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1 Failure modes in FRCC systems with dry and pre-impregnated carbon yarns: 2 experiments and modeling

3
4 **Jacopo Donnini¹, Giovanni Lancioni², Valeria Corinaldesi³**

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6 ¹ Dipartimento di Scienze e Ingegneria della Materia, dell'Ambiente ed Urbanistica,
7 Università Politecnica delle Marche, Ancona, Italy, j.donnini@univpm.it

8 ² Dipartimento di Ingegneria Civile, Edile e Architettura, Università Politecnica delle Marche,
9 Ancona, Italy, g.lancioni@univpm.it

10 ³ Dipartimento di Scienze e Ingegneria della Materia, dell'Ambiente ed Urbanistica,
11 Università Politecnica delle Marche, Ancona, Italy, v.corinaldesi@univpm.it
12

13 **ABSTRACT**

14 Fiber Reinforced Cementitious Matrix (FRCC) systems have emerged in recent years as an
15 effective tool for strengthening and retrofitting of the existing built heritage.

16 The effectiveness of FRCC systems is strongly related to the bond developed at the interface
17 between inorganic matrix and fabric reinforcement and between the inorganic matrix and the
18 substrate. However, since the major weakness is often located at the matrix-to-fiber interface, the
19 study of stress-transfer mechanisms between fibers and matrix, and a better understanding of the
20 failure processes become of fundamental importance.

21 In this paper, interface bond-slip relations are derived from pull out tests on multifilament carbon
22 yarns embedded in a cementitious matrix. Two different types of yarns are investigated, dry and
23 pre-impregnated with epoxy resin. A variational model able to reproduce the mechanisms of
24 interface debonding and frictional slippage observed in experiments is developed, where the
25 constitutive parameters are calibrated on the base of the experimental data available from pull
26 out tests. The model is implemented in a finite elements code, and pull out and double shear
27 bond tests on masonry substrates are simulated. Finally, numerical results are compared with
28 experimental evidences.
29

30 **Keywords:**

31 FRCC, carbon yarn, bond-slip law, debonding, slippage, variational modeling.
32

33 **1. Introduction**

34 In the last few years, the growing need to recover, reinforce and strengthen existing masonry or
35 concrete buildings led to the development of innovative materials and new repair techniques with
36 low environmental impact and high efficacy. Fiber Reinforced Cementitious Matrix (FRCC),
37 also called Textile Reinforced Mortar (TRM), applied as external reinforcement and acting as an
38 additional tensile-resistant element, proved to be very effective in increasing both mechanical
39 strength and ductility of masonry and concrete structures.

1 FRCM are generally constituted by uni-directional or bi-directional fabrics, consisting of
2 multifilament yarns made of carbon, glass, basalt or PBO fibers, disposed along two orthogonal
3 directions. If compared to organic matrix systems (e.g. FRP, SRP) the use of a cementitious or
4 lime-based matrix guarantees better compatibility with concrete or masonry substrates, higher
5 breathability and vapor permeability, high temperature resistance, applicability on wet and
6 irregular surfaces, more security for operators during installation [1-4].
7 FRP and FRCM are similar in the way they are designed as external reinforcement for structures
8 or elements that need to be strengthened. However, while the performances of FRP are strongly
9 influenced by the effectiveness of the stress transfer mechanisms from the substrate to the
10 composite, since the failure often occurs before the ultimate tensile strength of the composite is
11 reached [5-6], FRCM performances are more affected by the stress transfer mechanisms at the
12 interface between fabric reinforcement and inorganic matrix, since the failure is often expected
13 at this interface.
14 The FRCM mechanical behavior and failure mode are strongly dependent on the type of fabric
15 reinforcement used. Fabrics are usually constituted by yarns made of dry filaments but often they
16 are coated or pre-impregnated with organic compounds in order to improve the adherence with
17 the inorganic matrix. In this context, the term coating is intended for a superficial impregnation
18 of the yarn fibers, with the inner filaments of the bundle left dry, while pre-impregnation
19 indicates a complete impregnation of the yarn with organic resins.
20 In case of fabric reinforcement made of multifilament dry yarns, FRCM properties are
21 influenced by the ability of the inorganic matrix to penetrate within the filaments. Since the
22 matrix is able to engage only the external fibers of the yarns, the so called 'telescopic' behavior,
23 due to slippage of the internal filaments of the yarn, is activated [7].
24 In case of coated or pre-impregnated yarns, the penetration of the mortar within the filaments is
25 prevented by the presence of the organic resin. As a consequence, the bond between yarn and
26 matrix is no longer dependent on the ability of the mortar to penetrate within the filaments but
27 rather from the bond at the interface between coating and matrix [8,9]. In this case it is possible
28 to precisely identify a contact surface between yarn and matrix, while in case of dry yarns it is
29 not easy to establish how many filaments are engaged in the stress transfer mechanisms. In fact,
30 the ability of the mortar to penetrate between the filaments depends on many factors, such as
31 mortar viscosity, size of the aggregates or fillers that constitute the mortar, space between
32 filaments and also by the technique used to apply the FRCM to the substrate (trowel, plastering
33 machine). Thus two different types of bond are distinguished in multifilament dry yarns: bond
34 between filaments and matrix and bond between filaments. In case of pre-impregnated yarns the
35 slippage between filaments is prevented by the resin and only the bond between yarn and
36 cementitious matrix is considered.
37 In this scenario, the definition of an appropriate bond-slip law (BSL) for a given FRCM system
38 is fundamental to allow the development of analytical or numerical models able to simulate the
39 FRCM mechanical behavior and failure modes.
40 The bond between multifilament yarns and cementitious matrix has been studied by several
41 authors by using different methodologies. Banholzer and coworkers proposed two different
42 methods: the so called 'direct method', which requires the knowledge of an idealized BSL or the
43 measurement of the longitudinal strain along the interface between yarn and matrix [10] and the
44 'indirect method', that is able to predict a BSL based on experimental parameters obtained by
45 means of pull out, single or double shear bond tests on FRCM systems [11].
46 An indirect method to calibrate the cohesive material law of FRCM-concrete joints was proposed
47 by Focacci and coworkers [12]. The method is based on single-lap shear bond tests on FRCM-

1 concrete joints with different bonded lengths, using the peak load versus bonded length to
2 calibrate the BSL. Other models based on the analysis of the shear stress at the interface yarn-
3 matrix in pull out [13,14] or single shear tests [15-19] have been proposed in the literature.
4 However, all these studies have shown that measuring the fiber strain in FRCM composites is a
5 difficult task, due to the presence of the matrix embedding the fabric yarns, while using idealized
6 BSL is not always able to accurately describe the FRCM mechanical behavior.
7 In this work a different approach has been proposed, by using simple pull out tests on single
8 yarns to calibrate the BSL, for different FRCM systems.
9 During pull out test on a single yarn embedded in a cement matrix, force and slip at the loaded
10 end can be easily measured. As a result, the relationship between the shear stress and the slip
11 along the interface (BSL) can be derived by considering the shear stress to be constant along the
12 interface with no need to apply strain sensors on the yarn surface. This assumption is acceptable
13 for short embedded lengths (equal to 20 mm), as confirmed by experimental and numerical
14 results, while it is no longer reliable for higher lengths.
15 Two types of carbon yarns, dry and fully pre-impregnated with an organic resin, have been used,
16 these two representing the antipodes of what is currently on the market of FRCM systems. As a
17 consequence, two different BSL have been obtained from pull out tests.
18 A variational model has been developed and implemented in a finite element code to simulate
19 the possible failure mechanisms of FRCM systems reinforced with these two types of carbon
20 fabrics.
21 Following a phase-field approach [20], the proposed model is based on an energy functional
22 which depends on a damage parameter α ranging from 0 to 1 ($\alpha=0$ means sound material, and
23 $\alpha=1$ means totally damaged material), which accounts for possible fracture of the constituents
24 (cementitious matrix and carbon reinforcement), or debonding and slippage at the yarn-to-matrix
25 interface. Different energies have been assigned to the system constituents and to the interface.
26 Regarding matrix and yarn, the energy proposed in [21,22] has been considered. It incorporates a
27 linear local damage term and a non-local damage contribution in order to describe the behavior
28 of brittle materials. The energy assigned to the yarn-to-matrix interface is constituted by three
29 terms: (1) an elastic contribution, which accounts for a distribution of elastic springs whose
30 elastic stiffness degrades when damage increases, (2) a local damage term, accounting for
31 debonding, and (3) a frictional energy, expended in the phase of slippage that follows the
32 detachment. This third term is similar to that proposed in [23,24] to model plastic slip flow, and
33 it only depends on one parameter, τ_0 , which represents the frictional shear stress. The
34 corresponding parameter in [23] represented the plastic yielding stress.
35 It follows that the three energy terms are directly related to the three basic phases of the failure
36 process at the yarn-to-matrix interface observed in experiments, i.e., the initial elastic shear, the
37 debonding in a regime of stress-softening, and the slippage occurring at constant frictional shear
38 stress. This strict correlation is a distinguishing feature of the proposed formulation, with respect
39 to the model proposed in [25], where a unique damage energy was considered to describe the
40 post-elastic evolution.
41 Two different failure modes, brittle and ductile with a softening phase, have been observed for
42 dry and pre-impregnated carbon yarns, respectively. To account for these differences, two
43 expressions of the damage energy have been considered: a concave function of α , to capture
44 brittle damaging, and a linear function to describe the softening phase.
45 Evolution of strain and damage, as the external load increases, is determined by solving a
46 constrained incremental minimization problem, that has been implemented in a sequential

1 quadratic programming algorithm. Incremental minimization was used to determine local
2 minima in many different evolution problems concerning fracture [26], plasticity [27,28] or
3 crystal plasticity [29].

4 Experimental data have been used to calibrate the constitutive parameters that characterize the
5 interface, such as the shear elastic stiffness k , the peak shear stress τ_e and the frictional shear
6 stress τ_0 . They have been easily derived from the experimental force-displacement curves of pull
7 out tests. Once the model has been calibrated, numerical simulations have been carried out. Both
8 pull out and double shear bond tests have been reproduced by considering different bond lengths.
9 Interface debonding and slippage have been investigated in the two cases of dry and pre-
10 impregnated yarns, and comparisons with experimental results have been carried out.

11
12 The paper is organized as follows:

13 Experimental results of pull out tests on specimens made of dry or pre-impregnated carbon yarns
14 embedded in a cementitious matrix are presented in Section 2. The effect of varying the bond
15 length is also discussed.

16 In Section 3 a phase-field model has been developed in order to reproduce the FRCM
17 mechanical behavior, taking into account the possible failure of the constituent materials, the
18 slippage at the yarn to the matrix interface and the frictional stress in the post-elastic phase. The
19 bond parameters have been calibrated, based on experimental results, and implemented within
20 the model.

21 Pull out tests and double shear bond test have been simulated in Section 4. The effect of varying
22 the bond length of the carbon yarn or fabric within the inorganic matrix and the use of dry or pre-
23 impregnated carbon yarns was also considered. Finally, numerical results have been compared
24 with experimental outcomes in terms of load-displacements curves.

26 **2. Experiments**

27 *2.1 Pull out test*

28 A commercially available cementitious mortar with a maximum aggregates size of 0.5 mm has
29 been used as FRCM matrix. Mechanical properties were investigated by means of three-point
30 bending tests on standard prisms (40x40x160 mm) and compressive tests on the two remaining
31 broken parts. Tests were performed after 28 days of casting at 20 °C and 50% RH. Two different
32 types of carbon yarn were used: the first one constituted by multifilament dry carbon fibers
33 (DRY) and the second realized by applying a complete organic pre-impregnation (IMP). The
34 pre-impregnation consisted on a fully saturation of the yarn with flexible epoxy resin and a thin
35 layer of quartz sand (maximum sand size of 1 mm) applied to the surface. Tensile strength and
36 elastic modulus of the carbon yarn were evaluated by means of tensile tests, according to ISO
37 10618. Material properties of cementitious mortar and carbon yarn are collected in Table 1.

38 A total of 25 pull out tests were carried out on specimens made of dry or coated carbon yarns
39 embedded in a cylinder of cementitious mortar (diameter $D=50$ mm) with different embedded
40 lengths ($h=20\div 50$ mm). The pull out test setup is illustrated in Figure 1. Specimens have been
41 labelled by indicating the type of carbon yarn, DRY or pre-impregnated (IMP), followed by the
42 embedded length (h). The free length l is kept constant and equal to 80 mm. Pull out tests were
43 performed using a tensile testing machine with a load bearing capacity of 50 kN. The specimen
44 is fixed at the top by a metallic frame anchored to the testing machine and the yarn is gripped
45 and pulled in displacement control at 0.5 mm/min. The displacement s_e , corresponding to the
46 maximum force f_{max} , was calculated by subtracting the elastic deformation of the bare yarn to the

total displacement d_e , measured at the head of the testing machine: $s_e = d_e f_{max} l / (A_f E_f)$, where E_f is the Young's modulus of the carbon yarn, and A_f is the yarn cross-section area.

Table 1 - Material properties

Material	Compressive strength (MPa)	Tensile strength σ_e (MPa)	Elastic modulus E (GPa)	Unit weight (kg/m ³)	Poisson ratio ν
Cementitious mortar (m)	45	6.2	34.5	1320	0.2
Carbon fiber	-	4900	240	-	0.3
Carbon yarn (f)	-	1850	150	-	0.3

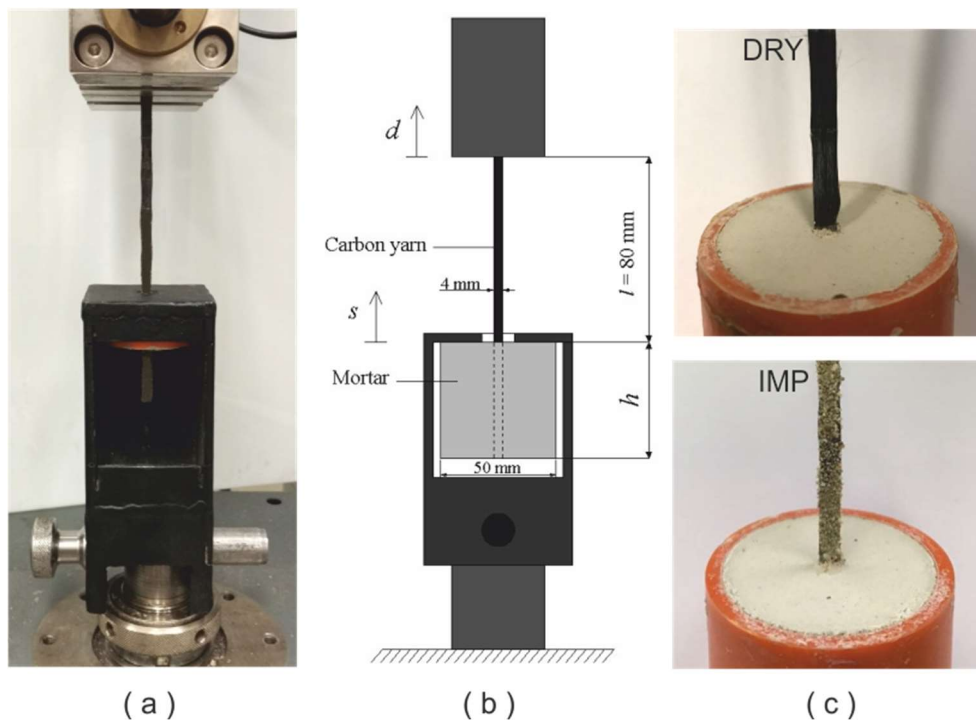


Figure 1 - Pull out test set up (a), geometrical scheme (b), dry and pre-impregnated carbon yarns (c).

Experimental results of pull out tests are reported in Table 2. The average maximum load f_{max} was evaluated on 5 specimens for each type of geometry while s_e represents the corresponding displacement. Three experimental pull out curves for each type of specimen are reported in Figure 2, while the grey region represents the envelope of all experimental curves which have been used to derive the bond parameters: τ_e , τ_0 , k .

The maximum shear stress τ_e at the interface is supposed to be constant along the embedded length. This value is estimated through the formula $\tau_e = f_{max} / (h \cdot p)$, where p is the perimeter of the yarn equal to 8.64 mm and h is the embedded length.

The frictional shear stress τ_0 has been evaluated as the stress corresponding to the pure frictional phase of the pull out curve. τ_0 represents the shear stress at $s = 3$ mm, since at this point it is sure that the yarn is completely debonded from the matrix and the pull out is governed only by frictional stresses at the interface. In case of dry yarns τ_0 is assumed equal to zero.

The shear modulus is $k = \tau_e / s_e$.

Table 2 - Experimental results of pull out tests

Specimen		f_{max} (N)	s_e (mm)	τ_e (MPa)	τ_0 (MPa)	k (N/mm ³)
DRY_20	Average	242	0.44	1.40	0	3.10
	CoV (%)	14.2	11.5	14.2	-	14.3
DRY_50	Average	479	0.72	1.11	0	1.54
	CoV (%)	6.0	15.1	6.0	-	13.2
IMP_20	Average	342	0.5	1.98	1.25	3.95
	CoV (%)	7.9	13.7	7.9	7.3	9.2
IMP_30	Average	501	0.77	1.93	1.28	2.51
	CoV (%)	6.8	17.3	6.8	4.7	14.7
IMP_50	Average	748	0.95	1.73	1.25	1.82
	CoV (%)	8.9	14.3	8.9	3.2	11.8

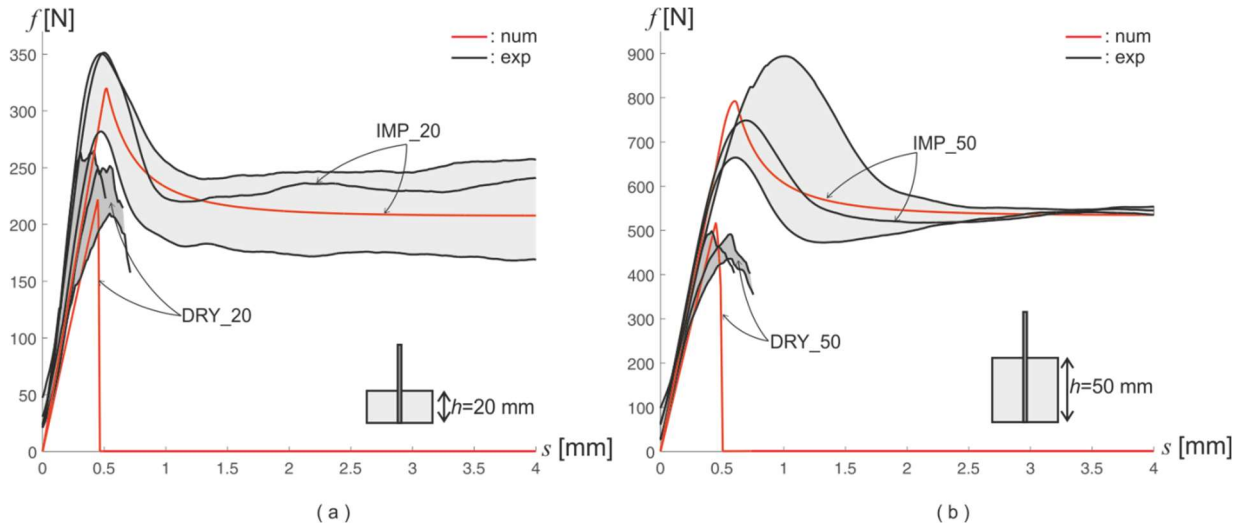


Figure 2 - Experimental and numerical load-displacement curves of pull out tests.

Load-displacement curves reported in Figure 2 show a substantial increase of the maximum load f_{max} passing from dry to pre-impregnated (IMP) carbon yarn. The pre-impregnation has a major influence on the quality of the adhesion between yarn and matrix and allows for a more uniform distribution of the stresses all over the yarn cross section. The pull out resistance increases about 40% and 55% for an embedded length of 20 and 50 mm, respectively.

It can be observed that the maximum load f_{max} increases by increasing the embedded length h while the shear stress τ_e calculated over the total length of the interface decreases. The reduction of the stress value τ_e by increasing the embedded length explains in part why the distribution of the shear stress along the interface cannot be considered constant for longer (>20 mm) embedded

1 lengths. The debonding at the interface yarn-to-matrix propagates slowly from the top of the
2 specimen. In case of short embedded length this propagation is fast, and stress distribution along
3 the interface, once the maximum load is reached, can be considered almost constant. This
4 phenomenon has been confirmed by numerical simulations.

5 In the case of dry yarns no frictional shear stress in the post-elastic phase was observed: the
6 behavior is elastic-brittle with a sudden drop of the load after exceeding the maximum load.

7 For pre-impregnated carbon yarns (IMP), when the maximum shear stress τ_e is reached, the
8 softening phase takes place and the shear stress decreases. During this phase the yarn begins to
9 detach from the matrix at the top of the specimen, up to reach the complete debonding through
10 the interface. After that, the shear stress becomes constant, while the slippage of the yarn within
11 the matrix is governed only by the frictional stress τ_0 . The value of τ_0 is about 35% lower than the
12 maximum shear stress τ_e .

13 It can be observed that τ_0 does not vary by changing the embedded length and so it can be
14 considered as a material parameter which depends only on the bond quality of the interface
15 between yarn and matrix.

16 17 *2.2 SEM analysis*

18
19 The interface between dry (Figures 3a,b) or pre-impregnated (Figures 3c,d) carbon yarn and the
20 surrounding mortar was observed by SEM (at magnifications ranging from 20× to 100×), on
21 specimens after pull out test. It is clear to observe that the inorganic matrix doesn't penetrate
22 within the dry carbon filaments and it is able to engage only the external carbon filaments. The
23 inner filaments are pulled out of the matrix while only few external filaments remain attached to
24 the matrix. In pre-impregnated carbon yarns, the penetration of the mortar within the filaments is
25 prevented by the resin (Figure 3c). However, the mortar is able to completely wrap the yarn,
26 ensuring a better interface quality. After the specimen has been tested in pull out it can be
27 observed a clear debonding at the yarn to mortar interface. A non-perfect impregnation of the
28 inner filaments of the yarn can also be observed in Figure 3d. In some cases, the sand grains used
29 to improve the adhesion within yarn and matrix remain attached to the surrounding mortar more
30 than to the yarn surface, due to their considerable size, if compared to the yarn cross section.
31 This might suggest to apply a sand or powder of smaller size to increase the adhesion at the
32 interface.

33
34

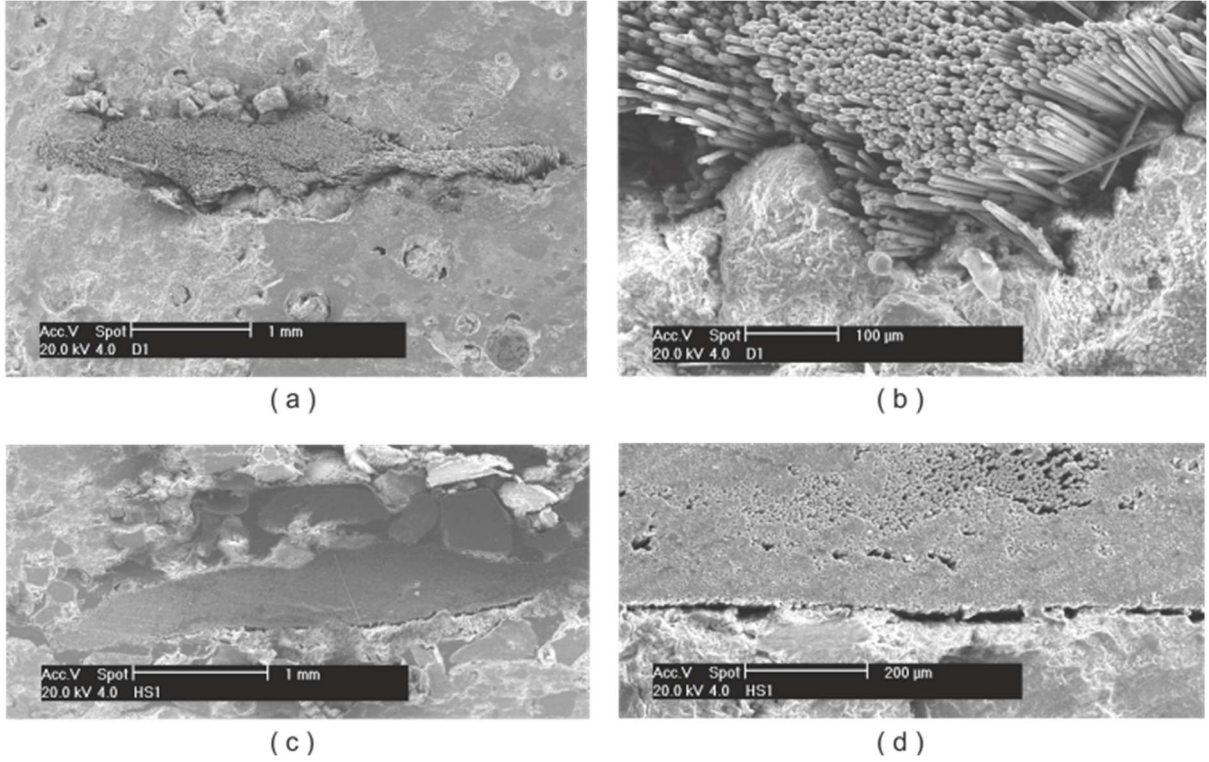


Figure 3 - SEM analysis at the yarn to matrix interface after pull out test, at different magnifications: Dry yarn (3ab); Pre-impregnated yarn (3cd).

3. Variational model

A composite body constituted by the subdomains Ω_m and Ω_f , representing the matrix and the yarn, has been considered, with Λ which represents the matrix-to-yarn interface (see Figure 6 for two-dimensional geometrical schemes of pull out and double shear bond tests). The multifilament yarn is considered as an homogeneous body, so that the shear between filaments and the so-called telescopic behavior are neglected. A monotonically increasing displacement s is applied on a portion $\partial_s \Omega_f$ of the yarn domain, and it constitutes the evolution parameter of the problem. The problem unknowns are the displacement $\mathbf{u}_s(x)$ and the damage $\alpha_s(x)$, which depend on the position x and on the imposed displacement s . Damage $\alpha_s(x)$ is a scalar field ranging from 0 to 1: $\alpha=0$ means that the material is sound, and $\alpha=1$ that it is totally damaged.

3.1 Energies

As in [22], the internal energy of the system is assumed to be sum of three contributions

$$\mathbf{E}(\mathbf{u}_s, \alpha_s) = \int_{\Omega_m} \varphi_m(\mathbf{u}_s, \alpha_s) dx + \int_{\Omega_f} \varphi_f(\mathbf{u}_s, \alpha_s) dx + \int_{\Lambda} \varphi_{\Lambda}(\boldsymbol{\delta}_s, \alpha_s) da, \quad (1)$$

which correspond to the energies of matrix, yarn and interface. In the third integral, $\boldsymbol{\delta}_s$ is the displacement jump at the interface, that is, $\boldsymbol{\delta}_s(x) = \mathbf{u}_s^f(x) - \mathbf{u}_s^m(x)$, with $\mathbf{u}_s^m(x)$ and $\mathbf{u}_s^f(x)$ the displacement on the yarn and matrix boundaries, respectively. Since volume loads and surface

1 loads at the boundary are assumed to be null, energy (1) represents the total energy of the
 2 system.

3

4 (i) *Matrix and yarn energy densities*

5 The matrix and yarn energy densities φ_m and φ_f have the shape

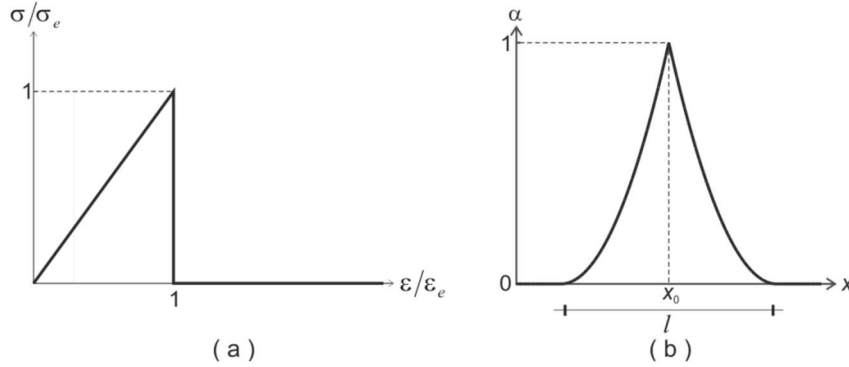
$$6 \quad \varphi(\mathbf{u}_s, \alpha_s) = (1 - \alpha_s)^2 W(\nabla \mathbf{u}_s) + \frac{\sigma_e^2}{E} \left(\alpha_s + \frac{l^2}{16} (\nabla \alpha_s)^2 \right), \quad (2)$$

7 where the first term represents the elastic strain energy, with

$$8 \quad W(\nabla \mathbf{u}) = \frac{E}{2(1 + \nu)} \left(\text{sym}^2 \nabla \mathbf{u} + \frac{\nu}{(1 - 2\nu)} \text{tr} \nabla \mathbf{u}^2 \right),$$

9 the elastic energy density of linearly elastic isotropic materials, depending on the Young's
 10 modulus E and the Poisson's ratio ν . The second term in (2) is the damage energy as proposed in
 11 [21,22]. It is composed of a local term linearly depending on α , and a quadratic non-local
 12 function of the damage gradient $\nabla \alpha$. The energy density (2) is proper of brittle materials, which
 13 exhibit an initial linearly elastic stage followed by brittle failure. The local stress-strain curve
 14 obtained with the energy (2) is drawn in Figure 4a, and the analytical justification of the brittle
 15 failure occurring when $\sigma = \sigma_e$ is given in the Appendix, where the equilibrium equations are
 16 deduced. The functional (2) depends on the peak stress σ_e , attained at the end of the initial elastic
 17 phase, on the Young's modulus E , and on the internal length l , which corresponds to the size of
 18 the damage localization zone at fracture. A profile of α in a line x normal to the crack surface is
 19 shown in Figure 4b, (see the Appendix for the analytical expression of $\alpha = \alpha(x)$, and [30] for a
 20 thorough study).

21



22

23 *Figure 4 - (a) Stress-strain curve for brittle materials (cementitious matrix and carbon yarn); (b)*
 24 *damage profile on a transversal section of a crack surface.*

25

26 (ii) *Matrix-to-yarn interface energy density*

27 The third term of (1) is the energy of the matrix-to-yarn interface Λ , which has the expression

$$28 \quad \varphi_\Lambda(\delta_s, \alpha_s) = (1 - \alpha_s)^2 \frac{1}{2} \mathbf{K} \delta_s \cdot \delta_s + w(\alpha_s) + \tau_0 \alpha_s^2 |\delta_s \cdot \mathbf{t}|. \quad (3)$$

29 The first term is the elastic energy density, where the elastic tensor \mathbf{K} is supposed to be diagonal,
 30 i.e., $\mathbf{K} = k\mathbf{I}$, with k the elastic coefficient and \mathbf{I} the identity tensor. The degradation function
 31 $(1 - \alpha_s)^2$ is equal to that assumed in (2).

32 The second term is the damage energy density, which assumes the following two possible forms

$$w_B(\alpha_s) = \frac{\tau_e^2}{k} \left(\alpha_s - \frac{1}{2} \alpha_s^2 \right), \quad w_D(\alpha_s) = \frac{\tau_e^2}{k} \alpha_s. \quad (4)$$

They are quadratic and linear polynomials of α whose graphs are plotted in Figure 5a, and they depend on the maximum elastic shear stress τ_e , and on the elastic coefficient k . In section 4.2, it is shown that the quadratic expression properly describes brittle debonding, as observed in experiments with dry yarns, while the linear expression is suitable for reproducing ductile debonding, as exhibited by impregnated yarns. Accordingly, indices B and D in (4) stand for brittle and ductile.

Finally, the third term in (3) represents the frictional energy density, whose expression is similar to that proposed in [23,24] to model plastic slip flow. It depends on α and on the absolute value of the tangential slip $\delta_s \cdot \mathbf{t}$, being \mathbf{t} the unit vector tangent to the interface. The constitutive parameter to be assigned in the frictional energy density is the frictional shear stress τ_0 .

3.2 Debonding and frictional sliding at the interface

In order to describe the damage evolution associated to the energy density (3), the equilibrium equations in Λ have been determined, when the two faces in contact are subjected to the relative constant load \mathbf{f} . In this case, the total energy of Λ is

$$\mathbf{L}(\delta, \alpha) = \int_{\Lambda} \varphi_{\Lambda}(\delta, \alpha) da - \int_{\Lambda} \mathbf{f} \cdot \delta da, \quad (5)$$

where the second integral is the potential energy of the external load, and index s is omitted. Equilibrium is obtained by requiring the first variation of the total energy (5) to be non-negative for any perturbation, that is,

$$\delta \mathbf{L}(\delta, \alpha; \mu, \beta) = \int_{\Lambda} \left[\frac{\partial \varphi_{\Lambda}}{\partial \delta}(\delta, \alpha) \cdot \mu + \frac{\partial \varphi_{\Lambda}}{\partial \alpha}(\delta, \alpha) \beta \right] da - \int_{\Lambda} \mathbf{f} \cdot \mu da \geq 0, \quad \forall \mu, \beta, \quad (6)$$

from which the equilibrium relations are

$$\mathbf{f} = (1 - \alpha)^2 \mathbf{K} \delta + \tau_0 \alpha^2 \frac{\delta \cdot \mathbf{t}}{|\delta \cdot \mathbf{t}|} \mathbf{t}, \quad (7)$$

$$\frac{dw(\alpha)}{d\alpha} - (1 - \alpha) \mathbf{K} \delta \cdot \delta + 2\tau_0 \alpha |\delta \cdot \mathbf{t}| \geq 0.$$

The first equation is the force-displacement balance equation which is sum of an elastic term and a frictional contribution. When $\alpha=0$, only the elastic response is active, and (7)₁ reduces to $\mathbf{f}=\mathbf{K}\delta$. When $\alpha=1$, springs are totally broken, and only the frictional force $\mathbf{f} = \tau_0 \text{sgn}(\delta \cdot \mathbf{t}) \mathbf{t}$ survives.

Inequality (7)₂ represents the yield condition for damage, according to which damage evolves only when it is satisfied as equality, while elastic evolution is expected when it is satisfied as strict inequality.

If the brittle and ductile damage energy densities (4) are substituted into (7)₂, we obtain, respectively,

$$\frac{\tau_e^2}{k} (1 - \alpha) - (1 - \alpha) \mathbf{K} \delta \cdot \delta + 2\tau_0 \alpha |\delta \cdot \mathbf{t}| \geq 0 \quad (\text{Brittle}), \quad (8)$$

$$\frac{\tau_e^2}{k} - (1 - \alpha) \mathbf{K} \delta \cdot \delta + 2\tau_0 \alpha |\delta \cdot \mathbf{t}| \geq 0 \quad (\text{Ductile}).$$

If a process in which \mathbf{f} monotonically increases is considered, with $\mathbf{f}=\mathbf{0}$ at the initial instant, and the interface is unstressed and undamaged, that is, $\delta=\mathbf{0}$ and $\alpha=0$, then the interface experiences an initial elastic phase whose solution is $\delta=\mathbf{f}/k$ and $\alpha=0$. In this phase, the

1 inequalities (8) assume the simplified form $\mathbf{f} \cdot \mathbf{f} \leq \tau_e^2$, which is strictly satisfied, until the
 2 magnitude of \mathbf{f} reaches the value τ_e . Interface debonding initiates when $\mathbf{f} \cdot \mathbf{f} = \tau_e^2$, and it advances
 3 in the subsequent instants of the evolution.

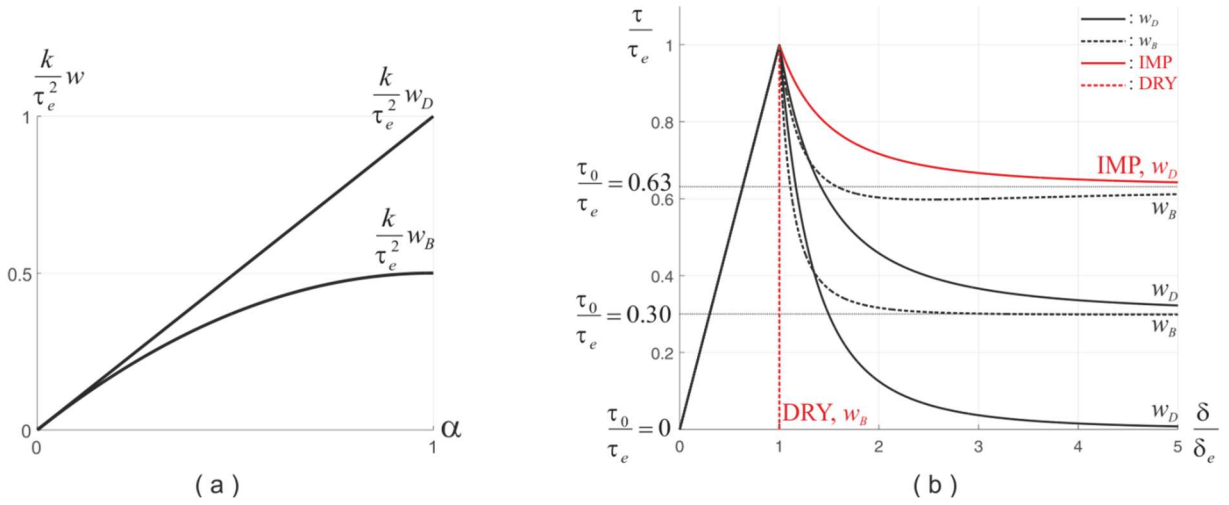
4 Since FRCM systems are usually subjected to shear stresses, we restrict the above problem to
 5 pure shear, by assigning the shear stress $\mathbf{f} = \boldsymbol{\tau} \geq 0$, and determining the displacement slip $\boldsymbol{\delta} = \boldsymbol{\alpha}$.
 6 Under this assumption, relations (7) simplify as follows

$$7 \quad \tau = (1 - \alpha)^2 k \delta + \tau_0 \alpha^2, \quad \frac{dw(\alpha)}{d\alpha} - (1 - \alpha)k\delta^2 + 2\tau_0\alpha\delta \geq 0. \quad (9)$$

8 By solving equations (9) with respect to δ and τ for varying α in (0,1), it is possible to represent
 9 the shear stress versus displacement slip relations. They are plotted in Figure 5b for three
 10 different values of τ_0 , considering the two energies (4).

11 After the initial linear elastic phase, the curves exhibit stress-softening, in correspondence of the
 12 interface debonding, and damage growth. Then the curves asymptotically tend to the frictional
 13 shear stress τ_0 . Exception is made by the curve where w_B is implemented and $\tau_0=0$ (red dashed
 14 line). In this case, the stress drops down to zero when τ_e is reached. A distinction between brittle
 15 and ductile models is that the softening branches (when present) are steeper for w_B than for w_D .
 16 The red curves in Figure 5b describe the local responses assigned to FRCM reinforced with dry
 17 (DRY) and pre-impregnated (IMP) yarns. The red curves will be recalled in the next Section 4.4,
 18 where the calibration issue is faced.

19



20

21 *Figure 5 - (a) Graphs of dimensionless damage energy densities w_B and w_D of formula (4). (b)*
 22 *Local shear stress-slip displacement curves in case of energy density w_D (solid line) and w_B*
 23 *(dashed line).*

24

25 3.3 Evolution problem

26 In a process where s increases, the evolution of \mathbf{u}_s and α_s is determined by incremental energy
 27 minimization [26,27,29]. The loading parameter s is increased through finite steps of size δs ,
 28 and, within each step, \mathbf{u} and α are supposed to be linear functions of the increment δs

$$29 \quad \mathbf{u}_{s+\delta s} = \mathbf{u}_s + \frac{d\mathbf{u}_s}{ds} \delta s, \quad \alpha_{s+\delta s} = \alpha_s + \frac{d\alpha_s}{ds} \delta s, \quad (2)$$

30

and the energy \mathbf{E} is developed up to the second order

$$\mathbf{E}(\mathbf{u}_{s+\delta}, \alpha_{s+\delta}) = \mathbf{E}(\mathbf{u}_s, \alpha_s) + \frac{d\mathbf{E}(\mathbf{u}_s, \alpha_s)}{ds} \delta + \frac{1}{2} \frac{d^2\mathbf{E}(\mathbf{u}_s, \alpha_s)}{ds^2} \delta^2. \quad (3)$$

The rates $(d\mathbf{u}_s/ds, d\alpha_s/ds)$ are determined by minimizing the above functional, under the constrain $d\alpha_s/ds \geq 0$. A finite element code has been developed which implements an alternate iterative minimization procedure, consisting in minimizing the energy functional with respect to the two unknown fields separately.

3.4 Parameters calibration

The constitutive parameters for matrix and yarn are assigned in Table 1. They are the Young's modulus E , the Poisson's ratio ν and the tensile strength σ_e , which are included in the energy density (2).

For the constitutive parameters of the yarn-to-matrix interface, a distinction was made for the dry and pre-impregnated yarn, since the surface treatment strongly influences the bond mechanics. The brittle and ductile energy densities (4) are assigned to DRY and IMP yarns, respectively, and different constitutive parameters are considered, in order to account for the different behaviors observed in pull out tests, as described in Subsection 3.1. The values assigned to τ_e , τ_0 , and k are those deduced in Subsection 3.1 from pull out experiments on samples with $h=20$ mm. To analytically derive them, the hypothesis of homogenous shear stress along the yarn-to-matrix interface is assumed, motivated by the fact that the embedded length is very short. The constitutive parameters assigned to the interface are recall in the following:

- DRY yarn: $\tau_e=1.40$ MPa, $\tau_0=0$ MPa, $k=3.10$ MPa;
- IMP yarn: $\tau_e=1.98$ MPa, $\tau_0=1.25$ MPa, $k=3.95$ MPa.

The τ - δ curves obtained by assigning the above parameters are the red curves plotted in Figure 5b. In the case of dry yarn, the behavior is brittle, and a drastic failure is expected when τ_e is reached. In the case of IMP yarn, a pseudo-ductile response is obtained, characterized by a long softening tail asymptotically tending to τ_0 .

4. Numerical simulations

In this section, numerical results of pull out and double shear bond tests are presented and compared with experimental evidences. For both pull out and double shear tests, simplified two-dimensional geometries are considered, and the hypothesis of plane strain state is assumed.

The simulations presented below reproduce debonding and slippage at the yarn-to-matrix interface as the unique mechanism leading to failure, which prevails over cracking of the constituents (cementitious matrix and carbon yarn). However, breakage of the constituents has been reproduced in simulations not reported in the paper, by considering different experimental setups (different geometries, loading and boundary conditions).

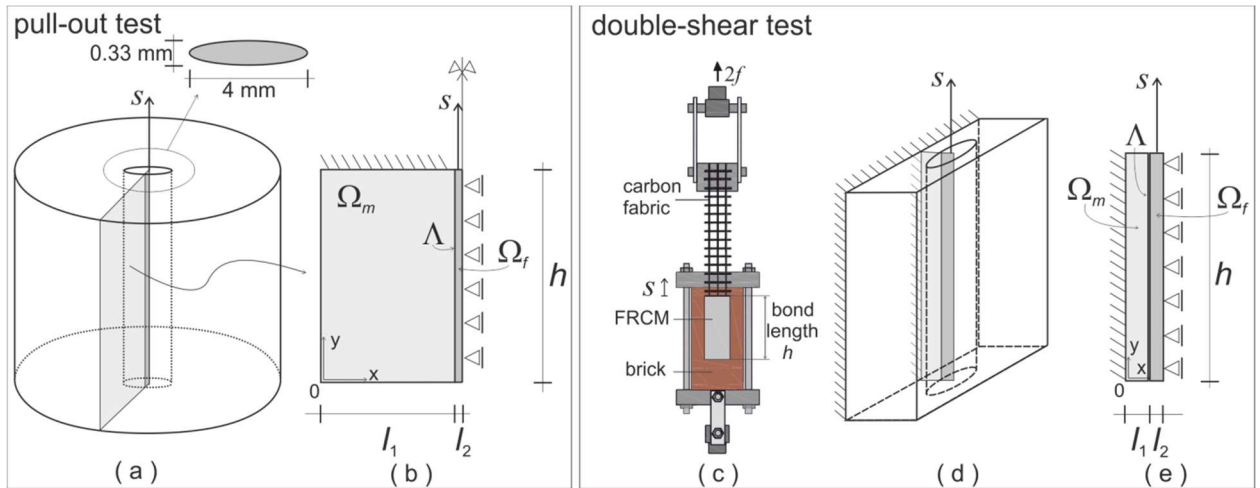


Figure 6 - Sample geometry and two-dimensional scheme of pull out (a,b) and double shear test (c,d,e).

4.1 Pull out tests

The sample geometry of pull out tests is drawn in Figure 6a. One-half sample is considered for symmetry reasons, and the longitudinal section drawn in gray tones is assumed as the two-dimensional domain considered in numerical simulations. Its plane representation is given in Figure 6b. The top side of the matrix domain Ω_m is clamped, and an upward displacement s is applied to the upper side of the yarn Ω_f . The geometrical dimensions are $l_1=25$ mm, $l_2=0.16$ mm, and two bond lengths, i.e., $h=20, 50$ mm, are considered. Simulations are performed for each length, for dry and pre-impregnated carbon yarns.

Force-displacement curves are plotted in Figure 2 (red line). Comparison with experimental results shows excellent agreement between numerical and experimental curves. Simulations accurately capture the peak forces and the different evolution phases observed in experiments.

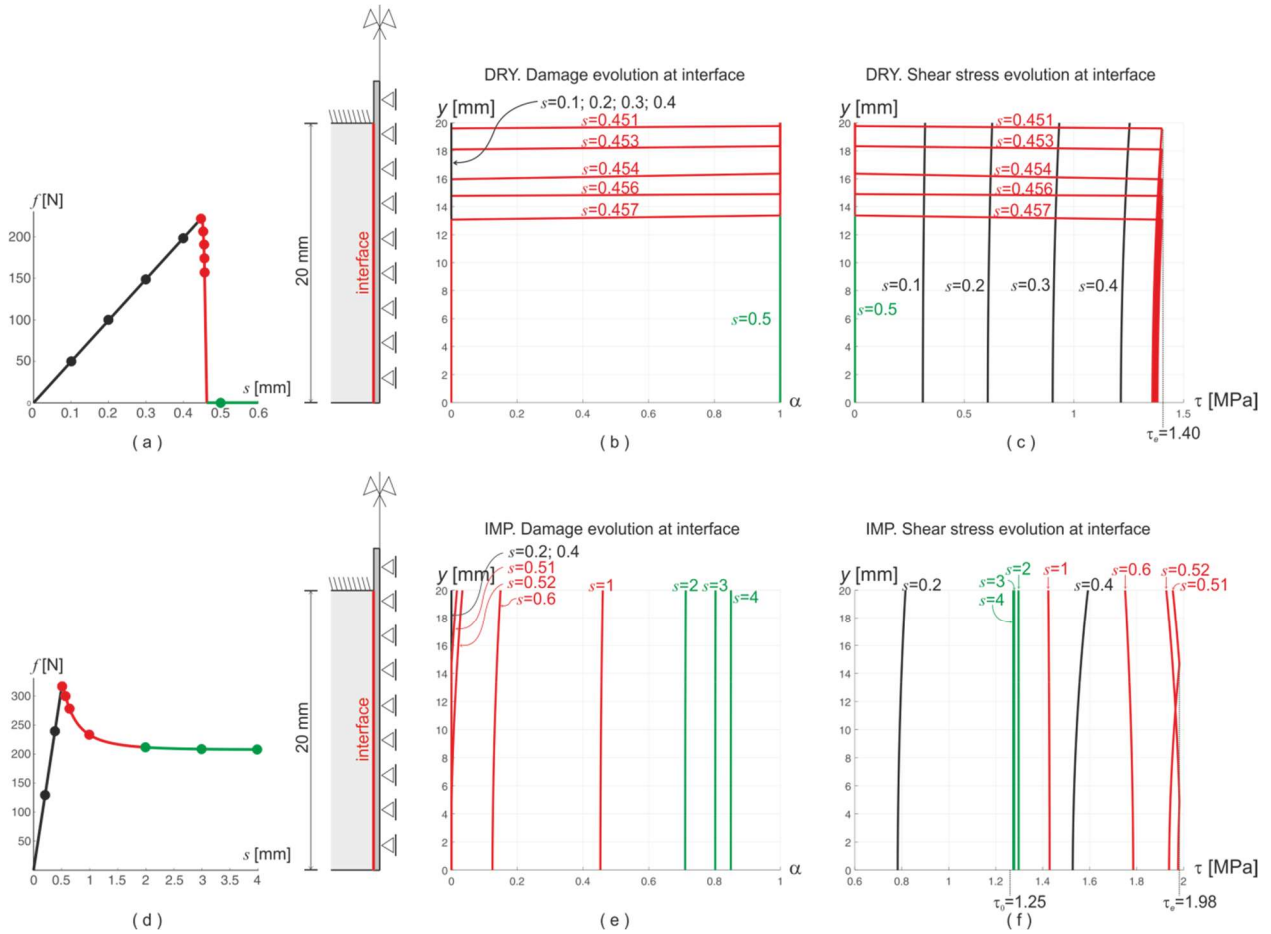
Evolution of damage and shear stress at the interface of 20 mm thick samples is analyzed in Figure 7, for DRY and IMP yarns, where profiles of α and τ are plotted for different value of s , and different colours are used to distinguish the different phases of the evolution. Dots in the response curves of Figures 7a,d indicate points at which snapshots of α and τ are drawn.

Firstly the brittle failure of the DRY sample is described (Figures 7a,b,c). Looking at the coloured response curve of Figures 7a, three phases can be distinguished: the elastic phase (black), the brittle debonding phase (red) and the final sliding stage (green).

In the initial elastic phase, $\alpha=0$, and τ grows homogeneously through the interface. When the limit value $\tau=\tau_e$ is reached at the top endpoint of the interface, the debonding process starts, and advances downward. When $\alpha=1$, any capability of carrying shear stresses is lost. This is due to the fact that the shear stress is null. The debonding front continuously advances up to 1/3 of the interface length, then it dramatically evolves over the entire interface, leading to the complete failure of the sample.

A different evolution process is reproduced in the case of IMP yarn (Figures 7d,e,f). After the initial elastic phase ($\alpha=0$), which ends when τ has reached the limit value τ_e at the top of the interface, a process of progressive debonding takes place. First, a small amount of damage advances downward. In Figures 7e,f, profiles at $s=0.51, 0.52$ account for the evolution process of the debonding front. The cusps in the curves of τ , where the shear stress attains the maximum value $\tau=\tau_e$, correspond to the positions assumed by the debonding front. For each profile, points

1 of the interface above the cusp are damaged, while those below are sound. Then α progressively
 2 grows homogeneously across the interface. Since shear stress decreases as damage increases, a
 3 regime of stress-softening is established. When the asymptotic value τ_0 is approached, the third
 4 phase of frictional slippage takes place. In this last stage, damage is practically homogeneous
 5 through the interface, assuming the value $\alpha \approx 0.9$. This result is in agreement with the analytical
 6 results of Subsect. 3.2, according to which the values $\tau = \tau_0$ and $\alpha = 1$ are asymptotically reached.
 7 The transition from debonding to slippage is gradual, with slippage that progressively prevails
 8 over debonding.
 9 Finally, brittle (DRY yarn) and ductile (IMP yarn) behaviors can be summarized as follows: a
 10 brittle interface experiences only two possible states, sound interface ($\alpha = 0$) and completely
 11 debonded interface ($\alpha = 1$); differently, a ductile interface exhibits a cohesive damage evolution
 12 where α progressively grows.
 13 Notice that the hypothesis of homogeneous shear stress through the sample, which has been
 14 assumed in Section 3.1 to determine τ_e , τ_0 and k (Table 2), is confirmed by simulations, at least
 15 in the case of 20 mm in thickness samples. Indeed, as shown by the shear stress profiles of
 16 Figure 7, τ is practically constant throughout the interface, with the exception of a very short
 17 phase of damage propagation from the top to the bottom of the interface.
 18



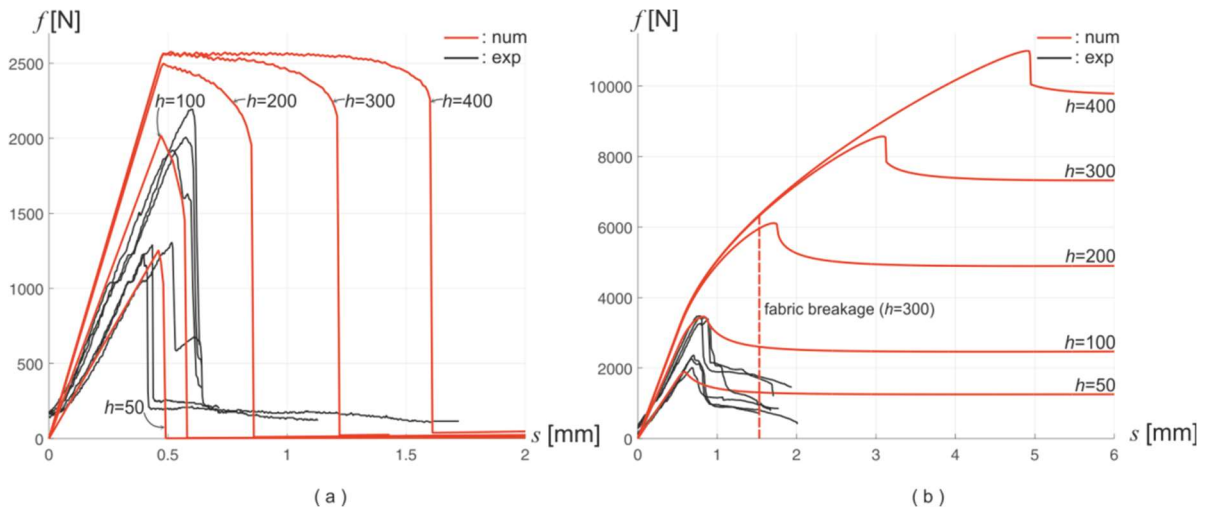
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1 *Figure 7 - Pull out tests: profiles of damage α and shear stress τ at the interface for different*
 2 *values of δ (corresponding to the dots in the force-slip curves (a) and (d)). Profiles of α and τ in*
 3 *the case of DRY yarns (b-c); profiles of α and τ in the case of IMP yarns (e-d). [s]=mm.*

4.2 Double shear bond tests

7 The experimental set-up of double shear bond test is shown in Figure 6c, where two FRCM
 8 strips are applied on the two sides of a clamped brick. The FRCM system is constituted by the
 9 same mortar used in pull out tests, reinforced with dry or pre-impregnated bi-directional carbon
 10 fabrics, with 3 longitudinal yarns embedded in the matrix. The carbon fabric is wrapped around a
 11 steel cylinder, and it is subjected to a tensile displacement. A sketch of a single yarn surrounded
 12 by cementitious mortar is drawn in Figure 6d. The tensile displacement s induces shear stresses
 13 at the yarn-to-matrix interface, flowing towards the brick substrate. As discussed in [25], the
 14 largest stresses are attained in the sample portion placed between the yarn internal side and the
 15 matrix clamped face. For this reason the longitudinal section placed in the middle of this zone is
 16 considered. It is highlighted with different gray shades in Figure 6d (light gray for the matrix and
 17 dark gray for the yarn), and drawn in Figure 6e. The hypothesis of plane strain state is assumed,
 18 justified by symmetry reasons. The lengths $l_1=5$ mm and $l_2=0.22$ mm are assigned. Only 2/3 of
 19 the total yarn thickness is considered, since it has been assumed that 2/3 of the total tensile force
 20 acting on the yarn flows to the matrix through the internal yarn surface, and 1/3 through the
 21 external surface. A similar assumption was made in [19].

22 Simulations are performed by considering DRY and IMP yarns with different bond lengths h .
 23 Force-displacement curves are plotted in Figure 8 (red line), where experimental curves for $h=50$
 24 and 100 mm are added for comparison (black line). Experimental tests with longer bonded
 25 lengths will be carried out in future studies.



27
 28 *Figure 8 – Double shear bond tests. Experimental and numerical load-displacement curves for*
 29 *different values of the sample bond length. (a) DRY yarns; (b) IMP yarns.*

31 In the case of dry yarn (Figure 8a), curves show an initial linear elastic branch, an intermediate
 32 decreasing branch, corresponding to progressive debonding, and a final dropdown branch.
 33 Compared with experimental curves where $h=50$ and 100 mm (three curves for each case),
 34 numerical curves provide accurate peak force values, catching the brittle response resulting from

1 experiments. As the sample length increases, the peak force and the slope of the elastic branch
2 increase till the limit value $h \approx 250$ mm. For larger h , neither the maximum force nor the elastic
3 stiffness grow, meaning that 250 mm can be considered the minimum bond length required to
4 properly transfer stresses from the carbon yarn to the substrate.

5 Curves of Figure 8b are obtained when IMP yarns are considered. In this case, after the initial
6 elastic phase, the debonding stage is described by a stress-hardening curve followed by a shorter
7 stress-softening branch. This latter branch is smooth when low values of h are considered, while
8 it becomes discontinuous for high values of h , exhibiting a dropdown. Curves conclude with
9 frictional plateaus. Comparison with experimental curves ($h=50, 100$ mm) shows an
10 overestimation of the force in the phase of frictional sliding, especially for $h=100$ mm. This
11 might be because of transversal yarns that constitute the fabric, which do not contribute to the
12 shear transfer mechanisms at the fabric to mortar interface, remaining attached to the mortar
13 during the slippage of the longitudinal yarns, and thus reducing the effective contact area
14 between longitudinal yarns and matrix. Furthermore, the experimental setup may have induced
15 some instabilities in the post-peak phase, due to the rapid drop of the load carried by the fabric.

16 Since the peak force increases as h increases, when $h > 210$ mm the tensile stress on a single yarn
17 exceeds the value $\sigma_e = 1850$ MPa (see Table 1) and the yarn breaks on the top cross-section,
18 where the displacement s is applied. The dashed curve of Figure 8b corresponds to the yarn
19 breakage in a sample of length $h=300$ mm. Since we are interested in the study of debonding and
20 sliding at the interface, the yarn tensile strength has been increased in order to perform
21 simulations even for longer bond lengths.

22 Profiles of α and τ at the interface of DRY and IMP samples with bond length $h=400$ mm are
23 drawn in Figure 9 for different values of s . The evolutions are qualitatively similar to those of
24 pull out tests (Section 5.1). The main difference lies in the fact that τ is no more homogeneous,
25 because of the longer bond length. Looking at the shear stress profiles in the elastic phase, the
26 value of τ decreases going downward through the interface, and the bond length required to
27 transmit the whole tensile force to the matrix is about 250 mm. It is the same bond length
28 deduced from the curves of Figure 8.

29 In the case of DRY yarn (Figures 9a,b,c), debonding starts at the top endpoint of the interface
30 when the shear stress value $\tau = \tau_e$ is reached, and then it advances downward. The debonding
31 front proceeds up to $2/3$ of the interface length and then it suddenly jumps to the bottom
32 endpoint, leading to the complete separation of the yarn from the matrix.

33 In the case of IMP yarn (Figures 9d,e,f), the interface is progressively damaged, and the damage
34 front moves down while α grows (see red profiles in Figure 9e). As damage increases, shear
35 stress reduces. In the final phase, damage spreads through the interface, with $\alpha = 0.80-0.85$, and τ
36 tends τ_0 .

37
38
39

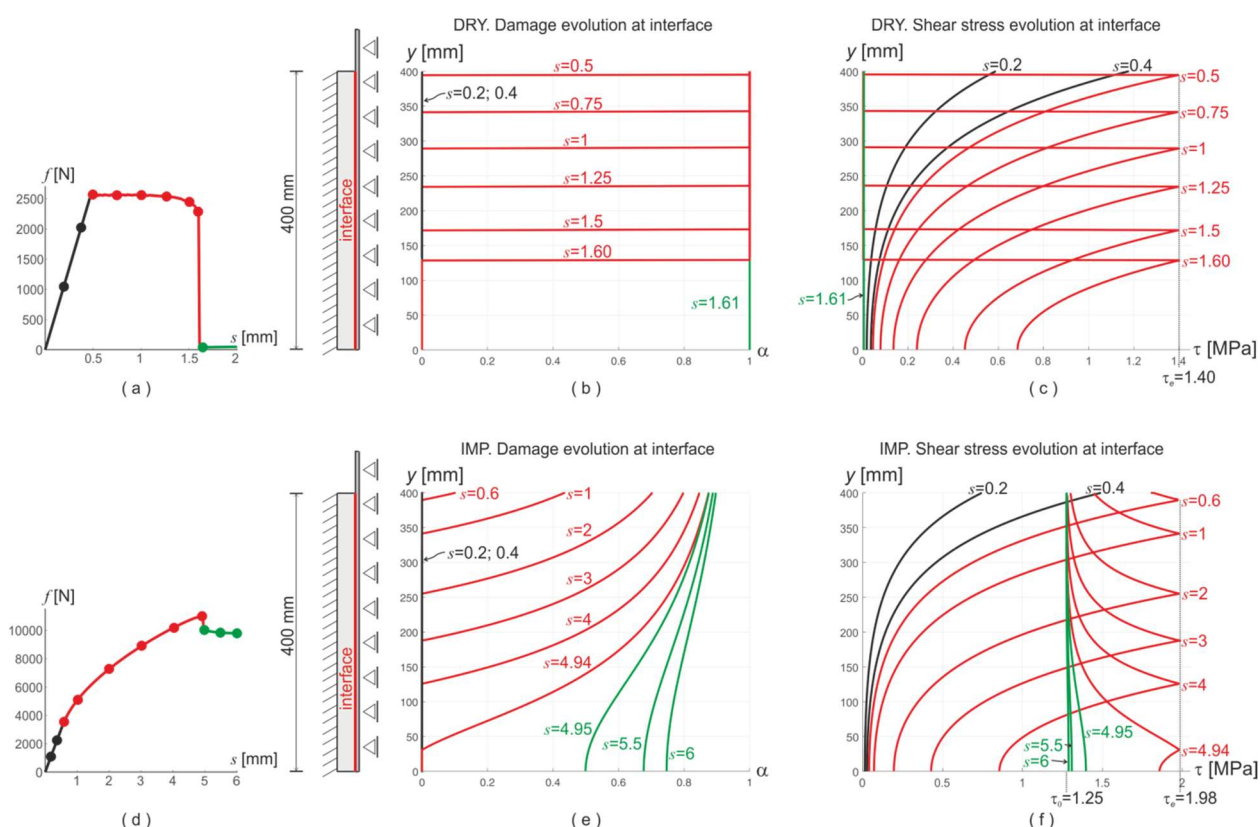


Figure 9 - Double shear bond tests. Profiles of damage α and shear stress τ at the interface for different values of δ (corresponding to the dots in the force-slip curves (a) and (d)). (b-c) profiles of α and τ in the case of DRY yarns; (e-d) profiles of α and τ in the case of IMP yarns. [d]=mm.

5. Conclusions

In this paper, the mechanical behavior and failure modes of multi-filament dry or pre-impregnated carbon yarns embedded in a cementitious matrix are investigated. The bond behavior at the yarn-to-matrix interface is analyzed by means of pull out tests, carried out on single carbon yarns embedded in a cement based mortar for different lengths.

On the basis of the experimental results it was observed a great difference on the mechanical behavior of the samples subjected to pull out tests, depending on the type of carbon yarn used (dry or pre-impregnated with epoxy). The main difference has been observed in the post-elastic phase, when the debonding process starts and evolves. Dry yarns failed in a brittle way once reached the peak load, while pre-impregnated yarns showed a more ductile failure, due to high friction developed at the yarn to matrix interface.

A variational model has been developed in order to provide a thorough understanding of the failure mechanisms for the two different systems. A phase field approach has been followed to formulate the model, where specific internal energies have been assigned to the different components of the system, with the aim of describing the failure modes observed in experiments. Energies accounting for brittle fracture have been assigned to the two FRCM constituents (cementitious matrix and carbon fibers yarn). Special attention was devoted to the definition of the interface energy, in order to reproduce debonding process and slippage at the yarn-to-matrix

1 interface. Three energetic terms have been assumed, each one contributing to describe a different
 2 phase of the evolution; the elastic shear stage, the debonding phase and the final frictional
 3 slippage process. Each contribution has a straightforward mechanical meaning, and it has been
 4 assigned as simple as possible, with few constitutive parameters, directly related to specific
 5 mechanical properties easily measurable from experiments. Thus, the proposed numerical model
 6 combined great predictive capability, being able to describe the different stages of the failure
 7 processes observed in experiments, and ease of calibration, since the constitutive parameters are
 8 related to the experimental data through simple analytical relations.

9 Numerical simulations have reproduced the FRCM behavior in single or double shear bond tests.
 10 In the case of a fabric reinforcement constituted by dry yarns the model allowed to establish an
 11 effective bond length of about 200 mm. By further increasing the bond length the maximum load
 12 does not increase. FRCM systems reinforced with pre-impregnated carbon fabrics experienced a
 13 different behavior. The load increased by increasing the bond length, due to the elevated friction
 14 developed in the slippage phase. In this case the failure occurred in the fabric, outside the bonded
 15 area.

16 This study highlighted the potentiality of a variational model to get insight on the mechanical
 17 behavior of FRCM systems, in particular at the interface between yarn and matrix, providing a
 18 useful numerical tool for the investigation of the effects of geometry and material properties on
 19 the mechanical performances of FRCM systems, and on their failure mechanisms.

22 APPENDIX

24 **Brittle failure of matrix and yarn.** In order to better understand the damage evolution
 25 associated to the energy density (2) (assigned to matrix and yarn), we consider the simple one-
 26 dimensional problem of a bar of length L , subjected to a tensile strain s , and we deduce and
 27 discuss the equilibrium equations, referring to [22] for a detailed study including the evolution
 28 problem and the stability analysis.

29 The longitudinal displacement $u=u(x)$ and damage $\alpha=\alpha(x)$, with $x \in (0, L)$, are the problem
 30 unknowns. The boundary conditions are $u(0)=0$, $u(L)=sL$, and $\alpha(0) = \alpha(L) = 0$, and the energy
 31 density is

$$32 \quad \varphi(u, \alpha) = (1-\alpha)^2 \frac{1}{2} E u'^2 + \frac{\sigma_e^2}{E} \left(\alpha + \frac{l^2}{16} \alpha'^2 \right), \quad (a)$$

33 where $f'(x) = df(x)/dx$. The density (a) is obtained by specializing (2) to the one-dimensional
 34 problem of a tensile bar. The tensile stress is $\sigma = (1-\alpha)^2 E u'$, defined as the derivative of the
 35 strain energy with respect to the deformation u' . If we impose the first variation of the energy
 36 $\int_0^L \varphi(u, \alpha; \delta u, \delta \alpha) dx$ to be non-negative for any admissible perturbation δu and $\delta \alpha \geq 0$, we obtain
 37 the equilibrium relations

$$38 \quad \sigma' = 0, \quad \sigma \leq \sigma_y = \sigma_e \sqrt{(1-\alpha)^3 (1-l^2 \alpha'')}, \quad (b)$$

39 where the first equation is the stress balance equation, which states that σ is constant trough the
 40 bar. The second relation is the damage yield condition, and it states that σ can never be larger
 41 than the yield stress σ_y . As described in [31], damage evolves when $\sigma = \sigma_y$, while the bar
 42 elastically deforms when $\sigma < \sigma_y$.

1 If the initial configuration of the bar is undamaged and undeformed, and s is monotonically
 2 increasing, then the bar experiences an initial elastic stretching, whose solution is $u(x) = sx$,
 3 $\alpha=0$, and $\sigma = Es < \sigma_e$. When the stress attains the value σ_e , inequality (b)₂ is satisfied as an
 4 equality, and damage can form. As shown in [22], for sufficiently long bars, failure brutally
 5 appear in the bar when $\sigma = \sigma_e$, and α jumps from 0 to 1 in a point x_0 , decreasing to zero in a
 6 neighborhood of x_0 of size l , according to the profile drawn in Figure 1a. The analytical
 7 expressions of the profile of α is

$$8 \quad \alpha = \begin{cases} \left(1 - 2 \frac{|x-x_0|}{l}\right)^2, & \text{if } |x-x_0| \leq l/2, \\ 0, & \text{if } |x-x_0| > l/2, \end{cases}$$

9 which is determined by integrating (b)₂, as described in [30].
 10
 11

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