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Direct computation of aeroacoustic fields in laminar flows: solver development and assessment of wall temperature effects on radiated sound around bluff bodies

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1 Abstract

2 This work presents results of a direct computation of acoustic fields produced by
3 several laminar flow configurations. A solver specifically developed for compress-
4 ible mass, momentum and energy equations, named `caafoam`, is presented. Low-
5 storage high-order Runge-Kutta schemes were used for time integration, and an un-
6 structured colocated finite-volume method for space discretization. A sponge-layer-
7 type non-reflective boundary treatment was adopted to avoid spurious numerical
8 reflections at the far-field boundaries. These techniques were chosen and tested to
9 see if they enable a broad range of physical phenomena, with a particular emphasis
10 on aeroacoustic problems, to be solved. The reliability, efficiency and robustness of
11 `caafoam` was demonstrated by computing several benchmarks concerning far-field
12 aerodynamic sound. After proving the direct simulation capabilities of `caafoam`, it
13 was used to analyze the effect of the wall temperature conditions on the aeroacoustic
14 sound produced by laminar flows over bluff bodies.

15 *Key words:*

16 OpenFOAM, Aeroacoustics, Direct Numerical Simulation, Bluff body, Active
17 sound reduction

18 **1 Introduction**

19 The study of noise radiated from objects is a key engineering problem because
20 the noise itself can have significant negative effects on our daily lives.

21 From the engineering standpoint, it is essential to understand the mechanisms
22 of aeroacoustic noise generation and propagation in order to achieve its con-
23 trol/reduction. A number of experimental efforts have been devoted to this
24 issue, but they have met with a few problems relating to aeroacoustic noise. It
25 is really difficult, for instance, to remove background noise that contaminates
26 the aeroacoustic field.

27 Computational aeroacoustic (CAA) techniques can be a reliable way to study
28 aerodynamically-produced sound [1]. They involve several approaches; how-
29 ever our interests are devoted to the direct numerical simulation (DNS) of the
30 aeroacoustic sound, where the flow generating the sound and its propagation
31 are both solved computationally.

32 DNS can encounter several difficulties, largely because the sound pressure
33 is usually much smaller than the ambient pressure [2]. In addition, acous-
34 tic waves are reflected at the far boundaries of the domain when standard
35 boundary conditions are employed and, for DNS computations, ad hoc non-
36 reflecting boundary conditions are needed to fix this issue [3]. To prevent
37 numerical dissipation and dispersion from overshadowing sound production,
38 DNS computations have traditionally been done using high-order methods,
39 such as finite difference (FD) [4], finite volume (FV) [5] or, more recently, dis-
40 continuous Galerkin (DG) methods [6]. For the same reasons, Runge-Kutta
41 (RK) methods are used for time integration. It is worth noting that high-order
42 FD (based on compact schemes) and FV methods carry a loss of parallel effi-
43 ciency due to a non-compact stencil. On the other hand, the theoretical order
44 of accuracy is not preserved when dealing with irregular grids, or at the phys-

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45 ical boundaries. DG methods are more flexible than FV or FD approaches,
46 but they carry a huge computational resource demand [7]. High-order meth-
47 ods have been also employed by CAA investigators since they allow to resolve
48 waves propagation phenomena with the minimum number of mesh points per
49 wavelength [3]. Differently, standard second-order schemes require a grater
50 number of mesh points per wavelength to ensure adequate accuracy. Thus,
51 they are not considered as the cutting-edge solution strategy in CAA.

52 All the above-mentioned high-resolution methods are typically adopted in
53 academic codes with a very limited dissemination to the general public. That
54 is why we have developed an open-source solver for aeroacoustic DNS to
55 publicize the feasibility of performing such computations. Our CAA solver,
56 named `caafoam`, is free to download on GitHub at the following address:
57 <https://github.com/vdalessa/caafoam>. It employs low-storage high-order Runge-
58 Kutta (RK) schemes for time integration, with an accurate artificial sponge-
59 layer-type, non-reflective boundary treatment. The governing equations are
60 space-discretized using an unstructured colocated FV method in order to ex-
61 ploit the solver's flexibility in handling complex geometries. Moreover, our
62 second order approach is also intended as extending the OpenFOAM library
63 capabilities for CAA and compressible flows and it is also conceived as a step-
64 ping stone to higher order implementations in OpenFOAM.

65 The solver has been validated, also by comparing its performance with other
66 freely-available tools, to demonstrate its reliability, efficiency and robustness.
67 Particularly, in the considered cases the sound radiated from bluff bodies in a
68 uniform undisturbed flow is directly simulated.

69 The impact of the thermal boundary conditions on sound propagation is also
70 investigated. It was shown that the wall temperature increment can reduce
71 the lift and drag pulsations and increase the drag generated by the Karman
72 vortex street that is shed over bluff bodies in laminar flows. In the available
73 literature, similar effects had already been noted by Lecordier et al. [8, 9]. In
74 the present context, however, any reduction in lift pulsations is very important

75 because it leads to a decay in aeroacoustic perturbations.

76 This paper is organised as follows: the governing equations are presented in
 77 Section 2, while the adopted numerical discretization techniques are discussed
 78 in Section 3; Section 4 is devoted to numerical results. Lastly, Section 5 con-
 79 tains the conclusions.

80 2 Governing equations

81 The flow model adopted in this work concerns the unsteady **mass,momentum**
 82 **and energy** equations. Let $t \in [0, T]$ be a given instant in the temporal domain,
 83 $\mathbf{x} \in \Omega \subset \mathbb{R}^d$ (with $d = 2, 3$) a given point in the spatial domain, and $Q =$
 84 $\Omega \times [0, T] \subset \mathbb{R}^d \times \mathbb{R}^+$. The initial boundary value problem consists in finding
 85 the solution vector $\mathbf{u} : Q \rightarrow \mathbb{R}^{d+2}$ that, for the given Dirichlet boundary
 86 conditions $\mathbf{u}_D : \Gamma_D \times [0, T] \rightarrow \mathbb{R}^{d+2}$, Neumann boundary conditions $\mathbf{h}_N :$
 87 $\Gamma_N \times [0, T] \rightarrow \mathbb{R}^{d+2}$, and initial conditions $\mathbf{u}_0 : \Omega \rightarrow \mathbb{R}^{d+2}$, satisfy the **governing**
 88 equations:

$$\begin{aligned}
 & \frac{\partial \mathbf{u}}{\partial t} + \frac{\partial \mathbf{f}_{\mathbf{c},j}}{\partial x_j} = \frac{\partial \mathbf{f}_{\mathbf{v},j}}{\partial x_j} && \text{in } Q, \\
 & \mathbf{u} = \mathbf{u}_D && \text{on } \Gamma_D \times [0, T], \\
 & \frac{\partial \mathbf{u}}{\partial x_j} n_j = \mathbf{h}_N && \text{on } \Gamma_N \times [0, T], \\
 & \mathbf{u} = \mathbf{u}_0 && \text{in } \Omega \subset \mathbb{R}^d, t = 0,
 \end{aligned} \tag{1}$$

90 where $\Gamma = \Gamma_D \cup \Gamma_N$ is the boundary of the domain Ω ; Γ_D and Γ_N are the
 91 Dirichlet and Neumann boundaries, respectively; and n_j are the components
 92 of the outward-facing unit normal vector on Γ .

93 Relying on the vector $\mathbf{u} = (\rho, \rho u_1, \rho u_2, \rho u_3, \rho E)^T$, the j -th component of the

94 convective and diffusive fluxes reads:

$$95 \quad \mathbf{f}_{\mathbf{c},j} = \begin{pmatrix} \rho u_j \\ \rho u_1 u_j + p \delta_{1j} \\ \rho u_2 u_j + p \delta_{2j} \\ \rho u_3 u_j + p \delta_{3j} \\ \rho u_j H \end{pmatrix}, \quad \mathbf{f}_{\mathbf{v},j} = \begin{pmatrix} 0 \\ \tau_{1j} \\ \tau_{2j} \\ \tau_{3j} \\ \tau_{ji} u_i - q_j \end{pmatrix}. \quad (2)$$

96 In these relations, ρ denotes the density, u_i is the generic Cartesian component
 97 of the velocity vector \mathbf{v} , and p is the pressure. E is the total internal energy,
 98 while the total enthalpy is obtained from $H = E + p/\rho$. The viscous stress
 99 tensor is computed using the standard constitutive relation for Newtonian
 100 fluids:

$$101 \quad \tau_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \mu \frac{\partial u_k}{\partial x_k} \delta_{ij} \quad (3)$$

102 and the heat flux vector components by means of the Fourier postulate:
 103 $q_i = -\lambda \frac{\partial T}{\partial x_i}$. Note that μ is the dynamic viscosity and λ the thermal con-
 104 ductivity **which in this work are modeled as temperature independent**. The
 105 fluid temperature, T , is measured starting from the total internal energy as
 106 follows: $c_v T = E - \frac{1}{2} \mathbf{v} \cdot \mathbf{v}$, where c_v is the specific heat at constant volume.
 107 Lastly, the pressure is computed by adopting the ideal gas equation of state
 108 as a thermodynamic model: $p = \rho(\gamma - 1) \left(E - \frac{1}{2} \mathbf{v} \cdot \mathbf{v} \right)$, where $\gamma = c_p/c_v$ is
 109 the specific heat ratio of the fluid.

110 2.1 Non-reflective boundary treatment

111 As discussed in Section 1, to compute acoustic wave propagation phenom-
 112 ena we need to avoid spurious numerical sound waves **produced by external**
 113 **boundaries of the domain**. An artificial sponge layer [10, 11] is used for this

114 purpose. The sponge treatment has been widely used because it is simple, ro-
 115 bust and flexible in handling complex geometries [12]. Taking this approach,
 116 the **governing** equations are modified as follows:

$$117 \quad \frac{\partial \mathbf{u}}{\partial t} + \frac{\partial \mathbf{f}_{\mathbf{c},j}}{\partial x_j} - \frac{\partial \mathbf{f}_{\mathbf{v},j}}{\partial x_j} = \sigma (\mathbf{u}_{ref} - \mathbf{u}) \quad \text{in } Q \quad (4)$$

118 The new non-physical term on the right-hand side of eq. 4 is only active
 119 near the external boundaries, where it dampens the flow variables to a known
 120 reference solution, \mathbf{u}_{ref} . In eq. 4, the scalar field $\sigma : \Omega \rightarrow \mathbb{R}$ is:

$$121 \quad \sigma = \sigma_0 \left(\frac{L_{sp} - d}{L_{sp}} \right)^n \quad (5)$$

122 where L_{sp} is the thickness of the layer, d is the minimum distance from the
 123 nearest far-field boundary, σ_0 is a constant value, and n is an integer parameter
 124 controlling the shape of the sponge's profile. An optimal sponge layer design
 125 is not trivial: larger sponges perform better than equally-strong smaller ones.
 126 In other words, they dampen flow features more quietly [13]. Larger sponges
 127 demand larger computational domains; **indeed** they must be positioned far
 128 enough away from the sound sources to avoid interference phenomena with
 129 the flow/acoustic fields.

130 **Another possible non-reflective approach consists in the adoption of sponge-**
 131 **layers which exploit the numerical dissipation produced by the grid stretching.**
 132 **Despite its conceptual simplicity this technique poses difficulties with regard**
 133 **to the evaluation of the grid stretching entity and grid cells' number needed**
 134 **to be applied in the buffer zone. The specific choice is often related to the**
 135 **computational experience gained on a particular code [14]. For this reason in**
 136 **the following we prefer polynomial sponge-layers.**

137 Mani [13] recently ran a theoretical and numerical analysis on non-reflecting
 138 boundary treatments based on **polynomial** sponge layers. The Author pro-
 139 vided several practical guidelines for CFD/CAA practitioners on how to avoid
 140 sponge failure. In particular, the non-reflective boundary implementation is

141 based on the following parameter:

$$142 \quad \eta_{target} = -\frac{40 \log_{10} e}{1 - M_{\infty}^2} \int_{L_{sp}} \sigma d\mathbf{x}, \quad (6)$$

143 where η_{target} is the sponge's strength expressed in dB, and M_{∞} is the Mach
144 number of the undisturbed flow. As an example, a sponge with a strength of
145 40 dB would dampen the amplitude of an incident sound wave by a factor of
146 100 under one-dimensional conditions. The sponge's thickness must also be
147 established with the following constraint:

$$148 \quad 0.5 \leq \frac{L_{sp} \cdot f}{c_{\infty}} \leq 2 \quad (7)$$

149 where f is the sound disturbance frequency, and c_{∞} is its phase speed [13].
150 For all the computations presented in this paper, we have observed that
151 $\eta_{target} = 40$ dB is needed, so $n = 2$ in eq. 5 has been selected. Indeed, Mani [13]
152 investigated the effect of n on the sponge performance and it showed that
153 quadratic sponge has best overall performance for η_{target} ranging from 20 dB
154 to 60 dB. Lastly, the dimensionless parameter $(L_{sp} \cdot f) / c_{\infty}$, strictly needed to
155 evaluate sponge width, is fixed equal to 0.5 to limit the computational load.

156 2.2 Computing the distance from far-field boundaries

157 For the purpose of establishing the distance from far-field (non-reflective)
158 boundaries, we have solved the Eikonal differential equation:

$$159 \quad \frac{\partial \varphi}{\partial x_j} \frac{\partial \varphi}{\partial x_j} = 1 \quad \text{in } \Omega, \quad (8)$$

160 where $\varphi : \Omega \rightarrow \mathbb{R}$ is the distance field. A homogeneous Dirichelet condition
161 is imposed on the non-reflective boundaries, and a homogeneous Neumann
162 boundary condition elsewhere.

163 The Eikonal equation computes the exact distance, defined as the distance

164 from the boundary normal direction. In other words, the distance can be seen
 165 as an advancing front with a unit velocity in the direction of the boundary
 166 normal. The main advantage of this technique is its good scalability on larger
 167 meshes.

168 The solution for eq. 8 has thus been obtained by converting it into a hyperbolic
 169 problem, adding a pseudo-time term:

$$170 \quad \frac{\partial \varphi}{\partial \tau} + u_{\varphi,j} \frac{\partial \varphi}{\partial x_j} = 1 \quad \text{in } Q \quad (9)$$

171 with $u_{\varphi,j} = \partial \varphi / \partial x_j$. The solver for computing far-field distance, named `eikonal`,
 172 is free to download at <https://github.com/vdalessa/eikonal>. It only has to be
 173 run once in the pre-processing stage because we rely on non-moving meshes.

174 3 Numerical approximation

175 3.1 Finite volume discretization

176 In the unstructured, collocated, cell-centered FV method adopted in this work,
 177 the computational domain Ω is divided into a set of non-overlapping polyg-
 178 onal cells. Finite volume discretization is briefly recalled here as it is crucial
 179 to discussing the approximation techniques for each term appearing in the
 180 discrete equations. In the following expressions, the values of the variables at
 181 the center of the cell faces are indicated with the subscript $(\cdot)_f$. The term \mathbf{S}_f
 182 is the surface area vector of each mesh face; see Fig. 1 for a schematic repre-
 183 sentation.

184 Starting from the integration of eq. 4 over each mesh element, K (having
 185 boundary ∂K), we obtain:

$$186 \quad \int_K \frac{\partial \mathbf{u}}{\partial t} \, d\Omega + \int_{\partial K} (\mathbf{f}_{c,j} - \mathbf{f}_{v,j}) n_j \, d\Gamma = \int_K \sigma (\mathbf{u}_{ref} - \mathbf{u}) \, d\Omega. \quad (10)$$

The non-linear convective term is discretized as follows:

$$\int_{\partial K} \mathbf{f}_{\mathbf{c},j} n_j \, d\Gamma = \sum_{f=1}^{N_f} \left(\mathbf{f}_{\mathbf{c},j} \right)_f n_j |\mathbf{S}_f| \quad (11)$$

187 where N_f is the number of faces belonging to the mesh element K . Rewriting
188 the Eulerian terms vector as:

$$189 \quad \mathbf{f}_{\mathbf{c},j} = u_j \mathbf{u} + \mathbf{f}_{\mathbf{c}_p,j} + \mathbf{f}_{\mathbf{c}_e,j} \quad (12)$$

190 with $\mathbf{f}_{\mathbf{c}_p,j} = (0, p\delta_{1j}, p\delta_{2j}, p\delta_{3j}, 0)^T$ and $\mathbf{f}_{\mathbf{c}_e,j} = (0, 0, 0, 0, u_j p)^T$, it can be ap-
191 proximated as follows:

$$192 \quad \sum_{f=1}^{N_f} \left(\mathbf{f}_{\mathbf{c},j} \right)_f n_j |\mathbf{S}_f| = \sum_{f=1}^{N_f} \phi_f \mathbf{u}_f + \sum_{f=1}^{N_f} \Lambda_f^{(1)} |\mathbf{S}_f| + \sum_{f=1}^{N_f} \phi_f \Lambda_f^{(2)} \quad (13)$$

193 where $\Lambda_f^{(1)} = (0, p, p, p, 0)^T$ and $\Lambda_f^{(2)} = (0, 0, 0, 0, p)^T$. A first way to handle
194 the three terms on the right-hand side of eq. 13 that we consider here follows
195 the Kurganov-Noelle-Petrova (KNP) approach [15]:

$$196 \quad \begin{aligned} \sum_{f=1}^{N_f} \phi_f \mathbf{u}_f &= \sum_{f=1}^{N_f} \frac{(\psi \phi \mathbf{u})_f^+ - (\psi \phi \mathbf{u})_f^-}{\psi_f^+ + \psi_f^-} + \frac{\psi_f^+ \psi_f^-}{\psi_f^+ + \psi_f^-} (\mathbf{u}_f^+ + \mathbf{u}_f^-), \\ \sum_{f=1}^{N_f} \Lambda_f^{(1)} \mathbf{S}_f &= \sum_{f=1}^{N_f} \frac{\psi_f^+}{\psi_f^+ + \psi_f^-} |\mathbf{S}_f| (\Lambda^{(1)})_f^+ + \frac{\psi_f^-}{\psi_f^+ + \psi_f^-} |\mathbf{S}_f| (\Lambda^{(1)})_f^-, \\ \sum_{f=1}^{N_f} \Lambda_f^{(2)} \mathbf{S}_f &= \sum_{f=1}^{N_f} \frac{(\psi \phi \Lambda^{(2)})_f^+ - (\psi \phi \Lambda^{(2)})_f^-}{\psi_f^+ + \psi_f^-}. \end{aligned} \quad (14)$$

197 Note that the term ϕ_f in the above equation represents the velocity flux
198 through the cells' face, and it is evaluated as: $\phi_f = \mathbf{v}_f \cdot \mathbf{S}_f$. In eq. 14, the
199 superscript $+$ denotes the face value of the element placed in the direction
200 parallel to the \mathbf{S}_f vector depicted in Fig. 1; and the superscript $-$ the oppo-
201 site direction. These values are obtained by means of a linear interpolation;

202 for example, the + interpolation for \mathbf{u}_f , *i.e.* \mathbf{u}_f^+ , is simply:

$$203 \quad \mathbf{u}_f^+ = \left(1 - \frac{\mathbf{S}_f \cdot \mathbf{d}_{fN}}{|\mathbf{S}_f| |\mathbf{d}_{fN}|}\right) \mathbf{u}_P + \frac{\mathbf{S}_f \cdot \mathbf{d}_{fN}}{|\mathbf{S}_f| |\mathbf{d}_{fN}|} \mathbf{u}_N, \quad (15)$$

204 the meaning of \mathbf{d}_{fN} is depicted in Fig. 1. ψ_f^+ and ψ_f^- are associated with the
 205 local speed of propagation, and they are calculated as reported in Greenshields
 206 et al. [16]:

$$207 \quad \begin{aligned} \psi_f^+ &= \max \left(|\mathbf{S}_f| \sqrt{\gamma RT_f^+} + \phi_f^+, |\mathbf{S}_f| \sqrt{\gamma RT_f^-} + \phi_f^-, 0 \right), \\ \psi_f^- &= \max \left(|\mathbf{S}_f| \sqrt{\gamma RT_f^+} - \phi_f^+, |\mathbf{S}_f| \sqrt{\gamma RT_f^-} - \phi_f^-, 0 \right), \end{aligned} \quad (16)$$

208 where R is the gas constant.

209 **KNP scheme was selected since: (i) there are no Riemann solvers and charac-**
 210 **teristic decomposition involved [15]; (ii) it is was already implemented within**
 211 **OpenFOAM package and repeatedly tested; so it produces a reliable approxi-**
 212 **mate solution of the Riemann problem.**

213 In this paper, we also consider a second approach to approximating the Eu-
 214 lerian numerical flux in which we split $\mathbf{f}_{\mathbf{c},j}$ into a convective and a pressure
 215 part:

$$216 \quad \mathbf{f}_{\mathbf{c},j} = \mathbf{f}_{\mathbf{cH},j} + \mathbf{f}_{\mathbf{cP},j} \quad (17)$$

217 with $\mathbf{f}_{\mathbf{cH},j} = u_j (\rho, \rho u_1, \rho u_2, \rho u_3, \rho H)^T$; so FV approximation for $\mathbf{f}_{\mathbf{c},j}$ is:

$$218 \quad \sum_{f=1}^{N_f} (\mathbf{f}_{\mathbf{c},j})_f n_j |\mathbf{S}_f| = \sum_{f=1}^{N_f} (\mathbf{f}_{\mathbf{cH},j})_f n_j |\mathbf{S}_f| + \sum_{f=1}^{N_f} (\mathbf{f}_{\mathbf{cP},j})_f n_j |\mathbf{S}_f|. \quad (18)$$

219 The convective part of the Eulerian flux is computed here by following Piroz-
 220 zoli's energy-conserving scheme [17]:

$$221 \quad \mathbf{f}_{\mathbf{cH},j} = \frac{1}{8} (\rho^+ + \rho^-) (u_n^+ + u_n^-) (\varphi^+ + \varphi^-) \quad (19)$$

222 where $\boldsymbol{\varphi} = (1, u_1, u_2, u_3, H)^T$ and $u_n = u_j n_j$. The pressure flux is obtained
 223 from:

$$224 \quad \mathbf{f}_{\mathbf{c}_p, j} = \frac{1}{2} \left(\mathbf{f}_{\mathbf{c}_p, j}^+ + \mathbf{f}_{\mathbf{c}_p, j}^- \right) + \mathbf{f}_{\mathbf{c}_p, j}^D. \quad (20)$$

225 The diffusive part in the numerical flux of eq. 20, $\mathbf{f}_{\mathbf{c}_p, j}^D$, is activated to increase
 226 the stability of the discretization technique in computations on unstructured
 227 or distorted meshes. In particular, to activate $\mathbf{f}_{\mathbf{c}_p, j}^D$ we rely on a classical shock
 228 sensor, [18]:

$$229 \quad \theta = \max \left(-\frac{\nabla \cdot \mathbf{v}}{\sqrt{(\nabla \cdot \mathbf{v})^2 + |\nabla \wedge \mathbf{v}|^2 + u_0^2/L_0^2}}, 0 \right) \quad \theta \in [0, 1] \quad (21)$$

230 where u_0 and L_0 are suitable velocity and length scales [19]. In the cases
 231 considered in this paper, as in Modesti and Pirozzoli [20], the artificial diffusion
 232 term is designed to be proportional to $\theta_f = (\theta^+ + \theta^-) / 2$:

$$233 \quad \left(\mathbf{f}_{\mathbf{c}_p, j}^D \right)_f = \alpha \theta_f \left(\mathbf{f}_{\mathbf{c}_p, j}^{\text{AUSM}} \right)_f. \quad (22)$$

234 Note that α is a flag controlling the activation of the diffusive pressure flux,
 235 while $\mathbf{f}_{\mathbf{c}_p, j}^{\text{AUSM}}$ is obtained using the AUSM⁺-up formula (eqs. (69) to (77) of
 236 Liou [21]).

237 We also wish to mention that the Courant number, Co , is computed in this
 238 work using the following equation:

$$239 \quad \text{Co} = \max \left(|\psi_f^+|, |\psi_f^-| \right) \frac{\delta \Delta t}{|\mathbf{S}_f|} \quad (23)$$

240 with:

$$241 \quad \delta = \frac{1}{\max \left(\mathbf{d} \cdot \frac{\mathbf{S}_f}{|\mathbf{S}_f|}, 0.05 |\mathbf{d}| \right)}, \quad (24)$$

242 \mathbf{d} as shown in Fig. 1.

243 Standard approximation schemes are used for the diffusive fluxes, $\mathbf{f}_{\mathbf{v}, j}$. Since
 244 discussing such techniques is beyond the scope of this manuscript, we refer

245 readers to the textbook by Ferziger and Peric [22] for more details.

246 It is worth noting that flow problems with shock-waves are not considered in
247 the presented numerical methodology. From here on, we refer to the KNP-
248 based solver as `caafoam-m1`, while `caafoam-m2` is used to indicate the solver
249 based on Pirozzoli’s scheme.

250 Lastly, we want to point out that Eikonal equation is solved in its hyperbolic
251 form, eq. 9, using a fully explicit approach. Standard central schemes have
252 been employed for this purpose for structured grids, while upwind techniques
253 have been used for unstructured meshes since in this case the former approach
254 is unstable.

255 3.2 Time integration schemes

256 For each FV, the interpolation coefficients obtained from the discretization
257 process are used to form the following system of ODEs:

$$258 \quad \frac{d\mathbf{U}}{dt} = \mathbf{R}(\mathbf{U}) \quad (25)$$

259 where \mathbf{R} is the residual of the space discretization including the convective,
260 diffusive and source terms; and \mathbf{U} is the degrees of freedom (DoFs) vector.

261 Explicit Runge-Kutta (ERK) schemes were used to solve eq. 25 in the present
262 work. The ERK Williamson formula [23] was implemented to contain memory
263 usage. The integration strategy in the k -th RK stage can be summarized as:

$$264 \quad \begin{aligned} \Delta\mathbf{U}^{(k)} &= A_k \Delta\mathbf{U}^{(k-1)} + \Delta t \mathbf{R}(\mathbf{U}^{(k-1)}), \\ \mathbf{U}^{(k)} &= \mathbf{U}^{(k-1)} + B_k \Delta\mathbf{U}^{(k)}. \end{aligned} \quad (26)$$

265 In eq. 26, the A_k and B_k coefficients are functions of the standard Butcher
266 matrix entries, $\mathbf{R}^{(k)}$ is the residual at the k -th intermediate RK stage, and
267 $\mathbf{U}^{(k)}$ is the DoFs vector at the same RK stage. It is important to note that
268 $\mathbf{U}^{(k)}$, $\Delta\mathbf{U}^{(k)}$ and $\mathbf{R}^{(k)}$ must be stored, so only three storage registers for each

269 variable are needed for this kind of scheme. This enables us to obtain a good
 270 performance in large-scale computations too [24].

271 We considered ERK schemes having an order of accuracy ranging from 2 to
 272 4; the tables of the coefficients A_k and B_k are given below. For the 2nd-order
 273 scheme (with 2 stages), named RK 2-2 in the text, we have:

| A_k | B_k |
|-------|-------|
| 0 | 1.0 |
| -1.0 | 0.5 |

275 The 3rd-order low-storage ERK scheme (with 4 stages), called RK 3-4 in
 276 the paper, is based on the following coefficients proposed by Carpenter and
 277 Kennedy [25]:

| A_k | B_k |
|--------------------|----------------|
| 0 | 8/141 |
| -756391/934407 | 6627/2000 |
| -36441873/15625000 | 609375/1085297 |
| -1953125/1085297 | 198961/526383 |

279 Lastly, a 4th-order accurate approach (with 5 stages) was also adopted, as
 280 proposed by Kennedy et al. [24], and called RK 4-5 in our work:

| A_k | B_k |
|-----------------|-----------------|
| 0 | 0.1496590219993 |
| -0.4178904745 | 0.3792103129999 |
| -1.192151694643 | 0.8229550293869 |
| -1.697784692471 | 0.6994504559488 |
| -1.514183444257 | 0.1530572479681 |

282 3.3 Implementation aspects

283 The solution algorithm is implemented in the OpenFOAM environment [26],
 284 which is an open-source library for numerical simulations in continuum me-
 285 chanics. Thanks to an object-oriented structure, the package is extremely flex-
 286 ible and it allows for outside users to develop complex physical models with
 287 relatively little effort.

288 The basic OpenFOAM classes, *i.e.* `scalarField`, `vectorField` and `tensorField`,
 289 have been conceived to mimic the main mathematical tools needed in tra-
 290 ditional continuum mechanics. Data type can also be specified in the cells
 291 or face centers. We also have two different types of tensor-derivative class:
 292 `finiteVolumeCalculus` or `fvc`, and `finiteVolumeMethod` or `fvm`. The for-
 293 mer performs explicit estimates of tensorial operators, while the latter can
 294 return a matrix representation of a given operation. More details about the
 295 above-mentioned data types can be found in [26, 27, 28].

296 In this paper, we only use the basic classes and the `fvc`-derived class because
 297 we opted for an explicit time integration approach. The hyperbolic version

298 of the Eikonal equation was also solved in a fully explicit way using the `fvc`
299 class.

300 3.4 Parallel performance

301 To investigate the parallel scalability of `caaf Foam`, we considered a widely-used
302 benchmark, i.e. the lid-driven cavity problem of a laminar flow with a low
303 Mach number in a 3D cubic domain [29, 30, 31]. All the boundaries were
304 treated as walls except for the top, which was a moving wall. The strong scal-
305 ing tests were run on a suite of three evenly-spaced grids with a number of
306 cells N_c amounting to: 320^3 , 240^3 and 160^3 . We also set the Reynolds number
307 at 20, and the Mach number relating to the wall velocity at 0.2.

308 In our specific case, the simulations were conducted on two different super-
309 computers: MARCONI-A2 hosted by CINECA; and MareNostrum hosted
310 by BSC. MARCONI is a NeXtScale cluster consisting of 3600 nodes with
311 a Knights Landing (KNL) 68-core, 1.40 GHz Intel processor. Each node is
312 equipped with 96 GB of RAM and 16 GB of multi-channel dynamic ran-
313 dom access memory (MCDRAM). MareNostrum comprises 3456 nodes with
314 two Intel Xeon Platinum 24-core processors of the Skylake (SKL) generation
315 operating at 2.1 GHz for each node. There are 96 GB of RAM available in
316 standard nodes (as used in this work). Both systems are of the Tier-0 type
317 forming part of the PRACE initiative, [32].

318 The scalability tests discussed below were conducted as part of a prepara-
319 tory PRACE project aiming to examine the parallel performance of `caaf Foam`
320 on massively parallel supercomputers. Access to the machines was limited, so
321 these tests could not be performed using all the solvers considered in this work.
322 Only `caaf Foam-m1` was therefore considered at this stage because it shares the
323 same spatial discretization approach as standard OpenFOAM solvers.

324 The tests were conducted without any I/O for 100 time-steps to cancel the
325 starting overhead, and using 64 CPU cores for each MARCONI node, while 48

326 CPU cores were used for each MareNostrum node. The code was built using
327 Intel compilers and the MPI library version developed by Intel.

328 Fig. 2 shows the effect of grid size scalability in terms of speed-up and par-
329 allel efficiency. It is worth noting that inter-node scalability is good on both
330 systems until the latency due to node communications becomes predominant.
331 It is also very obvious that, on MARCONI, smaller grids have a better par-
332 allel performance with fewer cores, while grids with more cells perform better
333 using a larger number of CPU cores. A clearly different trend is apparent on
334 MareNostrum, where performance is almost always super-linear due to cache
335 effects. In this case, smaller grids perform better than larger ones until com-
336 munications issues override the parallel effects. A super-linear behavior is only
337 achieved on MARCONI up to 2048 CPU cores, using the finest grid, on which
338 we obtain a good parallel performance up to 8192 CPU cores. A good effi-
339 ciency was achieved on MARCONI up to $4 \cdot 10^3$ cells for each core, while on
340 MareNostrum we obtained an efficiency of about 88 % with 2250 cells per
341 core.

342 As concerns the above results, it is important to note that adopting an explicit
343 time integration approach is particularly appealing from the parallel efficiency
344 standpoint. Appropriately selecting the scheme coefficients also enables us to
345 obtain good stability limits, as shown in Section 4.1.1. These are the reasons
346 why we consider `caafoam` an appealing tool for massively parallel aeroacoustic
347 simulations.

348 4 Results

349 Several literature benchmark problems were considered to test the reliability of
350 the `caafoam` solver. We considered the far-field aerodynamic sound generated
351 by bluff bodies in a flow with a uniform inlet velocity, in various arrange-
352 ments, at low Mach numbers. The cases of a single circular cylinder and of
353 two square cylinders placed side by side, as well as in a tandem configuration,

354 were analyzed to test the capabilities of our approach. Then a numerical study
 355 was conducted on the effect of the wall's thermal boundary conditions on the
 356 aeroacoustic field by addressing the sound generated by the flow over isolated
 357 square and circular cylinders.

358 In all the above-mentioned cases, the Mach number of the undisturbed flow
 359 was $M_\infty = 0.2$, $\gamma = 1.4$, and the Prandtl number, Pr , was 0.75. We present
 360 the results below in terms of standard parameters relating to fluid dynamic
 361 and acoustic fields, i.e. (i) drag and lift coefficients; (ii) the Strouhal number;
 362 (iii) fluctuations in pressure and its root mean square; and (iv) dilatation rate
 363 field. The dimensionless drag and lift coefficients are given by eq. 27:

$$364 \quad C_D = \frac{2D'}{\rho u_\infty^2 A_{ref}}, \quad C_L = \frac{2L'}{\rho u_\infty^2 A_{ref}}. \quad (27)$$

365 Standard statistics are used to analyze force coefficients behavior: the mean
 366 drag coefficient $\langle C_D \rangle$, the root mean square of the lift coefficient $C_{L,rms}$, and
 367 the amplitudes of oscillation of the force coefficients ($\Delta C_D = (C_{D,max} -$
 368 $C_{D,min})/2$, and $\Delta C_L = (C_{L,max} - C_{L,min})/2$). The Strouhal number is defined
 369 as:

$$370 \quad St = \frac{f L_{ref}}{u_\infty} \quad (28)$$

371 where f is the vortex-shedding frequency found from spectral analysis of the
 372 time history of the fluctuating lift coefficient, and L_{ref} is the reference length.
 373 The acoustic results are presented below in terms of dimensionless fluctuating
 374 pressure, defined as:

$$375 \quad p' = \frac{p - \langle p \rangle}{\rho_\infty c_\infty^2} \quad (29)$$

376 where $\langle p \rangle$ is the average pressure field and c_∞ is the speed of sound of the
 377 undisturbed flow. Polar plots containing the root mean square of p' are shown
 378 to elucidate the sound features in the far field. For the purpose of a comparison
 379 with the literature, the acoustic statistics were sampled over a dimensionless
 380 time $u_\infty T/D = 100$. Unless stated otherwise, the plots are built at $r/D = 75$.
 381 The dilatation rate field, $\partial u_j / \partial x_j$, is also used to visualize the acoustic wave

382 because, taking the mass conservation equation into account, it equates to the
 383 negative rate of change of the density which is directly linked to p' .
 384 Finally, the acoustic power output, defined as the acoustic intensity flux through
 385 a closed circle surrounding the source and having a radius r' , is examined to
 386 estimate the wall heating effects on the sound produced. The analytical ex-
 387 pression of the acoustic power is as follows:

$$388 \quad W = \int_0^{2\pi} I_a(r = r', \theta) R d\theta \quad (30)$$

389 where $I_a = (p'_{rms})^2 / \rho c$ accounts for the mean acoustic intensity in the far-field
 390 region. The sound power level is obtained as:

$$391 \quad L_w = 10 \log_{10} \frac{W}{W_0} \quad (31)$$

392 where W_0 is the reference acoustic power.

393 All the solutions were obtained on distributed-memory parallel machines: the
 394 computations requiring a lower load were run on a Linux Cluster, with 16 Intel
 395 Xeon E5-2603v3-based nodes, for a total of 192 CPU cores operating at 1.6
 396 GHz. Larger cases were run on a MARCONI-A2 system. Intel's `libhbm` library,
 397 which can be downloaded from the OpenFOAM-dev-Intel branch on Github
 398 (<https://github.com/OpenFOAM/OpenFOAM-Intel/tree/master/libhbm>), was
 399 used to enable access to the MCDRAM. Adopting `libhbm` enabled us to speed
 400 up the computations by up to 20%.

401 *4.1 Validation cases*

402 *4.1.1 Circular cylinder*

403 The first test case in this work concerns the sound generated by the Karman
 404 vortex street that is shed behind a circular cylinder. The Reynolds number
 405 based on the cylinder's diameter is $Re = 150$. The problem had already been

406 considered in the context of sound generation computation [33, 34, 35, 36], so
407 it is an appropriate benchmark for `caafoam`.

408 Two different suites of computational meshes were generated to test the perfor-
409 mance of `caafoam`. A first group included three fully-structured O-type grids.
410 The coarser structured grid, named G1, was created with $N_c = 3.5 \cdot 10^5$ ($500 \times$
411 700); the G2 grid was generated by starting from G1 and increasing the num-
412 ber of cells in the radial direction, $N_c = 5.25 \cdot 10^5$ (750×700). The last grid,
413 G3, was the result of a further refinement in the radial direction: $N_c = 7 \cdot 10^5$
414 (1000×700). It is important to remark that the G series grids have a number
415 of cells per wavelength equal to about: 90 for G1, 135 for G2 and 180 for G3.
416 Note also that the previous data are compatible with recent literature refer-
417 ences [36, 37, 38]. Our second set of computational meshes consisted of two
418 fully-unstructured (triangular cells) grids: the U2 grid had about $2 \cdot 10^6$ cells
419 and was obtained by refining a starting grid, named U1 with $N_c \simeq 5.2 \cdot 10^5$,
420 in order to have a wavelength resolution comparable to G2 grid. In all the
421 above cases, the far-field boundaries were placed at 150 times the cylinder's
422 diameter, D , and the height of the first cell next to the wall, y_c , was set at
423 $y_c/D = 5 \cdot 10^{-3}$. The sponge's strength was set at 40 dB. The different space
424 discretized domains were tested using both versions of `caafoam`, i.e. `m1` and
425 `m2`.

426 An instantaneous representation of the pressure wave generated by vortex
427 shedding, computed with our low-dissipation approach, is shown in Fig. 3.
428 It contains the positive and negative pressure pulses, alternately produced
429 from the upper and lower sides of the cylinder, as also noted by Inoue and
430 Hatakeyama, [34].

431 Fig. 4 shows the polar plots of the root mean square of the fluctuating pres-
432 sure, p'_{rms} , obtained using the RK 4-5 scheme and the maximum allowable Co
433 number. The nature of the sound field clearly emerges, also confirming that
434 the lift dipole dominates. Fig. 4(a) clearly shows that good reconstruction of
435 the acoustic far field can be obtained with the G2 grid. In fact, solutions G2

436 and G3 are almost indistinguishable, while some little wiggles appear for in the
437 case of G1. It is important to note that `caafoam-m1` and `caafoam-m2` (without
438 the dissipative term on the pressure flux, *i.e.* $\alpha = 0$) produce very similar re-
439 sults on the structured grids. On the other hand, `caafoam-m1` proved unstable
440 on our unstructured grids without any limiters on the interpolation schemes
441 for the DoFs, which in turn cause acoustic wave depletion. Only `caafoam-m2`
442 with $\alpha = 1$ proved capable of directly simulating the acoustic field on the U1
443 and U2 grids (see Fig. 4(b)). The main drawback of adopting unstructured
444 grids, however, lies in the dramatic increase in the number of cells needed to
445 obtain acceptable predictions. Our investigations were consequently limited to
446 structured meshes from this point on.

447 Fig. 5 is worthy of careful attention because it shows the comparison (per-
448 formed on the G2 grid) between our approach and the results obtained with
449 `rhoCentralFoam`. The solver is density-based and available in the official
450 OpenFOAM release. It adopts the KNP scheme for the space discretization of
451 the convective terms [16]. In this particular case, `rhoCentralFoam` was submit-
452 ted to the non-reflective boundary treatment described in Section 2.1. Fig. 5
453 shows that `rhoCentralFoam` is unable to properly reconstruct the acoustic
454 field. This is due to the significant amount of numerical dissipation intro-
455 duced by the solver, as also noted by Modesti and Pirozzoli [20] in a different
456 context. We might also add that the “backward” scheme, available in the offi-
457 cial OpenFOAM releases, was used in `rhoCentralFoam` for time integration.
458 The RK-based approaches proposed in this work show a very good agree-
459 ment with the reference data in both `caafoam-m1` and `caafoam-m2` modes (see
460 Fig. 4). They also show a directivity of 83° , which differs from Inoue and
461 Hatakeyama, who found 78.5° , by 5.7%. We can therefore conclude that, on
462 structured grids, the space discretization needed to handle Eulerian numerical
463 fluxes is not the crucial issue. Our results demonstrate that, in the FV frame-
464 work, the solution strategy of space discretized equations has a central role
465 in the correct prediction of acoustic waves. For the sake of completeness, we

466 must add that applying `rhoCentralFoam` to unstructured grids suffers from
 467 the same problems as those described for `caafoam-m1`, which also use the KNP
 468 approach. In our computational experience, we found that a Co_{max} of about 1
 469 can be used when the RK 4-5 technique was adopted. The RK 3-4 scheme only
 470 proved stable for $Co_{max} \simeq 0.6$, whereas for RK 2-2 the maximum allowable
 471 Courant number was around 0.4. Fig. 5 suggests that the RK 2-2 approach is
 472 the best choice for solving **the governing** equations because, in both `m1` and
 473 `m2` modes, it enables us to obtain results comparable with the RK 3-4 and
 474 RK 4-5 schemes using only 2 stages. We have to emphasize, however, that the
 475 RK 2-2 and RK 4-5 schemes are less costly because they have the same ratio
 476 between the number of computational operations and the stability limits. In
 477 the following cases, we preferred to adopt the RK 4-5 technique because it
 478 provided slightly better results than the RK 2-2 method. As shown in Fig. 6,
 479 the increase in the size of the Δt does not significantly affect the accuracy of
 480 the solution for either of the schemes for approximating the convective terms
 481 considered in this paper.

482 Finally, we wish to confirm that a non-reflective boundary treatment is indis-
 483 pensable for DNS cases. Sponge-layer-free numerical solutions produce com-
 484 pletely nonphysical results (see Fig. 7). A sponge strength of 40 dB suffices, as
 485 also noted by Mani [13], to properly suppress spurious wave reflections near
 486 the boundary field. The sponge layer thickness was computed using the crite-
 487 rion discussed in Section 2.1.

488 Table 1 and Table 2 show the aerodynamic parameters regarding the effect of
 489 the time-step's size, including the `rhoCentralFoam` solutions. The maximum
 490 dimensionless time-step size, $u_\infty \Delta t / D$, was chosen in order to overcome the
 491 stability limit of the scheme considered. For `rhoCentralFoam`, Co_{max} has to be
 492 less than 0.2 to avoid the computation blowing up. For the sake of compact-
 493 ness, the above-mentioned results only refer to the G2 grid, and they almost
 494 converge. The force coefficients established with `caafoam` are very consistent
 495 with the results reported by Inoue and Hatakeyama [34], and by Muller [33]

496 high-order finite difference data (see Table 3). The Strouhal number was also
497 computed, obtaining $St = 0.182$ for all the cases considered. Here again, our
498 results are very consistent with the main references in the literature. Com-
499 pared with `caafoam`, the `rhoCentralFoam` solver slightly overestimates the
500 amplitudes of the aerodynamic coefficients, but it has a good overall fit with
501 the data in the literature.

502 4.1.2 Square cylinders arranged side by side: $L/D = 3$

503 In this subsection we discuss the results concerning the flow field and sound
504 generation around two square cylinders placed side by side, as shown in Fig. 8.
505 The ratio L/D was set at 3, where L is the spacing between the centers of
506 the two cylinders and D is the diameter. The Reynolds number, based on a
507 single cylinder's diameter, is $Re = 150$. Depending on the initial condition,
508 a bifurcation of the wake patterns appears for this flow configuration, as for
509 circular cylinders [39]. Different sound patterns are generated in response to
510 this phenomenon [40]. In this work, a symmetrical initial field (with respect to
511 the $y = 0$ plane) was imposed by two vortices, one moving clockwise and the
512 other counterclockwise, behind the upper and lower cylinders, respectively.
513 The resulting flow field is described as in-phase because it exhibits synchro-
514 nized lift coefficients (Fig. 9(b)). Fully-structured orthogonal computational
515 grids were used, adopting a sponge layer with a strength of 40 dB. The grid
516 cells were clustered near the cylinder walls, whereas the far field was placed
517 at $200 D$ from the midpoint of the two cylinders (see Fig. 8). N_c was set at
518 $1.11 \cdot 10^6$. It should be noted that we had to extend the domain due to the
519 lower frequency of vortices shed behind the square cylinders than behind the
520 single circular one. For this reason, L_{sp} was increased to keep the dimension-
521 less extent of the sponge layer, $L_{sp}f/c_\infty$, at 0.5. The inflow/outflow conditions
522 were consequently set at $200 D$ to avoid interference phenomena between the
523 acoustic far field and the layer. A finer version of the grid was also generated:

524 it has a number of cells equating to half that of the previous grid, with a total
525 number of $4.44 \cdot 10^6$, and the height of the dimensionless first cell bordering
526 on the walls is 10^{-2} . Note that the finest grid guarantees about 180 cells per
527 wavelength. Time integration was performed using the RK 4-5 scheme and
528 the size of the dimensionless time step was set at $1.8 \cdot 10^{-3}$. This enabled us to
529 obtain a maximum Courant number of around 1 for the finer grid. The larger
530 case was run on the MARCONI-A2 system using 256 CPU cores.

531 The main aims of this benchmark are to further validate `caafoam`, and also to
532 investigate the role of the solution strategy involving space-discretized equa-
533 tions in the context of the structured grids. We consequently limit our efforts
534 to the `m1` version of our code because it uses the same space discretization ap-
535 proach as `rhoCentralFoam`, and it is equivalent to `m2` in terms of the results
536 on structured grids.

537 Table 4 shows the aerodynamic parameters predicted from the above-mentioned
538 computations. An overall good agreement emerged between our data and those
539 in the literature. We might also add that it is hard to say whether the intrinsi-
540 cally dissipative nature of `rhoCentralFoam` could affect the forces predicted.
541 Finally, Fig. 10 shows the acoustic results. The overall results show a good
542 agreement with the findings of Inoue et al. [40], but our grid refinement clearly
543 improved the agreement between the data presented here and those in the lit-
544 erature. Fig. 10(a) shows the p'_{rms} polar plot, which is very similar to the
545 case of the single circular cylinder: a directivity of 80.2° was obtained. It is
546 important to note that the sound wave is always symmetrical to the $y = 0$
547 plane, and of a similar nature to the longitudinal quadrupole, as discussed in
548 Blake [41]. Once again, the non-reflective `rhoCentralFoam` version does not
549 properly reconstruct the acoustic far field, as shown in Fig. 10.

550 This confirms that, here too, the space discretization schemes adopted for the
551 governing equations are not the main factor responsible for correctly predict-
552 ing acoustic waves.

553 4.1.3 Square cylinders in tandem: $L/D = 2$

554 With the same aims as for the side-by-side arrangement, we also considered
555 the flow and sound generation around two square cylinders in a tandem config-
556 uration with $L/D = 2$, where L and D have the meaning expressed in Fig. 11.
557 The computational domain was generated to place the far field $200 D$ away
558 from the origin of the Cartesian frame in Fig. 11. Quadrilateral orthogonal
559 cells were used to discretize the flow domain. The total number of cells, N_c ,
560 was about $4.2 \cdot 10^6$, and a grid refinement was performed near the walls of
561 the cylinders adopting $y_c/D = 10^{-2}$. This grid allow to have about 190 cells
562 per wavelength for the specific configuration. As for the side-by-side config-
563 uration, we tested a coarser grid with a quarter of the N_c of the finer one.
564 The `caafoam-m1` solutions are based on the RK 4-5 time integration scheme to
565 obtain a maximum allowable Courant number of around 1. So $u_\infty \Delta t/D$ was
566 set at $9 \cdot 10^{-3}$ for the finer grid. Acoustic wave reflections at the far boundaries
567 were removed using a configuration derived from the previous test cases; the
568 sponge layer's strength was 40 dB, while its dimensionless thickness was 0.5
569 to limit the computational resources required. Finer grid computations were
570 run using 256 CPU-cores MARCONI-A2 HPC system.

571 Table 5 shows the aerodynamic parameters for the square cylinders in tan-
572 dem. The picture is similar to the one seen for the side-by-side cylinders. The
573 features of the sound field generated by the interaction of the flow and the two
574 cylinders are shown in Fig. 12. The p'_{rms} plot in Fig. 12(a) refers to a circle
575 having a r/D of 80. `caafoam-m1` results, achieved with the finer grid, are con-
576 sistent with the reference data in the literature [42], and reveal a directivity
577 of 71.2° . Lastly, we wish to add that, for this test case too, the non-reflective
578 `rhoCentralFoam` is inappropriate for far-field sound computation. Fig. 12(a)
579 clearly shows that, in the far zone of the acoustic field, `rhoCentralFoam` is
580 not consistent with the data reported in [42].

581 4.2 Thermal effects on the aeroacoustic field

582 Two different configurations were considered to analyze the effect of the ther-
583 mal boundary conditions at the wall on the acoustic field generated by the
584 laminar flows around bluff bodies: a single circular cylinder at $Re = 150$, and
585 a single square cylinder at the same Re number. In both problems, the base-
586 line configuration involved an adiabatic wall; then cases having $T_w = 2T_\infty$
587 and $T_w = 3T_\infty$ were also investigated. $M_\infty = 0.2$ was used to conduct this
588 assessment. Given the results presented in the previous sections, the following
589 data were based on `caafoam-m1`.

590 The first case we mention, represented in Fig. 13, is the sound radiated by
591 the Karman vortex street shed behind a square cylinder. A fully-structured
592 orthogonal grid was used: the grid cells were clustered near the cylinder walls
593 using a dimensionless first cell height, y_c/D , of $5 \cdot 10^{-3}$, and far-field boundaries
594 placed at a distance of $200 D$ from the center of the cylinder. The resulting
595 computational mesh had a total number of cells, N_c , amounting to $4.4 \cdot 10^6$ with
596 about 170 cells per wavelength. The polar plot containing the p'_{rms} is shown
597 in Fig. 16(a). The data were collected over a circumference built around the
598 square cylinder having a dimensionless radius $r/D = 75$, as in Inoue et al. [42].
599 Our approach clearly ensures a good reconstruction of the acoustic far field. It
600 is important to note that, for the flow regimes considered, the acoustic field is
601 generated by periodical vortical structures shedding. This phenomenon causes
602 a pressure fluctuation on the cylinder's surfaces, leading to the generation an
603 unsteady lift/downforce. The drag is influenced by the Karman vortex street
604 as well, showing a downstream/upstream pulsation. These perturbations have
605 a sound quadrupole nature, but the dominance of the lift fluctuation yields a
606 typical dipolar acoustic field [43].

607 Note that, due to thermal effects, in order to estimate the changes in the
608 acoustic field we present our data on a circle having $r = 40 D$ as this prevents
609 an excessive sound wave decay in the far field. This condition enabled us to set

610 the extent of the computational domain at $r = 150 D$, reducing the number of
 611 cells to: $N_c \simeq 3 \cdot 10^6$. Fig. 14 shows that the rise in wall temperature increases
 612 $\langle C_D \rangle$ and reduces $C_{L,rms}$. Fig. 15 shows that the force coefficient pulsations,
 613 ΔC_D and ΔC_L , are reduced as a result of the increase in wall temperature.
 614 These results are in agreement with the reports from Lecordier et al. [8, 9],
 615 who experimentally found vortex shedding dumping behind a heated circular
 616 cylinder. Similar effects were also found on heated airfoils operating at low
 617 Re, [44, 45, 46], which revealed a higher drag force and lower lift force in steady
 618 conditions. Looking at the results in Fig. 15, it is easy to see that the sound
 619 sources, *i.e.* ΔC_D and ΔC_L , are damped, producing a far-field sound abate-
 620 ment at higher wall temperatures. It is worth noting that this phenomenon
 621 is not limited to a specific Re but holds throughout the range of 90-150, as
 622 shown in the figures from Fig. 16 to Fig. 19. The radiated sound decay is also
 623 less significant for higher Re numbers, as shown in Fig. 20(b). In particular,
 624 at Re = 90 the maximum sound power level decay ($T_w = 3T_\infty$) is slightly less
 625 than 5 dB, while at Re = 150 it is ~ 3 dB, which is still significant. The St
 626 number is reduced by wall heating as well, as shown in Fig. 20(a), consistently
 627 with the findings of Lecordier et al..

628 The second case considered in this context is the sound radiated by the Kar-
 629 man vortex street that is shed behind a circular cylinder. The fully-structured
 630 G2 grid was used for this analysis. All the numerical settings mentioned in
 631 Section 4.1.1 were confirmed here to investigate the effects of wall heating on
 632 the radiated sound. As for the square cylinder, we present the p'_{rms} data on a
 633 circle having $r = 40 D$.

634 In this case, increasing the wall temperature produced an evident reduction
 635 in $C_{L,rms}$, while $\langle C_D \rangle$ was increased up to Re = 130. Fig. 21 clearly shows,
 636 however, that the thermal boundary conditions at the wall have a more signif-
 637 icant effect on the lift coefficient in this flow configuration, whereas the effect
 638 on $\langle C_D \rangle$ is almost negligible. As for the square cylinder, the force coefficient
 639 fluctuations are dumped.

640 In short, the aeroacoustic sound emitted in the far-field region is lower when
641 the wall temperature is higher, as shown in Fig. 23 to Fig. 26. It is also im-
642 portant to note that the sound decay is less significant at higher Re numbers
643 in this configuration, as confirmed by Fig. 27(b). It is also worth noting that
644 overall L_w abatement is greater for the circular cylinder than for the square
645 one. In fact, we obtained $\Delta L_w \simeq 8\text{dB}$ at $\text{Re} = 90$, and $\sim 5\text{dB}$ at $\text{Re} = 150$.
646 Lastly, the St number shows the same behavior vis-à-vis Re and wall temper-
647 ature as for the square cylinder.

648 At the time of writing this paper, there were no papers available in the open
649 literature dealing with the reduction of emitted aeroacoustic sound based on
650 wall heating. The above-mentioned phenomenon was analyzed on two com-
651 pletely different geometries, showing that it is not limited to a particular flow
652 configuration.

653 5 Conclusions

654 This paper addresses the development and application of an open-source solver
655 for compressible **mass, momentum and energy** equations, named `caafoam`,
656 which is able to capture a wide range of flow phenomena. Particular atten-
657 tion was devoted to computing aeroacoustic sound. Our solver was devel-
658 oped within the FV OpenFOAM library and it adopts explicit low-storage
659 Runge-Kutta schemes for time integration. KNP and Pirozzoli energy con-
660 serving schemes were used to handle Eulerian numerical fluxes, while stan-
661 dard central schemes were considered for diffusive terms. Only the Pirozzoli
662 schemes proved capable of predicting acoustic waves on fully unstructured
663 computational grids, while the two different approaches performed equally
664 well on structured grids. An appropriate non-reflective boundary treatment
665 was achieved using an artificial sponge layer because it is simple to code, ro-
666 bust, and not stiff; and it proved flexible in handling complex geometries. The
667 solver also showed a very good parallel performance on two completely differ-

668 ent architectures, making it suitable for use in massively parallel aeroacoustic
669 computations.

670 A broad range of far-field aeroacoustic sound configurations, emitted from
671 bluff-bodies in a flow with uniform velocity inlet, was investigated for vali-
672 dation purposes. In all the benchmarks considered, `caafoam` performed well
673 in predicting the sound produced by the flow-body interaction. We found
674 `rhoCentralFoam` unable to capture acoustic wave propagation phenomena
675 correctly, even though we had introduced a proper non-reflective boundary
676 treatment. On the other hand, the `caafoam-m1` version can produce a direct
677 solution of aeroacoustic fields. This goes to show that the inviscid numerical
678 flux is not the key ingredient on structured grids; the solution algorithm is the
679 primary issue to address.

680 Another novelty of this paper concerns our assessment of the impact of ther-
681 mal boundary conditions at the wall on the sound produced by the interaction
682 of a bluff body with a uniform laminar flow. Two different cases were consid-
683 ered, with square and circular cylinders. In both cases, we found that heating
684 the wall reduces the vortex shedding developing in the wake region, as noted
685 experimentally by Lecordier et al. [8]. This is of considerable interest in aeroa-
686 coustics because the pulsations of the lift and drag forces for these objects
687 are directly related to the aerodynamically-produced sound. In fact, reducing
688 them by heating the wall in turn reduces the production of acoustic pertur-
689 bations. In other words, increasing the wall temperature reduces the sound
690 power level. This finding has important practical implications since it can be
691 considered as a method for actively controlling aeroacoustic sound.

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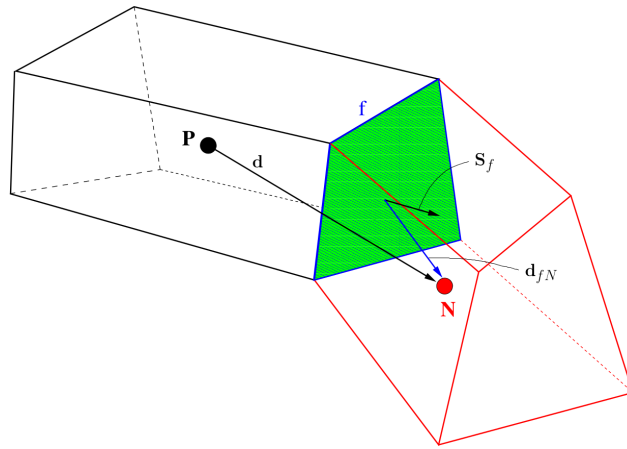
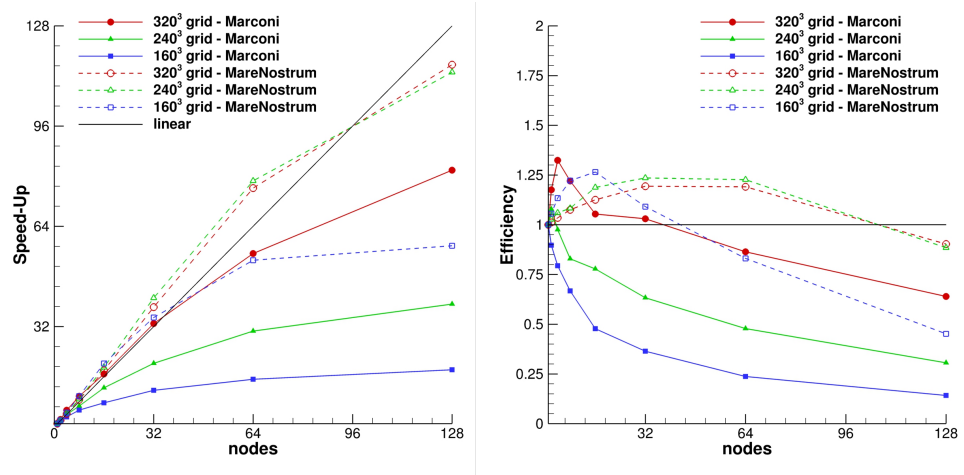


Figure 1. The computational cells.



(a) Speed-up

(b) Efficiency

Figure 2. Parallel performance.

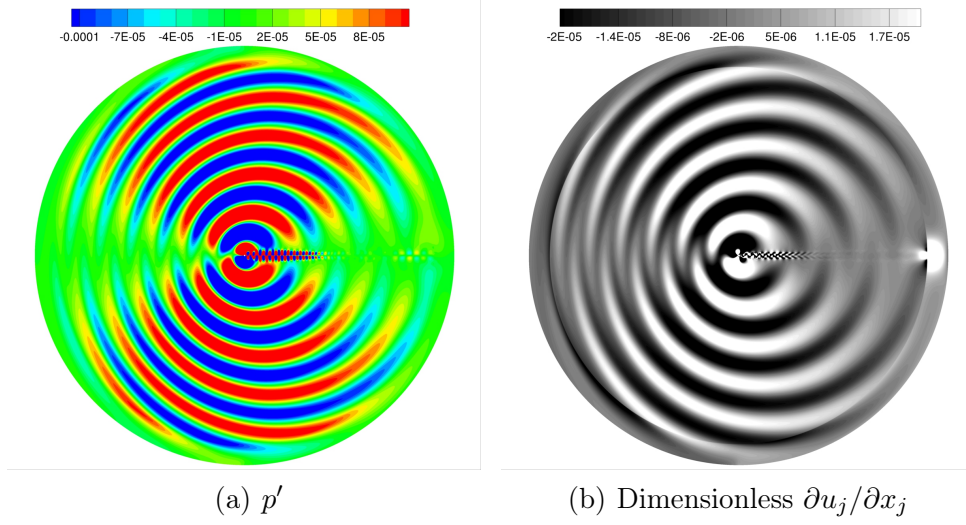
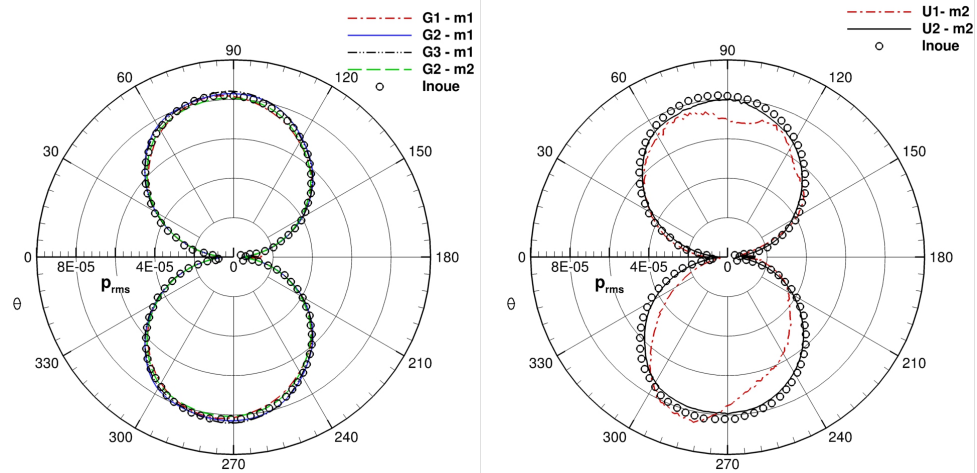


Figure 3. Sound wave generated by the flow past a circular cylinder at $Re = 150$.



(a) Structured grids

(b) Unstructured grids

Figure 4. Flow past a circular cylinder at $Re = 150$. Grids effect.

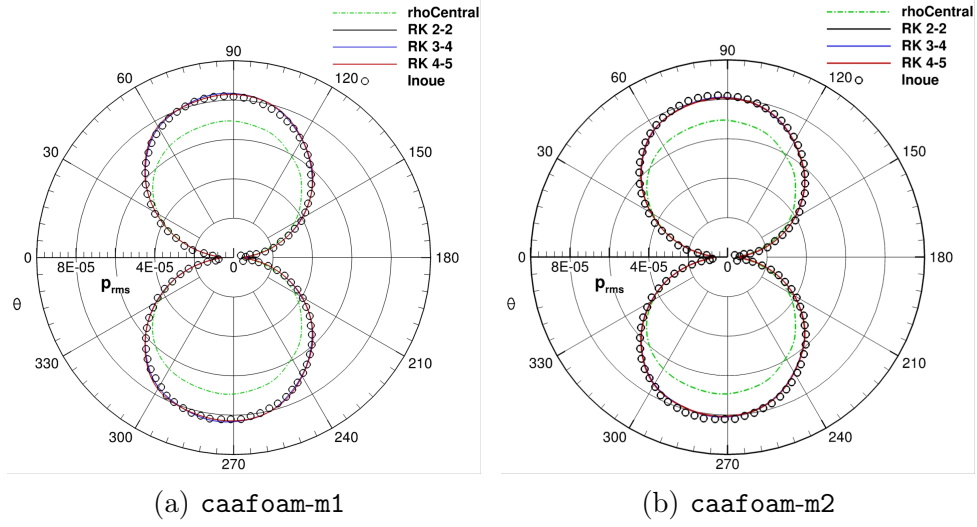


Figure 5. Flow past a circular cylinder at $Re = 150$. G2 grid. RK scheme effect; $Co_{max} \simeq 0.2$.

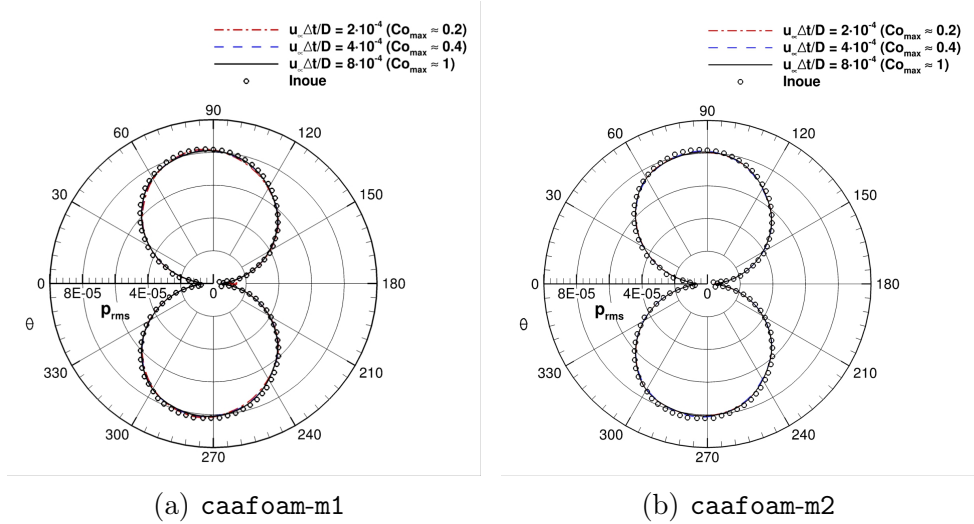


Figure 6. Flow past a circular cylinder at $Re = 150$. G2 grid. Time-step size effect.

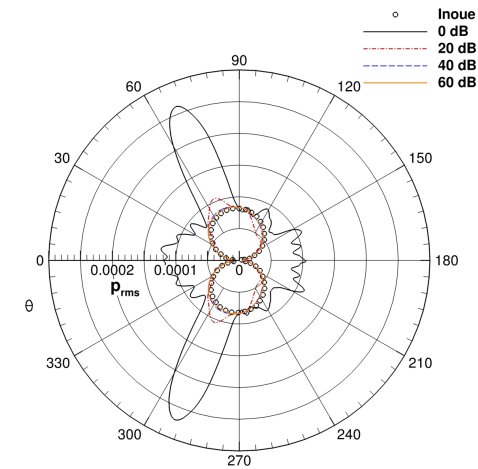


Figure 7. Flow past a circular cylinder at $Re = 150$. G2 grid. Sponge layer strength impact.

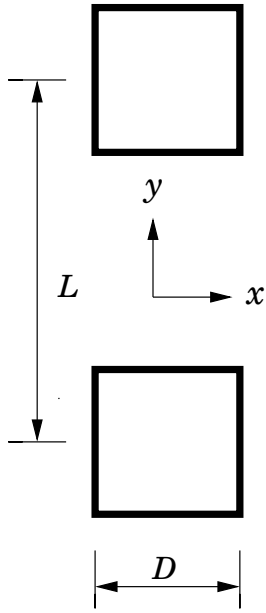
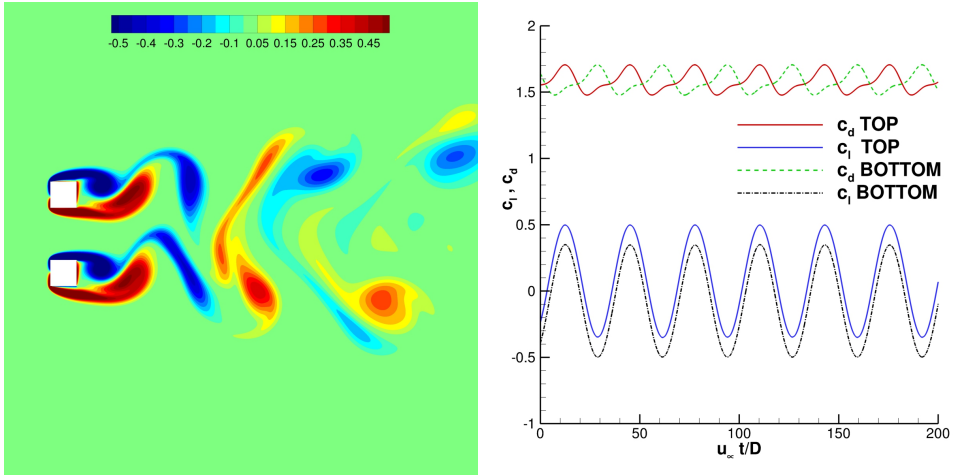


Figure 8. Cylinders arranged side by side.



(a) Dimensionless vorticity field

(b) Force coefficients time history

Figure 9. Square cylinders side by side at $Re = 150$, $M_\infty = 0.2$, $L/D = 3$. Finer grid results.

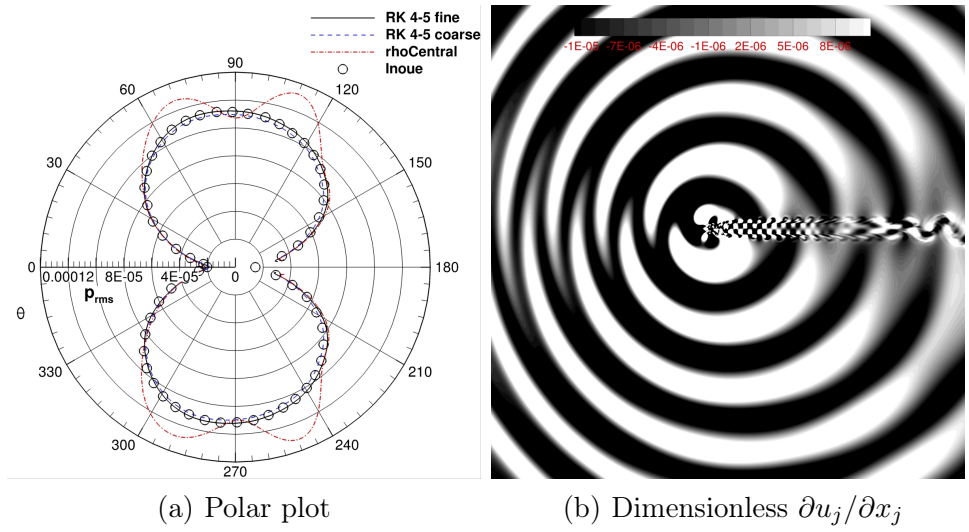


Figure 10. Square cylinders side by side at $Re = 150$, $M_\infty = 0.2$, $L/D = 3$.

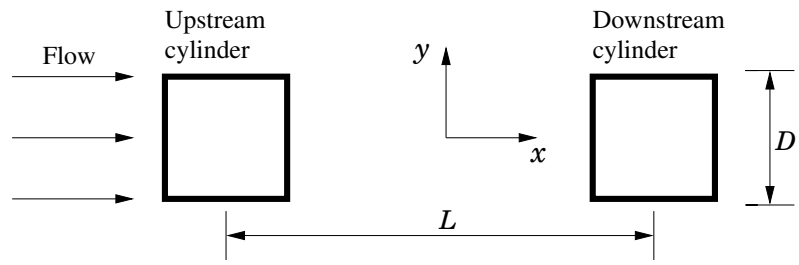


Figure 11. Square cylinders in tandem configuration.

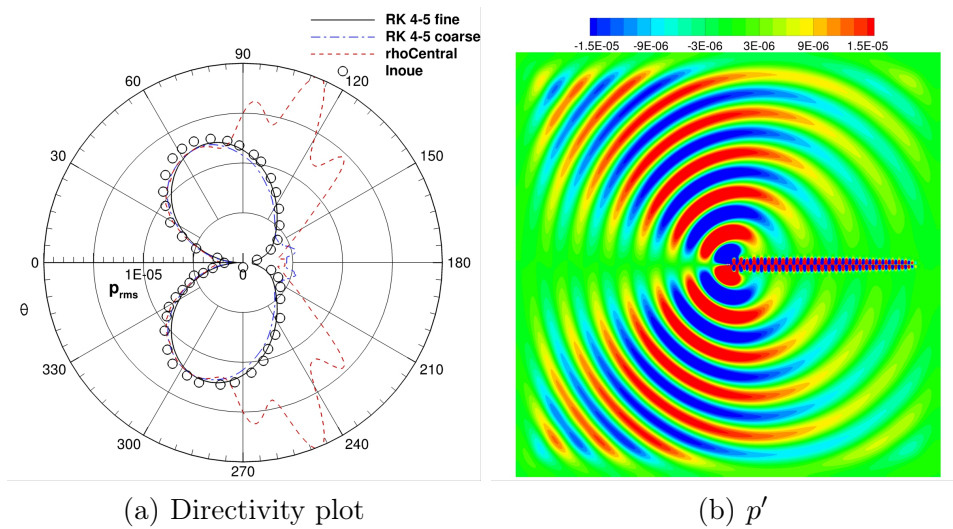
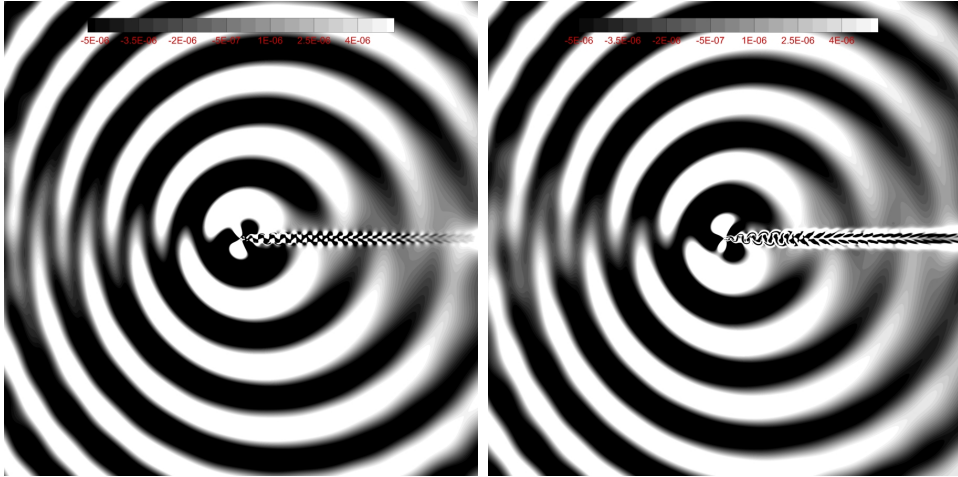
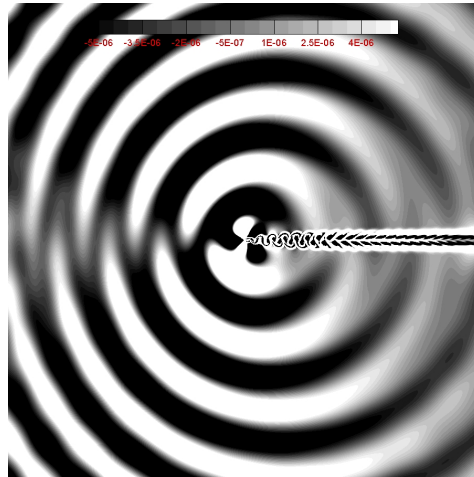


Figure 12. Square cylinders in tandem at $Re = 150$, $M_\infty = 0.2$, $L/D = 2$.



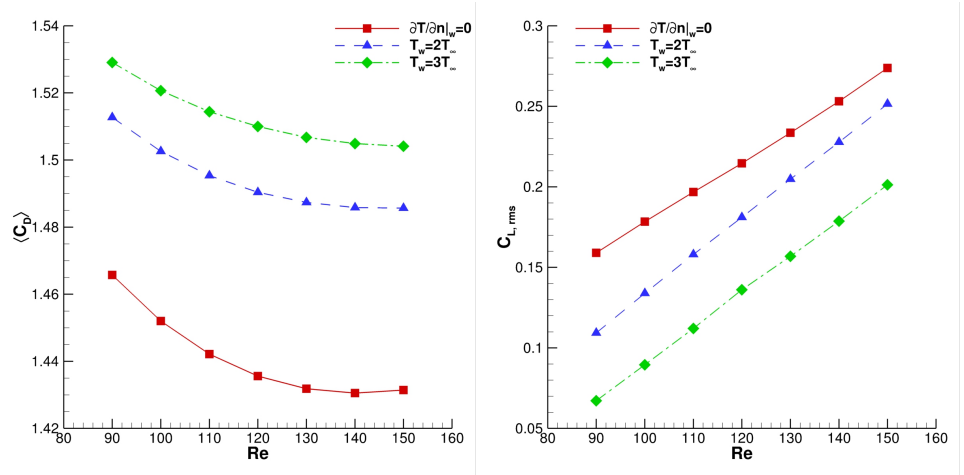
(a) $\partial T/\partial n|_w = 0$

(b) $T_w = 2T_\infty$



(c) $T_w = 3T_\infty$

Figure 13. Wall temperature effect, square cylinder $Re = 150$. Dimensionless $\partial u_j/\partial x_j$.



(a) Average drag coefficient

(b) Root mean square of the lift coefficient

Figure 14. Wall temperature effect, square cylinders. Force coefficients.

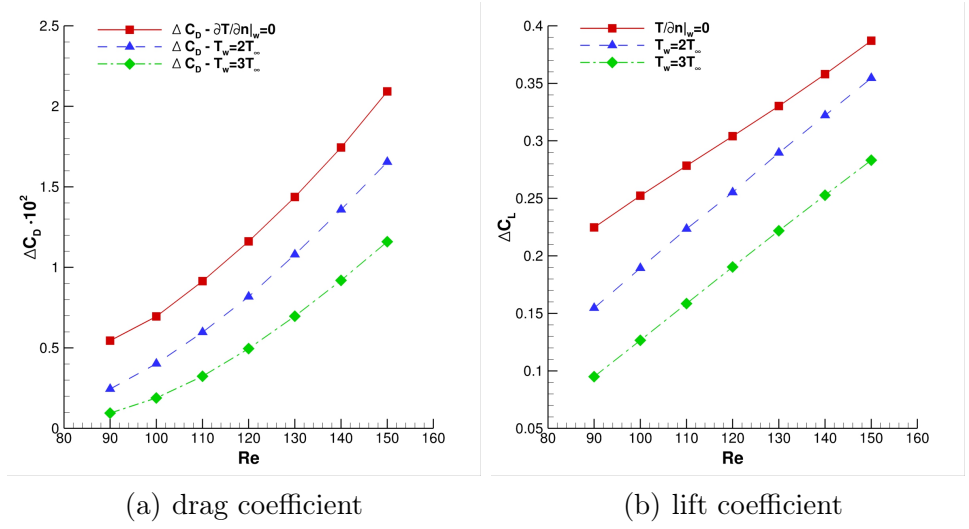
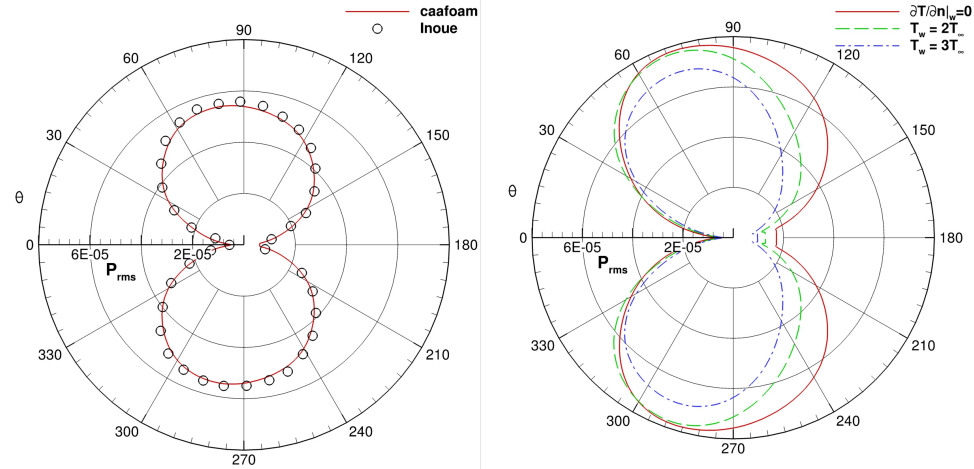


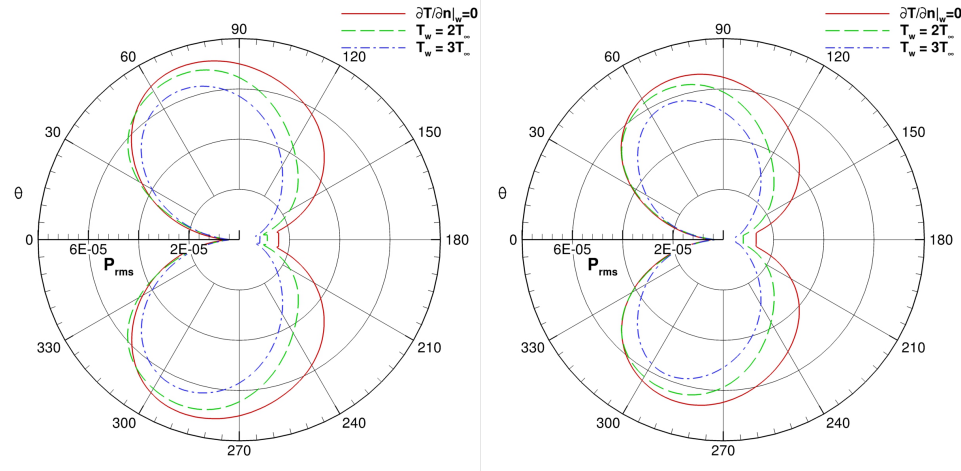
Figure 15. Wall temperature effect, square cylinders. Force coefficients fluctuations.



(a) Validation, $r/D = 75$

(b) $r/D = 40$

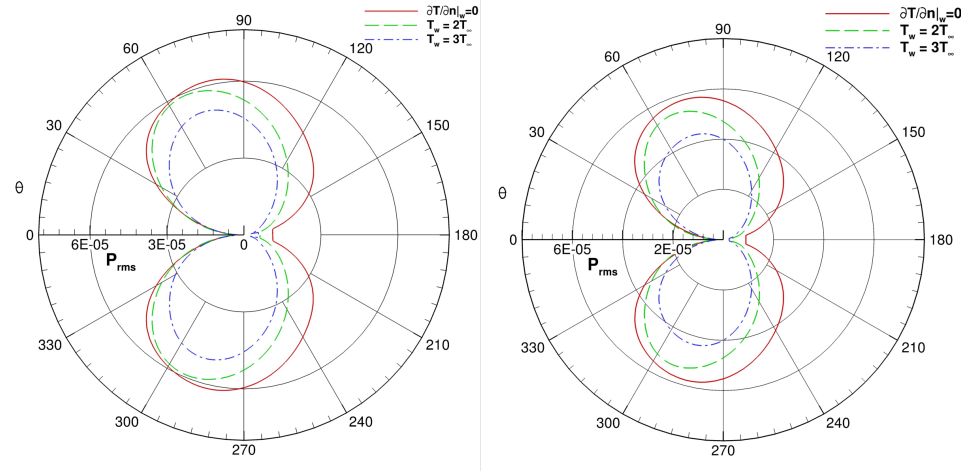
Figure 16. Wall temperature effect, square cylinder at $Re = 150$.



(a) $Re = 140$

(b) $Re = 130$

Figure 17. Wall temperature effect, square cylinder. p'_{rms} , $r/D = 40$.



(a) $Re = 120$

(b) $Re = 110$

Figure 18. Wall temperature effect, square cylinder. p'_{rms} , $r/D = 40$.

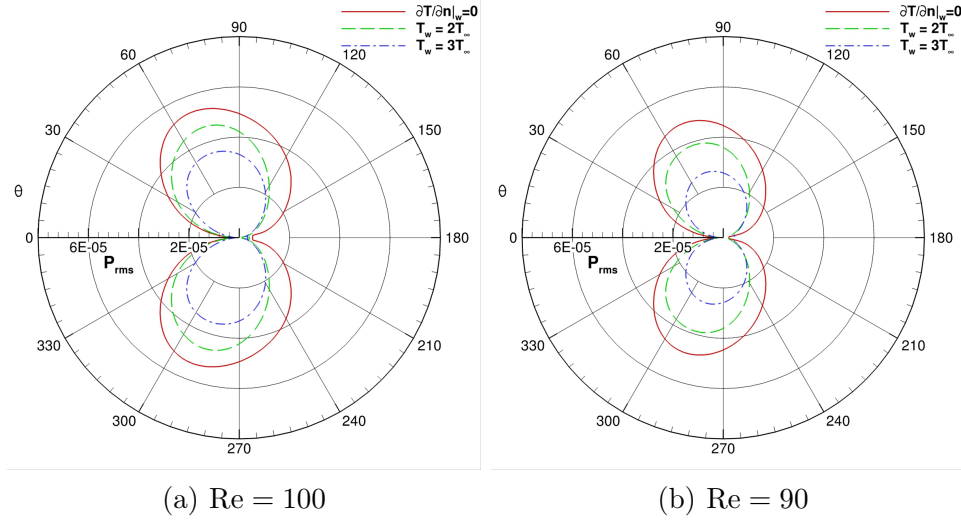
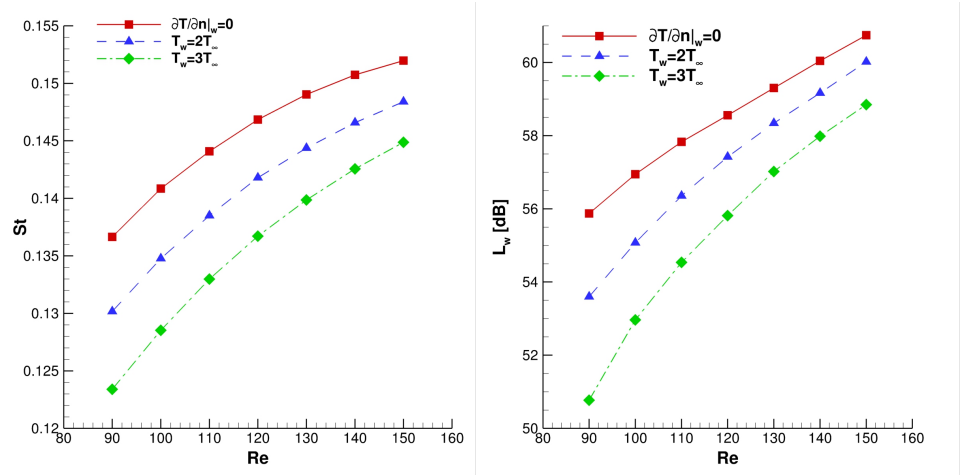


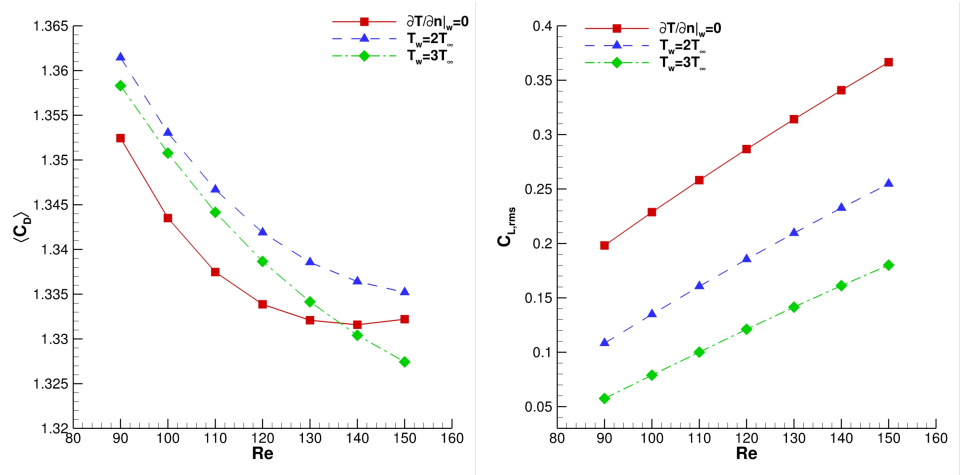
Figure 19. Wall temperature effect, square cylinder. p'_{rms} , $r/D = 40$.



(a) Strouhal number

(b) Sound power level

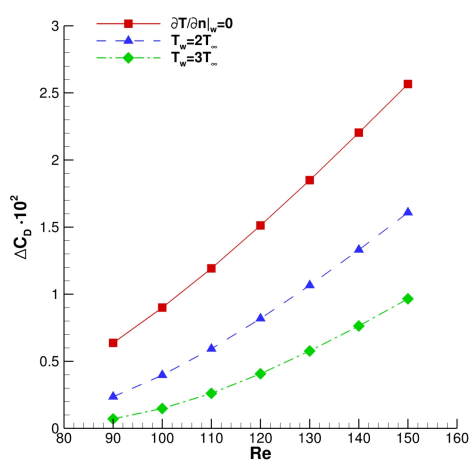
Figure 20. Wall temperature effect, square cylinder.



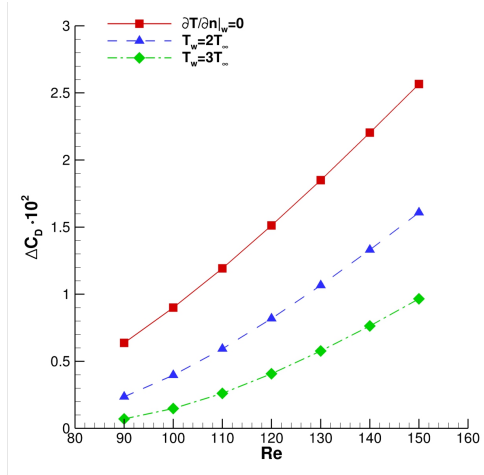
(a) Average drag coefficient

(b) Root mean square of the lift coefficient

Figure 21. Wall temperature effect, circular cylinder. Force coefficients.

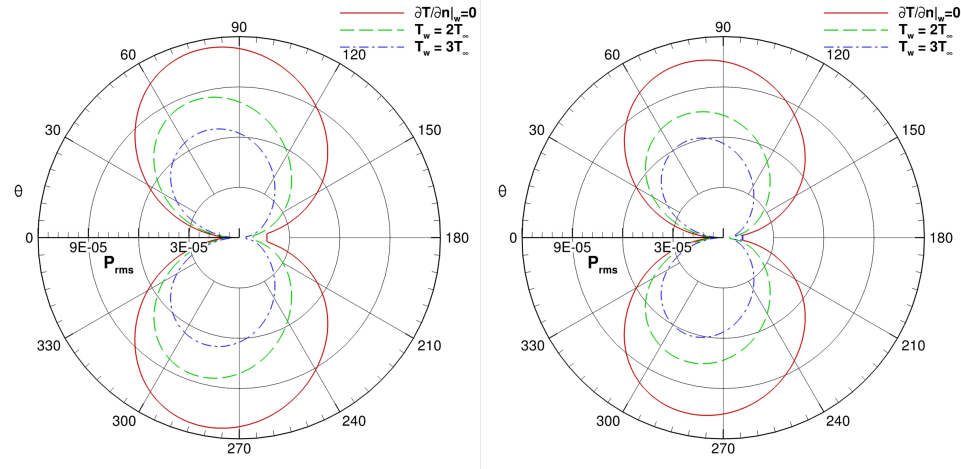


(a) drag coefficient



(b) lift coefficient

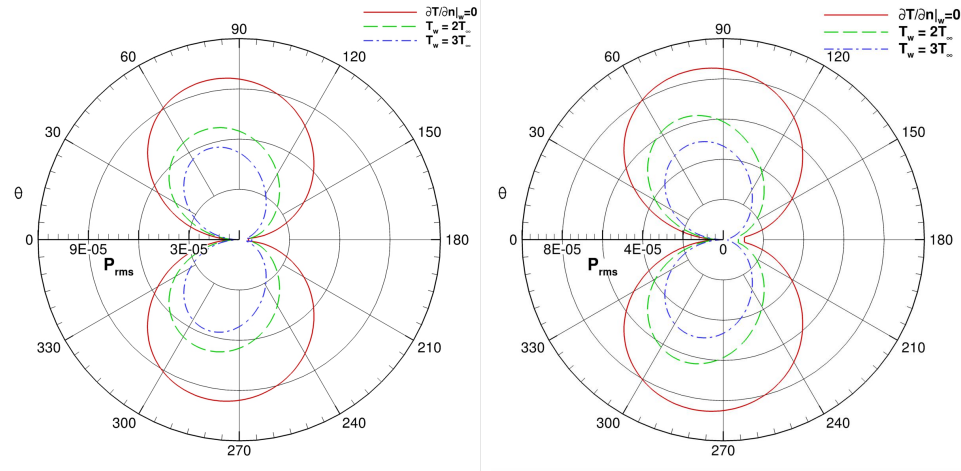
Figure 22. Wall temperature effect, circular cylinder. Force coefficients fluctuations.



(a) $Re = 150$

(b) $Re = 140$

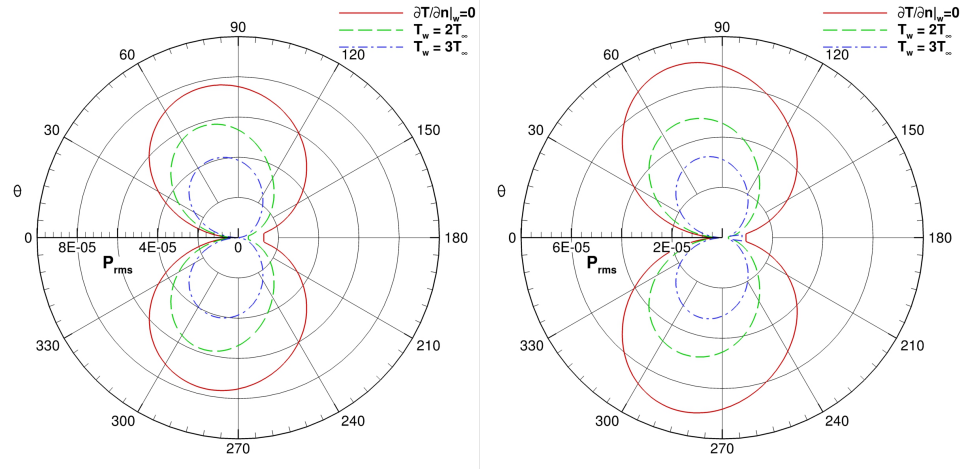
Figure 23. Wall temperature effect, circular cylinder. p'_{rms} , $r/D = 40$.



(a) $Re = 130$

(b) $Re = 120$

Figure 24. Wall temperature effect, circular cylinder. p'_{rms} , $r/D = 40$.



(a) $Re = 110$

(b) $Re = 100$

Figure 25. Wall temperature effect, circular cylinder. p'_{rms} , $r/D = 40$.

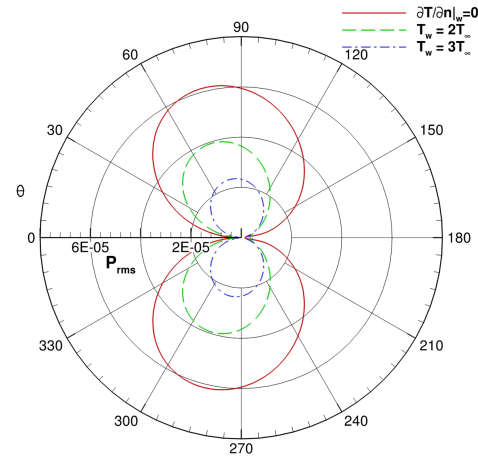
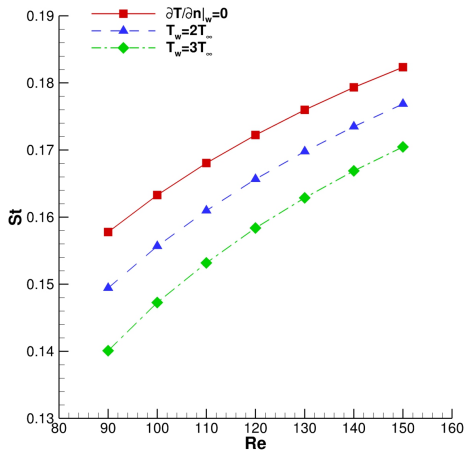
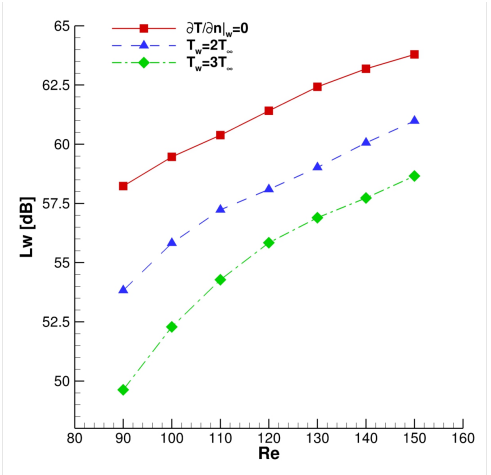


Figure 26. Wall temperature effect, circular cylinder. p'_{rms} , $r/D = 40$.



(a) Strouhal number



(b) Sound power level

Figure 27. Wall temperature effect, circular cylinder.

Table 1

Cylinder at $Re = 150$, $M_\infty = 0.2$, G2-grid results. `caafoam-m1`

| Case | $u_\infty \Delta t / D$ | $\langle C_D \rangle$ | $\Delta C_D \cdot 10^2$ | ΔC_L | St |
|--|-------------------------|-----------------------|-------------------------|--------------|-------|
| RK 2-2 ($Co_{max} \simeq 0.2$) | $2 \cdot 10^{-4}$ | 1.3326 | 2.580 | 0.5203 | 0.182 |
| RK 2-2 ($Co_{max} \simeq 0.4$) | $4 \cdot 10^{-4}$ | 1.3325 | 2.560 | 0.5200 | 0.182 |
| RK 3-4 ($Co_{max} \simeq 0.2$) | $2 \cdot 10^{-4}$ | 1.3329 | 2.570 | 0.5203 | 0.182 |
| RK 3-4 ($Co_{max} \simeq 0.4$) | $4 \cdot 10^{-4}$ | 1.3329 | 2.575 | 0.5201 | 0.182 |
| RK 3-4 ($Co_{max} \simeq 0.6$) | $6 \cdot 10^{-4}$ | 1.3325 | 2.580 | 0.5199 | 0.182 |
| RK 4-5 ($Co_{max} \simeq 0.2$) | $2 \cdot 10^{-4}$ | 1.3329 | 2.580 | 0.5203 | 0.182 |
| RK 4-5 ($Co_{max} \simeq 0.4$) | $4 \cdot 10^{-4}$ | 1.3328 | 2.575 | 0.5201 | 0.182 |
| RK 4-5 ($Co_{max} \simeq 1.0$) | $8 \cdot 10^{-4}$ | 1.3325 | 2.570 | 0.5199 | 0.182 |
| rhoCentralFoam ($Co_{max} \simeq 0.2$) | $2 \cdot 10^{-4}$ | 1.3347 | 2.580 | 0.5215 | 0.182 |

Table 2

Cylinder at $Re = 150$, $M_\infty = 0.2$, G2-grid results. caafoam-m2

| Case | $u_\infty \Delta t / D$ | $\langle C_D \rangle$ | $\Delta C_D \cdot 10^2$ | ΔC_L | St |
|--|-------------------------|-----------------------|-------------------------|--------------|-------|
| RK 2-2 ($Co_{max} \simeq 0.2$) | $2 \cdot 10^{-4}$ | 1.3321 | 2.565 | 0.5183 | 0.182 |
| RK 2-2 ($Co_{max} \simeq 0.4$) | $4 \cdot 10^{-4}$ | 1.3321 | 2.565 | 0.5183 | 0.182 |
| RK 3-4 ($Co_{max} \simeq 0.2$) | $2 \cdot 10^{-4}$ | 1.3321 | 2.565 | 0.5183 | 0.182 |
| RK 3-4 ($Co_{max} \simeq 0.4$) | $4 \cdot 10^{-4}$ | 1.3321 | 2.566 | 0.5183 | 0.182 |
| RK 3-4 ($Co_{max} \simeq 0.6$) | $6 \cdot 10^{-4}$ | 1.3321 | 2.564 | 0.5183 | 0.183 |
| RK 4-5 ($Co_{max} \simeq 0.2$) | $2 \cdot 10^{-4}$ | 1.3321 | 2.564 | 0.5183 | 0.182 |
| RK 4-5 ($Co_{max} \simeq 0.4$) | $4 \cdot 10^{-4}$ | 1.3347 | 2.565 | 0.5182 | 0.182 |
| RK 4-5 ($Co_{max} \simeq 1.0$) | $8 \cdot 10^{-4}$ | 1.3321 | 2.564 | 0.5183 | 0.182 |
| rhoCentralFoam ($Co_{max} \simeq 0.2$) | $2 \cdot 10^{-4}$ | 1.3347 | 2.580 | 0.5215 | 0.182 |

Table 3
 Cylinder at $Re = 150$. Literature data.

| Case | $\langle C_D \rangle$ | $\Delta C_D \cdot 10^2$ | ΔC_L | St |
|---------------------------|-----------------------|-------------------------|--------------|-------|
| Muller [33] | 1.34 | 2.6 | 0.52 | 0.183 |
| Inoue and Hakateyama [34] | 1.32 | 2.6 | 0.52 | 0.183 |
| Williamson [47] (Exp.) | – | – | – | 0.18 |

Table 4

Side-by-side square cylinders at $Re = 150$, $M_\infty = 0.2$. Force coefficients.

| | RK 4-5 coarse | RK 4-5 fine | rhoCentralFoam (fine) | Inoue [40] |
|-----------------------|---------------|--------------|--------------------------|--------------|
| $\langle C_D \rangle$ | 1.5806 | 1.5920 | 1.5859 | 1.5519 |
| $\langle C_L \rangle$ | ± 0.0759 | ± 0.0753 | ± 0.0749 | ± 0.0689 |
| $2\Delta C_D$ | 0.2216 | 0.2282 | 0.2281 | 0.2377 |
| $2\Delta C_L$ | 0.8286 | 0.8479 | 0.8456 | 0.8575 |
| St | 0.153 | 0.144 | 0.155 | 0.150 |

Table 5

Square cylinder in tandem configuration at $Re = 150$, $M_\infty = 0.2$. Force coefficients.

Upstream cylinder

| Case | $\langle C_D \rangle$ | $2\Delta C_D \cdot 10^4$ | $2\Delta C_L$ | St |
|-----------------------|-----------------------|--------------------------|---------------|-------|
| RK 4-5 coarse | 1.2753 | 2.0 | 0.0378 | 0.134 |
| RK 4-5 fine | 1.2803 | 2.1 | 0.0384 | 0.134 |
| rhoCentralFoam (fine) | 1.2805 | 2.1 | 0.0383 | 0.134 |
| Inoue et al. [42] | 1.2794 | — | — | 0.133 |

Downstream cylinder

| Case | $\langle C_D \rangle$ | $2\Delta C_D \cdot 10^3$ | $2\Delta C_L$ | St |
|-----------------------|-----------------------|--------------------------|---------------|-------|
| RK 4-5 coarse | -0.1936 | 1.50 | 0.106 | 0.134 |
| RK 4-5 fine | -0.1959 | 1.54 | 0.1068 | 0.134 |
| rhoCentralFoam (fine) | -0.1961 | 1.53 | 0.1065 | 0.134 |
| Inoue et al. [42] | -0.1945 | — | — | 0.133 |