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A methodology to forecast the main non-dimensional performance parameters of Pumps-as-Turbines (PaTs) operating at Best Efficiency Point (BEP)

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Abstract

This work presents a model based on analytical equations to identify the Best Efficiency Point (BEP) of Pumps-as-Turbines (PaTs). The equations are developed exploiting an experimental data-set of 59 PaTs, obtained in both pump and turbine modes, in form of non-dimensional parameters. Data analysis shows a linear correlation between specific speeds in pump and turbine modes, as well as between specific diameters in both operating modes. An additional indicator of the PaT performance, which is often disregarded in literature works, is presented: a second order polynomial equation to forecast the mechanical efficiency of PaTs in turbine mode is developed using the values of specific speed and mechanical efficiency in pump mode as independent variables. Performance data obtained through experiments and numerical simulations of six PaTs, which were not used in the development of the model, are employed to validate and assess the accuracy of the proposed analytical equations. The prediction capability of the model is also compared to other four models available in literature. Results demonstrate a good forecast capability and a general better agreement with respect to experimental and numerical data. A further improvement of the model can be achieved by extending the experimental data-set with additional PaTs typologies.

Keywords: Pumps-as-Turbines, Best Efficiency Point, Prediction model, Non-dimensional analysis, Energy recovery

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Nomenclature

Physics magnitudes

Q	Volumetric flow rate [m^3/s]
H	Head [m]
ρ	Density [kg/m^3]
g	Gravitational acceleration [m/s^2]
P	Mechanical power [W]
D	Impeller outer diameter [m]
ω	Rotating speed [rad/s]
Φ	Flow coefficient $[-]$
Ψ	Head coefficient $[-]$
η	Mechanical efficiency $[-]$
Λ	Power coefficient $[-]$
N_s	Specific speed $[-]$
D_s	Specific diameter $[-]$

Subscripts

p	Pump mode operation
t	Turbine mode operation
BEP	Best Efficiency Point
$Exp.$	Experimental
CFD	Computational Fluid Dynamics
$Model$	Presented prediction model

1. Introduction

Large-scale hydropower has experienced a reduction of both installations and economical investments in developed countries since the main available natural water sources have been exploited and new civil works required for these power plants would have a significant environmental impact. As a consequence, the hydropower sector is currently shifting from large-scale plants to small-scale ones [1, 2]. Besides the production of electricity in remote and rural zones, where the development of an electric grid is not easy [3], small-scale hydropower also includes energy recovery applications such as Water Distribution Networks (WDNs) [4, 5], wastewater plants [6] and irrigation systems [7, 8]. The technologies used in small-scale applications have the same operating principle of the large-scale ones, even though several important modifications on hydraulic machines are essential to obtain high performance in each single installation site.

Among them, Pumps-as-Turbines (PaTs) are dominating the field due to their cheaper cost and larger spare parts availability compared to traditional hydraulic turbines [9, 10]. Nevertheless, the evaluation of PaTs performance in off-design operating conditions, as well as the forecast of their Best Efficiency Point (BEP), is still an issue due to the small number of operating data used to develop most of the models available in literature [11, 12].

With regards to the performance prediction of Pumps-as-Turbines (PaTs), when dealing with off-design operating conditions, published work has only appeared recently. For instance, Huang et al. [13] proposed a theoretical approach to forecast both flow rate and head obtained when a PaT operates in pump or in turbine mode. The model is based on the characteristic matching between rotor and stator, but the knowledge of the main geometrical parameters of a PaT are required.

In order to bypass the need for the availability of the geometrical parameters, which are not easy to get from pump manufacturers, other studies proposed methodologies to forecast PaT performance from given design parameters in turbine mode. Delgado et al. [14] developed a new methodology with the aim of predicting the performance curves of a PaT and its rotational speed through a Hermite polynomial chaos expansion. This tool is intended to improve the management of the operation of PaTs in specific hydropower applications. Novara et al. [15] proposed a model, which is based on fixed-coefficient polynomials, to forecast the performance curves of a PaT from a given BEP. The model was validated through experimental data of some PaTs available in literature. Furthermore, the authors of this paper have proposed an analytical model that

was validated through experimental and numerical data related to three different centrifugal PaTs [16] not included in the development of the model itself. The model, which was presented in [16], relies on experimental data of 32 PaTs [9, 17, 18, 19, 20, 21, 22, 23, 24, 25] and it is based on both non-dimensional analysis and a normalization process to predict the off-design performance of PaTs. The model provides simple analytical equations that can be used in a range of specific speeds between 0.09 and 2.24. However, also in this case, the knowledge of the BEP flow rate and head coefficients, as well as the efficiency in turbine mode, is needed a priori.

Several authors proposed analytical equations that correlate the main pump data with those in turbine mode, to predict the BEP in turbine mode, since the knowledge of pump data is easily available from pump manufacturers. Among them, Alatorre-Frenk [26], Schmiedl [27] and Grover [28] developed analytical methodologies aimed at evaluating flow rate, head and mechanical efficiency of PaTs operating in turbine mode. However, their empirical correlations were obtained using the experimental results of a single or a limited number of PaTs, showing good prediction capabilities only for narrow ranges of specific speeds.

Recently, other predictive models were developed with the aim of forecasting both BEP and off-design operating conditions. Barbarelli et al.[24] proposed a one-dimensional code capable of reconstructing the performance curves of a PaT, BEP included, using information related to the main geometrical characteristics of pumps. The validation of the code was carried out from experimental results of 12 different PaTs as reported in [29]. However, also this code requires specific geometrical input parameters, which are not always available nor freely provided by the manufacturers, thus resulting in a complex tool for predicting the performance of a PaT. Some of the authors of this work also developed a new model, based on Artificial Neural Networks (ANNs) capable of reconstructing off-design operating conditions of PaTs, including BEP values [30]. The data used for training the networks refer to the same experimental values used in [16]. The predicted results were compared with the experimental ones obtained by a centrifugal PaT that was not involved in the training process, showing a general good agreement for both off-design operating points and for the BEP. Venturini et al. [31] developed a physics-based simulation model able to forecast PaTs performance curves over the entire operation range based on pump characteristics, coupled with an optimization tool to define the main geometrical parameters of the machine in case these design data are not available.

Regarding the prediction of the mechanical efficiency at BEP in turbine mode, several prediction models suggest to use the same value obtained in pump mode

[32, 33]. However, this assumption might be a limitation because the mechanical efficiency of a PaT at BEP can vary depending on the internal fluid dynamic behaviour of the flow and on the external operating conditions [16]. To the authors' knowledge, only some works supplied a model for the evaluation of the mechanical efficiency in turbine mode, such as the one of Tan et al. [25]. Alatorre-Frenk [26] supplied an equation where the efficiency in pump mode is used. Schmiedl [27] relates the efficiency in turbine mode to the one in pump mode, as well as to the specific speed in turbine mode, while Grover [28] requires to input a priori the specific speed value of a PaT in turbine mode.

The authors of this work used a large experimental data-set composed of 59 different BEP values in both pump and turbine modes, whose data were collected in [16] and [34]. This large data-set allowed the range of validity of the model to be widened and the performance prediction accuracy. Therefore, the final aim of this work is to provide a new set of equations, which are developed to identify the BEP of PaTs, able to predict flow and head coefficients, as well as the mechanical efficiency at BEP, with a high accuracy and with a better generalization capability, using only the main design data of the pump mode operation. After a non-dimensional analysis of the used data-set, two correlations are proposed to forecast the specific speed and the specific diameter of the PaT at BEP in turbine mode, knowing the corresponding values in pump mode. Once these two values are known, both flow and head coefficients are evaluated using well-known formulations for turbomachines and, subsequently, their dimensional values are computed. Moreover, a new correlation for the prediction of the mechanical efficiency at BEP in turbine mode is supplied by linking two parameters at BEP in pump mode, namely the specific speed and the mechanical efficiency.

The present paper is structured as follows: Section 2 describes i) the experimental data-set and its elaboration using the non-dimensional analysis and ii) the mathematical formulas used for the prediction of the main magnitudes of the PaTs at BEP in turbine mode. Section 3 resumes the description of both the experimental apparatus and the Computational Fluid Dynamics (CFD) models employed to obtain experimental and numerical data of six different PaTs studied by the authors of this work and by other scientists, whose some of them were provided by the Brno University of Technology (BUT) and the Polytechnic University of Bari (POLIBA). Subsequently, these data were used in Section 4 to validate the proposed model that is finally compared with those available in literature, such as [25, 26, 27, 28], in order to evaluate and assess its accuracy. Finally, Section 5 reports the conclusions of the work.

2. Research and methods

2.1. Experimental data-set and non-dimensional analysis

Experimental data of 59 PaTs operating at BEP in both pump and turbine modes were collected from the literature and rearranged by means of the non-dimensional analysis.

These data were used in the proposed prediction model in order to provide correlations between the operating modes of PaTs. Specifically, the formulations aim to predict specific speed, specific diameter and mechanical efficiency of PaTs, operating in turbine mode, using the same parameters in pump mode as input values. The hydraulic machines involved in this work are included in a wide range of operating conditions and dimensions as listed in in Table 1. Most of them belong to the categories of radial and mixed-flow hydraulic machines.

Figure 1 shows the distribution of the 59 BEP values in terms of flow rate vs head (left) and flow rate vs efficiency (right), showing that the majority of data are within the range of flow rate $0.00 \text{ [m}^3/\text{s]} - 0.10 \text{ [m}^3/\text{s]}$.

Table 1: Range of BEP values related to 59 PaTs operating in pump mode

Flow rate [m^3/s]	Head [m]	Rotating speed [rpm]	Impeller diameter [m]	Specific speed [$-$]	Mechanical efficiency [$-$]
[0.005 – 0.222]	[4.00 – 135.00]	[750 – 2445]	[0.139 – 0.330]	[0.09 – 2.24]	[0.44 – 0.87]

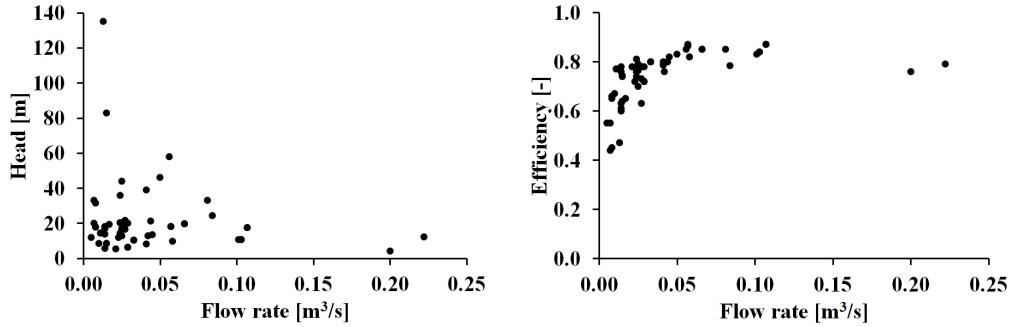


Figure 1: Flow rate vs head (left) and flow rate vs efficiency (right) of 59 PaTs operating in pump mode

The non-dimensional analysis of the experimental data-set, commonly used for extending operating data to other hydraulic machines that operate with the

same fluid dynamic similarity conditions, involves the evaluation of the flow coefficient (Φ) and the head coefficient (Ψ). Knowing the values of both flow and head coefficients, the specific speed (N_s) and the specific diameter (D_s) are obtained by means of Equations (1) and (2), respectively:

$$N_s = \frac{\sqrt{\Phi}}{\sqrt[4]{\Psi^3}}, \quad (1)$$

$$D_s = \frac{\sqrt[4]{\Psi}}{\sqrt{\Phi}}, \quad (2)$$

Both N_s and D_s are mainly used for classifying the hydraulic machines according to their design and, subsequently, their fluid path (e.g. radial, mixed-flow and axial).

2.2. Methodology used to predict PaTs performance at BEP

Both N_s and D_s parameters, which were evaluated by means of the experimental data-set related to 59 PaTs operating in both pump (p) and turbine modes (t), were used to develop the following functions:

$$N_{st} = f(N_{sp}), \quad D_{st} = f(D_{sp}). \quad (3)$$

The first function is expressed by Equation (4), which expresses N_{st} as a function of N_{sp} :

$$N_{st} = 0.9051 \cdot N_{sp}. \quad (4)$$

This is a first order equation that fits accurately the collected experimental data-set, achieving an R^2 -value equal to 0.9488. It is worth noticing that an increase to a second order would have led to a minimal improvement (+1.6%) in terms of R^2 -value with respect to Equation (4); for this reason, a first order equation was chosen. Such a result allows us to conclude that there is a good correlation between the specific speed in pump and in turbine modes considering the investigated hydraulic machines and operating ranges of non-dimensional parameters. Figure 2 shows the correlation expressed by Equation (4) together with the experimental points available in the data-set (black dots).

Similarly, the second function is reported in Equation (5), which expresses D_{st} as a function of D_{sp} :

$$D_{st} = 0.9436 \cdot D_{sp}. \quad (5)$$

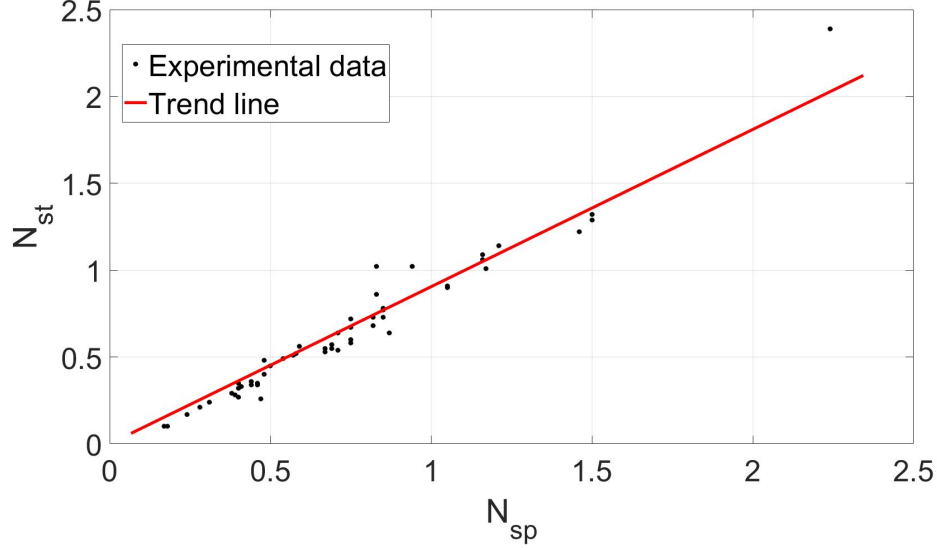


Figure 2: Correlation between N_{sp} and N_{st} values at respective BEPs

145 This is a first order equation that fits accurately the collected experimental
 146 data-set, achieving an R^2 -value equal to 0.986. Also in this case, since the lin-
 147 ear equation fits accurately the experimental values, an increase of the equation
 148 order would have led to a minor improvement of the R^2 -value (+0.2%) with
 149 respect to Equation (5); thus, a linear equation was chosen. Figure 3 displays
 150 the correlation expressed by Equation (5) together with the experimental points
 151 available in the data-set (black dots). The trend line in Figure 3 shows a similar
 152 behaviour as the one in Figure 2. It is worth noticing that the majority of the
 153 points that are shown in Figures 2 and 3 refer to radial and mixed-flow hydraulic
 154 machines. For this reason, it is suggested to apply the proposed model only for
 155 PaTs having an N_{sp} lower than 1.5, as well as a D_{sp} lower than 10. As previously
 156 mentioned in Section 1, also Tan et al. [25] proposed linear correlations between
 157 N_{sp} and N_{st} , as well as between D_{sp} and D_{st} , obtaining different coefficients.
 158 However, the number of data used by [25] was lower than the one reported in
 159 this work since only four PaTs were employed to develop the correlations, thus
 160 resulting in a narrow range of both N_s [0.58 – 1.52] and D_s [1.24 – 2.80].

161 Knowing the data of N_{sp} and D_{sp} at BEP of a generic pump, both N_{st} and D_{st}
 162 at BEP in turbine mode are evaluated by means of Equations (4) and (5). Once
 163 N_{st} and D_{st} are predicted, the values of Φ_t and Ψ_t are calculated by solving the

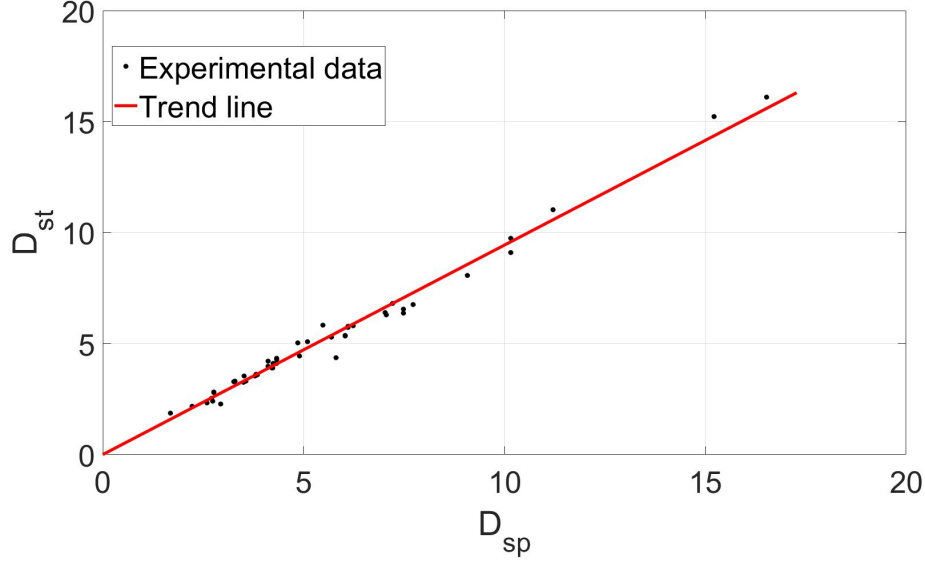


Figure 3: Correlation between D_{sp} and D_{st} values at respective BEPs

164 following two-equation system:

$$\begin{cases} N_{st} = \frac{\sqrt{\Phi_t}}{\sqrt[4]{\Psi_t^3}} \\ D_{st} = \frac{\sqrt[4]{\Psi_t}}{\sqrt{\Phi_t}} \end{cases} \quad (6)$$

165 In particular, solving the first equation of the system for $\sqrt{\Phi_t}$ and inserting
166 it in the second one, Ψ_t is:

$$\Psi_t = \frac{1}{(N_{st} D_{st})^2}. \quad (7)$$

Then, once Ψ_t has been computed, Φ_t is evaluated through one of the two equations of System (6). The corresponding dimensional magnitudes, i.e. flow rate (Q) and head (H), are obtained by the following equations:

$$Q = \Phi \cdot \omega \cdot D^3 \quad (8)$$

$$H = \frac{\Psi \cdot \omega^2 \cdot D^2}{g} \quad (9)$$

167 As reported in Section 1, the proposed prediction model supplies a further in-
168 dication of the mechanical efficiency of PaTs at their BEP. In the literature, some

169 correlations between the mechanical efficiency of a PaT operating in both pump
 170 and turbine modes have already been proposed. Alatorre-Frenk [26] suggested a
 171 direct correlation between mechanical efficiencies in both operating modes, as-
 172 suming η_t equal to η_p minus 0.03. Other authors provided correlations between
 173 two variables, such as Schmiedl [27], that correlate η_t with η_p and N_{st} , while
 174 Grover [28] requires to identify N_{st} from experimental tests.

175 However, a priori knowledge of the N_{st} values needs a model to predict the
 176 BEP in turbine mode or data obtained by experimental campaigns. For instance,
 177 N_{st} could be obtained by means of a correlation that uses non-dimensional pa-
 178 rameters in pump mode at BEP, but its prediction inevitably affects the accuracy
 179 of the mechanical efficiency forecast in turbine mode η_t . In order to find out an
 180 accurate correlation to predict η_t , several combinations of non-dimensional pa-
 181 rameters in pump mode were used. In particular, polynomial equations with two
 182 parameters were studied in order to obtain higher forecast capabilities.

183 Table 2 reports the forecast accuracy, in terms of R^2 -values, achieved by using
 184 second order polynomial equations with different non-dimensional parameters
 185 as independent variables. For sake of conciseness, only the top three results are
 186 reported. Among them, the best correlation was obtained by expressing η_t as a
 187 function of N_{sp} and η_p , achieving an R^2 -value equal to 0.7857.

188 The trend of η_t , which is described by the second order Equation (10), is
 189 shown in Figure 4.

Table 2: R^2 -values obtained by correlating two different non-dimensional parameters

1 st variable	2 nd variable	R^2 -value
N_{sp}	η_p	0.7857
Φ_p	η_p	0.7649
Λ_p	η_p	0.7452

$$\eta_t = 0.7933 \cdot N_{sp} + 0.605 \cdot \eta_p - 0.09246 \cdot N_{sp}^2 - 0.8254 \cdot (N_{sp} \cdot \eta_p) + 0.3936 \cdot \eta_p^2. \quad (10)$$

190 The R^2 -value obtained through Equation (10) is lower than the ones achieved
 191 by the linear Equations (4) and (5) for predicting both specific speed N_s and
 192 specific diameter D_s , respectively. However, since the mechanical efficiency is

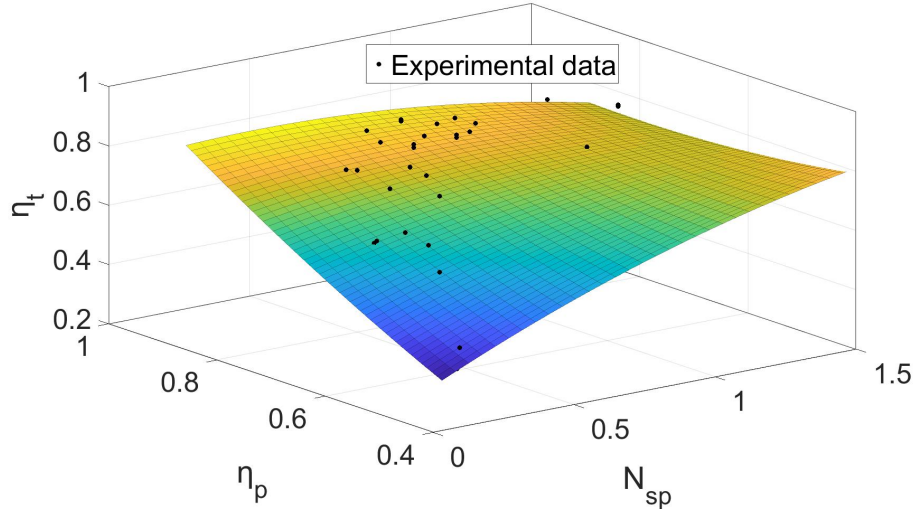


Figure 4: 3D plot of η_t as a function of N_{sp} and η_p

linked not only to the operating principle of the machine but also to its specific design and typology, the result could be considered, either way, satisfactory.

Finally, it is worth noticing that an increase in the forecast capability of the model could be obtained by increasing the number of PaTs involved in the experimental data-set, as well as widening the range of N_s by adding data of other PaTs typologies.

3. Experimental tests and numerical analysis

An experimental apparatus was designed to obtain the operating data of a centrifugal pump (PaT A) operating in both direct and reverse modes. In addition, a numerical analysis was performed on the same hydraulic machine. The comparison between experimental and numerical data of this PaT has been used to validate the numerical CFD simulations. The same CFD numerical models have been employed to assess the operating performance of another two PaTs (PaT B and PaT C), since experimental data on these two machines were not available. Furthermore, an additional three pumps (PaT D, PaT E and PaT F), whose data were provided by Brno University of Technology (BUT) and Polytechnic University of Bari (POLIBA), were used to further assess the prediction capability of the proposed model.

211 The experimental campaign and the numerical simulations have already been
 212 performed and discussed in detail in a previous work by the authors. Therefore,
 213 for the sake of conciseness, this Section has the aim to briefly describe only the
 214 main equipment that composes the test bench and the main numerical methods
 215 used to perform the CFD simulations. For further details, please refer to [16].

216 The experimental apparatus is constituted by: i) a flow meter, ii) two pres-
 217 sure probes, which are located in proximity to the volute's openings and iii) the
 218 torque meter that links the shaft of the PaT to the electric motor/brake. It is
 219 worth noticing that the electric generator is constituted by a two pole pairs syn-
 220 chronous motor. A frequency converter was used in the laboratory set-up but,
 221 for the current tests, the frequency was kept at the rated value of 50 Hz . No
 222 additional closed loop controller was used, therefore the rotational speed was
 223 equal to 1450 rpm . The electric power produced by the PaT was then dissipated
 224 using appropriate electric resistors to avoid introducing harmonic frequencies
 225 in the grid that would have affected its reliability and safety. Table 3 lists the
 226 characteristics of the equipment previously mentioned.

Table 3: Technical data of the equipment that constitute the test rig

Measured quantity	Measurement equipment	Range	Accuracy	Signal
Flow rate	Endress+Hauser Promag 50 W	$0.3 - 10\text{ m/s}$	0.5% (0.2% optional)	$4 - 20\text{ mA}$
Pressure	Keller PA33X	$0 - 10\text{ bar}$	0.1% FS Error band ($10 - 40\text{ }^{\circ}\text{C}$)	$0 - 10\text{ V}$
Torque	Kistler Type 4503A50	$0 - 50\text{ Nm}$	0.1 - 0.2%	$0 - 5\text{ V}$

227 Figure 5 shows the experimental set-up, highlighting the equipment in order
 228 to have a general overview of the used test bench, while Figure 6 displays a
 229 general scheme of the used test bench.

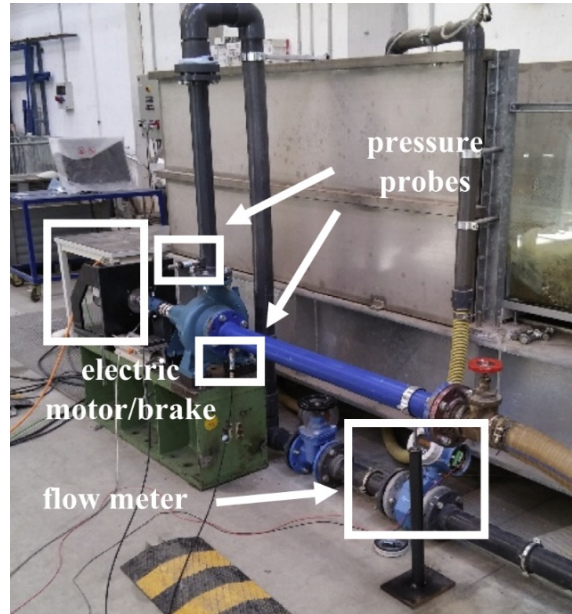


Figure 5: Used test bench [16, 30]

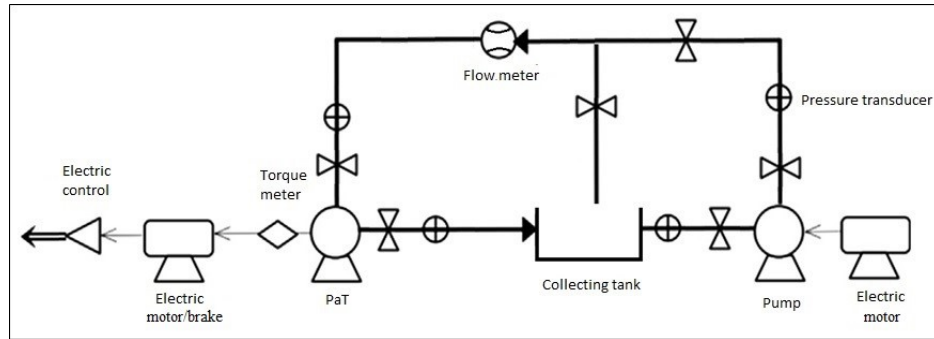


Figure 6: Scheme of the test bench [16, 30]

230 The overall measurement inaccuracy, considering the error propagation, is
 231 0.72% for the hydraulic power, 0.32% for the mechanical power and 0.85% for
 232 the mechanical efficiency [16]. Laboratory tests were performed by keeping the
 233 rotating speed of the PaT constant at 1450 *rpm*, as previously said, and vary-
 234 ing the flow rate exploited by the hydraulic machine with a 3-way valve. This
 235 procedure has been performed for both operating modes of PaT A, allowing a
 236 reconstruction of the entire performance curves, such as characteristic, power

237 and mechanical efficiency curves. Table 4 lists the experimental data of PaT A,
 238 operating at BEP in both pump and turbine modes.

Table 4: Experimental data of PaT A at BEP in both pump and turbine modes

Magnitude	Pump mode	Turbine mode
Flow rate [m^3/s]	0.014	0.021
Head [m]	10.0	15.0
Efficiency [—]	0.76	0.76
Rotating speed [rpm]		1450
Specific Speed [—]	0.57	0.53
Impeller Diameter [m]		0.193
Specific Diameter [—]	5.15	4.66
Number of blades		7
Liquid processed		H_2O ($\rho \approx 1000 \text{ kg}/m^3$)

239 The numerical analysis was performed with the commercial ANSYS® soft-
 240 ware package and, in particular, using the CFX® solver [35] based on the Finite
 241 Volume Method (FVM). The mesh grid independence was ensured by successive
 242 mesh refinements until a maximum difference of 0.7%, in terms of two contigu-
 243 ous results of head, was achieved [16].

244 The experimental data of PaT A, which are reported in Table 4, were used
 245 to validate the accuracy of the numerical results obtained for the same machine.
 246 Considering the values of head and efficiency obtained by the CFD simulations
 247 in both pump and turbine modes, a maximum absolute relative error of 2.00%
 248 was achieved, which represents a very good result for the engineering practice
 249 [16]. After the validation of the simulations, the same numerical CFD models
 250 were used to perform the CFD simulations on 3D CAD geometries of both PaT
 251 B and PaT C.

252 The authors of this work have already validated the CFD simulations of PaT
 253 B and PaT C by means of an analytical model, based on experimental data, which
 254 was described and applied in [16], showing a good forecast capability both at BEP
 255 and in off-design operating conditions ($\pm 40\%$ with respect to BEP) in turbine
 256 mode.

257 Since PaTs belong to the turbomachinery category, an interface between sta-
 258 tionary and rotating domains was created in the definition of the computational

fluid domain. In this work, the Frozen Rotor Model (FRM) [35], which is a steady-state method that allows a reduction in the computational effort using rotating reference frame, was used. Regarding diffusive and convective terms, both of them were discretized with High-Resolution schemes [36]. Reynolds Averaged Navier-Stokes (RANS) equations, coupled with the standard $k - \omega$ two-equation model, were used for running steady-state simulations of the turbulent flow. For the near wall-treatment, the automatic wall function was employed, which is a blending between viscous sublayer and log-law relation. Finally, the normalized residuals drop, considering all the simulations, was between 10^{-5} and 10^{-10} . As a result, y^+ values below 30 were obtained along the impeller walls. However, y^+ values close to 250 – 300 were detected in tip gap regions, which are considered still reasonable if the global performance of a hydraulic machine has to be evaluated [37], without focusing the attention on the leakage flow field.

To have an overview of the spatial discretization for the simulations of PaTs A, B and C, the numbers of elements of the mesh in the single fluid domains are reported in Table 5.

Table 5: Mesh elements of PaTs A, B and C

Geometry	n° of elements of PaT A	n° of elements of PaT B	n° of elements of PaT C
Impeller	1, 398, 917	743, 544	1, 514, 004
Inlet/Exit pipes	195, 342	84, 319	107, 752
Volute	431, 669	277, 643	182, 814
Total	2, 025, 928	1, 055, 506	1, 804, 670

The numerical results obtained in both operating modes of PaTs A, B and C at BEP are instead shown in Table 6.

Table 6: Numerical data of PaTs A, B and C at their BEPs in both direct and reverse modes

Characteristics	PaT A		PaT B		PaT C	
	Pump mode	Turbine mode	Pump mode	Turbine mode	Pump mode	Turbine mode
Flow rate [m^3/s]	0.014	0.021	0.077	0.109	0.120	0.228
Head [m]	10.20	14.70	21.59	20.80	32.00	70.40
Efficiency $[-]$	0.75	0.76	0.80	0.79	0.66	0.67
Rotating speed [rpm]	1450		1450		1500	
Specific speed $[-]$	0.57	0.51	0.76	0.93	0.73	0.56
Impeller Diameter [m]	0.193		0.281		0.340	
Specific diameter $[-]$	5.16	4.62	3.86	3.22	4.13	3.65
Number of blades	7		6		10	
Liquid processed	H_2O ($\rho \approx 1,000 \text{ kg/m}^3$)		H_2O ($\rho \approx 1,000 \text{ kg/m}^3$)		H_2O ($\rho \approx 1,000 \text{ kg/m}^3$)	

Along the same line, the experimental data of PaT D, PaT E and Pat F in both operating modes are shown in Table 7.

Table 7: Experimental provided data of PaTs D, E and F at their BEPs in both direct and reverse modes

Characteristics	PaT D		PaT E		PaT F	
	Pump mode	Turbine mode	Pump mode	Turbine mode	Pump mode	Turbine mode
Flow rate [m^3/s]	0.032	0.041	0.064	0.093	0.122	0.177
Head [m]	34.70	49.01	26.80	33.40	21.81	34.92
Efficiency $[-]$	0.79	0.79	0.82	0.80	0.84	0.84
Rotating speed [rpm]	2900		1000		1000	
Specific speed $[-]$	0.69	0.60	0.41	0.41	0.65	0.55
Impeller Diameter [m]	0.174		0.419		0.405	
Specific diameter $[-]$	4.15	4.03	6.66	5.86	4.44	4.15
Number of blades	6		6		7	
Liquid processed	H_2O ($\rho \approx 1,000 \text{ kg/m}^3$)		H_2O ($\rho \approx 1,000 \text{ kg/m}^3$)		H_2O ($\rho \approx 1,000 \text{ kg/m}^3$)	

4. Results and comments

The operating data of the six centrifugal PaTs described in the previous section were not involved in the development of the presented model; therefore, they were used as a validation data-set for validating the proposed BEP prediction model with a fair comparison. BEP values obtained by laboratory tests (PaTs A, D, E and F) and numerical simulations (PaTs B and C) are also compared with the forecast values obtained using the models developed by other researchers,

such as [26, 27, 28, 25], in order to assess their singular and global prediction accuracy.

Table 8 lists the BEP data of the six centrifugal PaTs and the forecast ones using the proposed model, while Table 9 shows the comparison between its accuracy and one of the other models available in the literature. The comparison, which is performed by using the relative percentage error between experimental/numerical data and the forecast ones at BEP, is also shown in Figure 7 for PaT A, B and C, and in Figure 8 for PaT D, E and F.

Table 8: Experimental/Numerical vs predicted BEP values for the six PaTs using the presented model

PaT A	Φ	Ψ	η	Λ	N_s	D_s
BEP _{Exp.}	0.0197	0.1713	0.76	0.0026	0.53	4.58
BEP _{Model}	0.0167	0.1565	0.75	0.0020	0.52	4.87
PaT B	Φ	Ψ	η	Λ	N_S	D_S
BEP _{CFD}	0.0324	0.1121	0.79	0.00286	0.93	3.22
BEP _{Model}	0.0303	0.1595	0.77	0.00369	0.69	3.63
PaT C	Φ	Ψ	η	Λ	N_s	D_s
BEP _{CFD}	0.0369	0.24	0.67	0.00594	0.54	3.01
BEP _{Model}	0.0468	0.2358	0.70	0.00773	0.64	3.22
PaT D	Φ	Ψ	η	Λ	N_s	D_s
BEP _{Exp.}	0.0255	0.1722	0.79	0.0042	0.60	4.03
BEP _{Model}	0.0268	0.1693	0.78	0.0045	0.62	3.92
PaT E	Φ	Ψ	η	Λ	N_s	D_s
BEP _{Exp.}	0.0120	0.1702	0.80	0.0021	0.41	5.86
BEP _{Model}	0.0109	0.1852	0.79	0.0020	0.37	6.28
PaT F	Φ	Ψ	η	Λ	N_s	D_s
BEP _{Exp.}	0.0254	0.1899	0.84	0.0044	0.55	4.15
BEP _{Model}	0.0230	0.1636	0.81	0.0038	0.59	4.19

Table 9: Relative percentage errors between experimental/numerical and predicted BEP values

Error(%) PaT A	Φ	Ψ	η	Λ	N_s	D_s
This paper	-15.23	-8.64	-1.32	-23.08	-1.90	+6.33
Alatorre-Frenk [26]	-43.78	+11.01	-3.95	-45.08	-30.67	+36.90
Schmiedl [27]	+10.53	+25.96	+3.65	+32.19	-11.58	+0.77
Grover [28]	+57.69	+78.74	-13.12	+124.33	—	-7.92
Tan et al.[25]	-35.41	-25.52	+6.86	-52.91	+0.24	+15.59
Error(%) PaT B	Φ	Ψ	η	Λ	N_s	D_s
This paper	-6.48	+42.28	-2.53	+29.02	-25.81	+12.73
Alatorre-Frenk [26]	-45.52	+56.43	-2.53	-75.346	-47.23	+51.52
Schmiedl [27]	+5.96	+79.05	+0.69	-43.29	-33.50	-12.37
Grover [28]	+66.33	+177.32	-13.95	+17.81	—	+0.06
Tan et al.[25]	-26.68	+28.44	+3.57	-71.05	-29.03	+24.33
Error(%) PaT C	Φ	Ψ	η	Λ	N_s	D_s
This paper	+26.83	-1.75	+4.48	+30.13	+18.52	+6.98
Alatorre-Frenk [26]	-43.25	-0.40	-5.97	-91.09	-23.12	+30.34
Schmiedl [27]	+16.32	+16.87	+0.04	-77.97	-4.05	-3.59
Grover [28]	+28.72	+31.18	-14.61	-76.64	—	-5.67
Tan et al.[25]	-46.02	-44.24	+1.41	-95.06	+13.85	+17.62
Error(%) PaT D	Φ	Ψ	η	Λ	N_s	D_s
This paper	+5.10	-1.68	-1.27	+7.14	+3.33	-2.73
Alatorre-Frenk [26]	-37.25	+9.52	-3.80	-28.57	-26.67	+29.53
Schmiedl [27]	+22.35	+24.91	+1.27	+59.52	-6.87	-4.22
Grover [28]	+87.84	+89.72	-13.92	+271.43	—	-14.14
Tan et al.[25]	-19.61	-15.97	+3.80	-28.57	+1.67	+6.95
Error(%) PaT E	Φ	Ψ	η	Λ	N_s	D_s
This paper	-9.17	+8.81	-1.25	-4.76	-9.76	+7.17
Alatorre-Frenk [26]	-48.33	+14.57	-1.25	-42.86	-34.15	+44.54
Schmiedl [27]	-0.83	+32.31	+10.00	+28.57	-18.70	+7.85
Grover [28]	+64.17	+115.33	-10.00	+242.86	—	-5.46
Tan et al.[25]	-39.17	-25.03	+13.75	-57.14	-2.44	+19.45
Error(%) PaT F	Φ	Ψ	η	Λ	N_s	D_s
This paper	-9.45	-13.85	-2.99	-13.64	+7.27	+0.96
Alatorre-Frenk [26]	-50.39	-14.43	-3.59	-54.55	-20.00	+36.39
Schmiedl [27]	-5.12	-0.42	+1.80	+2.27	-1.66	+2.41
Grover [28]	+62.99	+67.40	-13.77	+200.00	—	-11.08
Tan et al.[25]	-31.10	-26.65	+5.39	-45.45	+5.45	+11.33

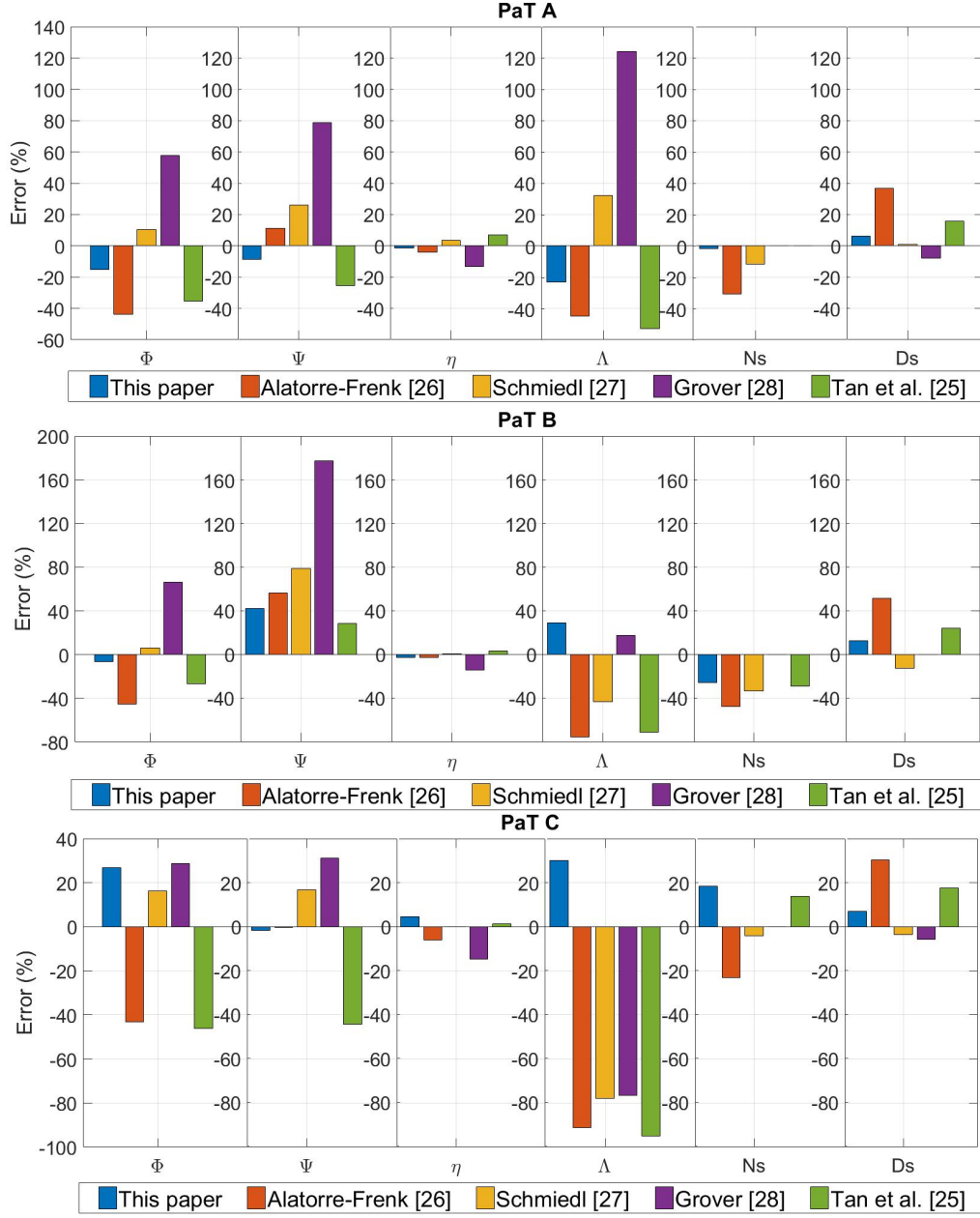


Figure 7: Comparison between the errors obtained by this paper and by another four models available in literature for PaT A (top row), PaT B (middle row) and PaT C (bottom row)

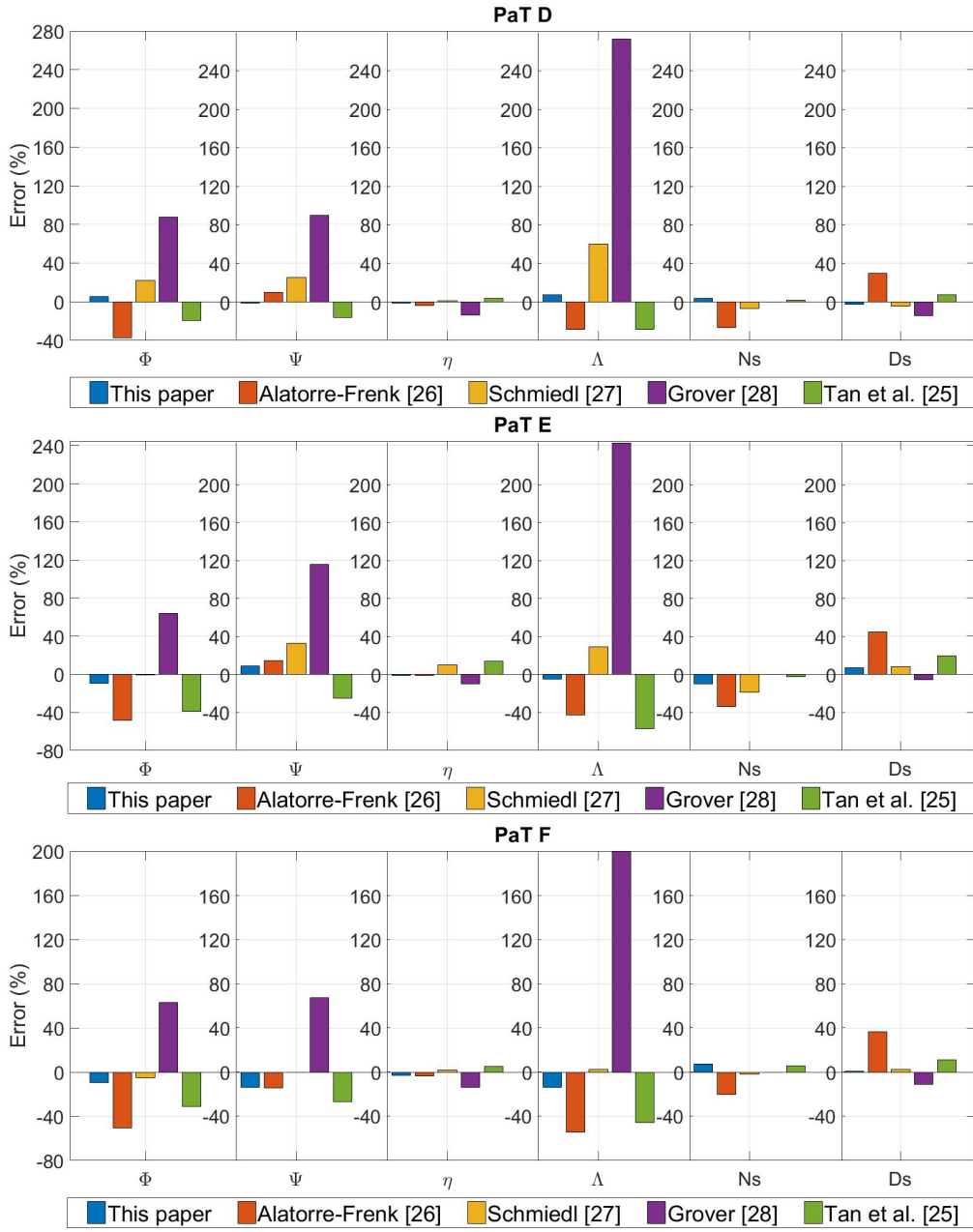


Figure 8: Comparison between the errors obtained by this paper and by another four models available in literature for PaT D (top row), PaT E (middle row) and PaT F (bottom row)

Table 8 shows that the values obtained by using the presented model are close

295 to the experimental/numerical ones globally.

296 Furthermore, as highlighted in Table 9, Figures 7 and 8, even if some forecast
 297 values present relatively high differences with respect to the reference ones, they
 298 show a general good prediction capability when compared to other analytical
 299 methodologies available in the literature.

300 In order to compare the accuracy of the considered models globally, the av-
 301 erage error for each method, taking into account the results obtained by all the
 302 six PaTs, is evaluated as:

$$\overline{\text{Error } a} (\%) = \frac{\sum_{k=1}^6 |\text{Error } a_{PaTk} (\%)|}{6} \quad (11)$$

303 where "a" is a generic variable. In Table 10 and Figure 9, the average errors
 304 obtained by each model related to the main operating parameters are reported
 305 and shown. It is worth noticing that both flow rate and head coefficient errors
 306 are higher than the ones related to the mechanical efficiency due to the error
 307 propagation linked to the the developed correlations between specific speeds
 308 and specific diameters of PaTs operating in both pump and turbine modes, re-
 309 spectively.

Table 10: Average errors between experimental/numerical and considered models data

Error (%)	Φ	Ψ	η	Λ	N_s	D_s
This paper	12.04	12.84	2.31	17.96	11.10	6.15
Alatorre-Frenk [26]	44.75	17.73	3.52	56.25	30.31	38.20
Schmiedl [27]	10.19	29.92	2.91	40.64	12.73	5.20
Grover [28]	61.29	93.28	13.23	155.51	—	7.39
Tan et al.[25]	33.00	27.64	5.80	58.36	8.78	15.88

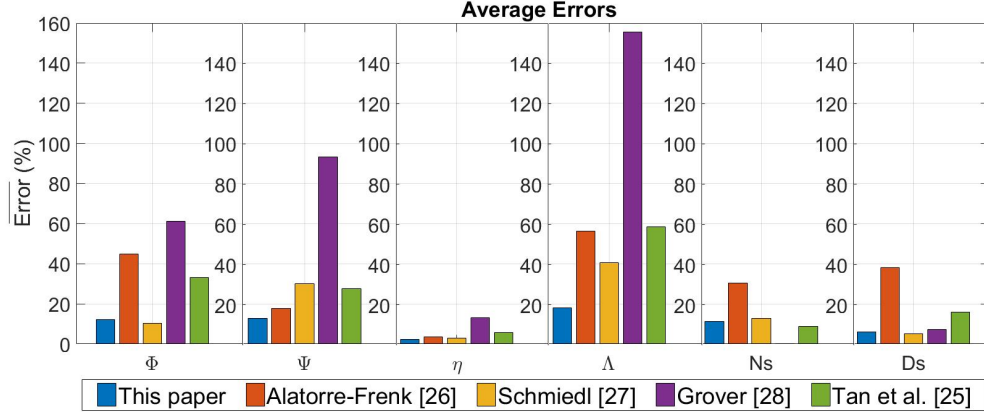


Figure 9: Comparison between the average errors obtained by this paper and selected models available in the literature

Considering the outcomes of this analysis (see Table 9, Figures 7 and 8, Table 10 and Figure 9), a detailed analysis of the prediction capability related to the model described in this work is discussed. Moreover, a comparison between the results obtained by this model and the ones of the four models available in the literature, chosen as terms of comparison, is performed.

In particular, it is possible to highlight the following aspects:

- Flow coefficient (Φ): the model of Schmiedl [27] obtained the best result, reaching a maximum error of 22.35% for PaT D and an average one of 10.19%. The second best model is the presented one that had a maximum error of 26.83% for PaT C, while the average error was equal to 12.04%, being very close to the one achieved by Schmiedl [27]. Regarding the other models, they achieved both maximum and average errors higher than 33%.
- Head coefficient (Ψ): the presented model is the most accurate, showing an average error of 12.84%. It follows the model of Alatorre-Frenk [26] with an average error of 17.73%, whereas the average errors obtained by the other models were higher than 27%.
- Mechanical efficiency (η): the presented model obtained the best results globally, achieving a maximum error equal to 4.48% for PaT B and an average one of 2.31%. Along the same line, the model of Schmiedl [27], together with the ones of Alatorre-Frenk [26] and Tan et al. [25], got indicatively the same results, even though the maximum and average errors

were slightly higher than the presented model. On the other hand, Grover [28] obtained the worst results, showing errors higher than 10% for each PaT and, consequently, an average error equal to 13.23%.

- Power coefficient (Λ): the presented model gained the best results globally, reaching a maximum error of 30.13% for PaT C and an average error equal to 17.96%. All the other models obtained maximum errors higher than 77%. Regarding the average errors, the other models showed values higher than 40%. The reason for these results the fact that the power coefficient is a derived variable, which is evaluated as $\Lambda = \eta \cdot \Phi \cdot \Psi$; thus, the error propagation contributes to increase the difference between the reference data and the predicted ones.
- Specific speed (N_s): the model of [28] has no error since it is based on a priori knowledge of this parameter in turbine mode. On the other hand, the model of Tan et al. [25], the presented model and the one of Schmiedl [27] gave the best results, showing average errors around 10%. The model of [26] reached a maximum error of -47.23% for PaT B and an average error of about 30%.
- Specific diameter (D_s): in this case, Schmiedl [27] obtained the best result with an average error of 5.20%. It follows the presented model and the one of Grover [28] that obtained good results globally. In fact, even if maximum errors close to $\pm 13\%$ were achieved by both models, the average errors were equal to 6.15% and 7.39%, respectively. Regarding the other models, both maximum and average errors were higher than 15%.

Considering the above-mentioned evaluations, it can be stated that the accuracy of the proposed model, as well as of those available in literature, is affected by the non-dimensional parameters used for their development. Nevertheless, the accuracy of the BEP prediction obtained through the presented model was sometimes better or, at least comparable to the best results obtained by the other models. Furthermore, examining the entries of Table 9, it is important to highlight that the presented model shows average errors that are always lower than 17.96%, whereas the other models showed a broader range of errors, even though sometimes they resulted in more accurate predictions of single non-dimensional parameters. Certainly, an increase of the experimental data-set on PaTs performance, as well as widening the typology of tested machines, would lead to a higher reliability of the proposed model and, subsequently, to predicted values closer to the real ones.

5. Conclusions

In this paper, a model used to predict BEP values of PaTs was developed by analysing and rearranging the experimental data collected by laboratory tests on 59 PaTs. This goal was achieved by correlating the specific speed (N_s) and the specific diameter (D_s) in pump mode with the ones in turbine mode. Moreover, a novel approach regarding the prediction of the mechanical efficiency (η) at BEP in turbine mode was proposed in order to supply a complete definition of the PaT performance. The comparison between experimental/numerical and forecast BEP values, taking into account six different centrifugal PaTs that were not used in the development of the presented model, has shown a good general agreement. The proposed model was subsequently compared to other four models chosen from the literature. This analysis showed an improvement in the prediction accuracy in terms of obtained maximum and average errors for some of the non-dimensional variables involved in the study of PaTs. Furthermore, the presented model showed a very narrow range of errors with respect to the other investigated ones. A further improvement in the forecasting accuracy would be possible by increasing the number of experimental data, as well as by widening the typology of PaTs used to develop the presented model.

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