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Three Essays on Income Distribution and Economic Growth

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1 Outline

Income distribution has always been one of the main topic of economic thought since its origin. In the second half of the last century, with the marginalist revolution, mainstream economics seems to have lost interest in this topic (for example, think of endogenous growth models in which income distribution is not considered at all). In recent years, however, maybe because of the economic crisis or because increasing inequality in most of developed countries, many scholars have rediscovered the distribution of income as a central topic in economic analysis. On this topic even the general public has been involved (think about Occupy Wall Street movement in 2011) thanks to dissemitation books (above all Piketty, 2014 but also Rajan, 2010; Milanovic, 2011; Krugman, 2012; Stiglitz, 2012). The institutions have also acknowledged the importance of this issue: one of the pillars of European Union stategy (the so-called Europe 2020 Agenda, the results of which one may judge more or less satisfactory) is a more inclusive growth through increasing participation in the labor market and the fight against poverty.

Dissertation is organized as follow: in chapter 2 I will review both theoretical and empirical literature on the relationship between income distribution and economic growth; in next two chapters I will develop two original models pertinent to the topic. Model of chapter 3 is a Post-Keynesian one in which I analyse how education and increasing labour productivity affect economic growth and the distribution of income within the working class. In chapter 4 I move the focus of the analysis from fuctional distribution to personal distribution of income, an agent-based model has been the most appropriate solution to do it.

2 Literature review

In this chapter I will analyze literature on relationship between economic distribution and economic growth both from a theoretical and empirical point of view.

2.1 Theoretical literature

Mainstream economics has identified as source of economic growth substantially two factors: technological progress and education. In post-keynesian literature, however, income distribution itself is that determines growth. In next sections I will review most relevant literature that has dealt with the issue of growth separating them according to the source.

2.1.1 Technological change

The effects of technological change on economic growth is the main topic of Shumpeterian economists. In this research line technological change can constitute in two different ways: as increase of the number of intermediate goods (Romer, 1990) or as introduction of a better quality new intermediate good (Aghion Howitt, 1992).
Both models are characterized by the presence of three sector: the consumption good sector, the intermediate good one and the research one. While the first and the latter are competitive the second one is a monopolistic competition system. Even if the notion of technological change is different between the two models the results are quite similar:

- economy will reach an equilibrium in which the growth rate, the shares of skilled workers in the final and intermediate good and firms investments in R& D are constant over time;
- higher preference of consumer to future consumption leads skilled worker move from the production of final good to the R& D sector [1] and a higher growth rate;
- higher incentives to the firms in the R & D sector to innovate (higher stock of actual intermediate goods or lower cost of innovation sector in the first model; higher probability of success in the second) induces to higher growth rate.

[1] in both models skilled workers are needed for the production of the final good

While these models investigate the effects of technological change in the supply side of the economy they do not take into account the demand side, the distribution of income and the market of labor.

The latter has been treated by Acemoglu (1998, 2002). He builds a model that is able to explain how the increase of the supply of skilled workers (due to an increase of graduate students since the 70s) [2] is compatible with the increase of skill premium rather then a fall as intuition suggests. His answer is skill-biased technological change.
In his model, according to neoclassical condition, the wage level of each typology of worker is equal to his marginal productivity so an exogenous increase of skilled workers decreases the marginal productivity[3], the skilled workers' wages and skill premium. This is just the short run effect. However this fall in high-skilled worker wages lets firms invest money in research and development and adjusts the productivity of each typology of workers: he hypothesizes an aggregate production function which utilize two kinds of input: an input prdoduced by skilled workers and another input produced by unskilled ones, both industries utilize specific machinery. Machinery are produced by a monopolist that has incentive in invest in one or another type of technology because of two different incentives that behave in different directions:

- Price effect. Technologies producing more expensive goods will be improved faster. Since goods using the scarce factor will command a higher price innovation is directed at the scarce factor (unskilled sector from the 70s)

- Market size effect. A large clientele for a technology leads to more innovation. Since customers for a technology is the number of workers who use it, the market size effect encurages innovation in markets where the more abundant factor is (skilled sector from the 70s)

The final direction is the sum of two effects and depends on the substitutability of the two intermediate goods: if two intermediate goods are substitutes the market effect prevails and a higher supply of skilled workers leads to a skilled biased technology change; if intermediate goods are complemetaries

[2] even if he doesn't use an appropriate index to measure the supply of skilled labour: indeed he uses the weeks worked by college equivalents divided by weeks worked of non-college equivalents that, in my opinion, measures the equilibrium level of labour market and not the supply curve. A more appropriate measure for the relative supply of skilled labour could have been the number of graduates within a certain cohort of people.

[3] under the condition of decreasing return

technological change in addressed to development of low-skilled sector. This technological change process determines a growth in demand of both kind of workers and in the long run the skill premium is higher then the starting point in case of substitutes inputs or lower (but always higher than the short-run equilibrium with no innovations) in the case of complementarities inputs. From his estimations for the States it results that intermediate goods are substitutes so his model is coherent with facts he wanted to explain.

Dahlby e Ferede (2013) find same results: "The widespread adoption of computer technologies in the workplace has increased the productivity, and therefore the demand, for skilled (educated) labour relative to unskilled or low-skilled workers. This skill-biased technological change has increased pay differentials and contributed to the rise in earnings inequality."

Jacobs (2016) finds a further channel through inequality affects growth: she argues that child of poor parents will have not only lower future income and lower education level (as Bowles; Corak, 2006; Piraino, 2006; Mocetti, 2007; Alesina *et al.*, 2017; Acciari *et al.*, 2017) but also a lower propensity to entrepreneurship and lower probability to obtain a patent. Her conclusion is that a more unequal society produces a less innovative economy and, under the assumption that the prime engine of modern economies is technological innovation, a lower growth rate.

2.1.2 Education and human capital

One of the first model that investigates the role of accumulation of knowledge is Romer (1986). His model is not focused on education and human capital in the strict sense but rather in the effect of learning by doing. This phenomenon generates increasing return to scale and therefore a growth rate not decreasing over time.

The effects of education and increase in human capital on economic growth are properly investigated by Lucas (1988) model. In his model individuals choose the optimal path of consumption and time devoted to accumulation of human capital[4] in order to maximize their discounted utility. The accumulation of human capital generates positive economies of scale so the solution of the model is a non-decreasing growth rate which depends both on physical capital accumulation and human capital one.
Balducci (2009) changes the type of constraint agents face: while in Lucas (1988) the constraint is the time individuals have to allocate in accumulation

[4]in this way indiviudals' choise is also about the amount of savings and time devoted to production.

of human capital or in producing the final good, his constraint is the financial resources individuals dedicate to accumulation of human capital. The results are in any case the same: lower discount rate induces individuals to invest in accumulation in human capital rather than consume in the current period and an higher growth rate of the economy.
Galor and Zeira (1988) show that in the presence of an imperfect financial market, if the marginal productivity of workers is constant then inequality in the distribution of wealth is crucial for the definition of income distribution. According to the authors, individuals with a wealth above a certain amount have the incentive to invest in education and - thanks to the skills they have acquired - in the later periods they will get a high income; On the contrary, people with a wealth lower than that amount will not invest in education and will get lower wages. Assuming that people get some usefulness by transferring their wealth through inheritance to their children, this condition of inequality will also flow to successive generations according to a vicious cycle: people with higher incomes will leave their children a wealth that will allow them to invest in education and in turn get high salaries; low-income people will leave their children a fortune that will not allow them to invest in education and will therefore be forced to work as unskilled workers by earning lower wages.
Galor (2006) proposes a model in which the effects of income distribution on economic growth depend on the stage of development of the economy. His model could be imagined as divided into four periods:

1. after a long phase characterized by a Malthusian equilibrium ended with the industrial revolution, the starting point of development process is a situation in which physical capital is scarce, therefore its rate of return is higher than the rate of return of human capital (wages). At this stage individuals who own no capital (the poors) have low income, they consume it all and transfer nothing to future generation as bequest; individuals who own the physical capital (the rich) instead have high income, they invest it only in physical capital because there is no incentive to invest in human one and are able to transfer to next generation their capital. In this situation poors are in a poverty trap while rich group becomes richest over time.

2. The process of accumulation of physical capital by rich people continues and generates a gradually fall in the rate of return of physical capital and a rise of wages. This rise in wages is not sufficient to poors to exit from the poor trap but it is sufficient to give an incentive to rich people to switch a part of their investment from physical capital accumulation

to human capital one. In this phase rich group is still the only group who makes transfers to next generation.

3. The continuance of accumulation of physical capital by the rich further increases wages until wages exceed the critical level in which poor people becomes able to invest in human capital but given the fact that there are financial contraints their investment is sub-optimal. Rich individuals who have no financial constraint invest an higher amount of income in education. This situation in which investments in education are different among the two classes causes different marginal return on investment in human capital, higher for the poor, lower for the riches.

4. When wages level lets credit constraints become non-binding investments in human capital become equal across groups and thus also the rate of return to human capital is equalized across groups, and inequality therefore has no effect on economic growth.

To summarise, according to this model, in early stage of development when physical capital is scarce and its accumulation is the prime engine of economic growth inequality is functional for growth because it channels resources towards the owners of capital whose marginal propensity to save is higher. In later stages of development, as the return to human capital increases due to capital-skill complementarity, human capital replaces physical one as prime engine of economic growth; since human capital is inherently embodied in individuals and individuals' investment in human capital is subjected to diminishing marginal returns, the aggregate return to investment in human capital is maximized if investment in human capital is widely spread among individuals in society. For this reason equality, in the presence of credit constraints, stimulates investment in human capital and promotes economic growth. In my opinion Galor's view is too optimistic: in the future, what it will matter is mainly human capital so, implicitly, inequality can be ignored.

Fershtman *et al.* (1996) propose a model in which an inefficient allocation of resources emerges: persons belonging to the higher social class but with little innate skills invest too much in education, while people in lower social classes invest not enough. This misallocation discourages growth.
Staffolani, Valentini (2006) to relieve this inefficient allocation propose a proportional taxation on bequests which would allow an higher average utility.

Tadeu Lima and Carvalho (forthcoming) propose a post-keynesian model in which economic growth is due by both accumulation of physical capital and

accumulation of human one. They use a fixed coefficients production function as typically used by post-keynesian schoolars[5] whose inputs are capital and labour. While the productivity of capital is constant over time, education provided by government lets labour productivity grows. This hypothesis on production function generates a paradox: under the same conditions the growth of labour productivity results in a fall in demand of labour. Additional hypotesis of this model is that firms operate in oligopolistic market and, given his objective income share (based on the state of demand), adjust price to reach it. Workers have as well an objective about their wage share (based in their bargaining power in the labour market and the productivity growth) and bargain wages in order to obtain it. The model shows how in the long-run public investments in education (financed by taxation) leads to an increase of human capital accumulation, this increase let the bargaining power of workers rises and therefore their wage share. This growth in the wage share (in a context of wage-led growth) generates higher consumption and higher capacity of utilization so higher investment by capitalists and higher growth.

Dutt and Veneziani (2011) propose a marxian approch. Unlike Tadeu Lima and Carvalho, their production function requires the use of three inputs: capital, skilled labour and unksilled one. They impose a perfect complementarity between capital and aggregate labour but allow a certain degree of substitutability between the two kinds of labour. Firms' labour demand of each typology is increasing in the level of capital and decreasing on his relative cost (with rispect to the other typology of labour). Their main assumption are that: the two labour productivity grow at the same rate (increasing function of the amount of skilled workers); that skilled workers growth rate is a increasing function of the level of skilled workers and the skill premium; that profit rate is obtained as a residual of production less total wages and, finally, that workers consume all their income. The results they obtain are: a greater openess in the education system increases the stock of skilled workers, reduces the skill premium and generates a substitution of low-skilled workers with high-skilled ones. Even if the real wages increases, profit share increases because the growth of productivity is higher than growth of wages, this generates the exploitation of working class by capitalists that is main result of this kind of literature.

A feature that two last models shares is the presence of unemployment, while in the neoclassical ones this aspect is not considered. The main difference between two models (and more generally between the two approaches) is the

[5]because in this way the production process is not affected by changes in relative factor prices

investment function: while in Tadeu Lima and Carvalho's models investment are driven by profit rate and capacity of utilization in Dutt and Veneziani's model the only determinant of investments is profit rate, in this way the issue of aggregate demand is completely omitted.

2.1.3 Post-keynesian literature

In Post-Keynesian literature distribution is not the result of the growth as in mainstream literature just passed in review, but it is the source itself of the growth.
The main characteristics of keynesian are the presence of social classes, the distribution of the income between them and the role played by aggregate demand.

Reference model for most economists from the second post-war is Harrod and Domar's model. This model, taking on Keynes's investment function, which considers saving as a function of income ($S = sY$) and putting the equilibrium condition that savings and investments are equal in each period ($S = I$), predicts that the system will grow in a balanced manner both in terms of the goods market (ie, it ensures that the increased production capacity generated by the investments is absorbed entirely by aggregate demand) and the labor market (that is, there is no unemployment) when the following equality occurs:

$$a + n = \frac{s}{v} \tag{2.1}$$

where a is the growth rate of labor productivity, n the population growth rate, s the average propensity to save and v the ratio, considered stable over time, between used capital and production obtained.
However there is no mechanism guaranteeing equality between natural rate of growth rate (LHS of equation 2.1) and warranted one (RHS of equation 2.1) since the four parameters are independent of each other.
In fact, if the natural growth rate was higher than the warranted one then there would be a situation of structural unemployment and inflationary tensions (saving would be insufficient to allow full growth). In the opposite situation, however, it would create an economy characterized by excessive savings or, in other words, poor demand that would lead the system to a depressive phase.

To solve this problem of randomness of equilibrium (the so called growth on *"knife blade"*), it is necessary to deal on one of the four parameters and make it endogenous with respect to the model. Neo-malthusian economists have focused on the parameter n , neoclassical economists on v[6] and a[7] and, finally, the keynesian have investigated the effects of investments in terms of increase in production capacity, so on the parameter s.
In particular Kaldor (1957) has proposed a model in which a process of adjustment during time of parameter s is performed through the change in the functional distribution of income. His hypothesis is as follows: given the propensity to save of capitalists (s_c) and workers (s_w; with $s_c > s_w$), total savings would be equal to the weighted average savings of the two groups:

$$S = s_c \Pi + s_w W \tag{2.2}$$

where Π stands for capitalists' profits and W stands for workers' salary. The total amount of savings would therefore depends on the functional distribution of income. The average propensity to save of the entire economy (s) would be equal to:

$$s = s_c \frac{\Pi}{Y} + s_w \frac{W}{Y} \tag{2.3}$$

In this way, an automatic rebalancing mechanism would work by adjusting the relative income shares held by each of the two social classes: if the guaranteed growth rate is less than the natural one, growth would be accompanied by unemployment; in this case, wages would decrease or grow less profits, and consequently increase total savings (S) through increased propensity to overall savings (s) and, given v, the growth rate guaranteed would increase and reach level of natural growth rate. If the situation were the opposite inflation would increase wages and the W/Y ratio would increase. This increase would decrease the propensity to save until the two growth rates are equal. This growth model in equilibrium results from the fact that, given the propensity to save capitalists and workers, the share of national product destined for profits depends solely on the decisions of the entrepreneurs themselves. In fact, since eq. 2.2, putting the same savings on

[6]Neoclassical analysis uses a production function whose productive factors, with decreasing marginal productivity, are, in a perfect competition context, perfectly interchangeable between them. In this way, production technology is not given, but it can adapt to re-balance the system.

[7]The solution adopted by neoclassical economists is to make endogenous productivity growth (the so-called endogenous growth models). What does not deal with neoclassical is the distribution, the income each individual faces within the economic system is assumed to equal the contribution offered in production (marginal productivity).

investment ($S = I$) and replacing wages with the difference between national product and profits ($W = Y - \Pi$) it is obtained:

$$\frac{\Pi}{Y} = \frac{1}{s_c - s_w}\Big(\frac{I}{Y} - s_w\Big) \tag{2.4}$$

It is possible to criticize this model that the parameter s can not be adjusted to market freely from 0 to 1, but that it is bound by the values of s_c and s_w[8]. Pasinetti(1962) extends further Kaldor model considering the possibility that even workers can save some of their income and thus have a capital to invest. In this situation, there are changes in the functional distribution of income but do not change the distributive quotas allocated to labor and capital therefore does not change the result that the profit rate would only depend on the investment choices.
Another economist who has dealt with the topic is Simon Kuznets. The relation between income inequality and economic development postulated by Kuznets (1955) is known as inverted U-shaped[9] : economies with a low level of income per capita are characterized by an egalitarian distribution of income, with economic development inequality grows, and, finally, it falls when economies reach a more mature level of development. The mechanism he thought is: in the in the early stages of a country development, "modern" sectors, such as industry, are minority compared to traditional ones, such as agriculture; they need skilled labor and being more productive than traditional ones are willing to pay higher salaries to their employees; these higher wages paid to a few workers would increase inequalities. This process would be further reinforced by the willingness to invest and thus receive higher incomes in subsequent periods, limited to people with high income. With the progress of development, compensatory mechanisms (such as increased demand for traditional sectors because of high wages in the modern sectors) will be created, and along with the continued change in the production structure towards innovative sectors, the inequalities will be reduced. The effects

[8]or even more limited since it is unlikely the hypothesis of a society in which all income is destined for capitalists or workers

[9]Contrary to Pareto's hypothesis that the distribution of income would be constant over time and institutional changes. This distribution could be represented according to the following power rule:

$$lnx = A - \alpha lny$$

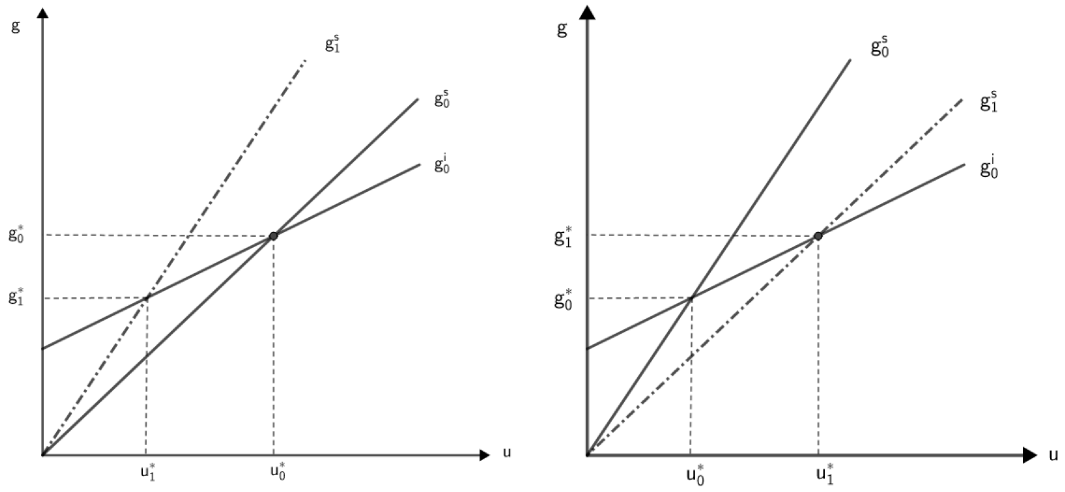
where x is the number of families with an income lower of y. His estimation of the parameter α was between 1.4 and 1.5 (Impulliti, Rebman, 2002; Milanovic, 2011). This result means that in order to obtain a 10% increase in average income from a certain population, 15% of its components should be excluded.

of this structural change would also be reinforced by other phenomena: the interaction between demographic factors and the labor market and the slow territorial spread of economic development. The demographic growth that accompanied economic development in the first phase would have contributed to the increase in inequalities as the greater supply of youth work (resulting from an increase in the importance of the youth population inevitably due to the inevitable phase of demographic growth) would have met a demand for lower wages than adults (it is demonstrated, in fact, that, under other conditions, young people receive lower wages than adults) that would increase inequalities. When the demographic transition reaches its full potential, the relative importance of the juvenile population will diminish and wages will be more evenly distributed. With regard to the second phenomenon one may think that the development of a certain country begins in a restricted area, increasing the income of the inhabitants of this area and, therefore, the total inequalities. With the continuation of the development process, the other areas of the country will also be interested in this phenomenon, and they will see, as well, an increase in the income of their inhabitants, which will reraise the situation by reducing inequalities at national level. What Kuznets does not demonstrate, and perhaps not even in its intentions, is the causal link between distribution and growth and inequality is a favorable condition for growth.
Kuznets' theory would seem to be questioned by the data concerning the increased concentration of income in developed countries in recent years. The U-reversed relationship would thus become in a tilde relationship ($\sim$). Possible explanation is that Kuznets' relation is only valid up to a certain level of development, now overtaken in all western countries, and that researchers need to find a proper relationship for this advanced stage of development. Another possible explanation is that Kuznets' relation is valid but that we have entered a new phase of economic development characterized by a different paradigm[10] by that of the last two centuries and that we should therefore expect the inequalities to increase and then decrease according to the U-reversed relationship that, when compared to that of the previous paradigm, forms a double U-reversed.

Kaleckian model Canonical Kaleckian growth model is composed by three equations:

[10]represented by the shift of most of the workforce from the secondary to the tertiary sector

Figure 1: Paradox of thrift and paradox of costs in the Kaleckian model



(a) Effects of an increase in the propensity to save out of profits

(b) Effects of an increase in wages

1. Investment function:
$$g^i = \gamma + \gamma_u u + \gamma_r r \tag{2.5}$$

2. Saving function:
$$g^s = s_p r \tag{2.6}$$

3. Decomposition of the profit rate:
$$r \equiv \frac{\Pi}{K} = \frac{\Pi}{q}\frac{q}{q^*}\frac{q^*}{K} = \pi u \frac{1}{\nu} \tag{2.7}$$

Substituting 2.7 into saving and investment functions and equating the latter two, economy would reach the following equilibrium with endogenous distribution:
$$u^* = \frac{\gamma\nu}{(s_p - \gamma_r)\pi - \gamma_u\nu} \tag{2.8}$$
$$r^* = \frac{\gamma}{s_p - \gamma_r - \gamma_u\frac{\nu}{\pi}} \tag{2.9}$$

Two paradoxes characterize the model: the so-called paradox of thrift and the paradox of costs. In Fig. 1a and Fig. 1b I show them.

The first paradox is about the effect of an increase in the propensity to save out of profits that generates an upward rotation of the saving function

and thus a decrease of capacity of utilization and growth as well rather than an increase as one could imagine. The second paradox considers the effects of an increase in wages, this also generates a rotation of the savings function but downward this time getting an increase of utilization rate and growth.

Bhaduri and Marglin model Bhaduri, Marglin (1990) uses the same framework of Kalecki model, with respect to it they change the investment function: instead of profit rate they build investment function as function of profit share:

$$g^i = \gamma + \gamma_u u + \gamma_\pi \pi \tag{2.10}$$

That for a firts glance may seem marginal, it has determining effects on the model. In fact an increase of profit share does not produce solely the rotation of saving function but a movement to the top of investment function. This determines, as desctibed by Fig. 2a-2c three possible results: an increase of profit share generates a decrease both of utilization rate and growth rate(2a);an increase(2b) of both; or, finally, a decrease of utilization rate but an increase of utilization rate(2c). These three different situations were defined by the authors as "cooperative stagnationism", "exhilarationism" and "conflict stagnationism". Note that Fig. 2a shows the same results of Fig. 1b[11]: an increase of wages causes both an increase of utilization rate and an increase of growth rate. So Kaleckian model can be interpreted as a particular case of Bhaduri and Marglin one.

Mark-up In an oligopolistic market, where firms set good price as mark-up (z) over average labour cost ($\frac{LC}{q} = \frac{wL}{q} = \frac{wL}{aL} = \frac{w}{a}$), wage share ($\sigma$) would become function of mark-up itself:

$$\sigma = \frac{w}{pa} = \frac{1}{z\frac{w}{a}}\frac{w}{a} = \frac{1}{z} \tag{2.11}$$

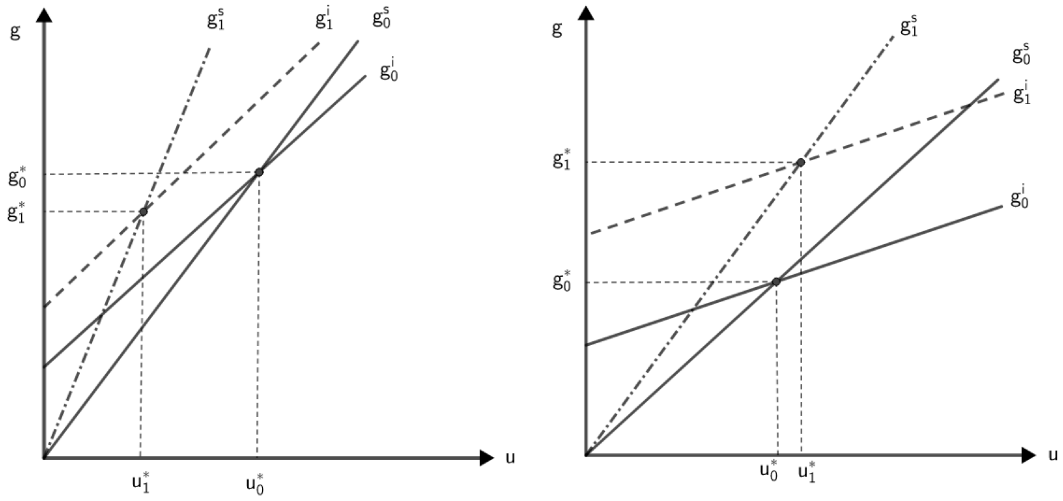
and the profit share would be, computed as residual:

$$\pi = \frac{z-1}{z} \tag{2.12}$$

so, if you consider mark-up as index of bargaining power between workers and capitalists, income distribution depends on this bargaining power: lower is the bargaining power of workers higher are the mark-up and profit share.

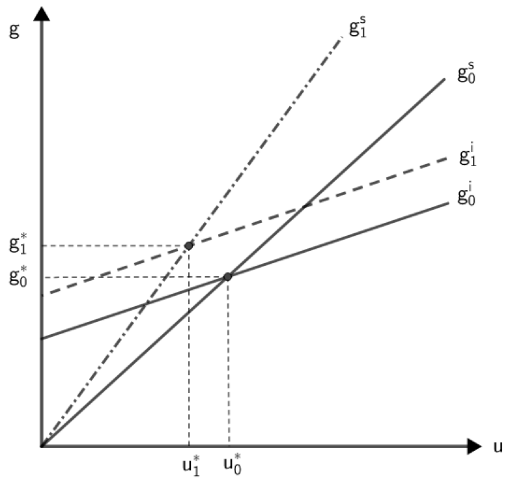
[11]Note that in Fig. 1b there is an increase of wages while in Fig. 2a there is an increase of profits

Figure 2: Impact of an increase in the profit share



(a) Cooperative stagnationism

(b) Exhilarationism



(c) Conflict stagnationism

2.2 Empirical literature

Dahlby, Ferede (2013) present a very complete survey on empirical literature. Table 1 and Table 2 summarize it.

Table 1: Empirical Studies of Inequality and Growth Based on International Cross-section and Time Series Data

Key Studies Arranged by Date	**Data Used**	**Conclusion Regarding Effect of Inequality on Growth**	**Other Conditioning Variables**
Alesina and Rodrik (1994)	Cross-section data on 46 to 70 countries from 1970 to 1985. Gini coefficients on land distribution (for 54 countries around 1960) and income.	Greater initial inequality in the distribution of land and income is associated with slower growth over the subsequent 25 years.	Initial per-capita income, primary school enrolment ratio, and a democracy dummy variable.
Persson and Tabellini (1994): Historical Evidence	Nine countries: Austria, Denmark, Finland, Germany, the Netherlands, Norway, Sweden, the US and the UK, 1830 to 1985, growth rates calculated over 20-year periods (57 observations). Inequality measure is the share of income of the top 20 per cent of the population (38 observations).	Coefficient on the income share of the top 20 per cent is negative and statistically significant.	Political participation (share of the population that is not eligible to vote), schooling (a weighted average of education levels of the population), a measure of economic development (the gap between GDP per capita and highest GDP per capita at that date in the nine countries).
Persson and Tabellini (1994): Post-WW II Data	56 countries, growth rates calculated from 1960-1985; income inequality measure is the income share of the third quintile.	A larger income share of the middle quintile is associated with a higher growth rate in democracies but not significant in non-democracies.	Percentage of population attending primary schools, dummy variable for whether country is democracy, initial GDP per capita in 1960.
Perotti (1996)	Cross-section data for 67 countries, 1960 to 1985. Inequality measure is the share of the income in the third and fourth quintiles.	In the baseline model, a one standard deviation increase in the middle-income groups share is associated with a 0.6 per cent increase in the growth rate. (Table 4, page 160).	Basic model includes per-capita GDP, the average years of secondary schooling in the male and female population, and the purchasing power parity value of the investment deflator relative to the US (a proxy for capital market distortions) all measured in 1960. Other variables include share of population over age 65, urbanization rates, regional dummies, and life expectancy.

Table 1: Empirical Studies of Inequality and Growth Based on International Cross-section and Time Series Data

Key Studies Arranged by Date	**Data Used**	**Conclusion Regarding Effect of Inequality on Growth**	**Other Conditioning Variables**
Li and Zou (1998)	Panel data from 46 countries, five-year averages for growth rates and other variables, 217 observations.	"Our baseline estimations and the sensitivity analysis have shown that income inequality is positively, and very often significantly, associated with economic growth." p.332	Initial or lagged GDP, urbanization rate, population growth rate, investment-to-GDP ratio, financial market development, exports to GDP, primary school enrolment ratio, back market currency premium, and democracy dummy variable.
Barro (2000)	Panel data from roughly 100 countries from 1960 to 1995, with growth rates calculated for three time periods 1965-75, 1975-85, and 1985-95. Gini coefficient used as measure of inequality.	Inequality retards growth in poor countries (less than $2000 US per-capita GDP in 1985) but inequality promotes growth in high-income countries.	Log of GDP per capita at beginning of period and squared value, government consumption to GDP, rule of law index, democracy index and squared value, inflation rate, years of schooling, log of the fertility rate, growth rate of the terms of trade. Country and time period dummy variables.
Forbes (2000)	Panel data for 45 countries with growth rates calculated over five-year periods from 1966-95. A total of 180 observations. Gini coefficient used as measure of inequality.	"A 10-point increase in a countrys Gini coefficient is correlated with a 1.3 per cent increase in average annual growth over the next five years" p.878 See Table 3 page 877. "Countries may face a trade-off between reducing inequality and improving growth performance." p.885	Log of real GNP per capita in 1987 dollars, measures of female and male education levels, price level of investment.

Source: Dahlby, Ferede (2013), page 8.

Table 2: Empirical Studies of Inequality and Growth Based on Panel Data from US States

Key Studies Arranged by Date	**Data Used**	**Conclusion Regarding Effect of Inequality on Growth**	**other Conditioning Variables**
Partridge (1997)	Panel data for 48 US states, 1960-90, growth rates of states calculated over 10-year periods, (144 observations). Inequality measures are Gini coefficients of before-tax family income and the income share of the third quintile based on census data.	Gini coefficient is positive and significant. "States with greater overall economic inequality subsequently experienced greater economic growth" p. 1022 Income share of the third quintile is associated with faster economic growth. (Both variables have positive coefficients either separately or together in the same regression.)	Labour force skills (percentages with high school and four-year college degrees), initial real per-capita income, measures of the states industrial mix from employment shares, state and local public welfare expenditures, state and local tax rates, percentage of population employed in agriculture, time dummy variables, regional dummy variables.
Panizza (2002)	Panel data for the US states, 1940-90, with 10- and 20-year growth rates. Gini coefficients based on tax data on incomes and income share of the third quintile.	"A one standard deviation decrease in the Gini index is associated with a 0.2- percentage-point increase in average annual growth over the next 10 years." p.34 (based on col. 5 and 6 of Table 7)	Percentages of adults with high school and college degrees, fraction of the population living in urban areas, percentage of the population over age 65, regional dummy variables, state and time dummy variables, initial income.
Partridge (2005)	Panel data for 48 US states, 1960- 2000, growth rate of per-capita personal income over 10-year periods and over a 40-year cross-section. Gini coefficient and income share of the third quintile.	"The middle income-class share and overall inequality are positively related to longrun growth." p.363	Same as in Partridge (1997)
Frank (2009)	Annual panel data for 48 US states, 1945-2004. (2,784 observations). Log of real state per-capita income. Various measures of inequality (top decile share, top one per cent share, Gini, Atkinson, and Entropy indices) based on tax data.	"The long-run relationship between inequality and growth is positive." p.55 "Top one per cent share, Atkinson indices Entropy, and Gini coefficient all show positive effects of inequality on long-run growth." Table 3, p.63	Real income per capita, high school and college attainment, industrial composition of employment, fixed time and state effects.
Huang and Yeh (2012)	Same data set as in Frank (2012) but with growth rate of real per-capita income by state.	A two standard deviation increase in the income share of the top decile is associated with a 2.4 per cent increase in the growth rate.	Same as in Frank (2009).

Source: Dahlby, Ferede (2013), page 8.

Table 3: The summary of the effects of a 1 % point increase in the profit share

	C/Y	I/Y	NX/Y	Private excess demand/Y
	(a)	(b)	(c)	(a+b+c)
Euro area-12	-0.439	0.299	0.057	-0.084
Germany	-0.501	0.376	0.096	-0.029
France	-0.305	0.088	0.198	-0.020
Italy	-0.356	0.130	0.126	-0.100
United Kingdom	-0.303	0.120	0.158	-0.025
United States	-0.426	0.000	0.037	-0.388
Japan	-0.353	0.284	0.055	-0.014
Canada	-0.326	0.182	0.266	0.122
Australia	-0.256	0.174	0.272	0.190
Turkey	-0.491	0.000	0.283	-0.208
Mexico	-0.438	0.153	0.381	0.096
Korea	-0.422	0.000	0.359	-0.063
Argentina	-0.153	0.015	0.192	0.054
China	-0.412	0.000	1.986	1.574
India	-0.291	0.000	0.310	0.018
South Africa	-0.145	0.129	0.506	0.490

source: Onaran, Galanis (2013)

What it evident from a reading of previous tables is that the impact of inequality on economic growth is not unambiguous. While all papers mentioned above refer to distribution as personal distribution, Onaran and Galanis (2014) refer to distribution as funtional distribution. In particular they estimate the effects of a 1% point increase in the profit share on demand obtaining result contained in Table 3.

Obviously, an increase in wages has positive effects on consumption and negative effects on investment end net exports. Total results, as sum of two types of effects, say that most of the countries considered are wage-led economies.

Social mobility Social mobility can be considered as an index of equality of opportunity. It is possible to estimate it formalizing the income of a person as:

Table 4: Estimation for coefficient β

Country	Estimates for cross country comparisons		
	Preferred	Lower Bound	Upper Bound
Denmark	0.15	0.13	0.16
Norway	0.17	0.15	0.19
Finland	0.18	0.16	0.21
Canada	0.19	0.16	0.21
Sweden	0.27	0.23	0.30
Germany	0.32	0.27	0.35
France	0.41	0.35	0.45
United States	0.47	0.40	0.52
UnitedKingdom	0.50	0.43	0.55

source: Corak (2006)

$$Y_i^s = \alpha + \beta Y_i^f + \epsilon_i \tag{2.13}$$

where Y_i^s is the logarithm of the child's income, Y_i^f the logarithm of the parent one and β is a coefficient that measures intergenerational elasticity namely how much the income of the child depends on that of the parent.

Results of estimation for coefficient β are summarized in table 4

Piraino (2006) and Mocetti (2007) estimate coefficent β for Italy obtaining results between 0.479 and 0.509 and 0.49 and 0.61 depending on criteria used for the fathers-son association[12]

Low inter-generational mobility in Italy is not only related to income, Table 5 and Table 6 will show the data on social mobility as regards employment status and education level. Acciari *et al.* (2017), thanks to the availability of new data, build their own "real" dataset. Their results say that income mobility is not a linear phenomenon: while intergenerational mobility is high, both upward and downward, for the children of parents of the middle class, economic opportunities become more unequal to the extremes of income distribution.

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[12]In the absence of a complete dataset in which data on income of real couples fathers and children of are reported, the TS2SLS method was used. This method allows you to associate people to hypothetical fathers according to income, education level, and type of employment.

Table 5: Education mobility. Italy, Early 2000s

Parent education	Child education				
	None	Elementary school	Middle school	High School	University
None	3.5	19.0	47.6	28.3	1.6
Elementary school	0.1	6.0	50.0	38.3	5.6
Middle school	0.0	1.3	26.2	56.7	15.8
High School	0.0	1.0	6.8	59.2	33.0
University	0.0	0.4	9.2	35.8	54.5
Average	0.4	6.2	39.3	42.5	11.6

source: Mocetti (2007)

Table 6: Occupation mobility. Italy, Early 2000s

Child job	Father job				
	Factory worker	Employee, teacher	Manager	Self-employed	Entrapreneur
Factory worker	47.6	24.4	5.1	3.2	19.8
Employee, teacher	13.8	43.3	14.9	13.3	14.7
Manager	5.1	31.1	32.1	16.1	15.6
Self-employed	5.5	17.5	17.3	25.9	33.7
Entrapreneur	21.4	17.1	6.1	8.2	47.3
Average	32.1	26.00	8.9	7.1	25.9

source: Mocetti (2007)

3 A post-keynesian model of growth and distribution with education

3.1 Introduction

The objective of this chapter is to develop a model that analyzes how human capital accumulation affects functional distribution and economic growth. This topic has been studied by many scholars of different theoretical inspiration (such as Acemoglu; 1998, 2002 or Dutt and Veneziani; 2011)
My model would like to be an attempt to investigate the effects of education on distribution within the working class (as Dutt and Veneziani, 2011) but with an approach that considers the role of demand (as post-Keynesian models). The main extension is the change in the hypothesis about the distribution of human capital: while in their model it is homogeneously distributed among all workers, in mine there are two different kinds of workers: skilled and unskilled ones with the first ones that benefit more than the latter of human capital accumulation. In this way it is possible to study the effect of education not only in the distribution between capitalists and workers but also within the working class.

3.2 The model

3.2.1 The baseline model

A "static" model (with constant labor-productivities, price and wages) has been presented by Tavani and Vasudevan (2014)[13] [66].

Firms produce a consumption good y with a fixed coefficients production function using capital (K) and two kinds of labour, namely unskilled labour (L_u) and skilled one (L_s) . In symbols:

$$y = min\Big\{a_k K; a_u L_u; a_s L_s\Big\} \tag{3.1}$$

where a_K, a_u and a_s are respectively the productivity of capital, unskilled workers and skilled ones.

Income share of unskilled workers is defined as the ratio of total salary over gdp; after the mild condition entrepreneurs behave in an optimizing way,

[13]Their model analyze the role played by managers in running firms. They conclude that an increase of managers' supervision productivity (that means a manager can supervise a higher number of workers), reduce profits through compesation payments to managers but increase the profit share.
Here I simply change perspective in skilled and unskilled workers rather than managers and workers.

you get:

$$\sigma_u = \frac{\frac{w_u}{p} L_u}{y} = \frac{\frac{w_u}{p} L_u}{a_u L_u} = \frac{w_u}{p a_u} \tag{3.2}$$

Similarly, income share of skilled workers is:

$$\sigma_s = \frac{w_s}{p a_s} \tag{3.3}$$

By defining the skill premium (the ratio between the skilled workers wages and unskilled workers ones) as $\omega = \frac{w_s}{w_u}$ and the productivity gap as $\alpha = \frac{a_s}{a_u}$, equation 3.3 could be rewritten as:

$$\sigma_s = \frac{\omega}{\alpha} \sigma_u \tag{3.4}$$

Profit share, computed as a residual, is: $\pi = 1 - \sigma_u - \sigma_s$. It could be written, using the previous notation:

$$\pi = 1 - \sigma_u \Big(1 + \frac{\omega}{\alpha} \Big) \tag{3.5}$$

From this expression it is easy to understand the effect that increasing productivity gap has on the income distribution if wages were constant over time, differentiating (3.5) with rispect to α:

$$\frac{\partial \pi}{\partial \alpha} = \frac{\omega}{\alpha^2} \sigma_U > 0 \tag{3.6}$$

the result says that an increase of skilled workers productivity, if there is no wage adjustment, causes an increase of profit share.
Let's examine now the effects on unskilled workers wage shares:

$$\frac{\partial \sigma_s}{\partial \alpha} = -\frac{\omega}{\alpha^2} \frac{\partial \sigma_u}{\partial \alpha} < 0 \tag{3.7}$$

An increasing productivity of skilled workers damages the skilled workers themselves. This unexpected result is a direct consequence of the type of aggregate production function that has been chosen: in the short run a higher productivity results in a lower demand of workers in the short run.

My goal is to build a model that can overcome this paradox. More in detail I endogenize the labour productivity growth through the introduction of knowledge and investment in it by the government. Moreover I let capitalists and workers adjust over time wages in order of reacting to productivity growth.

3.2.2 The "dynamic" model

Firms produce goods with a fixed coefficients production function [14] [15] :

$$y = min\left\{a_k K; a_u(H)L_u; a_s(H)L_s\right\} \tag{3.8}$$

The difference with respect to eq. 3.1 is that labour productivities depend on human capital H [16].
In an analogue way to the previous section, two labour shares are:

$$\sigma_i = \frac{w_i}{pa_i} \quad i = u, s \tag{3.9}$$

Firms set price as mark-up z over unit labor cost[17]:

$$p = z\left(\frac{w_u}{a_u} + \frac{w_s}{a_s}\right) = z\frac{w_s}{a_s}\left(1 + \frac{\alpha}{\omega}\right) \tag{3.11}$$

Substituing the definition of price two wage shares become:

$$\sigma_u = \frac{w_u}{a_u p} = \frac{w_u}{a_u}\frac{1}{z\frac{w_s}{a_s}\left(1+\frac{\alpha}{\omega}\right)} = \frac{w_u}{a_u}\frac{1}{z}\frac{a_s}{w_s}\frac{\omega}{\alpha+\omega} = \frac{w_s}{a_s}\frac{\alpha}{\omega}\frac{1}{z}\frac{a_s}{w_s}\frac{\omega}{\alpha+\omega} = \frac{1}{z}\frac{\alpha}{\alpha+\omega} \tag{3.12}$$

[14] As, among others, You (1994), Lima (2000), Dutt and Veneziani (2011), Tavani and Vasudevan (2014), Tavani and Zamparelli (2015).

[15] The value of each variable should be considered at its time; so, for example, for a generic time-varying variable z should be specified the time when is evaluated: $z(t)$. For easier reading I omit the index t.

[16] For the same reason of previous note I omit, when it does not make confusion, the term H in labour productivities.

[17] Unit labour cost, under the assumption firms hire workers in order to maximize their profits ($y = a_u L_u = a_s L_s$), is:

$$\begin{aligned}\frac{LC}{y} &= \frac{w_s L_s}{y} + \frac{w_u L_u}{y} \\ &= \frac{w_s L_s}{a_s L_s} + \frac{w_u L_u}{a_s L_s} \\ &= \frac{w_s}{a_s} + \frac{w_u}{a_s}\end{aligned} \tag{3.10}$$

and

$$\sigma_s = \frac{w_s}{a_s p} = \frac{w_s}{a_s} \frac{1}{z\frac{w_s}{a_s}\left(1+\frac{\alpha}{\omega}\right)} = \frac{w_s}{a_s}\frac{1}{z}\frac{a_s}{w_s}\frac{\omega}{\alpha+\omega} = \frac{1}{z}\frac{\omega}{\alpha+\omega} \tag{3.13}$$

Obviously, the two wage shares sum up to $\frac{1}{z}$.
Profit share is computed as residual:

$$\pi = 1 - \sigma_u - \sigma_s = 1 - \sigma_s\left(1+\frac{\alpha}{\omega}\right) = 1 - \frac{w_s}{a_s p}\left(1+\frac{\alpha}{\omega}\right) \tag{3.14}$$

Substituing the definition of price, profit share becomes:

$$\pi = 1 - \frac{w_s}{a_s}\left(1+\frac{\alpha}{\omega}\right)\frac{1}{z\frac{w_s}{a_s}\left(1+\frac{\alpha}{\omega}\right)} = \frac{z-1}{z} \tag{3.15}$$

In this model wages and two labour productivities are not constant over time; in next sections it is explained in details how they evolve.

3.2.3 Labour market

Wages are not constant over time but change with labour productivity.
As Dutt (1984), Dutt (1990) and Lima (2000), I assume workers have a target wage share and they are able to bargain their nominal wages in order to achieve them. Wages grow (fall) according the following rule[18]:

$$\hat{w}_i = \mu[\sigma_i^d - \sigma_i] \qquad i = u, s \tag{3.16}$$

where $0 < \mu < 1$ is a positive parameter that represents the adjustment speed of wages, or in other terms, the bargaining power of workers, and σ_i^d is the labour share desired by each group of workers.
Desired wage shares depend on how workers have increased their productivity:

$$\sigma_i^d = \eta_i + \delta_i \hat{a}_i \qquad i = u, s \tag{3.17}$$

where $0 < \eta_i < 1$ and $0 < \delta_i < 1$ are positive parameters that represent a minimum wage share exogenously determined and a request for an increase in wages resulting from increased productivity.

Substituing eq.(3.17) into (3.16) two nominal wages growth rate could be rewritten as:

$$\hat{w}_i = \mu\Big[\eta_i + \delta_i \hat{a}_i - \sigma_i\Big] \qquad i = u, s \tag{3.18}$$

[18]given a generic variable $z(t)$ I call $\dot{z} = \frac{\partial z(t)}{\partial t}$ and $\hat{z}$ the growth rate $\hat{z} = \frac{\dot{z}}{z}$

3.2.4 Labour productivity and human capital

Main assumption of the model is that human capital is not uniformly distributed among workers. Skilled workers and unskilled ones benefits in a different way from an increase of human capital with the first that benefit more than the latter.
I assume unskilled workers' productivity, a_u, is constant over time while skilled workers' one has a one-to-one corrispondence with the stock of human capital:

$$a_s = H \tag{3.19}$$

Its growth is thus:

$$\hat{a}_s = \hat{H} \tag{3.20}$$

3.2.5 Saving and investment

The canonical Kaleckian/post-Keynesian saving function of previous chapter has been modified in order to take into account two groups of workers and taxation:

$$\begin{aligned} g^s = \frac{S}{K} &= \Big[s_p(1-\tau)\pi + s_u(1-\tau)\sigma_u + s_s(1-\tau)\sigma_s\Big]a_k u \\ &= (1-\tau)\Big[s_p + (s_s - s_p)\sigma_s + (s_u - s_p)\sigma_u\Big]a_k u \end{aligned} \tag{3.21}$$

where s_i is the propensity to save out of income of each of three social groups and τ the tax rate out of income. As in the standard literature I assume that propensity to save out of profits is higher than the one out of wages, and, moreover, that propensity to save of skilled workers (with higher wages) is higher than that ofunskilled ones: $s_p \geq s_s \geq s_u$.
Investment function is:

$$g^i = \frac{I_K}{K} = \gamma + \gamma_u u + \gamma_r r \tag{3.22}$$

Substituing the net profit rate in the equation 3.22 I have[19]:

$$g^i = \gamma + \Big[\gamma_u + \gamma_r(1-\tau)(1-\sigma_u - \sigma_s)a_k\Big]u \tag{3.23}$$

Together these two hypotheses quite standard on savings and investment I include a role for the public sector. As investment and saving fuctions

[19] $r = u\pi$

hint, I assume Government taxes income; its whole budget is invested in accumulation of human capital:

$$\begin{aligned} g^h \equiv \frac{I_H}{K} \equiv \frac{\dot{H}}{K} &= \Big[\tau\sigma_u + \tau\sigma_s + \tau\pi\Big] a_k u \\ &= \Big[\tau\sigma_u + \tau\sigma_s + \tau(1-\sigma_u-\sigma_s)\Big] a_k u = \tau a_k u \end{aligned} \quad (3.24)$$

Human capital growth rate is thus:

$$\hat{H} = \frac{\dot{H}}{H} = \tau a_k u \frac{K}{H} \quad (3.25)$$

3.3 Results

3.3.1 The short term equilibrium

In the short term the levels of physical and human capital are given.
The equilibrium in the goods market can be expressed by:

$$Y = C + I_K + I_H \quad (3.26)$$

since $I_H = T$ therefore all saving are allocated to investments in physical capital $S = I_K$. For simplicity reason I assume capitalists completely save their income ($s_p = 1$) while workers consume consume it all ($s_s = s_u = 0$). By equating g^i and g^s I obtain the utilization rate:

$$u^* = \frac{\gamma}{(1-\gamma_r)(1-\tau)(1-\sigma_u-\sigma_s)a_k - \gamma_u} \quad (3.27)$$

and the short term profit rate is:

$$r^* = (1-\tau)(1-\sigma_u-\sigma_s)u^* \quad (3.28)$$

3.3.2 Long term equilibrium

In the long run accumulations of physical and human capital take places.

I define k as the capital per efficient skilled worker:

$$k = \frac{K}{a_s L_s} \quad (3.29)$$

where L_s is the amount of skilled workers. Normalizing the amount of skilled workers ($L_s = 1$), capital per capita growth is:

$$\hat{k} = \hat{K} - \hat{a}_s \quad (3.30)$$

Substituing capital growth rate, $\hat{K}$, with $g(u^*)$[20] and productivity growth rate with (3.25) you obtain[21]:

$$\hat{k} = (1-\tau)\Big[1-\sigma_u-\sigma_s\Big]a_k u^* - \tau a_k u^* k \tag{3.31}$$

Rewriting the unskilled workers income share in function of skilled workers one, equation (3.31) becomes:

$$\hat{k} = (1-\tau)\Big[1-\sigma_s\Big(1+\frac{\alpha}{\omega}\Big)\Big]a_k u^* - \tau a_k u^* k \tag{3.32}$$

Given the definition of skilled workers wage share (eq. 3.13) its growth rate is[22]:

$$\hat{\sigma_s} = \frac{\alpha}{\alpha+\omega}\Big(\hat{\omega}-\hat{\alpha}\Big) - \hat{z} \tag{3.34}$$

[20] $g(u^*) = g^s(u^*) = g^i(u^*)$

[21] It is possible to replace $\frac{K}{H}$ with k since normalization of amount of skilled workers and the one-to-one corrispondence of skilled workers productivity with human capital.

[22] Given the definition of σ_s (eq. 3.13) its deviation is:

$$\begin{aligned}
\dot{\sigma_s} &= \frac{\dot{\omega}(\alpha+\omega)-\omega(\dot{\alpha}+\dot{\omega})}{(\alpha+\omega)^2}\frac{1}{z} + \frac{\omega}{\alpha+\omega}\Big(-\frac{1}{z^2}\dot{z}\Big) \\
&= \frac{1}{z}\left[\frac{\dot{\omega}}{\alpha+\omega} - \omega\frac{\dot{\alpha}+\dot{\omega}}{(\alpha+\omega)^2} - \frac{\omega}{\alpha+\omega}\frac{\dot{z}}{z}\right] \\
&= \frac{1}{z}\frac{\omega}{\alpha+\omega}\left[\frac{\dot{\omega}}{\omega} - \frac{\dot{\alpha}+\dot{\omega}}{\alpha+\omega} - \frac{\dot{z}}{z}\right] \\
&= \sigma_s\left[\frac{\dot{\omega}}{\omega} - \frac{\dot{\alpha}}{\alpha+\omega} - \frac{\dot{\omega}}{\alpha+\omega} - \frac{\dot{z}}{z}\right] \\
&= \sigma_s\left[\dot{\omega}(\frac{1}{\omega} - \frac{1}{\alpha+\omega}) - \frac{\dot{\alpha}}{\alpha+\omega} - \frac{\dot{z}}{z}\right] \\
&= \sigma_s\left[\dot{\omega}\frac{\alpha}{(\alpha+\omega)\omega} - \frac{\dot{\alpha}}{\alpha+\omega} - \frac{\dot{z}}{z}\right] \\
&= \sigma_s\left[\frac{\alpha}{\alpha+\omega}\Big(\frac{\dot{\omega}}{\omega} - \frac{\dot{\alpha}}{\alpha}\Big) - \frac{\dot{z}}{z}\right]
\end{aligned}$$

Dividing both sides by σ_s and replacing the ratio of deviation on the level of variables with their growth rates, growth rate of skilled workers income share becomes:

$$\hat{\sigma}_s = \frac{\alpha}{\alpha+\omega}(\hat{\omega}-\hat{\alpha}) - \hat{z} \tag{3.33}$$

As Marques Ribeiro (2016) I assume firms do not vary their mark-up ($\hat{z} = 0$); in this way the distribution of income between capitalists and workers considered as a whole do not change[23]. Under this assumption and substituing $\hat{\omega}$ with the difference of growth of two nominal wages (3.18) and $\hat{\alpha}$ with the difference of growth of two labour productivities (3.34) becomes[24]

$$\hat{\sigma_s} = \frac{\alpha}{\omega + \alpha}\Big[\mu(\eta_s - \eta_u) + \mu\delta_s\tau a_k u^* k - \mu\sigma_s\Big(1 - \frac{\alpha}{\omega}\Big) - \tau a_k u^* k\Big] \tag{3.35}$$

Equation (3.32) toghether with equation (3.35) constitute a system of dynamical equations.
Considering two equations at steady state, namely when two growth rates are nil ($\hat{k} = 0$, ank $\hat{\sigma_s} = 0$), they simply say that the capital per efficient worker does not change if capital and productivity grow at the same rate and that the distribution of the total wage share between skilled and unskilled workers does not change if the wage premium grows at the same rate as the productivity gap. Nullcline of equation (3.32) is:

$$k = \frac{1 - \tau}{\tau}\Big[1 - \sigma_s\Big(1 + \frac{\alpha}{\omega}\Big)\Big] \tag{3.36}$$

Nullcline of equation (3.35) is:

$$k = \mu\frac{\tilde{\eta} - \Big(1 - \frac{\alpha}{\omega}\Big)\sigma_s}{\tau a_k u^*\big(1 - \mu\delta\big)} \tag{3.37}$$

where $\tilde{\eta} = \eta_s - \eta_u$. First nullcline (equation 3.36) is clearly a straight line with a negative slope; instead shape of second nullcline (equation 3.37) is less intuitive since equilibrium utilization rate contains wage share (equation 3.27). To determine the shape of second nullcline I compute its slope:

$$\frac{\partial k}{\partial \sigma_s} = \mu\frac{-(1 - \frac{\alpha}{\omega})\tau a_k u^*(1 - \mu\delta_s) - [\tilde{\eta} - (1 - \frac{\alpha}{\omega})\sigma_s]\tau a_k \frac{\partial u^*}{\partial \sigma_s}(1 - \mu\delta_s)}{\tau^2 a_k^2 u^{*2}(1 - \mu\delta_s)^2} \tag{3.38}$$

For the assumption of invariance of mark-up utilization rate does not depend on distribution of income within working class ($\frac{\partial u^*}{\partial \sigma_s} = 0$), indeed an increase of income share of a group of workers due to an increase of nominal wage

[23]This topic is addressed by Dutt, 1987 (Cfr §2.1.3)

[24]ω is defined as the ratio of two wages ($\frac{w_s}{w_u}$) so its growth rate is: $\hat{\omega} = \hat{w}_s - \hat{w}_u$. At the same way α is defined as the ratio of two productivities ($\frac{a_s}{a_u}$); considering that unskilled workers productivity is constant the growth rate of the ratio of two productivity is: $\hat{\alpha} = \hat{a}_s$;

or productivity is perfectly counterbalanced by fall of income share of other group of workers (see equations 3.12 and 3.13). Therefore the second nullcline is also a straight line, however its slope is not a priori determinable, it depends on the initial[25] values of α and ω. I distinguish two cases:

- Case A. If $\alpha > \omega$ nullcline $\hat{\sigma}_s = 0$ is positively inclined;
- Case B. On the contrary, if $\alpha < \omega$ nullcline $\hat{\sigma}_s = 0$ is negatively inclined. In this case it is necessary to identify two subcases because it is not possible to compare two lines' slope:
  - Case B1. Nullcline $\hat{\sigma}_s = 0$ is negatively inclined and its slope is lower than nullcline $\hat{k} = 0$;
  - Case B2. Nullcline $\hat{\sigma}_s = 0$ is negatively inclined and with a larger slope[26]

Equating two nullclines it is possible to obtain the value of growth rate and high-skilled workers wage share at steady state:

$$k^{SS} = \frac{1-\tau}{\tau}\Big[1 - \frac{\mu\tilde{\eta} - A}{\frac{\omega-\alpha}{\omega+\alpha} - A}\Big] \tag{3.39}$$

where $A = (1-\tau)a_k u^*(1-\mu\delta_s)$ and

$$\sigma_s^{SS} = \frac{\mu\tilde{\eta} - A}{(1-\frac{\alpha}{\omega}) - A(1+\frac{\alpha}{\omega})} \tag{3.40}$$

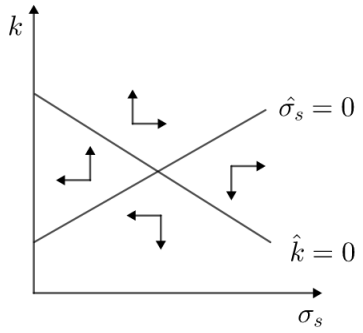
To determine if (k^{SS}, σ_s^{SS}) is a stable equilibrium and under which conditions a graphical analysis has been conducted. Figure 3 represents phase planes on three cases.

From these phase planes it is evident that $(k^{SS}; \sigma^{SS})$ is an unstable equilibrium in case A and case B1, indeed for any initial couples of values phase arrows diverge from the equilibrium. In case B2, $(k^{SS}; \sigma^{SS})$ is a saddle point: the attractiveness of equilibrium depends on the initial conditions, if initial growth rate is sufficiently high and skilled workers income share is sufficiently low (in the upper left area of the graph 4c) or the opposite (in the lower left area) the attainment of equilibrium is guaranteed.
In order to understand possible effects of an intervention of policy making

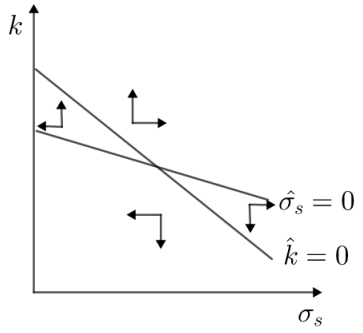
[25]For the equation 3.34 in the steady state α and ω in the steady state equilibrium grow at the same growth rate.

[26]As a further condition in case A and B1 I have assumed that the value of the intercept of nullcline $\hat{k} = 0$ is greater than which of nullclince $\hat{\sigma}_s = 0$; the opposite relation has been assumed in case B2.

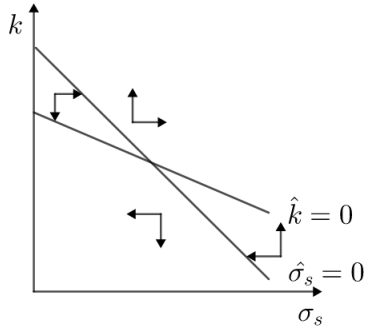
Figure 3: Phase planes of under different scenarios



(a) Case A



(b) Case B1

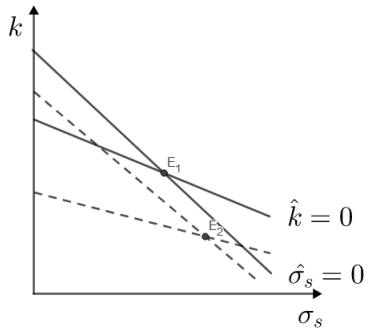


(c) Case B2

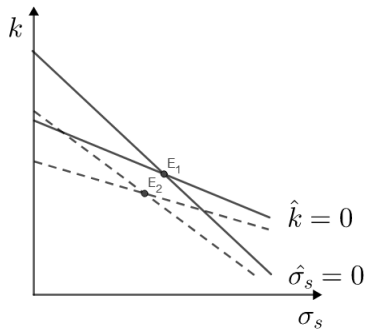
an exercise in comparative statics has been carried out, in particular I suppose an exogenous increase in the tax rate τ. This policy intervention affects on one side investments and savings of firms and households with shrinkage effects and on the other side the education system and the accumulation of human capital with opposed effects. Specifically, limiting the analysis in the relevant case (B2), an increase of τ decreases the intercept and the slope of both equation (3.36) and equation (3.37). In Figure 4 I report the effects of an increase of tax rate on final equilibrium.

The effects of an increase of tax rate is not unambiguous, indeed it depends on the slopes of two nullcline. Three kinds of results emerge: a) an increase of tax rate increases skilled workers income share (as already explained, to the detriment of low-skilled workers, so inequality between workers of two groups increases) and decreases of the ratio physical capital-human capital; b) a decrease of skilled workers income share together with a decrease of physical capital to human capital ratio and c) a decrease of skilled workers income share and an increase of capital per capita. These possible scenarios stress the role of Government on the performance of economy and distribution of income. By estimating parameters, or in a subordinate position guessing their relationships, Government is able to identify which state the economy is (a, b or c) and chooses the priority between economic growth and equality.

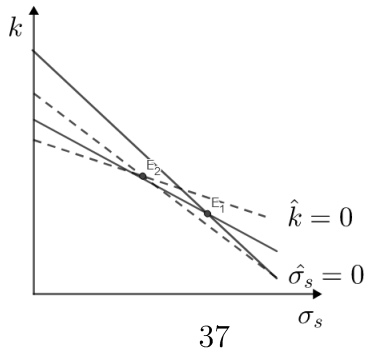
Figure 4: Effects of an increase in the tax rate on the ratio between physical capital and human capital and skilled workers wage share



(a) Case B2.a



(b) Case B2.b



(c) Case B2.c

4 A SFC-AB model of income distribution and social mobility

4.1 Intoduction

In this chapter I develop a model that tries to investigate the role of inequality and social mobility in an economy. The metodology used is a mixture of agent-based (AB) modelling and stok-flow consistent (SFC) one. This recent, but increasingly used[27], metodology combines the advantages of AB modelling[28], namely microfondation, eterogeneity of economic agents, their relations and the possibility to have *emerging* resulting from this interactions, with advantages of SFC modelling[29], in particular the perfect connection of sectors' balance sheets leaving no "black holes" in the model, in this case, since it deals with AB-SFC model the balance sheets connection is at agent level rather than sector level.

In this model, as already existing AB-SFC models, such as Caiani et al. (2015), the household sector is divided in different classes[30]. I introduce a fundamental innovation: individuals may change their social status[31] and type of employment.

4.2 The model

The model represents a closed economy with a homogenous consumption good.
Five sectors are present: houshold sector, production sector, bank sector, government and central bank.
Household sector consists in N individuals belonging to the same number of

[27]Dosi et al.(2010)[18]; Kinsella et al.(2011)[34]; Dosi et al.(2013)[19]; Caiani et al.(2015)[12]; Caiani et al.(2016)

[28]Epstein, Axtell (1996)[22]; Delli Gatti, Gallegati (2005)[15]; Terna et al. (2006)[67] Delli Gatti et al. (2010)[16]; Gallegati et al. (2016)[26]

[29]Zezza, Dos Santos(2006)[69], Godley, Lavoie M., Godley W. (2007)[27] and Lavoie M., Godley W. (2002) [38]

[30]in their model the classes are workmen, office workers, researchers and managers; while in my model the classes are capitalists, low skilled workers and high-skilled ones.

[31]Russo (2017)[57] develops a model in which there is a switch process from the status of worker to the status of capitalist. However in his model the switch is a random process, higher the individual's wealth higher the probability to become capitalist; in my model social mobility is a more "realistic" process: the class change happens through the stock market and the education system.

generation: with the passage of time, each individual grow old, once he reach his maximum age the individual is replaced by a new family member who inherit wealth but not qualities that are embodied in the individual himself that is its education. Household sector is further divided in three social classes: unskilled workers, skilled workers and pure capitalists. Furthermore skilled workers class is divided in three sub-classes: "factory workers", teachers and researchers. Unskilled and skilled factory workers are complementarily employed in production department of firms to produce final goods, teachers are hired by government in order to provide education to unskilled workers who ask for it and researchers are employed in research department of firms in order to improve firm's technology. Pure capitalists are individuals who decide not working at all if their capital income is higher than a *social acceptable* threshold[32]. With the passing of time individuals may change their social classes switching, for example, from unskilled workers class to skilled workers one by attendig an education course or from skilled workers class to pure capitalists one by purchasing more firm's shares. Other sectors consist of Nf firms, Nb banks, Government and central bank.

The events of the model happen according the following list, while the stock-flow consistency of the model is represented in tables 7 and 8.

Time of events:

1. firms update their capital and computes the desired output level based on their sales expectation (paragraph 4.2.1);
2. firms compute their optimal number of machineries and desired investment;
3. uneducated individuals evaluate the opportunity to ask for education and if they do (paragraph 4.2.2);
4. government hires teachers;
5. firms invest in R&D hiring researchers (paragraph 4.2.3);
6. firms demand labour taking into account workers need to produce machineries and consumption goods (paragraph 4.2.4);
7. workers sets their reservation wages;
8. skilled workers offer their work;

[32]See Mohun(2016)

9. firms hire skillled workers;
10. unskilled workers and unemployed skilled ones offer their work as unskilled;
11. firms hire skillled workers;
12. production takes place. Firms produce first machineries and then final goods (paragraph 4.2.1);
13. price setting. Firm sets price of goods as mark-up over unit labor cost (paragraph 4.2.5);
14. government spending. Government demands goods and consume them (paragraph 4.2.9);
15. households demand goods and consume them (paragraph 4.2.5);
16. firms compute sales, inventories and profits;
17. firms and government pay wages (paragraph 4.2.4).
18. firms distribute profits;
19. government taxes income (paragraph 4.2.9);
20. individuals trade shares in the stock market (paragraph 4.2.7);
21. firms fire all workers;

4.2.1 Expectation, Production and Investments

According to Dosi et al. (2010) and Caiani et al. (2015) firms desired output depend on the firms' sales expectaions s_t^e and their inventories:

$$q_{j,t}^d = (1+\nu)s_{j,t}^e - inv_{j,t-1} \quad j = 1, ..., Nf \tag{4.1}$$

where ν is the amount of inventories firms want to have (in percentage of the expected sales).
I assume firms want *"to hold a certain amount of real inventories, expressed by a share ν of expected real sales, as a buffer against unexpected demand swings (Steindl, 1952) and to avoid frustrating customers with supply constraints (Lavoie, 1992)"*[33]. Sales expectations is computed as:

$$s_t^e = s_{j,t-1}^e + \lambda(s_{j,t-1} - s_{j,t-1}^e) \tag{4.2}$$

[33]Caiani et al. (2015)[12]

Table 7: Stock matrix

	Households					Firm	Commercial bank	Government	Central bank	Σ
	Unskilled workers	Skilled workers			Pure capitalists					
		"Factory workers"	Teachers	Researchers						
Deposits	$+D_u$	$+D_s$	$+D_t$	$+D_r$	$+D_c$	$+D_f$	$-D$			0
Loans						$-L_f$	$+L$			0
Inventories						$+inv \cdot p$				$+inv \cdot p$
Capital						$+K$				$+K$
Equity	$+p_e e_u$	$+p_e e_s$	$+p_e e_t$	$+p_e e_r$	$+p_e e_c$	$-p_e e$				0
Government bonds								$-B$	$+B$	0
Dep and loans CB							$+/-M$		$-/+M$	0
Σ	W_u	W_s	W_t	W_r	W_c	W_f	W_b	W_g	W_{cb}	

Table 8: Flows and transactions matrix

	Households					Firm	Commercial bank	Government	Central bank	Σ
	Unskilled workers	Skilled workers			Pure capitalists					
		"Factory workers"	Teachers	Researchers						
Wages	$+w_u$	$+w_s$	$+w_t$	$+w_r$		$-w_u - w_s - w_r$		$-w_t$		0
Dividends	$+DP_u$	$+DP_s$	$+DP_t$	$+DP_r$	$+DP_c$	$-DP$				0
Consumption	$-C_u$	$-C_s$	$-C_t$	$-C_r$	$-C_c$	$+C$		$-G$		0
Education	$-I_H$					$+I_H$				0
Taxes	$-T_u$	$-T_s$	$-T_t$	$-T_r$	$-T_c$			$+T$		0
Interests on deposits	$+i_d d_u$	$+i_d d_s$	$+i_d d_t$	$+i_d d_r$	$+i_d d_c$	$+i_d d_f$	$-i_d d$			0
Interests on loans	$-i_l l_u$	$-i_l l_s$	$-i_l l_t$	$-i_l l_r$	$-i_l l_c$	$-i_l l_f$	$+i_l l - i_{CB} M$	$-i_{CB} B$	$+i_{CB} B + i_{CB} M$	0
Advances							$+/-A$		$-/+A$	0
Change on deposits	Δd_u	Δd_s	Δd_c	Δd_f	Δd					0
Change on loans	Δl_u	Δl_s	Δl_c	Δl_f	Δl_f					0
Change on equity	$p_e \Delta e_u$	$p_e \Delta e_s$	$p_e \Delta e_t$	$p_e \Delta e_r$	$p_e \Delta e_c$					0
Capital gain	$e_u \Delta p_e$	$e_s \Delta p_e$	$e_t \Delta p_e$	$e_r \Delta p_e$	$e_c \Delta p_e$	$e \Delta p_e$				0
Change on bonds								ΔB	ΔB	0
Σ	0	0	0	0	0	0	0	0	0	

Given the desired output, firms' desired investment is:

$$I_t^d = \frac{q_{j,t}^d}{a_{k,j}} - (1-\delta)K_{j,t-1} \tag{4.3}$$

where a_k is the capital productivity and $0 < \delta < 1$ is the depreciation rate. The first term is the desired number of machineries and the second one is the effective number of machineries firm owns. If desired investment is positive firm hires workers in order to self-produce machineries it needs as well as those needed for the consumption goods production (paragraph 4.2.5). Production of goods and machineries takes place in two different firm departments. First firms produce new machineries, the techology used is:

$$I_{j,t} = min\{A_{u,j}L_{u,j,t}; A_{s,j}L_{s,j,t}\} \tag{4.4}$$

where $L_{u,j,t}$, $L_{s,j,t}$, $A_{j,u}$and $A_{j,u}$ are unskilled labour, skilled labour and their productivity in machineries department. The resulting new amount of machineries (K) is so:

$$K_{j,t} = (1-\delta)K_{j,t-1} + I_{j,t-1} \tag{4.5}$$

Then, firms start production of consumption goods, the technology each firm uses is:

$$y_t = min\{a_k K; a_u L_{u,t}; a_s L_{s,t}\} \tag{4.6}$$

where a_u and a_s are the productivity of unskilled and skilled workers in final good production department and $L_{u,j,t}$ and $L_{s,j,t}$ are the remaining workers after machineries production.

4.2.2 Switching processes

The main feature of the model is the division of household sector in three social classes: unskilled workers, skilled ones and pure capitalists.
Unskilled workers may work only as unskilled factory workers while skilled ones may work as skilled factory workers, as researchers, as theachers or, as an alternative if they are unemployed, as unskilled factory workers. Pure capitalists instead decide to not work at all.
There are two swithcing process from a social class to another: the first from the class of unskilled workers to the one of skilled workers through education. The second from the working class (both skilled and unskilled workers) to the pure capitalists class, and viceversa, through the process of wealth accumulation.

Education system. Similarly to Kinsella et al.(2011)[34], young individuals may choose to invest in education if the expected future income is higher than the actual one and if their wealth is higher than the amount requested by government.
Government estimates the cost of education per student as the cost of previous period:

$$CE_t = \frac{\sum_{p=1}^{P} y_{p,t-1}}{n_{t-1}} \tag{4.7}$$

where $y_{t,t-1}$ is the salary of teachers in previous period and n_{t-1} is the number of students in previous period.
Government asks prospective students $(1-\theta)CE_t$ as tax to benefit for education and takes from general taxation the remaining part of cost θCE_t.
The part just described can be summarized with the following formula:

$$I_{Hut} = \begin{cases} (1-\theta)CE_t & \text{if } W_{u,t} \geq (1-\theta)CE_t \\ & \text{and } age_i \leq age_{ED} \\ & \text{and } \bar{w}_{s,t}(1-u_{s,t}) \geq \bar{w}_{u,t}(1-u_{u,t}), \\ 0 & \text{otherwhise} \end{cases} \tag{4.8}$$

where W_{u,t_1} is the wealth of prospective student in previous period; $age_{u,t}$ is his age; age_{ED} is an exogenous age limit beyond which it is no longer possible to attend the education system; $\bar{w}_{s,t}$, $\bar{w}_{u,t}$, $u_{s,t}$ and $u_{u,t}$ represent the average wage of skilled and unskilled workers and the two unemployment rates. Once unskilled workers have made decision about education Government hire teachers and education take place. Teachers are randomly selected among educated individuals; however individuals who have been teacher in the previous period have an higher "propensity" to be teacher again in current period (a higher probability to be selected). The salary of teachers is the sum of average unskilled workers in previous period plus the reservation premium of last teachers employee (see paragraph 4.2.4 for more details).

Pure capitalists and labour supply. Households own shares and deposits which produce dividends and interests. Taking inspiration by Mohun

[34]I use only two education level (not educated and educated) while the authors mentioned uses a more exhaustive scenario in which there are different education levels and agents can choose to achieve them depending not only on financial constraint but innate abilities also. Instead Pluchino et al. (2018) focus mainly on the role of talent distribution and leave out the financial constraint.

(2016) I model the supply of labour: if previous period capital income of a household is higher than a *social acceptable threshold*, household behaves as pure capitalist and decides to do not supply labour. I define this threshold as the 90^{th} percentile of income.

4.2.3 Research and development

I assume firm invests a part ϕ of previous revenue in R& D activity:

$$R\&D_{j,t} = \phi s_{j,t-1} p_{j,t-1} \tag{4.9}$$

The R&D activity is realized hiring a number R of researchers:

$$R_{j,t} = \frac{R\&D_{j,t}}{\bar{y}_{R,j,t-1}} \tag{4.10}$$

where $\bar{y}_{R,t-1}$ is the wage of researchers in previous period. As teachers, researchers are randomly selected among educated individuals (that are not already teacher in the current period); however individuals who has been researcher in the previous period have a higher "propensity" to be researcher again in current period (an higher probability to be selected). The salary of researchers is the sum of average unskilled workers in previous period plus the reservation premium of last researchers employee (see paragraph 4.2.4 for more details).
Once researchers are hired they work on labour-saving innovation. Similarly[35] in Dosi *et al.* (2010) the innovation is a two steps process: (i) first, it is determined whether firm obtains or not an access to innovation through a draw from a Bernoulli distribution, whose probability of success is $1 - e^{-\xi R_t}$, with $0 < \xi < 1$; (ii)if firm innovates, it may draw a new machine embodying technology $(a_{u,t}, a_{s,t})$. This process can be summarized by the following formula:

$$a_{w,j,t} = \left[(1 + B(1 - e^{-\xi R_{j,t}})\epsilon_a)\right] a_{w,j,t-1} \qquad w = s, u \tag{4.11}$$

4.2.4 Labour market

Once firms have computed desired output (paragraph 4.2.1) and desired investments (paragraph 4.2.2) their labour demand are:

$$L^D_{w,j,t} = \frac{I^d_{K,j,t}}{A_{w,j}} + \frac{q^d_{j,t}}{a_w} \qquad w = u, s \tag{4.12}$$

[35] I use the number of researchers rather than the amount invested in research.

where $A_{j,w}$ is the labour productivity for production of machineries. The first term is the need of workers to produce the amount of new machineries firm desires and the second one is the need of workers to produce the desired quantity of final good.

While amount of workers is decided mostly in the demand side of labour market, wages are determined in the supply side. Each individual has a reservation wage. Uneducated workers set it based on the employment condition in the previous period:

$$w_{u,t}^{MIN} = \begin{cases} w_{u,t-1}^{MIN}(1+\epsilon_w) & \text{if employed,} \\ w_{u,t-1}^{MIN}(1-\epsilon_w) & \text{if unemployed.} \end{cases} \tag{4.13}$$

where ϵ_w is a folded normal random variable with mean m_w and variance v_w. Educated workers, instead, set their reservation wage on the basis of their employment condition in previous period and the wage of uneducated workers:

$$w_{s,t}^{MIN} = \begin{cases} w_{u,t-1} + sp_{s,t}^{MIN}(1+\epsilon_w) & \text{if employed,} \\ w_{u,t-1} + sp_{s,t}^{MIN}(1-\epsilon_w) & \text{if unemployed.} \end{cases} \tag{4.14}$$

where $sp_{s,t}^{MIN}$ is their individual "reservation skill-premium".

The supply of skilled labour is simply the number of educated workers who are not already teachers or researchers or pure capitalists. Firms hire the minimum amount between labour demand and labour supply of skilled workers.

Once firm has hired qualified workers if an educated worker has not been hired by any firm he, sacrificing the skill premium, apply for unskilled job; the number of unskilled workers supply is so the total number of workers net of actually employed skilled workers..

Once both kinds of workers are hired by firms, the wage firms pay them is the higher reservation wage of each group of workers in each firm.

Individuals' labour income is:

$$ly_{i,t} = w_{i,t} \qquad i = u, s, r, p \tag{4.15}$$

According to this approch, there is not a clearing condition: unemployment occurs.

4.2.5 Consumption behavior

Firms set price of goods as a mark-up $\mu_{j,t}$ over unit labour cost:

$$p_{j,t} = (1+\mu)\frac{LC_{j,t}}{q_{j,t}} \tag{4.16}$$

where labour cost is computed as sum of wages of unskilled workers, skilled ones and researchers ($LC_{j,t} = \sum_u w_{u,j,t} + \sum_s w_{s,j,t} + \sum_r w_{r,j,t}$).
As Riccetti et al. (2014) and Caiani at al. (2015) households demand for goods to a single firm, however they do not interact during the whole simulation with the same firm: once each firm sets the price (p^{new}), correspondent buyer compares this price with previous one (p^{old}) and decides to change supplier or not according following rule:

$$Pr_s = \begin{cases} 1 - e^{\epsilon(p^{new}-p^{old})/p^{new}} \\ \\ 0 \qquad\qquad\qquad\qquad \text{otherwhise} \end{cases} \tag{4.17}$$

where Pr_s is the probability to switch supplier and $\epsilon > 0$ is an exogenous parameter. If the household decides to switch supplier, it is allowed to observe prices of a sub-samples of firms and it chooses the one with lower prices.

Households' desired consumption (in quantity) depends on expected disposable income and previous wealth:

$$C^d_{i,t} = c_y \frac{yd^e_{i,t}}{p_t} + c_W \frac{W_{i,t-1}}{p_t} \tag{4.18}$$

where $y^e_{i,t}$ is the expected income (defined as firms' expected sales). Effective consumption ($C_{i,t}$) corrisponds to desired one if available quantity of goods of his supplier is higher or equal to the sum of its demand; otherwise there is a rationing and goods are divided among demanders using as weights the desired consumption.

Households' wealth ($W_{i,t}$) is computed as sum of deposits and the values of shares owned:

$$W_{i,t} = d_{i,t} + \underline{s}'_{i,t}\underline{p}_{e,t} \tag{4.19}$$

where $d_{i,t}$ is the amount of deposits (positive or negative), $\underline{s}_{i,t}$ is a vector containing the number of shares of each firm or bank owned by household i and $\underline{p}_{e,t}$ is the vector of shares prices.

4.2.6 Profits

Firm profits are[36]:

$$\pi_{j,t} = q_{j,t}p_{j,t} - LC_{j,t} + i_d d_{j,t-1} - i_l l_{j,t-1} \tag{4.20}$$

[36] $q_t = s_t + \Delta inv = s_t + (inv_{t-1} - inv_t)$

Bank profits are computed as the sum of interests on loans of firms and interests on deposits of households and firms less the amount of non-performing loan:

$$\pi_{b,t} = +i_l l_{j,t-1} - i_d d_{i,t-1} - i_d d_{j,t-1} - NPL_{j,t-1} \tag{4.21}$$

At each period firms not able to pay salaries to its workers because banks are not able to lend them money (see next section) fail and its counterpart bank has a loss equal to its loan.

Companies (firms producing goods and banks) distribute a share ρ of its profits to shareholders:

$$DP_{h,f} = \rho \pi_{h,f} \quad h = j, b \tag{4.22}$$

The reteined profits are: $RP_t = \pi_t - DP_t$.

Individuals' capital income is:

$$cy_{i,t} = i_d d_{i,t-1} + \sum_{j=1}^{Nb+Nf} \frac{e_{i,j,t-1}}{e_j} DP_t \qquad i = u, s, p, r, c \tag{4.23}$$

where $e_{i,t-1}$ and e_j are respectively the number of share that each shareholder owns and the total amount of shares. The first term indicates dividends shareholders receive from the firm and the other two terms are interests on deposits and loans.

4.2.7 Equity market

Households make decisions on savings and how invest their wealth following Davidson's two steps decision process: first, they decide how much to save $(y_{i,t} - C_{i,t} p_t)$ and then make decision on portfolio choice.

Unlikely the labour and good markets, the equity market is always in equilibrium. Indeed, the equity price p_e is the price that equals supply and demand. The aggregate demand of shares of each company is:

$$E^D_{j,t} = \sum_{i=1}^{N} E^D_{i,j,t} \tag{4.24}$$

where $E^D_{i,j,t}$ is the demand of each individual. In the same way the aggregate supply of shares is:

$$E^S_{j,t} = \sum_{i=1}^{N} E^S_{i,j,t} \tag{4.25}$$

where $E^S_{i,t}$ is the individual supply.
Individual supply and demand are defined as follows: households have a risk aversion index $(1 - \alpha_i)$ that indicates the share of total wealth housholds desire to retain as deposit, the remaining amount of wealth is invested on equity. If the sum households currently detain as shares exceed that threshold households supply shares otherwise demand shares, shares supplied are those for which the profit-price ratio is lower, contrarily households that want to invest in equity market demand shares of firm with higher profit-price ratio.

The number of total shares is constant over time (companies do not issue new equity).

4.2.8 Bank sector

Banks collect deposit from household and firms with excess of liquidity and provide loans to firms in order to pay wages. However the amount of money each bank supply is not unrestricted, banks have to comply with a compulsory reserve r: once each bank has computed loans it has already provided and deposits it has collected, the current supply of money is a multiple of the difference of loans and deposits, in formula:

$$L^s_{b,t} = \left[\sum_j (d_{j,t-1} - l_{j,t-1}) + \sum_i d_{i,t-1} \right] \frac{1}{r} \tag{4.26}$$

4.2.9 Public sector: Government and Central bank

Individuals' total income is:

$$y_{i,t} = ly_{i,t} + cy_{i,t} \tag{4.27}$$

Government establishes a proportional tax system on income of households and company:

$$T_t = \tau \sum_i y_{i,t} + \tau \sum_j \pi_{j,t} \tag{4.28}$$

Government uses this tax revenue for two aims: to pay wages to teachers and to purchase goods. Government deficit is so:

$$deficit_t = T - G_t + (1 - \theta)CE_t - \sum_{p=1}^{P} y_{p,t} - i_{CB} B_{t-1} \tag{4.29}$$

where G_t is the public spending in goods and B is the public debt.
Government deficit is financed by issue of new bonds.
The role that central bank plays in this model is marginal: the key interest

rate i_{CB} on the base of which commercial banks set interests on deposits and loans is exogenous. Its task is merely to print money: it always buys Government bonds and lends (with restriction on quantity due to reserve rate) money to commercial banks.

4.3 Calibration

The model has been initialized using the parameters and initial values of vaiables contained in Table 9. The first part of the table shows time invariant parameters, instead the latter shows variables' initial value.

Particular importance is the role of adjustment time, indeed it allow initial value of variable to endogenously adjust if it is necessary.

4.4 Results

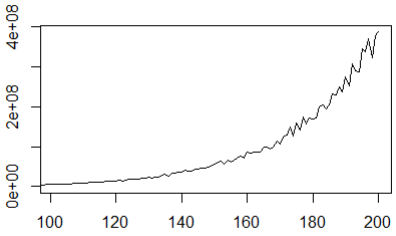
Main results of a rapresentative simulation[37]are summarized in Fig. 5; in particular I have focused my attention on five indicators (nominal gdp, distribution of income and wealth, unemployment rate and utilization rate) and two tables reporting the intergenerational income mobility and intergenerational class mobility.

[37]The model has been simulated using the software R[55].

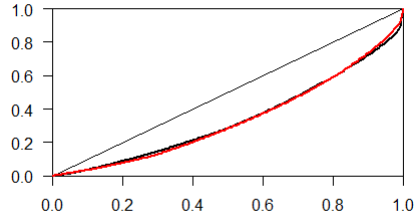
Table 9: Calibration and initial values.

Symbol	Description	Value	Source
N	Number of individuals	200	arbitrary
Nf	Number of firms	50	arbitrary
Nb	Number of banks	10	Caiani et al(2016)
T	Number of periods	200	arbitrary
T_{adj}	Number of adjustment periods	100	Riccetti et al. (2014)
δ	Depreciation rate	0.05	arbitrary
ν	Desired inventories	0.1	Dosi et al. (2010) Caiani et al(2016)
a_k	capital productivity	1	arbitrary
c_y	consumption out of income	0.8	Godley, Lavoie (2007)
c_W	consumption out of wealth	0.2	Godley, Lavoie (2007)
i_{CB}	Central bank interest rate	0.04	arbitrary
i_d	Interest rate on deposits	0.03	arbitrary
i_l	Interest rate on loans	0.05	arbitrary
μ_0	Mark-up	0.5	Dosi et al. (2010)
ρ	Percentage of distributed profits	0.5	arbitrary
τ	tax rate on household income	0.3	Riccetti et al. (2014)
τ_f	tax rate on firm income	0.3	arbitrary
τ_b	tax rate on bank income	0.3	arbitrary
m_w	mean of r.v.	0.05	arbitrary
v_w	variance of r.v.	0.005	arbitrary
ϕ	Firm search capabilities parameter	0.3	Dosi et al. (2010)
λ	Adaptive expectations parameter	0.25	Caiani et al(2016)
α	risk propensity index	0.5	arbitrary
$K_{f,0}$	physical capital	100	arbitrary
$d_{i,0}$	Deposits	0	arbitrary
$d_{f,j,0}$	Firm deposit	0	arbitrary
$l_{f,j,0}$	Firm loan	0	arbitrary
$a_{u,j,0}$	unskilled labor productivity	1	arbitrary
$a_{s,j,0}$	skilled labor productivity	1	arbitrary
$p_{e,0}$	Equity price	1	Caiani et al(2016)
$E_{i,j,0}$	Number of shares	100	B(10, .1)

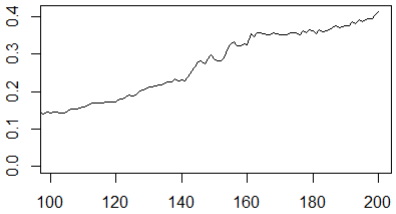
Figure 5: Main indicators of a rapresentative simulation



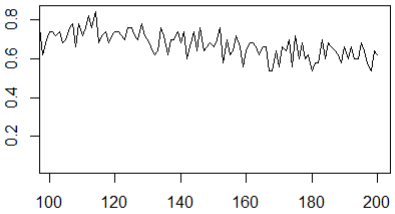
(a) Nominal Gdp



(b) Lorenz curve of disposable income (red) and wealth (black)



(c) Unemployment rate



(d) Utilization rate

Income quintile of son	Parent status					Tot.
	first	second	third	forth	fifth	
first	15	10	13	2	0	40
second	9	12	4	7	8	40
third	8	4	8	12	8	40
forth	7	8	6	8	11	40
fifth	1	6	9	11	13	40

(e) Intergenerational income mobility matrix

Son status	Parent status			Tot.
	Uneducated	Educated	Pure capitalists	
Uneducated	92	43	1	136
Educated	28	30	0	58
Pure capitalists	0	0	6	6
Tot.	120	73	7	250

(f) Intergenerational class mobility matrix

Table 11: Average result of baseline scenario. Mean of 100 Monte Carlo simulations

	Final level of gdp (thousands)	Av. growth rate	Av. Gini index of income	Av. unemployment rate	Av. utilization rate	Av. income mobility index	Av. class mobility index
Min.	336231	0.04920	0.3004	0.0716	0.6441	0.3613	0.1353
1st Qu.	367336	0.05016	0.3309	0.1093	0.6600	0.3660	0.1768
Median	399281	0.05073	0.3480	0.1188	0.6632	0.3675	0.1911
Mean	406016	0.05079	0.3478	0.1203	0.6642	0.3680	0.1875
3rd Qu.	421614	0.05142	0.3597	0.1302	0.6702	0.3696	0.1970
Max	686401	0.05305	0.4084	0.1667	0.6870	0.3769	0.2184
Sd	59857	0.0008	0.0222	0.0196	0.0091	0.0032	0.0157

Graphical results of a single simulation are consistent with AB literature and empirical data, in particular: i) nominal gdp has a positive long-term growth with short-term cycles (Fig. 5a); ii) income and wealth have an "emerging" distribution with wealth more concentrated than income (Fig. 5b); iii) economy does not use at the maximum of availability it resources, there is a persistence in both unemployment rate (Fig. 5c) and utilization rate is not at its maximum potentiality (Fig. 5d). A novelty, at the best of my knowledge, in this kind of literature, is the study of mobility through mobility matrix. Two mobility matrixes show, coherently with chapter 2, how the position of son, whether in term of position in distribution of income or class status, depends largel on parent situation, most of the individuals are in the same quintile of income of their parent (Fig. 5e) and in the same social class (Fig. 5f).

In order to control for the consistency of the results presented in Fig. 5 and be sure that they are not the result of the case the model has be run 50 times with the same parameters of simulation whose results are shown in the figure 5 but with different seeds for generation of random numbers. Summary of results of those simulations are summarized in Table 11.

To compare different mobility matrix I have computed two mobility indexes as the percentage of population with a different social status or income quintile with respect his parent. That is as the ratio between the elements not in the diagonal of Tables (5e -5f) and the total population:

$$MI_c = \frac{1}{N} \sum_{i=1}^{3} \sum_{p=1}^{3} c_{i,p} \quad when\, i \neq p \tag{4.30}$$

where c_i and c_p are the three possible classes to which individuals and their parents belong.

$$MI_q = \frac{1}{N} \sum_{i=1}^{5} \sum_{p=1}^{5} q_{i,p} \quad when\, i \neq p \tag{4.31}$$

where q_i and q_p are the five possible quintiles in the distribution of income of individuals and their parents.

Results of multiple simulations seem to confirm results of the first simulation.

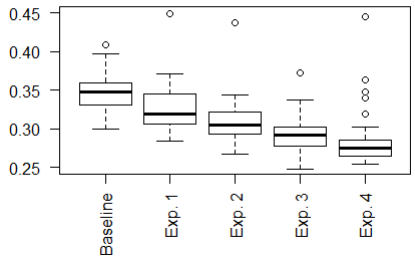
4.4.1 Sensitivity analysis

After this analysis of results it is interesting to evaluate how different parameters affect those results. To evaluate them, a sensitivity analysis has be run: the model has been run again with same parameters of initial simulation but the investigated parameter has been changed. The parameter on which I focus my attention is the one Government can affect directly: tax system. While in baseline scenario the tax system at work is a proportional system, I have carried out four experiments with a system increasingly more progressive. This progressivity in tax system has been obtained dividing the distribution of income in four quartiles and taxing households in different income brackets with different marginal tax rates. Tax rate I have used are reported in table 12, τ_1 is the tax rate on income below first bracket, τ_2, τ_3 and τ_4 are the marginal tax rates on income in excess of three thresholds.

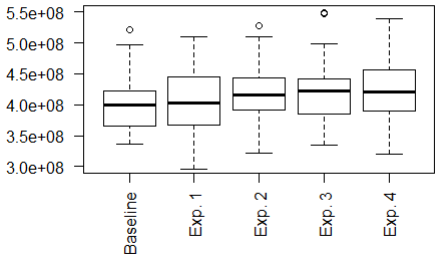
Table 12: Values of tax rates

	Baseline (1:50)	Exp. 1 (51:100)	Exp. 2 (101:150)	Exp. 3 (151:200)	Exp. 4 (2011:250)
τ_1	0.3	0.25	0.2	0.15	0.1
τ_2	0.3	0.275	0.25	0.2	0.15
τ_3	0.3	0.325	0.35	0.4	0.45
τ_4	0.3	0.35	0.4	0.45	0.5

Figure 6: Effects of an increase in tax progressivity inequality and gdp level.



(a) Average Gini index of disposable income



(b) Final level of gdp

Among many variables on which it is possible to identify the effects of a different parameters setting I have selected the most relevant ones: the final level of gdp, the average of income concentration index and the average of class mobility index. Boxplots[38] of results of these experiments[39] are shown in Fig. 6.

The results of four experiments suggest that introducing a progressive tax system (6a) causes a decrease of income concentration; same results for more progressive tax system with respect to less progressive: Gini index goes from

[38]The bottom and top of the boxes are the first and third quartiles, the bands inside the boxes are the medians. The ends of the whiskers represent the lowest datum still within 1.5 interquartile range of the lower quartile, and the highest datum still within 1.5 interquartile range of the upper quartile. Dots represent outlier (any data not included between the whiskers). Red triangles represent the average.

[39]The same group of seeds has been used for each scenario.
Some outliers are eliminated from the graphs in order to have readable graphical representation.

about 0.348 in the proportional tax system to 0.275 in the more progressive tax system.

Along with this result, in some ways obvious, more interesting is the effect of the change of tax system on growth of gdp. A more progressive tax system stimulates growth, indeed gdp level goes from about 399.3 million in the baseline scenario to about 421.08 millions in the last experiment with a growth of more than 5%. The explanation of this result is: on one side, higher taxes directly stimulates public spending, on the other side, it generates higher demand for firms and with the well-known multiplicative mechanism higher production, higher expectations and eventually higher demand of household with higher propensity to consume.

This result is (partially) consistent with that of previous chapter: while in that model a decrease of functional inequality could produce a growth in capital per capital, in this simulation model a decrease of personal distribution is positively correlated to higher level of output.

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