



UNIVERSITÀ POLITECNICA DELLE MARCHE

DOCTORAL THESIS

Ageing and Secular Stagnation in Italy

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"The world of reality has its bounds; the world of the imagination is boundless."

Jean-Jacques Rousseau, 1763

"Pessimism of the intellect, optimism of the will."

Romain Rolland, 1920

Acknowledgements

When I look back to three years ago, I can clearly recognise how far my perception of the world has changed. It has been a holiday of knowledge and thought, full of doubts and uncertainties but crucial in shaping my life. Many people have been around me, beginning with my family. My mother Simonetta, my father Lorenzo, and my brother Samuel, have always been present, helping me find my own direction in life without making judgements but rather encouraging me. Deserve a special thanks Alessandro Celani, Giordano Margotti, Giulio Sciacchitano, Damiano Meloni, Matteo Battagli, Federico De Angelis, Sofia Garruto, Bisrat Kura, Francesca Minelli, Feliziano Copponi, Federico Marinelli, Giacomo Teodori, and Veton Mustafi. People who are always looking on the bright side of life. Discussions with them have been a pleasure, an opportunity to challenge my ideas, consolidate principles, and never forget to look at the world with pluralism. I am particularly grateful to my home supervisors, Giulia Bettin and Federico Giri, who have welcomed and believed in me since the beginning. They supported my roadmap, respecting my ideas and attitude. It's been a pleasure, and I'm looking forward to keep working with them. Finally, my abroad supervisor Fabio Mariani, who, along with my home supervisors, gave me the opportunity to experience what has become a sensational period at the Université Catholique de Louvain in Belgium, where I met many fantastic people from all over the world.

Thank you all!

UNIVERSITÀ POLITECNICA DELLE MARCHE

Abstract

Faculty of Economics

Department of Economics and Social Sciences

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by Alex CRESCENTINI

The Great Financial Crisis of 2008 forced the economic community to reconsider the economic fundamentals as well as the tools to be used for policy reasons. In the last two decades, many interesting theories have been developed, and institutions expanded their actions with unconventional measures to fight slow economic growth. The Secular Stagnation Hypothesis (Summers, 2015a), which identifies ageing and income inequality as structural determinants of the sluggish economic environment, is one of the most credited developments. Building on this field of research, the work presented here contributes along four dimensions. The first chapter presents the macroeconomic debate fueled by the bursts of the Great Financial Crisis, by focusing primarily on the Secular Stagnation Hypothesis and the various orthodox and heterodox alternatives that have been proposed. The second chapter deals with a replication of the model by Eggertsson et al. (2019), by employing a different computational technique included in the well-known Matlab/Julia package Dynare (Adjemian et al., 2022), with the aim of contributing to the field of replication exercises, as well as lowering the entry barriers for those interested in working with quantitative models. The final two chapters consider Italy as a case study to examine some aspects of the Secular Stagnation Hypothesis. The third chapter builds a computational overlapping generation model (OLG) to determine whether migration dynamics can be quantitatively relevant as a dampening factor to the declining pattern of interest rates. The fourth chapter use a Shift-Share methodology to investigate and quantify the effects of ageing and income inequality in driving the savings dynamics in Italy and determine which is dominant.

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Abstract

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La crisi finanziaria del 2008 ha forzato la comunità economica a riconsiderare i fondamenti teorici e gli strumenti d'azione sulla quale si è appoggiata per anni. Negli ultimi due decenni, con l'obiettivo di spiegare e affrontare la bassa crescita economica, il pensiero accademico ha prodotto nuove teorie economiche mentre le istituzioni governative hanno ampliato gli strumenti d'azione dispiegando misure non convenzionali. Circa il pensiero accademico, la teoria della Stagnazione Secolare (Summers, 2015a), identificando l'invecchiamento e la disuguaglianza reddituale come determinanti strutturali della bassa crescita economica, è una delle ipotesi più accreditate. Il seguente lavoro, inserendosi in questo quadro di ricerca economica, contribuisce lungo quattro dimensioni. Il primo capitolo presenta una rassegna del dibattito macroeconomico frutto della Grande Crisi Finanziaria, partendo dalla teoria della Stagnazione Secolare e analizzando le varie alternative proposte, ortodosse e non. Il secondo capitolo replica il modello di Eggertsson et al. (2019) sfruttando una diversa tecnica computazionale inclusa nel software Dynare (Adjemian et al., 2022), con l'obiettivo di contribuire al campo delle replicazioni, nonché di abbassare le barriere d'ingresso nel lavorare con modelli quantitativi. I due capitoli finali considerando l'Italia come caso studio, esaminano alcuni aspetti della Stagnazione Secolare. Il terzo, utilizzando un modello overlapping (OLG), determina se il fattore migratorio risulta quantitativamente rilevante nello smorzare il declino dei tassi d'interesse. Il quarto invece, sfruttando una metodologia Shift-Share, indaga gli effetti d'invecchiamento e disuguaglianza reddituale sul risparmio aggregato e determina quale risulta dominante.

Contents

Acknowledgements	v
Abstract	vii
1 Review of the Macroeconomic Debate since the Great Financial Crisis	1
1.1 Introduction	1
1.2 Growth Factors: Summers and Gordon	4
1.2.1 The Secular Stagnation Hypothesis	4
1.2.2 The Technology (TFP) and the Headwinds	7
1.3 International Factors: Bernanke, Caballero, Farhi and Gourinchas	13
1.3.1 The Global Saving Glut	13
1.3.2 The Safe Assets Shortage and the Safety Trap	15
1.4 Financial Factors: Rogoff, Borio and Koo	17
1.4.1 The Debt Supercycle	17
1.4.2 The Financial Drag Hypothesis	21
1.4.3 The Balance Sheet Recession	23
1.5 System Factors: Heterodox Economics	26
1.5.1 The Destruction of Shared Prosperity	28
1.5.2 The Stagnation and Financialization Trap	37
1.6 Conclusions	42
2 Dynare Replication of Eggertsson et al. (2019)	45
2.1 Introduction	45
2.2 Replication of the original Matlab code	46
2.2.1 Matlab equations	47
2.2.2 Results	52
2.3 Paper replication	55
2.3.1 Paper Equations	56
2.3.2 Results	61

2.4	Conclusions	63
3	Secular Stagnation: a Quantitative analysis of Migration in Italy	65
3.1	Introduction	65
3.2	Immigration in Italy	69
3.3	Model	76
3.3.1	Population	77
3.3.2	Households	78
3.3.3	Firms	79
3.3.4	Government	81
3.3.5	Law of Motion of Capital	82
3.3.6	Asset Market Clearing Condition	82
3.3.7	Equilibrium	82
3.4	Calibration	83
3.4.1	Baseline Calibration	85
3.4.2	Alternative Specification 1	86
3.4.3	Alternative Specification 2	86
3.5	Simulation and Policy Implications	88
3.5.1	Role played by Migrant Population till 2020	88
3.5.2	Expected role by Migrant Population in the future	89
3.6	Conclusions	91
4	The Challenge of Ageing and Income Inequality for Italian Savings	93
4.1	Introduction	93
4.2	Methodology	97
4.2.1	National Accounting Framework	97
4.2.2	The Income Less Consumption method	99
4.2.3	Shift-Share Decomposition	101
4.3	Dataset	103
4.3.1	Survey on Household Income and Wealth (SHIW)	103
4.3.2	Age Cohorts	104
4.3.3	Income Shares	105
4.4	Results	106
4.4.1	Saving rates across income and age groups	106
4.4.2	Changes in income shares	107
4.4.3	Shift-Share Analysis	108

4.4.4	Intergenerational Inequality	111
4.5	Conclusions	111
A	Technical Details of Chapter 1	113
A.1	Debt Overhang Literature Review	114
B	Technical Details of Chapter 2	117
B.1	Model Derivations	118
B.1.1	Demographics	118
B.1.2	Households Problem	119
B.1.3	Firms Problem	123
B.1.4	Government	126
B.1.5	Aggregates	126
C	Technical Details of Chapter 3	127
C.1	Model Derivations	128
C.1.1	Households Problem	128
C.1.2	Firms Problem	130
C.1.3	Stationary Equilibrium	133
D	Technical Details of Chapter 4	135
D.1	Proof of the Shift-Share Decomposition	136
	Bibliography	137

List of Figures

1.1	Capacity Utilization (Top Left), Inflation CPI Rate (Top Right) and Unemployment Rate (Bottom). Source: Authors' elaboration from FRED data.	5
1.2	G7-Economies Natural Population Growth Rates (left) and Long-Term Interest Rates (right). Source: Authors' elaboration from UN and JST data.	6
1.3	TFP annual growth rate for 10-years for United States. Source: Authors' elaboration from Gordon (2015) data.	10
1.4	Public debt (Left), Private debt (Center) and External debt (Right). Source: Authors' elaboration from Reinhart et al. (2012b) data.	19
1.5	US Business and Financial cycles. Source: Borio (2019, p. 5).	22
1.6	TARGET2 Balances in Millions of Euros. Source: Authors elaboration from ECB data.	25
1.7	The economics of secular stagnation: the demand-side view. Source: Storm (2022, p. 51).	37
2.1	Aggregate variables dynamics from Eggertsson et al. (2019) Matlab code (blue line) and the Dynare implementation (dashed red line)	54
2.2	Natural Interest Rate (HP Trend)	56
2.3	Aggregate variables dynamics from Eggertsson et al. (2019) Matlab code (blue line) and the paper Dynare implementation (dashed red line)	62
2.4	HP Trend Interest Rate	63
3.1	G7-Economies Natural Population Growth Rates (left) and Long-Term Interest Rates (right). Authors elaboration from UN and JST data.	66
3.2	Italy Natural and Total Population Growth, and Net Migration Growth. Authors elaboration from UN data.	69

3.3	Italy Natives, migrants and total population TFRs from ISTAT (left) and Total population TFR from World Bank (right). Authors elaboration from ISTAT and World Bank data.	72
3.4	Migrant Population Pyramid (upper) and Native Population Pyramid (lower), 2002, 2010, 2021. Authors elaboration from ISTAT data.	73
3.5	Labor Market: percentage of labor force over total population. Authors elaboration from ISTAT data.	74
3.6	Authors elaboration from ISTAT data.	89
3.7	Transition path of real interest rate (left-hand) and spread between scenarios (right-hand). Authors model elaboration and Bank of Italy, ECB and FRED data.	90
3.8	Full transition path of real interest rate (left-hand) and full spread between scenarios.	91
4.1	Income Inequality (left) and Population Ageing (right). Authors' elaboration from WID and UN database.	95
4.2	Crude Birth and Death Rates and Net Migration Rates for Italy. Authors elaboration from UN database.	96
4.3	Within-cohort Income Distribution (left) and Age Distribution (right). Sample period: average annualized saving rates 1980-2020.	107
4.4	Income shares along the income distribution (left) and among birth cohorts with median age between 18-54 years and 55-95+ years (right) over time.	108
4.5	Shift-Share by Income Group (left) and Shift-Share by Age Group (right). Sample period: average (2000-2020) - average (1980-2000). . .	109

List of Tables

1.1	Classification of Macroeconomic Theories after the GFC.	3
1.2	Authors' elaboration from Storm (2017) data.	36
2.1	1970 steady state for the baseline calibration	53
3.1	Percentage 65+/20-64 age-dependency ratio (upper-part), average total factor productivity growth (central-part), and migration ratio (bottom-part) by specified group of countries and years. Authors' elaboration from OECD, The Conference Board Total Economy Database, Statistics Korea, Japan's National Institute of Population and Social Security Research, and Italian Istat.	68
3.2	Parameters from Statistical Data that do not change with the Scenario	84
3.3	Parameters from Statistical Data: TFRs	85
3.4	Parameters from Literature	85
3.5	Parameters chosen to match Targets	86
3.6	Parameters from Statistical Data: TFRs	86
3.7	Parameters from Literature	87
3.8	Parameters chosen to match Targets	87
4.1	Descriptive Statistics: Age Cohorts	105
4.2	Shift-Share Analysis by Income Groups	110
4.3	Shift-Share Analysis by Age Groups	110
A.1	Debt Overhang Literature Review. Authors elaboration from Salmon and Ruyg (2020) and Reinhart et al. (2012a) data.	114
A.1	Continued.	115

Dedicated to those full of doubts...

Chapter 1

Review of the Macroeconomic Debate since the Great Financial Crisis

1.1 Introduction

The Great Financial Crisis (GFC) made visible the failure of classical theories in macroeconomics to achieve sustainable growth with financial stability. In a speech at the London School of Economics in 2008, Queen Elisabeth II summed up the failure by asking leading economists: "Why did nobody notice it?" Today, 15 years later, there's still something in the air that doesn't make us say that we have recovered from the GFC. Clearly, we are missing something, and since 2008, there has been a wide reflection in macroeconomic thought about theory and tools to be used for policy purposes. The GFC can be thought of as a turning point in economic history for two reasons. First, it has been the greatest financial crisis ever, with unprecedented economic and social repercussions. It led to a massive credit crunch in the banking system, a loss of more than 2 trillion dollars, and a rise of 30 million jobless two years after the outbreak of the bubble, globally. The response from governments and central banks was an unprecedented fiscal and monetary stimulus that stabilised the economies from the crisis' hangover in 2010 at a cost of a debt level of more than 200% of GDP for the first time globally. However, governments were not able to push economic growth back to pre-recession levels. Indeed, the economy still had an open output gap in 2013, five years after the bubble burst, and its pattern was reducing only for the downward revision of potential GDP, so no progress was made in restoring actual GDP to its potential level. Second, it called into question institutions' belief that they are always capable of reviving the economy. Indeed, during the period between 2007 and 2013, in conjunction with the high

debt levels, nominal interest rates fell to zero for the first time in history, and central banks in Western countries quickly found themselves unable to further stimulate the economies because their monetary policies were bound at the zero lower bound (ZLB). The economies have been further challenged by the outbreak of the Eurozone sovereign debt crisis, which forced Mario Draghi to use the famous "whatever it takes" stance to prevent a collapse in November 2012. With reduced fiscal spaces and limited monetary policies, unconventional measures and a wide reflection on macroeconomic thought have been inescapable, both about theories and policies to be undertaken. However, the latter depends on the former, so many international meetings were organised, looking for ways out. Summers (2013) in a speech at the IMF, dusted off the old idea of Hansen (1939) called "Secular Stagnation" (SS) to explain what was going on, becoming one of the earliest and most credited developments in economic theory. Since then, a great debate around the concept of the SS has arisen, becoming much more comprehensive than what Summers stated in 2013. The words of Eichengreen (2014, p. 41) explain it well: "While the term Secular Stagnation is widely repeated, it is not widely understood. Secular Stagnation, we have learned, is an economist's Rorschach test. It means different things to different people". Indeed many economists from various schools of thought, including the most radical, such as Marxists and Post-Keynesians, have joined the debate, either because they felt called upon or because they were already emphasising stagnation. The importance of secular stagnation is therefore in the confusion and the heterogeneous debate that have followed, a necessity that institutions must quickly ride to take back the helm of the economy. Given this premise, the purpose of the following work is to reorganize, in the simplest way possible, the macroeconomic literature born or revived after the Great Financial Crisis, allowing the reader to build an organised map through which to navigate the current macroeconomic debate. The pressing need to understand what's going on in our economies requires not leaving anything aside; therefore, both orthodox and heterodox economic theories will be considered, and the focus will be on Western countries. The various contributions will be grouped into narrow categories, reflecting the main factors stressed by each theory. Table 1.1 illustrates the division placed upon theories.

The rest of this paper proceeds as follows. Section 2 reviews theories stressing growth or structural factors like ageing and innovation. Section 3 reviews theories stressing international factors such as the international position of emerging

economies in the global financial market. Section 4 reviews theories stressing financial factors such as debt overhangs, the financial cycle, and the private sector's balance sheets. Section 5 reviews theories stressing factors at the system level, in particular the neoliberal economic growth model. Conclusions are given in the final section.

Growth Factors	Financial Factors
Secular Stagnation Hypothesis (Summers, 2015a)	Debt Supercycle (Rogoff, 2016)
Technology and Headwinds (Gordon, 2015)	Financial Drag Hypothesis (Borio, 2017)
	Balance Sheet Recession (Koo, 2011)
International Factors	System Factors
Global Saving Glut (Bernanke, 2005)	Destruction of Shared Prosperity (Hein, 2012; Palley, 2012; Storm, 2017)
Safe Assets Shortage and Safety Trap (Caballero et al., 2017)	Stagnation and Financialization Trap (Foster and McChesney, 2012)

TABLE 1.1: Classification of Macroeconomic Theories after the GFC.

1.2 Growth Factors: Summers and Gordon

1.2.1 The Secular Stagnation Hypothesis

Larry Summers, riding up to Krugman (2011)'s arguments based on a series of thoughts by economic professionals in many blog posts, formalised and opened "officially" the debate on secular stagnation in a speech at the IMF in November 2013. To understand the hypothesis stated by Summers (2013) consider the following observation, on which it is based: the GFC may not be the cause of the sluggish output growth, but as a burden, it may emphasise something that, behind the scenes, has been acting long before it. Observing the pre-GFC period (2003-2007) among G7 economies, even with financial carelessness, possibly sustained by too much expansionary fiscal policy and too much loose monetary policy, capacity utilisation was not under great pressure, the consumer price index was not that high, and the unemployment rate was not that low, as shown in Figure 1.1. Hence, the economy wasn't overheating. But with the largest bubble in history, it seems reasonable that we should have expected an excess of aggregate demand, isn't it? But we didn't get it. Thus, even the GFC, to some extent, wasn't able to produce an overheating in the economy. How is that possible? Summers writes that it would be possible that the economy is settling into a new steady state, long before the outbreak of the GFC. In other words, it is not a matter of going back to equilibrium because the equilibrium itself has changed. "That's why it seems that one has to contemplate macroeconomic theories of a very different kind than suggested by the conventional business cycle theory" (Summers, 2015b). This point is well noted by Krugman (2013): "it seems reasonable that this hasn't just been true since the GFC, it has arguably been true, although perhaps with increasing severity, since the 1980s. In other words, you can argue that our economy has been trying to get into the liquidity trap for a number of years, and that it only avoided the trap for a while thanks to successive bubbles, and if that's how you see things, when looking forward you have to regard the liquidity trap not as an exceptional state of affairs, but as the new normal". Why is this happening? What are the forces behind this kind of tendency? To answer to these questions, Summers (2013) explicitly invokes the notion of Secular Stagnation, going back to Hansen (1939).

In 1939, Alvin Hansen stressed demographic factors, in particular the slowing population growth, for a future persistent aggregate demand shortage. A lower population growth would mean less labour to use for each unit of capital, implying a

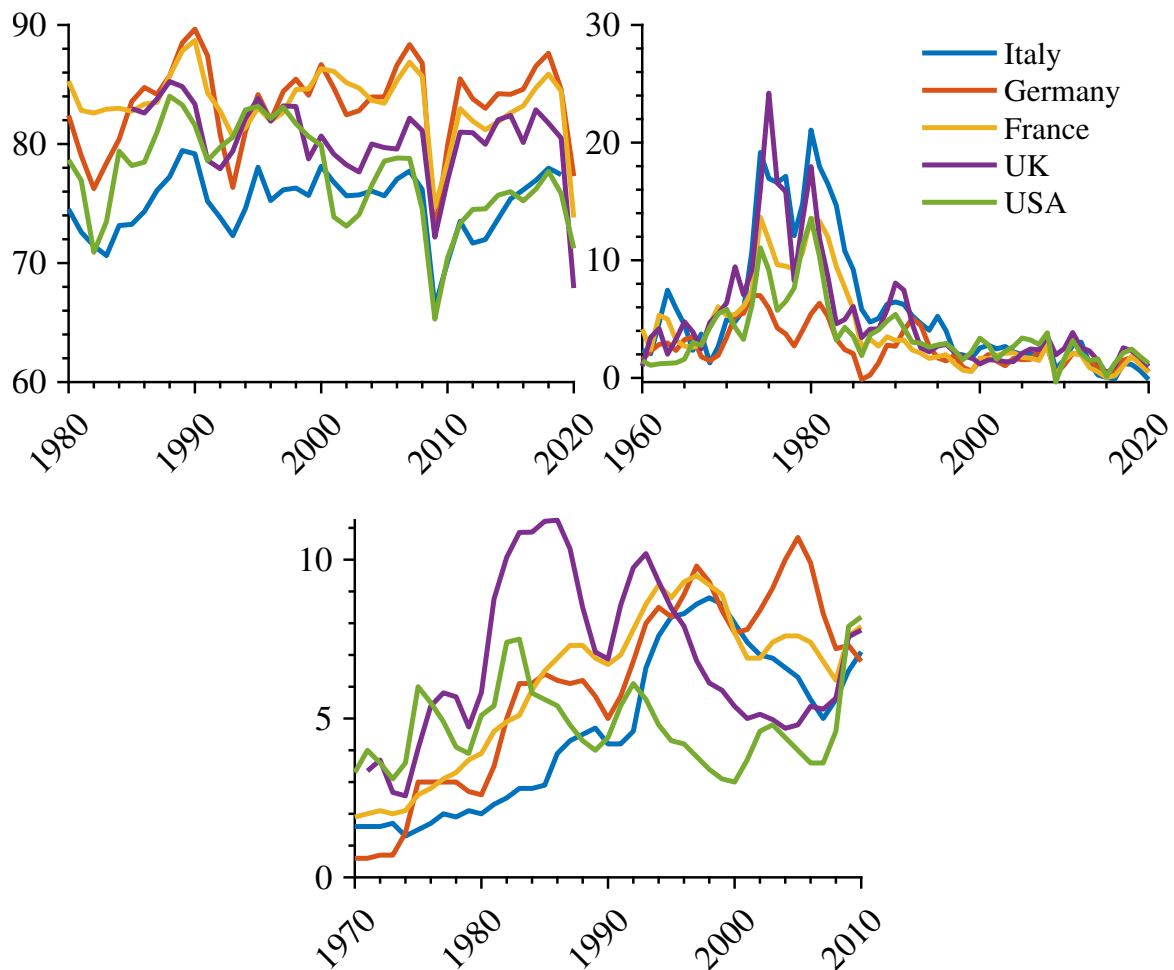


FIGURE 1.1: Capacity Utilization (Top Left), Inflation CPI Rate (Top Right) and Unemployment Rate (Bottom). Source: Authors' elaboration from FRED data.

lower investment demand and, as a consequence, a lower aggregate demand. However, a few years later, the baby boom came out, and he was proved wrong. "But this time around the slowdown is here, and looks real" (Krugman, 2013) and today there's space for his theory to be proven right. Furthermore, according to the classic Ando and Modigliani (1963)'s life-cycle theory, an ageing population implied by lower fertility rates means a greater propensity to save relative to a younger population. Eventually, the increased life expectancy (Carvalho et al., 2016), the pressure on public finances that make the pension systems unsustainable at the current replacement level, and the health-related expenses which have become heavily compressed in our final years, as we live longer and face expensive end-of-life care (DeNardi and Borella, 2020) are other factors that contribute to the accumulation of precautionary

savings. Summing up, reflecting on Hansen's (1939) ideas and observing the current evolution in the ageing process of Western populations, Summers concludes that an excess of saving, which alters the real interest rate, could be the driving force behind the new equilibrium. This point is crucial in the Secular Stagnation Hypothesis, because it explains a particular stylized fact observed in Western countries, namely, the declining pattern of real interest rates since 1980. Figure 1.2 depicts both processes, ageing and interest rates for G7 economies. Population ageing is not the only factor

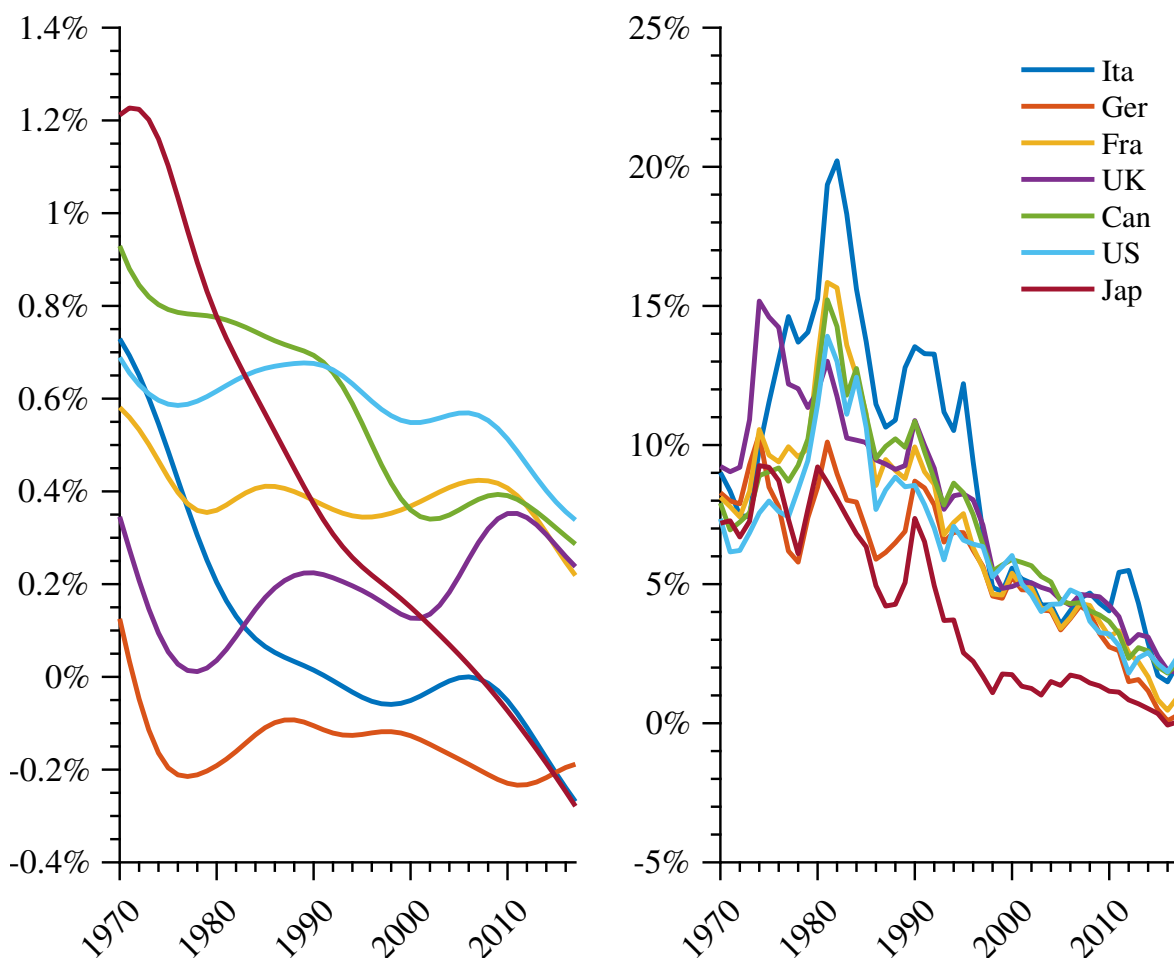


FIGURE 1.2: G7-Economies Natural Population Growth Rates (left) and Long-Term Interest Rates (right). Source: Authors' elaboration from UN and JST data.

emphasized, but it is the most important in the view of Summers (2015a), which in its original formulation reflects the permanent and long-run feature of the Secular Stagnation. Indeed, ageing is a slow and long-term process that tends to act subtly over the years, constantly pushing the economy into a less innovative and

slow one. The reason why the SS gained a lot of attention is that almost all the industrialised economies are facing an ageing process and a declining pattern in real interest rates (Figure 1.2), so one concept is able to explain many different common situations. Japan, used as a case study by Summers, seems to be the first example for the theory to be proven right, and Italy could be the second one.

In summary, what the Secular Stagnation Hypothesis states is that due to some factors, industrialised economies have an excess of saving that doesn't merge into the demand for investment as in the past; the old IS curve is decreasing, and the same does the aggregate demand. The result is a shortage of aggregate demand given an excess of savings. But, why didn't the equilibrium between saving and investment restore itself automatically? Prices have traditionally been thought to be vent variables in economics, which means they are called upon to restore equilibrium between the forces they represent by changing their values. Why didn't it happen to restore aggregate demand? A possible explanation is that the jammed mechanism comes from the zero lower bound (ZLB) over the official interest rate. The excess of savings may have pushed the short-term real rate required for full employment well below zero. Because nominal interest rates were already quite low, central banks were unable to lower them further, and given the low "natural" or "forced" inflation rates (Top Right graph of Figure 1.1),¹ real rates were unable to fall sufficiently to clear the loanable funds market.² As a consequence aggregate saving didn't flow into the investment demand, quantities instead of prices reduced and, the economy slowed operating below its potential.

1.2.2 The Technology (TFP) and the Headwinds

Another perspective of the same story passes by the supply side and the role of innovation. Although a kind of interplay between aggregate demand and potential GDP is not denied by Neoclassical economics, it has not been theorized neither, and as we will see later, it is a point well-remarked by the Post-Keynesians as a critique to orthodox economists. For the Neoclassicals, aggregate demand as well as the potential GDP in fact have their own distinct specifications, and they are linked

¹For "forced" inflation, I mean a sort of "repressed inflation" achieved through "disinflation policies engineered first in the US and the UK, and a bit later in many other advanced countries" (Blanchard et al., 2014) making use of Inflation Target regimes. Good insights on the target in various countries around the world can be found in Bernanke and Mishkin (1997).

²We will see in the following sections that, one of the point which differentiates the orthodox with the heterodox view is about the existence of the Loanable Funds Market in the economy.

by the concept of "output gap". Potential GDP represents the long-run growth of the economy given its structural determinants (population growth and technology, principally) and the aggregate demand the short-run, that is, the expenditures made in a country in a specific year. In this way, using a concept like the output gap, we have a sort of idea about the state of the economy at a specific time t . Returning to Gordon (2015), although he assumes a long-run perspective like Larry Summers, he considers the supply side of the economy, thus the potential GDP. The following quote grasps his idea: "since Solow's seminal work in the 1950s, economic growth has been regarded as a continuous process that will persist forever. But there was virtually no economic growth before 1750, suggesting that the rapid progress made over the past 250 years could well be a unique episode in human history rather than a guarantee of endless future advance at the same rate" (Gordon, 2012, p. 1). The belief about the uniqueness of the economic progress humankind have made in the last two centuries, the subsequent belief on a retracement of the pace of innovation (measured by the total factor productivity, TFP) at its long term average, and the evidence of a series of headwinds which may be added up to the conceivable mild effect of innovation, brings Robert Gordon to a pessimistic outlook about future growth.

Using a growth accounting approach, as specified in Gordon (2010), the potential GDP growth is expressed by the following identity:

$$Y \equiv \frac{Y}{H} \cdot \frac{H}{N} \cdot N \tag{1.1}$$

with:

Y = Real Potential GDP

$\frac{Y}{H}$ = Output per Hour (labor productivity)

$\frac{H}{N}$ = Labor Supply (in hours of work)

N = Working-Age Population.

If we logarithmically differentiate equation (1.1), we rewrite the result in difference-form and, we leave aside the demography component for a while, we get the same

expression in growth rates:

$$\hat{Y} \equiv \hat{\lambda} \tag{1.2}$$

with:

\hat{Y} = Potential GDP Growth

$\hat{\lambda}$ = Labor Productivity Growth ("technology")

From a supply-side perspective, potential GDP growth is the result of labour productivity growth or simply the TFP, because once the output is described using a classic production function (such as the Cobb-Douglas), the labor productivity becomes completely defined by the TFP and, the latter is usually measured as a Solow residual. Gordon therefore, analyses the dynamics of potential GDP, first by looking at the pattern of the TFP during the three industrial revolutions from the 19th century to today in the United States, and second considering some headwinds in the current economic environment. Looking at Figure 1.3, the period 1920-1970 represents the making process of the modern society associated with the 2nd industrial revolution and the higher TFP ever recorded, and recalling the David (1990)'s "Delay Hypothesis", he claims the reasons why the world awaited around 30 years (since the 1st industrial revolution) before showing off on the data. Further, he also explains how, around 1970, the 2nd revolution started to be challenged by diminishing returns with a clear effect over the TFP and, how the 3rd industrial revolution that came out around the same period, offset the dissolution of the 2nd one. Then, turning to today's world, in the mind of Gordon, the current secular stagnation comes from the fact that "the economy has also already benefitted from the internet and web revolution - the 3rd industrial revolution -, and in this dominant sphere of economic activity methods of production have been little changed over the past decade". More precisely, he explains how a combination of "statis in the office", "statis in retailing" and a "decline in business dynamism" have contributed to the slowdown in TFP and, therefore, labour productivity and potential GDP. About the first, "by 2005 flat screens had completed the transition to the modern office. But then progress stopped. Throughout the world the equipment used in office work and the productivity of office employees closely resembles that of a decade ago". About the second, "payment methods gradually changed from cash and checks to credit and

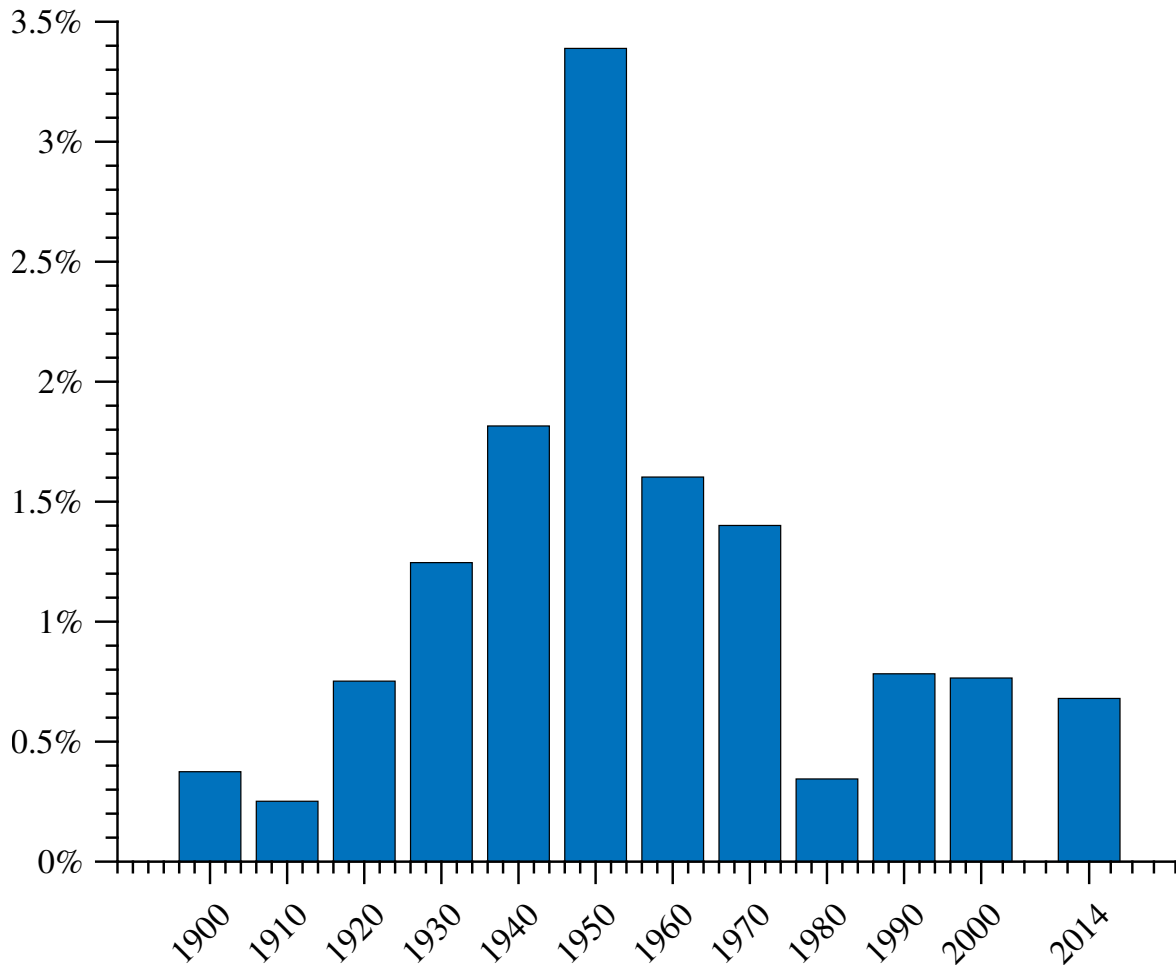


FIGURE 1.3: TFP annual growth rate for 10-years for United States.
Source: Authors' elaboration from Gordon (2015) data.

debit cards, and the process of card authorization became almost instantaneous by the late 1990s. Wal-Mart and other big-box chain transformed supply chains, wholesale distribution, inventory management, pricing, and product selection, but that productivity-enhancing shift away from traditional small-scale retailing is largely over". Concerning the third, the term "business dynamism," which has been used to describe a process of creative destruction in which productivity gains are derived from new best-practice technologies introduced by new start-ups and young firms, is declined with the arrival of the new century. "This decline was pervasive across retailing and services, and after 2000 even the high-tech sector experienced a large decline in startups and fast-growing young firms". In a nutshell, Gordon believes it will be difficult that the role that technology played in the middle of the last century will happen again, implying lower potential GDP growth from technology, at least

when compared to the period 1920-1970.

Once the dynamic of technology has been accounted, in the second part of his analysis, Gordon (2015) looks at some headwinds which have been playing since many years ago, that may offset the already expected mild pace in innovation. Six headwinds have been identified as concerns about future economic growth:

1. *Demographic Dividend*: There is evidence that the population growth rate in the West is declining due to a decrease in birth rates and an increase in life expectancy (see Figure 1.2). The effect is inevitable; the growth of the working-age population is shrinking. Further, the retirement of the baby-boom generation (which started around 2004) is another stressing factor that put pressure over the working-age population. Simply using the output identity in equation 1.1, the overall effect of the "Demographic Dividend" is a further decrease in the real potential GDP growth.
2. *Education*: In a society, it has been usually associated education achievement with economic growth. In particular, the education (or human capital) has always played a pumping role for labour productivity, in the sense of knowledge. The more we know, the more we are able to exploit and produce, and so, the more productive we are in the same time period (hour per work). Technically, defining the potential output through the production function, the more human capital, the higher the output per hour (or labour productivity) in the output identity of the equation (1.1). Gordon (2015) emphasises that in the United States, education was a huge engine for economic growth during the last half of the twentieth century, with high school graduation rates rising from 10% of youth in 1900 to 80% in 1970. After that, there has been a stagnation in high-school graduation rates, and if the trend continues, the labour productivity gain over the potential growth can't be the same as before. Two points must be made clear over this concern, one qualitative and another quantitative: 1) among the 34 OECD countries, the United States ranks 17th in reading, 20th in science, and 27th in mathematics in terms of secondary education quality; 2) quality has been hampered by affordability and student debt. "The cost of a university education has risen since 1972 at more than triple the overall rate of inflation. Even when account is taken of the discounts from the full-tuition made possible by scholarships and fellowships, the current level of American

college completion has been possible only by a dramatic rise in student borrowing - and an increased fraction of the next generation may choose not to complete college as they are priced out of the market for higher education" (Gordon, 2015, p. 57).

3. *Income Inequality*: The polarisation of income has produced, between 1980 and 2020, an average change in the Top 1% income share of 1.5% per year relative to the -0.2% of the Top 99%.³ "If what we care about when we talk about consumer well-being is the bottom 99% percent", then for almost all of the population, the real GDP per capita has declined. (Gordon, 2012, p. 17)
4. *Government Debt*: Starting in the 1980s and continuing through the GFC in 2008 and the COVID-19 spike in 2020, the federal debt over GDP increased from 30% in 1980 to 134% in 2020, with foreign holders increasing from 4% to 35% during the same period. "As a matter of arithmetic the ratio of government debt to GDP can be reduced by a mix of higher taxes, lower expenditures, and lower entitlement benefits (including higher retirement ages). But the same arithmetic implies that higher taxes and/or lower transfers reduces the growth rate of real household disposable income relative to that of real GDP" (Gordon, 2012, p. 18).
5. *Globalization*: It opened the doors all over the world, which means that as well as the outsourcing process, there is also the importation of low-cost labour from emerging nations, which, in Gordon's words, means the full work of "Factor Price Equalization's Theorem" of Samuelson (1948, p. 169): "as long as there is partial specialisation, with each country producing something of both goods, factor prices will be equalised, absolutely and relatively, by free international trade" and before two countries integrate resulting in one single market, the lowest-priced factor will tend to become more expensive, while the highest-priced factor will become cheaper. The overall results will be a decrease in the real household disposable income of Western economies.
6. *Energy and Environment*: The exploitation of the soil dated back since the 1st industrial revolution caught up the bill. With high debt, as explained in the fourth headwind above, there's a little margin to cope with climate change

³Data are taken from World Inequality Database (WID) "<https://wid.world>". Series: "sptinc_992_j_US"; Pre-tax National Income; Top 1% and Bottom 99%.

without getting our hands dirty on taxes at all. "The consensus recommendation of economists to impose a carbon tax will reduce the amount that households have left over to spend on everything else (unless it is fully rebated in lump-sum or other payments)" (Gordon, 2012, p. 17).

1.3 International Factors: Bernanke, Caballero, Farhi and Gourinchas

1.3.1 The Global Saving Glut

One of the criticisms levelled at Summers (2015a) is the lack of an international dimension. In this respect, the Global Saving Glut Theory (GSG) of Bernanke (2005), is the most influential alternative explanation to the Secular Stagnation Hypothesis. In a series of back and forth blog posts with Larry Summers, Bernanke (2015a,b,c) renovated his idea about the GSG, which dates back to a speech held at the Virginia Association of Economists in 2005. For what concerns the interest rates path, Bernanke (2015a) considers the monetary policy as a follower rather than a leader in determining real interest rates, meaning that the central banks are able to steer the short-term interest rates but have just a partial influence over the long-term ones. Therefore, the pattern of real interest rates is mainly the by-product of the state of the economy and, just residually, of monetary policy decisions. Of course, central banks played an important role in keeping interest rates low in advanced countries following the GFC, but they are not as important as the current state of the economy. Relative to this Bernanke is sceptical of Summers' narrative of the SS along four dimensions: 1) around 2015, the unemployment rate was around 5%, therefore near full-employment, hardly a symptom of stagnation; 2) he denied that the real interest rate could be so negative for a prolonged period of time to produce an investment dearth, as said by Summers. Recalling Paul Samuelson's quote: "if the real interest rate were expected to be negative indefinitely, almost any investment is profitable. For example, at a negative (or even zero) interest rate, it would pay to level the Rocky Mountains to save even the small amount of fuel expended by trains and cars that currently must climb steep grades". It's therefore questionable that the economy's equilibrium real rate can really be negative for an extended period; 3) the slowing recovery after the GFC is less due to secular factors but more

to temporary "headwinds" such as slow recovery of housing, credit conditions, and too restrictive fiscal policies; 4) most importantly, recalling the GSG, the SS lacks an international perspective, and that is where all the problem comes out. The SS could be correct as well, but "the availability of profitable capital investments anywhere in the world should help defeat the SS at home" by weakening the value of the dollar and encouraging exports (Bernanke, 2015b). But since this has not happened, other forces, placing aside the capital restrictions given that the capitals do not move freely all over the world, could have played a role. In particular, the GSG stressed in Bernanke (2005) could have been the reason. The excess of global saving pushed down interest rates globally, kept the dollar appreciated, produced a large trade deficit, and aggregate demand shortage.

The GSG assumes a global perspective, where "core business" is the observation of the large current account deficit accumulated by the United States since the 1990s. Just before becoming chairman of the Federal Reserve in 2006, Bernanke stressed the role of ageing in the major industrial economies as being the most important factor in making up the GSG domestically but not the main point quantitatively. What he considers relevant is the shift in the position of emerging economies (EMEs) in the global financial market from borrowers to lenders, due to a series of financial crises, the shock in oil prices, and the promotion of export-led economies between 1990 and 2005.⁴ First, the financial crises in Mexico (1994), Asian countries (1997), Russia (1998), and the sharp rise in oil prices in Venezuela (2002) and Argentina (2004) produced large capital outflows from those countries, inducing the buildup of large amounts of reserves for self-insurance against future capital outflows. Second, during the early 2000s in Asian countries (primarily China), "reserves were accumulated in the context of foreign exchange interventions intended to promote export-led growth by preventing exchange-rate appreciation" (Bernanke, 2005, p. 6). The reserve accumulation processes have been pursued by channelling the domestic savings of those countries (mainly by their respective central banks and just residually by the private sector itself) into foreign purchases of United States and European assets.⁵ The majority was accumulated in fixed-income assets; as a result, prices rose and yields declined. Overall, this process resulted in a switch by EMEs' international position from borrowers to lenders.

⁴About these arguments see also Mann (1999), Cooper (2001), Edwards (2005) and, Obstfeld and Rogoff (2005).

⁵See also Higgins and Klitgaard (2004) "Impact on local financial market" for details on how these operations have been technically pursued by central banks.

What about nowadays? "The Savings Glut interpretation of current events as providing a bit more reason for optimism than the Stagnationist perspective" (Bernanke, 2015c). The current account deficit of United States changed tendency, moving from a 6% of GDP in 2005 to something like 2% in 2015. The other part of the GSG story, the current account surplus of EMEs, decreased, mostly due to the reduction in China, Latin American and, Middle-east North Africa (mostly due to the fall in oil prices). Japan and Canada, among other developed countries, contributed to this. Conversely, the improvement in the current account surplus of Europe reduced this tendency, although the principal factors behind the improvement in the current account of European countries are the results of the Sovereign Debt Crisis that occurred in 2010, in particular in the European periphery. If China continues to reduce its export-led economic strategy, implying a reduction in the buildup of reserves, and if oil prices continue to remain low and the cyclical factor of European countries disappears, the current account of the United States should improve as well as interest rates and the economy as a whole.

To sum up, "because the dollar is the leading international reserve currency, and because some emerging-market countries use the dollar as a reference point when managing the values of their own currencies, the saving flowing out of the developing world has been directed relatively more into dollar-denominated assets, such as U.S. Treasury securities. The effects of the saving outflow may thus have been felt disproportionately on U.S. interest rates and the dollar" (Bernanke, 2005, p. 8). "The global saving glut is about weaker exports and a larger trade deficit" (Bernanke, 2015c), implying a shortage of aggregate demand as in Summers (2015a) although the shortage comes from external circumstances and just a residual part is accounted for domestically due to population ageing.

1.3.2 The Safe Assets Shortage and the Safety Trap

With the same international perspective, from the Global Saving Glut Theory of Bernanke (2005), takes ride the Safety Trap of Caballero and Farhi (2018) which, differently from the GSG, details the type of instruments that have been used by the EMEs to channel the saving glut globally. Bernanke's early work considered just a general kind of asset, while Caballero (2006) underlines how the saving glut has been mainly channelled through the so-called "safe assets", that is, financial instruments engineered by central banks, governments, banks, and shadow banks that

tend to preserve value during shocks. First, the production of safe assets depends largely on "constraints in the financial sector, the level of financial (under-) development, the fiscal capacity of the sovereign, and the track record of the central bank for exchange rate and price stability" (Caballero et al., 2017, p. 30). Historically, this primate has been in the hands of countries such as the United States and, more recently, Europe, as reflected in the huge amount of dollar- and euro-denominated assets holdings around the world. Second, the higher growth achieved by EMEs, mixed with their inability to produce a proper supply of safe assets, put pressure on the prices of those assets both in the United States and Europe, with a consequent decline in yields, as has been the case since the 80s.

Although "the first-order macroeconomic implications of this shortage could be explained without the additional subtlety of isolating various characteristics or identifying the particular assets that were in chronic scarcity" (Caballero et al., 2017, p. 30),⁶ the distinction becomes essential because the shortage of safe assets, which came up in the United States and Europe around the early 2000s, contributed to what would later become known as the Great Financial Crisis and the European Sovereign Debt Crisis. Indeed, the shortage of safe assets has distorted the financial market, resulting in the building up of high-rated (AAA) securitized assets, such as CDOs, as well as the issuance of "pseudo-safe" Treasury assets in countries with limited fiscal space, such as Greece and Italy. As the bubbles burst, the huge contraction in the supply of those "pseudo-safe" assets, along with a massive demand for purely safe ones (liquidity and gold) to be used as a store of value, kicked up interest rates down to the ground, reaching the ZLB. In fact, although the trend toward an increase in the demand for safe assets coming from the EMEs has been gradually increasing since before the new century and has been producing a decrease in interest rates, the process has been masked by the distortive effects that itself has produced. Once the bubbles burst, the mask slipped off, leaving the economy with interest rates at zero in a "safety trap". This mechanism essentially reminds us of the words of Krugman (2013): "it seems reasonable that this hasn't just been true since the GFC, it has arguably been true, although perhaps with increasing severity, since the 1980s. In other words, you can argue that our economy has been trying to get into the liquidity trap for a number of years, and that it only avoided the trap for a while thanks to successive bubbles". Indeed, the safety trap, as defined by Caballero

⁶Read "the first-order macroeconomic implications could be explained with the Global Saving Glut of Bernanke (2005)".

et al. (2017) is a subset of the classical "liquidity trap" à la Krugman et al. (1998). The latter indeed arises when there's a general increase in the supply of savings or a reduction in the demand for investment, regardless of which assets are involved; the former, instead, specifically calls for the role of safe assets.

To summarize, like in Summers (2015a) the jammed mechanism passes through an aggregate demand shortage given a constraint over rates. Before the ZLB became a tipping point, the economy kept full employment via changes in prices mainly driven by central banks' tightening policies, but once the ZLB had been reached, "nominal interest rates were already quite low and central banks around the world quickly found themselves unable to decrease nominal or real rates further. With real safe rates unable to decrease so as to clear markets, the demand for safe assets remained too elevated and the economy had to slow down and operate below its potential. This is a modern version of the paradox of thrift: faced with elevated safe real rates, households prefer to save and postpone consumption; simultaneously, faced with low demand and elevated risk premia, firms prefer to postpone investment. Aggregate demand suffers and a recession ensues. In short, unable to clear markets via prices -the safe real rate-, the economy clears by adjusting quantities" (Caballero et al., 2017, p. 34).⁷

1.4 Financial Factors: Rogoff, Borio and Koo

1.4.1 The Debt Supercycle

A different angle relative to the previous theories is taken by the Debt Supercycle (DS) theory of Rogoff (2016), which considers the temporary and financial features of the factors behind the sluggish growth. As Rogoff argues, many of the long-run factors mentioned by Summers and Gordon, such as population ageing and innovation, are important, but the current and future economic scenario is less pessimistic than SS suggests. "Unlike secular stagnation, the debt supercycle is not forever" (Rogoff, 2016, p. 25). The SS is unable to explain why house prices rose during the GFC, the crisis itself in fact, may not have been the result of a long-term trend, as argued in Summers (2013), but rather the end of a Debt Supercycle. Since the birth of

⁷The Secular Stagnation Hypothesis by Summers (2015a) and the Safety Trap Theory of Caballero et al. (2017) belong to the so-called ZLB economics and, the mechanism, once interest rates approach the ZLB, by which deflation comes out has been modelled almost in the same way in both theories by Eggertsson et al. (2019) and Caballero and Farhi (2018), respectively.

capitalism, the economy has emphasised the role of financial variables in pursuing growth, which build up cycles that are sometimes disconnected from real production. This environment was exacerbated in the 1980s, particularly in the United States, by so-called "deregulation", which made the link between money and real activity highly unstable and made economic growth even more influenced by the financial system. Relying on the works of Schularick and Taylor (2012) and Reinhart et al. (2012b), Rogoff explains how since 1800, "the rich and poor countries alike have been lending, borrowing, crashing and recovering" and that before the crisis ended, leveraging tended to fall and often disappear. Nowadays, this mechanism is well-documented:⁸ "as credit booms, asset prices rise, which raises their value as collateral, thereby helping to expand credit and raise asset prices even more and, when the bubble ultimately bursts, often catalyzed by an underlying adverse shock to the real economy, the whole process spins into a harsh and precipitous reverse" (Rogoff, 2016, p. 20). However, this was clear even in the past: the Debt-Deflation Theory of Fisher and the Financial Instability Hypothesis of Minsky are two important examples. Fisher (1933, p. 338) argued that "instead of one cycle, there are many co-existing cycles, constantly aggravating or neutralizing each other," with over-indebtedness and subsequent deflation being the most important factors; Minsky (1992, p. 7-8), on the other hand, described modern economies as an environment equipped with complex financial systems that tend to deviate from stable growth, "over periods of prolonged prosperity, the economies transit from financial relations that make for a stable system to financial relations that make for an unstable system" and "the greater the weight of speculative and Ponzi finance, the greater the likelihood that the economy is a deviation amplifying system" resulting in a deep fall. It is in this respect that some responsibilities for the current sluggish growth can be attributed to policymakers' responses, which have been inadequate given the "inadequate understanding of how advanced economies have dealt with banking and debt crises in the past" (Rogoff, 2016, p. 21) and not fully to secular factors.

With this in mind, Rogoff calls for an understanding of the role that the global debt overhang is actually playing in the global economy, especially in Western countries. Many types of debt are out there, and it is evident that advanced economies are facing unprecedented levels of each type of debt. Figure 1.4 shows this. The surge in debt use is a global phenomenon, especially for public debt, which in both advanced and emerging economies has reached previously unlikely peaks. Along with public

⁸See Borio et al. (2013) for example.

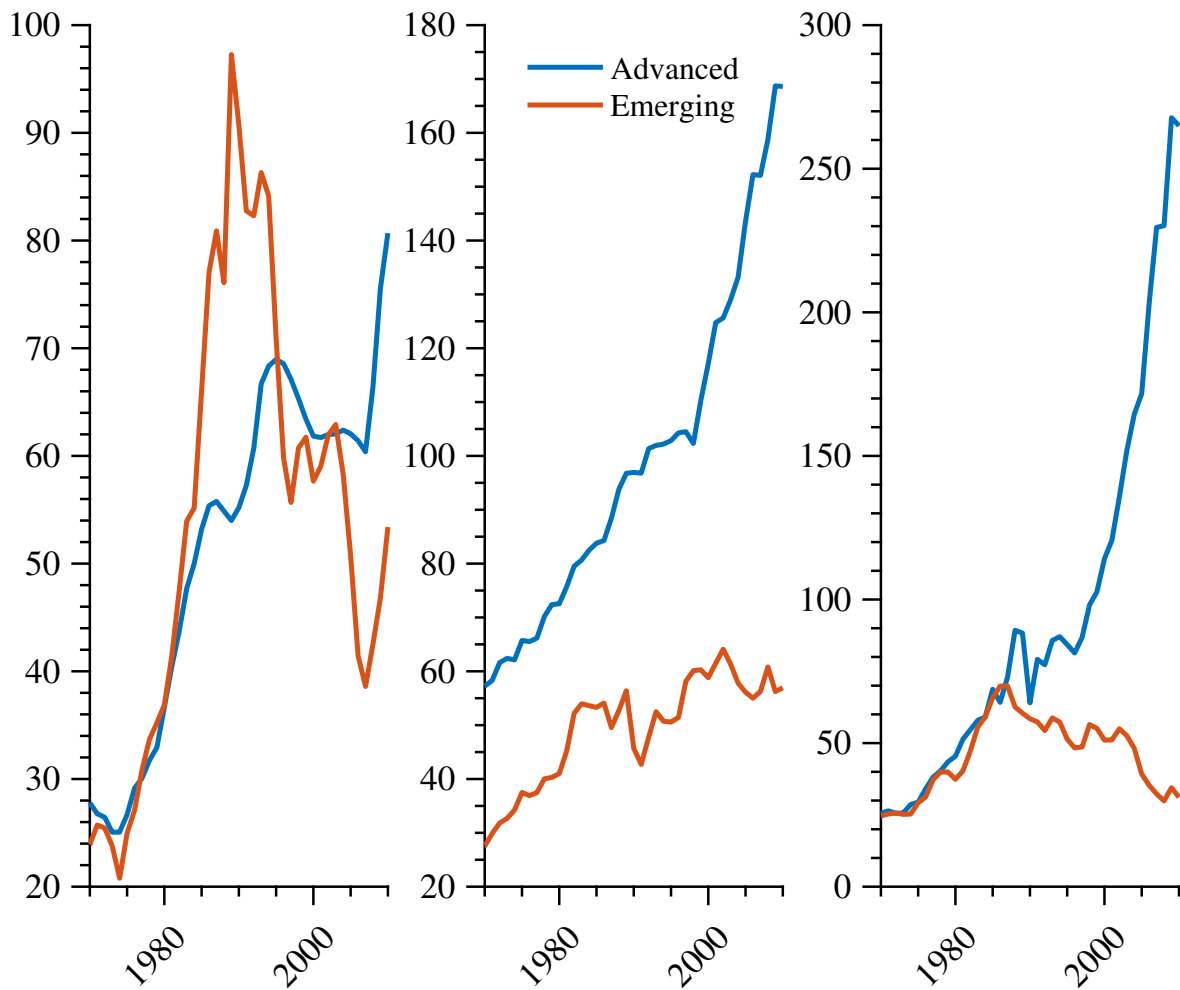


FIGURE 1.4: Public debt (Left), Private debt (Center) and External debt (Right). Source: Authors' elaboration from Reinhart et al. (2012b) data.

debt, also private and external debts peaked up to unprecedented levels in Western countries with financialization and globalization, following the arguments above of Bernanke (2005). Eventually, a dramatic spillover is represented by the implicit debt in the current unfunded (or underfunded) pension and medical systems. The impact of overall indebtedness on economic growth, whether from the public, businesses, or households, is also amplified if more than one sector is heavily indebted (Bornhorst, 2014).⁹ Although more research is needed to determine the precise effects of each debt type and their interconnections on growth, the causal relationship between debt and growth has been the subject of a decade of research, and we now know a little more about it; a good review on the topic is summarised by Salmon

⁹Other worsening factors are the so-called "hidden debts", see Reinhart and Rogoff (2009).

and Rugg (2020) and Reinhart et al. (2012a). Table A.1 shows the findings. The evidence is mixed, but the majority finds a threshold for the debt-to-GDP ratio above 75-100% to be negatively related to economic growth, using public debt as the main variable. A minority either does not see a specific threshold as a tipping point in the relationship between debt and growth (but still sees a negative relationship overall) or does not see a negative relationship at all. However, overall, the literature agrees on the idea that a debt-burdened economy (especially one with high public debt) has a negative impact on economic growth. Turning to theoretical arguments, long-term high public debt, for example, increases the probability of future tax buildups and expenditure cuts, resulting in slow economic growth. As seen in many countries (particularly in Europe), corporate debt overhang has resulted in an increase in non-performing loans, limiting banks' net profit margins and ability to provide credit and slowing aggregate demand growth. High external debt is often more challenging because the tools usually used to manage it are less feasible. For example, think about the feasibility of capital controls (a form of financial repression) in a globalised world. Eventually, about the interconnections among various debt types: the high private debt for example, if results in a high level of defaults, could "lead to weaker growth, affecting the sustainability of government debt; if households are suffering debt problems, this can lower demand and can lead to strains in corporate debt, etc." (Lo and Rogoff, 2015, p. 9). Moreover, private debt is usually linked with external debt; as explained in Mendoza and Terrones (2012), domestic credit booms are sometimes the result of large capital inflows, that is, borrowing from the rest of the world.¹⁰ In summary, the findings and arguments above have pushed Reinhart et al. (2012b) to characterise the current environment as a quadruple (private, public, external, and pension debts) debt overhang, which made the post-GFC economy's recovery slow and U-shaped.

Policymakers, just after the GFC, should have undertaken a massive debt write-down before private-sector mistakes were assumed by governments; the latter were already affected by a huge debt overhang at that time, and now the situation is worse. The private sector deleveraging that should have accompanied the end of the crisis has been quite low around the world, and public and external debts have also reached their highest levels in history. This could implies "that many countries are potentially caught in a vicious circle between debt overhang and deleveraging.

¹⁰This could be why in Figure (1.4), we see how private and external debts tend in some way to follow the same trend, both in advanced and emerging economies.

Debt overhang implies slower growth, which makes deleveraging more difficult, feeding back into continued slow growth" (Lo and Rogoff, 2015, p. 10), preventing the financial cycle from reviving and economic growth from coming back. In the long run, governments should pursue a higher level of precautionary saving behaviour to preserve their role as the last resort,¹¹ "it is important to maintain the option value of being able to issue sudden large bursts of debt in response to catastrophes (war, financial or otherwise)" (Reinhart et al., 2015, pag. 53).

1.4.2 The Financial Drag Hypothesis

The Financial Drag (FD) hypothesis of Borio (2017) is a sort of subset of the Debt Supercycle of Rogoff (2016), whereby using proxies like private debt and housing prices to characterise a measure of the financial cycle, it studies the impact on economic growth. Borio argues that, when it comes to the United States, the high current-account deficit (pre and post-crisis), the spectacular economic growth of the Great Moderation, and recent declines in unemployment rates close to full employment are difficult to interpret as symptoms of a domestic demand shortage, as stated by Summers (2015a), while a room for a supply side appears more evident:

1. relying on Borio et al. (2016), financial booms driven by credit expansion (private debt), such as during the GFC, have an impact on productivity growth. The debt overhang from the private sector, combined with a broken banking system, resulted in a spillover that misallocated resources towards lower-productivity growth sectors, particularly construction. If the deleveraging process is not completed, an uncleaned private sector "make it harder to reallocate resources away from bloated sectors during the bust". (Borio, 2017, p. 90);
2. the financial cycle, as proxied by data on private debt and housing prices, shows that the output gap was positive at the time of GFC; actual output was indeed above and not below the potential pre-crisis, indicating an excess of aggregate demand rather than a shortage. More importantly, the Phillips curve, which links an excess of aggregate demand with raising inflation, was hiding the expansion because the excess was coming not from business factors but from financial imbalances, as in Figure 1.5;

¹¹Take a look at the several policy remedies, both orthodox and heterodox, in Reinhart et al. (2015).

3. as for the financial cycle, Juselius et al. (2017) including financial proxies for the measurement of the natural interest rate, discovered that it has not reached negative levels as stated by Summers (2015a). Instead the pattern of real interest rates over time is high dependent on the monetary policy regime: "adopting an inflation targeting regime lowers average real rates by 1.05% relative to the post-Bretton Woods regime, and by 1.86% relative to the gold standard regime". As a result, "repeated attempts to push inflation towards target *lowering the nominal interest rates over time* could lead to persistent declines in real interest rates" (Borio et al., 2017, p. 25-30).

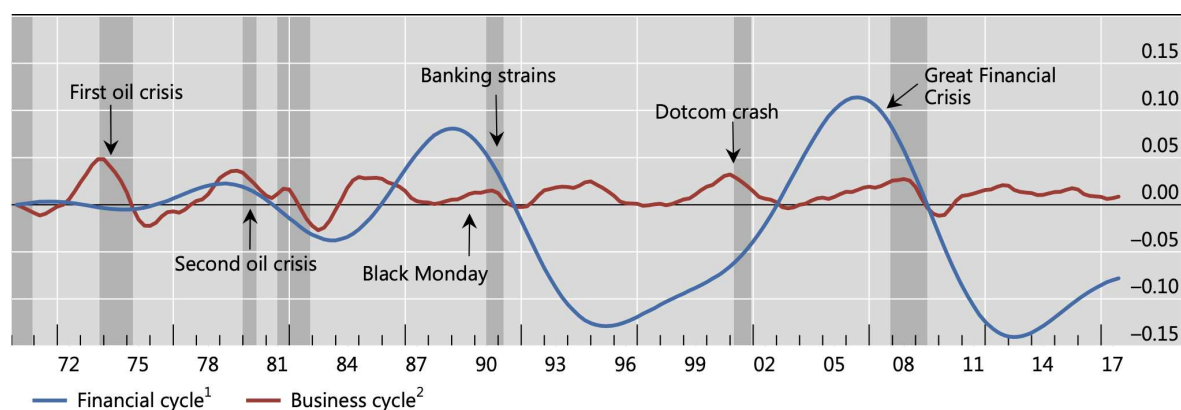


FIGURE 1.5: US Business and Financial cycles. Source: Borio (2019, p. 5).

Putting the pieces together, a form of supply-side hysteresis rather than purely demand-side hysteresis seems to better fit the environment for Claudio Borio. However, in such a scenario, in which aggregate demand pre-crisis is over potential output, inflation should have been rising instead of remaining stable at a low level of growth. Borio (2017) calls for secular disinflationary pressures in the economy due to supply factors such as globalisation. The liberalisation of markets by emerging economies populated the market with low-cost producers, especially in advanced economies, which "have weakened the pricing power of firms and, above all, the bargaining power of labour, as markets have become more contestable. During the cost convergence process, this would result in persistent disinflationary winds, especially in advanced economies, where wages are higher. More generally, it would also make the wage-price spirals of the past less likely. This would have helped central banks during the long pre-crisis disinflationary phase, but would have complicated their task post-crisis, as they sought to boost inflation back towards their

inflation objectives. Tailwinds become headwinds" (Borio, 2017, p. 90). In other words, as discussed by Gordon (2012), a sort of "Factor Price Equalization's Theorem" of Samuelson (1948) coming from globalisation could have produced disinflationary pressures, making the inflation less responsive to monetary policy.

1.4.3 The Balance Sheet Recession

To conclude the section on financial factors, Koo (2011) over the same line as the previous two contributions, stressed the role played by private debt. Relying on the experience of Japan in the 90s, the author calls for a Balance Sheet Recession (BSR) to mark the difference from an ordinary recession. At least two types of recessions are out there: those driven by the business cycle and those driven by private sector deleveraging or debt minimization due to a large debt accumulation or bubble. The latter produces a kind of environment in which "a large portion of the private sector is actually minimizing debt instead of maximizing profits following the bursting of a nation-wide asset price bubble. When a debt-financed bubble bursts, asset prices collapse while liabilities remain, leaving millions of private sector balance sheets underwater" (Koo, 2011, p. 19). The result is straightforward: households and businesses are forced to reduce their aggregate demand by increasing savings and paying down debt in order to revitalise their balance sheets, credibility (credit ratings), and financial health. In doing so, the economy descends into a spiral of stagnation until the deleveraging process is complete. In such a scenario, monetary policy becomes ineffective, at least in a traditional sense; the classical lending channel is dampened because, at any given interest rate, the private sector is not interested in increasing borrowing, and the pass-through stops working as a healing channel. According to Richard Koo, the only viable option is fiscal policy, which entails governments engaging in large fiscal stimulus in the spirit of New Deal programs, such as those implemented in the United States during the Great Depression and in Japan beginning in 1990; in this environment, governments should assist the private sector in paying down debt; although a spike in public debt is unavoidable, fiscal action does not result in a crowding out of private capital, nor a spike in inflation or interest rates. In particular, the amount that governments should "borrow and spend to avert a deflationary spiral should be at least equal to the un-borrowed and un-invested savings in the private sector that is sitting somewhere in the financial

system" (Koo, 2011, p. 27). However, governments in the United Kingdom and Europe in 2010 have fallen into the trap of premature fiscal consolidation, as happened in the United States in 1937 and Japan in 1997, causing a "double-dip" recession. The more governments become aware that the recovery takes time after such a crisis, the more will realise that a full support to the financial sector will be essential and any premature fiscal consolidation will delay the recovery. When the economy loses the momentum, many ideas and products are still put in place through R&D, but firms' managers may not be able to produce them, given the financial constraints they are facing; moreover, a kind of collective debt-related trauma or psychological block to operate as before may delay the recovery as well and fiscal stimulus becomes essential.

However, it is not all that rosy. The fallacy of composition triggers the recovery. In such a scenario, households and firms are doing right, saving to clean up their balance sheets, but the collective results will be a sort of paradox of thrift: the more agents save as opposed to invest, the lower the GDP, and the total amount of savings in the economy that, as a result, contracts. If the government will not be persuaded to do the opposite, a deflationary spiral is inevitable. However, in democracies, many people must be persuaded that the stimulus is the right way to operate, and many individuals, including citizens, the media, and experts, will turn up their noses, considering the stimulus a waste of taxpayers' money. In contrast, in an autocratic state, only a few people must be persuaded to implement fiscal stimulus. Further, in a special country such as the Eurozone, other problems arise and deserve attention. The major problem among European countries is that they do not have a fiscal union, but they share a common currency at the same time. The purchases of assets by European investors are not affected by the exchange rate risk. As a result, funds that are worried about future perspectives can simply switch positions and buy other governments' debt. When times are good, funds flow into booming economies (usually in the South of Europe) that have higher yields, but in contrast, when times become bad, the funds just flow out of those countries, making these shifts pro-cyclical. Figure 1.6 depicts the TARGET2 balances of the Eurozone's seven most important countries by GDP, and it is clear that Germany and the Netherlands have always been creditors to the south of Europe, primarily Italy and Spain, since the GFC burst. They switched their position's tendency with the outbreak of the Eurozone sovereign debt crisis without being affected by exchange rate risk, and once the European Central Bank started the Quantitative Easing programme in 2015, they

changed their position again. Therefore, "countries facing balance sheet recessions

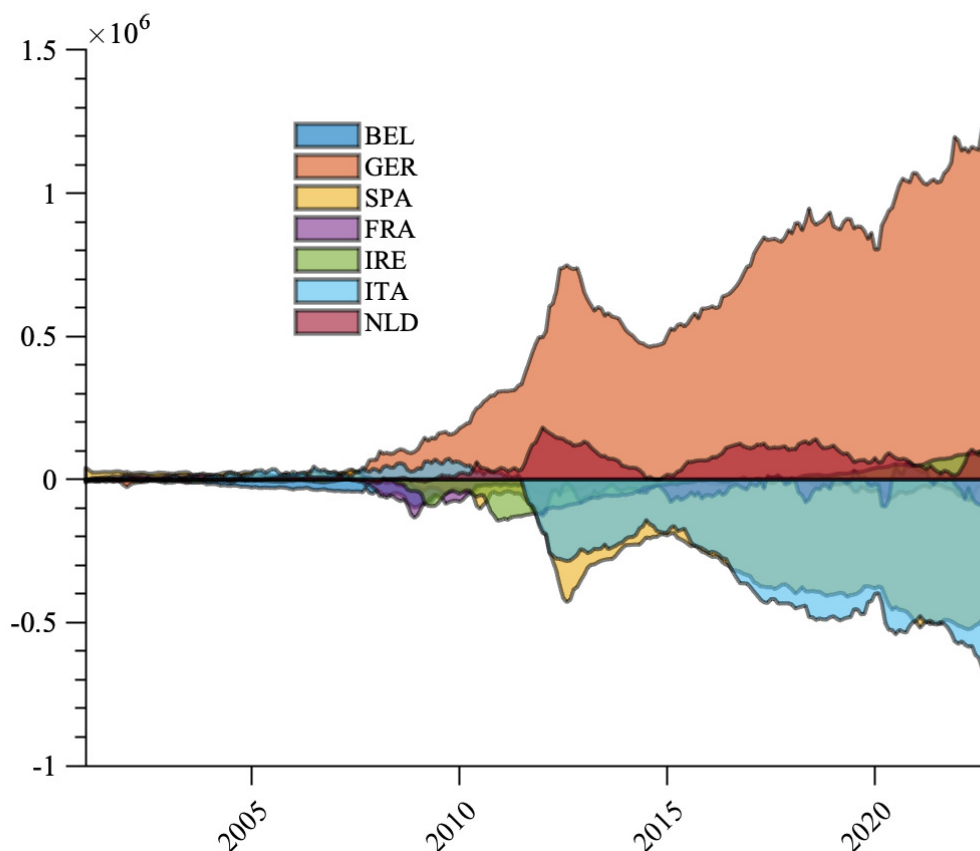


FIGURE 1.6: TARGET2 Balances in Millions of Euros. Source: Authors elaboration from ECB data.

and in need of funds can only watch as money flows abroad, preventing their governments from implementing the fiscal stimulus needed to stabilize the economy" (Koo, 2011, p. 30). This is called a capital flow problem, and it emphasised the balance sheet recession that occurred after the GFC and the Eurozone Sovereign Debt Crisis in 2010, mainly driven by the high deficit in some European countries' public accounts, primarily Greece. These three problems are interrelated, requiring on the one hand a fiscal stimulus and on the other a fiscal consolidation (the so-called "austerity"). Obviously, there's a contrast. Koo (2012) proposed a solution called "Nationals-Only". It is based on selling government bonds to citizens of the issuing country only. Only German citizens will be able to purchase Bunds, only Italian citizens will be able to purchase BTP, etc. With this rule, the high deficit should have been avoided because it would have been financed only by each country's private

sector savings; further, the balance sheet recession will not be emphasised by the capital flow problem, but governments instead, if they could convince their citizens that a fiscal stimulus is needed, could have made a high deficit on their own, ensuring responsibility among citizens and flexibility given that the debt would be domestic. Finally, the rule will also challenge the capital flow problem. The fund manager will either buy national bonds or take the currency risk of investing in assets outside of Europe. Buying national bonds would produce "low bond yields and high bond prices and, talk of a Sovereign Debt Crisis would disappear" (Koo, 2012, p. 3).

The takeaway from Richard Koo is therefore that "there is no need to suffer Secular Stagnation if the government offsets private sector deleveraging with fiscal stimulus. If the GDP level is maintained, the private sector will have the income to pay down debt. Since asset prices will not fall below zero, as long as the private sector has the income to pay down debt, the balance sheet problem will eventually be resolved" (Koo, 2014, p. 142). Two issues arise as a result of this: one in democracies due to the fallacy of composition, which is difficult to address, and the other, a special case for the Eurozone, which could be addressed, forcing fiscal issues to be internalised by each individual nation unless proper fiscal union is undertaken.

1.5 System Factors: Heterodox Economics

As underlined above, the outbreak of the GFC has represented a turning point in macroeconomic thought, and a wide debate among different schools of thought has followed. In this respect, heterodox economics got relevance, as is evident just by looking at the expansion of publications by heterodox economists in many important journals, given the failure of classic receipts of macroeconomics both theoretically and methodologically. Theoretically, there has been a need to find an alternative explanation to the GFC, and heterodox economics already had an alternative theoretical foundation about the workings of modern societies with the contributions of Marx, Kalecki, Steindl, Baran, and Sweezy, just to mention a few. Furthermore, the speech at the IMF of Summers (2013), renewed interest about stagnation with the accent on the role of aggregate demand, all arguments that piqued the interest by heterodox economists, particularly Post-Keynesians, who believe that, "demand drives growth in the short, medium and long run" (Wray and Dantas, 2022, p. xviii), and Neo-Marxists, who never left the stagnation debate off, as the quote

of the Marxist economist Paul Sweezy at Harvard in 1982 makes clear: "it is my impression that the economics profession has not yet begun to resume the debate over stagnation which was so abruptly interrupted by the outbreak of the Second World War. I have the feeling that if you ask an economist how we got into the mess we are in, he or she, while not denying that it is indeed a mess, will reply by giving advice as to how to get out of it but will not have anything very enlightening to say about how we got into it" (Sweezy, 2004, p. 4).¹² Methodologically, the work-horse tool in orthodox economic modelling, the dynamic stochastic general equilibrium (DSGE), have failed in understanding what was going on during the new century. The predictions of those kinds of models at the peak of the financial bubble were still forecasting positive growth in the economy because "the standard models were designed for non-crisis periods" (Bernanke, 2010, p. 17), "there was no room in the prevailing models for such things as bubbles and banking-system collapse" (Krugman, 2009). This opened up a room for the development of alternatives, and again, heterodox economics already had an alternative approach in addition to the birth of agent-based computational economics (ACE), principally used by Post-Keynesians. Furthermore, in terms of methodology, mainstream economics, which takes a top-down approach with rational expectations and representative agents, has proven to be unsuitable for gaining insights into the origins of financial cycles and bubbles, which emerges as a deviation from purely rational expectations. The mainstream approach considers the aggregation as the sum of the microfoundations, and this is true if rational expectations are fulfilled. During financial bubbles, the deviation from rational expectations produces a story in which the aggregation of the microfoundation does not produce the expected aggregation of neoclassical theory. The need for a zoom-in through a pair of "scientific" lenses was needed, and the heterodox approach was already equipped with such a tool. The heterodox economics have joined the debate mainly with the contributions of Post-Keynesians like Palley (2012), Hein (2016) and Storm (2017), and Neo-Marxists such as Foster and McChesney (2012) and Magdoff and Foster (2014). We will discuss them in the next two subsections.

¹²About this point, take a look at the consistent literature on stagnation by the Neo-Marxian school, which followed: Taylor (1985), Foster (1987), Cowling (1995), Foster (1997) and, Foster (2006).

1.5.1 The Destruction of Shared Prosperity

The two most important contributions from the Post-Keynesian School are Palley (2012) and Hein (2012), a structural Keynesian and a Kaleckian/Steindlian. Although there are some differences in their roots, when it comes to stagnation, the divisions disappear, as Hein (2022, p. 2) observed: "what is contained in Tom's work is very much in line with my own post-Keynesian assessment of the Macroeconomics of Finance-dominated Capitalism (Hein, 2012), although I have to admit that I made the immanent post-crises stagnation tendencies contained in that approach only explicit somewhat later in Hein (2016, 2019)".

The contribution of the Post-Keynesian School is twofold. First, it helped to contextualise the current post-GFC sluggish growth in a broader (system level) economic context, taking into account aspects of orthodox economics that are completely absent, such as the role of finance-dominated capitalism. Although the Post-Keynesians share with Summers and Gordon the idea that the GFC represents the triggering event of a subtle, long-lasting economic environment started in the last century, they do not agree about the ultimate factors. In two different books, Palley (2012) and Hein (2012), argue how the new liberal economic order born at the end of the 1970s, symbolised by the election of Ronald Reagan to the presidency of the United States, represented a turning point in West economics, leading to the destruction of shared prosperity, the origin of the GFC, and the sluggish growth that followed. The new model, known as finance-dominated capitalism, has resulted in a massive shift in income distribution in favour of a polarised niche with rising profit share, retained profits, dividends, and interest payments, and thus a declining labour income share, as well as increasing wage and top management salary inequality. The rise of the financial corporate sector has increased shareholder power by enhancing management's short-termism, decreasing animal spirits and real investment, as well as "drained internal means of finance for real investment purposes from the corporations, through increasing dividend payments and share buybacks in order to boost stock prices and thus shareholder value" (Hein, 2012, p. 2). As Kalecki has long explained in his final remark on "Theory of Economic Dynamics", the development of capitalistic societies is not an inherent feature and thus must be engineered through some forces. "We singled out innovations in the broadest sense as the most important promoter of development. Another long-run influence considered, rentiers' savings, was found to be an obstacle rather than a stimulus

to development. If the effect of the increase in the degree of monopoly upon the distribution of national income is not counteracted by other factors there will be a relative shift from wages to profits and this will constitute another reason for the slowing down of the long-run rise in output" (Kalecki, 1954, p. 161). Therefore, finance-dominated capitalism "has generated increasing potential for wealth-based and debt-financed consumption, thus creating the potential to compensate for the depressing demand effects of financialization, which were imposed on the economy via redistribution and the impact on real investment" (Hein, 2012, p. 3). In other words, "after 1980, with the advent of the new growth model, the commitment to full employment was abandoned as inflationary, with the result that the link between productivity growth and wages was severed. In place of wage growth as the engine of demand growth, the new model substituted borrowing and asset price inflation. Adherents of the new orthodoxy made controlling inflation their primary policy concern, and set about attacking unions, the minimum wage, and other worker protections. Meanwhile, globalization brought increased foreign competition from lower-wage economies and the prospect of offshoring of employment" (Palley, 2012, p. 34). Second, relying on the ideas of Kalecki and Steindl, it built up theoretical critiques and alternatives to two important contributions by orthodox economists, like Summers (2015a)¹³ and Gordon (2015). Three main critiques have been raised. The first is about the natural interest rate mechanism of the Secular Stagnation Hypothesis of Summers (2015a) or, more generally of the so-called "ZLB economics", where Hein (2016) and Palley (2019), using a Keynesian perspective as well as recalling the "Cambridge controversies in the theory of capital", deny the existence of a loanable funds market and, as a consequence, of the natural interest rate. The former do not determine the interest rate as the by-product of the supply and demand of loanable funds but as the result of economic agents' preference for liquidity, the price of money holdings. The higher the preference for liquidity, the lower the interest rate, and vice versa. And more profoundly, investment determines savings, not the other way around. The latter instead, questions the always downward sloping investment demand in a more than one good economy. The second critique is about the role of finance capitalism, which is absent in the orthodox approach. The latter calls current economies as modern or advanced market systems, which to some extent resemble what Galbraith in "The Economics of

¹³Post-Keynesians have also put forward the idea if the Summers (2015a)'s view is really demand-sided, take a look at Di Bucchianico (2019) about this point.

Innocent Fraud" called a fraudulent renaming from capitalistic to "market system" economies with aim of hiding the adverse history. "It is of the market system we teach the young. No individual or firm is thus dominant. No economic power is evoked. There is nothing here from Marx or Engels. There is only the impersonal market, a not wholly innocent fraud" (Galbraith, 2004, p. 16). The most important consequence is that "the growing likelihood of a severe crisis and a long-term slowdown in the economy was systematically hidden from view by this fraudulent displacement of the very idea of capitalism, and even of the corporate system" (Foster and McChesney, 2012, p. 9). Therefore, although absent in orthodox economics, for Post-Keynesians, financialization served as a purpose "to fuel demand growth - which was faltering given - the increasing negative effects of the model of growth and global economic engagement" Palley (2012, p. 5). *The third critique*, relies on the independence between potential and actual growth in orthodox theories, that do not allow the possibility for potential GDP growth to be endogenous and shaping the aggregate demand growth. For the dominant view in macroeconomics, the difference between potential GDP and actual demand is just a measure of the output gap, and the interplay between them is denied. Although both Summers (2015a, p. 63) and Gordon (2015, p. 58) have emphasised the importance of the interplay between demand and supply, there is still no theoretical foundation for the mechanisms in orthodox economics, at least in the current debate about stagnation. As a consequence of this, Hein (2016) and Storm (2017) have provided alternative theoretical and empirical analyses of how stagnation may arise and what the relevant factors are actually playing in the current economic environment.

Hein (2016), making use of a demand-driven model with a Steindlian framework, discusses the first two critiques and partially the third. The model, in fact, does not rely on the natural interest rate, incorporates some important features regarding the evolution of capitalism into a finance-dominated one, and shows how aggregate demand can be modelled as TFP-dependent. The model has two main elements: a microeconomic component that enters into a second component of general equilibrium. The first component reflects the classic Kaleckian/Steindlian arguments about the firms' pricing behaviour, where prices (p) are set based on a constant mark-up (m) applied over unit labour costs (wa), which are assumed constant up to full capacity and determine the profit share (h) as well as the profit rate (r). Modern economies are assumed to be populated by oligopolistic goods markets, and the mark-up is the

result of: a) the degree of price competition, b) overhead costs and c) the bargaining power of workers and trade unions. The following equations summarise the microeconomic part:

$$p = (1 + m)wa \quad (1.3)$$

$$h = 1 - \frac{wa}{p} \quad (1.4)$$

$$r = \frac{h\mu}{v} \quad (1.5)$$

where: $\mu = \frac{Y}{Y^p}$ represents the capacity utilisation as the ratio between the actual output (Y) and the potential output (Y^p) and $v = \frac{Y^p}{K}$ the ratio between the potential output (Y^p) and the stock of capital used (K). The second component, instead, has two main elements: an investment function and a saving function, which describe the choices of consumption, investment, and saving of economic agents. Firms choose to invest based on a) the expected demand, b) the level of internal means (or undistributed profits),¹⁴ c) the level of "animal spirits" in the firm, and d) the level of technological progress. The first, $\beta(\mu - \mu_0)$, is represented as the difference between realised and targeted (or planned) capacity utilisation and it is the primary Steindlian mechanism arising from firms' behaviour in goods market dominated by oligopolies. The second, $\theta(r - \rho\gamma)$, is defined as the difference between total profits (r) and profits distributed to capitalists (a combination of the interest rate and the dividend rate paid for financing with bonds and equity, ρ , and the amount of outside finance-capital ratio γ). The third, α , is modelled with a constant. The fourth, $\omega\hat{y}$, is exogenously determined, where technological progress depends on innovation and potential labour productivity growth, which was a missing point in the original framework of Steindl (1952). Here it is assumed that: a) workers do not save, b) capitalists save with a propensity of s_r and, c) firms save all undistributed profits. The saving function is thus determined by the undistributed profits, $\Pi_f = r - \rho\gamma$ and the amount saved by capitalists, $s_r\rho\gamma$. Both functions are described by the following

¹⁴This comes from Kalecki (1937) "principle of increasing risk", where in imperfectly competitive financial markets firms to attract external capital need internal capital's accumulation in advance; moreover internal accumulation is also the result of minimising the risk of illiquidity and insolvency coming from the use of external capital.

equations:

$$i = \alpha + \omega \hat{y} + \beta(\mu - \mu_0) + \theta(r - \rho\gamma) \quad (1.6)$$

$$s = (r - \rho\gamma) + s_r \rho\gamma \quad (1.7)$$

As an alternative to Summers (2015a) and Caballero and Farhi (2018)'s mechanisms based on ZLB economics, the setup above is used to describe the dynamic by which an economy could set itself in a period of stagnation. The following conditions, affecting the investment function as well as the saving function, would lower the aggregate demand and produce stagnation: 1) the rise/decrease in the profit/wage share, which is the direct result of imperfect competition, where industries tend to rise mark-up sucking away surpluses of the labour share and therefore changing the national income distribution. As a result, the lower the degree of competition, the higher the overhead costs, and the lower workers' and trade unions' bargaining power would result in a higher markup and a more distorted national distribution of income from workers to capitalists. The greater the distortion, the greater the negative effects on equilibrium rates of capacity utilisation, capital accumulation, and profit, and thus investment and aggregate demand; and 2) the rise in firms' target rate (planned) of capacity utilisation. This is the main point of Steindl (1952)'s arguments. In oligopolistic industries, firms abandon price competition as a tool for defending market share in favour of a quantity-cutting strategy (a planned excess of capacity utilisation meant as an incomplete reinvestment of profit), because the former would simply lead to a price war, harming all market participants. As a result, the more oligopolistic the markets, the greater the excess capacity held by firms to compete, and the greater the profits not reinvested, resulting in a chronic shortage of aggregate demand and the rise of stagnation; 3) the effect of financial variables (the combination of interest rate and dividend rate, ρ , and the amount of outside finance-capital ratio, γ) will depend on capitalists' propensity to consume (s_r) and on firms' investment responsiveness towards internal funds (θ). If capitalists' propensity to consume ($1 - s_r$) is less than firms' investment responsiveness towards internal funds (γ), we get the "normal case" as in Lavoie (1995) and also the "debt-burdened" case as in Taylor (2021). An increase in the capitalists' rate (interest rate and/or dividend rate) or debt-capital ratio will shift funds towards economic agents with less propensity to consume/invest, triggering a shortage of aggregate demand.

Conversely, if the capitalists' propensity to consume ($1 - s_r$) is greater than firms' investment responsiveness towards internal funds (γ), we get the "puzzling case" as in Lavoie (1995) and also the "debt-led" case as in Taylor (2021). An increase in capitalists' rate (interest rate and/or dividend rate) or debt-capital ratio will shift funds towards economic agents with an high propensity to consume/invest triggering an improvement of aggregate demand; 4) the fall in autonomous investment growth (or autonomous consumption, government expenditures, or exports) and/or a fall in "animal spirits" of firms as the result of the rise in shareholder power enhancing short-termism on management which directly affect the investment function and the aggregate demand; 5) the fall in innovation; 6) the rise in the capitalists' propensity to save (or in the workers' propensity to save set equal to zero in the current model version) that suck away resources for consumption and investment purposes, thus reducing aggregate demand.

Storm (2022) instead provides a theoretical and empirical discussion of why a lower level of aggregate demand could feedback into a lower potential GDP, in a vicious cycle that reinforces itself. To look at this, he has challenged the supply-side view of Gordon (2015), or more generally, the dominant view, which does not consider the interplay between potential GDP and aggregate demand. Two points have been stressed. First, if we measure the technology growth (TFP) as a Solow residual, the potential GDP growth becomes the by-product of just exogenous supply-side factors, such as population growth and the technology growth, and no interaction between potential GDP and aggregate demand is allowed. Second, as a result of the first point, the causality between labour productivity and TFP is derived from equation 1.11, which is read from left to right, as is common among orthodox economists, and from which also comes out the causality between wage growth and labour productivity growth. To make clear the points, recall equation 1.2 from Gordon (2010) and assume that the output is produced with a simple Cobb-Douglas with constant-returns-to-scale, as it follows:¹⁵

$$x = A \cdot L^\theta K^{1-\theta} \quad (1.8)$$

dividing both sides by x^θ , and solving for labor productivity $\lambda = \frac{x}{L}$, we obtain:

$$\lambda = A^{\frac{1}{\theta}} K^{\frac{1-\theta}{\theta}} \quad (1.9)$$

¹⁵We follow Storm (2017) closely.

then, differentiating as we did for 1.2 we obtain:

$$\hat{\lambda} = \frac{1}{\theta}\hat{A} + \frac{1-\theta}{\theta}\hat{K} \quad (1.10)$$

Using the Kaldor (1961)'s stylised fact where capital-output ratio is equal to zero (or like in a steady-state of a neoclassical model) we obtain that the labor productivity is fully determined by the TFP (A) and the labor share (θ):

$$\hat{\lambda} = \frac{1}{\theta}\hat{A} \quad (1.11)$$

and by the same token, substituting equation 1.11 into equation 1.2, also the potential GDP growth becomes fully derived by the TFP:

$$\hat{Y} = \frac{1}{\theta}\hat{A} \quad (1.12)$$

Eventually, differentiating equation 1.8 and solving for the technology we can estimate the TFP as a residual:

$$\hat{A} = \hat{x} - \theta\hat{L} - (1-\theta)\hat{K} \quad (1.13)$$

As we can see from equation 1.11-1.12, labor productivity growth as well as potential GDP growth are fully determined by the TFP growth, which is in turn measured as an exogenous object, the Solow residual. Storm (2022) proposes to estimate the TFP in two different ways, both of which make use of observable data:

1. following (Rada and Taylor, 2006): it considers the TFP as endogenously determined by the weighted average of the growth rates of labour and capital productivities. We just need to use the following two definitions $\hat{\lambda} = \hat{x} - \hat{L}$ and $\hat{\lambda} = \hat{x} - \hat{K}$ into the formula of the Solow residual, equation 1.13, and exploit again the Kaldor (1961)'s stylized fact for the capital to output ratio, to get the TFP growth becomes directly a function of the labor productivity growth and not anymore "unexplained":

$$\hat{A} = \theta\hat{\lambda} \quad (1.14)$$

$$\hat{Y} = \hat{\lambda} + \hat{l} \quad (1.15)$$

The TFP and potential GDP become endogenous in this approach, determined by observable labour productivity growth. Furthermore, in equation 1.11, the causality between labour productivity and TFP is reversed, because we are now determining TFP with labour productivity, which is directly observable.

2. using the Dual Approach (Simon and Levy, 1963): it is based on the National Accounting framework, and considers real GDP growth as the by-product of wage and capital income, both directly observables. In this way, the TFP is a function of both:

$$\hat{A} = \theta\hat{w} + (1 - \theta)\hat{r} \quad (1.16)$$

of course here the Kaldor (1961)'s stylised fact cannot be exploited, but a simple estimate of equation 1.16, Table 1.2, shows that "the secular decline in real wage growth is, in other words, another key factor playing a role in the fading of TFP growth and potential output growth" (Storm, 2022, p. 45).

To summarize, for Storm (2022), the dominant narrative, by measuring TFP with the Solow residual and excluding more intuitive methods, comes to the incorrect conclusion that potential GDP is declining solely due to ageing and a declining level of technology. On the other hand, if we look at other measures of TFP instead, the conclusions and the interpretations can be considerably different relative to what Gordon (2015) shows. As summarised in Table 1.2: 1) using the first approach, TFP growth decline is mainly the result of declining labour productivity growth and 2) using the second approach, real wage growth represents a considerable factor in TFP decline. "We therefore have two separate accounts of the secular stagnation of potential output growth—one centered on the slowdown of labor productivity growth and the other focusing on stagnating real wage growth. How can these two explanations be aligned?" (Storm, 2017, p. 17) The dominant view is to not allow any influence of wage-setting on productivity growth because it reads the causality in the reverse way. In a neoclassical setting, in fact, when technology falls, so does labour productivity, and profit-maximizing firms that hire workers until the marginal productivity is equal to the real wage rate will lower their offer for the real wage. "However, the problem with this simple 'intuition' is that it is wrong because it fails to recognise that the relationship between wage growth and productivity growth must go both ways" (Storm, 2017, p. 17). Measuring the TFP with

	\hat{w}	$\hat{\lambda}$	\hat{r}	\hat{K}
1948-1972	2.68	2.32	-0.06	0.46
1972-1995	1.15	1.38	0.88	0.52
1995-2008	1.92	1.92	0.49	0.49
1948-2008	1.94	1.88	0.41	0.49
1972-2008	1.43	1.57	0.73	0.51
2008-2015	0.58	0.91	0.93	0.46
1948-2015	1.80	1.78	0.46	0.49

TABLE 1.2: Authors' elaboration from Storm (2017) data.

different approaches, Storm gets evidence that labour productivity and real wages are important determinants of the TFP. Furthermore, they have a solid theoretical foundation as contributors to stagnation; in fact, many post-Keynesians have attempted to include the TFP either exogenously or endogenously in Steindlian models (see, for example, Dutt (2005) and Hein (2014)), providing theoretical arguments for how lower aggregate demand reinforces itself with lower potential GDP growth in a vicious cycle. For example the model of Hein (2016) can be extended with an endogenous mechanism which reads the causality from wage growth to labor productivity growth, in the sense of Storm (2017), and where the lower aggregate demand feedback into a decreasing potential GDP, labor productivity and again aggregate demand. Determining the labor productivity as endogenous in the investment function (ϵi), to the inverse of the profit share or positive related with the wage share (γh) and, to a constant η which represents the innovation or "learning by doing", we get a new investment function:

$$\hat{y} = \eta + \epsilon i - \gamma h \quad (1.17)$$

$$i = \alpha + \omega(\eta + \epsilon i - \gamma h) + \beta(\mu - \mu_0) + \theta(r - \rho\gamma) \quad (1.18)$$

where "rising real wages, as in the U.S. economy during the period 1948-1972, provide an incentive for firms to invest in labor-saving machinery - read η - and productivity growth - read \hat{y} - surges as a result; but when wage growth is low - read $-\gamma h$ - , as in the U.S. during 1972-2015, businesses have little incentive to invest in the modernization of their capital stock and productivity growth falters" (Storm, 2022, p. 52). The following mechanism is described in Figure 1.7, from which Storm (2022) argues that permanently low real wage growth give rise to stagnation with three

channels: 1) with the classical Kaleckian's argument - on profit share -, depressing the aggregate demand permanently; 2) destroying the potential output through the decline of labor productivity growth; 3) "because the 'observed' output gap is small (which makes the risk of inflation look relatively large), monetary policy authorities (inflation-adverse) will be inclined to step on the brakes and raise interest rates in response to a revival of actual growth — nipping the recovery in the bud and creating a 'sick recovery' which 'dies in its infancy'" (Storm, 2022, p. 51). This vicious cycle is self-reinforcing and thus may cause long-run stagnation as shown in Figure 1.7, which sketches the main mechanisms by which stagnation may arise by a Post-Keynesian perspective.

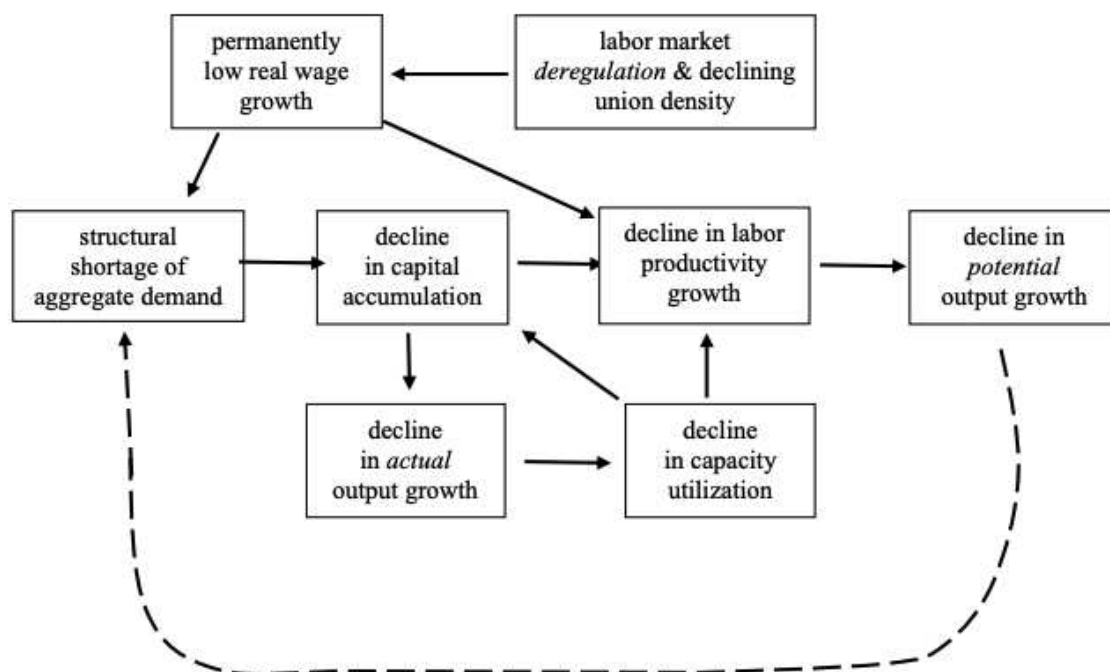


FIGURE 1.7: The economics of secular stagnation: the demand-side view. Source: Storm (2022, p. 51).

1.5.2 The Stagnation and Financialization Trap

Like for the Post-Keynesian School, the contributions of Kalecki and Steindl have been particularly important for the Neo-Marxist School. However, the book of Baran and Sweezy (1966), from which the name "the school of Monopoly Capitalism" derives, has largely shaped them and still represents the foundation for Neo-Marxist

arguments about stagnation. Nowadays, what Foster and McChesney (2012) refers to as the "Stagnation and Financialization Trap" is the main contribution to post-GFC growth from a Marxist's perspective. However, to understand what Foster and McChesney mean, we should go back to the ideas of Baran and Sweezy. In a nutshell, Baran and Sweezy (1966) relying on Kalecki and Steindl, argued that modern economies are structured in oligopolistic markets where the presence of a price system in the form of the traditional monopoly theory,¹⁶ and the use of price competition as an instrument of action are denied. The latter, in fact, would just produce a price war, leaving all the firms in a worse position. The resulting procedure is "banning price cutting as a legitimate weapon of economic warfare" (Baran and Sweezy, 1966, p. 58), where, like in Steindl (1952), a downward rigidity of prices arises. Therefore, "with price competition banned, sellers of a given commodity or of close substitutes have an interest in seeing that the price or prices established are such as to maximize the profits of the group as a whole" (Baran and Sweezy, 1966, p. 59). Here it is, where the term "Monopoly Capitalism" is referred to. That is to a group known as "Big Business," which consists of profit maximizers and capital accumulators such as large corporations. However, in such an environment, banned price competition doesn't mean the end of all competitions among firms, but the replacement with another form of it, called cost competition. This new form of competition along with many other advantages, "stems from the exigencies of non-price competition in the producer goods industries. Here, as in industries producing consumer goods, sellers must be forever seeking to put something new on the market. But they are not dealing with buyers whose primary interest is the latest fashion or keeping up with the Joneses. They are dealing with sophisticated buyers whose concern is to increase profits. Hence the new products offered to the prospective buyers must be designed to help them increase their profits, which in general means to help them reduce their costs. If the manufacturer can convince his customers that his new instrument or material or machine will save them money, the sale will follow almost automatically" (Baran and Sweezy, 1966, p. 70). The result of this form of competition with the aim of increasing profits along with the monopolistic structure of the market, is a "law of the tendency of the surplus to rise",¹⁷ that is, "a strong and

¹⁶Lowering the price to the point where the addition to the revenue from selling an extra unit exactly equals the addition to the costs involved in producing an extra unit.

¹⁷"This law immediately invites comparison, as it should, with the classical-Marxian law of the falling tendency of the rate of profit. Without entering into an analysis of the different versions of the latter, we can say that they all presuppose a competitive system. By substituting the law of

systematic tendency for surplus to rise, both absolutely and as a share of total output" (Baran and Sweezy, 1966, p. 79), where the surplus is defined as the difference between what a society produces and the costs of producing it, the alter ego of the excess of capacity in Steindl. This kind of tendency, Baran and Sweezy argued to be the by-product of an intrinsic feature of Monopoly Capitalism: its incapacity to absorb all the excess of surplus produced, with consequences for aggregate demand. In other words, the surplus tends to increase under Monopoly Capitalism as the result of oligopolistic competition, and given the incapacity to absorb it, what follows is a persistent stagnation tendency, what is known as the "Sweezy normal state". The question then becomes, why isn't Monopoly Capitalism capable of absorbing all of the excess surplus produced? For Baran and Sweezy, the surplus can be consumed, invested, or wasted. Leaving aside the waste component for a while, the question becomes: "does capitalists' consumption tend to rise as a share of surplus? If not, the investment-seeking part of surplus must rise relatively to total income, and the possibility that capitalists' consumption might provide a solution to the problem is excluded" (Baran and Sweezy, 1966, p. 80). Following this logic, stagnation arises because the consumption component is not able to absorb all the surplus and the investment component is not able to keep up with the pace of surplus production. Even assuming that capitalists tend to consume all of the profit distributed (as dividends, for example) by firms, consumption is not able to absorb all the surplus. The reason comes from the fact that "large companies have a target dividend payout rate which remains remarkably constant over long periods of time. When profits rise, they do not immediately adjust dividends to maintain the target rate. If this pattern is adhered to-and there is every indication that it is a deeply rooted aspect of corporate behavior-it follows that a continuous rise in earnings would be accompanied by an equally continuous decline in the payout rate. Under these circumstances, capitalists' consumption would increase absolutely, which of course is to be expected, but it would decline as a proportion of surplus and even more as a proportion of total income" (Baran and Sweezy, 1966, p. 80-81). Therefore, the last option is the investment component. Ruling out an accelerating growth process for the investment component of surplus, as totally unrealistic, "one is left with the inescapable

rising surplus for the law of falling profit, we are therefore not rejecting or revising a time-honored theorem of political economy: we are simply taking account of the undoubted fact that the structure of the capitalist economy has undergone a fundamental change since that theorem was formulated. What is most essential about the structural change from competitive to monopoly capitalism finds its theoretical expression in this substitution." (Baran and Sweezy, 1966, p. 72).

conclusion that the actual investment of an amount of surplus which rises relatively to income must mean that the economy's capacity to produce grows more rapidly than its output. Such an investment pattern is certainly not impossible; indeed, it has frequently been observed in the history of capitalism. But what is impossible is that it should persist indefinitely. Sooner or later, excess capacity grows so large that it discourages further investment. When investment declines, so do income and employment and hence also surplus itself. In other words, this investment pattern is self-limiting and ends in an economic downturn—the beginning of a recession or depression" (Baran and Sweezy, 1966, p. 82). Again, "since surplus which cannot be absorbed will not be produced, it follows that the normal state of the monopoly capitalist economy is stagnation. With a given stock of capital and a given cost and price structure, the system's operating rate cannot rise above the point at which the amount of surplus produced can find the necessary outlets. And this means chronic underutilization of available human and material resources" (Baran and Sweezy, 1966, p. 108). In a nutshell, "what Baran and Sweezy sought to explain was not capitalism as such, the fundamental account of which was to be found in Marx's *Capital*, but rather a particular stage of capitalist development. Their stated goal was nothing less than to provide a brief "essay-sketch" of the monopoly stage of capitalism by examining the interaction of its basic economic tendencies, narrowly conceived, with the historical, political, and social forces that helped to shape and support them" (Foster, 2006, p. 1).

However, the rise of financialization represented a turning point, and the book of Magdoff and Sweezy (1987) has been an important development in updating what was becoming the new stage of capitalism, called "Monopoly-Finance Capitalism". Just above, we left aside the waste component; the developments made after the work of Baran and Sweezy (1966) have been substantially on figuring out how the waste component has changed over time. The waste component represents all those kinds of operations made up by the oligopolistic system to offset the stagnant tendency that would arise without them, cause the consumption and the investment are not able to keep up with the pace of surplus production, the latter sustained by the "law of the tendency of the surplus to rise". Baran and Sweezy, have identified three elements through which the surplus is absorbed by the waste component to offset the stagnant tendencies of the system: 1) the sales effort by firms, 2) the

civilian government spending, and 3) the military spending.¹⁸ Without going into details, what is important to underline is that financialization has formed a new form of waste, and it is here that we find recent contributions by Neo-Marxists, such as Magdoff and Sweezy (1987) and Foster and McChesney (2012). Prior to the 1970s, there was no financialization of the economy; of course, credit institutions and financial markets evolved as the economy grew and booming, but "traditionally financial expansion - in the form of credit-based supply of money by the banking system - has gone hand-in-hand with prosperity in the real economy. Is it really possible that this is no longer true, that now in the late twentieth century the opposite is more nearly the case: in other words, that now financial expansion feeds not on a healthy real economy but on a stagnant one? The answer to this question, I think, is yes it is possible, and it has been happening. And I will add that I am quite convinced that the inverted relation between the financial and the real is the key to understanding the new trends in the economy" (Sweezy, 1994, p. 8). This quote captures the grasp of the real contribution of the School of Monopoly Capitalism. Modern societies evolved in the 1970s, and financialization appeared in the form of "a drug or stimulant, akin to those sometimes used by athletes, that had emerged within the system to keep the economy going despite what they called creeping stagnation. Finance acts as an accelerator of the business cycle, pushing it farther and faster along on the way up and steepening the decline on the way down" (Foster and McChesney, 2012, p. 17). "Faced with a shortage of investment outlets, the surplus capital available to corporations and the wealthy increasingly flowed into the financial sector looking for speculative opportunities unrelated to the production of use values. Financial institutions found ways to absorb this increased demand for speculative outlets by supplying an alphabet soup of exotic instruments—all sorts of repackaged futures, options, derivatives, and money-market schemes, leveraged by ever-growing mountains of debt. Central banks took on the institutional role as lenders of last resort, expected to intervene quickly whenever the whole rickety system seemed to be in danger of a credit crunch or financial collapse. The new financial architecture was rapidly globalized and soon began to take on a logic of its own, dominating over production itself" (Magdoff and Foster, 2014, p. 4). Since the 1970s, financialization has been evidence of Baran and Sweezy (1966)'s correct analysis of capitalism's inability to absorb all of the excess of surplus produced by new investment

¹⁸These three elements, in particular the military and government expenditures, are the reasons why a stagnation tendency has been offset and thus didn't rise during the last century.

opportunities. In other words, financialization has been the evidence of "Sweezy's normal state" and the new form of waste, or surplus' absorption in the scheme of Baran and Sweezy or with another perspective, one of those "development factors" Kalecki was referring to about long-run economic growth: "long-run development is not inherent in the capitalist economy. Thus, specific 'development factors' are required to sustain a long-run upward movement" (Kalecki, 1954, p. 161) otherwise, a stagnant tendency could arise. Financialization, therefore, started to act as a boost to aggregate demand in the form of indebted demand. It is therefore during this period that the School of Monopoly Capitalism produced its main contribution, arriving at a detailed decline of the interplay between monopolization, stagnation, and financialization, under the name of Monopoly-Finance Capitalism or, as called by Foster and McChesney (2012), "Stagnation-Financialization Trap" still relevant today and visionary at the time.

1.6 Conclusions

This review summarises the work of several authors whose research falls into various fields of macroeconomics. Although there are numerous interconnections among them, we attempted to keep the review simple by dividing the literature into the most important factors highlighted. Going from the field economic growth, international macroeconomics, financial and monetary economics, and the more radical and political perspective of macroeconomics, we have shown the directions that macroeconomic thought, born and revived as a result of the Great Financial Crisis, has taken in the last 15 years.

Along with Krugman, Summers' most important contribution has been reviving an old debate about Stagnation, which has aroused interest and called into question so many different schools of thought. The reasons for this are many. First, about Stagnation per se, the debate dates back to the controversies of Hansen-Terborgh and Schumpeter-Sweezy in the first half of the last century, and relative to this, the Marxist School, for which Stagnation has always been the cornerstone of its thought, provided the most radical and deep view on the existence of such a stagnant tendency, stressing the internal fallacy of the current Monopoly-Finance Capitalism. Second, Summers' primary role in aggregate demand teased the Post-Keynesian School, which holds that demand is the main driver of the economy also in the

medium- to long-run, providing good insight on wage stagnation and the interconnection between potential GDP and actual GDP. Third, Summers' explanation of the aggregate demand shortage with an excess of saving and the ZLB mechanism brought out the contributions of Bernanke and Caballero, both with an accent on the international dimension of emerging economies and their role in causing a shortage of demand. Fourth, the accent for the current slow growth which Summers has been given to long-run factors, teased Robert Gordon's long-run perspective, focusing on the declining innovation growth rate relative to the second industrial revolution and prompted authors such as Rogoff, Borio, and Koo, who instead emphasise more cyclical of financial-type factors, in particular the quadruple debt overhang.

Nowadays, the debate still goes on, particularly with the recent developments by which global economies have been challenged, namely the COVID-19 pandemic and the geopolitical tensions in Ukraine, Iran and Taiwan; however, the importance still lies in its heterogeneity to shed light on the truth.

As usual, *in medio stat virtus*.

Chapter 2

Dynare Replication of Eggertsson et al. (2019)

2.1 Introduction

This paper replicates the large-scale overlapping generation model of "A Model of Secular Stagnation: Theory and Quantitative Evaluation" by Eggertsson et al. (2019) using the latest version (5.3) of Dynare (Adjemian et al., 2022), a standard software to simulate and estimate dynamic general equilibrium models widely used in academies, central banks, and other institutions. The original model builds a large-scale OLG model with several occasionally binding constraints (OBCs) in order to capture the main features of the Secular Stagnation Hypothesis (Summers, 2015a), with a focus on the long-run decline of interest rates. The authors show the quantitative importance of key drivers such as population aging and slowing productivity growth over the interest rate for the United States, computing transitional dynamics from 1970 to 2015.

Our replication exercise is intended to provide a valuable contribution along three dimensions. First, it confirms the results of a significant contribution well-known in macroeconomics. Indeed, the Secular Stagnation Hypothesis (Summers, 2015a) and its quantitative relevance (Eggertsson et al., 2019), has been one of the most credited developments in macroeconomic thought since the Great Financial Crisis (GFC), especially for gaining insights on the causes of the GFC and on the role of monetary policy stuck at the Zero lower bound (ZLB). Second, the replication makes use of Dynare, which represents the state-of-the-art software for the simulation and estimation of dynamic general equilibrium models. Dynare provides a

user-friendly platform that is easier to use with respect to the complex original Matlab code of (Eggertsson et al., 2019), therefore reducing the entry barriers for those interested in large-scale OLG modeling and being helpful for the entire community of users. Third, the original model includes several OBCs that are particularly challenging to compute but extremely important from a policy standpoint. Modelers have traditionally dealt with the presence of OBCs by using toolkits such as Guerrieri and Iacoviello (2015) and Holden (2016). In this case, the presence of an OBC for each working-age generation makes them not feasible. By rewriting the constraints as in Swarbrick (2021), we show how to handle and treat several OBCs with a large-scale model and with the standard algorithms included in Dynare in an easier and faster way relative to the original Matlab code.

The results from the replication substantially confirm the original outcome of Eggertsson et al. (2019). On one hand, following the original Matlab code available on the *American Economic Journal: Macroeconomics* repository, we are able to fully replicate the original results with the exception of Figure 8 of the original paper. However, we found the equations of the original Matlab code to be slightly different from the ones reported in the text. Once we rewrite the model as in the paper, small differences between the transition dynamics of the two models emerge, especially in the first thirty periods of the simulation. All in all, the replication exercise confirms the original results of the paper.

This replication paper is organized as follows: In section 2.2, we compare the results obtained from the Dynare code with the original Matlab code, whereas in Section 2.3 we compare the results of the model presented in the paper with the original Matlab code. Section 2.4 concludes.

2.2 Replication of the original Matlab code

In this Section, we focus on the replication of the original Matlab code. A salient feature of the original model is the presence of several occasionally binding collateral constraints, one for every 40 working-generations (j) contained in the model. The collateral constraint is as follows:

$$a^j \geq D^j \cdot w \cdot hc^j \tag{2.1}$$

OccBin (Guerrieri and Iacoviello, 2015) and DynareOBC (Holden, 2016) toolkits, typically used in Dynare to deal with OCBs cannot deal with a large numbers of constraints. The simplest way to workaroud the problem is therefore using a brute-force approach with the **min/max** function in a perfect foresight enviroment along the lines proposed by Swarbrick (2021). Therefore, equation 2.1 can be transformed as follows:

$$\min \left(\lambda^j, a^j - D^j \cdot w \cdot hc^j \right) = 0 \quad (2.2)$$

where λ^j represents the lagrange multiplier, a^j the asset of each generation, $D^j \leq 0$ the individual debt limit of each generation, and hc^j the human capital profile which shapes the wage w profile among generations. However, in our Dynare code as well as in the Matlab implementation by Eggertsson et al. (2019), the debt limit is written with positive values, $D^j \geq 0$, therefore each constraint will be:

$$\min \left(\lambda^j, a^j + D^j \cdot w \cdot hc^j \right) = 0 \quad (2.3)$$

2.2.1 Matlab equations

The following block describe the equations for both the 1970 and the 2015 steady state, as well as the equations for the transitional dynamics. These are direct translations from the original Matlab code provided by the authors into Dynare notation.¹

¹The model's derivations are basically the same as detailed in the Appendix B.1 for the model written as in the paper, but with the discrepancies in the equations as outlined in Section 2.3.

Steady state equations

$$j \in \{26, \dots, J = 81\}$$

$$n^{26} = 1 \tag{2.4}$$

$$n^{j+1} = \frac{s^j \cdot n^j}{1+n} \tag{2.5}$$

$$\text{for } j \in \{26, J-1\}$$

$$\frac{1}{\beta} = \left(\frac{c^{j+1}}{c^j} \right)^{-\frac{1}{\gamma}} \cdot (1+r) + \lambda^{j+1} \frac{(c^j)^{-\frac{1}{\gamma}}}{su^j \beta^j} \tag{2.6}$$

$$\text{for } j \in \{26, J-1\}$$

$$x^J = \left(\frac{\Gamma}{\mu} \right)^{-\gamma} \cdot c^J \tag{2.7}$$

$$\text{for } j \in \{J\}$$

$$a^j = 0 \tag{2.8}$$

$$\text{for } j \in \{26\}$$

$$a^{j+1} = \frac{(1+r) \cdot a^j}{sv^j} + \left((1-\tau) \cdot w \cdot hc^j + \pi^j \right) \cdot (1 + AL_{growth})^j - c^j \tag{2.9}$$

$$\text{for } j \in \{26, \dots, 56\}$$

$$a^{j+1} = \frac{(1+r) \cdot a^j}{sv^j} + \left((1-\tau) \cdot w \cdot hc^j + \pi^j \right) \cdot (1 + AL_{growth})^j + \dots$$

$$\dots + q^{j+1} \cdot (1 + AL_{growth})^{j+1} - c^j \tag{2.10}$$

$$\text{for } j \in \{56\}$$

$$q^j = \frac{x^J \cdot \Gamma \cdot n^J}{n^j} \tag{2.11}$$

$$\text{for } j \in \{57\}$$

$$a^{j+1} = \frac{(1+r) \cdot a^j}{sv^j} + \left((1-\tau) \cdot w \cdot hc^j + \pi^j \right) \cdot (1 + AL_{growth})^j - c^j \tag{2.12}$$

$$\text{for } j \in \{57, \dots, 65\}$$

$$a^{j+1} = \frac{(1+r) \cdot a^j}{sv^j} - c^j \tag{2.13}$$

$$\text{for } j \in \{66, \dots, 80\}$$

$$c^j = \frac{(1+r) \cdot a^j}{sv^j} - \Gamma \cdot x^j \quad (2.14)$$

for $j \in \{81\}$

$$\min \left(\lambda^j, a^j + (D^j \cdot w \cdot hc^j) \cdot (1 + AL_{growth})^j \right) = 0 \quad (2.15)$$

for $j \in \{26, \dots, 65\}$

$$\min \left(\lambda^j, a^j \right) = 0 \quad (2.16)$$

for $j \in \{66, \dots, 81\}$

$$\pi^j = \frac{hc^j \cdot \Pi}{L} \quad (2.17)$$

for $j \in \{26, \dots, 65\}$

$$\frac{p^{int}}{P} = \frac{\theta - 1}{\theta} \quad (2.18)$$

$$A_{adj} = \frac{p^{int}}{P} \cdot \left(\alpha \cdot (AK \cdot K)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \cdot (AL \cdot L)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} \cdot \dots \quad (2.19)$$

$$\dots \cdot (1-\alpha) \cdot AL^{\frac{\sigma-1}{\sigma}} \cdot L^{-\frac{1}{\sigma}}$$

$$w = 1 \quad (2.20)$$

$$rk = \frac{\frac{p^{int}}{P} \left(\alpha (AK \cdot K)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) (AL \cdot L)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} \cdot \alpha \cdot AK^{\frac{\sigma-1}{\sigma}} K^{-\frac{1}{\sigma}}}{A_{adj}} \quad (2.21)$$

$$Y = \frac{\left(\alpha \cdot (AK \cdot K)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \cdot (AL \cdot L)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}}{A_{adj}} \quad (2.22)$$

$$r = \frac{rk + (1-\delta)\xi}{\xi} - 1 \quad (2.23)$$

$$\Pi = \frac{Y}{\theta} \quad (2.24)$$

$$gov^{deficit} \cdot gov^{rev} = \left((1 + AL_{growth}) \cdot (1 + n) - 1 \right) \cdot (gov^{debt} \cdot K) \quad (2.25)$$

$$gov^{debt} = b \cdot \frac{Y}{K} \quad (2.26)$$

$$gov^{rev} = (g \cdot Y + r \cdot gov^{debt} \cdot K) \quad (2.27)$$

$$\tau = \frac{\left(gov^{rev} \cdot (1 - gov^{deficit}) \right)}{w \cdot L} \quad (2.28)$$

$$N = \sum_{j=26}^J n^j \quad (2.29)$$

$$L = \sum_{j=26}^J n^j hc^j \quad (2.30)$$

$$C = \sum_{j=26}^J n^j c^j (1 + AL_{growth})^j \quad (2.31)$$

$$K = \frac{\left(\sum_{j=26}^J \frac{n^j a^j}{sv^j \cdot (1 + AL_{growth})^j} \right)}{\xi + gov^{debt}} \quad (2.32)$$

Transitional Dynamics

$$j \in \{26, \dots, J = 81\}$$

$$n_t^{26} = \frac{n_{t-1}^{25}}{su_{t-1}^{25}} \cdot \Gamma_t \quad (2.33)$$

$$n_t^{j+1} = s_{t-1}^j \cdot n_{t-1}^j \quad (2.34)$$

$$\text{for } j \in \{26, J-1\}$$

$$\frac{1}{\beta} = \left(\frac{c_{t+1}^{j+1}}{c_t^j} \right)^{-\frac{1}{\gamma}} \cdot (1 + r_{t+1}) + \lambda_t^{j+1} \frac{(c_t^j)^{-\frac{1}{\gamma}}}{su_t^j \beta^j} \quad (2.35)$$

$$\text{for } j \in \{26, J-1\}$$

$$x_t^J = \left(\frac{\Gamma_{t-J+26}}{\mu} \right)^{-\gamma} \cdot c_t^J \quad (2.36)$$

$$\text{for } j \in \{J\}$$

$$a_t^j = 0 \quad (2.37)$$

$$\text{for } j \in \{26\}$$

$$a_{t+1}^{j+1} = \frac{(1 + r_t) \cdot a_t^j}{sv_t^j} + (1 - \tau_t) \cdot w_t \cdot hc^j + \pi_t^j - c_t^j \quad (2.38)$$

$$\text{for } j \in \{26, \dots, 56\}$$

$$a_{t+1}^{j+1} = \frac{(1 + r_t) \cdot a_t^j}{sv_t^j} + (1 - \tau_t) \cdot w_t \cdot hc^j + \pi_t^j + q_{t+1}^{j+1} - c_t^j \quad (2.39)$$

$$\text{for } j \in \{56\}$$

$$q^j = \frac{x_{t-1}^j \cdot \Gamma_{t-56} \cdot n_{t-1}^j}{n_t^j} \quad (2.40)$$

for $j \in \{57\}$

$$a_{t+1}^{j+1} = \frac{(1+r_t) \cdot a_t^j}{sv_t^j} + (1-\tau_t) \cdot w_t \cdot hc^j + \pi_t^j - c_t^j \quad (2.41)$$

for $j \in \{57, \dots, 65\}$

$$a_{t+1}^{j+1} = \frac{(1+r_t) \cdot a_t^j}{sv_t^j} - c_t^j \quad (2.42)$$

for $j \in \{66, \dots, 80\}$

$$c_t^j = \frac{(1+r_t) \cdot a_t^j}{sv_t^j} - \Gamma_{t-55} \cdot x_t^j \quad (2.43)$$

for $j \in \{81\}$

$$\min \left(\lambda_t^j, a_t^j + (D_{t+1}^j \cdot w_{t+1} \cdot hc^j) \right) = 0 \quad (2.44)$$

for $j \in \{26, \dots, 65\}$

$$\min \left(\lambda_t^j, a_t^j \right) = 0 \quad (2.45)$$

for $j \in \{66, \dots, 81\}$

$$\pi_t^j = \frac{hc^j \cdot \Pi_t}{L_t} \quad (2.46)$$

for $j \in \{26, \dots, 65\}$

$$\frac{p_t^{int}}{P_t} = \frac{\theta - 1}{\theta} \quad (2.47)$$

$$w_t = \frac{\frac{p_t^{int}}{P_t} \left(\alpha (AK_t K_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) (AL_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} \cdot (1-\alpha) \cdot AL_t^{\frac{\sigma-1}{\sigma}} L_t^{-\frac{1}{\sigma}}}{A_{adj}} \quad (2.48)$$

$$rk_t = \frac{\frac{p_t^{int}}{P_t} \left(\alpha (AK_t K_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) (AL_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} \cdot \alpha \cdot AK_t^{\frac{\sigma-1}{\sigma}} K_t^{-\frac{1}{\sigma}}}{A_{adj}} \quad (2.49)$$

$$Y_t = \frac{\left(\alpha (AK_t K_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) (AL_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}}{A_{adj}} \quad (2.50)$$

$$r_t = \frac{rk_t + (1 - \delta)\zeta_t}{\zeta_{t-1}} - 1 \quad (2.51)$$

$$\Pi_t = \frac{Y_t}{\theta} \quad (2.52)$$

$$gov_t^{rev} = g_t \cdot Y_t + r_t \cdot gov_t^{debt} \cdot K_t \quad (2.53)$$

$$gov_t^{debt} = \frac{\left(gov_{t-1}^{debt} K_{t-1} (1 + r_{t-1}) + g_{t-1} Y_{t-1} - gov_{t-1}^{rev} (1 - gov_{t-1}^{deficit}) \right)}{K_t} \quad (2.54)$$

$$gov_t^{deficit} = \frac{(b_{t+1} \cdot Y_{t+1} - gov_t^{debt} \cdot K_t)}{gov_t^{rev}} \quad (2.55)$$

$$\tau_t = \frac{gov_t^{rev} \cdot (1 - gov_t^{deficit})}{w_t L_t} \quad (2.56)$$

$$N_t = \sum_{j=26}^J n_t^j \quad (2.57)$$

$$L_t = \sum_{j=26}^J n_t^j h^j \quad (2.58)$$

$$C_t = \sum_{j=26}^J n_t^j c_t^j \quad (2.59)$$

$$K_t = \frac{\left(\sum_{j=26}^J \frac{n_t^j a_{t-1}^j}{sv_t^j} \right)}{\zeta_{t-1} + gov_t^{debt}} \quad (2.60)$$

2.2.2 Results

We start our replication exercise by computing the 1970 steady-state values for the baseline calibration.² Results are reported in Table 2.1 where the last four rows should be compared with the ones reported in Table 5 on page 39 of the original paper. The other variables are not presented in the original paper but can be easily retrieved from the Matlab code.

Figure 2.1 instead, reports the comparison of the transitional dynamics of several endogenous variables of our Dynare code with the original Matlab code, again for the main calibration made by the authors. There are basically no discrepancies between the two simulations, where many differences arrive to a magnitude of 10^{-13} .

²Inside the replication kit, the user will also find the code to replicate the calibration of the model using Dynare.

TABLE 2.1: 1970 steady state for the baseline calibration

Variables	steady state 1970
Capital	48.69
Labor	36.58
Population	34.03
Income	52.72
Consumption	31.49
Aggregate Profit	6.65
Investment	10.03
Rental rate	0.195
Wage tax	0.301
Bequest	2.504
Population growth rate	0.014
Debt	0.458
Public expenditure	11.22
Investments to Output ratio	19.0%
Interest rate	2.55%
Labor share	72.4%
Consumer-debt-to-output-ratio	4.20%

Note: bold variables are presented in Table 5 on page 39 of the original paper.

The dynamics of these aggregate variables are not reported in the original paper but can be found on the Matlab code.³

³In the folder "Alt_Calibrations" of our replication material, you also have access to the alternative calibrations made by Eggertsson et al., 2019 (look at the Online Appendix from page A.28) and the relative comparisons with our Dynare code. Again, the results are identical with many differences in the order of 10^{-13} .

FIGURE 2.1: Aggregate variables dynamics from Eggertsson et al. (2019) Matlab code (blue line) and the Dynare implementation (dashed red line)

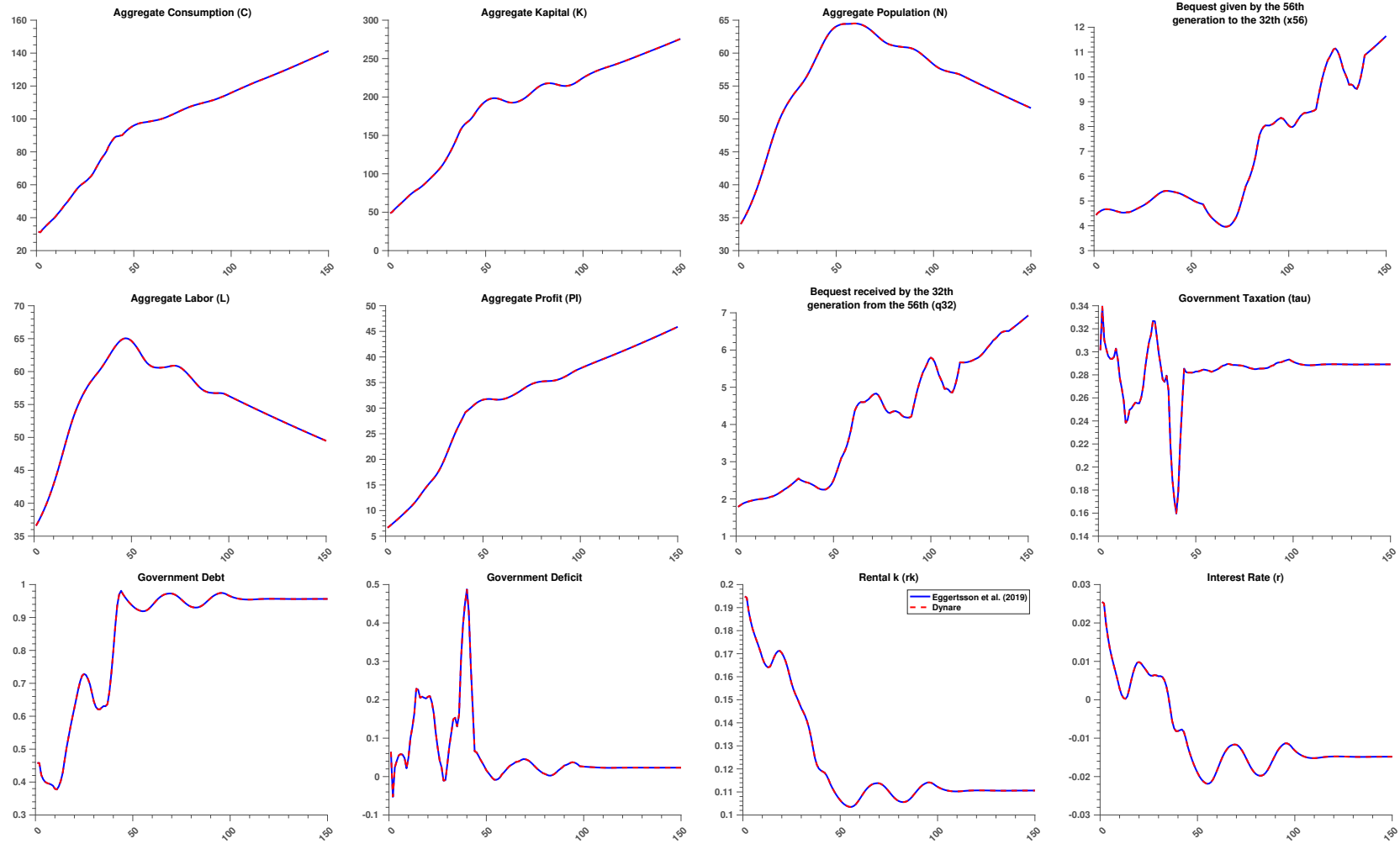


Figure 2.2 is the replication of Figure 8 on page 42 of the original paper. The simulations obtained from the Dynare code and the Matlab code are filtered using the two-side HP filter and they are identical to each other. However, the two simulations differ from Figure 8 in the original paper (the dotted line in Figure 8, Model national rate). The original paper does not report any details on the filtering methodology or if the data are filtered at all. Our intuition is that, since the Fed fund rate is plotted using the HP trend, the simulated series are also filtered in order to maintain comparability. However, also by doing this, we find some discrepancies in Figure 8, it seems to us that the dynamics reported are shifted down by a kind of constant, relative to what is written in the text and what comes from the Matlab code of the authors. In particular, the description of Figure 8 on page 41 basically does not correspond with Figure 8 itself. The steady-state at 1970 which is around 2% in Figure 8, is lower than what is reported in the text and what is coming from the Matlab code of the authors, 2.55%. Moreover, data about the figure and the plot of the figure itself are not present in any part of their replication kit material.

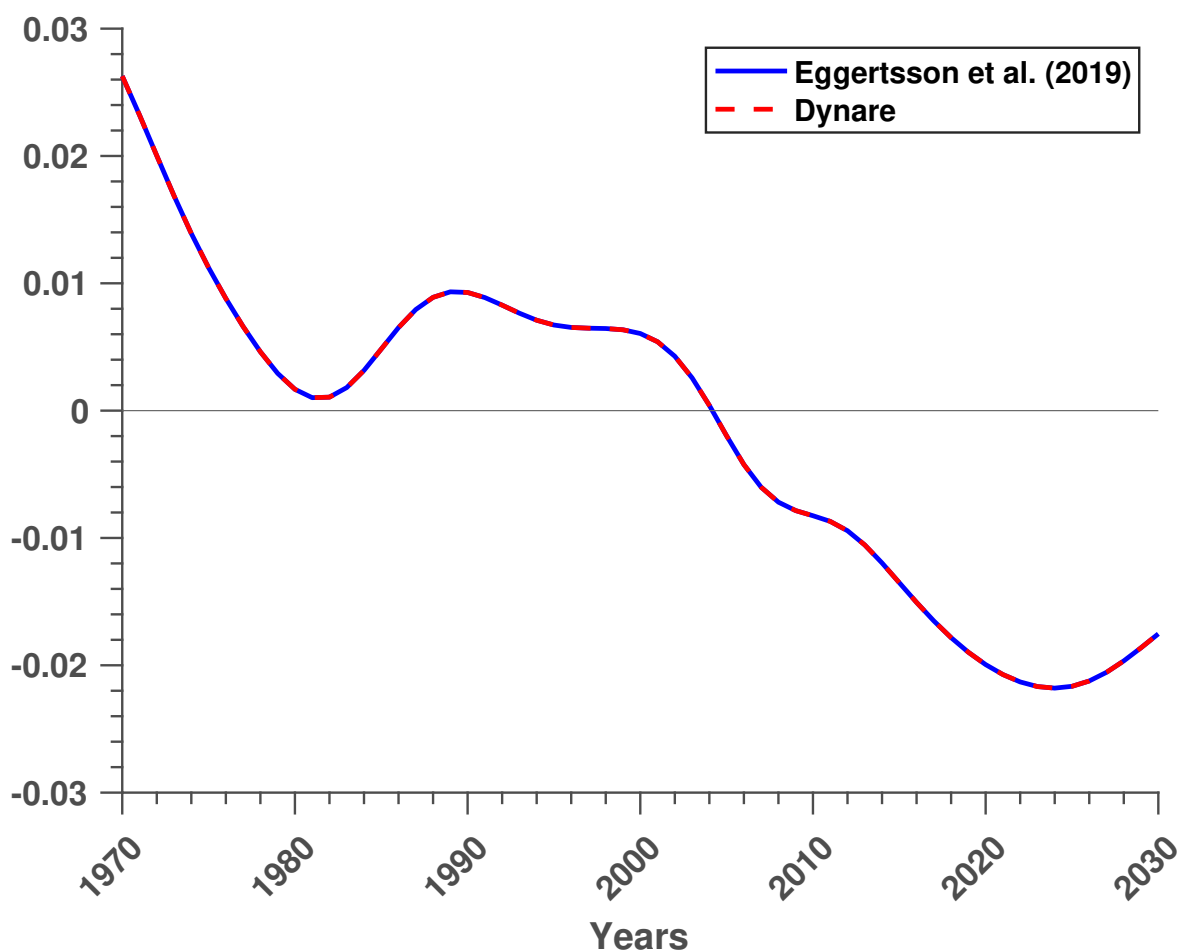
2.3 Paper replication

In this section, we implement the Dynare code using the equations as written in the original paper, both for the steady state and the transition dynamics. We highlight the differences in red. Appendix A provides the full derivation of the model.

Assuming as a benchmark their Matlab code, we find differences about some points:

1. the equations 2.63, 2.67-2.72 and 2.93, 2.96-2.101 for the various euler equations and budget constraints written in the original paper differ from the ones written in the function "opt_lb_alt.m" of the Matlab code with respect to: a) the relative price of capital goods (e) which is not presents at all in the Matlab function and cannot be simplified with the no-arbitrage condition, at least when considering the transitional dynamics for what is written in the paper; b) the bequest received (q^j) differently from what is written in the paper, is left out of the multiplication with $(rk + \xi(1 - \delta))$ in Matlab;
2. the equations 2.73 and 2.102 for the borrowing constraints used in the functions "opt_lb_alt.m" and "create_profile.m" of the Matlab code, is different from

FIGURE 2.2: Natural Interest Rate (HP Trend)



what is written in the paper. Instead of using $a^j \geq \frac{D^j}{1+r}$, they used $a^j \geq D^j \cdot w \cdot hc^j$;

- the equations 2.90 and 2.118 for the asset market clearing conditions are different from what is written in "repeatfunc.m" in the Matlab code.

2.3.1 Paper Equations

The following block describe the equations for both the 1970 and the 2015 steady state, as well as the equations for the transitional dynamics coming from the model as written in the original paper.

Steady state equations

$$j \in \{26, \dots, J = 81\}$$

$$n^{26} = 1 \tag{2.61}$$

$$n^{j+1} = \frac{s^j \cdot n^j}{1 + n} \tag{2.62}$$

for $j \in \{26, J - 1\}$

$$\frac{1}{\beta} = \left(\frac{c^{j+1}}{c^j} \right)^{-\frac{1}{\gamma}} \cdot (1 + r) + \lambda^{j+1} \cdot \frac{sv^{j+1}(c^j)^{\frac{1}{\gamma}}}{su^j \beta^j \xi} \tag{2.63}$$

for $j \in \{26, J - 1\}$

$$x_{t+80}^{81} = \left(\frac{\Gamma_{t-80}^{26}}{\mu} \right)^{-\gamma} c_{t+80}^{81} \tag{2.64}$$

$$x^J = \left(\frac{\Gamma}{\mu} \right)^{-\gamma} \cdot c^J \tag{2.65}$$

for $j \in \{J\}$

$$a^j = 0 \tag{2.66}$$

for $j \in \{26\}$

$$\xi \cdot a^{j+1} = \frac{(rk + \xi(1 - \delta)) \cdot a^j}{sv^j} + \left((1 - \tau) \cdot w \cdot hc^j + \pi^j \right) \cdot \dots$$

$$\dots \cdot (1 + AL_{growth})^j - c^j \tag{2.67}$$

for $j \in \{26, \dots, 56\}$

$$\xi \cdot a^{j+1} = \frac{(rk + \xi(1 - \delta)) \cdot (a^j + sv^j q^{j+1} \cdot (1 + AL_{growth})^{j+1})}{sv^j} + \dots$$

$$\dots + \left((1 - \tau) \cdot w \cdot hc^j + \pi^j \right) \cdot (1 + AL_{growth})^j - c^j \tag{2.68}$$

for $j \in \{56\}$

$$q^j = \frac{x^J \cdot \Gamma \cdot n^J}{n^j} \tag{2.69}$$

for $j \in \{57\}$

$$\xi \cdot a^{j+1} = \frac{(rk + \xi(1 - \delta)) \cdot a^j}{sv^j} + \left((1 - \tau) \cdot w \cdot hc^j + \pi^j \right) \cdot \dots$$

$$\dots \cdot (1 + AL_{growth})^j - c^j \tag{2.70}$$

for $j \in \{57, \dots, 65\}$

$$\xi \cdot a^{j+1} = \frac{(rk + \xi(1 - \delta)) \cdot a^j}{sv^j} - c^j \quad (2.71)$$

for $j \in \{66, \dots, 80\}$

$$c^j = \frac{(rk + \xi(1 - \delta)) \cdot a^j}{sv^j} - \Gamma \cdot x^j \quad (2.72)$$

for $j \in \{81\}$

$$\min \left(\lambda^j, a^j + \frac{D^j}{1+r} \cdot (1 + AL_{growth})^j \right) = 0 \quad (2.73)$$

for $j \in \{26, \dots, 65\}$

$$\min \left(\lambda^j, a^j \right) = 0 \quad (2.74)$$

for $j \in \{66, \dots, 81\}$

$$\pi^j = \frac{hc^j \cdot \Pi}{L} \quad (2.75)$$

for $j \in \{26, \dots, 65\}$

$$\frac{p^{int}}{P} = \frac{\theta - 1}{\theta} \quad (2.76)$$

$$A_{adj} = \frac{p^{int}}{P} \left(\alpha(AK \cdot K)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha)(AL \cdot L)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} (1 - \alpha) AL^{\frac{\sigma-1}{\sigma}} L^{-\frac{1}{\sigma}} \quad (2.77)$$

$$w = \frac{\frac{p^{int}}{P} \left(\alpha(AK \cdot K)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha)(AL \cdot L)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} (1 - \alpha) AL^{\frac{\sigma-1}{\sigma}} L^{-\frac{1}{\sigma}}}{A_{adj}} = 1 \quad (2.78)$$

$$rk = \frac{\frac{p^{int}}{P} \left(\alpha(AK \cdot K)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha)(AL \cdot L)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} \cdot \alpha \cdot AK^{\frac{\sigma-1}{\sigma}} K^{-\frac{1}{\sigma}}}{A_{adj}} \quad (2.79)$$

$$Y = \frac{\left(\alpha(AK \cdot K)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha)(AL \cdot L)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}}{A_{adj}} \quad (2.80)$$

$$r = \frac{rk + (1 - \delta)\xi}{\xi} - 1 \quad (2.81)$$

$$\Pi = \frac{Y}{\theta} \quad (2.82)$$

$$b \cdot Y \cdot \left((1 + AL_{growth}) \cdot (1 + n) - 1 \right) = g \cdot Y + (1 + r) \cdot b \cdot Y - \tau \cdot w \cdot L \quad (2.83)$$

$$g\sigma v^{rev} = (g \cdot Y + r \cdot b \cdot Y) \quad (2.84)$$

$$g\sigma v^{deficit} = \frac{\left((1 + AL_{growth}) \cdot (1 + n) - 1 \right) \cdot (b \cdot Y)}{g\sigma v^{rev}} \quad (2.85)$$

$$g\sigma v^{debt} = b \cdot \frac{Y}{K} \quad (2.86)$$

$$N = \sum_{j=26}^J n^j \quad (2.87)$$

$$L = \sum_{j=26}^J n^j h c^j \quad (2.88)$$

$$C = \sum_{j=26}^J \frac{n^j c^j}{(1 + AL_{growth})^j} \quad (2.89)$$

$$\xi \cdot K = \left(\sum_{j=26}^J \frac{\xi \cdot n^j a^j}{(1 + AL_{growth})^j} \right) - b \cdot Y \quad (2.90)$$

Transitional Dynamics

$$j \in \{26, \dots, J = 81\}$$

$$n_t^{26} = \frac{n_{t-1}^{25}}{su_{t-1}^{25}} \cdot \Gamma_t \quad (2.91)$$

$$n_t^{j+1} = s_{t-1}^j \cdot n_{t-1}^j \quad (2.92)$$

$$\text{for } j \in \{26, J-1\}$$

$$\frac{1}{\beta} = \left(\frac{c_{t+1}^{j+1}}{c_t^j} \right)^{-\frac{1}{\gamma}} \cdot (1 + r_{t+1}) + \lambda_t^{j+1} \cdot \frac{sv_t^{j+1} (c_t^j)^{\frac{1}{\gamma}}}{su_t^j \beta^j \xi_t} \quad (2.93)$$

$$\text{for } j \in \{26, J-1\}$$

$$x_t^J = \left(\frac{\Gamma_{t-J+26}}{\mu} \right)^{-\gamma} \cdot c_t^J \quad (2.94)$$

$$\text{for } j \in \{J\}$$

$$a_t^j = 0 \quad (2.95)$$

for $j \in \{26\}$

$$\xi_t \cdot a_{t+1}^{j+1} = \frac{(rk_t + \xi_t(1 - \delta)) \cdot a_t^j}{sv_t^j} + (1 - \tau_t) \cdot w_t \cdot hc^j + \pi_t^j - c_t^j \quad (2.96)$$

for $j \in \{26, \dots, 56\}$

$$\begin{aligned} \xi_t \cdot a_{t+1}^{j+1} = & \frac{(rk_t + \xi_t(1 - \delta)) \cdot (a_t^j + sv_t^j \cdot q_{t+1}^{j+1})}{sv_t^j} + (1 - \tau_t) \cdot w_t \cdot hc^j + \dots \\ & \dots + \pi_t^j - c_t^j \end{aligned} \quad (2.97)$$

for $j \in \{56\}$

$$q_t^j = \frac{x_{t-1}^j \cdot \Gamma_{t-56} \cdot n_{t-1}^j}{n_t^j} \quad (2.98)$$

for $j \in \{57\}$

$$\xi_t \cdot a_{t+1}^{j+1} = \frac{(rk_t + \xi_t(1 - \delta)) \cdot a_t^j}{sv_t^j} + (1 - \tau_t) \cdot w_t \cdot hc^j + \pi_t^j - c_t^j \quad (2.99)$$

for $j \in \{57, \dots, 65\}$

$$\xi_t \cdot a_{t+1}^{j+1} = \frac{(rk_t + \xi_t(1 - \delta)) \cdot a_t^j}{sv_t^j} - c_t^j \quad (2.100)$$

for $j \in \{66, \dots, 80\}$

$$c_t^j = \frac{(rk_t + \xi_t(1 - \delta)) \cdot a_t^j}{sv_t^j} - \Gamma_{t-55} \cdot x_t^j \quad (2.101)$$

for $j \in \{81\}$

$$\min \left(\lambda_t^j, a_t^j + \frac{D_t^j}{1 + r_t} \right) = 0 \quad (2.102)$$

for $j \in \{26, \dots, 65\}$

$$\min \left(\lambda_t^j, a_t^j \right) = 0 \quad (2.103)$$

for $j \in \{66, \dots, 81\}$

$$\pi_t^j = \frac{hc^j \cdot \Pi_t}{L_t} \quad (2.104)$$

for $j \in \{26, \dots, 65\}$

$$\frac{p_t^{int}}{P_t} = \frac{\theta - 1}{\theta} \quad (2.105)$$

$$w_t = \frac{\frac{p_t^{int}}{P_t} \left(\alpha (AK_t K_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) (AL_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} \cdot (1-\alpha) \cdot AL_t^{\frac{\sigma-1}{\sigma}} L_t^{-\frac{1}{\sigma}}}{A_{adj}} \quad (2.106)$$

$$rk_t = \frac{\frac{p_t^{int}}{P_t} \left(\alpha (AK_t K_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) (AL_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} \cdot \alpha \cdot AK_t^{\frac{\sigma-1}{\sigma}} K_t^{-\frac{1}{\sigma}}}{A_{adj}} \quad (2.107)$$

$$Y_t = \frac{\left(\alpha (AK_t K_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) (AL_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}}{A_{adj}} \quad (2.108)$$

$$r_t = \frac{rk_t + (1-\delta)\xi_t}{\xi_{t-1}} - 1 \quad (2.109)$$

$$\Pi_t = \frac{Y_t}{\theta} \quad (2.110)$$

$$b_{t+1}Y_{t+1} = g_t Y_t + (1+r_t) \cdot b_t Y_t - \tau_t w_t L_t \quad (2.111)$$

$$gov_t^{rev} = g_t Y_t + r_t b_t Y_t \quad (2.112)$$

$$gov_t^{deficit} = \frac{b_{t+1}Y_{t+1} - b_t Y_t}{gov_t^{rev}} \quad (2.113)$$

$$gov_t^{debt} = \frac{b_t Y_t}{K_t} \quad (2.114)$$

$$N_t = \sum_{j=26}^J n_t^j \quad (2.115)$$

$$L_t = \sum_{j=26}^J n_t^j hc^j \quad (2.116)$$

$$C_t = \sum_{j=26}^J n_t^j c_t^j \quad (2.117)$$

$$\xi_t \cdot K = \left(\sum_{j=26}^J \xi_t n_t^j a_{t-1}^j \right) - b_t Y_t \quad (2.118)$$

2.3.2 Results

The results obtained from the paper equations are reported in Tables 2.3 and 2.4. The results are essentially the same compared to the ones obtained with the original Matlab code. However, there are a few discrepancies at the beginning of the sample, especially for the bequest variables (q32 and x56), but at the end of the day the endogenous variables' dynamics are fully captured by our Dynare implementation.

FIGURE 2.3: Aggregate variables dynamics from Eggertsson et al. (2019) Matlab code (blue line) and the paper Dynare implementation (dashed red line)

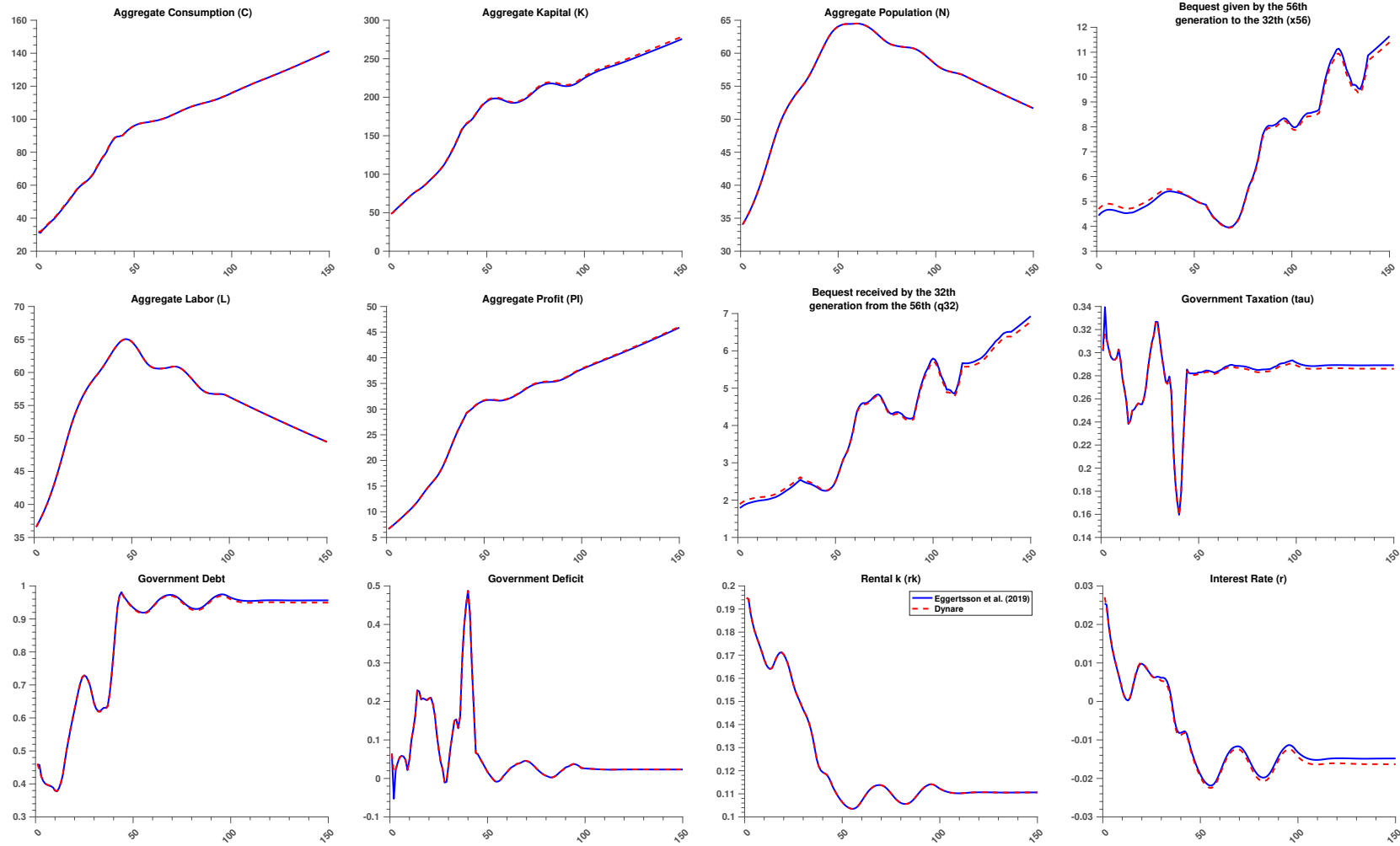
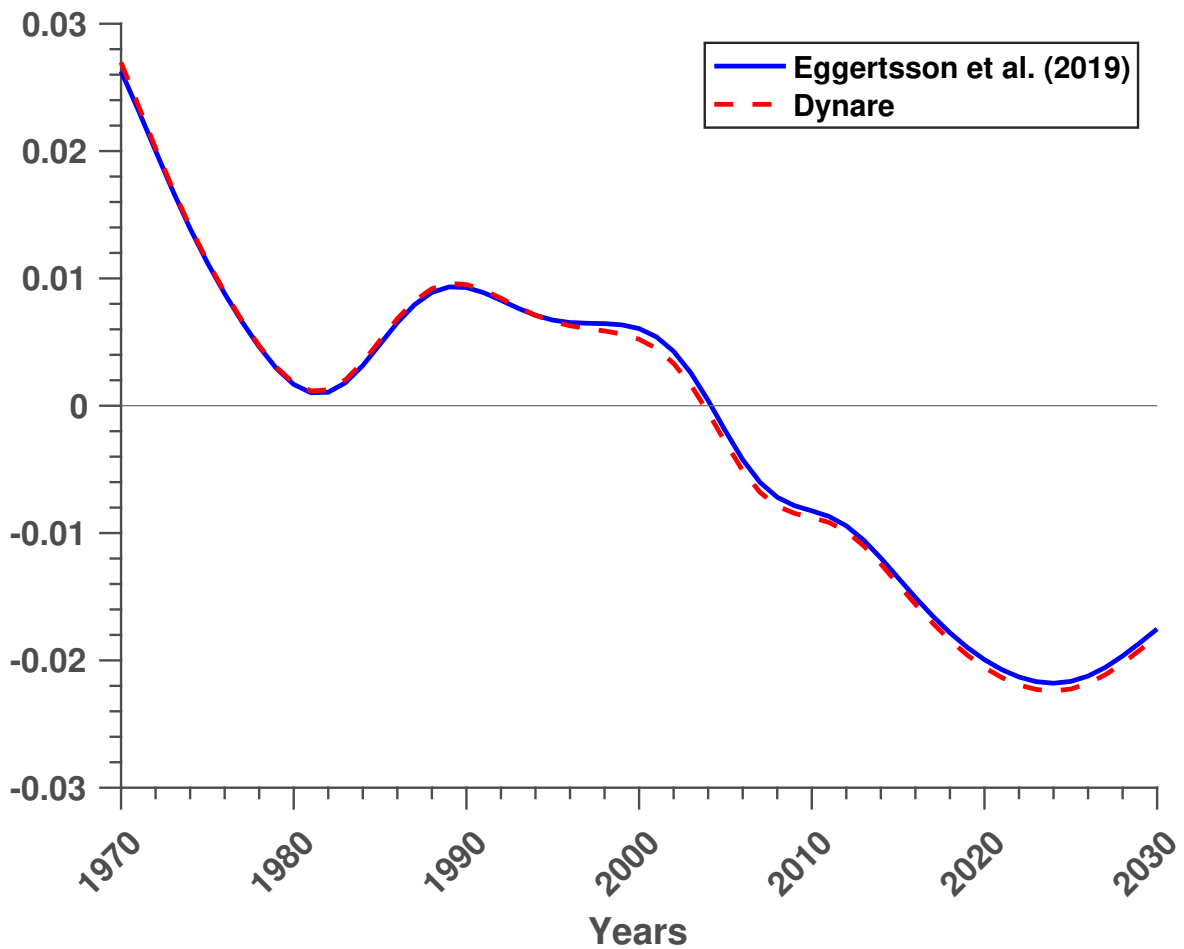


Figure 2.4 compares the dynamics of the interest rates filtered using the two-side HP filter obtained from the original Matlab implementation and our Dynare code written with the paper equations. The replication is not as perfect as in Figure 2.2 but the dynamic is basically the same. The problems emerged in the previous Section remain.

FIGURE 2.4: HP Trend Interest Rate



2.4 Conclusions

In this article, we provide a successful replication of the paper "A Model of Secular Stagnation: Theory and Quantitative Evaluation" by Eggertsson et al., 2019 using the free software Dynare. The results are almost identical to the ones presented in the original paper. A few discrepancies emerge between the equations presented in

the paper and the ones available in the original Matlab code. However, these small differences have negligible effects on the dynamics of the aggregate variable.

Our Dynare implementation has several strengths with respect to the original Matlab implementation. In particular, the easiness of our code can facilitate the works of other scholars in the field of OLG modeling tearing down the entry barriers usually very high, helping them in developing a complete quantitative research going from the calibration to the simulation procedures. Finally, our code shows how to deal with several OCBs in Dynare providing a fast, yet accurate, way to produce consistent results.

Chapter 3

Secular Stagnation: a Quantitative analysis of Migration in Italy

3.1 Introduction

Since 2013, a significant literature has emerged in Western countries to investigate the impact of population ageing on the economy as an explanation for the observed declining pattern in real interest rates. Indeed, in the advanced economies (G7), population growth accelerated with the "baby boom" generation and began to slow in the 1970s, as shown in Figure 3.1. If we consider the normal functioning of an economy as primarily dragged by the working middle-aged population, the ageing process is a serious problem. After being trained for the current working environment, the young generation becomes worker who help and finance the older generation. With lower fertility or higher life expectancy, the normal equilibrium is affected, and something must be done to avoid pressure on public finances (such as pensions and social benefits) and young unemployment (a longer working life limits the role of the youngs in the society). A prominent role in the literature has been played by the Secular Stagnation Hypothesis (SSH). Key ingredients of the theory are declining fertility rates (Summers, 2014) and declining TFP growth rates (Gordon, 2015). The former contributes to lower real interest rates due to a mismatch between saving and investment and, in a zero lower bound (ZLB) context, aggregate demand. The latter contributes to directly reducing the aggregate supply through the production function. As a result, the country affected by the Secular Stagnation Hypothesis experiences slow growth.

For what concern ageing, the migration process, a fundamental component of

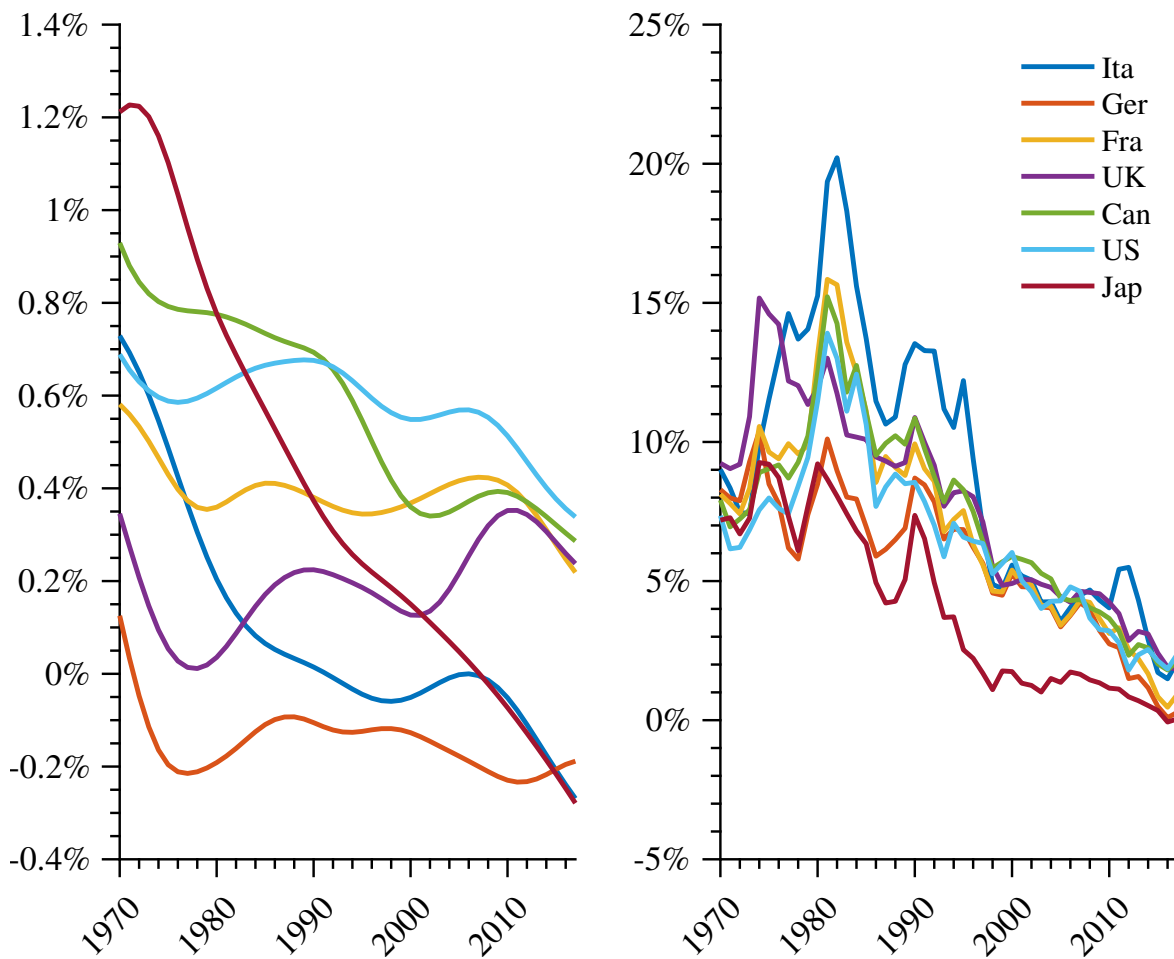


FIGURE 3.1: G7-Economies Natural Population Growth Rates (left) and Long-Term Interest Rates (right). Authors elaboration from UN and JST data.

population dynamics, has been considered a theoretical solution by the related literature (Crafts, 2014; Eggertsson et al., 2019; Gordon, 2012), but usually discarded cause temporary, or of low magnitude relative to the inertia of the natural population ageing dynamics (Jimeno et al., 2014). The aim of this study is therefore to quantitatively explore the possibility that such a factor is relevant in the discussion. Our main point is the following: By definition, the ageing process is given by the annual increase in the population size of a given geographic area, calculated as the sum of two differences, the difference between native births and deaths (natural population growth) and the difference between immigrants and emigrants (net migration flow). As a result, if demographic changes are important in explaining the dynamics of current slump growth (Carvalho et al., 2016; Cooley and Henriksen,

2018; Eggertsson et al., 2019; Papetti, 2019), migration may play a significant role too, especially during an economic downturn. About the connection between migration and secular stagnation, something have been already done in the literature, such as in Miyagiwa and Ono (2019), and Alves and Morgado (2022). However, the contributions mentioned do not take into account the different qualitative features of the migrant population, such as preferences, age and fertility, which are of particular relevance in studying such a complex and heterogeneous process like migration. People in fact, move to another country for different reasons, and they also differ from the natives in terms of age, skill level, cultural features and beliefs, economic situation, and so on. All of this affects their behaviour, which in economic terms means preferences over consumption and saving. As a consequence, even the aggregate variables of the host country are affected. In our exercise therefore, we try to include those features, being supported by some stylized facts that will be detailed later on, in order to widening the understanding about the macroeconomic impact of migration over the host country.

In doing so, we focus on the Italian economy which appears to be the most appropriate case study to analyze the effect of migration in a context of secular stagnation. The SSH is about ageing process and slowdown in productivity growth, and mostly referred to developed countries, in particular United States, Europe, and Japan. Among these countries, Italy seems to be the most suitable candidate, because as shown in Figure 3.1 and studied in many contributions in the literature (Calligaris et al., 2016; Comin et al., 2020; Giordano and Zollino, 2021; Hassan and Ottaviano, 2013; Huo et al., 2020; Mistretta and Zollino, 2018), it is highly affected by both factors (aging and declining TFP growth). Moreover, as shown in Table 3.1, the projected weight of migration over the total population is the most significant among those developed countries treated by the secular stagnation literature. In 2020, in terms of age-dependency ratio, Italy was the second most-aged country in the world after Japan, and in 2050, it is projected to be the third after Japan and South Korea. For what concern the TFP, Italy among the most-aged countries (Japan and South Korea), is the worst-positioned, with an average annual decrease of 0.5% from 1990 to 2020. Lastly, about migration, Italy has been challenged by a large flow of migration starting in the 1990s. As shown in Figure 3.2, it has been just because of the positive net migration rate that the Italian population growth hasn't become negative. Moreover, the projections among the most-aged countries (Japan and South Korea), are again the most significant ones for our study, with the foreign population

Period	Japan	Korea	Italy	EU-28	G-20
<i>Age-dependency ratio 65+/20-64 (%)</i>					
1990	19	9	24	22	15
2020	52	24	40	34	21
2050	81	79	74	56	38
<i>Average total-factor-productivity (TFP) growth (%)</i>					
1990 - 2019	-0.4	1.7	-0.5		
<i>Foreign population (%)</i>					
2000	1	0.4	2		
2019	2	2	9		
-	11 (2070)	4 (2040)	23 (2065)		

TABLE 3.1: Percentage 65+/20-64 age-dependency ratio (upper-part), average total factor productivity growth (central-part), and migration ratio (bottom-part) by specified group of countries and years. Authors' elaboration from OECD, The Conference Board Total Economy Database, Statistics Korea, Japan's National Institute of Population and Social Security Research, and Italian Istat.

projected to be 23% of the total Italian population in 2065. As a result, Italy appears to be the best case study to shed a light about the role of migration over population dynamics as well as about its macroeconomic impact on the host economy. This is what we study here.

The rest of this paper proceeds as follows. Section 2 shows a brief quantitative and qualitative summary of the dynamics of Italian migration from the 1990s to today and introduces a representative migrant agent. Section 3 shows the theoretical framework used in the analysis, a quantitative large-scale OLG model with migration. Section 4 shows the calibration for the Italian country. Section 5 shows the simulated impact of migration over the real interest rate since the 1990s and its policy implications. Conclusions are given in the final section.

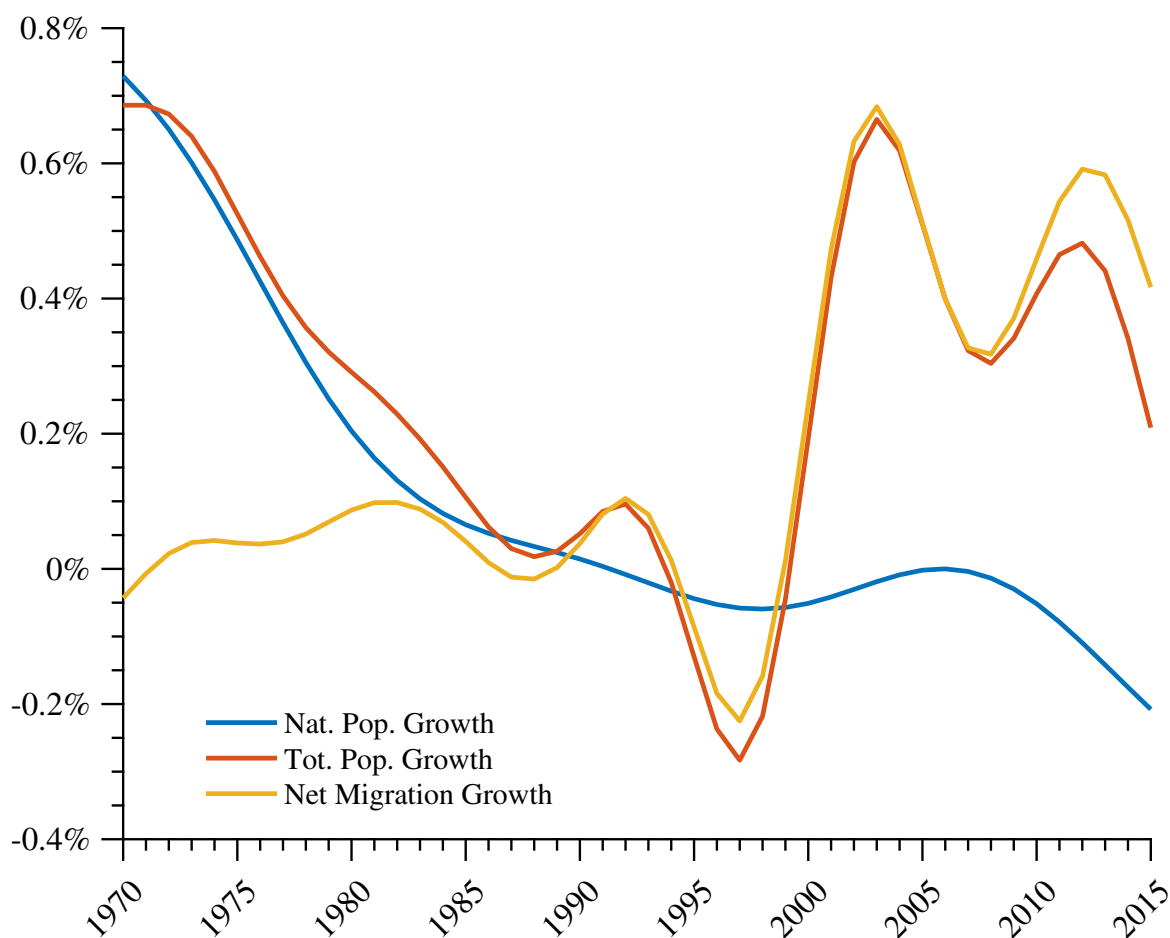


FIGURE 3.2: Italy Natural and Total Population Growth, and Net Migration Growth. Authors elaboration from UN data.

3.2 Immigration in Italy

A brief summary

Italy is a country with a long history of emigration and a very short experience with immigration. Mass emigration started with Italian unification: during the period 1861–1976, over 26 million people emigrated, half of them to other European countries, the rest to North and South America (Del Boca and Venturini, 2003). Emigration decreased in the period 1970–1980, and Italy changed from being a sender country into a host country, receiving immigrants largely from developing countries and Eastern Europe (1972 was the first year of positive signs in immigration). The following events are behind the immigration process in Italy: 1) Since World War II, many Italians known as "rimpatriati" have returned home from South and Central

America; 2) according to King (1993), the peculiar character and evolution of the Italian labour market, as well as the ways in which the economy has been restructured in the post-industrial era towards greater specialization, have created a dual economic system and labour market in which the informal or underground economy and the secondary labour market of casual, unorganised workers have arisen. The secondary labour market has traditionally consisted of insecure, part-time, and seasonal work, which has attracted marginal workers such as women and immigrants primarily from the Philippines, Middle East, and Sub-Saharan Africa who emigrated to Italy in the 1980s; 3) restriction policies in migration flows in classical host countries such as Germany, France, and Belgium diverted the flow towards more marginal areas of Western Europe such as Italy, Spain, Greece, and Portugal during the crisis; 4) during the 1980s and 1990s, Italy was growing in prosperity, GDP per capita and wages for manual workers were rising, and the economic and social gap with countries in North Europe was reducing, making Italy an attractive destination for labour migrants; 5) the fall of the communist regimes in eastern Europe in the 1990s has driven the migration flow of Albanians towards Italy; 6) the supply factor, such as the demographic setting of the less-developed countries (mainly Morocco and Egypt), which had annual rates of population increase of around 2–3 percent. The necessary annual rates of GDP growth to merely keep the unemployment rate at its average level weren't obtained, and the result has been an increase in migration from those countries. These factors contributed to large flows of immigrants during the 1990s that didn't break with the new millennium. Although Albanians and Moroccans were the main ethnic groups for many years, Romanians quickly surpassed them after Romania joined the European Union in 2006. Furthermore, the Arab Spring and the collapse of regimes in Tunisia and Libya in 2011 resulted in a significant increase in the number of asylum seekers, a phenomenon known as the "North Africa Emergency," resulting in an unprecedented number of asylum seekers in Italy. Recently, a far greater jump in the numbers of migrants and asylum seekers arriving in Europe occurred in 2015, as the civil war in Syria and other humanitarian crises drove more than 1 million people to the European continent.

Summing up, all these migration waves led to a large increase in the foreign-resident population in Italy, ranging from 781.100 in the 1990s (1.37% of the total population) to nearly 5.039.637 in 2020 (nearly 9% of the total population). We considered the official residency to distinguish migrants from natives because it is one

of the most reliable statistical indicators on migration; "however, it can detect only those regular immigrants who have already registered as residents, thus overlooking other significant components of immigrant population. Taking into account the gap between arrivals and registrations and including an estimate of people still in the registration process, the number of regular migrants (holders of a valid residence permit) present in the country increases of an additional 200.000 people in 2018. In addition, ISMU(2018), estimated the number of migrants without any form of legal status (illegal immigration) to be around 490.000 people", (Caritas, 2019). As a result, a broad assessment of the composition of migrants (regulars + pendings + irregulars) yields nearly 10 percent of the total population at the time of writing.

Stilyzed facts

Given the heterogeneity of Italian immigration, it's not easy to capture a clear pattern in the qualitative features of the immigrant population. However, some stylized facts can be extrapolated from the available data. First, immigration in Italy has been characterised principally by people coming from less-developed and developing countries, which are usually associated with higher total fertility rates (TFR). "In developing countries children are needed as a labour force and to provide care for their parents in old age. In these countries, fertility rates are higher due to the lack of access to contraceptives and generally lower levels of female education" (Nargund, 2009, p. 1). Data on them are only available in Italy since 2002, as shown in the left-hand panel of 3.3. However, looking at the TFRs of the countries most present in Italy and the increase in TFR during the 1990s (right-hand panel of Figure 3.3) we can get an idea of the trend before 2002. Data from 1995 for the most prevalent ethnic groups in Italy show a high level of TFRs, particularly for Morocco (3.4), the Philippines (3.7), and Albania (2.6), compared to the Italian (1.2) (Del Boca and Venturini, 2003). Romanians arrived in Italy in large numbers only since 2005, outpacing Albanians and Moroccans, and they are typically associated with lower TFR (1.4 in 1995); this could explain why the pattern in foreign-resident TFR began to decline slightly in 2006 (left-hand panel of Figure 3.3). Second, looking at the population pyramids of both populations in Figure 3.4, we observe that the migrant population is much younger than the native one. To capture the waves of immigration, we selected three main years: 2002 (before the Romanians' wave), 2010 (before the "North Africa Emergency" in 2011 and the civil war in Syria in 2015), and 2021. Regardless

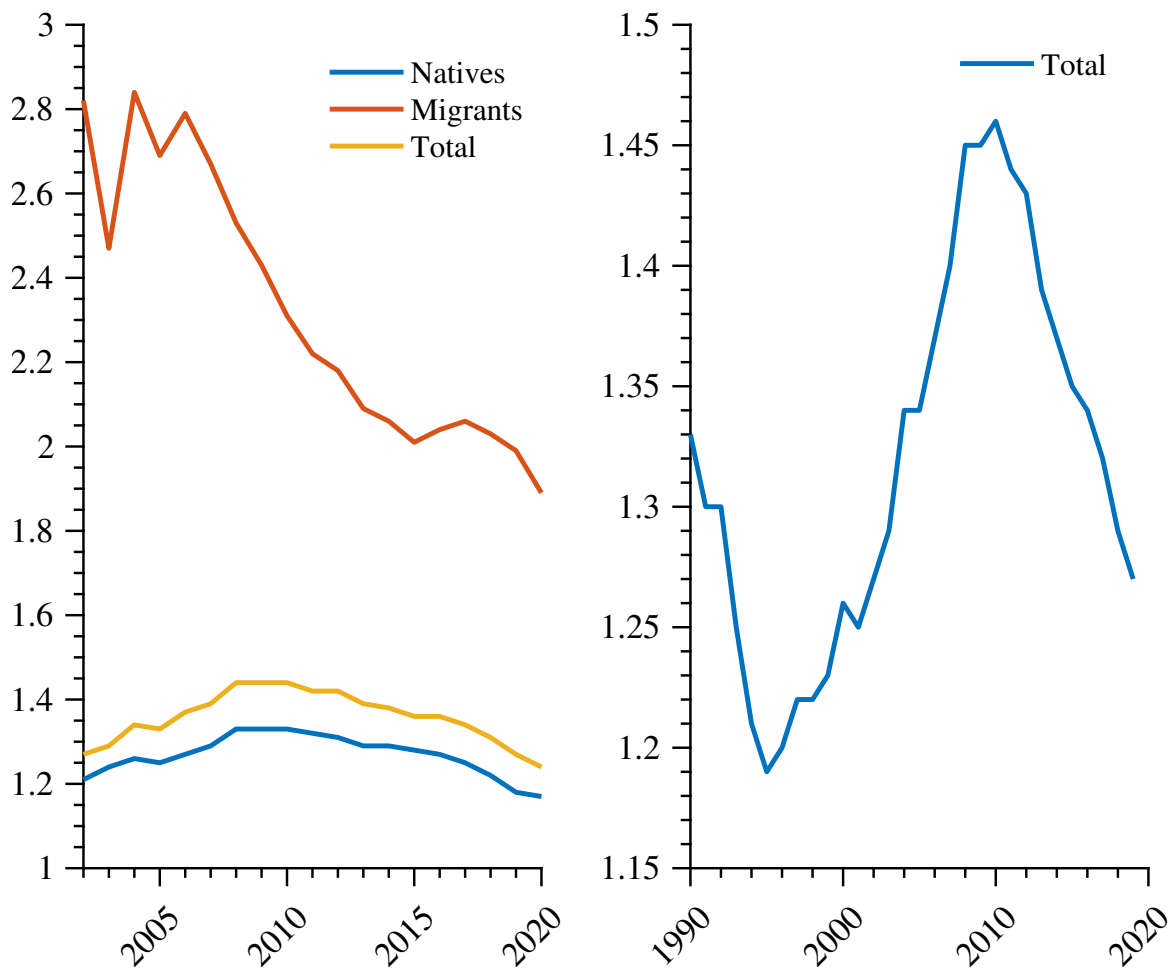


FIGURE 3.3: Italy Natives, migrants and total population TFRs from ISTAT (left) and Total population TFR from World Bank (right). Authors elaboration from ISTAT and World Bank data.

of which immigration wave occurred during those years, the migrant population was always younger in comparison to the Italian population. Comparing the mean and the median ages between both populations during the years selected, we see how the Italian population is more than ten years older than the migrant one. The mean age of the migrant population has been 31, 31, and 35 years, respectively. Instead, it has been 42, 44, and 46 years for the Italian one. As a result, the first two factors can potentially be dampening factors, reversing the ageing process and avoiding some strain on the host economy's public finances. Third, the reasons for the move allow us to know something about their behavior. However, they are difficult to retrieve from the data because the way to join the Italian country changes

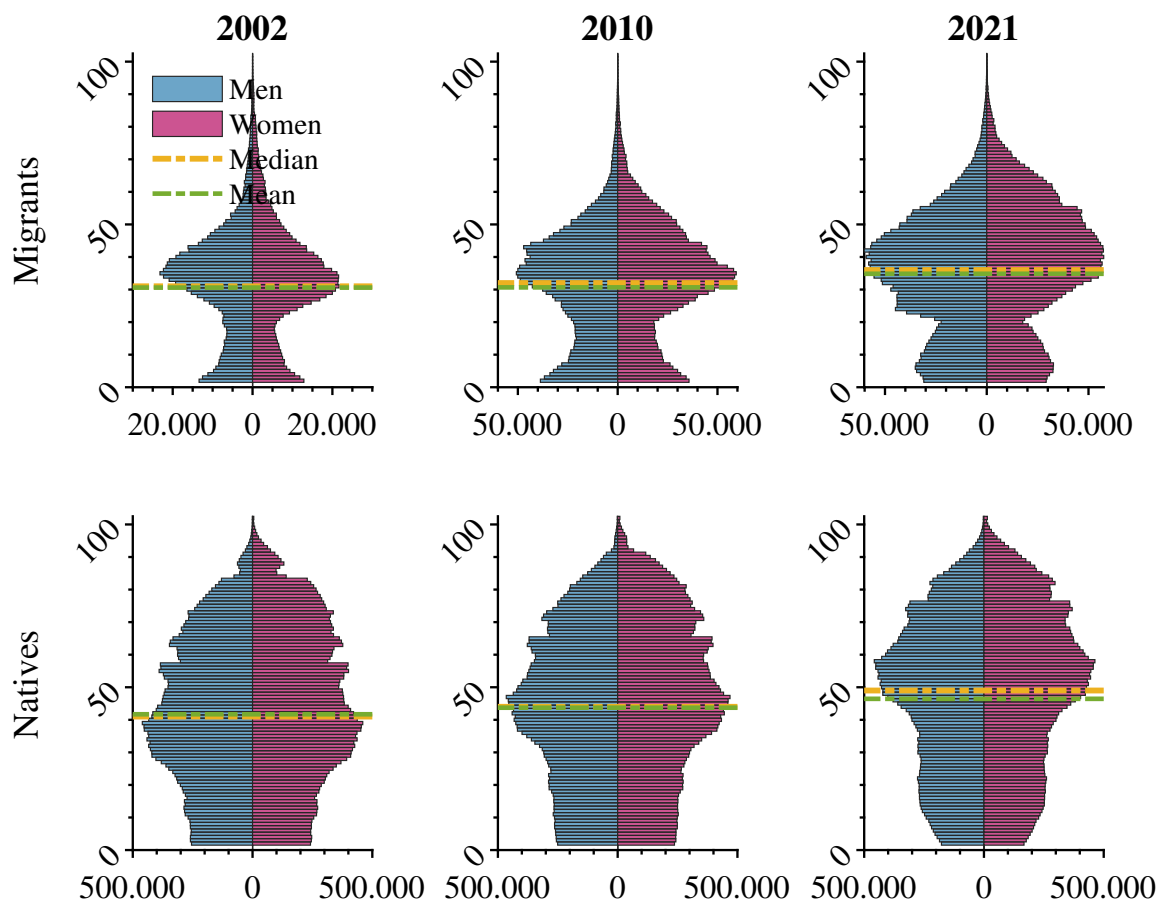


FIGURE 3.4: Migrant Population Pyramid (upper) and Native Population Pyramid (lower), 2002, 2010, 2021. Authors elaboration from ISTAT data.

depending on the country of citizenship and also changes over time due to different government laws. A residence permit is required for non-EU citizens. This one would help us understand the reasons behind immigration; indeed, it can be released for three main reasons: 1) work; 2) family reunion; 3) asylum seekers and refugees. Between the non-EU citizen migrants, the types of residence permits in the 1990s were principally for labour; "the Italian migration was at its initial phase in which one family member alone usually immigrates" (Del Boca and Venturini, 2003, p. 20). However, during the years, migration flow developed, and we started observing a clear decline in labor-type residence permits in favour of family-type permits. "Already one of the major destinations in Europe for third-country migrants seeking employment, Italy has more recently seen a spike both in regular arrivals due to family reunification and in irregular entries of undocumented migrants and asylum seekers - the so-called "mixed-flows" - mainly from sub-Saharan

African countries" (Caritas, 2019, p. 15). However, because a residence permit is not required for EU citizens (but not Italians), we lack data on why they are moving to Italy. Anyway, we know that in 2021 the migrant population (with EU and

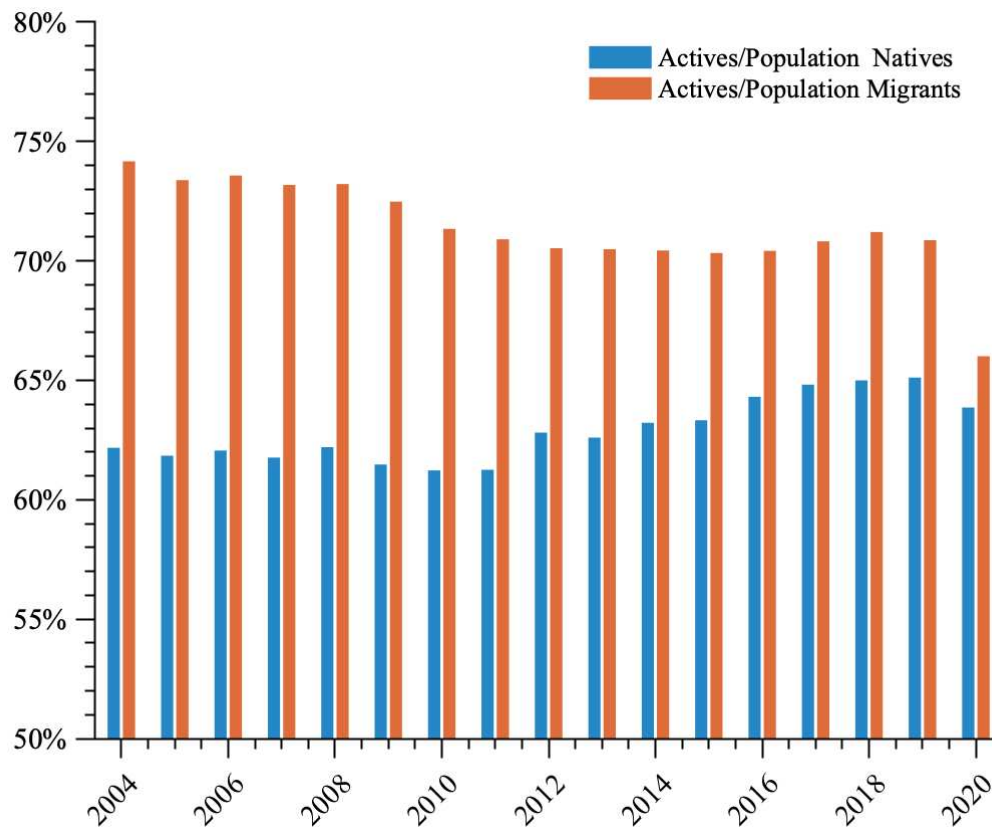


FIGURE 3.5: Labor Market: percentage of labor force over total population. Authors elaboration from ISTAT data.

non-EU citizenship) was principally composed of working-age humans. 67 percent are between the ages of 25 and 64, 28 percent are between the ages of 0 and 24, and 5 percent are between the ages of 65 and 100+. So, if we look further at the labour market, we are able to extrapolate some key features about what they do once they move to Italy, independently of the "unknown" reasons for their movement. Looking at Figure 3.5, we can see that both migrant populations (non-EU and EU) have a higher labour force (activity) to total population ratio than the Italian population. Migrants have always participated in the labour market at a higher rate than Italians, ranging from 12 to 6 percent higher. This is a key aspect because it allows us to say that, independently of the reason for the move, once the migrant

population comes into the Italian country, they become workers to a greater extent than Italians do. Therefore, it is reasonable to model them as a classic working agent in a neoclassical framework. Fourth, the consumption/saving pattern of the migrant population is more inclined towards consumption. In other words, their propensity to consume is higher than Natives given that they receive less income on average. Almost all income is spent on primary goods (house rent and children's education) and remittances. In 2010, a foreign family's average income in Italy was less than 45 percent of that of a native family, and their average consumption was less than 30 percent (Moressa, 2012). Therefore, just a small amount remains saved. The estimated marginal propensity to consume is around 96.6 percent, compared to 76.6 percent for natives. Therefore, an important quantity of migration can alter the normal capital accumulation in the life-cycle of the host economy and, as a consequence, even the fundamentals. All these features are captured through a different calibration of the structural parameters for the migrant population in the household problem and in the equations for population dynamics.

An Italian Representative Migrant Agent

In modelling the migrant population, some difficulties arise due to factors such as age, culture, beliefs, habits, economic situation, and so on. However, from the data available, we are able to trace out an Italian representative migrant, which can be modelled with the following features: 1) He has a higher total fertility rate (TFR); 2) He is younger than the average Italian person; 3) He participates more in the labour market than the average Italian person; therefore, he is a working-age person; 4) He has a specific consumption and saving pattern that is more inclined towards consumption than that of Italians. This is what we want to capture with the theoretical model built in the next section to explain the role played by migration in the host economy. Overall, migration dynamics affect the host country's demographic structure, causing changes in both the ratio of savers to non-savers and the size of households, which in turn affect the aggregate savings and interest rate. The Italian country has been impacted by these dynamics and is likely to face challenges in the future, given its importance in southern Europe as one of the first host countries for Mediterranean migration, as well as estimates of environmental refugees (UN, 2014). The current estimate of the ratio of the resident migrant population without Italian citizenship in 2065 is 23 percent over the total population (ISTAT,

2011), without taking the expected role climate change will play in the future into account. Therefore, we find it important to introduce the role played by migration in a framework of Secular Stagnation, to allow policy-makers to better shape the policy, particularly during an economic downturn and from their perspective.

3.3 Model

The baseline framework used to introduce migration is the one from Eggertsson et al. (2019). They have been the first to model the Secular Stagnation hypothesis in a 56-generations OLG model based on Auerbach and Kotlikoff (1987) and Ríos-Rull (1996). We propose an extension to the baseline model using the well-known Dynare 5.3 (Adjemian et al., 2022), which introduces a source of heterogeneity among households, namely the representative migrant agent, allowing us to capture a more realistic and detailed dynamics of the demographic process, including a wider range of factors such as different fertility rates or time preferences between migrants and natives, as well as other related quantitative and qualitative features, as outlined in the previous section. A closely related paper is Storesletten (2000) however, differently from it, we do not account for different labor skills (labor productivity)¹ and legal status among the migrant population, but we focus on a representative migrant agent, which has different time preferences and intertemporal elasticity of consumption relative to the representative native agent. Moreover, we do not analyze the quantitative impact on the fiscal side of the economy but on the monetary side instead. In particular, we study the impact on interest rates over the Italian's life cycle, which is central to the Secular Stagnation Theory. On the same line as Storesletten (2000), we account for different fertility rates between natives and migrants. The three most important innovations (in terms of modelling) we bring are: 1) with respect to Eggertsson et al. (2019), the introduction of population heterogeneity and therefore the possibility for the model to be driven by two different total fertility rates (TFRs) and a flow of migration. Therefore, we will be able to study the Secular Stagnation Theory in a specific country with a wider view of population dynamics, which is the key aspect of the theory; 2) The setup of migration dynamics mixed with the native ones is general, therefore it can be applied further to

¹In other words, the labor skills of natives as well as of migrants are assumed to be homogeneous, conditionally on their age.

other economic theories that make use of the overlapping-generation (OLG) framework; 3) We modelled the framework using the widely used user-friendly Dynare 5.3 (Adjemian et al., 2022), which is easier to use, interpret, and replicate compared to Fortran (in which the original code of Auerbach and Kotlikoff (1987) has been written) or Matlab (in which the original code of Eggertsson et al. (2019) has been written), bringing down the entry barriers for those interested in such analyses. One of the mathematical problems we faced when switching to Dynare is the presence of multiple occasionally binding constraints (OBCs). We bypassed the problem by rewriting them as in (Swarbrick, 2021). Given that the firm and government sectors are identical to Eggertsson et al. (2019), the notation used is broadly the same.

3.3.1 Population

The economy is composed of the native (n) and migrant populations (m). The population growth rate is determined by the total fertility rate of every family in the two populations (Γ^n, Γ^m) and by the amount of immigration the economy faces, denoted by (f). Furthermore, they have a stochastic chance of dying before reaching the maximum age J , which is set at 81 years as in (Eggertsson et al., 2019). The probability of surviving between ages j and $j + 1$ is given by s_j^i and is known as conditional probability; the probability of arriving at age j , on the other hand, is given by $s^{j,i}$ and is known as unconditional probability. The total population, N_t , is the sum of the populations of different ages and breeds, $N_t^{j,i}$ with $i = n, m$. Except for the generation $j = 26$ years, which is the first generation in the model, the population size of a given generation $N_t^{j,i}$ is the population of the generation from the previous year that has survived. That is given by the total population of their parents, which reached economic maturity at time $t - 25$, multiplied by the total fertility rate of their parents' generation at that time (Γ_{t-25}^i) and discounted for the unconditional probability of survival. Furthermore, the migrant population varies even due to the exogenous immigration flow process (f_t); in this case, we assume that the immigration flow consists of people approaching economic maturity, i.e., $j = 26$, which is not a strong assumption given that the migrant population's average age is around 30 years, as shown in the previous section. As a result, the flow of immigration will increase the population of generation $j = 26$ at each time interval t . The total population evolves in the model according to the laws of motion and aggregates given

below:

$$\begin{aligned}
N_t^n &= \sum_{j=26}^J n_t^{j,n} \\
n_{t+1}^{j+1,n} &= s_j^n n_t^{j,n} \quad \text{for } j \in \{27, J-1\} \\
n_t^{26,n} &= \frac{n_{t-25}^{26,n} \Gamma_{t-25}^n}{s^{26,n}} \\
N_t^m &= \sum_{j=26}^J n_t^{j,m} \\
n_{t+1}^{j+1,m} &= s_j^m n_t^{j,m} \quad \text{for } j \in \{27, J-1\} \\
n_t^{26,m} &= \frac{n_{t-25}^{26,m} \Gamma_{t-25}^m}{s^{26,m}} + f_t
\end{aligned}$$

where:

$$\begin{aligned}
\Gamma_{t-25}^n &= (1 + n_{t-25}^n)^{\frac{1}{25}} \\
\Gamma_{t-25}^m &= (1 + n_{t-25}^m)^{\frac{1}{25}}
\end{aligned}$$

Thus the total population at a given time t is given by:

$$N_t = N_t^n + N_t^m \quad (3.1)$$

Households do not receive wage income after retirement, set at age $j = 65$. The labour supply is determined inelastically by an individual age-specific exogenous labour productivity $hc^{j,i}$, as follows:

$$L_t = \sum_{j=26}^J n_t^{j,n} hc^{j,n} + \sum_{j=26}^J n_t^{j,m} hc^{j,m} \quad (3.2)$$

3.3.2 Households

Both populations, native and migrant, face identical intertemporal utility functions of consumption. They enter economic maturity at age 26, after which they work, consume, have children, and trade in asset markets. Households pass away with certainty at the age of $J = 81$ years, and they have children at the age 26. As a result, the following intertemporal utility function is maximised by each generation j of

both populations ($i = n, m$):

$$\max_{\{c_{t+j-1}^{j,i}\}} U_t^i = \frac{1}{(1 - \frac{1}{\gamma})} \left(\sum_{j=26}^J s^{j,i} \beta^{j-1,i} u(c_{t+j-1}^{j,i}) \right) \quad (3.3)$$

A household of age j can trade in a real asset $a_t^{j,i}$ at time t , which is used as productive capital. At time $t + 1$, capital will pay a return of r_t , which is the real interest rate. Each household in each population has the same exogenous labour productivity process, or human capital profile, which varies with age and is denoted by $hc^{j,i}$. Households receive no wage income after retirement, which occurs after age 65. Assuming an inelastic labour supply, wage income equals the wage multiplied by the household's age-specific labour productivity $hc^{j,i}$, minus labour taxes $(1 - \tau)$. Households also receive income from firms' pure profits, denoted by $\Pi_t^{j,i}$, and aggregate profits are assumed to be distributed proportionally to labour income. The flow budget constraint of a household of population i of age j at time t is:

$$c_t^{j,i} + a_{t+1}^{j,i} = (1 - \tau)w_t hc^{j,i} + \pi_t^{j,i} + (1 + r_t) \frac{a_t^{j,i}}{s_j^i} \quad (3.4)$$

Households can borrow against future income, and we impose an occasionally binding borrowing constraint expressed as a percentage of the wage income:

$$a_t^{j,i} \geq D_t^i w_t hc^{j,i} \quad (3.5)$$

Finally, we impose other constraints and initial conditions as follow:

$$c_t^{j,i} \geq 0, \quad a_t^{26,i} = 0, \quad a_t^{J,i} = 0 \quad (3.6)$$

The utility is a CES form function:

$$u(c) = c^{1 - \frac{1}{\gamma}}$$

3.3.3 Firms

There are two types of firms: producers of final goods (Y_t^f) and producers of intermediate goods (Y_t^m). A continuum of firms of type i of measure one produce final goods (Y_t^f) by costlessly differentiating an intermediate good (Y_t^m) and reselling it

to the representative household via a linear technology, $Y_t^f = Y_t^m$. They buy the intermediate good at a price determined in the perfectly competitive market of the intermediate firms, and they differentiate the intermediate good, arriving at a final good to sell to the representative household. These firms are monopolistically competitive, so they set prices in each period. This allows for a time-varying markup given by $\frac{\theta_t}{\theta_t - 1}$. Pure profits due to monopoly rents are returned to households. The final good composite is the CES aggregate:

$$Y_t = \left(\int_0^1 Y_t^f(i)^{\frac{\theta_t - 1}{\theta_t}} di \right)^{\frac{\theta_t}{\theta_t - 1}}$$

The markup condition is given by:

$$\frac{p_t^{int}}{P_t} = \frac{\theta_t - 1}{\theta_t} \quad (3.7)$$

Aggregate profits in equilibrium will be given by:

$$\Pi_t = \frac{Y_t}{\theta_t} \quad (3.8)$$

Intermediate goods (Y_t^m) are produced by a perfectly competitive intermediate goods sector that sells its output to the final goods sector. These firms hire workers at wage rate w_t and rent capital at rate rk_t . They employ a CES production function in labour and capital, with an elasticity of substitution given by σ . The production function is:

$$Y_t^m = \left(\alpha(K_t)^{\frac{\sigma - 1}{\sigma}} + (1 - \alpha)(AL_t L_t)^{\frac{\sigma - 1}{\sigma}} \right)^{\frac{\sigma}{\sigma - 1}} \quad (3.9)$$

where AL_t is the exogenous labor-augmenting technological progresses, and $0 < \alpha < 1$.

Taking w_t as a numeraire, we define $A_{adj} = w$ as a parameter at its steady-state value, and we divide w_t, rk_t, Y_t for A_{adj} to get the first-order conditions for

the intermediate-firms problem:

$$A_{adj} = \frac{p_t^{\text{int}}}{P_t} (1 - \alpha) (AL_t)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{L_t} \right)^{\frac{1}{\sigma}} \quad (3.10)$$

$$w_t = \frac{\frac{p_t^{\text{int}}}{P_t} (1 - \alpha) (AL_t)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{L_t} \right)^{\frac{1}{\sigma}}}{A_{adj}} \quad (3.11)$$

$$rk_t = \frac{\frac{p_t^{\text{int}}}{P_t} (\alpha) \left(\frac{Y_t}{K_t} \right)^{\frac{1}{\sigma}}}{A_{adj}} \quad (3.12)$$

$$Y_t = \frac{\left(\alpha (K_t)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha) (AL_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}}{A_{adj}} \quad (3.13)$$

Eventually, we have the no-arbitrage condition which relates the risk-free real rate with the return on capital taking into account the role of the relative price of capital goods:

$$1 + r_t = \frac{rk_t + (1 - \delta)\zeta_t}{\zeta_{t-1}} \quad (3.14)$$

3.3.4 Government

A fiscal policy rule is governed by an exogenous process where the government spends an exogenous sum $g_t Y_t$ (as a percentage of gdp), may accumulate debt $b_t Y_t$ (as percentage of gdp), and collects labour income tax τ . The budget constraint is given by:

$$b_t Y_t = g_t Y_t + (1 + r_t) \cdot b_{t-1} Y_{t-1} - \tau_t w_t L_t \quad (3.15)$$

where:

$$G_t = g_t \cdot Y_t \quad (3.16)$$

$$T_t = \tau_t \omega_t L_t \quad (3.17)$$

$$gov_t^{rev} = g_t Y_t + r_t b_t Y_t \quad (3.18)$$

$$gov_t^{deficit} = \frac{b_{t+1} Y_{t+1} - b_t Y_t}{gov_t^{rev}} \quad (3.19)$$

$$gov_t^{debt} = \frac{b_t Y_t}{K_t} \quad (3.20)$$

3.3.5 Law of Motion of Capital

The aggregate capital stock evolves according to a standard law of motion:

$$\zeta_t K_{t+1} = (1 - \delta) \zeta_{t-1} K_t + I_t$$

where δ is the rate of depreciation, I_t is investment, and ζ_t is the relative price of capital goods.

3.3.6 Asset Market Clearing Condition

Asset markets clear, thus:

$$K_t = \frac{\left(\sum_{j=26}^J \frac{n_t^j a_{t-1}^j}{sv_t^j} \right)}{\zeta_{t-1} + gov_t^{debt}} \quad (3.21)$$

3.3.7 Equilibrium

A competitive equilibrium is a set of household allocations, $\{c_t^{j,i}, a_t^{j,i}, \pi_t^{j,i}\}_{j=26}^J\}_{t=0}^\infty$, a set of aggregate quantities $\{Y_t, K_t, L_t, \Pi_t\}_{t=0}^\infty$, a set of prices $\{\omega_t, r_t^k, r_t, \frac{p_t^{int}}{p_t}\}_{t=0}^\infty$, a set of government variables $\{b_t, \tau_t\}_{t=0}^\infty$, and exogenous processes $\{N_t^{j,i}\}_{j=26}^J, hc^{j,i}\}_{j=26}^{65}\}_{t=0}^\infty$ that jointly satisfy:

1. Consumption maximizes 3.3 subject to 3.4, 3.5 and 3.6.
2. Profits are distributed proportionally to labor income, $\pi_t^{j,i} = hc^{j,i} \frac{\Pi_t}{L_t}$.
3. Output is given by aggregate production function 3.13.

4. Asset markets clear, and thus aggregate capital is given by 3.21.
5. Aggregate labor supply is given by 3.2.
6. Aggregate profits are given by 3.8.
7. There are perfect factor markets, and thus 3.11, 3.12, and 3.14 hold.
8. The monopolistic market set prices, and thus the markup condition 3.7 hold.
9. The government satisfies budget equation 3.15.
10. Population by age group is given by 3.1.

3.4 Calibration

The period we take into account is 1990–2020. However, because there are two different populations and thus two population growth rates, to ensure the convergence we assume that the TFR of migrants, Γ^m , begins to converge with the one of natives, Γ^n , beginning in 2020 and approaching it in 2030. We calibrate the model for 1990, and then we consider the transitional dynamics assuming the new steady state to be after 150 periods (about three times the number of generations as in Auerbach and Kotlikoff (1987)). Our parameters are divided into statistical data, literature parameters, match-target parameters. The first are data for mortality rates (s_j, s^j), total fertility rates (Γ^i), TFP (AL_{growth}), the income profile ($hc^{j,i}$), the government debt (b) and expenditure (g) and the borrowing limit (D). The capital share (α), capital/labor elasticity of substitution (σ), and consumption intertemporal elasticity of substitution (γ^i) are the second. The third is the migration flow (f), followed by the depreciation rate (δ), the time preferences (β^i), and the profit share (θ).

To quantify the role played by the migrant population, we used the model without migration as a benchmark, and we built up two other scenarios described as follows:

- **Baseline Calibration:** this is the benchmark case, in which the migrant population is exactly equal to the native population in terms of quantitative and qualitative features. Therefore, both populations have the same TFR ($\Gamma^n = \Gamma^m$), the same time preferences ($\beta^n = \beta^m$) and the same intertemporal elasticity of substitution ($\gamma^n = \gamma^m$). So, it's just like if there's not migration at all;

- Alternative Specification 1 (ALT1): this scenario assumes that the only difference between the two populations is their TFR ($\Gamma^n \neq \Gamma^m$). Therefore, we are able to capture the impact of the higher fertility rate in the migrant population;
- Alternative Specification 2 (ALT2): this scenario assumes that the two populations differ in terms of TFRs (as specified in ALT1), time preferences ($\beta^n \neq \beta^m$), and the intertemporal elasticity of substitutions ($\gamma^n \neq \gamma^m$). In this way, we are able to capture the overall effects of different quantitative and qualitative features between native and migrant populations.

For all the three scenarios, we calibrated the model at steady state in 1990, matching some targets in the economy, specifically the ratio between the migrant population and the total population ($N2/N$) observed in the data and projected by ISTAT, the real interest rate, the investment-output ratio, and the labour share. Parameters from statistical data are described in Table 3.2. They don't change with the type of scenario, except for the migrants' TFR, which is assumed to be equal to the one of the natives in the Baseline Calibration. The calibrated parameters for all the specifications will be detailed in the next tables.

Parameters	Symbol	1990	2020	Source
Survival rates	$s_j^i, s_j^{j,i}$	/	/	ISTAT
TFP	AL_{growth}	0.92%	0.03%	Conference Board Database
Income profile	$hc^{j,i}$	/	/	SHIW Bank of Italy
Government debt	b	91.6%	132.7%	JST Database
Government expend.	g	31.4%	31.4%	JST Database
Borrowing limit	$D^{j,i}$	/	/	Bank of Italy

TABLE 3.2: Parameters from Statistical Data that do not change with the Scenario

3.4.1 Baseline Calibration

In the Baseline Calibration the TFR of the migrant population is set to be equal to the one of the native population, as in Table 3.3. Table 3.4 describes parameters from

Parameters	Symbol	1990	2020	Source
TFR	$\Gamma^n = \Gamma^m$	2.44	1.11	ISTAT

TABLE 3.3: Parameters from Statistical Data: TFRs

the literature. The depreciation rate (δ) has been set to a level of 10% as in the Italian General Equilibrium Model (IGEM) by Annicchiarico et al. (2013); the capital/labor elasticity (σ) has been set to a level of 0.8 following Kopecna et al. (2020); and the intertemporal elasticity of substitution (γ^i) has been set to a level of 0.39, which is an average of what is found in the related literature for Italy. The structural parameters

Parameters	Symbol	Level	Source
Depreciation Rate	δ	10%	(Annicchiarico et al., 2013)
K/L Els	σ	0.78	(Kopecna et al., 2020)
C Eis	γ^n, γ^m	0.39	Average Literature

TABLE 3.4: Parameters from Literature

are chosen in order to match targets in the economy in 1990, described in Table 3.5.² These are the time preference parameters ($\beta^n = \beta^m$) set to match the real interest rate, the capital share (α) set to match the investment-to-GDP ratio, and the profit share (θ) set to match the labour share-to-GDP ratio.³

²In all the specifications the parameters chosen to match targets, have been calibrated by minimising an objective function between the data coming from the output of the model and our targets observed in real data, as in Ludwig et al. (2012).

³For the time preference $\beta^n = \beta^m$ we got a value greater than 1 which is possible in an overlapping setting and is close to Ríos-Rull (1996) and Constantinides et al. (2002). The capital share α instead, is equal to 0.39 and close to Ríos-Rull (1996) again.

Parameters	Symbol	Level	Targets	1990
Time pref.	β^n, β^m	1.01	Real Interest Rate	6.3%
Capital Share	α	0.39	Investment/GDP Ratio	21%
Profit share	θ	7.33	Labor Share/GDP	58%

TABLE 3.5: Parameters chosen to match Targets

3.4.2 Alternative Specification 1

In the Alternative Specification 1, the only parameter that changes from the Baseline Calibration is the TFR of the migrant population, which is set following ISTAT data as in Table 3.6. The remaining calibration is equal to the baseline (Table 3.4 and 3.5).

Parameters	Symbol	1990	2020	Source
Natives TFR	Γ^n	2.44	1.11	ISTAT
Migrants TFR	Γ^f	4.47	2.10	Interpolation from ISTAT data

TABLE 3.6: Parameters from Statistical Data: TFRs

3.4.3 Alternative Specification 2

In the Alternative Specification 2, two types of parameters change: the intertemporal elasticity of substitution (γ^i) and the time preference (β^i) which are set differently between natives and migrants to reflect the stylised facts discussed in Section 3.2. The first is shown in Table 3.7; specifically, the intertemporal elasticity of substitution of natives (γ^n) has been set to 0.39, which is an average of the elasticities found in the related literature for Italy, and the intertemporal elasticity of substitution of migrants (γ^m) has been set to a level of 2, which is a weighted average of the elasticities of the origin countries of the migrants most present in Italy. The second is shown in Table 3.8, where β^n is set to the level of the Baseline Calibration or Alternative Specification 1 because it represents the time preference of the natives. β^m

Parameters	Symbol	Level	Source
Natives C Eis	γ^n	0.39	Average Literature
Migrants C Eis	γ^m	2	Weighted Average Literature

TABLE 3.7: Parameters from Literature

instead is set to match the real interest rate in 1990 and is lower than that of natives, reflecting their higher propensity to consume (Moressa, 2012).

Parameters	Symbol	Level	Target	1990
Natives Time pref.	β^n	1.01	Baseline Calibration	/
Migrants Time pref.	β^m	0.97	Real Interest Rate	6.3%

TABLE 3.8: Parameters chosen to match Targets

3.5 Simulation and Policy Implications

In this section, we show the results we got through model simulation for the three scenarios. We computed the initial steady state in 1990, and then we simulated the transition dynamics until 2141 using the homotopy approach in Dynare 5.3; then we cut the sample period of interest, 1990-2020. In order to get a realistic quantitative effect of the migrant population in the host economy, for Alternative Specification 1 and Alternative Specification 2 we set the parameter that control for the migration flow (f) to match the dynamics of the ratio between the migrant population ($N2$) and the total population (N) from 1990 to 2065, as shown by the red line in Figure 3.6; after that the ratio is assumed to be constant at the 2065 level.⁴ In the Baseline Calibration that ratio is simply equal to 0, and the total population is matched.

3.5.1 Role played by Migrant Population till 2020

Given this premise, the left-hand panel of Figure 3.7 shows the results for the three scenarios. In all cases, the model accurately predicts the interest rate in 1990 and its declining trend. The presence of migration via higher fertility (Alternative Specification 1) and different qualitative features such as time preference and the elasticity of intertemporal substitution (Alternative Specification 2) results in a higher real interest rate, temporarily offsetting the impact of the native population's ageing process and the slowdown in productivity growth. We described this effect better by plotting the spread between the Baseline Calibration and the two Alternative Specification, in the right-hand panel of Figure 3.7. The effect of the higher fertility of the migrant population had an effect of around 0.6 percent over the real interest rate. When you factor in the higher propensity for consumption (due to a lower time preference and a higher elasticity of intertemporal substitution), the effect is greater, around 1.4 percent. As a result, the role of a younger population with a higher fertility rate and propensity to consume, such as migrants, serves as a temporary offsetting factor for the Secular Stagnation Hypothesis in Italy which otherwise would have been more severe.

⁴As we did with the other parameters, we chose f in order to minimise an objective function between the data coming from the output of the model and our targets observed in real data.

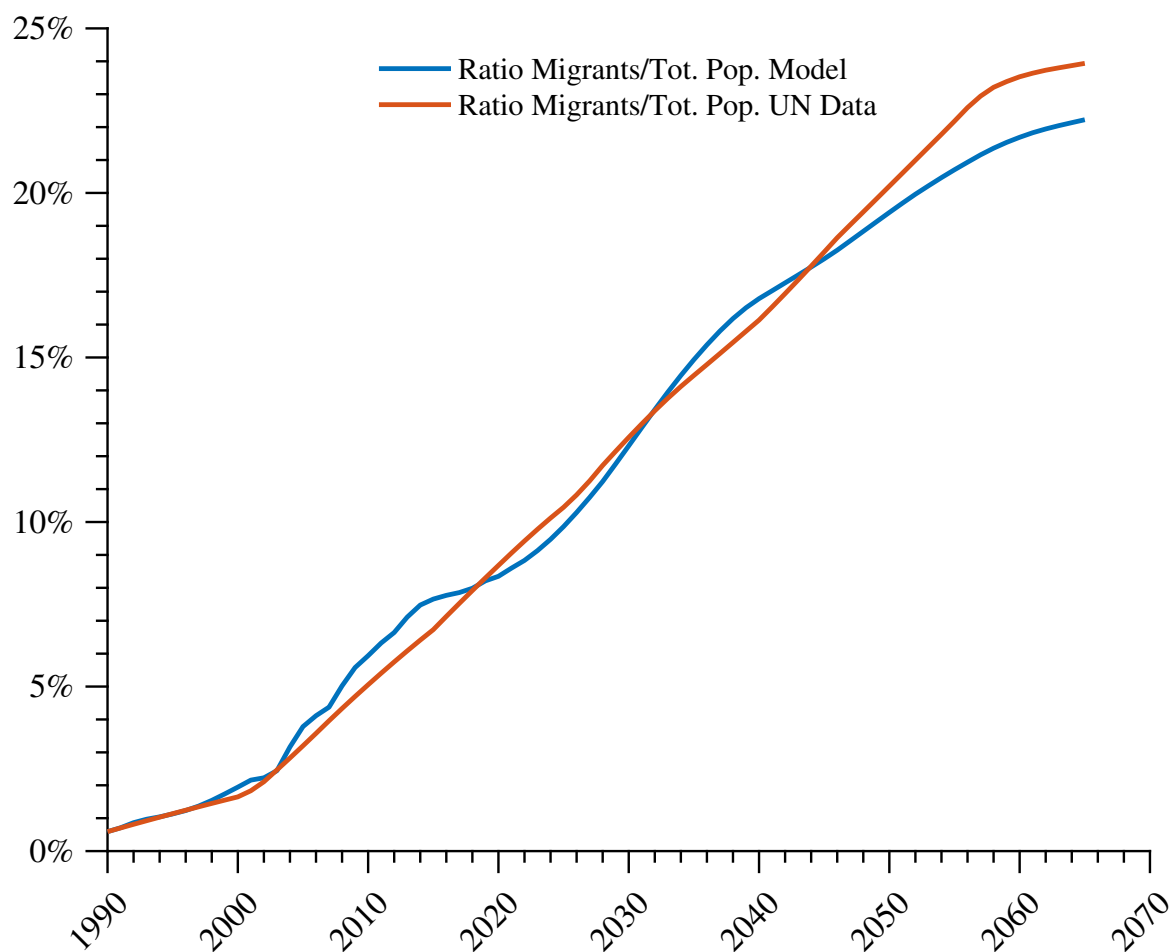


FIGURE 3.6: Authors elaboration from ISTAT data.

3.5.2 Expected role by Migrant Population in the future

Without cutting the simulation sample and taking the entire period from 1990 to 2141 and matching the projected ratio N_2/N by ISTAT until 2065, we can get some policy implications. Assuming the migrant population is going to be around 22 percent of the total population in 2065 (ISTAT, 2011) and assuming that the marginal propensity to consume and other economy variables (such as the TFP growth) remain constant at 2020 levels, we obtain the following forecasts, as shown in the left-hand and the right-hand panels of Figure 3.8. As we can see, Alternative Specification 1 is now useless because the fertility rate of the migrant population is going to converge with that of the native population, and in that scenario it was the only difference in parameters. Instead, what looks interesting is the Alternative Specification 2. The final steady state of the real interest rate, with a presence of the

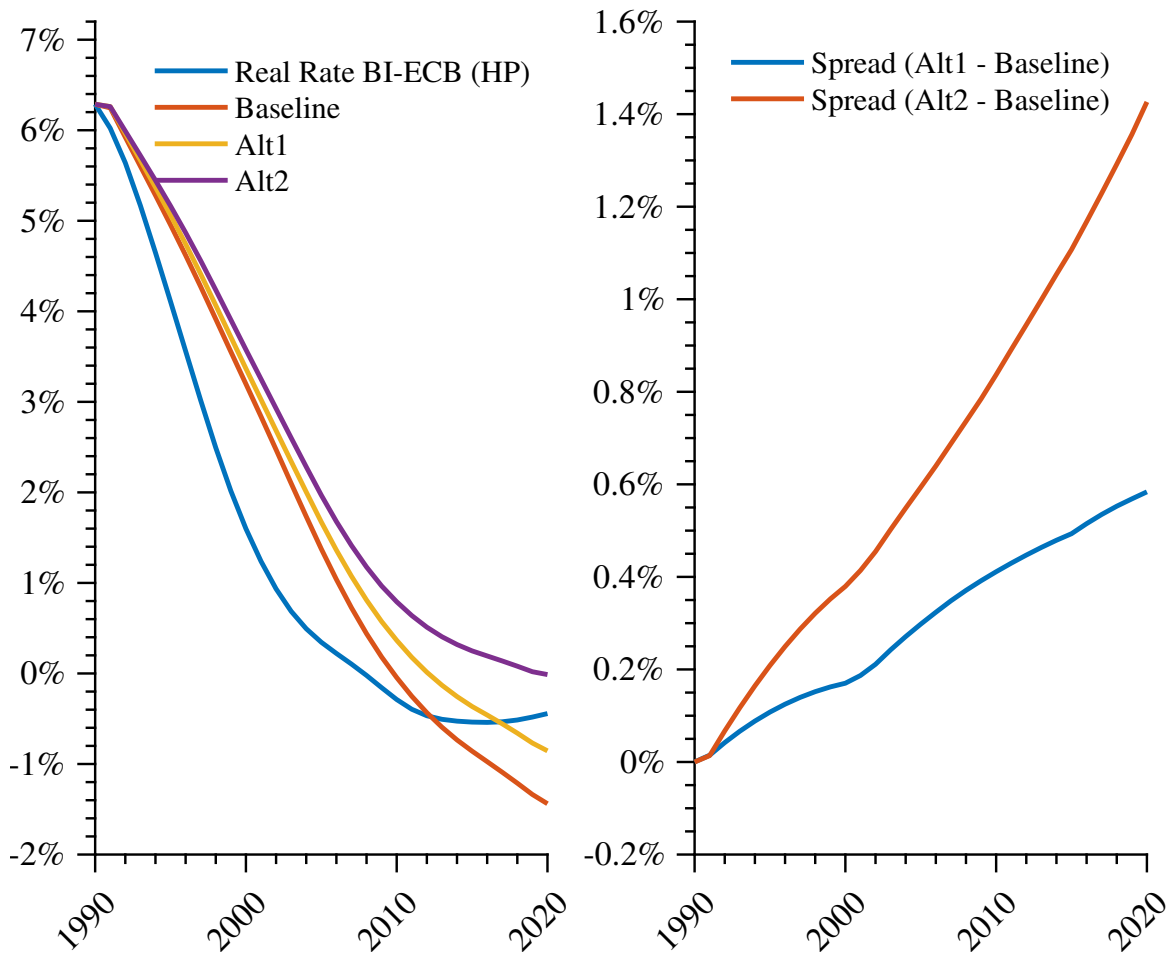


FIGURE 3.7: Transition path of real interest rate (left-hand) and spread between scenarios (right-hand). Authors model elaboration and Bank of Italy, ECB and FRED data.

migrant population of around 22 percent of the total and, with an higher propensity to consume it is around 2 percent, instead of the -1.5 percent in the economy without migration (Baseline Calibration). Therefore, assuming that those forces behind the Secular Stagnation Hypothesis, like ageing and slowing productivity growth, continue to remain relevant in a country like Italy, highly affected by both, the migrant population can play an important role in counterbalancing them. The higher fertility via reduction in aggregate savings and the higher propensity to consume via an increase in the investment function offset the decline pattern of interest rate in Italy in a way to push it away from the negative territory. From the model simulation, in 2065 we arrive at having a real interest rate higher than 2.5% respect to the case

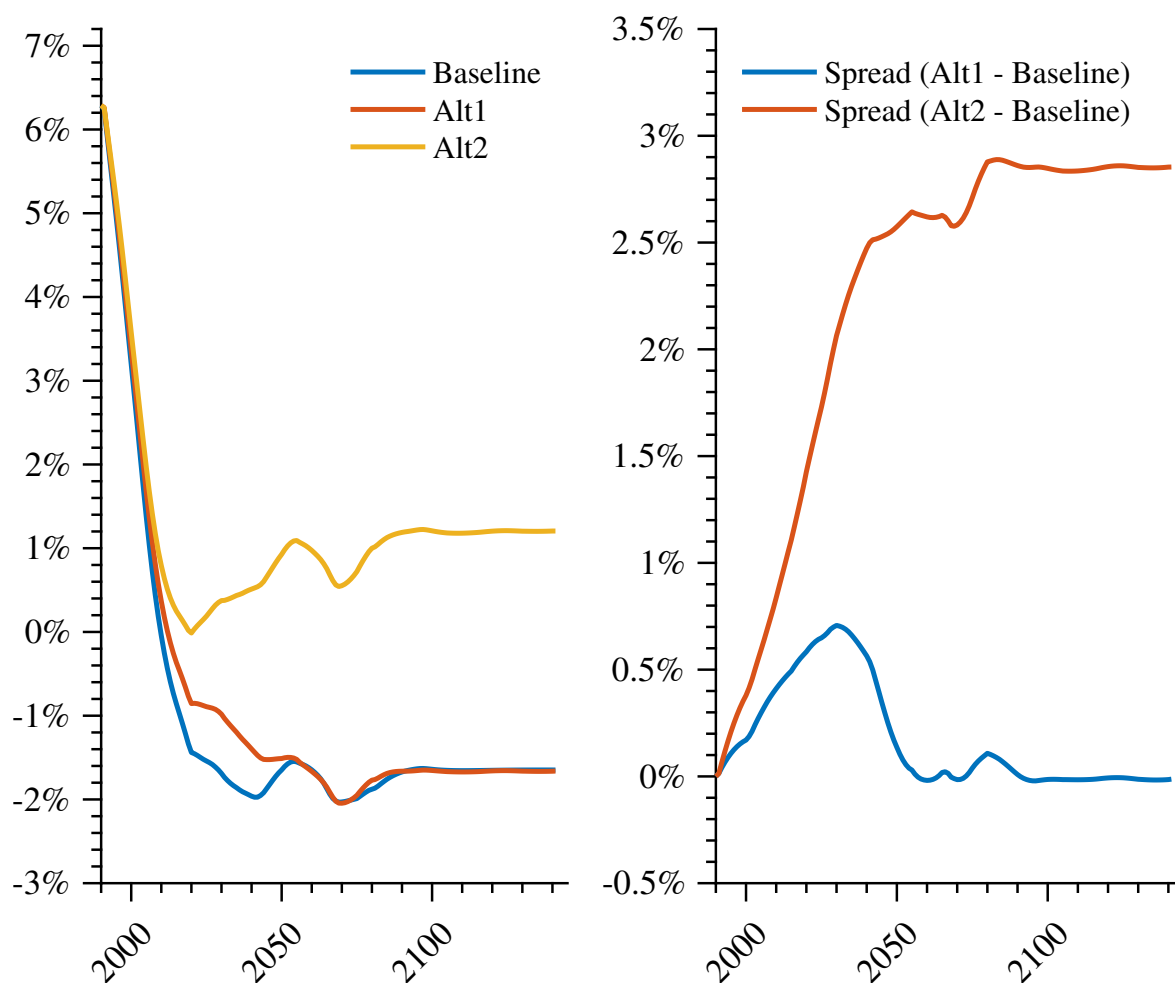


FIGURE 3.8: Full transition path of real interest rate (left-hand) and full spread between scenarios.

without migrants (right-hand panel of Figure 3.8). Therefore, without other significant changes in the economy, the presence of future migration will be a dampening factor for the tendency that a stagnant economy has to settle into a steady state with negative rates.

3.6 Conclusions

We have proposed an extension of Eggertsson et al. (2019)'s model to shed a light on the role that migrant population can have in shaping the dynamics of an economy. Although the literature on Secular Stagnation has accounted migration for a temporary solution, we have found that this turns to be true just in theory. With

a quantitative model that accurately decompose the ageing process to consider the flow and the qualitative features of migration, we have found that it play a crucial role, quantitatively relevant for the Italian economy. In particular, the higher fertility rate kept the interest rate higher of a magnitude around 0.5% till 2020, although its effect is then absorbed due to the assumption we made on the convergence of migrants fertility rate to the one of natives. If we sum to the fertility effect, the higher propensity to consume, migration contributes in keeping the interest rate higher of a magnitude around 1.5% till 2020, increasing to 2.5-3% in the new steady-state. The results are important: although a temporary channel, given the projections of the United Nations that see migrant population to be around 22% of the total Italian population in 2065, migration can be seen a factor which dampens the stagnant dynamics over the interest rate up to the point of pushing it away from the negative territory.

Chapter 4

The Challenge of Ageing and Income Inequality for Italian Savings

4.1 Introduction

The dynamic and distribution of aggregate saving have been central to the economic literature around the world, beginning with Bosworth et al. (1991), Jappelli and Modigliani (1998), Bernanke (2005), Gordon (2012), Summers (2015a), Neri et al. (2017) and, most recently, Mian et al. (2021a). About the dynamic, its significance is related to the role that saving plays in shaping the overall economic growth and public finances. On the one hand, without savings, households have few other mechanisms to smooth out unexpected variations in their income, and as a result, shocks may cause problems with human capital accumulation at a young age. On the other hand, the ability to save becomes one of the most important tools for social mobility and future earning potential. Furthermore, savings are one of the primary resources for financing investments, thereby facilitating the banking system and thus economic growth, particularly in countries dominated by banks. About the distribution, its importance has become apparent just recently, given the two major macroeconomic challenges that the industrialised economies are facing, population ageing and income inequality (Blanchard and Tirole, 2021). Both will be discussed in due course. The effects of population ageing on savings and public finances have been studied since the 1990s, for example, by Jappelli and Modigliani (1998), Sabelhaus and Pence (1999), and Baldini and Mazzaferro (2000), but they became visible only with the new century, particularly in countries that did not properly account for the consequences. The impact of population ageing is commonly associated

with Ando and Modigliani (1963)'s life-cycle theory and, more practically, its effect on public finance. According to it, the pattern of private savings tends to be hump-shaped over the life-cycle, and thus, the greater the share of the elderly and the lower the share of young people who dissave, the greater the decrease in private savings. However, this does not appear to be confirmed in general, as the elderly do not tend to dissave (Jappelli and Modigliani, 1998), making the theory highly diversified across countries. On this point, two recent works are worth mentioning. Carvalho et al. (2016, p. 209) showed how "for a given retirement age, an increase in life expectancy lengthens the retirement period and generates additional incentives to save throughout the life cycle". The effect is also strengthened in those countries with defined-benefit (DB), instead of defined-contribution (DC) pension systems. Consider also, that most pension systems are actually not able to guarantee a stable pension at the current replacement rate. DeNardi and Borella (2020) instead, found that the elderly tend to save more as a precaution for future "health-related expenses which have become heavily compressed in our final years, as we live longer and face expensive end-of-life care" (Horwich, 2022). For what concern the impact over public finances instead, it is determined by the demographic transition, that is the shift from a population with high birth and death rates to one with low birth and death rates; in particular, an increase in public expenditure for social contributions due to a higher number of retirees, without increasing revenues (taxes), tends to unbalance the public budget and negatively contributes to saving accumulation. As a result, the overall impact of population ageing on the economy is unclear and highly volatile across countries. Income inequality and savings instead, are typically associated with rich people's greater ability to accumulate savings in comparison to the middle and lower classes (Dynan et al., 2004); thus, countries with highly polarised income tend to have higher levels of private saving. In this regard, relevant literature for the American economy and our guideline for the current work are Mian et al. (2021a) and Mian et al. (2021b).

As previously underlined, two of the major macroeconomic challenges of advanced economies are population ageing and income inequality; both have a significant impact on the dynamics of aggregate savings and therefore on aggregate demand and economic growth, and they have been great sources of research in the last two decades. At the Jackson Hole Economic Symposium hosted by the Federal Reserve Bank of Kansas City in 2021, Mian et al. (2021b) explained the dominance of income inequality in relation to population ageing in shaping the dynamics of

savings for the American economy and the relation with decreasing real interest rates (Juselius et al., 2017). Our point is that, having the United States one of the highest Gini indices among industrialised countries and a not-so-pronounced ageing process, the result of Mian et al. (2021b) can be viewed as a specific feature of the American economy and not a more general trend in Western countries. The aim of this work is therefore to put this possibility to the test. In doing so, we take Italy as a case study; a country with opposing trends in income inequality and ageing processes.

Figure 4.1 illustrates the point. The left side depicts the income inequality pro-

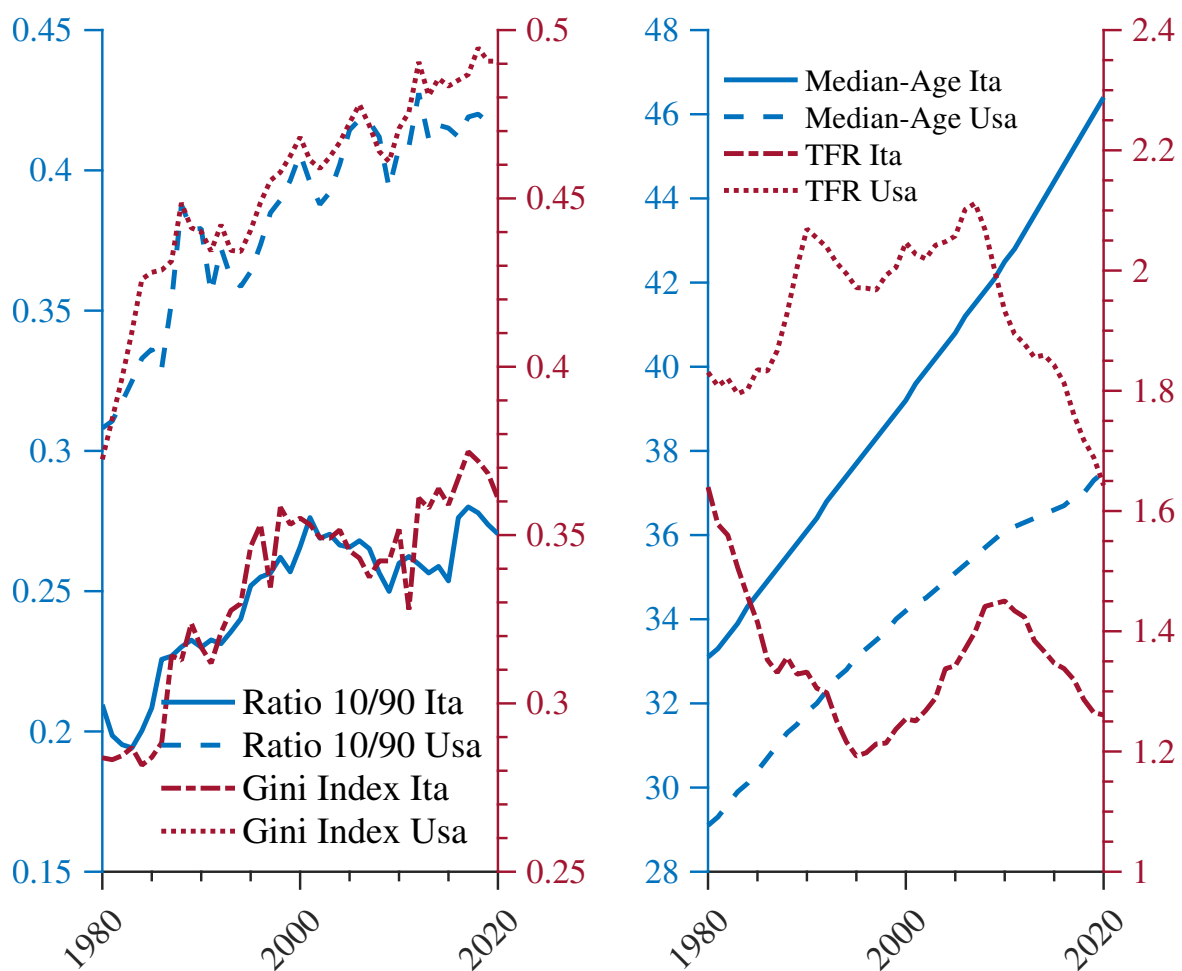


FIGURE 4.1: Income Inequality (left) and Population Ageing (right).
Authors' elaboration from WID and UN database.

cesses of both countries using two well-known measures: the ratio of the Top 10 to the Bottom 90 and the Gini Index. In comparison to Italy, income inequality in the United States is about 10-15% higher on average. On the other hand, the right

side depicts the population ageing processes. The median age of Italians is ten years older, and the number of children per woman (total fertility rate, TFR) in Italy is 1.26 versus 1.70 in the United States. To remark the point, given the number of people at least 65 years old (around 23% of the total population (UN, 2020)), Italy is considered the second oldest country in the world after Japan. This characteristic is clearly

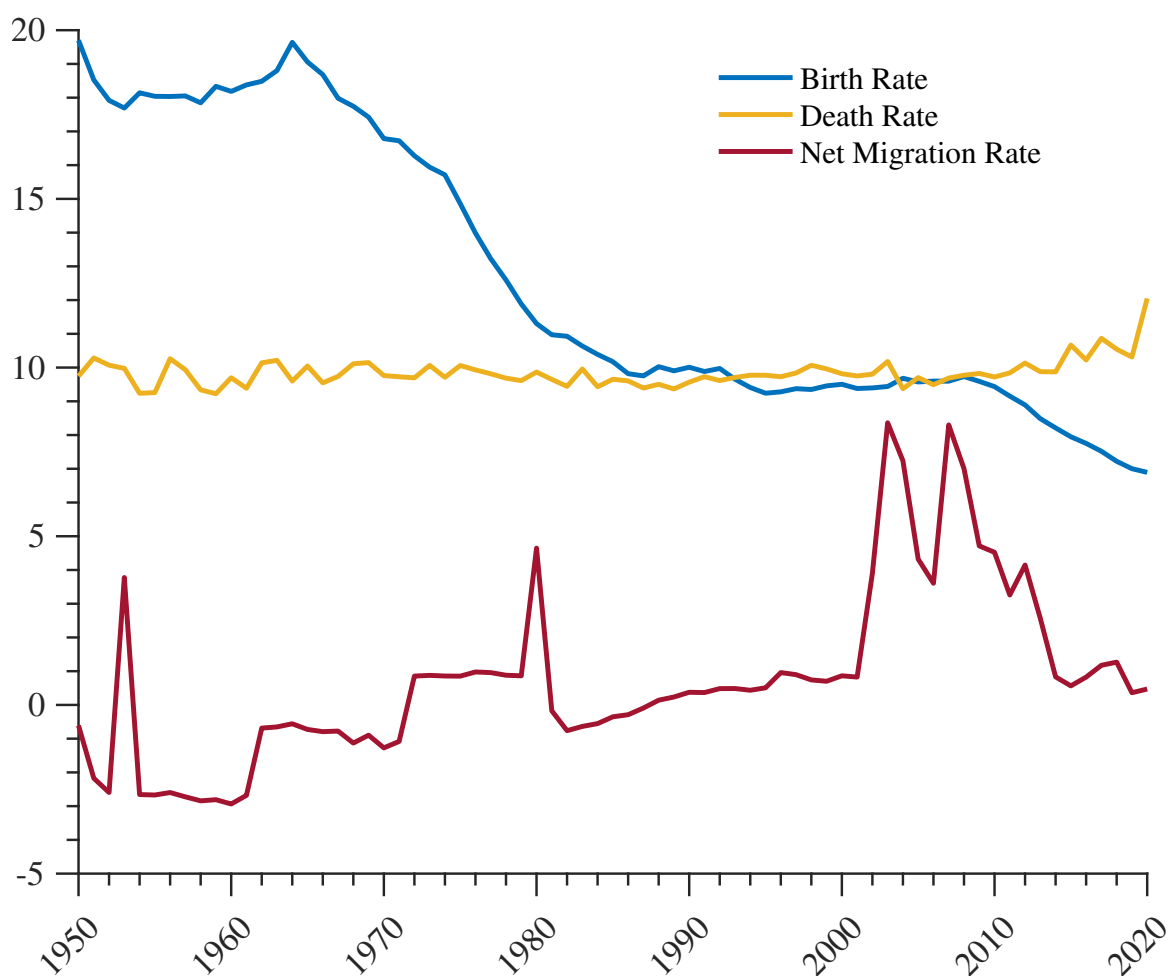


FIGURE 4.2: Crude Birth and Death Rates and Net Migration Rates for Italy. Authors elaboration from UN database.

the result of a declining process in birth rates, as illustrated in Figure (4.2), where the replacement level is well below the threshold of 2.1 children per woman for industrialised economies since 1975.¹ Moreover, in Italy, the literature on the dynamic of savings is typically concerned with changes in income inequality with various

¹Demographers use the replacement statistic to represent the second phase of a demographic transition. It denotes the level of fertility at which a population exactly replaces itself from one generation to the next, resulting in a stable population that does not increase or decrease. In developed

wealth estimations (Cannari et al., 2008), (Cannari and D'Alessio, 2018), (Acciari et al., 2021), (Guzzardi et al., 2022) and less weight is given to the effect of ageing over savings, which appear to be an important factor too. Eventually, as previously stated, the importance of savings dynamic in Italy is accentuated by the bank-centric banking system. As a result, the following exercise tests the weight that population ageing has for the dynamic of savings in Italy relative to income inequality.

The rest of this paper is organised as follows. Section 2 shows the methodology for the analysis. Section 3 displays the dataset used, the Bank of Italy's Survey on Household Income and Wealth (SHIW). Section 4 presents the findings of the analysis. Conclusions are given in the final section.

4.2 Methodology

With the aim of tracing out the effects of population ageing and income inequality for the dynamic of aggregate savings from the 1980s to the present, we exploit the core methodology used by Mian et al. (2021a) and Mian et al. (2021b) called Shift-Share, applying some differences due to data constraints. The various differences will be detailed later on. In a nutshell, first we use the Survey on Household Income and Wealth (SHIW) from the Bank of Italy, dividing the population into age and income groups, to calculate the share of savings of each group-type over the total, and we use each share to match the aggregate data on savings from the Italian National Accounts (NAs) which are based on the European System of Accounts, ESA (2010). In what it follows we describe the overall procedure and the differences with Mian et al. (2021a) and Mian et al. (2021b) in detail.

4.2.1 National Accounting Framework

The core idea is to exploit the micro-data of the SHIW survey, from which derive informations about the economic behaviour of Italian households' and then use its representativeness to infer the behaviour at national level, matching aggregate data from the NAs and be sure the methodology is correct. Given that the SHIW is a representative survey of Italian households, the most logical option is to match what

countries, replacement level for fertility is estimated to require 2.1 children per woman. Only if mortality rates remain constant and migration has no effect will replacement level fertility result in zero population growth.

the SHIW is designed for, which is the private sector; we therefore exclude public saving and we focus on private saving (S^{pr}) as a variable of interest. According to the European System of Account, ESA (2010), the gross aggregate saving (S) is measured as follows:

$$S = \underbrace{S^h + S^{npish} + S^{nfb} + S^{fb}}_{S^{pr}} + \underbrace{S^g}_{S^{pu}} \quad (4.1)$$

where:

S^h = households saving

S^{npish} = non-profit institutions serving households saving

S^{nfb} = non-financial business saving

S^{fb} = financial business saving

S^g = government saving

Rewriting 4.1 we get that the aggregate saving can be divided into private (S^{pr}) and public (S^{pu}) components:

$$S = S^{pr} + S^{pu} \quad (4.2)$$

where:

$$S^{pr} = S^h + S^{npish} + S^{nfb} + S^{fb}$$

$$S^{pu} = S^g$$

Furthermore, we know that public saving is determined by the difference between revenue (taxes) and spending (public consumption and transfers), so we get:

$$S = S^{pr} + T - G - R \quad (4.3)$$

where:

$$S^{pu} = T - G - R$$

G = public consumption

T = taxes

R = transfers

We also know that gross national disposable income (NI) may be utilised for both national consumption (C) and national saving (S), so:

$$NI = C + S \tag{4.4}$$

where the national consumption is defined by the sum between private consumption (C^{pr}) and public consumption (G):

$$C = C^{pr} + G \tag{4.5}$$

Using 4.3-4.5 we can rewrite 4.4 arriving to:

$$NI = C^{pr} + G + S^{pr} + T - G - R \tag{4.6}$$

and rewriting we get:

$$S^{pr} = \underbrace{NI - T + R}_{\text{post tax and transf. nat. income}} - C^{pr} \tag{4.7}$$

Finally, we get our benchmark by dividing both sides by the income:

$$\frac{S^{pr}}{NI} = \frac{(NI - T + R - C^{pr})}{NI} \tag{4.8}$$

which represents the aggregate variables from the NAs that we want to match from the SHIW data.

4.2.2 The Income Less Consumption method

According to Mian et al. (2021a), starting from a survey two ways are available to measure and match the private saving of equation 4.8:

1. the Income Less Consumption approach;
2. the Wealth-Based approach.

In the present work we use the simplest approach, that is the former. Equation 4.8 is a ratio (the share of saving over national income), as a consequence the aim is to get that share for each age and income group from the SHIW survey, and then apply those shares to NAs data. Eventually, taking the summation among all the groups we get the same ratio at national level. We proceed as it follows:

$$\frac{S_t^{shiw,j}}{NI_t^{shiw}} = \frac{(NI_t^{shiw,j} - T_t^{shiw,j} + R_t^{shiw,j})}{NI_t^{shiw}} - \frac{C_t^{shiw,j}}{NI_t^{shiw}} \quad (4.9)$$

where:

$$j = \left[\begin{array}{l} \text{age groups (18-34, 35-44, \dots, 75-84, 85+);} \\ \text{income groups (Top 10\%, Next 40\%, Bottom 50\%)} \end{array} \right]$$

The choice about the specific age and income groups employed will be detailed in Sections 4.3.2 and 4.3.3. Rewriting in terms of ratios, we get:

$$s_t^{shiw,j} = i_t^{shiw,j} - c_t^{shiw,j} \quad (4.10)$$

We can multiply the shares we got, for the aggregate data from Italian NAs (ISTAT) to match the private aggregate saving and the gross national income:

$$S_t^{match,j} = s_t^{shiw,j} NI_t^{Istat} \quad (4.11)$$

$$NI_t^{match,j} = i_t^{shiw,j} NI_t^{Istat} \quad (4.12)$$

$$C_t^{match,j} = c_t^{shiw,j} NI_t^{Istat} \quad (4.13)$$

where each sum matches the ISTAT data:

$$\sum_j^J S_t^{match,j} = S_t^{pr,Istat} \quad (4.14)$$

$$\sum_j^J NI_t^{match,j} = NI_t^{Istat} \quad (4.15)$$

$$\sum_j^J C_t^{match,j} = C_t^{Istat} \quad (4.16)$$

In a nutshell, the procedure is just a simple disaggregation of the Italian NAs data exploiting micro-data and representativeness of the SHIW. Now that we know the aggregate data are correct, we can use the Shift-Share methodology, to assign a weight to each factor we are studying, population ageing and income inequality.

4.2.3 Shift-Share Decomposition

Early contributions to this methodology include Summers and Carroll (1987), Bosworth et al. (1991), Baldini and Mazzaferro (2000), and most recently, Mian et al. (2021b). The main idea is to explain the dynamics of an aggregate variable as the difference between two time periods (0 and T). In our case, we want to show how population ageing and income inequality contribute to the difference in the ratio of private aggregate saving to gross disposable national income (S^{pr}/NI) between time 0 (1980-2000) and, time T (2000-2020). To achieve this, we need our variable of interest to be divided into age income groups ($S^{pr,j}/NI^j$), with the sum of each group matching exactly the aggregate from NAs (S^{pr}/NI), as we described above. Assuming that the variable of interest has already been divided into age and income groups, the change in the two periods of time considered, can be decomposed by two deterministic components: 1) the change in income of each group (age or income group), and 2) the change in the saving rate of the same group (age or income group).² Simply changing notations and following Mian et al. (2021b) we get the following equation:

$$\frac{S_T^{pr}}{NI_T} - \frac{S_0^{pr}}{NI_0} = \sum_j^J \left(\alpha_T^j - \alpha_0^j \right) \frac{S_0^{pr,j}}{NI_0^j} + \sum_j^J \alpha_T^j \left(\frac{S_T^{pr,j}}{NI_T^j} - \frac{S_0^{pr,j}}{NI_0^j} \right) \quad (4.17)$$

²The proof is included in the Appendix D.

where:

$$j = \left[\begin{array}{l} \text{age groups (18-34, 35-44, \dots, 75-84, 85+);} \\ \text{income groups (Top 10\%, Next 40\%, Bottom 50\%)} \end{array} \right]$$

$$\alpha^j = \frac{NI^j}{NI}$$

Denoting $s = \frac{S^{pr}}{NI}$ we have:

$$s_T - s_0 = \underbrace{\sum_j (\alpha_T^j - \alpha_0^j) s_0^j}_{\text{shift-share}} + \underbrace{\sum_j \alpha_T^j (s_T^j - s_0^j)}_{\text{residual}} \quad (4.18)$$

Our main object of interest will be the shift-share component, which measures the change of our variable of interest between the period 0 and T, due to the change in income going to the specific age or income group j , $\alpha_T^j - \alpha_0^j$, while assuming the propensity to save remains constant at time 0, s_0^j . On the other hand, the residual component represents what remains from the total change of our variable, and can be thought as the change in the specific age or income group j 's deep preference between consumption and saving. In other words, it represents the change in the propensity to save, $s_T^j - s_0^j$, multiplied by the new level of income, α_T^j . The general purpose behind the use of the Shift-Share method is the following: if a specific age or income group j received more income than another group j and, the propensity to save did not change significantly between time 0 and time T, then that group's aggregate savings should have increased more. In this way, we can assign to each factor (population ageing and income inequality), the weight it has played in the overall change of our variable of interest and determine which one had the greater impact. More concretely, to apply the Shift-Share in equation 4.17 or 4.18, we use equations 4.11-4.12 as determined above. Before turning to the results, in the next section we detail the choices about the dataset (SHIW) and the specific division placed upon about the age groups (18 – 34, 35 – 44, . . . , 75 – 84, 85+) and the income shares (Top 10%, Next 40%, Bottom 50%), as well as the choice about the time period (1980-2020).

4.3 Dataset

The fundamental requirement for our work is a database that:

1. ask questions about the birth year, income, and saving at a household level;
2. has a panel component, crucial for a life-cycle analysis.

4.3.1 Survey on Household Income and Wealth (SHIW)

Being aware that "the use of different data sources for the study of inequality is essential, as each source is open to challenge and has different advantages and shortcomings" (Acciari et al., 2021, p. 2), the only ones available for Italy are the SHIW from the Bank of Italy and the Household Budget Survey (HBS) from ISTAT. However, given our interest in the period going from 1980 to 2020, we are forced to employ only the SHIW given the limited sample of the HBS, which unfortunately cover just years from 1997 to today. The time span considered, allows us to focus on two key aspects of the Italian's ageing process: the drop in birth rates that began in the mid-1970s (see Figure 4.1) and the dissaving of the baby-boom generation. The latter is a significant time effect over the life cycle of the Italian economy that cannot be overlooked in the analysis; it is associated with those born between 1946 and 1964. People from those years reached adulthood 18 years later (1964-1982) and retired 40 years later (2004-2022). However, SHIW data prior to 1977 and after 2020 are not available, so we chose our final sample to be from 1980 to 2020. In this way, we can also divide the sample into two equal-length periods: 1980-2000 (decrease in birth rate) and 2000-2020 (dissaving of baby-boom generation). The SHIW survey, includes data on income, consumption, and saving from approximately 8,000 households (20,000 individuals) spread across approximately 300 Italian municipalities. Furthermore, it includes a panel component of approximately 50% of the total households interviewed beginning in 1989; however, as explained in Section 4.3.2 we have to workaroud it, given that our time-span (1980-2020) is longer relative to the available panel component. The variables we use are:

- demographic variables such as, "ANASC" (birth year), "ETA" (age) and, "NORD" (identifier of the head of the family), from the database "comp.dta";
- the private income "Y1" from the database "rfam.dta" which comprises income from dependent (employees) and independent (self-employment and firms)

work, pension and transfer and capital (excluded financial capital which is not available before 1987), at the net of taxes. In this way, we have an equivalent measure of the income used in the National Accounts.

- the private consumption "C" from the database "cons.dta", which comprises expenditure of income for durable and non-durable goods.

We merged the databases using the following variables: "ANNO" (observation year), "NQUEST" (household identifier), and "NORD" (identifier of the head of the household) and we divided the database into age and income shares in order to capture the effects of population ageing and income inequality.

4.3.2 Age Cohorts

In terms of population ageing, the use of a limited sample (1980-2020) creates a structural problem for the analysis. As previously stated, the panel component of the SHIW became available after 1989 and is an important part of our analysis because it allows us to properly analyse the life-cycle theory. If we miss it, we basically fail to analyse the saving behaviour of the same household or individual over time, as required by the life-cycle theory. To get around the problem, we follow Baldini and Mazzaferro (2000) and create a pseudo-panel (also called synthetic-panel). Instead of following the same household, we select a different representative sample; in our case, all the households within the same birth cohort. In other words, for each survey available, we take and divide the households (8,000 for each survey) into birth-cohorts of 10 years. In this way, we can track the life-cycle of a representative birth cohort, which is a group of households born in the same range of years (cohort) at the same time (t). The median age will then be assigned to each cohort as the reference variable for the life cycle, where by "the age of the household" we refer to as "the age of the family's head". As described by Baldini and Mazzaferro (2000, p. 6) "the two cases should not produce significant differences in the results if the sampling procedure does not distort the social and demographic composition of the survey population compared to that of the real population", and the used the SHIW survey too. Descriptive statistics of age cohorts are presented in Table 4.1. It describes the cohort ID used to follow the life-cycle, the birth-year of each family's

head, and the median age in two different periods (the average between 1980-2000, and between 2000-2020), and the observations for each birth-cohort.³

Cohort ID	Birth Year of Head	Median Age 1991	Median Age 2010	Observations
190	1895-1904	89	/	82
191	1905-1914	79	/	881
192	1915-1924	70	88	1877
193	1925-1934	61	80	2918
194	1935-1944	51	70	4081
195	1945-1954	42	60	4472
196	1955-1964	32	50	3587
197	1965-1974	25	41	2249
198	1975-1984	/	32	1007
199	1985-1994	/	24	323
200	1995-2004	/	/	55

TABLE 4.1: Descriptive Statistics: Age Cohorts

4.3.3 Income Shares

For what concerns income inequality, we start by defining it. Given that household surveys are unsuitable "at the very top of the income distribution for a variety of reasons, including a lack of over-sampling of wealthy households and differential non-response and under-reporting rates across wealth classes" (Acciari et al., 2021, p. 2), we cannot define income inequality using high-income shares of the income distribution, such as the top 5% or top 1%. In other words: 1) the very wealthy are rarely surveyed, and 2) even when they are surveyed, the wealthy may under-report their consumption more than the non-wealthy. Because the SHIW is a survey with no oversampling, it is vulnerable to the problem. Typically, the solutions are as follows:

1. oversample rich households (or rich areas);
2. use robust statistics (i.e. median vs mean; winsorized estimates);
3. collect data throughout ad-hoc surveys on the wealthiest households or other sources (i.e. Forbes) and adjust your statistics;

³We chose 1991 instead of 1990 because the waves of the SHIW published by the Bank of Italy are usually every two years, starting from 1977.

4. wide the income shares (i.e. top 10%).

We choose the simplest solution to circumvent the problem by widening the income distribution shares, as shown in Mian et al. (2021a) and Mian et al. (2021b). As a result, the income shares used to define income inequality are the top 10% of the distribution, the next 40%, and the bottom 50%. At the end, to properly compare high and low-income households, we must eliminate the so-called "age effect", which confuses the true impact of income over savings. In other words, it is reasonable to believe that households in their middle years earn more than those in their early years, and thus the three income shares are biased by the age profile. It may happen that into the top 10% share, there are more people in their middle-age who earn more, respect to the mid 40% or the bottom 50%. To get around the problem, we decompose the income shares "within birth cohort," as Mian et al. (2021b) does. In this way, the true effect of income is clear. Otherwise, the impact on saving behaviour would be caused not only by that share's income but also by its specific age profile. This is exactly what we want to keep distinct in the analysis.

4.4 Results

4.4.1 Saving rates across income and age groups

Figure 4.3 is an average of the full sample 1980-2020, based on SHIW annualized savings rates by income (within birth-cohort) and age groups. Looking at the figure, we can see how the theories underlying our analysis are upheld. The left-panel depicts how wealthy households tend to save more (given their specific income share) than those with a lower share of income, as documented by Dynan et al. (2004). Furthermore, because the analysis is done within birth cohorts, the differences in income between groups are not influenced by life cycle factors. This means that higher-income households save a greater proportion of their income than lower-income households, even if they are in the same period of the life cycle. The Top 10% saved a rate that is 11% and 31% higher than the Mid 40% and Bottom 50%, respectively. This demonstrates that the bulk of the inequality primarily impacted the bottom of the distribution rather than the middle class. In fact, the difference between the two inequalities is about 20%. The right-panel, on the other hand, is a demonstration of Ando and Modigliani (1963)'s life cycle theory. People of working age tend to save more respect than those in their early and late stages of life. Looking at both

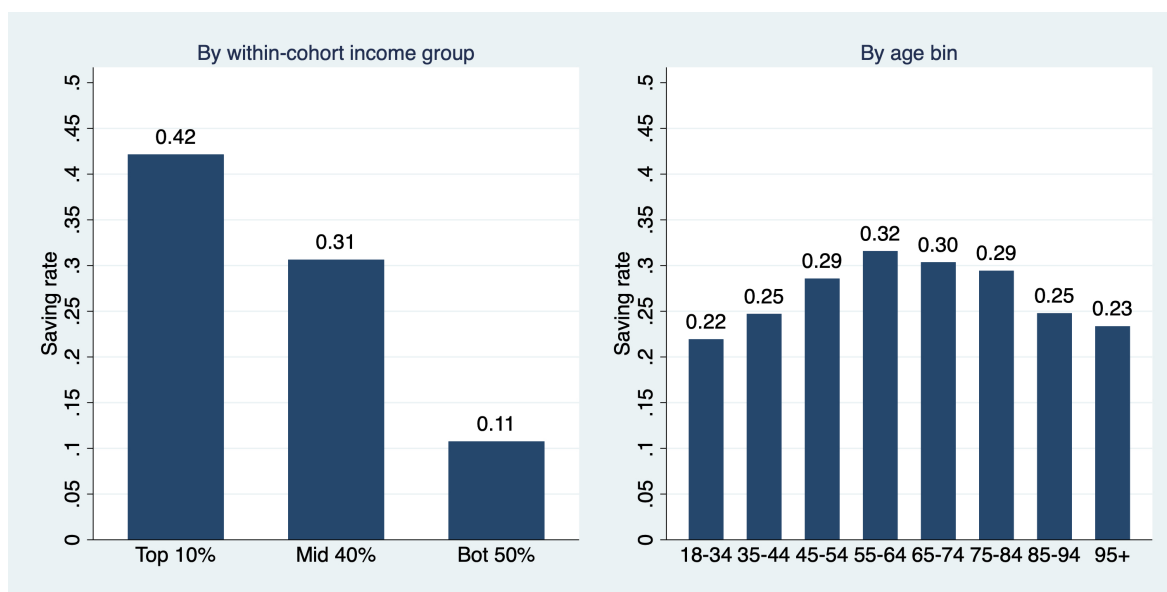


FIGURE 4.3: Within-cohort Income Distribution (left) and Age Distribution (right). Sample period: average annualized saving rates 1980-2020.

distributions, we can see that the differences in saving rates across the age distribution are much less prominent than those found in the income distribution. Indeed, the maximum difference between age bins is 10 %, between the 55-64 and 18-34 age groups.

To summarise, given the greater differences between income groups relative to age groups, a change in the income shares within the income distribution is reasonable to have a greater effect on savings which should go in favor of the thesis of Mian et al. (2021b). As a result, our initial hypothesis in favor of a greater power in population ageing could be confirmed only by a large change in the income shift across the age distribution. Differences in saving rates are too small between age groups (especially when compared to the difference between the Top 10% and the Bottom 50%) to offset income inequality effects, without a large shift in the income shares across the age distribution.

4.4.2 Changes in income shares

As equation 4.18 makes clear, the shift of the income share among the various income and age groups is an important driver of saving's dynamics. Figure 4.4 shows that the income shares among the income inequality distribution had an increase

but not as important as the shift of the income shares among the population ageing distribution. It turns out that people 60+ years old are the ones that gained the higher share of income from the 1980 to today, at the expense of the youngest generations who are the most affected by the lost of income. We turn to the results of the Shift-Share analysis to investigate this point.

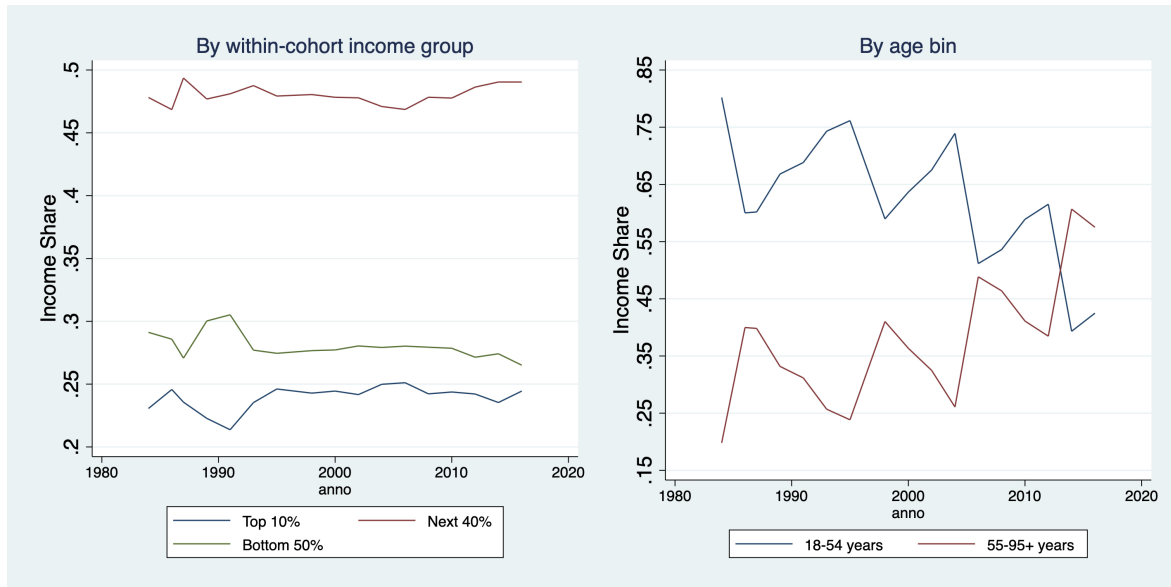


FIGURE 4.4: Income shares along the income distribution (left) and among birth cohorts with median age between 18-54 years and 55-95+ years (right) over time.

4.4.3 Shift-Share Analysis

We anticipated the potential greater effect of the income inequality factor due to greater differences in saving rates across the income distribution, unless a significant change in income across the age distribution occurs. Figure 4.5 shows the failure of this potential effect. We remind that the main object of the analysis is the Shift-Share component, because it represents the effect of the change in income towards a specific groups (which in our case are income inequality and population ageing). Looking at the Shift-Share component (red bars) of income inequality (left-side) and population ageing (right-side), the latter appears to be the most important factor in explaining the dynamics of private saving over income between 1980 and 2020. What exactly happened is a large portion of income shift to retired households (65+ years) producing a change in the ratio ($\frac{S^{pr}}{NI}$) which sums up to 3.9% instead of

the 0.5% produced by the Top 10% of the income distribution. More specifically, the methodology would have predicted an increase in private savings of 3.9% of national income due to population ageing if saving rates had remained constant over time, instead of the 0.5% predicted by income inequality. Moreover, considering also the change in the preferences of the saving rate, defined by the Residual component (green bars), the total change in the ratio $\frac{Spr}{NI}$ produced by population ageing (65+ years) sums up to 3.4% versus the contribution of 0.7% by the Top 10% of the income distribution. To make clear the point, we look at both exercises separately, making

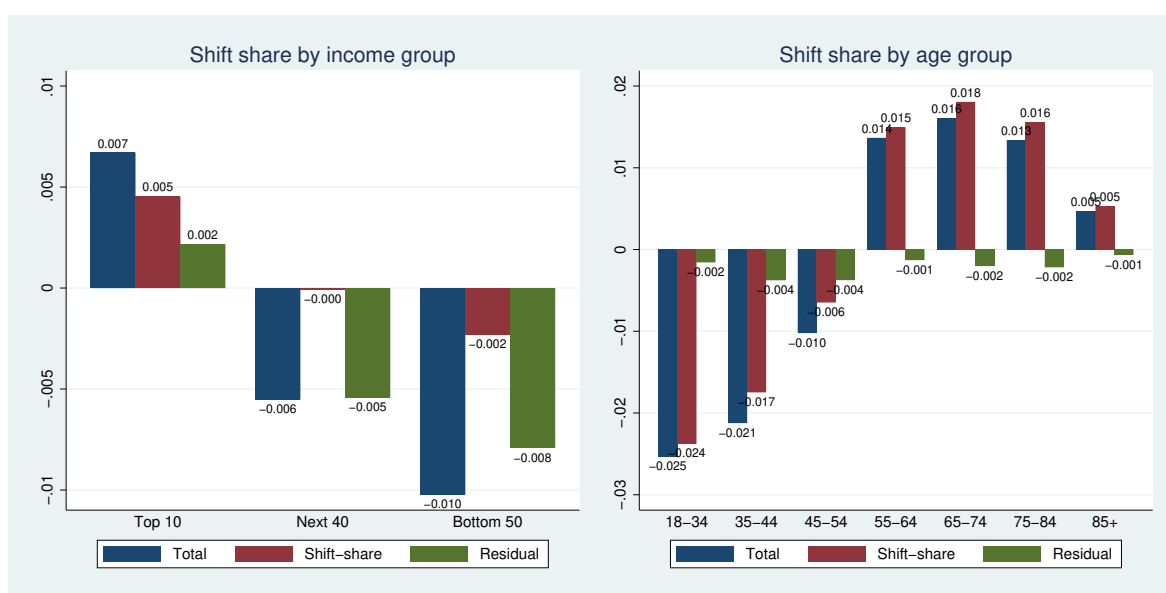


FIGURE 4.5: Shift-Share by Income Group (left) and Shift-Share by Age Group (right). Sample period: average (2000-2020) - average (1980-2000).

use of the Table 4.2 and Table 4.3. The reason why income inequality, given its large difference in saving rates (s^{10}, s^{40}, s^{50}), as depicted in Figure 4.3, didn't produce a large effect over private saving is that the Income Shift towards the Top 10% (the share that save the most), has been just of 0.9%, therefore very limited (Table 4.2). Instead, a great Income Shift towards the most aged share of the population (65+ years) occurred, with a magnitude that averaged around 3% (Table 4.3).

Overall, the impact of population ageing seems to be the most relevant factor relative to income inequality in explaining the dynamics of private saving over income in Italy, between 1980 and 2020. The dissaving of the baby-boom generation which would have occurred during the period $t = T$, 2000-2020, didn't happen and good insights can be traced out looking at Carvalho et al. (2016) and DeNardi and Borella

Income Group	Income Shift ($\alpha_T - \alpha_0$)	Shift-Share $s_0 \cdot (\alpha_T - \alpha_0)$	Δ SavRate ($s_T - s_0$)	Residual ($s_T - s_0$) $\cdot \alpha_T$	Total
Top 10%	0,009	0,005	0,009	0,002	0,007
Mid 40%	-0,000	0,000	-0,011	-0,011	-0,006
Bot 50%	-0,009	-0,002	-0,029	-0,029	-0,010
	s_0	s_T	α_0	α_T	
Top 10%	0,237	0,245	0,521	0,529	
Mid 40%	0,481	0,480	0,434	0,423	
Bot 50%	0,283	0,274	0,273	0,244	

TABLE 4.2: Shift-Share Analysis by Income Groups

Age Group	Income Shift ($\alpha_T - \alpha_0$)	Shift-Share $s_0 \cdot (\alpha_T - \alpha_0)$	Δ SavRate ($s_T - s_0$)	Residual ($s_T - s_0$) $\cdot \alpha_T$	Total
18-34	-0,065	-0,024	-0,020	-0,002	-0,025
35-44	-0,045	-0,017	-0,019	-0,004	-0,021
45-54	-0,016	-0,006	-0,014	-0,004	-0,010
55-64	0,034	0,015	-0,006	-0,001	0,014
65-74	0,042	0,018	-0,015	-0,002	0,016
75-84	0,036	0,016	-0,032	-0,002	0,013
85+	0,013	0,005	-0,038	-0,001	0,005
	s_0	s_T	α_0	α_T	
18-34	0,145	0,080	0,368	0,348	
35-44	0,244	0,199	0,389	0,370	
45-54	0,283	0,268	0,417	0,403	
55-64	0,195	0,229	0,438	0,432	
65-74	0,096	0,138	0,431	0,416	
75-84	0,032	0,068	0,431	0,400	
85+	0,004	0,017	0,410	0,372	

TABLE 4.3: Shift-Share Analysis by Age Groups

(2020). The longer life-expectancy at birth, the pressure over public finance and pension system and, the health-related expenses compressed in our final years, could be some good explanations behind the observed missing dissaving of the baby-boom generation.

4.4.4 Intergenerational Inequality

What come out from the analysis is an overall decline of the ratio between private saving and national income. If we sum the Total components (blue bars) in Figure 4.5 we see that the difference amount to -0.9% between 1980-2000 and 2000-2020. Just a small portion of this decline is explained by the change in saving preferences (measured by the Residual component), both between income and age groups.⁴ Instead, the greatest portion of the decline is due to an income loss by young population (18-44 years in column 2 of Table 4.3). The Income Shift have been of -6.5% and -4.5% for the shares 18-34 and 35-44, respectively. Even without considering the effect of change in preferences (Residual component), the overall impact of the young population over the ratio between private saving and income is the greatest, indeed resulting in a Shift Share component of -2.4% and -1.7%, respectively. In other words, without the high loss of these age shares in the Income shift, the resulting change in the ratio would have been probably positive. We call this dynamics Intergenerational Inequality. Between the two periods, we observed a polarisation of income among the age distribution, in particular towards old age groups at the expense of young ones.

4.5 Conclusions

Differently to what has been found by the literature for the United States, in Italy population ageing seems to be the most relevant factor in shaping the dynamics of private savings over national income. Although at first instance, the differences in savings rates among the income distribution are higher relative to those among the age distribution, if we look in detail, decomposing between the Shift-Share and the Residual components the impact of among age distribution results dominant. The reason in the dominance of the ageing relative to the income inequality effect, is the large shift of income towards the most aged people occurred between 1980 and 2020.

This large shift has been at the expense of the younger generations, which have seen a huge lost in income, especially the groups 18-34 years. Respect to this, at least for Italy, it should be more correct to talk about intergenerational inequality, that is the polarisation of income among the age distribution, respect to purely ageing or

⁴This can be seen as the change in the time preference parameter β or in the intertemporal elasticity of consumption in a standard neoclassical setting.

income inequality effects. Investigate the causes behind this point are of particular importance: indeed, without the loss of income by the young generations, the difference in the overall private savings over income between 1980 and 2020 would have been probably positive. We leave this point for future researches.

Appendix A

Technical Details of Chapter 1

A.1 Debt Overhang Literature Review

TABLE A.1: Debt Overhang Literature Review. Authors elaboration from Salmon and Rugey (2020) and Reinhart et al. (2012a) data.

Study	Sample	Debt Type	Debt Effect	Threshold
Reinhart and Rogoff (2010)	44 countries from 1946 to 2009	Pu_D	Negative	at 90%
Kumar and Woo (2010)	38 countries from 1970 to 2007	Pu_D	Negative	at 90%
Caner et al. (2010)	99 countries from 1980 to 2008	Pu_D	Negative	at 77%
Checherita et al. (2012)	12 countries from 1970 to 2011	Pu_D	Negative	at 95%
Herndon et al. (2014)	20 countries from 1946 to 2009	Pu_D	Negative	not found
Cecchetti et al. (2010)	18 countries from 1980 to 2010	Pu_D	Negative	at 85%
Cecchetti et al. (2010)	18 countries from 1980 to 2010	Pr_D	Negative	at 85-90%
Padoan et al. (2012)	28 countries from 1960 to 2011	Pu_D	Negative	at 82-91%
Baum et al. (2013)	12 countries from 1990 to 201	Pu_D	Negative	at 95%
Alfonso and Jalles (2013)	155 countries from 1970 to 2008	Pu_D	Negative	at 59%
Ghosh et al. (2013)	23 countries from 1970 to 2007	Pu_D	Negative	at 90-100%
Kourtellos et al. (2013)	82 countries from 1980 to 2009	Pu_D	Negative	not found
Alfonso and Alves (2015)	14 countries from 1970 to 2012	Pu_D	Negative	at 75%
Topal (2014)	12 countries from 1980 to 2012	Pu_D	Negative	at 71.6-80.2%
Mencinger et al. (2014)	25 countries from 1980 to 2010	Pu_D	Negative	at 80-94%

TABLE A.1: Continued.

Study	Sample	Debt Type	Debt Effect	Threshold
Eberhardt and Presbitero (2015)	118 countries from 1961 to 2012	Pu_D	Negative	not found
Égert (2015)	44 countries from 1960 to 2010	Pu_D	Negative	at 20-60%
Brida et al. (2017)	16 countries from 1977 to 2015	Pu_D	Negative	at 90%
Chudik et al. (2017)	40 countries from 1965 to 2010	Pu_D	Negative	not found
Yolcu Karadam (2018)	134 countries from 1970 to 2012	Pu_D	Negative	at 106.6%
Caner et al. (2019)	29 countries from 1995 to 2014	$Pu_D + Pr_D$	Negative	at 137%
Jacobs et al. (2020)	31 countries from 1995 to 2013	Pu_D	No Effect	not studied
Eberhardt (2019)	27 countries from 1800 to 2010	Pu_D	No Effect	not studied
Lim (2019)	41 countries from 1952 to 2016	$Pu_D + Pr_D$	Negative	not studied
Swamy (2019)	252 countries from 1960 to 2009	Pu_D	Negative	at 110%
Arkand et al. (2011)	44 countries from 1976 to 2005	Pr_D	Negative	at 104-110%
Balassoni et al. (2011)	Italy from 1861 to 2010	Pu_D	Negative	not studied
Balassoni et al. (2011)	Italy from 1861 to 1914	Ext_D	Negative	not studied
Patillo et al. (2011)	93 countries from 1969 to 1998	Ext_D	Negative	at 35-40%

Appendix B

Technical Details of Chapter 2

B.1 Model Derivations

We follow closely Eggertsson et al. (2019) and we report the main derivations of the model to help the reader. We invite interested readers to follow the original paper for a complete description of the model's equations.

B.1.1 Demographics

The population growth rate is determined by the total fertility rate of every household (Γ) and by the probability of dying before arriving at the maximum age $J = 81$ years, which is set stochastically. The probability of surviving between age j and $j + 1$ is given by s_j and it's called conditional, instead, the probability of arriving at age j is given by s^j and it's called unconditional probability. The total population alive at any given time, N_t , is the sum of the population of the individual ages, n_t^j . The population size of a given generation n_t^j is the population of the generation the previous year that has survived, except for the generation $j = 26$ years, which is the first generation in the model. That is given by the total population of their parents which entered the economic maturity at time $t - 25$, multiplied by the total fertility rate of their parent's generation at that time (Γ_{t-25}) and discounted for the unconditional probability of survival. In sum, the total population evolves in the model according to the law of motions and aggregates given below:

$$N_t = \sum_{j=26}^J n_t^j \quad (\text{B.1})$$

$$n_{t+1}^{j+1} = s_j n_t^j \quad \text{for } j \in \{26, J - 1\} \quad (\text{B.2})$$

$$n_t^{26} = \frac{n_{t-25}^{26} \Gamma_{t-25}}{s u^{26}} \quad (\text{B.3})$$

where:

$$\Gamma_{t-25} = (1 + n_{t-25})^{\frac{1}{25}}$$

Households do not receive wage income after retirement, set at age $j = 65$. Labor is supplied inelastically, but it depends on the individual age-specific exogenous

labor productivity hc^j . Thus the total labor supply at a given time t is given by:

$$L_t = \sum_{j=26}^J n_t^j hc^j \quad (\text{B.4})$$

B.1.2 Households Problem

Each generation j of the population maximizes the following intertemporal utility function:

$$\max_{\{c_{t+j-1}^j, x_{t+j-1}^j\}} U_t = \frac{1}{(1 - \frac{1}{\gamma})} \left[\left(\sum_{j=26}^J su^j \beta^{j-1} u(c_{t+j-1}^j) \right) + su^J \beta^{J-1} \mu v(x_{t+J-1}^J) \right]$$

subject to:

$$\begin{aligned} c_t^j + \zeta_t a_{t+1}^{j+1} + \Gamma_{t-j+26}^{26} x_t^j &= (1 - \tau^w) w_t hc^j + \pi_t^j + \dots \\ \dots + (r_t^k + \zeta_t(1 - \delta)) \cdot \left(a_t^j + q_{t+1}^{j+1} + \frac{1 - sv_j}{sv_j} a_t^j \right) & \end{aligned} \quad (\text{B.5})$$

$$a_t^j \geq \frac{D_t}{1 + r_t} \quad (\text{B.6})$$

$$c_t^j \geq 0 \quad (\text{B.7})$$

$$a_t^{26} = 0 \quad (\text{B.8})$$

$$a_t^{J+1} = 0 \quad (\text{B.9})$$

$$q_t^j = \frac{n_{t-1}^J x_{t-1}^J \Gamma_{t-J+26}^{26}}{n_t^{57}} \quad (\text{B.10})$$

where:

$$su^j = \prod_{m=26}^{j-1} sv_m$$

$$D_t^j \leq 0 \quad \text{for } j \leq 65$$

$$D_t^j = 0, hc^j = 0, \pi_t^j = 0 \quad \text{for } j > 65$$

$$q_t^j = 0 \quad \text{for } j \neq 57$$

$$x_t^j = 0 \quad \text{for } j \neq 81$$

The utility and bequest are CES function:

$$u(c_{t+j-1}^j) = (c_{t+j-1}^j)^{(1-\frac{1}{\gamma})}$$

$$v(x_{t+J-1}^J) = (x_{t+J-1}^J)^{(1-\frac{1}{\gamma})}$$

The non-negativity constraint for consumption B.7 can be omitted. Substituting the consumption c_{t+j-1}^j into the utility function C.1 using the equality constraint B.5, using the financial (occasionally binding) constraint B.6 and taking care of all the other conditions, we can form the lagrangian to be maximized as follows:

$$\begin{aligned} \max_{\{a_{t+j}^{j+1}, x_{t+j-1}^j, \lambda_{t+j-1}^j\}} \mathcal{L}_t = & \frac{1}{(1-\frac{1}{\gamma})} \left\{ \sum_{j=26}^J \left(\prod_{m=26}^{j-1} sv_m \right) \cdot \beta^{j-1} \cdot \dots \right. \\ & \left[-\xi_t a_{t+j}^{j+1} - \Gamma_{t-j+26}^{26} x_t^j + (1-\tau)w_t hc^j + \pi_{t+j-1}^j + \dots \right. \\ & \left. \dots + (rk_t + \xi_t(1-\delta)) \cdot \left(a_t^j + q_{t+1}^{j+1} + \frac{1-sv_j}{sv_j} a_t^j \right) \right]^{1-\frac{1}{\gamma}} \left. \right\} + \dots \\ & \dots + \frac{1}{(1-\frac{1}{\gamma})} \left\{ \left(\prod_{m=26}^{J-1} sv_m \right) \cdot \beta^{J-1} \mu \left[x_{t+J-1}^J \right]^{1-\frac{1}{\gamma}} \right\} + \dots \\ & \dots + \sum_{j=26}^J \lambda_{t+j-1}^j \left(a_{t+j-1}^j - \frac{D_t^j}{1+r_t} \right) \end{aligned}$$

subject to:

$$a_t^{26} = 0$$

$$a_t^{J+1} = 0$$

$$q_t^j = \frac{n_{t-1}^J x_{t-1}^J \Gamma_{t-J+26}^{26}}{n_t^{57}}$$

where:

$$D_t^j \leq 0 \quad \text{for } j \leq 40$$

$$D_t^j = 0, hc^j = 0, \pi_t^j = 0 \quad \text{for } j > 40$$

$$q_t^j = 0 \quad \text{for } j \neq 57$$

$$x_t^j = 0 \quad \text{for } j \neq 81$$

Deriving with respect to a_{t+j}^{j+1} , x_{t+j-1}^j and considering the complementary slackness conditions, we get the first-order conditions (FOCs):

$$\bullet \frac{\partial \mathcal{L}}{\partial a_{t+j}^{j+1}} :$$

$$su^j \beta^{j-1} (c_{t+j-1}^j)^{-\frac{1}{\gamma}} \cdot -\xi_t + su^{j+1} \beta^j (c_{t+j}^{j+1})^{-\frac{1}{\gamma}} \frac{(rk_{t+1} + \xi_{t+1}(1-\delta))}{sv_j} + \lambda_{t+j}^{j+1} = 0$$

$$\text{for } j \in \{26, \dots, 80\}$$

$$su^j \beta^{j-1} (c_{t+j-1}^j)^{-\frac{1}{\gamma}} \cdot 0 = 0$$

$$\text{for } j \in \{81\}$$

$$\bullet \frac{\partial \mathcal{L}}{\partial x_{t+j-1}^j} :$$

$$su^j \beta^{j-1} (c_{t+j-1}^j)^{-\frac{1}{\gamma}} \cdot -\Gamma_{t-j+1}^{26} + su^j \beta^{j-1} \mu (x_{t+j-1}^j)^{-\frac{1}{\gamma}} = 0$$

$$\text{for } j \in \{81\}$$

• *Slackness conditions:*

$$\lambda_{t+j-1}^j \left(a_{t+j-1}^j - \frac{D_t^j}{1+r_t} \right) = 0$$

$$\text{for } j \in \{26, \dots, 65\} \text{ and, } a_{t+j-1}^j \geq 0$$

$$\lambda_{t+j-1}^j \left(a_{t+j-1}^j \right) = 0$$

$$\text{for } j \in \{66, \dots, 81\} \text{ and, } a_{t+j-1}^j \geq 0$$

Rewriting using the no-arbitrage condition below (equation B.28) we get:

- $\frac{\partial \mathcal{L}}{\partial a_{t+j}^{j+1}}$:

$$\frac{1}{\beta} = \left(\frac{c_{t+1}^{j+1}}{c_t^j} \right)^{-\frac{1}{\gamma}} \cdot (1 + r_{t+1}) + \lambda_t^{j+1} \cdot \frac{sv_t^{j+1} (c_t^j)^{\frac{1}{\gamma}}}{su_t^j \beta^j \xi_t} \quad (\text{B.11})$$

for $j \in \{26, \dots, 80\}$

- $\frac{\partial \mathcal{L}}{\partial x_{t+j-1}^j}$:

$$x_{t+80}^{81} = \left(\frac{\Gamma_{t-80}^{26}}{\mu} \right)^{-\gamma} c_{t+80}^{81} \quad (\text{B.12})$$

for $j \in \{81\}$

- *Slackness conditions:*

$$\lambda_{t+j-1}^j \left(a_{t+j-1}^j - \frac{D_t^j}{1 + r_t} \right) = 0 \quad (\text{B.13})$$

for $j \in \{26, \dots, 65\}$ and, $a_{t+j-1}^j \geq 0$

$$\lambda_{t+j-1}^j \left(a_{t+j-1}^j \right) = 0 \quad (\text{B.14})$$

for $j \in \{66, \dots, 81\}$ and, $a_{t+j-1}^j \geq 0$

We follow Swarbrick (2021, p. 8) and we summarise conditions B.6, B.13 and B.14 making use of the minimum function to handle the financial OBCs. The resulting two expressions are the following:

$$\min \left(\lambda_{t+j-1}^j a_{t+j-1}^j - \frac{D_t^j}{1+r_t} \right) = 0 \quad (\text{B.15})$$

for $j \in \{26, \dots, 65\}$

$$\min \left(\lambda_{t+j-1}^j a_{t+j-1}^j \right) = 0 \quad (\text{B.16})$$

for $j \in \{66, \dots, 81\}$

B.1.3 Firms Problem

Final Goods Firms

The final goods firms choose real prices $\frac{p_t(i)}{P_t}$ to maximize real profits:

$$\max_{\left\{ \frac{p_t(i)}{P_t} \right\}} \Pi_t = \frac{p_t(i)}{P_t} y_t^f(i) - \frac{p_t^{int}}{P_t} y_t^f(i)$$

subject to the following demand curve constraint:

$$y_t^f(i) = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\theta_t}$$

where θ_t is a time-varying shock to the firm's market power. An increase in θ_t decreases a firm's market power and lowers equilibrium markups. Then, the lagrangian is given by:

$$\max_{\left\{ \frac{p_t(i)}{P_t} \right\}} \mathcal{L}_t = \frac{p_t(i)}{P_t} Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\theta_t} - \frac{p_t^{int}}{P_t} Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\theta_t}$$

Deriving with respect to $\frac{p_t(i)}{P_t}$ we get the first-order condition (FOC):

$$\frac{p_t(i)}{P_t} = \frac{\theta_t}{\theta_t - 1} \frac{p_t^{int}}{P_t} \quad (\text{B.17})$$

The nominal price index implies the following expression for the price of intermediate goods:

$$P_t = \left(\int p_t(i)^{1-\theta_t} di \right)^{\frac{1}{1-\theta_t}}$$

Since the price of intermediate good is the same, all final goods firms make the same pricing decisions (no pricing frictions), and thus $p_t(i) = P_t$, yielding to:

$$\frac{p_t^{int}}{P_t} = \frac{\theta_t - 1}{\theta_t} \quad (\text{B.18})$$

Substituting, $\frac{p_t^{int}}{P_t}$, $\frac{p_t(i)}{P_t}$, $y_t^f(i)$ into Π_t we get the aggregate profit:

$$\Pi_t = \frac{Y_t}{\theta_t} \quad (\text{B.19})$$

Profits from monopolistically competitive firms are distributed according to wage income, $\pi_t^j = hc^j \frac{\Pi_t}{L_t}$. In equilibrium, the total distributed profit must equal total profits:

$$\Pi_t = \sum_{j=26}^{65} n_t^j \pi_t^j \quad (\text{B.20})$$

Intermediate Goods Firms

This is a perfectly competitive market in which intermediate firms rent capital K_t from the capital market at rk_t , hire labor L_t from the labor market at w_t , and sell their production Y_t to the final firms at a real price $\frac{p_t^{int}}{P_t}$ taken as given. They maximize the following profit function:

$$\max_{\{K_t, L_t\}} \Pi_t^{int} = \frac{p_t^{int}}{P_t} Y_t - w_t L_t - rk_t K_t$$

subject to the production constraint, given by a CES production function:

$$Y_t = \left(\alpha (AK_t K_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) (AL_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

The lagrangian for the Intermediate Firms problem is:

$$\max_{\{K_t, L_t\}} \mathcal{L}_t = \frac{p_t^{\text{int}}}{P_t} \left(\alpha (AK_t \cdot K_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) (AL_t \cdot L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} - w_t L_t - rk_t K_t$$

Deriving with respect to L_t, K_t we get the first-order conditions (FOCs):

$$w_t = \frac{p_t^{\text{int}}}{P_t} (1-\alpha) (AL_t)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{L_t} \right)^{\frac{1}{\sigma}} \quad (\text{B.21})$$

$$rk_t = \frac{p_t^{\text{int}}}{P_t} (\alpha) (AK_t)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{K_t} \right)^{\frac{1}{\sigma}} \quad (\text{B.22})$$

$$Y_t = \left(\alpha (AK_t \cdot K_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) (AL_t \cdot L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad (\text{B.23})$$

Taking w_t as a numeraire, we define $A_{adj} = w$ as a parameter at its steady-state value, and we divide w_t, rk_t, Y_t for A_{adj} to get:

$$A_{adj} = \frac{p_t^{\text{int}}}{P_t} (1-\alpha) (AL_t)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{L_t} \right)^{\frac{1}{\sigma}} \quad (\text{B.24})$$

$$w_t = \frac{\frac{p_t^{\text{int}}}{P_t} (1-\alpha) (AL_t)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{L_t} \right)^{\frac{1}{\sigma}}}{A_{adj}} \quad (\text{B.25})$$

$$rk_t = \frac{\frac{p_t^{\text{int}}}{P_t} (\alpha) (AK_t)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{K_t} \right)^{\frac{1}{\sigma}}}{A_{adj}} \quad (\text{B.26})$$

$$Y_t = \frac{\left(\alpha (AK_t \cdot K_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) (AL_t \cdot L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}}{A_{adj}} \quad (\text{B.27})$$

Eventually, we have the no-arbitrage condition which relates the risk-free real rate with the return on capital:

$$1 + r_t = \frac{rk_t + (1-\delta)\xi_t}{\xi_{t-1}} \quad (\text{B.28})$$

B.1.4 Government

The government spends an exogenous G_t and may issue debt. The following equations describe the main government variables:

$$G_t = g \cdot Y_t \quad (\text{B.29})$$

$$T_t = \tau_t w_t L_t \quad (\text{B.30})$$

$$b_{t+1} Y_{t+1} = g_t Y_t + (1 + r_t) \cdot b_t Y_t - \tau_t w_t L_t \quad (\text{B.31})$$

$$gov_t^{rev} = g_t Y_t + r_t b_t Y_t \quad (\text{B.32})$$

$$gov_t^{deficit} = \frac{b_{t+1} Y_{t+1} - b_t Y_t}{gov_t^{rev}} \quad (\text{B.33})$$

$$gov^{debt} = \frac{b_t Y_t}{K_t} \quad (\text{B.34})$$

B.1.5 Aggregates

Besides the other aggregates, such as [B.1](#), [3.2](#), [B.19](#) and, [C.20](#), we have:

$$C_t = \sum_{26}^J n_t^j c_t^j \quad (\text{B.35})$$

$$\xi_t \cdot K_t = \left(\sum_{j=26}^J \xi_t n_t^j a_{t-1}^j \right) - b_t Y_t \quad (\text{B.36})$$

Appendix C

Technical Details of Chapter 3

C.1 Model Derivations

C.1.1 Households Problem

Given that we have two populations with identical behaviour we can solve the model just for the generic population i . Each generation j of both populations ($i = n, m$) maximizes the following intertemporal utility function:

$$\max_{\{c_{t+j-1}^{j,i}\}} U_t^i = \frac{1}{(1 - \frac{1}{\gamma^i})} \left(\sum_{j=26}^J s^{j,i} \beta^{j-1,i} u(c_{t+j-1}^{j,i}) \right) \quad (\text{C.1})$$

subject to:

$$c_t^{j,i} + a_{t+1}^{j+1,i} = (1 - \tau^w) w_t h c^{j,i} + \pi_t^{j,i} + (1 + r_t) \frac{a_t^{j,i}}{s_j^i} \quad (\text{C.2})$$

$$a_t^{j,i} \geq D_t^i w_t h c^{j,i} \quad (\text{C.3})$$

$$c_t^{j,i} \geq 0, \quad a_t^{1,i} = 0, \quad a_t^{J+1,i} = 0 \quad (\text{C.4})$$

where:

$$s^{j,i} = \prod_{m=1}^{j-1} s_m^i$$

$$D_t^i \leq 0 \quad \text{for } j \leq 65$$

$$D_t^i = 0, h c^{j,i} = 0, \pi_t^{j,i} = 0 \quad \text{for } j > 65$$

The utility function form is CES:

$$u(c) = c^{1-\frac{1}{\gamma}}$$

Substitute the consumption $c_{t+j-1}^{j,i}$ into the utility function 3.3 using the equality constraint, 3.4. Then form the lagrangian equation using the inequality constraint (OBC), 3.5. The non-negativity constraint for consumption 3.6 can be omitted.

The problem to be maximized becomes:

$$\begin{aligned} \max_{\{a_{t+j}^{j+1,i}, \lambda_{t+j-1}^{j,i}\}} \mathcal{L}_t = & \frac{1}{\left(1 - \frac{1}{\gamma^i}\right)} \left\{ \sum_{j=26}^J \left(\prod_{m=26}^{j-1} s_m^i \right) \beta^{j-1} \left[-a_{t+j}^{j+1,i} + \dots \right. \right. \\ & \left. \left. \dots + (1 - \tau^w) \bar{w}_t h c^{j,i} + \pi_{t+j-1}^{j,i} + (1 + r_t) \frac{a_{t+j-1}^{j,i}}{s_j^i} \right]^{1 - \frac{1}{\gamma^i}} \right\} + \dots \\ & \dots + \sum_{j=26}^J \lambda_{t+j-1}^{j,i} \left(a_{t+j-1}^{j,i} - D_t^{j,i} \bar{w}_t h c^{j,i} \right) \end{aligned}$$

subject to:

$$a_t^{1,i} = 0, \quad a_{t+1}^{J+1,i} = 0$$

Deriving with respect to $a_{t+j}^{j+1,i}$ and considering the complementary slackness conditions, we get the first-order conditions (FOCs):

$$\bullet \frac{\partial \mathcal{L}}{\partial a_{t+j}^{j+1,i}} :$$

$$\begin{aligned} & \left(1 - \frac{1}{\gamma^i}\right) \left[\left(\prod_{m=26}^{j-1} s_m^i \right) \beta^{j-1,i} \left(1 - \frac{1}{\gamma^i}\right) (c_{t+j-1}^{j,i})^{-\frac{1}{\gamma^i}} + \dots \right. \\ & \left. \dots + \left(\prod_{m=26}^j s_m^i \right) \beta^{j,i} \left(1 - \frac{1}{\gamma^i}\right) (c_{t+j}^{j+1,i})^{-\frac{1}{\gamma^i}} \frac{(1+r_t)}{s_{j+1}^i} \right] + \lambda_{t+j}^{j+1,i} = 0 \\ & \left(s_{j+1}^i \frac{s_j^i}{s_{j+1,i}^i} \right) \left(\frac{\beta^{j-1,i}}{\beta^{j,i}} \right) = \left(\frac{c_{t+j}^{j+1,i}}{c_{t+j-1}^{j,i}} \right)^{-\frac{1}{\gamma^i}} (1+r_{t+1}) + \lambda_{t+j}^{j+1,i} \frac{s_{j+1}^i}{s_{j+1,i}^i \beta^{j,i}} \left(\frac{1}{c_{t+j-1}^{j,i}} \right)^{-\frac{1}{\gamma^i}} \\ & \frac{1}{\beta^i} = \left(\frac{c_{t+j}^{j+1,i}}{c_{t+j-1}^{j,i}} \right)^{-\frac{1}{\gamma^i}} (1+r_{t+1}) + \lambda_{t+j}^{j+1,i} \frac{(c_{t+j-1}^{j,i})^{\frac{1}{\gamma^i}}}{s_j^i \beta^i} \end{aligned} \tag{C.5}$$

for $j \in \{26, \dots, 81\}$

• *Slackness conditions:*

$$\lambda_{t+j-1}^{j,i} \left(a_{t+j-1}^{j,i} - D_t^{j,i} w_t h c^{j,i} \right) = 0 \quad (\text{C.6})$$

for $j \in \{26, \dots, 65\}$ and, $a_{t+j-1}^j \geq 0$

$$\lambda_{t+j-1}^{j,i} \left(a_{t+j-1}^{j,i} \right) = 0 \quad (\text{C.7})$$

for $j \in \{66, \dots, 81\}$ and, $a_{t+j-1}^{j,i} \geq 0$

We follow Swarbrick (2021, p. 8) and we summarise conditions B.6, C.6 and, C.7 making use of the minimum function to handle the financial OBCs. The resulting two expressions are the following:

$$\min \left(\lambda_{t+j-1}^{j,i}, a_{t+j-1}^{j,i} - D_t^{j,i} w_t h c^{j,i} \right) = 0 \quad (\text{C.8})$$

for $j \in \{26, \dots, 65\}$

$$\min \left(\lambda_{t+j-1}^{j,i}, a_{t+j-1}^{j,i} \right) = 0 \quad (\text{C.9})$$

for $j \in \{66, \dots, 81\}$

C.1.2 Firms Problem

Final Goods Firms

The final goods firms choose real prices $\frac{p_t(i)}{P_t}$ to maximize real profits:

$$\max_{\left\{ \frac{p_t(i)}{P_t} \right\}} \Pi_t = \frac{p_t(i)}{P_t} y_t^f(i) - \frac{p_t^{int}}{P_t} y_t^f(i)$$

subject to the following demand curve constraint:

$$y_t^f(i) = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\theta_t}$$

where θ_t is a time-varying shock to the firm's market power. An increase in θ_t decreases a firm's market power and lowers equilibrium markups. Then, the lagrangian is given by:

$$\max_{\left\{\frac{p_t(i)}{P_t}\right\}} \mathcal{L}_t = \frac{p_t(i)}{P_t} Y_t \left(\frac{p_t(i)}{P_t}\right)^{-\theta_t} - \frac{p_t^{int}}{P_t} Y_t \left(\frac{p_t(i)}{P_t}\right)^{-\theta_t}$$

Deriving with respect to $\frac{p_t(i)}{P_t}$ we get the first-order condition (FOC):

$$\frac{p_t(i)}{P_t} = \frac{\theta_t}{\theta_t - 1} \frac{p_t^{int}}{P_t} \quad (\text{C.10})$$

The nominal price index implies the following expression for the price of intermediate goods:

$$P_t = \left(\int p_t(i)^{1-\theta_t} di \right)^{\frac{1}{1-\theta_t}}$$

Since the price of intermediate good is the same, all final goods firms make the same pricing decisions (no pricing frictions), and thus $p_t(i) = P_t$, yielding to:

$$\frac{p_t^{int}}{P_t} = \frac{\theta_t - 1}{\theta_t} \quad (\text{C.11})$$

Substituting, $\frac{p_t^{int}}{P_t}$, $\frac{p_t(i)}{P_t}$, $y_t^f(i)$ into Π_t we get the aggregate profit:

$$\Pi_t = \frac{Y_t}{\theta_t} \quad (\text{C.12})$$

Profits from monopolistically competitive firms are distributed according to wage income, $\pi_t^{j,i} = hc_{j,i} \frac{\Pi_t}{L_t}$. In equilibrium, the total distributed profit must equal total profits:

$$\Pi_t = \sum_{j=26}^{65} n_{j,t} \pi_{j,t} \quad (\text{C.13})$$

Intermediate Goods Firms

This is a perfectly competitive market in which intermediate firms rent capital K_t from the capital market at rk_t , hire labor L_t from the labor market at w_t , and sell their production Y_t to the final firms at a real price $\frac{p_t^{int}}{P_t}$ taken as given. They maximize the following profit function:

$$\max_{\{K_t, L_t\}} \Pi_t^{int} = \frac{p_t^{int}}{P_t} Y_t - w_t L_t - rk_t K_t$$

subject to the production constraint, given by a CES production function:

$$Y_t = \left(\alpha (K_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) (AL_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

The lagrangian for the Intermediate Firms problem is:

$$\max_{\{K_t, L_t\}} \mathcal{L}_t = \frac{p_t^{int}}{P_t} \left(\alpha (K_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) (AL_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} - w_t L_t - rk_t K_t$$

Deriving with respect to L_t, K_t we get the first-order conditions (FOCs):

$$w_t = \frac{p_t^{int}}{P_t} (1-\alpha) (AL_t)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{L_t} \right)^{\frac{1}{\sigma}} \quad (\text{C.14})$$

$$rk_t = \frac{p_t^{int}}{P_t} (\alpha) \left(\frac{Y_t}{K_t} \right)^{\frac{1}{\sigma}} \quad (\text{C.15})$$

$$Y_t = \left(\alpha (K_t)^{\frac{\sigma-1}{\sigma}} + (1-\alpha) (AL_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad (\text{C.16})$$

Taking w_t as a numeraire, we define $A_{adj} = w$ as a parameter at its steady-state value, and we divide w_t, rk_t, Y_t for A_{adj} to get:

$$A_{adj} = \frac{p_t^{\text{int}}}{P_t} (1 - \alpha) (AL_t)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{L_t} \right)^{\frac{1}{\sigma}} \quad (\text{C.17})$$

$$w_t = \frac{\frac{p_t^{\text{int}}}{P_t} (1 - \alpha) (AL_t)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{L_t} \right)^{\frac{1}{\sigma}}}{A_{adj}} \quad (\text{C.18})$$

$$rk_t = \frac{\frac{p_t^{\text{int}}}{P_t} (\alpha) \left(\frac{Y_t}{K_t} \right)^{\frac{1}{\sigma}}}{A_{adj}} \quad (\text{C.19})$$

$$Y_t = \frac{\left(\alpha (K_t)^{\frac{\sigma-1}{\sigma}} + (1 - \alpha) (AL_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}}{A_{adj}} \quad (\text{C.20})$$

Eventually, we have the no-arbitrage condition which relates the risk-free real rate with the return on capital:

$$1 + r_t = \frac{rk_t + (1 - \delta)\xi_t}{\xi_{t-1}} \quad (\text{C.21})$$

C.1.3 Stationary Equilibrium

A stationary equilibrium is found by removing the time dependence t from all equations, therefore all the variables are kept constant rather than subscripted by time. Moreover, having three types of explosive processes, that is their growth rate is > 0 , that make the economy not stationary, we scaled some variables to ensure a balanced growth path (BGP).¹ Aggregate variables are scaled for productivity and population growth $((1 + n^n)(1 + n^m)(1 + AL))^t$, cohort level variables as well as wages are divided by productivity growth $(1 + AL)^t$. The stationary equilibrium is represented by about 520 equations, the majority of which are from the household sectors.

¹Those are the fertility rates (Γ^n, Γ^m) which imply the population growth rates (n^n, n^m) and the labor-productivity growth rate (AL) .

Appendix D

Technical Details of Chapter 4

D.1 Proof of the Shift-Share Decomposition

We start from equation 4.17, we simplify the notation with $S^{pr} = S$ and $NI = Z$, and we define $\alpha^j = \frac{Z^j}{Z}$. We get:

$$\frac{S_T}{Z_T} - \frac{S_0}{Z_0} = \sum_j \left(\frac{Z_T^j}{Z_T} - \frac{Z_0^j}{Z_0} \right) \frac{S_0^j}{Z_0^j} + \sum_j \frac{Z_T^j}{Z_T} \left(\frac{S_T^j}{Z_T^j} - \frac{S_0^j}{Z_0^j} \right) \quad (\text{D.1})$$

$$= \sum_j \left(\frac{Z_T^j}{Z_T} - \frac{Z_0^j}{Z_0} \right) \frac{S_0^j}{Z_0^j} + \sum_j \frac{Z_T^j}{Z_T} \left(\frac{S_T^j}{Z_T^j} - \frac{S_0^j}{Z_0^j} \right)$$

$$= \sum_j \left(\frac{Z_T^j}{Z_T} \frac{S_0^j}{Z_0^j} - \frac{S_0^j}{Z_0} \right) + \sum_j \left(\frac{S_T^j}{Z_T} - \frac{Z_T^j}{Z_T} \frac{S_0^j}{Z_0^j} \right)$$

$$= \sum_j \left(\frac{S_T^j}{Z_T} - \frac{S_0^j}{Z_0} \right) + \sum_j \left(\frac{Z_T^j}{Z_T} \frac{S_0^j}{Z_0^j} - \frac{Z_T^j}{Z_T} \frac{S_0^j}{Z_0^j} \right)$$

$$= \frac{S_T}{Z_T} - \frac{S_0}{Z_0} + 0$$

$$\frac{S_T}{Z_T} - \frac{S_0}{Z_0} = \frac{S_T}{Z_T} - \frac{S_0}{Z_0} \quad (\text{D.2})$$

Bibliography

- Acciari, P., Alvaredo, F., and Morelli, S. (2021). *The Concentration of Personal Wealth in Italy 1995 - 2016*. Working Papers. Ministry of Economy and Finance, Department of Finance.
- Adjemian, S., Bastani, H., Juillard, M., Karamé, F., Mihoubi, F., Mutschler, W., Pfeifer, J., Ratto, M., Rion, N., and Villemot, S. (2022). *Dynare: Reference Manual Version 5*. Dynare Working Papers 72. CEPREMAP.
- Alves, J. and Morgado, S. (2022). *Secular Stagnation: Is Immigration part of the solution?* Working Papers REM 2022/0212. ISEG - Lisbon School of Economics and Management, REM, Universidade de Lisboa.
- Ando, A. and Modigliani, F. (1963). "The "Life Cycle" Hypothesis of Saving: Aggregate Implications and Tests". *The American Economic Review*. Vol. 53, no. 1, pp. 55–84.
- Annicchiarico, B., Dio, F. D., Felici, F., and Monteforte, L. (2013). *IGEM: a Dynamic General Equilibrium Model for Italy*. Working Papers 4. Ministry of Economy and Finance, Department of Finance.
- Auerbach, A. and Kotlikoff, L. (1987). "Evaluating Fiscal Policy with a Dynamic Simulation Model". *American Economic Review*. Vol. 77, no. 2, pp. 49–55.
- Baldini, M. and Mazzaferro, C. (2000). *Transizione demografica e formazione del risparmio delle famiglie italiane*. Working Papers 366. Università di Bologna, Dipartimento Scienze Economiche.
- Baran, P. A. and Sweezy, P. M. (1966). *Monopoly Capital: An Essay on the American Economic and Social Order*. NYU Press.
- Bernanke, B. (2005). *The global saving glut and the U.S. current account deficit*. Speech. Board of Governors of the Federal Reserve System.
- (2010). *Implications of the Financial Crisis for Economics*. Speech. Princeton University.
- (2015a). "Why are interest rates so low?" *Brookings Blog*.
- (2015b). "Why are interest rates so low, part 2: Secular stagnation". *Brookings Blog*.

- Bernanke, B. (2015c). "Why are interest rates so low, part 3: The Global Savings Glut". *Brookings Blog*.
- Bernanke, B. and Mishkin, F. (1997). "Inflation Targeting: A New Framework for Monetary Policy?" *The Journal of Economic Perspectives*. Vol. 11, no. 2, pp. 97–116.
- Blanchard, O., Furceri, D., and Pescatori, A. (2014). "A Prolonged Period of Low Real Interest Rates?" *Secular stagnation: facts, causes and cures*. Vol. 8, p. 101.
- Blanchard, O. and Tirole, J. (2021). *Major Future Economic Challenges*. Tech. rep. France Stratégie.
- Borio, C. (2017). "Secular stagnation or financial cycle drag?" *Business Economics*. Vol. 52, no. 2, pp. 87–98.
- (2019). *A Tale of Two Financial Cycles: Domestic and Global*. Speech. University of Zürich.
- Borio, C., Disyatat, F. P., and Juselius, M. (2013). *Rethinking potential output: Embedding information about the financial cycle*. BIS Working Papers 404. Bank for International Settlements.
- Borio, C., Disyatat, P., Juselius, J., and Rungcharoenkitkul, P. (2017). *Why so low for so long? A long-term view of real interest rates*. BIS Working Papers 685. Bank for International Settlements.
- Borio, C., Kharroubi, E., Upper, C., and Zampolli, F. (2016). *Labour reallocation and productivity dynamics: financial causes, real consequences*. BIS Working Papers 534. Bank for International Settlements.
- Bornhorst, F. (2014). "Chapter 2. Growth and the Importance of Sequencing Debt Reductions across Sectors". *Jobs and Growth*. International Monetary Fund.
- Bosworth, B., Burtless, G., and Sabelhaus, J. (1991). "The Decline in Saving: Evidence from Household Surveys". *Brookings Papers on Economic Activity*. Vol. 22, no. 1, pp. 183–256.
- Caballero, R. (2006). *On the Macroeconomics of Asset Shortages*. NBER Working Papers 12753. National Bureau of Economic Research.
- Caballero, R. J. and Farhi, E. (2018). "The Safety Trap". *The Review of Economic Studies*. Vol. 85, no. 1, pp. 223–274.
- Caballero, R. J., Farhi, E., and Gourinchas, P.-O. (2017). "The Safe Assets Shortage Conundrum". *Journal of Economic Perspectives*. Vol. 31, no. 3, pp. 29–46.

- Calligaris, S., Del Gatto, M., Hassan, F., Ottaviano, G., and Schivardi, F. (2016). *Italy's Productivity Conundrum. A Study on Resource Misallocation in Italy*. Discussion Papers 030. European Commission, Directorate General Economic and Financial Affairs (DG ECFIN).
- Cannari, L. and D'Alessio, G. (2018). *Wealth inequality in Italy: reconstruction of 1968-75 data and comparison with recent estimates*. Questioni di Economia e Finanza (Occasional Papers) 428. Bank of Italy, Economic Research and International Relations Area.
- Cannari, L., D'Alessio, G., and Gambacorta, R. (2008). *Capital Gains and Wealth Distribution in Italy*. MPRA Paper 15108. University Library of Munich, Germany.
- Caritas (2019). "Common Home, Migration and Development in Italy". *Common Home Series*.
- Carvalho, C., Ferrero, A., and Nechio, F. (2016). "Demographics and real interest rates: Inspecting the mechanism". *European Economic Review*. Vol. 88, pp. 208–226.
- Comin, D. A., Quintana, J., Schmitz, T. G., and Trigari, A. (2020). *Measuring TFP: The Role of Profits, Adjustment Costs, and Capacity Utilization*. Working Paper 28008. National Bureau of Economic Research.
- Constantinides, G., Donaldson, J. B., and Mehra, R. (2002). "Junior Can't Borrow: A New Perspective on the Equity Premium Puzzle". *The Quarterly Journal of Economics*. Vol. 117, no. 1, pp. 269–296.
- Cooley, T. and Henriksen, E. (2018). "The demographic deficit". *Journal of Monetary Economics*. Vol. 93, pp. 45–62.
- Cooper, R. N. (2001). "Is the U.S. Current Account Deficit Sustainable? Will It Be Sustained?" *Brookings Papers on Economic Activity*. Vol. 2001, no. 1, pp. 217–226.
- Cowling, K. (1995). "Monopoly Capitalism and Stagnation". *Review of Political Economy*. Vol. 7, no. 4, pp. 430–446.
- Crafts, N. (2014). "Secular stagnation: US hypochondria, European disease?" *Secular stagnation: facts, causes and cures*. Vol. 8, pp. 91–97.
- David, P. A. (1990). "The Dynamo and the Computer: An Historical Perspective on the Modern Productivity Paradox". *The American Economic Review*. Vol. 80, no. 2, pp. 355–361.
- Del Boca, D. and Venturini, A. (2003). *Italian Migration*. IZA Discussion Papers 938. Institute of Labor Economics (IZA).

- DeNardi, M. and Borella, M. (2020). *Why Does Consumption Fluctuate in Old Age and How Should the Government Insure it?* Opportunity and Inclusive Growth Institute Working Papers 40. Federal Reserve Bank of Minneapolis.
- Di Bucchianico, S. (2019). *A critical analysis of the secular stagnation theory*. Economic Department Working Papers of Economics 0245. University Roma Tre.
- Dutt, A. K. (2005). "Steindl's Theory of Maturity and Stagnation and Its Relevance Today". *Rethinking capitalist development: essays on the economics of Josef Steindl*. Routledge, New York, NY. Pp. 55–78.
- Dynan, K., Skinner, J., and Zeldes, S. (2004). "Do the Rich Save More?" *Journal of Political Economy*. Vol. 112, no. 2, pp. 397–444.
- Edwards, S. (2005). "Is the U.S. Current Account Deficit Sustainable? If Not, How Costly Is Adjustment Likely to Be?" *Brookings Papers on Economic Activity*. Vol. 36, no. 1, pp. 211–288.
- Eggertsson, G. B., Mehrotra, N. R., and Robbins, J. A. (2019). "A Model of Secular Stagnation: Theory and Quantitative Evaluation". *American Economic Journal: Macroeconomics*. Vol. 11, no. 1, pp. 1–48.
- Eichengreen, B. (2014). "Secular Stagnation: A Review of the Issues". *Secular stagnation: facts, causes and cures*. Vol. 8, pp. 41–46.
- ESA (2010). *European system of accounts, ESA*. Tech. rep. European Commission.
- Fisher, I. (1933). "The Debt-Deflation Theory of Great Depressions". *Econometrica*. Vol. 1, no. 4, pp. 337–357.
- Foster, J. (2006). "Monopoly-Finance Capital". *Monthly Review*. Vol. 58, p. 1.
- Foster, J. B. (1987). "What Is Stagnation?" *The Imperiled Economy: Macroeconomics from a Left Perspective*. Pp. 59–70.
- (1997). "The Long Stagnation and the Class Struggle". *Journal of Economic Issues*. Vol. 31, no. 2, pp. 445–451.
- Foster, J. and McChesney, R. (2012). *The Endless Crisis: How Monopoly-Finance Capital Produces Stagnation and Upheaval from the Usa to China*. Monthly Review Press.
- Galbraith, J. K. (2004). *The Economics of Innocent Fraud : Truth for Our Time*. Penguin Canada.
- Giordano, C. and Zollino, F. (2021). "Long-Run Factor Accumulation and Productivity Trends in Italy". *Journal of Economic Surveys*. Vol. 35, no. 3, pp. 741–803.
- Gordon, R. J. (2010). *Revisiting U. S. Productivity Growth over the Past Century with a View of the Future*. NBER Working Papers 15834. National Bureau of Economic Research.

-
- (2012). *Is U.S. Economic Growth Over? Faltering Innovation Confronts the Six Headwinds*. Working Paper 18315. National Bureau of Economic Research.
- (2015). “Secular Stagnation: A Supply-Side View”. *The American Economic Review*. Vol. 105, no. 5, pp. 54–59.
- Guerrieri, L. and Iacoviello, M. (2015). “Occbin: A Toolkit for Solving Dynamic Models with Occasionally Binding Constraints Easily”. *Journal of Monetary Economics*. Vol. 70, pp. 22–38.
- Guzzardi, D., Palagi, E., Roventini, A., and Santoro, A. (2022). *Reconstructing Income Inequality in Italy: New Evidence and Tax Policy Implications from Distributional National Accounts*. LEM Papers Series 06. Sant’Anna School of Advanced Studies.
- Hansen, A. H. (1939). “Economic Progress and Declining Population Growth”. *The American Economic Review*. Vol. 29, no. 1, pp. 1–15.
- Hassan, F. and Ottaviano, G. (2013). *Productivity in Italy: The great unlearning*. Tech. rep.
- Hein, E. (2012). *The Macroeconomics of Finance-Dominated Capitalism – and its Crisis*. Edward Elgar Publishing.
- (2014). *Distribution and Growth after Keynes*. Edward Elgar Publishing.
- (2016). “Secular stagnation or stagnation policy? Steindl after Summers”. *PSL Quarterly Review*. Vol. 69, no. 276, pp. 3–47.
- (2019). “Financialisation and Tendencies Towards Stagnation: The Role of Macroeconomic Regime Changes in the Course of and After the Financial and Economic Crisis 2007–09”. *Cambridge Journal of Economics*. Vol. 43, no. 4, pp. 975–999.
- (2022). *Financialisation, Varieties of Macroeconomic Regimes and Stagnation Tendencies in a Stylised Kaleckian Model*. Working Paper 193. Hochschule für Wirtschaft und Recht Berlin, Institute for International Political Economy (IPE).
- Higgins, M. and Klitgaard, T. (2004). “Reserve accumulation: implications for global capital flows and financial markets”. *Current Issues in Economics and Finance*. Vol. 10, no. 9,
- Holden, T. D. (2016). *Computation of solutions to dynamic models with occasionally binding constraints*. EconStor Preprints 144569. Leibniz Information Centre for Economics (ZBW).
- Horwich, J. (2022). *The Complex Economics of Growing Old*. Tech. rep. Federal Reserve Bank (Minneapolis).

- Huo, Z., Levchenko, A. A., and Pandalai-Nayar, N. (2020). *Utilization-Adjusted TFP Across Countries: Measurement and Implications for International Comovement*. Working Paper 26803. National Bureau of Economic Research.
- ISTAT (2011). *Il Futuro Demografico Del Paese*. Tech. rep. ISTAT.
- Jappelli, T. and Modigliani, F. (1998). *The Age-Saving Profile and the Life-Cycle Hypothesis*. CSEF Working Papers. Centre for Studies in Economics and Finance (CSEF), University of Naples.
- Jimeno, J., Smets, F., and Yiangou, J. (2014). "Secular stagnation: A view from the Eurozone". *Secular stagnation: facts, causes and cures*. Vol. 8, pp. 153–164.
- Juselius, M., Borio, C., Disyatat, P., and Drehmann, M. (2017). "Monetary Policy, the Financial Cycle, and Ultra-Low Interest Rates". *International Journal of Central Banking*. Vol. 13, no. 3, pp. 55–89.
- Kaldor, N. (1961). "Capital Accumulation and Economic Growth". *The Theory of Capital: Proceedings of a Conference held by the International Economic Association*. Palgrave Macmillan UK, pp. 177–222.
- Kalecki, M. (1937). "The Principle of Increasing Risk". *Economica*. Vol. 4, no. 16, pp. 440–447.
- (1954). *Theory of Economic Dynamics: An Essay on Cyclical and Long-Run Changes in Capitalist Economy*. NYU Press.
- King, R. (1993). "Recent immigration to Italy: Character, causes and consequences". *GeoJournal*. Vol. 30, no. 3, pp. 283–292.
- Koo, R. (2011). "The World in Balance Sheet Recession". *Ensayos Económicos*. Vol. 1, no. 63, pp. 7–39.
- (2012). *Revitalizing the Eurozone without Fiscal Union*. Institute for New Economic Thinking Working Paper. Nomura Research Institute.
- (2014). "Balance Sheet Recession Is the Reason for Secular Stagnation". *Secular stagnation: Facts, causes and cures*. Vol. 8, p. 131.
- Kopecna, V., Scasny, M., and Recka, L. (2020). *Estimating Elasticity of Substitution in CES Production Function: Examining Different Nesting Structures and EU Regions*. Working Papers IES 43. Charles University Prague, Faculty of Social Sciences, Institute of Economic Studies.
- Krugman, P. (2009). "How Did Economists Get It so Wrong". *The New York Times*.
- (2011). "The Return Of Secular Stagnation". *The New York Times*.
- (2013). "Secular Stagnation, Coalmines, Bubbles, and Larry Summers". *The New York Times*.

- Krugman, P. R., Dominquez, K. M., and Rogoff, K. (1998). "It's Baaack: Japan's Slump and the Return of the Liquidity Trap". *Brookings Papers on Economic Activity*. Vol. 1998, no. 2, pp. 137–205.
- Lavoie, M. (1995). "Interest Rates in Post-Keynesian Models of Growth and Distribution". *Metroeconomica*. Vol. 46, no. 2, pp. 146–177.
- Lo, S. and Rogoff, K. (2015). *Secular stagnation, debt overhang and other rationales for sluggish growth, six years on*. BIS Working Papers 482. Bank for International Settlements.
- Ludwig, A., Schelkle, T., and Vogel, E. (2012). "Demographic Change, Human Capital and Welfare". *Review of Economic Dynamics*. Vol. 15, no. 1, pp. 94–107.
- Magdoff, F. and Foster, J. (2014). "Stagnation and Financialization: The Nature of the Contradiction". *Monthly Review*. Vol. 66, p. 1.
- Magdoff, H. and Sweezy, P. M. (1987). *Stagnation and the Financial Explosion*. NYU Press.
- Mann, C. L. (1999). *Is the U.S. Trade Deficit Sustainable?* Peterson Institute Press: All Books 47. Peterson Institute for International Economics.
- Mendoza, E. G. and Terrones, M. E. (2012). *An Anatomy of Credit Booms and their Demise*. Working Paper 18379. National Bureau of Economic Research.
- Mian, A., Straub, L., and Sufi, A. (2021a). *The Saving Glut of the Rich*. NBER Working Papers 26941. National Bureau of Economic Research.
- (2021b). *What explains the decline in r^* ? Rising income inequality versus demographic shifts*. Working Papers. Princeton University.
- Minsky, H. P. (1992). *The Financial Instability Hypothesis*. eng. Working Paper 74. Annandale-on-Hudson, NY.
- Mistretta, A. and Zollino, F. (2018). *Recent trends in economic activity and TFP in Italy with a focus on embodied technical progress*. Temi di discussione (Economic working papers) 1204. Bank of Italy, Economic Research and International Relations Area.
- Miyagiwa, K. and Ono, Y. (2019). *Immigration and Secular Stagnation*. ISER Discussion Paper 1054r. Institute of Social and Economic Research, Osaka University.
- Moressa, F. L. (2012). *Rapporto Annuale Sull'economia Dell'immigrazione*. Il Mulino.
- Nargund, G (2009). "Declining birth rate in Developed Countries: A radical policy re-think is required". *Facts, views & vision in ObGyn*. Vol. 1, no. 3, pp. 191–193.
- Neri, S., Ferrero, G., and Gross, M. (2017). *On secular stagnation and low interest rates: demography matters*. Temi di discussione (Economic working papers) 1137. Bank of Italy, Economic Research and International Relations Area.

- Obstfeld, M. and Rogoff, K. S. (2005). "Global Current Account Imbalances and Exchange Rate Adjustments". *Brookings Papers on Economic Activity*. Vol. 36, no. 1, pp. 67–146.
- Palley, T. (2012). "From Financial Crisis to Stagnation: The Destruction of Shared Prosperity and the Role of Economics". *From Financial Crisis to Stagnation: The Destruction of Shared Prosperity and the Role of Economics*. Cambridge University Press.
- (2019). "The fallacy of the natural rate of interest and zero lower bound economics: why negative interest rates may not remedy Keynesian unemployment". *Review of Keynesian Economics*. Vol. 7, no. 2, pp. 151–170.
- Papetti, A. (2019). *Demographics and the natural real interest rate: historical and projected paths for the euro area*. Working Paper Series 2258. European Central Bank.
- Rada, C. and Taylor, L. (2006). "Empty sources of growth accounting, and empirical replacements a la Kaldor and Goodwin with some beef". *Structural Change and Economic Dynamics*. Vol. 17, no. 4, pp. 486–500.
- Reinhart, C. M., Reinhart, V., and Rogoff, K. (2015). "Dealing with debt". *Journal of International Economics*. Vol. 96, 37th Annual NBER International Seminar on Macroeconomics, S43–S55.
- Reinhart, C. M., Reinhart, V. R., and Rogoff, K. S. (2012a). *Debt Overhangs: Past and Present*. Working Paper 18015. National Bureau of Economic Research.
- Reinhart, C. M., Reinhart, V. R., and Rogoff, K. S. (2012b). "Public Debt Overhangs: Advanced-Economy Episodes since 1800". *Journal of Economic Perspectives*. Vol. 26, no. 3, pp. 69–86.
- Reinhart, C. M. and Rogoff, K. S. (2009). *This Time Is Different: Eight Centuries of Financial Folly*. Economics Books 8973. Princeton University Press.
- Rogoff, K. (2016). "Debt Supercycle, Not Secular Stagnation". *Progress and Confusion: The State of Macroeconomic Policy*. The MIT Press.
- Ríos-Rull, J.-V. (1996). "Life-Cycle Economies and Aggregate Fluctuations". *Review of Economic Studies*. Vol. 63, no. 3, pp. 465–489.
- Sabelhaus, J. and Pence, K. (1999). "Household Saving in the '90s: Evidence from Cross-Section Wealth Surveys". *Review of Income and Wealth*. Vol. 45, no. 4, pp. 435–453.
- Salmon, J. B. and Ruggy, V. de (2020). "Debt and Growth: A Decade of Studies". *ERN: Debt; Debt Management (Topic)*.

- Samuelson, P. A. (1948). "International Trade and the Equalisation of Factor Prices". *The Economic Journal*. Vol. 58, no. 230, pp. 163–184.
- Schularick, M. and Taylor, A. M. (2012). "Credit Booms Gone Bust: Monetary Policy, Leverage Cycles, and Financial Crises, 1870-2008". *American Economic Review*. Vol. 102, no. 2, pp. 1029–61.
- Simon, H. A. and Levy, F. K. (1963). "A Note on the Cobb-Douglas Function". *Review of Economic Studies*. Vol. 30, no. 2, pp. 93–94.
- Steindl, J. (1952). "Maturity and Stagnation in American Capitalism".
- Storesletten, K. (2000). "Sustaining Fiscal Policy through Immigration". *Journal of Political Economy*. Vol. 108, no. 2, pp. 300–323.
- Storm, S. (2017). *The New Normal: Demand, Secular Stagnation and the Vanishing Middle Class*. Working Papers Series 55. Institute for New Economic Thinking.
- (2022). "Chapter Three - The secular stagnation of productivity growth". *Handbook of Economic Stagnation*. Academic Press, pp. 37–58.
- Summers, L. (2013). *IMF Economic Forum: Crises, Yesterday and Today*. Speech. IMF Fourteenth Annual Research Conference in Honor of Stanley Fisher.
- (2014). "U.S. Economic Prospects: Secular Stagnation, Hysteresis, and the Zero Lower Bound". *Business Economics*. Vol. 49, no. 2, pp. 65–73.
- (2015a). "Demand Side Secular Stagnation". *American Economic Review*. Vol. 105, no. 5, pp. 60–65.
- (2015b). *Reflections on Secular Stagnation*. Speech. Princeton University.
- Summers, L. and Carroll, C. (1987). "Why Is U.S. National Saving So Low?" *Brookings Papers on Economic Activity*. Vol. 18, no. 2, pp. 607–642.
- Swarbrick, J. M. (2021). *Occasionally Binding Constraints in Large Models: A Review of Solution Methods*. Bank of Canada Staff Discussion Paper. Bank of Canada.
- Sweezy, P. M. (1994). "The Triumph of Financial Capital". *Monthly Review*. Vol. 46, pp. 1–11.
- (2004). "Why Stagnation?" *Monthly Review*. Vol. 56, pp. 69–77.
- Taylor, L. (1985). "A stagnationist model of economic growth". *Cambridge Journal of Economics*. Vol. 9, no. 4, pp. 383–403.
- (2021). *Reconstructing Macroeconomics*. Harvard University Press.
- UN (2014). *Should International Refugee Law Accommodate Climate Change?* Tech. rep. United Nations.
- (2020). *World Population Prospects 2019*. Tech. rep. United Nations.
- Wray, L. R. and Dantas, F. (2022). *Handbook on economic stagnation*. Academic Press.