed for point cloud analysis. For this reason, in [18] the authors enhanced the comparison between Machine Learning (ML) and DL methods for large 3D CH classification. They considered k-Nearest Neighbour (kNN) [19], Support Vector Machine (SVM) [20, 21], Decision Tree (DT) [22, 22], and Random 120 Forest (RF) [23] as ML approaches and DGCNN-Mod as DL model. Moreover, after the performance comparison of both techniques, they implemented an architecture named DGCNN-Mod+3Dfeat that combined the positive aspects and advantages of these two methodologies for semantic segmentation of CH point clouds. In [4] the approach relies on the application of machine learning techniques to semantically label 3D heritage data by identification of relevant geometric, radiometric and intensity features, then the use of the annotated data is dealt with the construction of Heritage-Building Information Modeling (H-BIM) systems. It is also interesting to cite [24], in which authors applied a Multi-Level and Multi-Resolution workflow for the automated segmentation and classification of surveying outputs. The research focused on the surveying of reticular, space grid structures, which might require high level of detail recognition, and relied on artificial intelligence (machine learning and deep learning).

PointNet [7] and PointNet++ [8] are architectures designed for point cloud processing. They are used for segmenting and classifying historical buildings represented as 3D point clouds obtained through laser scanning or photogrammetry. An example of the effectiveness of this DL framework is the work of Malinverni et al. [25]. They exploited PointNet++ to semantically segment 3D point clouds of CH dataset. A

newly dataset was specifically collected to deal with CH data and manually labelled by domain experts: ArCH dataset [26].

They have created domain-specific datasets for historical buildings, that are used for training and evaluating classification models specialized for historical architecture. They aim to address the issue of the lack of labeled data required to train deep learning algorithms. In fact, the lack of literature in this field is to be found in the need of a large scale well annotated dataset that limits the development of robust DL framework for processing CH point cloud data. To generate synthetic data for augmenting datasets and to create realistic 3D models of historical buildings for training segmentation and classification models, Generative models such as Generative Adversarial Networks (GANs) have been used. Pierdicca et al. [27] proposed a framework based on DL that synthetically generates architectural elements to increase segmentation accuracy. To generate novel scenes, they adopted three different generative networks: Point-Grow [28], PointFlow [29], and PointGMM [30].

With GAN that achieved the best performance, they trained DGCNN-Mod that classified the synthetically generate scenes. However, the generated objects are regular and not complex as a vault. In CH field, encouraging results have yielded by synthetic datasets as already done for urban point clouds [31].

In [32], using Blender, an open-source software which permits to access to each point in an object creating one in a new mesh, it is proposed a framework for automatic generation of synthetic dataset of point clouds. The described algorithms allowed to create a great number of point cloud synthetically, simulating a virtual laser scanner at a variable distance. Furthermore, the two algorithms not only work with a single object, but it was possible to create simultaneously many point clouds from a scene in Blender also with the use of an existing model of ancient architectures. Following this procedure, in [33], the authors released a dataset composed of synthetic annotated point clouds derived from curated 3D models of 10 different historical buildings and from 159 3D models of single architectural elements collected from open web archives. Furthermore, they proposed a variation of the DGCNN architecture [16], based on the use of radius distance, which was proven to increase performances on the test datasets.

With respect to the above mentioned state-of-the-art works, our approach consists in creating a novel synthetic point cloud dataset with complex shape and several kind of vaults, focusing on a more refined hierarchical level. Thus, this approach realizes vaults according to domain taxonomy and the grammar of historical shapes, which respect the variability and level of detail of the CH ones. To prove the effectiveness of this study, a real case study with sophisticated vaults has been taken in exam: in fact, the testing of this method is implemented in an exemplary building of the Renaissance age.

### 3. Materials and Methods

The methodology developed to perform our experiments is based on few steps, summarized in Figure 1. In step 1a the ontology discussion and the shape grammar analysis (3.2.1) support the creation of the synthetic training dataset of vaults (Figure 1-step 1b), based on a modeling procedure (3.2.2) with Visual Program Language and point clouds extraction (Figure 1-step 1b). Afterwards, pre-processing actions are performed (Figure 1 -step 2), as to provide data suitable for the subsequent classification (3.3.1) of vaults (Figure 1 -step 3a) in synthetic and real scenes, constituting the segmentation procedure (3.3.2) (Figure 1 -step 3b). In the following sections, these steps are detailed.

Figure 1: The methodology developed to perform our experiments is based on few steps, summarized in the chart.

### 3.1. Case Study

The proposed methodology has been tested in a real case study: the Ducal Palace of Urbino which houses the National Gallery of Marche. It is a unique site, one of the symbols of the Italian Renaissance, described as a “city in the shape of a Palace” [34]. As the residence of the eclectic duke of Montefeltro back in the 15th century, it presents a number of peculiar architectural elements and precious artworks as well as, a complex historical stratification which involved many centuries. Polytechnic University of Marche (UNIVPM) established a partnership with the museum through many initiatives and projects in the last years. For instance, within the strategic project CIVITAS [35], [36] the comprehensive point cloud of the Ducal Palace has been implemented. In this framework a complete survey campaign, mainly based on TLS and photographic data capturing, was carried out. At the current stage, the whole numerical model of the Palace consists in 1.790 mln of points. As mentioned in the state-of-the-art, a previous research work [33] leveraging a deep learning framework was already conducted. A neural network which employed an improved version of the DGCNN models has been tested. Such approach permitted an effective segmentation with notable values of F1-score and IoU. The architectural elements of the Ducal Palace’s court that have been segmented belong to lower detailed hierarchical level (i.e., architectural components level according to Getty) and consist of: Column, Window/Door, Wall, Pilaster, Floor, Moulding, Vault, Other. Starting from these promising results, the present work aims to perform a step forward by implementing an automatic segmentation of vault types. The experiment deals with part of the point cloud related to the Piano Nobile (first floor). It covers an area of 5073, 18 sq.m, and provides an exhaustive taxonomy of vaults that can be considered as representatives of the entire Ducal Palace (Figure 2). In particular, there are 41 vaults sorted in the following categories:

1. Barrel vaults (8 items)
2. Groined Vaults (7 items)
3. Mirror Vaults (7 items)
4. Barrel vaults with cloister heads and lunettes (18)
5. Barrel vault with lunettes (1)

Figure 2: Some internal pictures of the Ducal Palace of Urbino

As can be seen, the most common typology is the barrel vault with cloister heads with lunettes that counts 18 items. it consists of a combination between the compound vault “Barrel vault with cloister heads” and the vault component “Lunettes with groins on triangular plan”. Another parameter that must be considered is the dimensional gap. For instance, the lowest ridge height of a vault from the floor is 3.45 m while the tallest one reaches the height of 13,30 m. This profound size’s variability provided an even more rich and complex scenario for training the neural network. able to sufficiently stress the phase of classification.

### 3.2. Vaults synthetic dataset creation

#### 3.2.1. Vaults Shape Grammar

The first essential step was to identify, define and classify each type of vault. Such typologies have been selected, following some suitable criteria: grammar's accuracy according to available thesaurus, coherence with the language and rules of classical architecture and possible use for a large timespan of built heritage, usually mapped with point clouds. For the present work, the electronic thesauri produced by the Getty Research Institute (GRI), Art & Architecture Thesaurus (AAT)<sup>1</sup> has been taken as the main reference. A comprehensive ontological taxonomy has been developed in order to describe an univocal and robust architectural hierarchy (Figure 3).

Figure 3: The phylogenetic tree of vault types according to the Art & Architectural The-saurus. (a), (b), (c), were not originally included in the Getty vocabulary.

A first classification has been conducted, according to the research aims, and listed as follows: (vaults/simple vaults) barrel vaults; (vaults/compound vaults) groined vaults, cloister vaults; (domes/pendentive domes) sail vaults; (vault components) lunettes. As can be seen, the class of vaults presents three branches: "simple vaults", "compound vaults" and "vaults by construction". The latter was not examined since the present study is focused on the geometric features. The barrel vault type (child of simple vaults) includes many sub-typologies according to different features. The sail vault type belongs to the "domes" class as a child of the pendentive domes. Despite the presence of a large amount of terms, other typologies not mentioned in the Getty vocabulary have been added, in order to pursue an accurate correspondence with the typologies expected in the classical architecture. The lack of terms in the English bibliography is partly due to the absence of translations of some types of vaults which are more common in the Mediterranean panorama. Therefore, the following typologies have been included within the taxonomy, referred-to the scientific literature:

(vaults/compound vaults) barrel vault with cloister heads [37], occasionally mentioned as trough vaults [38];

(vaults/compound vaults) mirror vaults: such term is proposed for the first time in the present article and represents a translation from the Italian bibliography "volta a specchio" (often mentioned as "volta a schifo" [39]). It also appears in a German treatise from the 19th century as "Spiegelgewölbe" [40]. Such vaults are generated by sectioning a cloister vault with a horizontal plane above the impost level. Usually, the timber frame is supported by groins which represent the structural part.

(Vault component/lunettes) with groins on circular plan, with groins on triangular plan [37].

In particular, the groin, which Guarini described as generated from "a piece of cylinder cut triangular" [41], is actually generated by parallel and variable sections to secure the connection to the main surface. On the other hand, some typologies have been excluded. In fact, the typologies that have been chosen, are coherent with the main elements of the classical architectural language, massively used in Renaissance buildings (taken as reference case studies). For increased readability, the ontological taxonomy does not include pointed vaults, but they can be easily included extending the taxonomy itself. This part of the methodology then ensures a thorough re-reading and analysis of a standard taxonomy and to its reinterpretation in order to extend them to different use cases, with a greater level of detail.

<sup>1</sup> <https://www.getty.edu/research/tools/vocabularies/aat/>

### 3.2.2. Modelling and Point Clouds extraction

The necessity of testing automatized systems was given essentially by two points: the complexity of registering reality-based point cloud and the need of collecting a significant number of cases ready to be processed within the system. The procedural modelling approach, here adopted, follows the ideal shapes, according to the rules of descriptive geometry, previously presented, and is based on scripting for the generation of ruled surfaces (directrix and generator) [42] for the main classes of vaults. An iterative method, based on parametric modeling using the Rhinoceros software and the Visual Programme Language (VPL) tool Grasshopper, allowed the generation of the synthetic dataset and its extension, in order to have a significant quantity of labeled vaults. For the matter of synthesis, here below is described the script developed for the groined vault. The starting parameters for the generation of the surface are the dimensions of the room to be covered with the vault, thus representing the clearance of the vault.

Figure 4: The creation of the synthetic groined vault.

Figure 5: The creation of the synthetic sail vault.

In order to obtain suitable dimensions in line with the masonry architecture, the room geometry was set with sides A and B both ranging from 2 to 5 m as reported in Table 1.

Parameter	Min (m)	Max (m)	Step (m)
Side A /clearance of the vault	2	5	0.1
Side B /clearance of the vault	2	5	0.1
Vault height	2	5	0.1

Table 1: Input parameters for the vaults' synthetic generation.

Each side is increased by 0.1 m; this setting guarantees the generation of synthetic vaults with significant differences in the resulting geometries. The side A of the room is set with rounded groins ( $h=A/2$ ), the other radius of the homologous groins (which are set on side B) are modified according to the previously defined intervals (steps every 0,1 m). As a result, the groins set on B have a semi-elliptical directrix (Figure 4a). Hence, this part of the algorithm deals with the generation of the continuous vault (Figure 4b). As regards the sail vault, the script deals with the creation of a rectangle ( $A*B$ ) on horizontal plan which represents the impost level of the vault. As in the groined vault, such plan is further increased by 0,1 m. In order to obtain the rectangle's vertexes, a sphere which includes the vault's geometry is generated, isolating only the upper hemisphere (Figure 5a).

Finally, the surface of the sail vaults is isolated through the extrusion of the room's perimeter (Figure 5b).

Accordingly, the other geometries concerning the types of vaults were generated following the same procedure. The last part of the algorithm, valid for all scripts, deals with the generation of points on the vaulted surface starting from the implemented models. In particular, the Grasshopper Pop-Geo<sup>2</sup> component creates a scattered point cloud and has been set with the number of points value 100 points/m<sup>2</sup> (Figure 4c, 5c). This value has been evaluated as a good compromise between surface description and computational needs. Furthermore, it is considered comparable with reality-based point

<sup>2</sup> <https://github.com/popphp/pop-geo>



clouds, coming from TLS acquisition, after the decimation phase. The points have been further inserted in a list with X, Y, Z coordinates, and then exported in .csv extension. The name is defined from a fixed part (e.g., barrel-) and a variable part according to the modified dimension for the surface creation. Later, the entire algorithm ran in a loop in order to automatically generate points' coordinates of the synthetic vaults. This part of the methodology is aimed at obtaining a large number of labelled vaults, automatically but respecting geometric rules known from literature to architectural experts. Thus it was possible to make available a comprehensive dataset<sup>3</sup> and it includes 6 types of vaults according to the taxonomy discussed in paragraph 3.1 (see Appendix 1).

### 3.3. Deep Learning Models

State-of-the-art DNNs are specifically designed to deal with the irregularity of Point Clouds, directly managing raw Point Cloud data rather than using an intermediate regular representation. In this contribution, the performances obtained with the synthetic point cloud dataset of some state-of-art architectures are compared and then evaluated in the chosen case study. The DNNs selected are:

- **PointNet** [7]: is the pioneer of neural architectures for point cloud classification and segmentation. The network takes as input an unordered set of points. Each point is represented as a vector of coordinates (x, y, z) and is accompanied by any other features (e.g., color, normal). To handle an unordered set of points, the key module of the PointNet approach is the max pooling layer, which uses a symmetric function to aggregate information from all points. This makes the approach independent of the permutation of points. This part constitutes the global descriptor. For the classification task, two fully connected layers are added to the global descriptor in cascade. The fully connected layers combine the global features to predict the class of the input object. The output is a vector in which each value represents the probability of belonging to each of the m classes. For the semantic segmentation task, it concatenates the global and local features of individual points to predict the class at each point in the cloud. The output is an  $n \times m$  matrix, where for each of the n points, a vector of size m represents the probability of membership in each of the m classes. The main limit of PointNet architectures is that it neglects geometric relationships between points, which are important for capturing local features.
- **DGCNN** [16]: overcomes PointNet limit, since its architecture complements the basic PointNet architecture (without feature transformation operations) with the inclusion of the EdgeConv operation, which captures local geometric structure while maintaining permutation invariance. Instead of generating point features directly from their embeddings, EdgeConv generates edge features that describe the relationships between a point and its surroundings. EdgeConv is designed to be invariant with respect to the ordering of points in the surround, and thus invariant to permutation. Since EdgeConv explicitly constructs a local graph and learns embeddings at edges, the model is able to group points in both Euclidean space and semantic space.

These architectures were carefully selected to ensure robust and accurate performance in the given tasks. The classification experiments involved categorizing input data into different predefined classes, while the segmentation experiments aimed to identify and delineate specific objects or regions within the input data. We employed these architectures to evaluate their effectiveness in handling complex and diverse datasets. Furthermore, the same neural architectures were used to validate the findings of a comprehensive case study. This case study involved a detailed investigation into a specific problem or scenario, which provided valuable insights into the capabilities and limitations of the chosen architectures.

---

<sup>3</sup> <https://github.com/CarbatDCCA/Vaults-Synthetic-Dataset>

The results obtained from each experiment, along with comprehensive details of the experimental setups, parameters, and methodologies, have been compiled and thoroughly analyzed.

### 3.3.1. Classification Phase

In the first case, i.e., Classification Phase, “Synthetic Vault Dataset” is splitted into 2 portions: training set and test set, respectively 80%, 20% of the total dataset. In both portions, an attempt was made to maintain an equivalent ratio of individual classes between the training and test sets, with no balancing of classes. The architecture used is Pointnet [7]. The number of classes in output is fixed at 5 (for all classes in the dataset). The input size is set to process an object of 1024 points and 3 features per point (x,y,z). Since all objects in the dataset are conformal (x and y values distributed symmetrically to the left and right of the axis origin, z values always positive and limited superiorly), it is not necessary to preprocess (standardize) the input data.

### 3.3.2. Semantic Segmentation Phase

A second experiment is performed for a task of semantic segmentation. The same Pointnet architecture, used for the classification task, is used to segment the point clouds: this time scenes and no single objects (single times). For this, we created a synthetic scene dataset from the individual objects in the previous dataset. Specifically, the individual objects (vaults) used for classification were randomly selected and arranged in a 3D space (Figure 6). A label equal to the class of the object is assigned to each point of each object. The “Trough vaults with lunettes” class is excluded from the selection because they are difficult to arrange. Thus, in this segmentation experiment, we consider only 4 classes of vaults.

## 4. Results and Discussions

In this section, the results of the experiments conducted with “Synthetic Vault Dataset” are reported. Our experiments are separated in three phases. In the first one, we attempt at classifying the vaults created synthetically. Seg-mentation of the entire Point Cloud is a needed pre-processing step for all the analysed neural architectures. Finally, the evaluation of this approach has been done in a real case study. While this has in fact practical applications and could speed up the process of annotating an entire scene, our goal is to evaluate the automatic recognition of a vault in a complex scene never seen before. We address this more challenging problem, in the final experimental phase, where we train the networks with “Synthetic Vault Dataset” and then attempt at au-tomatically segmenting the ones in a real scene: the first floor of the Ducal Palace point cloud.

### 4.1. Classification Phase Results

As shown in Table 2 the mini-batch strategy with batch size set to 32 was used for training, the optimization algorithm used to minimize the loss function “sparse\_categorical\_crossentropy” is Adam, setting an initial learning rate of 0.001. Training was performed for 20 epochs.

Parameters	Value
Mini-batch strategy	Batch size 32
Optimization algorithm	Adam
Loss function	Sparse_categorical_crossentropy
Initial learning rate	0.001
Number of epochs	20

Table 2: DNN Parameters and Training Details for Classification Phase

To harden and generalize the learning by making the classification invariant to the transformations themselves and suitable for a larger dataset, random augmentation of the training data with the following transformations was used during training:

- Random rotation around the z-axis;
- random tilt around the x and y axes;
- Random extension (and reduction) of dimensions;
- Random offset on the xy plane;
- Random bias on the z axis;
- Random jitter of the points (x,y,z) and versors.

The results of the classification task achieve an overall accuracy of 99%. Table 3 shows the classification metrics for each of the 5 classes.

ClassId	Precision	Recall	F1-score	Support
0	1.00	1.00	1.00	130
1	1.00	0.99	1.00	145
2	1.00	0.50	0.67	2
3	1.00	0.98	0.99	145
4	0.97	1.00	0.98	145

Table 3: Classification results. Metrics across the classes.

Examples of classified objects and their prediction with ground-truth (label) classes are shown in Appendix 2. It can be noticed that, for almost all examples, there is match between prediction and label, confirming the good accuracy of the trained model. Further analysis was done by considering as input 6 features per point (x,y,z,x\_normal,y\_normal,z\_normal). However, the introduction of these features did not lead to better results. The network probably considering 3 input features and already achieving high performance, learned more discriminative local features by combining the 3 input features. The normals can be derived by considering the coordinates of the points of a surround, therefore, they are dependent on the same coordinates of the point (x,y,z).

#### 4.2. Segmentation Phase

Altogether, we generated 2000 scenes split between training set and test set, to the extent of 80%, 20% respectively (1600 training scenes and 400 test scenes). An acceptable balance of classes has been maintained over the whole dataset. Each scene contains 4096 points, equal to 1024 points for 4 objects. In this case, the input size was set to process a scene of 2000 points and 3 features per point. For this, a subsampling of the 2000-point scene is performed; in addition, a translation of the scene is performed such that its mean coordinate value is zero (cloud centered in the axis origin). The size of the output was set to 4 classes. The mini-batch strategy with batch size set to 32 was used for training; the optimization algorithm used to minimize the loss function was Adam, setting an initial learning rate of 0.001. Training was performed for 15 epochs., as shown in Tabel 4.

Parameter	Value
Total Number of Scenes	2000
Number of Points per Scene	4096
Batch Size	32
Optimization Algorithm	Adam
Initial Learning Rate	0.001
Epochs	15

Table 4: DNN Parameters and Training Details for Semantic Segmentation Phase

The results of the semantic segmentation task on synthetic scenes achieve a total accuracy of 90% on the 400 test scenes. Figure 6 shows some examples of scenes from the test set, comparing the ground-truth one and the one segmented by the trained model. It can be seen that in most cases there is match between prediction and label of individual points. However, some errors can be also noted. In 6a, (second line), in close proximity of a groin vault arch, some points are incorrectly predicted as being part of a vault. A likely explanation is the similarity between groin and fan vaults in the proximity of the arch, and that the network confuses.

Figure 6: Segmentation results on the synthetic scenes with 4 examples of the test set. (a), (c), (e), (g) are predictions while (b), (d), (f), (h) are Ground Truth

#### 4.3. Validation phases on Case Study

The same model used for segmentation of synthetic scenes (see Appendix 3) was retrained for segmentation of real scenes. For this, we used the dataset with individual vaults of the Ducal Palace and created synthetic scenes by randomly selecting individual vaults. The input size of the DNN was set at  $20 \times 20$  meters, which allows to analyze scenes containing more than one vault or at least a significant portion of them. Therefore, synthetic scenes were created by randomly placing in the xy plane one or more vaults (randomly rotated around the z axis and placed at a random height from 0 to 15 meters) as long as the center (in the xy plane) of each is within the square range from 0 to 20 meters and there is no overlap between them.

Dataset:

1. Barrel: 10
2. Groined: 5
3. Cloister: 0
4. Sail: 0
5. Barrel with lunettes: 19
6. Mirror: 7
7. Flat roof: 3
8. Barrel with lunettes: 1

It has been done a balance of the classes, avoiding that the points belonging to the majority class are more than three times those belonging to the minority class. The results obtained in the training phase achieve 71% accuracy on the validation set.

We performed the prediction on the main floor (which includes all the vaults with which the synthetic examples were generated), that can be seen in Appendix 4. The sliding window technique was used for the analysis of the whole scene. The DNN, with input size  $20 \times 20$ , slides in the xy plane analyzing a “segment” of the scene at each step. To obtain results less susceptible to in-stantaneous window position, the analysis is repeated  $n \times n$  times considering overlapping predictions (translation along the x-and y-axes equal to a fraction  $n$  of the window size) as can be seen in Figure 7; this overlap is strictly related to the stride of the

(a) Overlap: 1x1 -Accuracy=0.51

(b) Overlap: 1x1 -Accuracy=0.51

(c) Overlap: 3x3 -Accuracy=0.64

(d) Overlap: 3x3 -Accuracy=0.64

(e) Overlap: 5x5 -Accuracy=0.69

(f) Overlap: 5x5 -Accuracy=0.69

Figure 7: Results obtained with 3 tests on PointNet with different “degrees” of overlap. The images on the left show the predicted class of each point. The images on the right show the confidence (probability) of the predicted class of each point. The red points are classified with high confidence (about 1), while the blue points are classified with small confidence (about 0)

sliding window performed by the neural network over the entire scene of point cloud. Confidences of each prediction (for each class) were averaged, and the one with the highest confidence is chosen as the predicted class. To choose the best combination of hyper-parameters for both the DNNs chosen for the experiment, we have highlighted the ones that produced the best performance results; in Table 5 we advised that the Pointnet reached the best Overall Accuracy (OA) with an overlap of  $5 \times 5$ , whilst DGCNN with an over-lap of  $2 \times 2$ . This step was fundamental, since we went through the single classes, directly

Overlap	PointNet OA	DCGNN OA
1 x 1	0.49	0.65
2 x 2	0.60	0.74
3 x 3	0.64	0.72
4 x 4	0.67	0.73
5 x 5	0.69	0.72

Table 5: PointNet VS DGCNN overall accuracy results.

exploiting the best configuration for each of the tested DNNs. The results are reported in Table 6 and Table 7, respectively. The results are interesting and deserve further analysis; indeed, apart from those classes that were not present in the case study thus resulting as zero, there are some vaults which classification mostly depends on the complexity rather than the number of samples. This aspect will be studied in depth in future studies.

ClassId	Precision	Recall	F1-score	IoU	Support
0	0.66	0.54	0.59	0.35	40960
1	0.33	0.50	0.08	0.05	20480
4	0.68	0.98	0.80	0.67	77824
5	0.84	0.85	0.84	0.73	28672
6	0.00	0.00	0.00	0.00	12288
7	0.75	0.70	0.72	0.54	4096

Table 6: PointNet results (overlap  $5 \times 5$ )

ClassId	Precision	Recall	F1-score	IoU	Support
0	0.56	0.82	0.67	0.50	40960
1	0.79	0.59	0.67	0.51	20480
4	0.81	0.90	0.85	0.74	77824
5	0.95	0.71	0.81	0.68	28672
6	0.00	0.00	0.00	0.00	12288
7	1.00	0.30	0.47	0.30	4096

Table 7: DGCNN results (overlap  $2 \times 2$ )

## 5. Conclusions and Future Works

The paper presents and stresses a method for obtaining point clouds se-mantically aware: they are built upon a strong knowledge of geometrical rules and shape grammars exploiting procedural modelling and available software and add-on. The method is able to produce a sufficient amount of synthetic data for the training of DNN. A major achievement, enabled by this last, is thus the implementation and verification of a DL system for segmenting and distinguish-ing different types of architectural vaults. The results were satisfactory both for the classification and for the segmentation point of view. Above all good outcomes are noticeable in the real case scenario of the Ducal Palace first floor: in particular the main goal of refining the classification deepening the level of geometrical detail was obtained. As additional contribution the vaults' complete dataset, consisting in point clouds and 3D models, is made available in order to allow other scholars and researchers to compare with their case studies, to test further DNNs as well as to speed up the segmentation phase for different her-itage buildings. The dissertation about the shape grammar and the glossaries also allowed to present a reusable taxonomy. The work developed so far in regard to the vaults' procedural modelling should be enhanced also widening the timeframe of the involved historical ar-chitecture. In fact, even though classical architecture presents challenging complexities for scenes management and segmentation, it is known that in medieval architecture the peculiarities are even greater. In this light, a focus on pointed vault systems is considered an effective further step. From the validation of the automatic segmentation system also comes the need to link and integrate similar point cloud applications into scan2BIM processes. Considering the outlook of the present work, we should take into account different structural and geometrical vaulted systems such as the ribbed vaults. The system of scripting and generation of synthetic vaults also makes possible punctual geometric changes as addition of reinforcing elements of the ribs. The achievable results will thus be extendable to medieval vaulted surfaces or other historical periods. Regarding the semantic segmentation, it should be also tested if the

presence of ribs in PC should influence the typology classification or if the main geometries are in any case recognized.

Further investigations will include the evaluation of the influence of noise elimination and disturbance removal on the results obtained from our automatic segmentation approach. To address this concern, we will conduct an in-depth analysis of several preprocessing techniques commonly used in point cloud data preparation. We will explore different noise elimination methods, such as filtering algorithms and outlier removal techniques, to assess their effectiveness in enhancing the quality of the input data. Additionally, we will examine disturbance removal techniques, such as denoising and outlier rejection, to ascertain their impact on the segmentation process. Through extensive experimentation, we will quantitatively evaluate the performance of our automatic segmentation method under varying degrees of preprocessing. We will measure segmentation accuracy, robustness to noise, and the ability to handle disturbances in the data. Comparative studies will be conducted to compare the outcomes of the method with and without preprocessing, allowing us to quantify the improvements achieved through preprocessing. Furthermore, we will address the potential trade-offs between aggressive preprocessing, which may eliminate genuine data points, and preserving subtle but valuable information during segmentation. By striking a balance, we aim to identify the optimal preprocessing parameters that lead to improved segmentation results without compromising critical details. To validate the impact of preprocessing, we will also perform a qualitative analysis to visually inspect the segmentation outputs under different preprocessing configurations. This will allow us to identify scenarios where certain preprocessing methods may introduce artifacts or affect the delineation of specific heritage features. Ultimately, the insights gained from this investigation will enable us to provide a clearer explanation of how the pre-processing phase influences the application of our method. By understanding the interplay between preprocessing choices and segmentation performance, we can offer more informed guidelines for practitioners working in the cultural heritage domain. We believe that addressing these concerns will contribute to the robustness and reliability of our segmentation approach, ensuring its efficacy in diverse and challenging heritage site environments.

## Acknowledgements

The survey activities in the Ducal Palace in Urbino were authorized by MIC, Ministry of Culture -General Directorate for Museums -National Gallery of Marche. Thanks to the Director of the National Gallery of Marche, Dr Luigi Gallo, for his hospitality and for enabling access to the museum. The authors want to thank also Prof. Paolo Clini, Principal Investigator of the CIVITAS project, to have made available the point clouds for this experimentation.

## References

- [1] Mafkereseb Kassahun Bekele, Roberto Pierdicca, Emanuele Frontoni, Eva Savina Malinverni, and James Gain. A survey of augmented, virtual, and mixed reality for cultural heritage. *Journal on Computing and Cultural Heritage (JOCCH)*, 11(2):1–36, 2018.
- [2] George Pavlidis, Anestis Koutsoudis, Fotis Arnaoutoglou, Vassilios Tsioukas, and Christodoulos Chamzas. Methods for 3d digitization of cultural heritage. *Journal of Cultural Heritage*, 8(1):93–98, 2007.
- [3] Katja Malovrh Rebec, Boris Deanovic, and Laurens Oostwegel. Old build-ings need new ideas: Holistic integration of conservation-restoration process data using heritage building information modelling. *Journal of Cultural Heritage*, 55:30–42, 2022.

- [4] Valeria Croce, Gabriella Caroti, Livio De Luca, Kevin Jacquot, Andrea Piemonte, and Philippe Veron. From the semantic point cloud to heritage-building information modeling: A semiautomatic approach exploiting machine learning. *Remote Sensing*, 13(3):461, 2021.
- [5] Rita Machete, Joana R Silva, Rita Bento, Ana Paula Falcao, Alexandre B Goncalves, Jose Maria Lobo de Carvalho, and Daniel Vaz Silva. Information transfer between two heritage bims for reconstruction support and facility management: The case study of the chalet of the countess of edla, sintraportugal. *Journal of Cultural Heritage*, 49:94–105, 2021.
- [6] Xiucheng Yang, Pierre Grussenmeyer, Mathieu Koehl, Helene Macher, Arnadi Murtiyoso, and Tania Landes. Review of built heritage modelling: Integration of hbim and other information techniques. *Journal of Cultural Heritage*, 46:350–360, 2020.
- [7] Charles R Qi, Hao Su, Kaichun Mo, and Leonidas J Guibas. Pointnet: Deep learning on point sets for 3d classification and segmentation. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pages 652–660, 2017.
- [8] Charles Ruizhongtai Qi, Li Yi, Hao Su, and Leonidas J Guibas. Point-net++: Deep hierarchical feature learning on point sets in a metric space. *Advances in neural information processing systems*, 30, 2017.
- [9] Antonella Musicco, Rosella Alessia Galantucci, Silvana Bruno, Cesare Verdoscia, and Fabio Fatiguso. Automatic point cloud segmentation for the detection of alterations on historical buildings through an unsupervised and clustering-based machine learning approach. *ISPRS Annals of the Pho-togrammetry, Remote Sensing and Spatial Information Sciences*, 2:129–136, 2021.
- [10] Romina Nespeca and Livio De Luca. Analysis, thematic maps and data mining from point cloud to ontology for software development. *The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences*, 41:347–354, 2016.
- [11] Eleonora Grilli, Alessandro Daniele, Maarten Bassier, Fabio Remondino, and Luciano Serafini. Knowledge enhanced neural networks for point cloud semantic segmentation. *Remote Sensing*, 15(10):2590, 2023.
- [12] A S´anchez-Rodriguez, B Riveiro, B Conde, and M Soilan. Detection of structural faults in piers of masonry arch bridges through automated pro-cessing of laser scanning data. *Structural Control and Health Monitoring*, 25(3):e2126, 2018.
- [13] Yixiong Jing, Brian Sheil, and Sinan Acikgoz. Segmentation of large-scale masonry arch bridge point clouds with a synthetic simulator and the brid-genet neural network. *Automation in Construction*, 142:104459, 2022.
- [14] Eleonora Grilli, Fabio Menna, and Fabio Remondino. A review of point clouds segmentation and classification algorithms. *The International Archives of Photogrammetry, Remote Sensing and Spatial Information Sciences*, 42:339, 2017.
- [15] Roberto Pierdicca, Marina Paolanti, Francesca Matrone, Massimo Martini, Christian Morbidoni, Eva Savina Malinverni, Emanuele Frontoni, and An-drea Maria Lingua. Point cloud semantic segmentation using a deep learn-ing framework for cultural heritage. *Remote Sensing*, 12(6):1005, 2020.
- [16] Yue Wang, Yongbin Sun, Ziwei Liu, Sanjay E Sarma, Michael M Bronstein, and Justin M Solomon. Dynamic graph cnn for learning on point clouds. *Acm Transactions On Graphics (tog)*, 38(5):1–12, 2019.



- [17] E Grilli, E Ozdemir, and F Remondino. Application of machine and deep learning strategies for the classification of heritage point clouds. *International Archives of the Photogrammetry, Remote Sensing & Spatial Information Sciences*, 2019.
- [18] Francesca Matrone, Eleonora Grilli, Massimo Martini, Marina Paolanti, Roberto Pierdicca, and Fabio Remondino. Comparing machine and deep learning methods for large 3d heritage semantic segmentation. *ISPRS International Journal of Geo-Information*, 9(9):535, 2020.
- [19] Biwu Chen, Shuo Shi, Wei Gong, Qingjun Zhang, Jian Yang, Lin Du, Jia Sun, Zhenbing Zhang, and Shalei Song. Multispectral lidar point cloud classification: A two-step approach. *Remote Sensing*, 9(4):373, 2017.
- [20] Jixian Zhang, Xiangguo Lin, and Xiaogang Ning. Svm-based classification of segmented airborne lidar point clouds in urban areas. *Remote Sensing*, 5(8):3749–3775, 2013.
- [21] Pascal Laube, Matthias O Franz, and Georg Umlauf. Evaluation of features for Svm-based classification of geometric primitives in point clouds. In *2017 Fifteenth IAPR International Conference on Machine Vision Applications (MVA)*, pages 59–62. IEEE, 2017.
- [22] Pouria Babahajiani, Lixin Fan, and Moncef Gabbouj. Object recognition in 3d point cloud of urban street scene. In *Asian conference on computer vision*, pages 177–190. Springer, 2014.
- [23] Mariana Belgiu and Lucian Dragut. Random forest in remote sensing: A review of applications and future directions. *ISPRS journal of photogram-metry and remote sensing*, 114:24–31, 2016.
- [24] Dario Billi, Valeria Croce, Marco Giorgio Bevilacqua, Gabriella Caroti, Ag-nese Pasqualetti, Andrea Piemonte, and Michele Russo. Machine learning and deep learning for the built heritage analysis: Laser scanning and uav-based surveying applications on a complex spatial grid structure. *Remote Sensing*, 15(8):1961, 2023.
- [25] Eva Savina Malinverni, Roberto Pierdicca, Marina Paolanti, Massimo Martini, Christian Morbidoni, Francesca Matrone, and Andrea Lingua. Deep learning for semantic segmentation of 3d point cloud. *International Archives of the Photogrammetry, Remote Sensing & Spatial Information Sciences*, 2019.
- [26] F Matrone, A Lingua, R Pierdicca, ES Malinverni, M Paolanti, E Grilli, F Remondino, A Murtiyoso, and T Landes. A benchmark for large-scale heritage point cloud semantic segmentation. In *XXIV ISPRS Congress*, volume 43, pages 1419–1426, 2020.
- [27] Roberto Pierdicca, Marina Paolanti, Ramona Quattrini, Massimo Mar-tini, Eva Savina Malinverni, and Emanuele Frontoni. Generative net-works for point cloud generation in cultural heritage domain. In *ARQUEOLOGICA 2.0 -9th International Congress & 3rd GEORES -GEOmatics and pREServation*, 2021.
- [28] Yongbin Sun, Yue Wang, Ziwei Liu, Joshua Siegel, and Sanjay Sarma. Pointgrow: Autoregressively learned point cloud generation with self-attention. In *Proceedings of the IEEE/CVF Winter Conference on Applications of Computer Vision*, pages 61–70, 2020.
- [29] Guandao Yang, Xun Huang, Zekun Hao, Ming-Yu Liu, Serge Belongie, and Bharath Hariharan. Pointflow: 3d point cloud generation with con-tinuous normalizing flows. In *Proceedings of the IEEE/CVF International Conference on Computer Vision*, pages 4541–4550, 2019.
- [30] Amir Hertz, Rana Hanocka, Raja Giryes, and Daniel Cohen-Or. Pointgmm: A neural gmm network for point clouds. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pages 12054– 12063, 2020.
- [31] David Griffiths and Jan Boehm. Synthcity: A large scale synthetic point cloud. *arXiv preprint arXiv:1907.04758*, 2019.

- [32] Roberto Pierdicca, Marco Mameli, Eva Savina Malinverni, Marina Paolanti, and Emanuele Frontoni. Automatic generation of point cloud synthetic dataset for historical building representation. In *International Conference on Augmented Reality, Virtual Reality and Computer Graphics*, pages 203–219. Springer, 2019.
- [33] Christian Morbidoni, Roberto Pierdicca, Marina Paolanti, Ramona Quat-trini, and Raissa Mammoli. Learning from synthetic point cloud data for historical buildings semantic segmentation. *Journal on Computing and Cultural Heritage (JOCCH)*, 13(4):1–16, 2020.
- [34] Baldassare Castiglione. *Il libro del cortegiano*, volume 611. Bur, 1987.
- [35] Paolo Clini, Ramona Quattrini, Paolo Bonvini, Romina Nespeca, Renato Angeloni, Raissa Mammoli, Aldo Franco Dragoni, Christian Morbidoni, Paolo Sernani, Maura Mengoni, et al. Digit (al) isation in museums: Civi-tas project–AR, VR, multisensorial and multiuser experiences at the Urbino’s ducal palace. In *Virtual and augmented reality in education, art, and museums*, pages 194–228. IGI Global, 2020.
- [36] Romina Nespeca. Towards a 3d digital model for management and fruition of ducal palace at urbino. an integrated survey with mobile mapping. *SCIRES-IT-SCientific REsearch and Information Technology*, 8(2):1–14, 2019.
- [37] Roberta Spallone, Maria Concepcion Lopez Gonzalez, and Marco Vitali. *Integrazione di nuove tecnologie di rilevamento e modellazione per l’analisi dei sistemi voltati a fascioni*.
- [38] A Costa Rosado. Types and uses of vaults and timbrel vaults in interior alentejo: Data for a typological study. In *History of Construction Cultures*, pages 141–148. CRC Press, 2021.
- [39] Maurizio Berti. Funzioni proprie delle volte in mattoni: il caso della volta a schifo della loggia cornaro in Padova. 1987.
- [40] Georg Barkhausen. *Balkendecken*. Number 3. Bergstrasser, 1895.
- [41] Guarino Guarini. *Architettura civile*. Appresso G. Mairesse all’insegna di Santa Teresa di Ges`u, 1737.
- [42] Federico Fallavollita. Le superfici rigate. *Migliari, Riccardo, Geometria descrittiva-Tecniche e applicazioni, CittaStudi, Novara*, 2:153–224, 2009.