Damage detection in CFRP composite beams based on vibration analysis using proper orthogonal decomposition method with radial basis function and Cuckoo Search algorithm

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Abstract: This paper presents a new fast approach for crack identification using vibration analysis based on model reduction using proper orthogonal decomposition method with radial basis function (POD-RBF). The method is formulated as an inverse problem for detecting the position, length and depth of crack in composite Carbon Fiber Reinforced Polymer (CFRP) structure, where Genetic and Cuckoo search algorithms are used to minimize the cost function based on numerical and experimental frequencies. The results show that the POD-RBF combined with Cuckoo search algorithm is an efficient and a feasible methodology of predicting the width, depth and position of double rectangular notches in CFRP beams. The stability of this technique was tested by introducing a white Gaussian noise in the frequencies data input. The results show that the proposed approach is stable when the noise level is lower than 6%.

Keywords: Crack identification, Inverse problem, Carbon Fiber Reinforced Polymer (CFRP), Proper orthogonal decomposition, Radial basis functions, Cuckoo search, Genetic algorithm.

1. Introduction

Composite materials are nowadays increasingly used as an alternative to conventional materials, especially in the aerospace industry, because of their high strength, specific rigidity, and their mechanical properties being adjustable within wide limits. The composite materials are often subjected to various undetectable damages; i.e. cracks in fibers, matrix, and interfaces between fibers and matrix, which are very common as fatigue failure mode in composites [1]. The presence of cracks in beams of non-homogeneous material has been investigated both theoretically and experimentally in a number of works [2, 3].

Damage identification techniques, using model reduction based on the proper orthogonal decomposition (POD) method, have the main objective to estimate the crack length and its position in a structure using boundary displacements as input data. An optimization algorithm, such as Genetic Algorithm or Particle Swarm Optimization, is then applied for the minimization of the error function expressed as the difference between the boundary displacements of the actual crack and those of the estimated crack [4, 5]. The inverse damage detection and localization based on model reduction using a finite element model of bi-dimensional monolithic composite beam reinforced by a graphite-epoxy was used to define a numerical model of a tested structure, in which different scenarios of damage were considered by stiffness reduction [6]. The accuracy of the method was verified through different damage configurations. The proposed approach was the radial point interpolation method (RPIM) presented by [7] for the analysis of concrete structures using an elastic continuum damage constitutive model. A theoretical model of describing the behavior of CFRP cantilever beams was developed on the basis of previous research, which investigated free vibration of beams with single and multiples cracks [8] or notches [9, 10]. The dynamic response of structural elements was modified due to real damage resulting from defects, loss of integrity and cracking of FRP material by overloading during service life [11-13].

The application of three-dimensional spectral element method (SEM) into propagation problems in plate structures to predict damage location was reported in literature. The main purpose of the SEM is that the mass matrix is diagonal. This is because of the choice of Lagrange interpolation function supported on the Gauss–Lobatto–Legendre (GLL) points in conjunction with the GLL integration [14]. The proposed model can detect damages in those structures.

A large number of works based on cracks in composite materials and/or notched CFRP elements can be found in Ref. [15]. The damage detection and localization of damage in structures by applying concepts derived from the theory of proper orthogonal decomposition (POD) was investigated by simulating tests on two beams and provided promising results [16]. POD provides the most efficient way of capturing the dominant components of an infinite-dimensional process with only (often surprisingly) few modes. Various applications of POD to structural dynamics were carried out in the literature [17-19].

The above proposed approach has never been applied to composite structures and in this paper we present the first attempt to do so. The POD-RBF combined with Cuckoo Search and Genetic Algorithms is used for detecting double notches in CFRP composite beams. The POD-RBF approach is used to reduce the dimension of snapshot matrix calculated by finite element analysis with different notch positions and dimensions.

2. POD-RBF Procedure

POD is a powerful model reduction technique based on FEM, theoretical or experimental measured data. This procedure was used in different fields [20-22]. In this section, the procedure of using POD-RBF with FEM to calculate frequencies in terms of crack parameters. The theoritical results of different position of crack using frequencies of undamaged (u_{0und}) and damaged ($u_{(i)dam}$, where 'i' is for different scenarios, $i \geq 1$, to build snapshot matrix U and recording the corresponding damage parameters in the matrix P, P=[d, w, α]; with (p_{0und}) for undamaged and ($p_{(i)\,dam}$) for damaged structure:

$$\mathrm{U} = \begin{bmatrix} u_{0und} & u_{1dam} & u_{2dam} & & & & u_{9dam} \\ f_1^0 & f_1^1 & f_1^2 & \cdots\cdots & \cdots\cdots & \cdots & f_1^9 \\ f_2^0 & f_2^1 & f_2^2 & \cdots\cdots & \cdots\cdots & \cdots & f_2^9 \\ f_3^0 & f_3^1 & f_3^2 & \cdots\cdots & \cdots\cdots & \cdots & f_3^9 \end{bmatrix} \tag{1}$$

$$\mathrm{P} = \begin{matrix} \\ D \\ W \\ \alpha \end{matrix} \begin{bmatrix} p_{0und} & p_{1dam} & p_{2dam} & & & & p_{9dam} \\ D_1^0 & D_1^1 & D_1^2 & \cdots\cdots & \cdots\cdots & \cdots & D_1^9 \\ W_2^0 & W_2^1 & W_2^2 & \cdots\cdots & \cdots\cdots & \cdots & D_2^9 \\ \alpha_3^0 & \alpha_3^1 & \alpha_3^2 & \cdots\cdots & \cdots\cdots & \cdots & D_3^9 \end{bmatrix} \tag{2}$$

The covariance matrix C (C =U.U$^{\mathrm{T}}$), then its eigenvalues λ_i and eigenvectors v_i are calculated and the POD basis Φ by flip the eigenvector matrix is computed using the amplitude $\mathrm{A} = \Phi^{\mathrm{T}} \cdot \mathrm{U}$:

$$\Phi = \begin{bmatrix} \Phi_1^1 & \Phi_1^2 & \Phi_1^3 \\ \Phi_2^1 & \Phi_2^2 & \Phi_2^3 \\ \Phi_3^1 & \Phi_3^2 & \Phi_3^3 \end{bmatrix} \tag{3}$$

Truncating the matrix φ by keeping the first K columns to obtain $\hat{\phi}$, then calculating its amplitude matrix using: $\hat{A} = \hat{\phi}^T \cdot U$, gives:

$$\hat{\phi} = \begin{bmatrix} \Phi_1^1 \\ \Phi_2^1 \\ \Phi_3^1 \end{bmatrix} \tag{4}$$

$$\hat{A} = \begin{bmatrix} \hat{A}_1 & \hat{A}_2 & \hat{A}_3 & \cdots\cdots\cdots & \hat{A}_{10} \end{bmatrix} \tag{5}$$

The new snapshot matrix becomes :

$$U\ new \approx \hat{\Phi} \cdot \hat{A} \approx U \tag{6}$$

Choosing the interpolation function as RBF, and computation of the matrix G:

$$G = \begin{bmatrix} g_1(p_1) & \cdots & g_1(p_{10}) \\ \vdots & & \vdots \\ \vdots & \ddots & \vdots \\ g_{10}(p_{10}) & \cdots & g_{10}(p_{10}) \end{bmatrix} \tag{7}$$

The matrix B, which collects the interpolation coefficients, is calculated as $B = A \cdot G^{-1}$. Vector g(p) corresponds to the chosen parameters, where the approximating function g_i(p) is represented as a sum of N radial basis functions. Then calculate the corresponding amplitudes using a (p) ≈B · g (p):

$$g(p) = \begin{bmatrix} g_1(p) \\ g_2(p) \\ \vdots \\ g_3(p) \end{bmatrix} \tag{8}$$

The approximation of response can be calculated using following equation:

$$u(p) \approx \Phi \cdot a(p) \tag{9}$$

A summary of the POD-RBF procedure is illustrated in the flowchart shown in Figure 1. In this paper we used the POD approach with variable damage parameters based on frequencies of damaged and undamaged structure CFRP composite beams using theoretical and experimental data measured in reference [9]. The use of Radial Basis Function (RBF) allows to provide a continuous approximation, by generating responses corresponding to new parameters by interpolation between values of the initial selection.

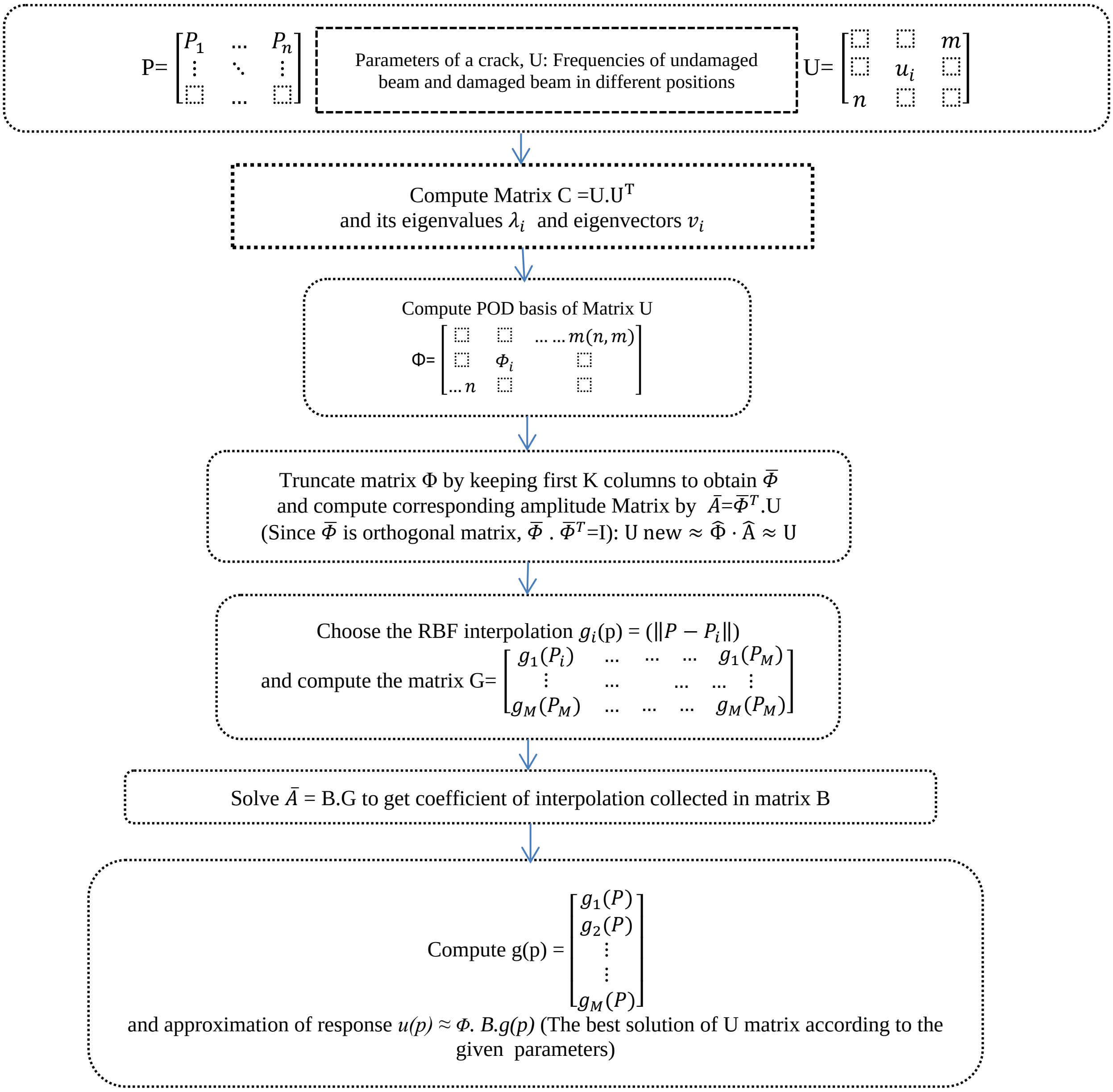

Figure 1. Procedure POD-RBF

3. Implementation for damage detection

In this section, we use the inverse identification of cracks using frequencies of undamaged and damaged structure to found parameters of unknown crack's depth, width and location P=[D ,W, α].the POD-RBF is coupled with GA and CS algorithm. The first three natural frequencies measured experimentally were used to build the snapshot matrix U. The main purpose of this approach is to compare the frequencies input measured by experiments with the frequencies both by both algorithm and POD-RBF using an objective function. The objective function is defined as the error between *u(f)* calculated and $u(f_0)$ measured using the following equation:

$$O_{BF} = \frac{\|u(f_0)-u(f)\|^2}{\|u(f_0\|^2} \quad (10)$$

Where the vector $u(f_0)$ contains the frequencies calculated by the optimization methods and the vector *u(f)* contains the reference measured frequencies measured of damaged or undamaged structures.

3.1. Cuckoo search algorithm

Cuckoo Search (CS) is a powerful meta-heuristic search algorithm, inspired by the reproduction strategy of cuckoo birds. Basically, cuckoos lay their eggs in the nests of other birds, who may discover that and either destroy the egg or abandon the nest. In this algorithm, it is assumed that each cuckoo lay only one egg at a time on a randomly chosen nest, and only the high-quality eggs are selected and marked as best nest for next generation, also the number of available host nests is fixed, and a host can discover an alien egg with a probability Pa between 0 and 1 [23-25].

Cuckoo search considered in this study employs levy flight instead of simple walk, which is random walk with step length drawn from Levy's distribution. The implementation can be done by considering the following; each nest represents a solution. Only one egg is laid on one nest by each cuckoo. The new solution X_i^{t+1} of cuckoo i at t^{th} iteration is generated by using the following equation:

$$X_i^{t+1} = X_i^t + \alpha \times \text{levy}(\lambda) \tag{11}$$

where X_i^t is the previous solution and α denotes the step size, which correspond to the scale of problem and should be greater than zero. If step length is chosen too large then the next solution will be too far from the current solution and if the step length is too short then the solution will be very close. Therefore in most cases $\alpha = 1$ is used because Levy (λ) is the random walk, its step length is calculated from levy distribution for levy flight.

$$\text{Levy} \sim u = t^{-\lambda} \qquad (1 \leq \lambda \leq 3) \tag{12}$$

The procedure Cuckoo Search (CS) can be summarized as following:

1- The first step based on initial population Generated of *n* host nests xi

while (t < Max-Generation)

2- Get a cuckoo randomly
   2.1. Generate a solution by Lévy flights [e.g., Eq. (12)]
   2.2. Evaluate the objective value
   2.3. Choose a nest among n randomly
            if (Fi < Fj),
   2.4. Replace j by the new solution i
   end
3- A fraction (Pa) of worse nests are abandoned
4- New solutions (nests) are generated by Eq. (11)
5- Keep best solutions
6- Rank the solutions and find the current best
     Update t ← t + 1
   end while
7- Come back to step 2
8- Results and visualization

The input parameters of CS algorithm are following in Table 1.

Table 1. Input parameters for CS algorithm

Input parameters	Values
Number of nests	200
Maximal number of generation N_{Gen}	100
probability of detecting a laid egg Pa	0.25
Lévy exponent λ	1.5

3.2. Genetic Algorithm

Genetic Algorithm (GA) is a general probabilistic algorithm inspired by Darwin's survival-of-the fittest theory. In GA, information about a problem to study, such as variable parameters, is coded into a genetic string known as an individual (chromosome). Each of these individuals has an associated fitness value, which is usually determined by the objective function to be minimized based on the description of the problem. GA has been shown to be able to found the optimization problem using mutation, crossover and selection operation applied to individuals in the population [26].

The procedure of GA approach can be summarized as follows:

1- Starting a population of 400 individual which was created randomly. Each individual has 3 parameters of crack (depth, width and alpha).
2- Each individual will be evaluating by introducing the proposed parameters in first step into proposal approach POD-RBF that generates the corresponding frequencies using u(p).
3- Next generation using crossover of individuals to produce a population.
4- Mutation of a specified percentage of the resulting population
5- The fitness value compares the frequencies put by calculation using POD-RBF-GA
6- Inverse the old population by new and coming back to step 2

The following GA parameters were chosen are presented in the Table 2.

Table 2. Input parameters of GA

Input parameters	Values
Population size N_{Pop}	200
Maximal number of generation N_{Gen}	100
Crossover fraction	0.8
Mutation fraction	0.01

3.3 Crack identification approach

The optimization algorithms are coupled with the POD-RBF to inversely estimate the damage parameters. The damage detection approach consists of two main stages:

1- The problem is defined by the unknown variables (in this case damage parameters) and the known data (in this case is the structural response in the form of natural frequencies.
2- The optimization algorithm is executed.

Figure 3 summarizes the procedure of the proposed approach. Using the process of obtaining the fitness value by calculating certain damage parameters in the trained POD-RBF model, the frequency vector is generated. The fitness function value is then calculated as the error between this vector and the reference frequency vector $u(P_0)$ caused by the real damage parameters.

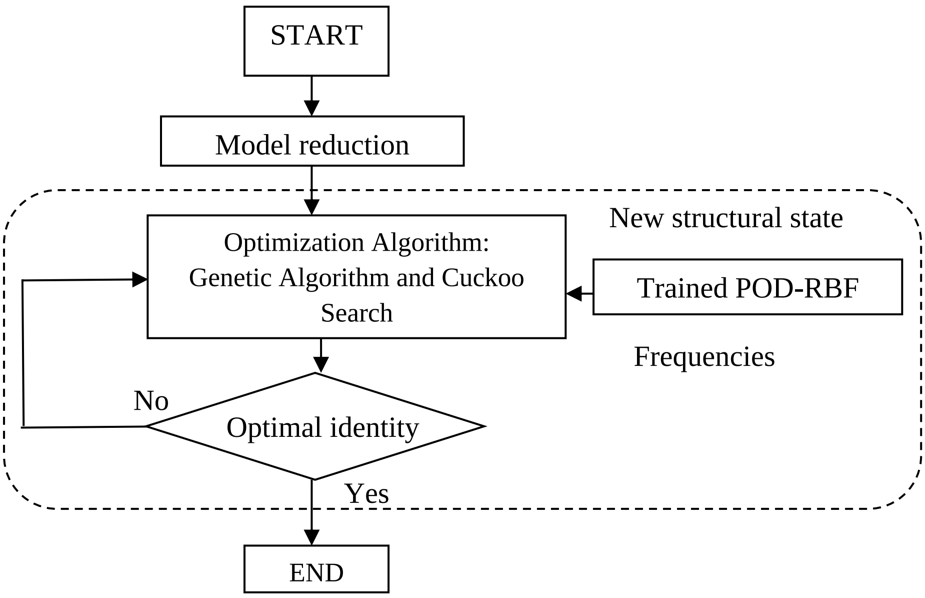

Figure 2. Algorithm of the damage detection approach

4. Experimental validation using a CFRP beam

4.1. Experimental setup

CFRP laminate specimens were subjected to local, increasing, reduction of bending stiffness due to double notches at different sections with different widths. These data are used as input to build snapshot Matrix. The geometric and mechanical parameters of CFRP laminates are shown in Table 3 [9]. In Figure 3, the setup of the free vibration test of a CFRP simply supported beam specimen is shown along with the accelerometers, data acquisition system and impact hammer. In the dynamic tests, the CFRP laminates were stroked at a distance of 20 mm, point M from the axis of the end bond, via an impact hammer (Type 8202 – Brüel & Kjaer), while a piezoelectric accelerometer (Type 4508) is used for measuring the responses. The theoretical analysis, FEM and experimental frequencies of free vibrations for a simply supported beam of undamaged CFRP laminate are shown in Table 4.

Table 3. Geometric and mechanical parameters of CFRP laminate [9]

Geometric and mechanical parameters	Value
Width b [mm]	35
Thickness t [mm]	2.4
Total length Lt [mm]	400
Length L [mm]	350
Young's modulus E [kN/mm^2]	93.85
Density ρ [Ns^2/mm^4]	$1.95 * 10^9$
Moment of inertia I [mm^4]	40.32

Table 4. Theoretical, FEM and experimental frequencies of free vibration simply supported beam [9]

Frequency values undamaged element – D0	Euler–Bernoulli uniform beam	Experimental average values	Finite Element Analysis
f1 [Hz]	71.10	69.40	73.61
f2 [Hz]	284.42	258.40	294.72
f3 [Hz]	640.00	564.20	663.95

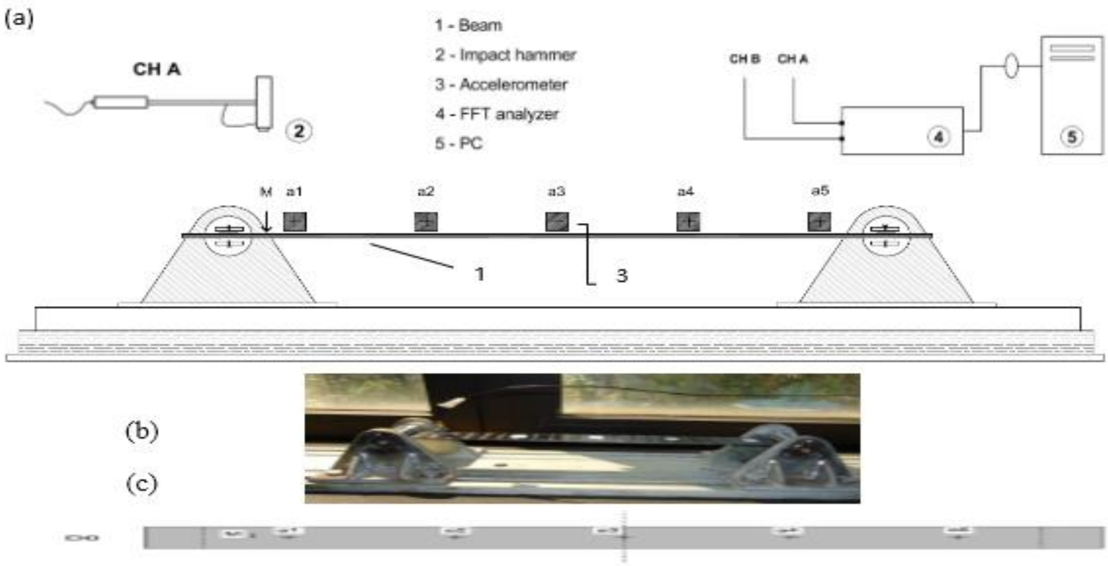

Figure 3. (a) Set up of dynamic tests [9]; (b) View of accelerometer and (c) Positions of accelerometers

The vibration behaviour of damaged CFRP laminates with double notches for a limited length, was investigated by experimental and theoretical studies. The theoretical part assumes that a rotation spring concentrated on the axis of notch with stiffness K, can describe, with sufficient approximation, the reduction in local stiffness. The theoretical and experimental average natural frequencies for damaged CFRP laminates are presented in Table 5.

Table 5. Theoretical and Experimental average natural frequencies for damaged CFRP laminates [9]

	Theoretical	Experimental	Theoretical	Experimental	Theoretical	Experimental
Parameters	α=0.05	α=0.05	α=0.15	α=0.15	α=0.35	
Depth =6.50mm Width =5 mm	70.52 284.37 635.03	64.40 255.75 540.20	70.55 283.94 636.94	66.00 254.60 551.20	70.88 284.13 642.29	– – 256.40 552.40
Depth =8 mm Width =7.75 mm	69.84 284.30 626.75	63.20 257.80 534.20	69.96 283.32 632.75	61.80 252.20 540.60	69.96 279.98 638.45	66.60 246.00 561.60
Depth =10.50 mm Width=11.50 mm	68.03 284.13 611.36	63.00 – – 522.40	68.16 281.92 624.53	65.40 253.80 544.20	68.81 275.07 630.04	69.40 245.40 – –

In this study, the damage identification of CFRP using POD-RBF coupled with Cuckoo Search and Genetic Algorithm is applied to three damage scenarios and the undamaged beam. The maximum number of iterations was set equal to 100. This identification method was implemented in MATLAB using PC, Intel I3 2.8 GHz and 8 GB RAM.

4.2. Prediction of damage parameters

As a first step, we tried to identify the health properties of the CFRP beam by introducing the frequencies of undamaged CFRP beam into the proposed algorithms POD-RBF-GA and POD-RBF-CS. The results are illustrated in Fig 4 and show that the correct parameters of crack (depth, width and alpha) have been found using both algorithms. The corrected results were reached at the second iteration for CS algorithm and ninth iteration for GA. The computation time is much less for CS algorithm compared with GA as presented in Table 6.

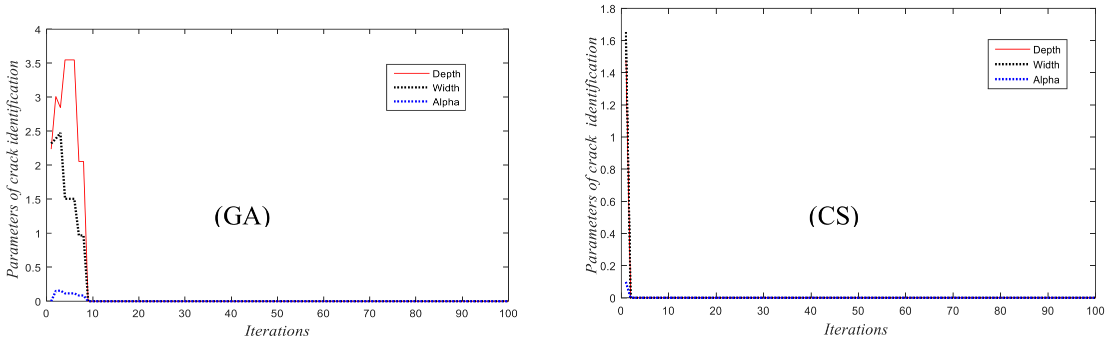

Figure 4. Results for crack parameters P= [0, 0, 0] using GA and CS coupled with POD-RBF

In a first damage scenario, we tried to identify a crack in the CFRP beam by introducing the frequencies of damaged CFRP beam based on three parameters P= [6.5 , 5, 0.05]. The results are illustrated in Figure 5 and show that the parameters found using POD-RBF-CS are more accurate and faster than those found using POD-RBF-GA.

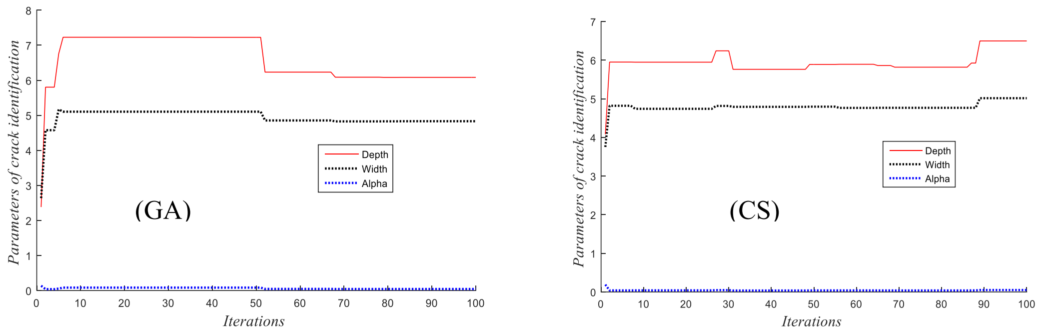

Figure 5. Results for crack parameters P= [6.5, 5, 0.05] using GA and CS coupled with POD-RBF

In a second damage scenario, the damaged beam has the following characteristics: depth =10.50 mm width =11.50 mm and α=0.05**.** The results are presented in the Figure 8. Again, the results show that the CS algorithm is better than GA based on computational time.

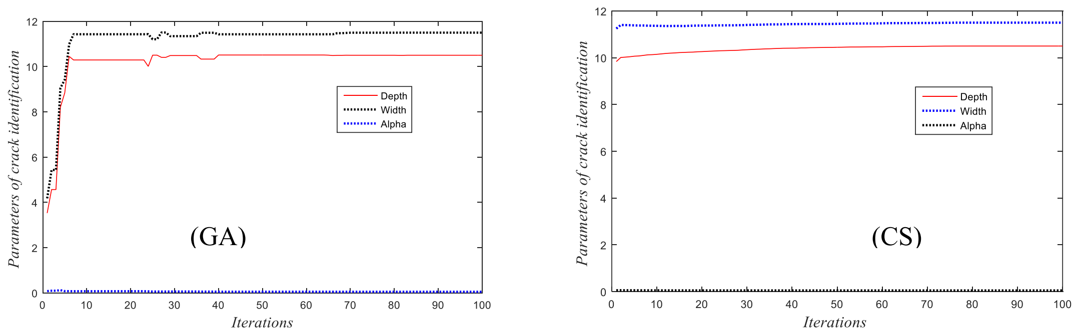

Figure 6. Results for crack parameters P= [10.5,11.5, 0.05] using GA and CS coupled with POD-RBF

In a last damage scenario, we used the frequencies of CFRP damaged based on the parameters P=[10.50, 11.50, 0.35]. The results are presented in Figure 7 and show that the damage parameters found by CS algorithm have small error compared with those found by GA. In Table 6, we present a comparative study between CS algorithm and GA combined with POD-RBF. The results show that the CS algorithm provide more accurate results than GA. The computational time presented in Table 6 shows that the CS algorithm is faster than GA using POD-RBF. Other methods, such as theoretical calculation of frequencies based on interpolation technique and FEM takes more than 4 hours for this composite CFRP beam.

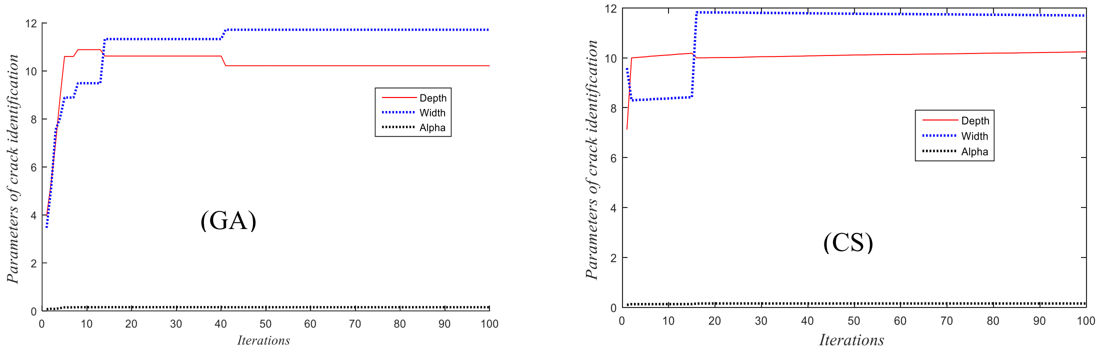

Figure 7. Results for crack parameters P= [10.5, 11.5, 0.35] using GA and CS coupled with POD-RBF

Table 6. CS algorithm and GA for crack identification using POD-RBF

Scenario	Optimization method	Depth		Width		Alpha		Calculation time (s)
		Real value	Predicted value	Real value	Predicted value	Real value	Predicted value	
Sc-1	GA	0	0.00054	0	0.0009	0	0.00067	476
	CS	0	0.000021	0	0.00	0	0.00	142
Sc-2	GA	6.5	6.2730	5	4.9452	0.05	0.05120	513
	CS	6.5	6.5481	5	5.0132	0.05	0.05113	154
Sc-3	GA	10.5	10.6400	11.5	11.5200	0.05	0.04750	492
	CS	10.5	10.5323	11.5	11.5163	0.05	0.05090	139
Sc-4	GA	10.5	10.2900	11.5	11.7519	0.35	0.3395	465
	CS	10.5	10.4301	11.5	11.5623	0.35	0.35200	147
Remark: The time calculation using theoretical method around [3-4 hours]								

To study the effect of noise on the stability of the crack detection algorithm, we introduced white Gaussian law applied into the data of the second damage scenario (depth =10.50 mm width =11.50 mm and α=0.05). We introduced three perturbation levels of 3, 6 and 10 % in the input frequencies. The results shown Table 7 illustrate the accuracy of the proposed crack identification algorithm. The 2 % noise shows no considerably effect on the precision of the predicted crack parameters.

Table 7. Crack identification using noise with three perturbation levels of 3, 6 and 10 % - damage scenario D3

Depth			Width			Alpha		
Noise 2%	Noise 6%	Noise 10%	Noise 2%	Noise 6%	Noise 10%	Noise 2%	Noise 6%	Noise 10%
10.54	10.58	10.77	11.546	11.61	11.74	0.0527	0.0599	0.612

5. Conclusions

In this paper, we presented POD-RBF-CS and POD-RBF-GA methods for damage detection in CFRP beams. The POD-RBF is used as model reduction of undamaged and damaged beam. The natural frequencies of double rectangular notches at different positions were measured in laboratory on simply supported beams. Snapshot matrix based on three parameters; location, depth and width P=[d, w, α], was build based on FEM. The results have clearly shown that the developed algorithm can predict the positions and dimensions of double rectangular notches crack using POD-RBF-CS and POD-RBF-GA. The CS algorithm was found to be more accurate and faster than GA. To examine the presented approach, noise was introduced to study the stability of the crack detection algorithm based on the white Gaussian law. The results show that the proposed approach is stable when the noise levels lower than 6%.

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