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# Linear depth inversion sensitivity to wave viewing angle using synthetic optical video

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## **Abstract**

The accuracy of bathymetry estimated by optical implementations of remotely sensed depth inversion algorithms is in part related to the presence of optical wave signal in the images, which depend nonlinearly on the water surface slope. The signal to noise ratio in video images of waves decreases under large azimuthal angles between the camera and wave propagation direction, which can result in poor bathymetry estimation. We quantified errors in depth estimation by analysing the sensitivity of the optical implementation of cBathy v1.1, a widely applied algorithm for depth inversion in coastal regions, to wave viewing angle using synthetic tests. We found relative root mean square errors between 0.02 and 0.08 when the azimuthal angle between the camera look direction and wave approach was less than 75°. However, for higher azimuthal angles, the wave signal was dominated by short wavelengths in the optical images lead in larger depth errors (relative root mean square error = 0.2). We also investigated the sensitivity of the initial guess of the wave direction in the nonlinear solution used by the cBathy v1.1 algorithm to estimate water depth. Observed water depth errors caused by wave viewing angle or initial guess of the wave direction are shown in part to be related to errors in the estimates of frequency and wavenumber. The synthetic methodology and the results of the sensitivity analysis can be generalized to test the accuracy of depth estimation in shore-based video monitoring systems, to design future fixed camera coastal video monitoring stations or to drive the choice of the better viewing angles using small Unmanned Aerial Systems (sUAS) using the Matlab Toolbox we developed.

## **Keywords**

Remote Sensing; Depth-inversion method; Bathymetry estimation; Video imaging.

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## 30 **1. Introduction**

31 Optical remote sensing techniques, especially video imagery, are widely employed to monitor the  
32 coastal evolution [1, 2]. Optical imagery offers the unique capability to collect high volumes of data at high  
33 temporal and spatial resolution with relatively low cost and over long periods. The underlying concept of  
34 video imaging in coastal regions is that any visually discernible physical phenomenon can be investigated [1,  
35 2]. Since many nearshore processes have optical signatures, video-monitoring techniques provide useful  
36 indirect measurements of the nearshore hydrodynamic and morphological processes.

37 Waves are imaged as sunlight reflected from the sloped sea surface reaches the camera sensor and is  
38 recorded as image intensity. The radiance reaching the camera sensor from a point on the sea surface,  $I$ ,  
39 depends on the sky conditions, the light reflected off the sea surface, as well as the light upwelled from  
40 below the sea surface [e.g. 3, 4, 5, 6]. The variation of the wave slope between wave crests and troughs  
41 produces the main time-dependent signal in optical imaging of surface gravity waves.  $I$  can be expressed as  
42 the sky radiance distribution,  $L$ , modified by the Fresnel reflection coefficient,  $R$ ,

$$43 \quad I = L R, \quad (1)$$

44 where  $L$  depends on the brightness of the sky and of the angle of incidence of the light. The sky radiance  
45 distribution may be modelled in different ways for different sky conditions [7]. In this manuscript, we  
46 consider a uniform sky condition for simplicity, which is equivalent to considering only the Fresnel  
47 coefficient,  $R$ , [5],

$$48 \quad I = R = \frac{1}{2} \left[ \frac{\sin^2(\omega - \omega')}{\sin^2(\omega + \omega')} + \frac{\tan^2(\omega - \omega')}{\tan^2(\omega + \omega')} \right]. \quad (2)$$

49 The Fresnel reflection coefficient describes the reflectivity of the surface for an unpolarised illumination  
50 source where  $\omega$  is the angle of incidence of the sky radiance with respect to the sea surface normal.  
51 Therefore,  $\omega$  is equal to the angle of the camera viewing direction with respect to the sea surface normal,  
52 while  $\omega'$  is the angle of refraction related to  $\omega$  by Snell's law,  $\sin(\omega) = 1.34 \sin(\omega')$ . The sea surface can  
53 be defined by the local wave slope, hence it is possible to calculate the vector normal to the wave sea  
54 surface,

$$55 \quad r_n = \frac{r_n'}{\|r_n'\|}, \text{ where } r_n' = \left[ \frac{\partial \eta}{\partial x}, \frac{\partial \eta}{\partial y}, 1 \right]. \quad (3)$$

56 The camera viewing direction,

$$57 \quad r_c = \left[ -\cos \tau \cos \alpha_c, -\cos \tau \sin \alpha_c, \sin \alpha_c \right], \quad (4)$$

58 depends on both camera tilt from horizontal,  $\tau$ , and azimuth,  $\alpha_c$ , where the latter is measured from the x-  
59 axis in the counter-clockwise direction. Therefore, the incident ray,  $r_i$ , can be defined knowing the surface  
60 normal and the extrinsic camera parameters as,

61 
$$r_i = 2r_n(r_n \cdot r_c) - r_c. \quad (5)$$

62 Then, the incidence angle,  $\omega$ , can be defined as[3],

63 
$$\omega = \cos^{-1}(r_n \cdot r_c). \quad (6)$$

64  
65 The highest optical contrast occurs when the camera looks in the direction of wave propagation ( $\theta - \alpha_c$   
66  $= 0^\circ$ , where  $\theta$  is the incident wave direction), while the waves are less visible when the camera looks along  
67 the crest ( $\theta - \alpha_c = 90^\circ$ ), where surface gravity wave slope is less than the direction of propagation. Images  
68 looking along the wave crest may be dominated by high frequency waves rather than the dominant  
69 component of the wave spectrum [3, 4].

70 The loss of wave signatures in the images may influence many algorithms that exploit imaging of waves.  
71 One of the most important morphological measurements that can be derived from optical determination of  
72 wave characteristics is the nearshore bathymetry. The importance of nearshore bathymetry stems from its  
73 influence on nearshore physical processes. For example, prediction skill of forecasting models increases  
74 with more accurate bathymetric boundary conditions [e.g. 8, 9, 10]. Quantifying bathymetric change is  
75 crucial to understand flood risk exposure [11] and erosion and accretion processes of the beach, as well as  
76 to support navigation and engineering projects. Monitoring the beach behavior under both seasonal and  
77 extreme events is also important to facilitate coastal management decisions [12]. Yet, traditional methods  
78 for surveying nearshore bathymetry are expensive and time-consuming, resulting in spatial and temporal  
79 resolution lower than necessary for observational and modelling needs. On the contrary, remote sensing  
80 techniques can indirectly estimate the water depth and fill spatial and temporal gaps in surveyed  
81 bathymetry [13].

82 Depth-inversion is one of the most frequently used video-based remote sensing methods to estimate  
83 nearshore bathymetry in the presence of surface gravity waves. The method is based on the inversion of  
84 the dispersion relationship and exploits the wave celerity observed by optical imagery in intermediate or  
85 shallow water depths. This approach is based on the linear [e.g. 14], nonlinear [e.g. 15], or extended  
86 Boussinesq dispersion equations [e.g. 16]. Wave celerity estimates needed for the inversion can be  
87 conducted in the time domain [e.g. 17] or the frequency domain [e.g. 14]. The temporal method computes  
88 a time-domain cross-correlation between neighboring positions to estimate the wave celerity [17], while  
89 the spectral method uses a cross-spectral correlation to estimate the wave celerity [18]. Both approaches  
90 result in depth estimates with similar accuracy using synthetic optical video data [19].

91 Optical applications of remotely sensed depth inversion methods require video images of waves.  
92 Therefore, accuracy of the bathymetric estimation depends partly on the ability to distinguish the wave

93 signal, which is dependent on viewing angle. Typically, shore based video monitoring stations have a fixed  
94 azimuthal direction that is nominally in the direction of wave propagation. However, shore based  
95 monitoring stations mounted at atypical locations (e.g., cameras mounted on a jetty, headland, or satellite  
96 video and unmanned aircraft system (UAS) looking perpendicular to the direction of wave propagation)  
97 may result in optical image with a lower signal to noise ratio.

98 The effects of azimuthal viewing angle on depth inversion algorithms are not documented in literature;  
99 therefore, this Short Communication aims to quantify the sensitivity of a widely used depth inversion  
100 algorithm, cBathy, to the wave-viewing angle. We chose to conduct the analysis using synthetic data to  
101 avoid the complexities of real imagery such as breaking waves, irregular bathymetry, currents, non-uniform  
102 lighting conditions and, sometimes, reflection or diffraction of waves and interaction with engineered  
103 structures, such as harbors and jetties that violate assumptions of cBathy. Synthetic tests simplify the  
104 problem and focus the analysis on the role of light reflection off the water surface and wave viewing angle  
105 on error in estimated water depth as well as estimated frequency and wavenumber. The method for  
106 creating synthetic imagery is presented in Section 2 along with a review of the cBathy algorithm. In Section  
107 3, we illustrate the application of synthetic tests to study the influence of wave viewing direction on water  
108 depth estimation and we discuss the results and the role of errors in frequency and wavenumber. General  
109 conclusions are provided in Section 4. In the appendix A we discuss the sensitivity of cBathy v1.1 to the  
110 initial guess of the wave direction necessary for the nonlinear solver. Additionally, we provide a freely  
111 available toolbox for generating synthetic imagery, so that users can generalize results to their own coastal  
112 video monitoring stations.

## 113 **2. Methods**

114 The procedure used to build synthetic data consists in four main steps. First, frequency-directional  
115 spectra are defined. Then, synthetic sea surface elevation time series are generated based on the  
116 previously defined input spectra. Afterwards, a time series of synthetic imagery from simulated reflected  
117 radiance are built for the simulated sea surface [7, 20]. Finally, the synthetic optical time series are used as  
118 input to a widely used depth inversion algorithm to estimate the bathymetry. The Matlab© functions we  
119 developed to model the synthetic optical time series, based on the work of Chickadel [21], are available  
120 (<https://github.com/Coastal-Imaging-Research-Network/station-design-toolbox>).

### 121 *2.1 Synthetic imagery*

122 We considered two types of input spectra to generate the synthetic imagery that included real spectra  
123 (Table 1) representative of protected seas (e.g., Adriatic Sea), characterized by bi-modal spectra [22], and  
124 analytic frequency-directional spectra (Table 2) to analyze different conditions (different camera height, tilt

125 or spreading parameter). The first type of spectra used to develop synthetic imagery comes from the  
 126 EsCoSed field experiment, performed at the Adriatic Sea [23], and are representative of the Adriatic winter  
 127 storm conditions. The observations were collected with a sentinel Acoustic Doppler Current Profile (ADCP)  
 128 deployed in about 7.3 m water depth and 850 m offshore of the mouth of the Misa River, Senigallia, Italy  
 129 (43° 43.588' N, 13° 13.941' E). The spectra were statistically estimated from ADCP velocity observations. We  
 130 focused on a storm that occurred on 25 January 2014, and we selected spectra around the peak of the  
 131 storm during which the wave height and energy were maximum (E01-E02-E03-E04). Then, we selected one  
 132 spectrum related to the mean storm energy (E07). We manually modified the peak direction of each  
 133 selected spectrum, shifting the wave angle but preserving the spectral shape and energy (Figure 1a). In our  
 134 analyses, the wave propagation direction,  $\theta$ , is measured from the x-axis (considered the cross-shore  
 135 direction) in the counter-clockwise direction. The second type of spectra was generated from an analytic  
 136 frequency-directional spectra,  $S(f, \theta)$ , which was expressed as

$$137 \quad S(f, \theta) = E(f)D(f, \theta), \quad (7)$$

138 where  $E(f)$  is the one dimensional, frequency dependent wave spectrum and  $D(f, \theta)$  is the directional  
 139 distribution, which depends on both frequency,  $f$ , and direction,  $\theta$ . The shape of the frequency spectrum ( $E$   
 140 ( $f$ )) is defined in terms of the significant wave height,  $H_s$ , and the mean zero-upcrossing period,  $T_z$ , by  
 141 fitting the JONSWAP spectrum. For  $E(f)$ , the formulation of Carter [24] was used, where  $T_p = 1.286T_z$  is  
 142 the spectral peak period,

$$143 \quad E(f) = G(f)0.0749H_s^2T_z(T_zf)^{-5}\exp[-0.4567/(T_zf)^4], \quad (8)$$

$$144 \quad G(f) = 3.3 \exp\left[-\frac{(1.286T_zf-1)^2}{2\sigma^2}\right], \quad (9)$$

$$145 \quad \sigma = \begin{cases} 0.07 & \text{for } 1.286T_zf < 1 \\ 0.09 & \text{for } 1.286T_zf > 1 \end{cases} \quad (10)$$

146 The direction distribution,  $D(\theta)$ , depends only on the wave direction,  $\theta$ ,

$$147 \quad \begin{cases} D(\theta) = D_0 \cos^{2s}[\theta - \theta_p] & \text{if } |\theta - \theta_p| < \pi/2 \\ 0 & \text{otherwise} \end{cases}, \quad (11)$$

$$148 \quad D_0 = \frac{1}{\pi^{0.5}\Gamma(s+1/2)}, \quad (12)$$

149 where  $\theta_p$  is the spectral peak direction,  $\Gamma$  is the Gamma Function,  $D_0$  is the normalization factor and  $s$  is the  
 150 spreading parameter [25, 26]. The parameters used for the analytical spectra are summarized in Table 2  
 151 and an example of the resulting frequency-directional spectra is shown in Figure 1b. We define two general  
 152 cases, characteristic of the Central Adriatic wave climatology, but the results may be generalized for other

153 sites. The first case used  $H_s = 3.0$  m and  $T_p = 7$  s, typical of storm waves in the Adriatic approaching the  
154 Italian coast from ESE (A10-A11-A12-A13). The second case used  $H_s = 2.5$  m and  $T_p = 10$  s, typical of storm  
155 waves approaching from NNE (A20 A21-A22-A23). We generated wave spectra for a range of peak  
156 directions.

157 For each defined spectrum, synthetic sea surface time series have been generated within a simulated  
158 camera field-of-view following Percival [27] and Scarsi [26]. The sea surface elevation time series,  $\eta(x,y,t)$ ,  
159 can be represented as

$$160 \quad \eta = \text{ifft}(W), \quad (13)$$

161 where  $\text{ifft}(W)$  is the inverse Fourier transform and the Fourier series,  $W$ , is defined as

$$162 \quad W = A_w(\cos(\varphi) + i\sin(\varphi)) + A_n(\cos(\varphi_n) + i\sin(\varphi_n)), \quad (14)$$

163 where  $A_w$  is the amplitude of the wave signal in the frequency domain and is related to the input spectral  
164 characteristics;  $A_n$  is the amplitude of the noise signal in the frequency domain and is proportional to the  
165 noise to signal ratio, and  $\varphi$  and  $\varphi_n$  are the phase of the harmonic variability of the waves and noise,  
166 respectively. Since the spectrum is independent of the phase of the harmonic variability, the phases,  $\varphi$  and  
167  $\varphi_n$ , are arbitrary, hence we computed them with a random function,

$$168 \quad \varphi = \varphi_r - k x \cos \theta - k y \sin \theta, \quad (15)$$

$$169 \quad \varphi_n = \varphi_{r,n}, \quad (16)$$

170 with random values,  $0 \leq \varphi_r < 2\pi$ ,  $0 \leq \varphi_{r,n} < 2\pi$ , and  $k$ , the wavenumber. The approach allows for an  
171 infinite number of possible time series to be generated with the same input spectrum. We generated a  
172 time series for each wave direction, then, we summed for all wave directions. We considered only the real  
173 part for the first  $N_s$  elements of the transformed series.

174 Considering the slope of the synthetic sea surface, we generated synthetic optical time series  
175 corresponding to the simulated wave time series using the radiance modulation model [7] described in (1) –  
176 (6). We simulated optical images of linear, non-breaking waves propagating over a flat bottom in  
177 intermediate water depth, where depth inversion algorithms were expected to work well.

## 178 2.2 Depth inversion

179 The optical time series generated in Section 2.1 were used as input to the well-known cBathy v1.1  
180 depth inversion algorithm [28]. We chose this algorithm because it is open source  
181 (<https://github.com/Coastal-Imaging-Research-Network/cBathy-toolbox>) and has become one of the most  
182 widely used depth inversion algorithms [e.g. 29, 30, 31, 32, 33, 6, 19, 34, 35, 36].

183 The cBathy algorithm is based on the inversion of the linear dispersion equation, that relates the water  
184 depth to the wave celerity, without a current present,

$$185 \quad \Omega^2 = gk \tanh(kh), \quad (17)$$

186 where  $\Omega$  is the radian wave frequency,  $k$  is the wavenumber,  $h$  is the water depth and  $g$  is the acceleration  
187 due to gravity. The local water depth was estimated from a suite of observed wave frequency and  
188 wavenumber pairs. Therefore, accurate bathymetry estimation is dependent upon accurate estimation of  
189 both frequency and wavenumber.

190 Execution of the cBathy v1.1 algorithm consists of three steps. The first step carries out a  
191 frequency-dependent analysis and estimates the (usually four) most coherent pairs of wave frequencies  
192 and wavenumbers. Following Plant et al. [18], for each analysis point, the algorithm considers a subgrid in  
193 which the dominant frequencies are estimated by Fourier transform of the input optical signal and the  
194 cross-spectral matrix is computed between all pixel pairs in the subgrid. The cross-spectral matrix is filtered  
195 using spatial eigenvector analysis to identify the dominant spatial phase of the waves. The corresponding  
196 wavenumbers are derived by fitting the observed spatial phase structure to a forward model. Initial guesses  
197 at the value of wavenumber and wave direction (seed angle) are necessary for this nonlinear fit. The  
198 second step in the cBathy v1.1 algorithm combines the frequency-wavenumber pairs from Step 1 to give a  
199 single depth estimate. At each analysis point, the algorithm chooses the  $f - k$  pairs from within the subgrid  
200 to use in the depth estimate by weighting by distance from the analysis point and skill of the modelled  
201 wave phase. Then, the algorithm calculates the depth as the value that yields the best weighted nonlinear  
202 fit between the first step  $f - k$  pairs and the dispersion (17). The third step uses a Kalman filter to smooth  
203 and average the estimated depth results. The third step is neglected in this analysis.

### 204 *2.3 Example imagery and depth inversion*

205 An example image and depth inversion is shown for a 1 km by 1 km region with 3 m resolution (Figure  
206 2). The camera height was 25 m and the water depth was constant and equal to 7 m. The camera was  
207 located at the origin of coordinate system and looks along the x-direction, but the tilt and azimuth changed  
208 over the synthetic image so that the angular difference between the wave and camera view directions  
209 varied. In the example imagery, we varied the direction of wave propagation that included,  $0^\circ$  or from the  
210 x-direction (Figure 2a and 2d),  $90^\circ$  or from the y-direction (Figure 2c and 2f) and  $45^\circ$  (Figure 2b and  
211 2e). Waves approaching from the x-direction have the convention,  $\theta - \alpha_c = 0^\circ$ .

212 The effects of the variation in camera tilt and camera azimuthal angles on the optical imaging of surface  
213 gravity waves were summarized in Section 1. The tilt variation effects manifest as variation of intensity  
214 magnitude so that when moving closer to the origin of the camera system, the tilt angle increased and the  
215 intensity magnitude decreased (Figure 2a, 2b, 2c). The azimuth variation effects have been observed by

216 changing the wave direction,  $\theta$ , that is equivalent to changes in azimuth direction,  $\alpha_c$ . Qualitatively,  
 217 synthetic imagery (Figure 2) demonstrates the effect of varying  $\theta - \alpha_c$  on both image intensity and  
 218 bathymetric estimation. By increasing the angular difference, the longer wavelength waves are less visible  
 219 in the optical image, and wave crests propagating parallel to the viewing direction are dominated by high  
 220 wavenumbers. Likewise, the estimated water depth is more variable in regions dominated by high  
 221 wavenumbers which fall closer to the deep water limit. For example, in Figure 2a and 2d, the error was  
 222 largest close to the y-axis, where the angular difference was maximum ( $90^\circ$ ), and the error decreases  
 223 towards the x-axis, where the angular difference was minimum ( $0^\circ$ ). In Figure 2b and 2e, the bathymetric  
 224 error was lower because the angular difference did not exceed  $45^\circ$ . In Figure 2c and 2f, the maximum error,  
 225 corresponding to the maximum angle difference, was close to the x-axis.

226

### 227 3. Results and Discussion

228 We used the synthetic procedure illustrated in Section 2 to perform a sensitivity analysis of wave  
 229 viewing direction on water depth estimation. We considered an analysis domain of 200 m by 200 m with 3  
 230 m resolution. The camera was located at the origin of coordinate system and looking along the x-direction.  
 231 Within the domain, we assumed a fixed camera tilt and azimuth angle to focus on the effects of the  
 232 variation of the azimuthal wave viewing direction. The azimuthal wave viewing angle,  $(\theta - \alpha_c)$  was  
 233 progressively increased from  $0^\circ$  to  $90^\circ$ , by changing the peak wave direction, over the small analysis  
 234 domain. The camera tilt was set to  $14^\circ$  or  $18^\circ$  and the camera height set to 25 m or 40m, respectively. In  
 235 one case the tilt was set to  $45^\circ$ . The input bathymetry had a constant depth of 7 m or 10 m. For each  
 236 combination of input parameters listed in Table 1-2, we computed ten random realizations of the sea  
 237 surface, optical image, and estimated the water depth, following the methodology outlined in Section 2.  
 238 Then, for each realization, the relative error in depth estimation was quantified by comparing the  
 239 estimated bathymetry to the water depth used to create the synthetic sea surface:

$$240 \quad \text{relative error} = \sqrt{\frac{\sum_{n=1}^N |(h_E - h_T)/h_T|^2}{N}}, \quad (18)$$

241 where  $h_E$  is the estimated water depth and  $h_T$  is the true water depth, and  $N$  is the number of comparison  
 242 values (number of grid points). Finally, the mean relative error and the corresponding standard deviation  
 243 were calculated over the ten realizations to reduce the noise due to the random phases (Figure 3-4).  
 244 Consistent with our understanding of the effects of azimuthal viewing angle on optical imaging of waves [4,  
 245 5], the variation of  $(\theta - \alpha_c)$  influenced estimates of water depth.

246 For analytical spectra, the relative errors for angular differences of less than  $75^\circ$  were almost constant  
247 and low (relative error order 0.02 – 0.08) over the horizontal viewing angle variation (Figure 3a, 3b). Within  
248 this range of viewing angles, the magnitude of error in bathymetry estimation was consistent with the error  
249 reported in observational studies when algorithm assumptions are not violated [14, 6, 34]. For larger  
250 angular differences, the waves are looked mainly along the crest and the optical images are dominated by  
251 high frequency waves rather than the dominant component of the wave spectrum heading to a noisy signal  
252 for the depth inversion algorithm (see Section 1). The presence of short wavelengths in the optical images  
253 lead to errors in depth estimation that rapidly increase until a relative error order 0.2 (Figure 3).

254 Again, using analytical spectra, we considered several other influences on estimated water depth  
255 including camera height, camera tilt angle, water depth and directional spreading of the analytical spectra.  
256 By considering a specific area of the field of view, changing the camera height is equivalent to changing the  
257 tilt angle and vice versa. The camera heights and the relative tilt angles considered here did not affect the  
258 general reconstruction of the bathymetry (see differences between case A20 and case A21 in Figure 3a).  
259 Considering a fixed camera height and a variable tilt angle is equivalent to modifying the distance from the  
260 camera location of the observed area in x direction. We analyzed several values of tilt for a fixed camera  
261 height (not shown) but we reported only the case in which the camera looks straight down (A23) because it  
262 could be relevant for sUAS. In all cases we did not find any relevant errors on bathymetry estimation in  
263 relation to the tilt variation. In fact, in the optical model that we used, the tilt variation affects only the  
264 intensity magnitude (Figure 2), which is then normalized by the depth inversion algorithm. The range of  
265 waters depths, and normalized water depths ( $kh$ ) considered here had a minimal effect on the relative  
266 depth error, in particular for  $(\theta - \alpha_c)$  less than  $75^\circ$  (compare cases A20 and case A22 Figure 3a). The water  
267 depths were in fact chosen deep enough to avoid breaking and nonlinear effects but not too deep to make  
268 the dispersion relationship insensitive to depth. Instead, the directional spreading somewhat affected the  
269 depth inversions, particularly as  $(\theta - \alpha_c)$  increased (Figure 3b). When directional spreading was small, the  
270 depth estimate from the inversion was insensitive to  $(\theta - \alpha_c)$  (see case A13 in Figure 3b).

271 Error analysis with experimental spectra produced similar results to the analytical spectra (rapidly  
272 increasing error for  $(\theta - \alpha_c)$  greater than  $75^\circ$ ), with a few notable differences (Figure 4). Our experimental  
273 input spectra were less directionally spread than the analytical spectra. In most cases the error magnitude  
274 of the experimental cases was similar to the error magnitude of the analytical cases with low directional  
275 spreading (A12-A13). The cases E01, E02, E03, E04 error increases as a function of  $(\theta - \alpha_c)$  in a way similar  
276 to the analytical cases while a different behavior has been observed for case E07 (Figure 4, green line). This  
277 last case is characterized by shorter peak wave period, and in turn by larger value of  $kh$ , than the other  
278 cases (see Table 1). We found that the anomalous shape is related to sampling problems inside the cBathy  
279 v1.1 due to the shorter waves of case E07. To avoid this problem, we used the cBathy v1.2 that improves  
280 the nonlinear fit for short waves (see Figure 4).

281 Since the depth inversion estimation depends upon accurate estimates of frequency and wave number  
282 pairs, we compared the  $f - k$  pairs, estimated from cBathy v1.1, with the linear dispersion relationship  
283 relative to the spectrum at the specific depth (Figure 5). The cBathy v1.1 derived frequencies and  
284 wavenumbers come from the four most coherent frequency-wavenumber pairs obtained in Step 1 of the  
285 algorithm that exceeded a minimum skill threshold. Errors in frequency and wavenumber pair estimations  
286 increase with increasing the wave viewing angle ( $\theta - \alpha_c$ ) and erroneous frequency and wavenumber pairs  
287 begin to dominate for angles exceeding  $75^\circ$ .

288

#### 289 **4. Conclusions**

290 We utilized synthetic tests to analyze the effects of wave direction on water depth estimation using the  
291 optical implementation of the linear depth inversion algorithm, cBathy v1.1. We found that the error in the  
292 water depth estimates where wave viewing angle is less than  $75^\circ$  were consistent with previous field  
293 observations (relative root mean square error = 0.02 - 0.08). Given that the synthetic tests were designed  
294 to adhere to algorithm assumptions, the result suggests the limit of accuracy that can be expected from the  
295 algorithm. When the wave viewing angle exceeded  $75^\circ$ , the wave slope associated with the dominant  
296 frequencies became obscured, leading to errors in both frequency and wavenumber estimation which in  
297 turn result to errors in depth. Errors were larger for directionally spread waves. Our results and the  
298 proposed procedure to build synthetic optical images can be applied to develop sampling schemes for fixed  
299 camera coastal video monitoring stations or for small Unmanned Aerial Systems (sUAS) with viewing waves  
300 different from the typically offshore-pointing azimuth direction.

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307

## 308 **Appendix A**

309       cBathy v1.1 requires an initial guess at the direction of wave propagation to seed the nonlinear fit used  
310 to obtain directions and wavenumbers in Step 2 of the algorithm. Therefore, the choice of the seed angle  
311 also impacts the final estimate of water depth. Usually, the cBathy v1.1 seed angle is set assuming that the  
312 waves come from the cross-shore direction (0°). However, in operational application the incoming wave  
313 may not be shore-normal directed and the direction of wave propagation may vary with space and time. As  
314 a result, the seed angle is a potential source of error in the estimated water depth using cBathy v1.1. Note  
315 that the new version of cBathy algorithm (cBathy v1.2) removes the need to specify the incoming wave  
316 angle by estimating the seed angle from the spatial phase structure and an initial guess at water depth.  
317 However, the cBathy v1.1 is still widely used and the seed angle problem is not yet addressed in the  
318 literature.

319       To quantify the sensitivity of the cBathy v1.1 to the seed angle, we present results with different initial  
320 guesses of the wave direction. This analysis has been performed using the 1 km by 1 km grid (Figure 2) and  
321 considering three directions of wave propagation (0°, 45°, 90°). cBathy was initialized using a range of seed  
322 angles (from 0° to 90°) and the parameters listed in Table 3. The relative error was computed with (18)  
323 (Figure A.1). Differences between seed angle and wave direction greater than 45° resulted in undulatory  
324 features in the estimated water depth (not shown) and relative errors order 0.1-0.4 (Figure A.1). Relative  
325 error was minimized when the seed angle was closest to the wave direction. In the analyses performed in  
326 Section 2-3, we cared to set the initial guess at the direction equal to the wave propagation direction to  
327 avoid that the error due to a mistake of the setting seed angle can be added to the error due to a large  
328 wave viewing angle.

329       Finally, we investigated the role of the seed angle in  $f - k$  estimated from Step 1 of the cBathy v1.1  
330 algorithm. For this analysis, we compared the linear dispersion relationship with the estimated frequency-  
331 wavenumber pairs that exceeded a minimum skill threshold in a way similar to the analysis performed in  
332 Section 3. Figure A.2 shows an example of this comparison for case E01 and wave direction equal to 0°. For  
333 cases with no error in seed angle (Figure A.2a), errors in frequency and wavenumber pair estimations were  
334 minimal while the errors increased when the seed angle was not correctly setting (Figure A.2b-c). In the last  
335 cases the underestimation of the depth was related to an overestimation of the wavenumber.

336

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**Table 1** - Summary of analysed sea states and parameters, from EsCoSed experiment source. For each case the table displays the index, the peak period, the significant wave height, the wave energy, the camera height, the camera tilt angle (fixed in the wave viewing analysis), the water depth and the kh computation.

case	Tp (s)	Hs (m)	Smax (m <sup>2</sup> s)	hc (m)	tilt (°)	h (m)	kh
E01	8.79	2.95	10.85	25	14	7	0.6427
E02	8.79	3.09	8.74	25	14	7	0.6427
E03	9.44	2.99	5.31	25	14	7	0.5933
E04	9.44	2.92	14.21	25	14	7	0.5933
E07	5.94	1.54	1.51	25	14	7	1.0304

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**Table 2** - Summary of analysed sea states and parameters, from analytical source. For each case the table displays the index, the peak period, the significant wave height, the spreading parameter, the camera height, the camera tilt angle (fixed in the wave viewing analysis), the water depth and the kh computation.

case	Tp (s)	Hs (m)	s	hc (m)	tilt (°)	h (m)	kh
A10	7.00	3.00	5	25	14	7	0.8384
A11	7.00	3.00	2	25	14	7	0.8384
A12	7.00	3.00	10	25	14	7	0.8384
A13	7.00	3.00	20	25	14	7	0.8384
A20	10.00	2.50	5	25	14	7	0.5567
A21	10.00	2.50	5	40	18	7	0.5567
A22	10.00	2.50	5	25	14	10	0.6798
A23	10.00	2.50	5	25	45	7	0.5567

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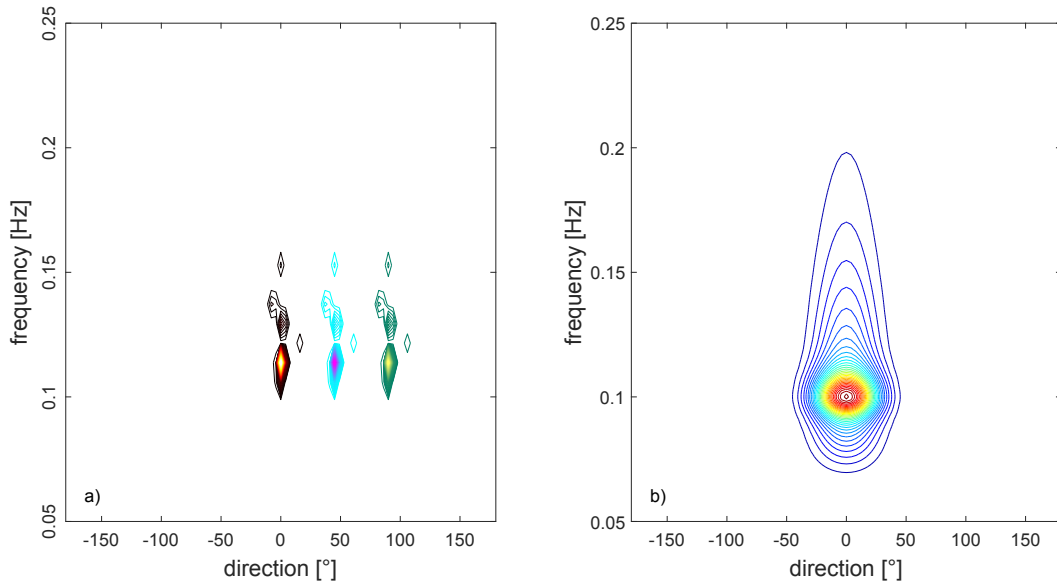
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**Table 3** - Summary of cBathy parameters. The x-axis is the cross-shore direction, and the y-axis is the alongshore direction.

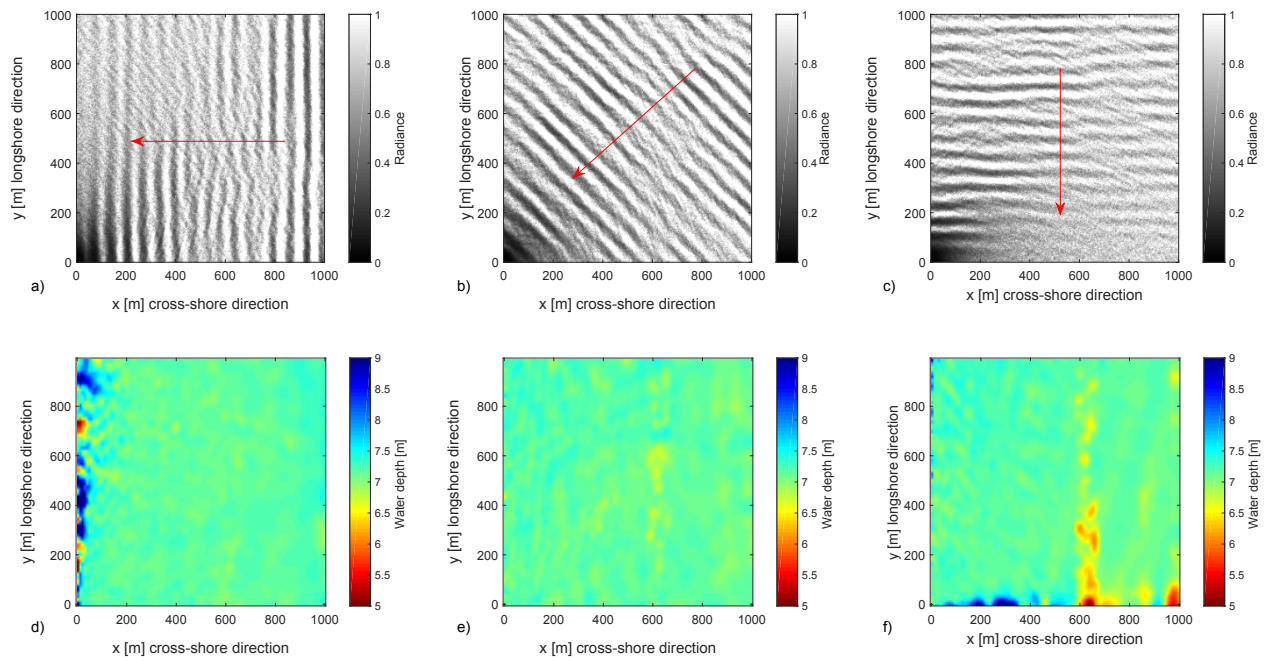
cBathy parameter name	value	description
params.dxm	9 m	Analysis domain spacing in x
params.dym	13 m	Analysis domain spacing in y
params.xyMinMax	[0 1000 0 1000] for Grids [50 250 50 250] for Patches	Spatial extent of the analysis grid
params.MINDEPTH	0.25 m	Min limit set for the nonlinear depth search in phase 2.
params.QTOL	0.5	Min skill
params.minLam	10	Min normalized eigenvalue to proceed
params.Lx	2*params.dxm	Smoothing length scales in x
params.Ly	2*params.dym	Smoothing length scales in y
params.kappa0	3	Multiplier that increase Lx seaward
params.maxNPix	80	Max number of pixels per subgrid
params.fb	[1/15 : 1/100 : 1/4]	List of frequencies for analysis
params.nKeep	4	Number of frequencies to keep
params.offshoreRadCCWFromx	Variable	Seed angle

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 431 **Figure 1** - a) Example of peak shifting for case E01 with peak directions of 0° (red), 45° (blue), and 90° (green). b) Example of  
 432 frequency directional spectrum (A20) designed using equations (7) to (12). (For interpretation of the references to color in this  
 433 figure, the reader is referred to the Web version of this article.)

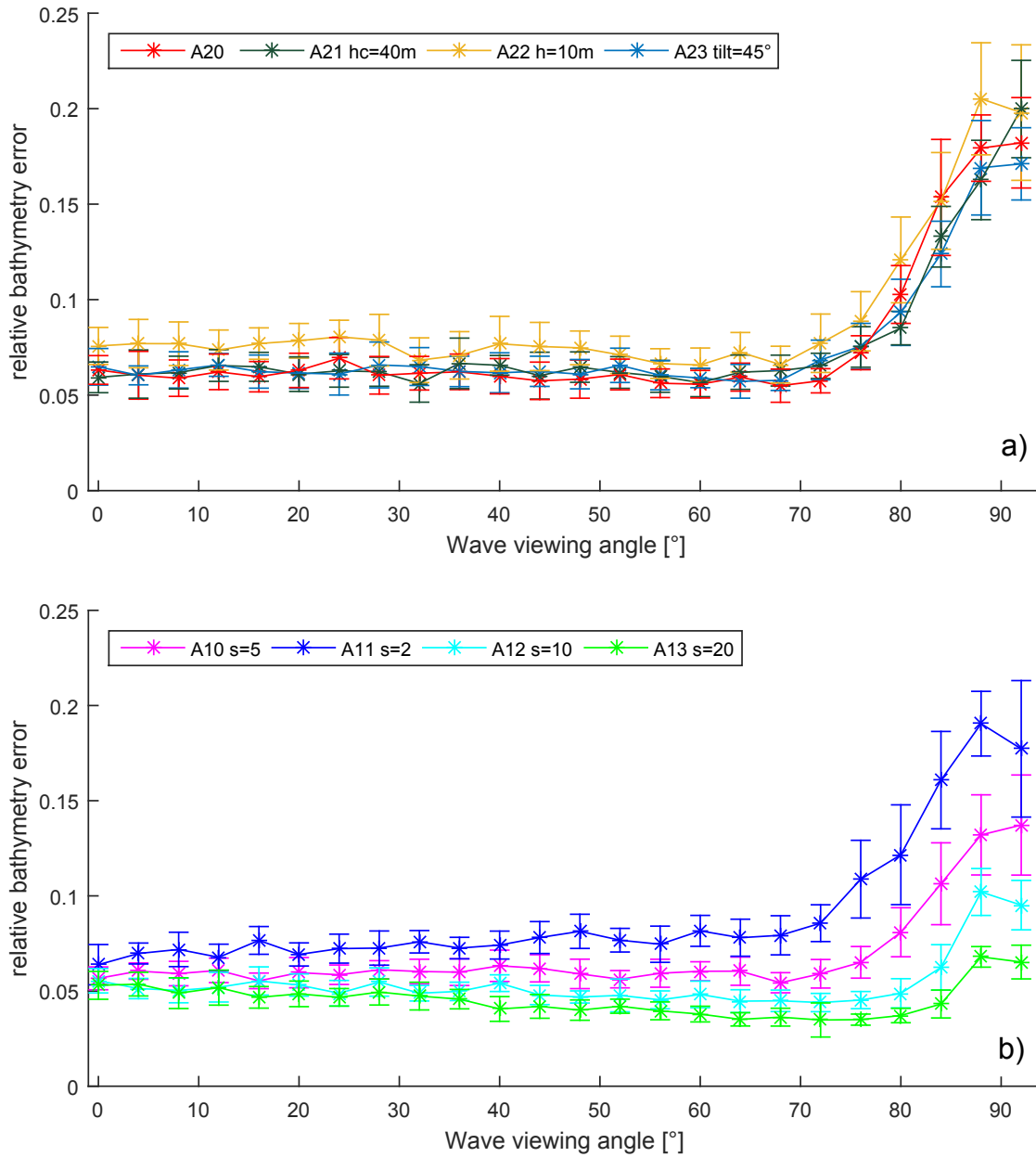
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 436 **Figure 2** -Synthetic optical images (upper panels) and estimated bathymetry (lower panels), for case E01, for wave angles equal to  
 437 0° (a,d), 45° (b,e), and 90° (c,f). The angles are positive in the counter-clockwise direction from the x-axis. The red arrows indicate  
 438 the wave direction. The seed angle was set coherent to wave propagation. (For interpretation of the references to color in this  
 439 figure, the reader is referred to the Web version of this article.)

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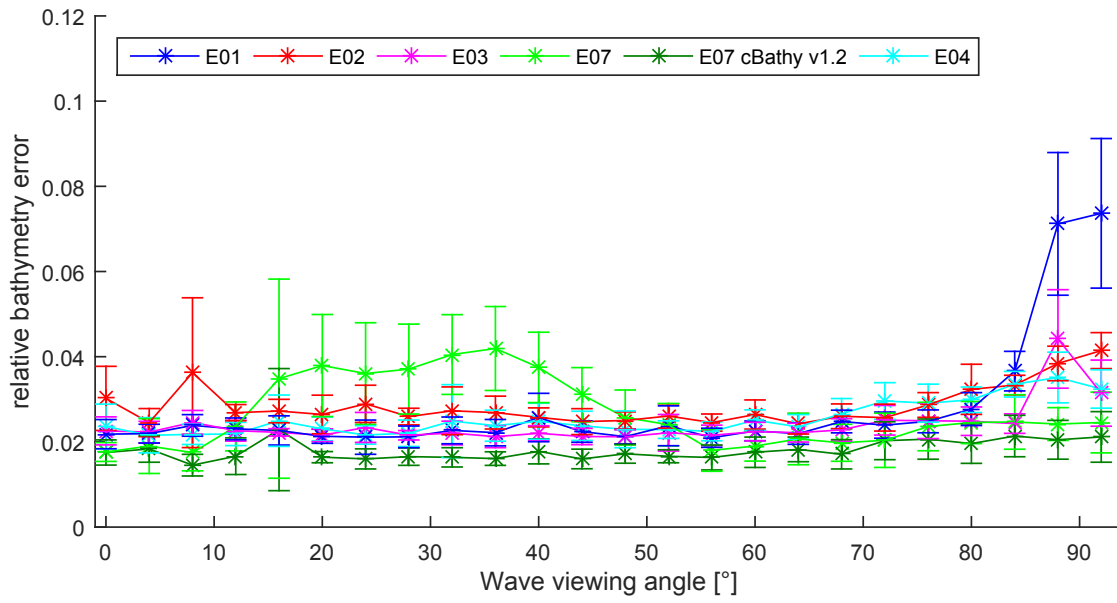
445 **Figure 3** - Mean (\*) and standard deviation (bars) of bathymetric error as function of difference between wave angle and camera  
446 viewing direction. a) analytical spectra A20, A21, A22 and A23; b) analytical spectra A10, A11, A12 and A13 with different  
447 directional spreading. (For interpretation of the references to color in this figure legend, the reader is referred to the Web version of  
448 this article.)

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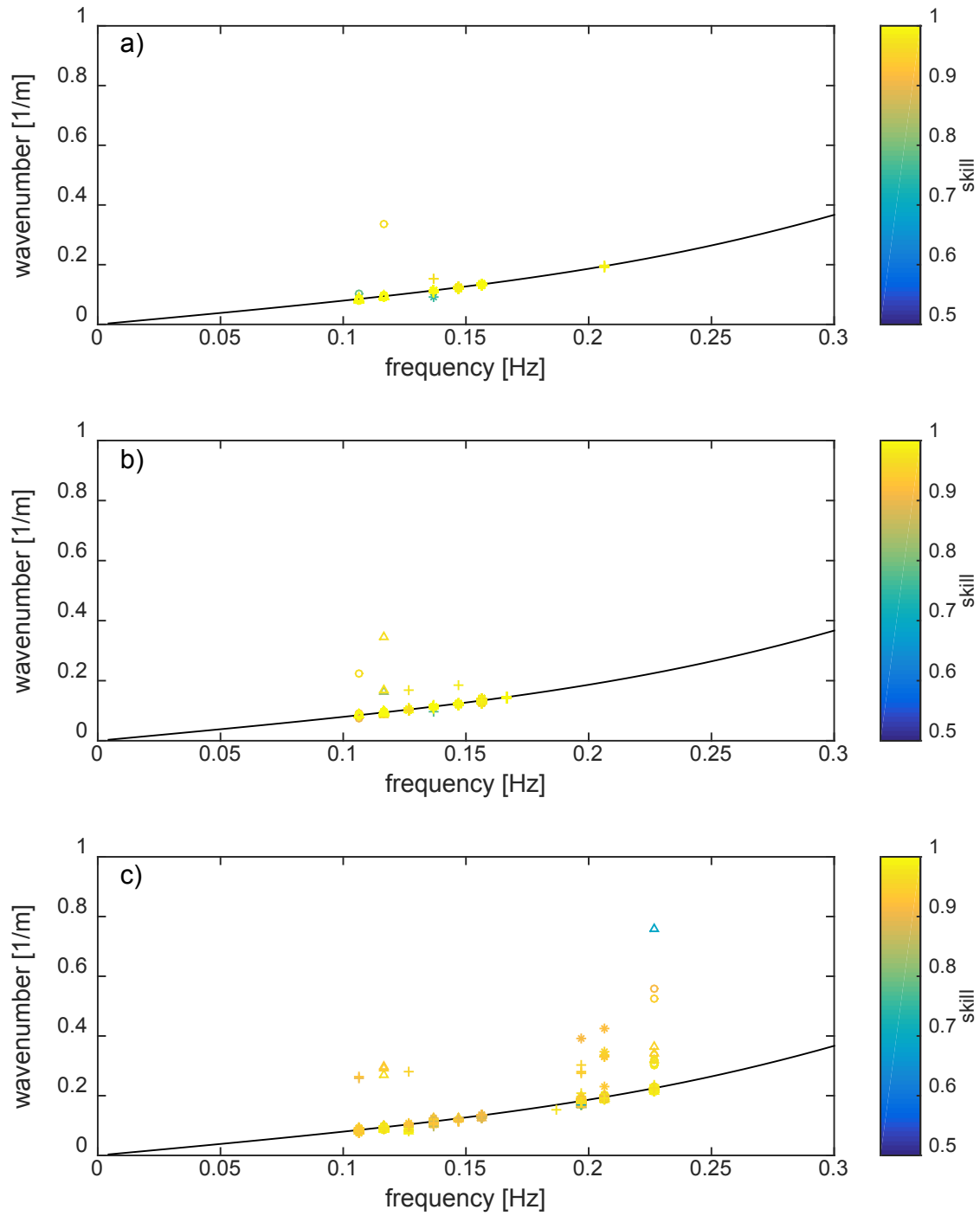
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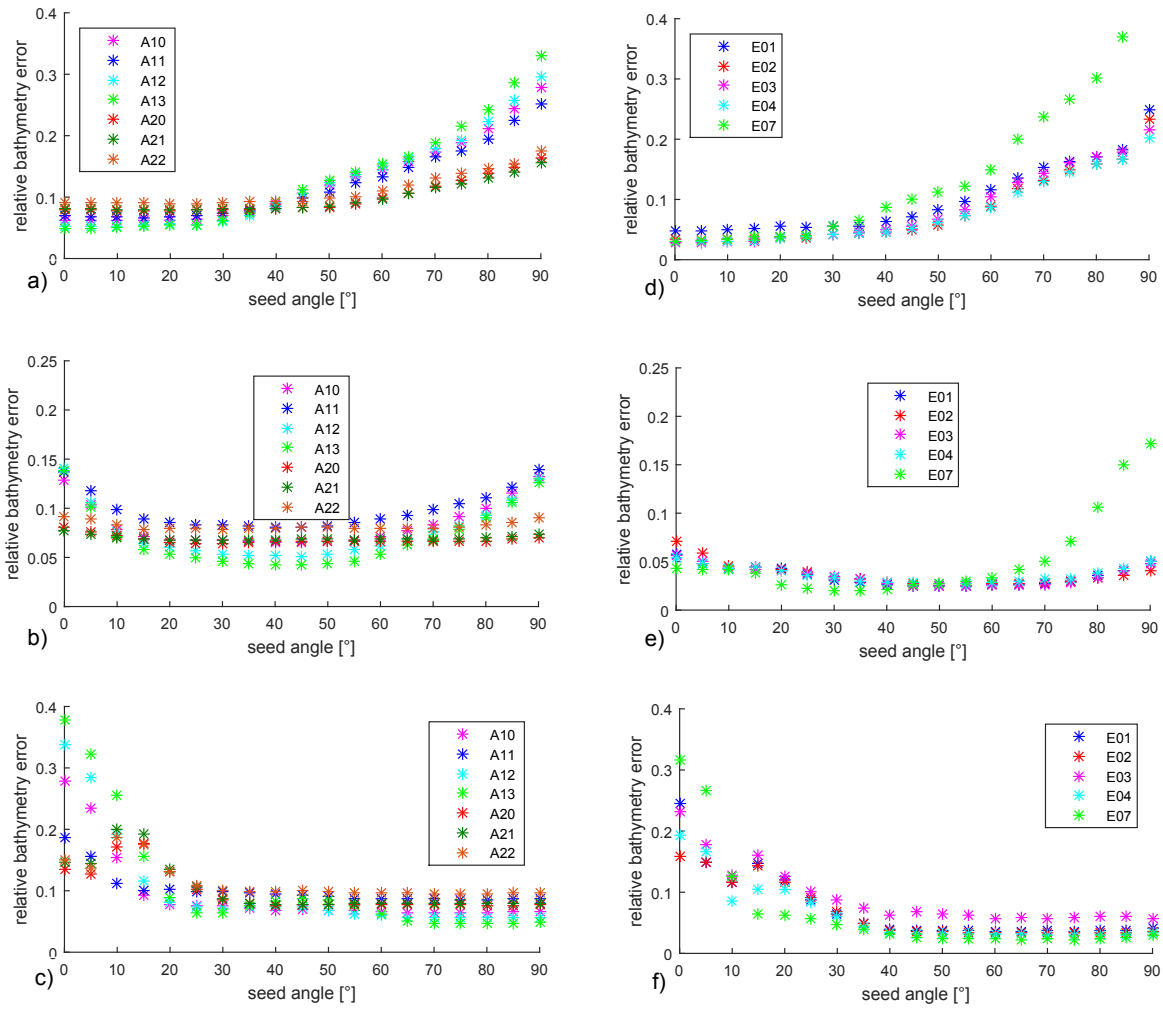
454 **Figure 4** - Mean (\*) and standard deviation (bars) of bathymetric error as function of difference between wave angle and camera  
455 viewing direction for observed spectra E01, E02, E03, E04 and E07. (For interpretation of the references to color in this figure legend,  
456 the reader is referred to the Web version of this article.)

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459 **Figure 5** –Frequency and wavenumber pairs estimated from cBathy for case E01 and relative to waves coming from 0° (a), 44° (b)  
 460 and 92° (c). The curve shows the linear dispersion relationship for the specified water depth (7m). The markers indicate the f-k pairs  
 461 estimated from cBathy in each point of the analysis grid and relative to the first (●), the second (△), the third (\*) and the fourth (+)  
 462 coherent frequency. The color gradient of the markers is proportional to the skill but only points that exceed the threshold are  
 463 plotted. Only one of the ten realizations is plotted for illustration. (For interpretation of the references to color in this figure legend,  
 464 the reader is referred to the Web version of this article.)

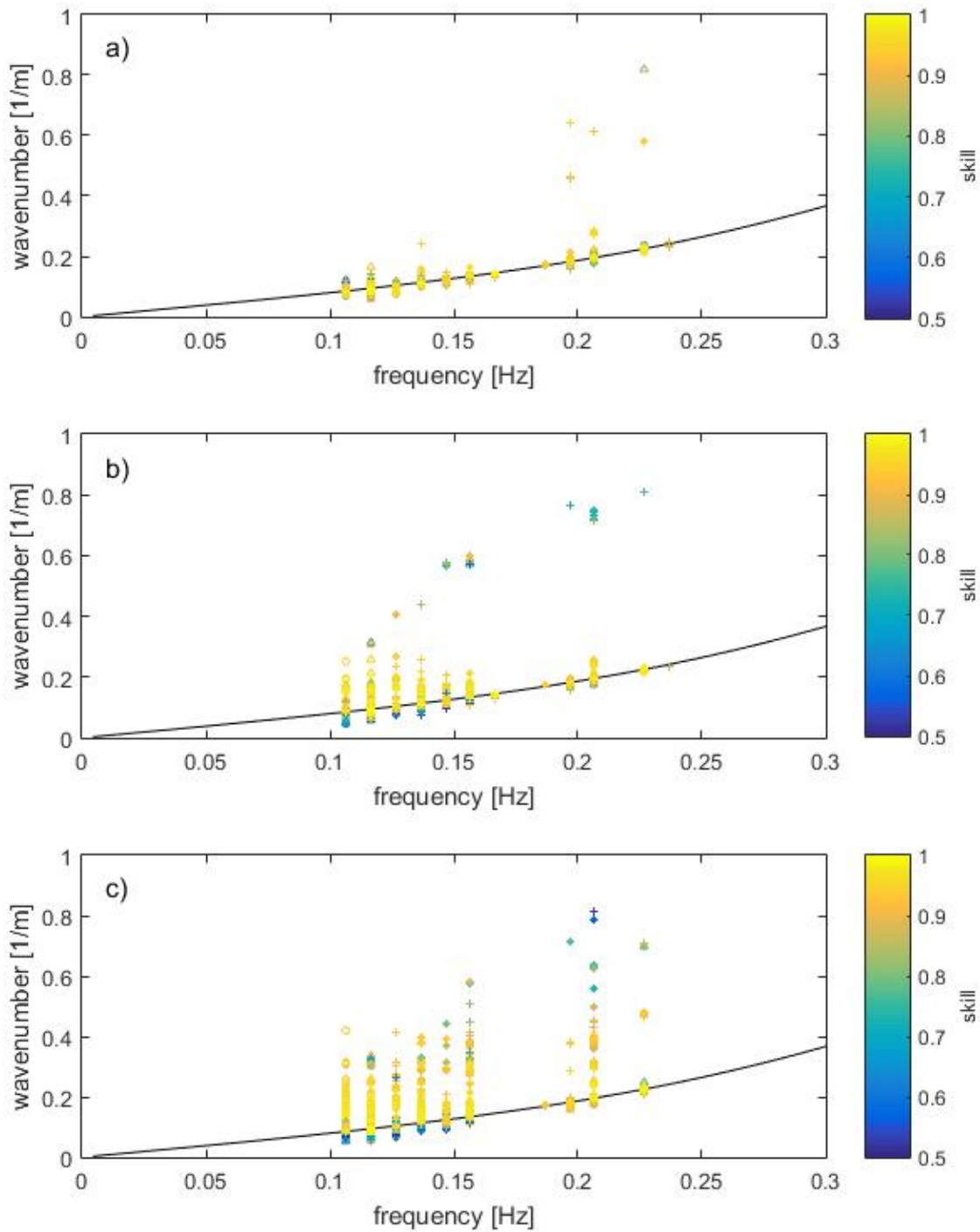


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467 **Figure A.1-** Seed angle sensitivity for analytical (a-b-c) and experimental (d-e-f) cases with wave angles of 0° (a,d), 45° (b,e), and 90°  
 468 (c,f). (For interpretation of the references to color in this figure legend, the reader is referred to the Web version of this article.)

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472 **Figure A.2** - Frequency and wavenumber pairs estimated from cBathy for case E01 and relative to waves coming from  $0^\circ$  and seed  
 473 equal to  $0^\circ$  (a),  $45^\circ$  (b) and  $90^\circ$  (c). The curve shows the linear dispersion relationship for the specified water depth (7m). The  
 474 markers indicate the f-k pairs estimated from cBathy in each point of the analysis grid and relative to the first (●), the second (△),  
 475 the third (\*) and the fourth (+) coherent frequency. The color gradient of the markers is proportional to the skill but only points that  
 476 exceed the threshold are plotted. (For interpretation of the references to color in this figure legend, the reader is referred to the Web  
 477 version of this article.)