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A novel Compressive Sampling method for ECG wearable measurement systems

Francesco Picariello, Grazia Iadarola, Eulalia Balestrieri, Ioan Tudosa, Luca De $\rm Vito^1$

Department of Engineering, University of Sannio, Piazza Roma, 21, 82100 Benevento, Italy

Abstract

The paper presents a novel method for the compressed acquisition of electrocardiographic (ECG) signals. The proposed method is intended to be applied to Internet-of-Medical-Things (IoMT) acquisition nodes (i.e. wearable measurement systems) as they can benefit from a reduction of the signal data rate to be transmitted, and the consequent reduction of energy consumption. Being based on Compressive Sampling (CS), the proposed method presents a very low computational complexity on the acquisition node. Moreover, since the sensing matrix is adapted to the acquired signal, it allows achieving a better reconstruction performance compared with the other CS-based methods available in literature.

Keywords: Internet-of-Medical-Things, Wearable Measurement Systems, ECG Signal, Compressive Sampling, Sampling Method

1 1. Introduction

In recent years, wearable measurement systems for measuring physiological signals and parameters are becoming more complex due to the integration of several sensors and electronic front-ends, allowing to observe and real-time transmit signals, such as electrocardiogram (ECG) and respiration wave. Moreover, they are going to be integrated in Internet-of-Things (IoT) systems where

¹Corresponding author e-mail: devito@unisannio.it

⁷ several acquisition nodes must be connected and managed. In particular, in
⁸ this field, Internet-of-Medical-Things (IoMT) systems have been proposed to
⁹ monitor and manage healthcare applications [1, 2].

One of the main challenges of IoMT nodes is to meet the energy consumption 10 and size requirements of wearable devices. Moreover, IoMT systems must collect 11 and store data from up to thousands of nodes, resulting in a significant data 12 rate to be handled. A possible compression of this data rate must however 13 guarantee a biosignal integrity, able to comply with medical standards, when 14 it is necessary. Such challenges becomes harder as IoMT nodes are going to 15 include not only Personal Area Network (PAN), but also Wireless Wide Area 16 Network interfaces, such as Wi-Fi, LoRa, Sigfox or Narrowband-IoT (NB-IoT), 17 which are characterized by higher power consumptions and lower data rates. 18

Compression of biosignals has been proposed as a way both to reduce the 19 data rate that the IoMT node has to transmit and to reduce the energy con-20 sumption of the node [3], since in many cases data transmission is the most 21 expensive activity in terms of energy. In particular, among the different biosig-22 nals, the compression of the ECG has been faced by many researchers. A first 23 classification of research contributions regarding ECG compression can be made 24 by distinguish hardware and hybrid methods from digital methods. Methods 25 in the former category exploit the sparsity of ECG signal in the time domain 26 to design optimized architectures of analog-to-digital conversion. A method 27 falling in this category is proposed in [4], where the authors designed a level-28 crossing Analog-to-Digital Converter (ADC) to efficiently sample analog ECG 29 signals. The advantage of solutions in this category is that they provide signal 30 compression without any computational load for the microcontroller of the ac-31 quisition node. Beside that, they rely on specific hardware designs that must 32 be integrated in the acquisition node. For this last reason, in most cases a com-33 plete digital solution is preferred. The papers falling in the category of digital 34 methods can be further classified into three types: direct methods, parameter 35 extraction methods and transform domain methods [5]. Direct methods achieve 36 compression by removing the redundancy of the ECG signal in the time domain. 37

A method belonging to this type, described in [6], proposed a protocol for ECG
data compression where ECG data are coded as ASCII characters.

In [7], the authors proposed a dynamic compression scheme for ultra-low 40 power and real-time wireless ECG applications. It consisted of a digital integrate-41 and-fire sampler, allowing to represent the ECG signal as a pulse train and a 42 lossless entropy encoder, allowing to encode the timestamps and sign phases of 43 the pulse train to a binary stream. Parameter extraction methods are based 44 on the extraction of some features of the ECG signal such as the P wave (the 45 earliest wave of the ECG cycle, corresponding to the atria depolarization), the 46 T wave (corresponding to the repolarization of the ventricles) or the QRS com-47 plex (corresponding to the depolarization of the right and left ventricles). A 48 method falling in this category was presented in [8]. This method is composed 49 by a preprocessing phase, where the signal is divided in beats and each beat 50 is further segmented to find the P section, the QRS section and the T section. 51 Each section is filtered with a different filter and the baseline is removed. Then, 52 an encoder searches the best match of the preprocessed beat with the entries of 53 a codebook and performs a Long-Term Prediction (LTP). Finally, the residue 54 between the output of the codebook and the predicted waveform from the LTP 55 is furtherly coded. In transform domain methods, the ECG signal is projected to 56 a transform domain by means of a linear orthogonal transformation. Then the 57 coefficients in the transformed domain are properly encoded. Some widely used 58 transforms include Discrete Fourier Transform (DFT), Discrete Cosine Trans-59 form (DCT), Walsh Transform and Discrete Wavelet Transform (DWT) [5], [9]. 60 In [10], the ECG is compressed by first evaluating the DWT based on the Db6 61 wavelet function. Then, the obtained coefficients are selected by the application 62 of a higher order statistics thresholding. The compressed signal is obtained by 63 linear prediction coding and Huffman coding. The transform domain methods 64 have gained a significant attention due to their capability of accurately repre-65 sent the ECG signal also at moderate compression ratios. However, most of 66 them are unsuitable for a real-time implementation in acquisition nodes with 67 constrained resources, due to their computational complexity and large buffer 68

memory requirements [11]. Hybrid approaches, combining methods from dif-69 ferent categories have been proposed in [11] and [12]. The method in [11] first 70 detects the beats, it divides each beat into plain and complex blocks, based on 71 their standard deviation. Then, specific compressions are applied to the two 72 block types. The method in [12] faces the problem of unacceptable distortion 73 of abnormal beats for high compression ratios. It uses a Support–Vector Ma-74 chine (SVM) binary classifier to identify abnormal beats. Then a wavelet-based 75 compression is used for abnormal beats, while normal beats are compressed in 76 groups by means of a hybrid encoder, employing a combination of wavelet and 77 Principal Component Analysis (PCA). 78

Alternatively, Compressive Sampling (CS) has been proposed for ECG signal 79 compression. The advantage of CS is its capability of achieving performance 80 comparable with transform-domain methods, while moving the computational 81 load from the acquisition node to the receiving node. Since the receiving node 82 is usually located on the Internet cloud, it has much greater computational 83 resources available. Therefore, CS methods allows to realize ECG compression 84 also on sensing nodes with constrained resources. For this reason, CS represents 85 a widely proposed solution for data compression in IoT systems [13], where the 86 acquisition and compression can be implemented in resource-constrained nodes, 87 while the reconstruction is actually carried out (where needed) in the cloud. 88

The literature regarding the application of CS for ECG compression is mostly 89 concentrated on the quality assessment and improvement of the reconstructed 90 signal by investigating the impact on the quality of a specific sensing matrix, 91 dictionary matrix, or reconstruction algorithm. (see Section 2 for further details 92 about CS) [14]. In [15], the authors compared the performance of the ECG 93 reconstruction with the application of different dictionary matrices, based on 94 the Discrete Wavelet Transform, belonging to several wavelet families, such as 95 Coiflet, Daubechies, Symlet, biorthogonal and reverse biorthogonal. Another 96 comparison of several dictionaries was reported in [14], where the dictionary 97 matrix was based on the Daubechies db4, Gabor, and mexican hat and spline 98 wavelets. Regarding the sensing matrix, often Gaussian and Bernoulli random 99

matrices are used as it has been proven their incoherence with Fourier and 100 wavelet dictionaries. In [16], the authors evaluated the Toeplitz, Circulant and 101 Triangular structured sensing matrices, because they do not require the random 102 number generation and therefore are easier to implement and less complex. In 103 [17] and [18], a CS-based sensing scheme using a Deterministic Binary Block 104 Diagonal matrix (DBBD) as sensing matrix is utilized. In particular, in [17] a 105 hardware implementation of the sensing scheme is also proposed and compared 106 with the random demodulator Analog-to-Information Converter architecture. 107 In [18], a digital CS scheme using the same sensing matrix is proposed and 108 compared with Gaussian random matrices. About reconstruction algorithms, 109 Basis Pursuit (BP) and Orthogonal Matching Pursuit (OMP) are mainly used in 110 the literature. In [16], also Compressed Sampling Matching Pursuit (CoSaMP) 111 and Normalized Iterative Hard Thresholding (NIHT) have been evaluated for 112 ECG reconstruction. Alternative approaches have been proposed in [19] and in 113 [20]. In the former, a parametric model of the ECG signal is used and the signal 114 recovery is achieved by means of the Differential Evolution algorithm. In the 115 latter, a time-normalized agnostic dictionary created by the PCA of training 116 signals is instead used as signal base. 117

This paper aims to propose a new digital CS-based ECG compression method 118 where a deterministic sensing matrix, adapted to the acquired signal, is utilized. 119 Differently from random sensing matrices, the sensing matrix does not require 120 the random number generation. Moreover, being adapted to the signal, the 121 sensing matrix contains more information on the signal features and therefore it 122 can guarantee a better reconstruction quality, than other deterministic matrices. 123 The present research aims to realize the ECG signal compression to be inte-124 grated in the smart wearable device of the Ambient-intelligent Tele-monitoring 125 and Telemetry for Incepting & Catering over hUman Sustainability (ATTI-126 CUS) project [21]. The project main objective is to develop a telemedicine 127 system where physiological signals are acquired by means of a smart T-shirt 128 and possible anomalies in the parameters are discovered. 129

130

A preliminary version of the proposed method was presented in [22]. In

the work presented here, the aforementioned method was further improved and assessed. In particular, the compression algorithm was modified such that the sensing matrix is not evaluated in each frame, but only whether a significant change in the signal distribution is found. Moreover, the proposed method has been compared against two state of the art methods performing CS-based ECG signal acquisition.

The rest of the paper is organized as follows. In Section 2, some background knowledge about CS is given. In Section 3, the proposed CS-based ECG compression scheme is presented. The experimental evaluation is presented in Section 4. Finally, Section 5 presents the conclusions and future work.

¹⁴¹ 2. Fundamentals of Compressive Sampling

¹⁴² CS is based on the assumption that the ECG signal is sparse in a transform ¹⁴³ domain, i.e. its representation in such domain is a vector with few elements. In ¹⁴⁴ particular, let us consider a vector \mathbf{x} of N samples acquired in a certain time ¹⁴⁵ window, at Nyquist rate, expressed as:

$$\mathbf{x} = \boldsymbol{\Psi}\boldsymbol{\theta},\tag{1}$$

where Ψ is a $N \times N$ matrix describing the domain transformation, often called *dictionary matrix*, and θ is the vector of the coefficients in the transformed domain. The vector θ is assumed to be K-sparse, i.e. it contains at most K elements. Under the sparsity assumption, the compression is actually performed by multiplying the acquired ECG samples by a $M \times N$ sensing matrix Φ , with M < N:

$$\mathbf{y} = \mathbf{\Phi} \mathbf{x},\tag{2}$$

where **y** is the *M*-size vector of the compressed samples. In order to guarantee the reconstruction, it is necessary that the sensing matrix $\boldsymbol{\Phi}$ presents a low coherence with the dictionary matrix, where the coherence is defined as [18]:

$$\mu(\mathbf{\Phi}, \mathbf{\Psi}) = \sqrt{N} \cdot \max_{i,j} \frac{\boldsymbol{\phi}_i^T \boldsymbol{\psi}_j}{\|\boldsymbol{\phi}_i\|_2 \|\boldsymbol{\psi}_j\|_2},\tag{3}$$

where ϕ_i^T and ψ_j are the *i*-th row of Φ and the *j*-th column of Ψ , respectively, and $\|\cdot\|_2$ indicates the ℓ_2 norm. The reconstruction of the ECG waveform is actually performed by solving the ℓ_1 minimization problem:

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \|\boldsymbol{\theta}\|_1, \quad \text{subject to:} \quad \mathbf{y} = \boldsymbol{\Phi} \boldsymbol{\Psi} \boldsymbol{\theta},$$
(4)

where $\|\cdot\|_1$ indicates the ℓ_1 norm.

¹⁵⁹ 3. The proposed method

In this Section, the proposed method is described. In particular, in Subsection 3.1 the compression procedure occurring in the acquisition node is discussed, while in Subsection 3.2 the signal reconstruction, executed at the receiving side, is presented. Furthermore, in Subsection 3.3 the compressed data rate is described, and in Subsection 3.4 the computational complexity is analyzed.

165 3.1. Compression

As mentioned in Section 1, in digital CS-based methods, the compression 166 is operated by multiplying the acquired samples by a sensing matrix. In this 167 paper, the sensing matrix Φ is chosen such that the vector **y** performs a sort 168 of cross-correlation between the vector \mathbf{x} , consisting of a frame of N samples 169 of the ECG signal acquired at the Nyquist rate, and the pulse train signal **p**, 170 consisting of ones where the signal \mathbf{x} has a high contribution and zero elsewhere. 171 In this way, the M samples of \mathbf{y} contain an information that is somehow related 172 to the auto-correlation coefficients of \mathbf{x} . 173

The proposed method operates on records of N samples acquired at Nyquist rate. For each record \mathbf{x} , the average of the samples in the record is performed to obtain the value x_{avg} . Then, the signal magnitude \mathbf{x}_a is evaluated as:

$$\mathbf{x}_a = |\mathbf{x} - x_{avg}|. \tag{5}$$

A threshold value x_{th} is computed in each frame, representing a certain percentile of the waveform amplitude, by means of a sorting-based algorithm operating on x_a . Then, x_{th} is obtained according to the defined percentile. An analysis of the method performance versus the considered percentile is reported in Section 4.2.

The Φ matrix is then updated whether a significant change is found in the x_{th} value. Otherwise, the Φ matrix of the previous frame is used. The change of x_{th} is considered significant when the distance between the value evaluated in the current frame and the one obtained in the previous frame is above a specified limit ε .

In the case the Φ matrix must be updated, an *N*-size binary vector **p** is constructed by quantizing the signal magnitude \mathbf{x}_a , with the resolution of 1 bit, according to the threshold x_{th} . Therefore, the *n*-th element p(n) of **p**, with $n = 0, \ldots, N - 1$, is evaluated as:

$$p(n) = \begin{cases} 1, & \text{if } x_a(n) \ge x_{th} \\ 0, & \text{if } x_a(n) < x_{th} \end{cases}.$$
 (6)

191 Once defined the vector \mathbf{p} , the sensing matrix $\mathbf{\Phi}$ is constructed as follows:

$$\Phi = \begin{bmatrix}
p(1) & p(2) & \dots & p(N) \\
p(N - USR + 1) & p(N - USR + 2) & \dots & p(N - USR) \\
\vdots & \vdots & \ddots & \vdots \\
p(USR + 1) & p(USR + 2) & \dots & p(USR)
\end{bmatrix}$$
(7)

where, USR = N/M is the Under-Sampling Ratio, expressing a measure of the data reduction respect to the number of samples acquired according to the Nyquist criteria. The sensing matrix Φ is built as a circulant matrix obtained by shifting the pulse train samples of USR positions in each row.

The whole compression procedure is summarized by a block scheme as depicted in Fig. 1. For the generic *n*-th frame, the \mathbf{x}_a vector is evaluated as in (5) and the $x_{th}(n)$ value is obtained, according to the chosen value of percentile.



Figure 1: Block diagram of the compression procedure. The Φ matrix is actually evaluated only whether a significant change is found in the value of x_{th} .

If the current frame is the first one or the absolute difference of $x_{th}(n)$ and $x_{th}(n-1)$ is greater than a certain value ε , a new **p** vector is evaluated, applying (6) and a new sensing matrix Φ is generated. In this case, it is also necessary to transmit the **p** vector. If instead n > 1 and no significant change occurred in x_{th} (i.e. $||x_{th}(n) - x_{th}(n-1)|| < \varepsilon$), the values of **p** and Φ of the previous frame are used. Once defined the sensing matrix to be used, the acquired samples are compressed by the (2) and the compressed samples are transmitted.

206 3.2. Reconstruction

As mentioned in Section 1, the ECG signal waveform is reconstructed by finding the coefficients in the sparsity domain by solving (4). To this aim, a key role is played by the dictionary matrix (i.e. Ψ) which defines the sparsity domain. In other words, the dictionary matrix allows transforming the vector of the acquired samples (see eq. (1)) to a sparse vector, i.e., having a reduced number of elements.

As reported in [14], the dictionary that achieves the highest reconstruction performance for the ECG signal is obtained by means of the Mexican hat wavelet kernel. In [23], the Mexican hat wavelet matrix Ψ_{base} is defined as:

$$\Psi_{base} = \left[\psi(2,0), \psi(2,2), \psi(2,4), \dots, \psi\left(2, 2\left\lfloor \frac{N-1}{2} \right\rfloor \right), \\ \psi(4,0), \psi(4,4), \psi(4,8), \dots, \psi\left(4, 4\left\lfloor \frac{N-1}{4} \right\rfloor \right), \quad (8) \\ \dots, \psi(N,0) \right]$$

where $\psi(a, b)$ is a *N*-size vector that describes the Mexican hat wavelet function, defined according to the following expression:

$$\boldsymbol{\psi}(a,b) = \frac{2}{\sqrt{3a} \cdot \pi^{1/4}} \cdot \left[1 - \left(\frac{\mathbf{n} - b}{a}\right)^2 \right] \cdot e^{-\frac{1}{2}\left(\frac{\mathbf{n} - b}{a}\right)^2} \tag{9}$$

with $\mathbf{n} = [0, \dots, N-1]^T$, *a* the scaling factor, $a = 2^m$, $m \in \{1, \dots, \lfloor \log_2(N) \rfloor\}$, and *b* the delay factor, which depends on *a*, i.e. $b \in \{0, a, 2a, \dots, a \lfloor \frac{N-1}{a} \rfloor\}$. Finally, in order to allow the reconstruction of the observed ECG signal in presence of a low-frequency baseline wander, the vector $\mathbf{u} = [1/N, \dots, 1/N]^T$ is added as last column of the matrix Ψ that is finally expressed as:

$$\Psi = \left[\Psi_{base}, \mathbf{u}\right]. \tag{10}$$

223 3.3. Compressed data rate

Since in the proposed CS method, the sensing matrix is adapted to the signal, it is necessary to provide, for the reconstruction purpose, a coded version of the vector \mathbf{p} together with the compressed samples, whenever the sensing matrix is updated. By assuming that each sample is provided with a resolution of b bits, the resulting compression ratio is slightly lower than the undersampling ratio:

$$CR = \frac{B_0}{B_c} = \frac{bNN_f}{bMN_f + N_pN} = \frac{USR}{1 + USR \frac{N_p}{N_f}},\tag{11}$$

where B_0 and B_c are the data rates before and after the compression, respectively, N_f is the number of frames and N_p is the number of frames where the matrix is updated.

232 3.4. Computational complexity

The proposed CS method presents a very low computational complexity for the acquisition node. For each acquired frame, the following operations are needed:

- $N \log_2 N$ for the evaluation of the percentile x_{th} , considering that a merge sorting algorithm is used;
- one subtraction and two comparisons to check for a significant change in the x_{th} ;
- if it is needed to update the Φ matrix, N further comparisons are needed
 to build the p vector;
- $N_1 1$ additions, where N_1 is the number of ones in the **p** vector, for each compressed sample are needed to evaluate the compressed frame.

The actual value of N_1 varies in each frame. However, its mean value depends on the considered percentile for the evaluation of the threshold x_{th} . Indicating with p% the value of the percentile:

$$E\{N_1\} = N \cdot P\{x(n) > x_{th}\} = N\left(1 - \frac{p\%}{100}\right),\tag{12}$$

where $E\{\cdot\}$ represents the expectation operator, and $P\{\cdot\}$ is the probability of finding x(n) samples higher than x_{th} . Therefore, for each frame, the following mean number of operations is needed in the worst case when the Φ matrix is updated:

$$E\{N_{ops}\} = N \log_2 N + N + 3 + M (E\{N_1\} - 1) =$$

$$= N \left[M \left(1 - \frac{p\%}{100} \right) + \log_2 N + 1 \right] - M + 3.$$
(13)

Considering, as an example, a record of N = 720 samples acquired at a sampling frequency of 360 Samples/s, USR = 4 and p% = 60, according to (13), the number of operations for each record is slightly less than 60 000, corresponding to about 30 000 operations per second. Such computational load can be easily sustained by a 32-bit microcontroller, based on the ARM Cortex M4F core, which provides at least 1.27 DMIPS (Dhrystone Millions of Instructions Per Second)/MHz and a clock frequency greater than 50 MHz.

At the receiving side, the ECG signal waveform needs to be reconstructed. At this stage, the computational complexity depends on the chosen reconstruction algorithm. BP has generally a complexity $\mathcal{O}(N^3)$. OMP has instead a much lower computational load. In its implementation based on the matrix inversion lemma, the number of operations is [24]:

$$N_{ops} = \mathcal{O}((N+M)S),\tag{14}$$

where, S is the number of iterations of the OMP algorithm which is in any case lower than N.

²⁶⁵ 4. Results and discussion

In this Section, the performance of the method presented in Section 3 is 266 analyzed through several tests implemented in MATLAB. For testing purposes, 267 the ECG signals from the PhysioNet MIT-BIH Arrhythmia Database (available 268 online [25]) have been considered. The MIT BIH database was chosen among the 269 Internet-available ones as it is the most widely used in the scientific literature. 270 Therefore, the results obtained on the ECG signals of this database can be 271 easily compared among several research papers. This database contains 48 272 half-hour excerpts of two-channel ambulatory ECG recordings, obtained from 273 47 subjects studied by the BIH Arrhythmia Laboratory. The recordings were 274

acquired at a 360 Hz sampling frequency per channel with 11-bit resolution over 275 a 10 mV range. The signals were segmented in N = 720-size frames, while M 276 depends on the adopted USR value. Before being processed by the proposed 277 CS method, in each frame the first three harmonics of the power-line signal, at 278 {60; 120; 180} Hz, respectively have been removed in the frequency domain [26]. 279 Preliminary assessment phases were conducted on the method with the fol-280 lowing aims: (i) to evaluate the method with different reconstruction algorithms, 281 (ii) to tune the amplitude percentile used in the method to build the **p** vector, 282 (iii) to verify the influence of the ε threshold, which controls the update of 283 the sensing matrix. In the above mentioned phases, two data-sets of ten ECG 284 signals from [25] have been used: 285

$$S_1 = \{100, 106, 107, 115, 117, 118, 119, 221, 223, 228\}$$
(15)

$$S_2 = \{101, 109, 122, 124, 200, 202, 205, 217, 219, 234\}$$
(16)

The first data-set contains the following main beat labels: atrial premature 286 beats, premature ventricular contractions, paced beats, isolated QRS-like arti-287 facts, and right bundle branch block beats. In the second data-set, the main 288 beat labels are: left bundle branch block beats, aberrated atrial premature 289 beats, non-conducted P-waves, and nodal (junctional) premature beats. For 290 each ECG signal, the Percentage of Root-mean-squared Difference (PRD) has 291 been evaluated as figure of merit [14, 27, 28, 29]. The PRD is computed as 292 follows: 293

$$PRD = \frac{\|\mathbf{x} - \hat{\mathbf{x}}\|_2}{\|\mathbf{x}\|_2} \times 100\%, \tag{17}$$

where \mathbf{x} and $\hat{\mathbf{x}}$ are the original and the reconstructed signal, respectively. Then, two evaluation phases have been carried out. In the former, the method is characterized on the above mentioned signal data-sets and compared with the method proposed in [18] (DBBD sensing matrix and DCT dictionary) and the method reporting the best result in [14] (Bernoulli sensing matrix and two scales Mexican hat based dictionary). Those methods were selected as they got the better accuracy of reconstruction within CS-based methods. In this analysis the following figures of merit were evaluated: PRD, PRD Normalized (PRDN), Mean Square Error (MSE), Root Mean Square Error (RMSE), Peak Signal to Noise Ratio (PSNR). The used expression of PRD is that reported in (17). The following expressions of PRDN, MSE, RMSE and PSNR have been used [12]:

$$PRDN = \frac{\|\mathbf{x} - \hat{\mathbf{x}}\|_2}{\|\mathbf{x} - \overline{\mathbf{x}}\|_2} \times 100\%, \tag{18}$$

$$MSE = \frac{\|\mathbf{x} - \hat{\mathbf{x}}\|_2}{N_s},\tag{19}$$

$$RMSE = \sqrt{\frac{\|\mathbf{x} - \hat{\mathbf{x}}\|_2}{N_s}},\tag{20}$$

$$PSNR = 10\log_{10}\frac{\max(\mathbf{x})^2}{MSE},\tag{21}$$

where $\overline{\mathbf{x}}$ is the average of the elements of \mathbf{x} , and N_s is the number of samples of the entire signal under observation.

In the latter phase, the method was tested on several types of abnormal beats, by evaluating the Weighted Diagnostic Distortion (*WDD*) [30], and the Wavelet Energy-based Diagnostic Distortion (*WEDD*) [31]. *WEDD* was evaluated according to [31] as a weighted sum of the Wavelet *PRD* (*WPRD*):

$$WEDD = \sum_{l=1}^{L+1} w_l WPRD_l \tag{22}$$

where L is the number of wavelet levels. The weights w_l and the WPRD are evaluated as:

$$w_l = \frac{\sum_{k=1}^{K_l} d_l^2(k)}{\sum_{m=1} L + 1 \sum_{k=1} K_l d_m^2(k)}$$
(23)

$$WPRD_{l} = \sqrt{\frac{\sum_{k=1}^{K_{l}} [d_{l}(k) - \tilde{d}_{l}(k)]^{2}}{\sum_{k=1}^{K_{l}} d_{l}^{2}(k)}}$$
(24)

where, K_l denotes the number of wavelet coefficients at the *l*-th level, $d_l(k)$ and $\tilde{d}_l(k)$ are the *k*-th wavelet coefficients of the original and of the reconstructed signal, respectively.

WDD was evaluated according to [12], by considering the following features: (i) QRS duration, (ii) P-wave height, (iii) P-wave duration, (iv) R-R interval, (v) QRS amplitude, (vi) PR interval, (vii) QT interval, and (viii) T-wave height. The features were evaluated by applying the method proposed in [32] for ECG delineation. For the evaluation of WDD, the following matrix of weights was used [12]:

$$\mathbf{\Lambda} = \text{diag}([2\ 2\ 2\ 2\ 1\ 1\ 1\ 1]). \tag{25}$$

322 Therefore, *WDD* is evaluated as:

$$WDD = \mathbf{\Delta}\boldsymbol{\beta}^T \cdot \frac{\mathbf{\Lambda}}{\operatorname{tr}(\mathbf{\Lambda})} \cdot \mathbf{\Delta}\boldsymbol{\beta} \times 100, \qquad (26)$$

where, $\Delta\beta$ is the vector of the normalized differences $\Delta\beta_i$ of the features evaluated on the original and reconstructed signal, respectively [30]:

$$\Delta \beta_i = \frac{|\beta_i - \hat{\beta}_i|}{\max\{\beta_i, \hat{\beta}_i\}}.$$
(27)

325 4.1. Influence of the reconstruction algorithm

In this test, the performance of the proposed method, by employing either 326 the OMP or BP algorithms in the signal reconstruction, has been evaluated in 327 terms of PRD averaged among all the signals of the S_1 data-set. The analysis 328 has been carried out on the first 5 min of the signal recordings, considering 329 values of the USR ranging from 2 up to 12 with a step of 2. The obtained 330 results are shown in Fig. 2. As it was expected, the proposed method exhibits 331 lower *PRD* if the BP algorithm is employed instead of OMP. In particular, by 332 considering a PRD less than 9%, which represents a good signal reconstruction 333 quality for medical applications [28], the OMP algorithm can be adopted with 334 a maximum USR of 6, while the BP algorithm allows to increase the USR at 335



Figure 2: Average PRD over the S_1 data–set using BP and OMP as reconstruction algorithms. BP achieves a better reconstruction quality for all the considered values of USR.

a maximum value of 9. However, the BP algorithm exhibits a computational
complexity higher than the OMP. Since it assures a better accuracy in terms of
signal quality reconstruction, BP was used in this paper.

339 4.2. Influence of percentile

This analysis aims at evaluating the influence of the percentile used for 340 the determination of the threshold x_{th} . To this aim, the average *PRD* over 341 the first $1 \min$ of the signals contained in the S_1 data-set was evaluated for 342 different percentiles and USR values. Each signal is reconstructed through the 343 BP algorithm. The analysis has been carried out according to percentile values 344 ranging from 20 % to 80 %, with a step of 10 % and for USR values equal to 345 $\{2, 4, 6, 8, 10\}$. In Fig. 3, the obtained results are depicted. From this figure, it 346 can be noted that the *PRD* does not show significant changes with the percentile 347 in the considered range, for all the USR values. A value of 60 % has been chosen 348 for the further tests. 349



Figure 3: Average PRD of the proposed method versus the percentile and for several values of the USR.

350 4.3. Influence of the ε threshold

In Fig. 4, the results in terms of average PRD, evaluated on the first 5 min of the signals in the S_1 data-set when different values of the USR and ε have been used, are shown. The PRD is reported versus the CR, which, as mentioned above, is slightly lower than the USR, due to the need of transmitting the



Figure 4: Average *PRD*, evaluated on the first 5 min of the signals in the S_1 data–set, for different values of *USR* and three values of the threshold ε : 0.00 mV (sensing matrix changes in all the frames), 0.05 mV, 0.15 mV, and 0.30 mV.

UGD	Method		PRD	PRDN	MSE	RMSE	SNR	PSNR
USh			[%]	[%]	$\left[\cdot 10^{-5} \mathrm{mV^2}\right]$	$[\cdot 10^{-3}\mathrm{mV}]$	[dB]	[dB]
	Proposed	2.00	1.42	2.30	2.39	4.87	37.72	50.41
2	DBBD+DCT	2.00	0.94	1.52	1.58	3.97	41.22	52.16
	Bernoulli+MexHat2	2.00	2.75	4.29	4.75	6.85	31.86	47.47
	Proposed	3.99	3.06	4.68	5.04	7.04	31.39	47.24
4	DBBD+DCT	4.00	3.35	5.00	5.47	7.23	31.21	47.15
	Bernoulli+MexHat2	4.00	4.91	7.49	8.32	9.04	27.09	45.09
	Proposed	5.97	4.61	7.24	8.16	8.93	27.34	45.22
6	DBBD+DCT	6.00	8.21	13.38	14.36	11.79	22.60	42.85
	Bernoulli+MexHat2	6.00	7.59	11.80	13.31	11.40	23.10	43.10
	Proposed	7.94	7.40	11.71	13.04	11.31	23.18	43.14
8	DBBD+DCT	8.00	14.46	23.72	25.10	15.57	17.78	40.43
	Bernoulli+MexHat2	8.00	11.41	17.88	20.05	14.01	19.49	41.29
	Proposed	9.91	10.62	17.21	18.99	13.66	19.91	41.50
10	DBBD+DCT	10.00	19.48	31.88	33.56	18.05	15.18	39.13
	Bernoulli+MexHat2	10.00	16.97	27.14	29.65	17.05	16.07	39.58

Table 1: Comparison of the results obtained with the proposed method (Proposed) vs. the method proposed in [18] (DBBD+DCT) and the method reporting the best results in [14] (Bernoulli+MexHat2) for the Data–set S_1 : (a) average, (b) standard deviation.

(a)	Data-set	S_1	average.
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UGD	Mathad	CD	PRD	PRDN	MSE	RMSE	SNR	PSNR
USA	Method	Ch	[%]	[%]	$\left[\cdot 10^{-5}\mathrm{mV}^2\right]$	$[\cdot 10^{-3}\mathrm{mV}]$	[dB]	[dB]
2	Proposed	2.00	0.60	1.00	0.39	0.40	3.86	6.23
2	DBBD+DCT	2.00	0.39	0.57	0.17	0.21	3.53	6.29
	Bernoulli+MexHat2	2.00	1.12	1.04	1.19	0.84	3.57	5.88
4	Proposed	3.99	1.64	1.66	1.28	0.91	4.63	5.81
4	DBBD+DCT	4.00	2.15	2.75	2.58	1.65	5.89	5.27
	Bernoulli+MexHat2	4.00	2.36	2.05	2.41	1.30	4.22	5.71
C	Proposed	5.97	1.80	1.84	2.72	1.45	3.51	5.86
0	DBBD+DCT	6.00	3.63	6.89	5.49	2.27	4.34	5.71
	Bernoulli+MexHat2	6.00	3.15	3.06	4.60	1.90	3.76	5.79
	Proposed	7.94	2.73	3.07	3.96	1.64	3.35	5.97
8	DBBD+DCT	8.00	7.21	12.44	9.88	3.06	4.37	5.98
	Bernoulli+MexHat2	8.00	4.38	4.75	6.16	2.15	3.64	5.60
10	Proposed	9.91	3.35	5.07	5.66	1.95	2.98	6.11
10	DBBD+DCT	10.00	9.73	15.84	12.12	3.30	4.32	6.08
	Bernoulli+MexHat2	10.00	6.39	10.36	8.59	2.52	3.78	5.58

(b) Data–set S_1 standard deviation.

UGD	Mathad		PRD	PRDN	MSE	RMSE	SNR	PSNR
USR	Method		[%]	[%]	$[\cdot 10^{-5} \mathrm{mV^2}]$	$[\cdot 10^{-3}\mathrm{mV}]$	[dB]	[dB]
	Proposed	2.00	1.83	2.47	2.66	5.06	36.10	51.24
2	DBBD+DCT	2.00	0.97	1.38	1.45	3.80	40.86	53.62
	Bernoulli+MexHat2	2.00	2.93	4.10	4.46	6.65	31.18	48.78
	Proposed	3.98	3.33	4.63	4.91	6.96	30.44	48.41
4	DBBD+DCT	4.00	3.29	4.47	4.93	6.85	31.02	48.70
	Bernoulli+MexHat2	4.00	5.48	7.58	8.17	8.98	25.99	46.19
	Proposed	5.96	4.98	7.02	7.53	8.60	26.77	46.58
6	DBBD+DCT	6.00	7.41	10.72	11.11	10.39	23.60	44.99
	Bernoulli+MexHat2	6.00	8.21	11.48	12.47	11.08	22.35	44.37
	Proposed	7.92	7.79	10.94	11.60	10.72	22.89	44.64
8	DBBD+DCT	8.00	12.57	18.26	18.46	13.34	19.34	42.86
	Bernoulli+MexHat2	8.00	11.75	16.70	18.04	13.33	19.15	42.77
	Proposed	9.88	10.63	15.06	16.12	12.64	19.99	43.19
10	DBBD+DCT	10.00	17.50	25.57	25.98	15.81	16.44	41.41
	Bernoulli+MexHat2	10.00	16.68	24.14	25.49	15.82	16.22	41.30

Table 2: Comparison of the results obtained with the proposed method (Proposed) vs. the method proposed in [18] (DBBD+DCT) and the method reporting the best results in [14] (Bernoulli+MexHat2) for the Data-set S_2 : (a) average, (b) standard deviation.

(a) Data–set	S_2	average.
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UGD		CD	PRD	PRDN	MSE	RMSE	SNR	PSNR
USR	Method	CR	[%]	[%]	$[\cdot 10^{-5} \mathrm{mV^2}]$	$[\cdot 10^{-3}\mathrm{mV}]$	[dB]	[dB]
	Proposed	2.00	1.22	1.11	1.24	1.03	4.92	2.40
2	DBBD+DCT	2.00	0.36	0.40	0.18	0.24	3.41	2.51
	Bernoulli+MexHat2	2.00	1.04	0.80	0.93	0.70	3.29	2.28
4	Proposed	3.98	1.57	1.52	1.31	0.91	4.23	2.40
4	DBBD+DCT	4.00	1.80	1.77	2.33	1.63	5.37	2.70
	Bernoulli+MexHat2	4.00	2.31	1.95	2.00	1.11	3.95	2.27
G	Proposed	5.96	2.15	2.19	2.13	1.20	3.79	2.48
0	DBBD+DCT	6.00	3.50	5.48	3.74	1.86	4.54	2.82
	Bernoulli+MexHat2	6.00	3.31	2.83	3.34	1.47	3.57	2.27
	Proposed	7.92	2.98	3.13	2.42	1.14	3.98	2.54
8	DBBD+DCT	8.00	6.84	10.75	6.81	2.70	5.26	2.92
	Bernoulli+MexHat2	8.00	4.21	4.79	4.65	1.77	3.39	2.46
10	Proposed	9.88	3.69	3.70	3.02	1.20	3.27	2.38
10	DBBD+DCT	10.00	9.40	14.04	9.92	3.33	5.25	3.07
	Bernoulli+MexHat2	10.00	6.63	9.68	6.65	2.29	3.64	2.65

(b) Data–set S_2 standard deviation.



Figure 5: Average PRD (solid line) and Maximum PRD (dashed line) obtained for the S_1 (a) and S_2 (b) for the proposed method (Proposed), the method proposed in [18] (DBBD+DCT), and the method reporting the best performance of [14] (Bernoulli+MexHat2).

³⁵⁵ **p** vector, whenever it is updated. In details, the number of times when **p** is ³⁵⁶ updated is controlled by the value ε . Higher ε is, lower is the probability that **p** ³⁵⁷ is updated and *CR* is closer to *USR*. It can be observed in Fig. 4 that the *PRD* ³⁵⁸ does not change significantly by increasing ε up to 0.30 mV, while a relevant ³⁵⁹ increase of *CR* is achieved. For this reason a value of ε of 0.30 mV is selected ³⁶⁰ for the further tests.

361 4.4. Experimental results

The results of the former evaluation phase are shown in Tab. 1 and Tab. 2, where the values of the considered figures of merit are reported versus the USRin the case of the proposed method (Proposed), of the method proposed in [18] (DBBD+DCT) and the method reporting the best results in [14] (Bernoulli+MexHat2). In particular, Tab. 1 reports the results for Data-set S_1 , while Tab. 2 reports the results for Data-set S_2 . In each table, the average and standard deviations of each figure of merit over the signals included in the Data-set are given.

The USR is therefore reported in the first column. The second column reports the method and the third column shows the obtained CR. It can be seen that the proposed method shows average values of the figures of merit that



(c) Fusion of ventricular and normal beat

(d) Left bundle branch block beat

Figure 6: Original waveforms (dashed grey line) and reconstructed waveform (solid black line) of some abnormal beats extracted from the MIT-BIH arrhythmia database signals, after a compression with USR = 8: (a) atrial premature beat (Signal no. 222), (b) premature ventricular contraction (Signal no. 105), (d) left bundle branch block (Signal no. 213), and (c) fusion of ventricular and normal beat (Signal no. 111).

		WDI	D [%]	WEDD [%]	
Abnormal beat	USD	Amonomo	Standard	Avenage	Standard
type	USh	Average	deviation	Average	deviation
	2	0.7	2.1	4.1	3.7
A (100 heats)	4	0.9	1.6	5.0	2.2
A (100 beats)	6	1.9	3.4	8.2	3.6
	8	3.3	4.1	13.3	ZEDD [%] Standard deviation 3.7 2.2 3.6 7.0 0.52 3.33 4.35 7.37 0.23 0.81 1.94 3.67 1.30 1.27 2.70 4.79
	2	2.08	4.05	0.96	0.52
V (41 heats)	4	3.91	5.22	4.75	3.33
V (41 Deats)	6	3.16	3.97	7.36	4.35
	8	3.78	5.02	12.72	7.37
	2	0.09	0.25	0.42	0.23
E(100 heats)	4	0.28	0.37	1.79	0.81
F (100 beats)	6	0.85	1.15	4.44	1.94
	8	1.82	2.78	8.94	3.67
	2	0.20	0.60	1.94	1.30
L (100 heats)	4	0.20	0.67	2.67	1.27
L (100 beats)	6	0.42	0.93	5.63	2.70
	8	1.06	1.22	10.54	4.79

Table 3: Results of *WDD* and *WEDD* for 4 types of abnormal beats for different values of the USR.

are better than the method Bernoulli+MexHat2 for all the considered values of CR. Compared with the method [18], the proposed method shows results that are slightly worse for USR = 2. Both the methods then give similar values for USR = 4. For USR > 4 the proposed method shows a much better accuracy. Looking at the standard deviations, it can be observed that, for both Data–sets S_1 and S_2 , the proposed method shows a lower variability, for almost all the figures of merit, for a CR higher than 2.

The superiority of the proposed method can be better observed in Fig. 5, 379 where the trend of the average PRD (solid line) and the maximum PRD (dashed 380 line), evaluated on the signals belonging to the Data-sets S_1 (Fig. 5a) and S_2 381 (Fig. 5b), respectively, are reported versus the CR for the three methods. It 382 can be seen that for CR > 4 the proposed method achieves a huge average 383 reduction of PRD compared with both the other methods on both the data-384 sets. Moreover, it shows also a lower variability than the other methods. The 385 higher accuracy of the proposed method than the other methods is due to the 386

adaptation of the sensing matrix to the acquired signal, realized by evaluating
and transmitting the p vector.

The latter evaluation phase was devoted to verify the behavior of the proposed method in the case of abnormal beats. To this aim, abnormal beats of 4 types where selected in the first 10 min of the signal recordings no. 222 (atrial premature beat - A), no. 105 (premature ventricular contraction - V), no. 213 (fusion of ventricular and normal beat - F), and no. 111 (left bundle branch block beat - L).

The results are reported in Tab. 3. The table reports in the first column 305 the type of abnormal beat and the number of analyzed beats. In the second 396 column, the USR is reported. The other columns report the average values and 397 the standard deviation of the WDD and WEDD figures of merit, respectively, 398 evaluated on the considered beats. As expected, the values of both WDD and 399 WEDD increase with the USR. However, for all the considered types of beats 400 and values of USR, the values of WDD and WEDD result in the ranges of the 401 highest quality categories reported in [30] and [31]. Slightly higher values were 402 found for A and V beat types, mainly for USR = 8. This can be due to a 403 non-perfect coverage of the used Mexican hat dictionary for specific beat types. 404 This will be analyzed in a further work where an automatic optimization of the 405 dictionary is planned, with the application of the method proposed in [33]. 406

The good reconstruction quality of abnormal beats can be seen in Fig. 6, 407 where the acquired waveforms (thick grey line) and the reconstructed ones (thin 408 black line), corresponding to 4 frames centered around abnormal beats of dif-409 ferent types, after the signal has been compressed with USR = 8. In particular, 410 atrial premature beat, premature ventricular contraction, fusion of ventricular 411 and normal beat and left bundle branch block beat are reported in Fig. 6a, 412 Fig. 6b, Fig. 6c and Fig. 6d, respectively. It can be seen a good overposition of 413 the thin black line (reconstructed signal) over the thick grey line (original ac-414 quired signal) that allows preserving all the characteristic features of the wave-415 form. 416

417 5. Conclusion

In this paper, a novel method for the compressive sampling of ECG signals, is 418 presented. The method is based on the idea of building a sensing matrix, which 419 is adapted on the acquired signal frame. In particular, it is a circulant matrix, 420 containing zeros and ones, obtained by quantizing (with the resolution of 1 bit) 421 the magnitude of the acquired signal. The adapted sensing matrix guarantees 422 that the significant portions of the signal waveform are actually contained in 423 the compressed version, thus allowing a more accurate reconstruction respect 424 to the methods already available in scientific literature. The sensing matrix is 425 then used in combination with a modified Mexican hat wavelet dictionary that 426 allows also the reconstruction of the signal wander for each processed frame. 427

The experimental results, obtained on signal recordings from the PhysioNet MIT-BIH Arrhythmia Database, showed that the proposed method achieves a better accuracy in ECG signal reconstruction than the other methods based on compressive sampling. The good ECG signal reconstruction accuracy was also confirmed on abnormal beats of several types.

Future work will be focused on a further improvement of the used dictionary, such to better adapt it to the different ECG signal waveforms. Moreover, the implementation and testing of the proposed method on an IoMT system is planned.

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443 References

- [1] D. Dias, J. P. S. Cunha, Wearable health devices-vital sign monitoring,
 systems and technologies, Sensors, MDPI (Basel) 18 (2018) 2414.
- [2] S. Shirmohammadi, K. Barbé, D. Grimaldi, S. Rapuano, S. Grassini, Instrumentation and measurement in medical, biomedical, and healthcare
 systems, IEEE Instr. & Meas. Magazine 19 (2016) 6–12.
- [3] E. Balestrieri, P. Daponte, L. De Vito, F. Picariello, S. Rapuano, I. Tudosa,
 A Wi-Fi IoT prototype for ECG monitoring exploiting a novel Compressed
 Sensing method, Acta IMEKO (2020).
- [4] M. Tlili, M. Ben-Romdhane, A. Maalej, F. Rivet, D. Dallet, C. Rebai,
 Level-crossing ADC design and evaluation methodology for normal and
 pathological electrocardiogram signals measurement, Measurement 124
 (2018) 413–425.
- [5] C. K. Jha, M. H. Kolekar, Electrocardiogram data compression using DCT
 based discrete orthogonal Stockwell transform, Biomedical Signal Processing and Control 46 (2018) 174–181.
- [6] S. Mukhopadhyay, S. Mitra, M. Mitra, An ECG signal compression technique using ASCII character encoding, Measurement 45 (2012) 1651–1660.
- [7] K. Luo, J. Li, J. Wu, A dynamic compression scheme for energy-efficient
 real-time wireless electrocardiogram biosensors, IEEE Transactions on In strumentation and Measurement 63 (2014) 2160–2169.
- [8] Y. Zigel, A. Cohen, A. Katz, ECG signal compression using analysis by
 synthesis coding, IEEE Transactions on Biomedical Engineering 47 (2000)
 1308–1316.
- ⁴⁶⁷ [9] N. Adochiei, V.David, F. Adochiei, I. Tudosa, ECG waves and features
 ⁴⁶⁸ extraction using wavelet multi-resolution analysis, in: Proc. of E-Health
 ⁴⁶⁹ and Bioengineering Conference (EHB), Iasi, 2011, pp. 1–4.

- [10] S. A. Chouakri, O. Djaafri, A. Taleb-Ahmed, Wavelet transform and Huff man coding based electrocardiogram compression algorithm: application
- to telecardiology, Journal of Physics: Conference Series 454 (2013).
- ⁴⁷³ [11] P. Bera, R. Gupta, Hybrid encoding algorithm for real time compressed
 ⁴⁷⁴ electrocardiogram acquisition, Measurement 91 (2016) 651–660.
- [12] P. Bera, R. Gupta, J. Saha, Preserving abnormal beat morphology in
 long-term ECG recording: an efficient hybrid compression approach, IEEE
 Transactions on Instrumentation and Measurement 69 (2020) 2084–2092.
- [13] E. Balestrieri, L. De Vito, F. Lamonaca, F. Picariello, S. Rapuano, I. Tudosa, Research challenges in measurement for Internet of Things systems,
 ACTA IMEKO 7 (2018) 82–94.
- [14] D. Craven, B. McGinley, L. Kilmartin, M. Glavin, E. Jones, Compressed
 sensing for bioelectric signals: a review, IEEE J. of Biomedical and Health
 Informatics 19 (2015) 529–540.
- [15] A. Mishra, F. Thakkar, C. Modi, R. Kher, Comparative analysis of wavelet
 basis functions for ECG signal compression through compressive sensing, International Journal of Computer Science and Telecommunications 3
 (2012) 23–31.
- [16] A. M. R. Dixon, E. G. Allstot, D. Gangopadhyay, D. J. Allstot, Compressed
 sensing system considerations for ECG and EMG wireless biosensors, IEEE
 Trans. on Biomedical Circuits and Systems 6 (2012) 156–166.
- [17] A. Ravelomanantsoa, S. Member, H. Rabah, S. Member, Simple and efficient compressed sensing encoder for wireless body area network, IEEE
 Transactions on Instrumentation and Measurement 63 (2014) 2973–2982.
- [18] A. Ravelomanantsoa, H. Rabah, A. Rouane, Compressed sensing: a simple
 deterministic measurement matrix and a fast recovery algorithm, IEEE
 Transactions on Instrumentation and Measurement 64 (2015) 3405–3413.

- ⁴⁹⁷ [19] L. Michaeli, J. Šaliga, P. Dolinský, I. Andráš, Optimization paradigm in
 ⁴⁹⁸ the signal recovery after compressive sensing, Measurement Science Review
 ⁴⁹⁹ 19 (2019) 35–42.
- [20] P. Dolinský, I. Andráš, J. Šaliga, L. Michaeli, Reconstruction for ECG
 compressed sensing using a time-normalized PCA dictionary, in: Proc. of
 the 12th Int. Conf. on Measurement, MEASUREMENT 2019, 2019, pp.
 30–33.
- E. Balestrieri, F. Boldi, A. R. Colavita, L. De Vito, G. Laudato, R. Oliveto,
 F. Picariello, S. Rivaldi, S. Scalabrino, P. Torchitti, I. Tudosa, The architecture of an innovative smart t-shirt based on the internet of medical things
 paradigm, in: Proc. of 2019 IEEE Int. Symp. on Medical Measurements
 and Applications, Istanbul, Turkey, 2019.
- E. Balestrieri, L. De Vito, F. Picariello, I. Tudosa, A novel method for
 compressed sensing based sampling of ecg signals in medical-iot era, in:
 Proc. of 2019 IEEE Int. Symp. on Medical Measurements and Applications,
 Istanbul, Turkey, 2019.
- [23] M. Burke, M. Nasor, ECG Analysis using the Mexican-Hat Wavelet, in:
 Advances in scientific computing, computational intelligence and applica tions, Athens, Greece, 2001, pp. 26–31.
- [24] B. L. Sturm, M. G. Christensen, Comparison of orthogonal matching pursuit implementations, in: Proc. of 20th European Signal Processing Conference, Bucharest, Romania, 2012, pp. 220–224.
- ⁵¹⁹ [25] MIT-BIH Arrhythmia Database, 2005. URL:
 https://www.physionet.org/physiobank/database/mitdb/.
- [26] I. Tudosa, N. Adochiei, LMS algorithm derivatives used in real-time filtering of ECG signals: A study case on performance evaluation, in: Proc. of
 Intern. Conf. and Exposition on Electrical and Power Engineering, 2012,
 pp. 565–570.

- ⁵²⁵ [27] H.Mamaghanian, N.Khaled, D.Atienza, P.Vandergheynst, Design and ex⁵²⁶ ploration of low-power analog to information conversion based on com⁵²⁷ pressed sensing, IEEE J. on Emerging and Selected Topics in Circuits and
 ⁵²⁸ Systems 2 (2012) 493-501.
- [28] H. Djelouat, X. Zhai, M. A. Disi, A. Amira, F. Bensaali, System-onchip solution for patients biometric: a compressive sensing-based approach,
 IEEE Sensors Journal 18 (2018) 9629–9639.
- [29] D. Mitra, H. Zanddizari, S. Rajan, Investigation of Kronecker-based recovery of compressed ECG signal, IEEE Trans. on Instr. and Meas. 69 (2020)
 3642–3653.
- [30] Y. Zigel, A. Cohen, A. Katz, The weighted diagnostic distortion (WDD)
 measure for ECG signal compression, IEEE Transactions on Biomedical
 Engineering 47 (2000) 1422–1430.
- [31] M. S. Manikandan, S. Dandapat, Wavelet energy based diagnostic distortion measure for ECG, Biomedical Signal Processing and Control 2 (2007)
 80–96.
- [32] J. P. Martínez, R. Almeida, S. Olmos, A. P. Rocha, P. Laguna, A waveletbased ECG delineator evaluation on standard databases, IEEE Transactions on Biomedical Engineering 51 (2004) 570–581.
- [33] E. Picariello, E. Balestrieri, F. Picariello, S. Rapuano, I. Tudosa,
 L. De Vito, A new method for dictionary matrix optimization in ECG
 compressed sensing, in: Proc. of 2020 Int. Symp. on Medical Measurements and Applications, Bari, Italy, 2020.